ABSTRACT

TEMPERATURE EFFECTS ON THE DYNAMIC
AND TRANSIENT MECHANICAL BEHAVIOR OF TENDON

by
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Elongation responses of cat extensor digitorum communis tendon to triangular wave loading and to constant loads achieved by ramp loading were studied over a temperature range from approximately 22° to 55°C.

The time-dependent portion of the elongation was found to be composed of two parts, a viscoelastic component and a component which at a constant load appeared plastic-like. This plastic-like elongation differed from classical plastic flow, however, since it represented a progressive decrease in stiffness with little or no apparent increase in unstressed length. Plastic-like elongation was observed only after specimens had been loaded to sufficiently large loads and, therefore, appeared to have a critical load similar to the yield stress for true plastic flow. This critical load decreased as plastic-like elongation proceeded, suggesting that the progressive decrease in stiffness might be the result of a gradual failure of the material. However, values for the critical load were relatively consistent from one experiment to another when determined from graphs of elongation rate as a function of load for tests performed shortly after plastic-like elongation had begun. Such 'early critical loads'

were found to be nearly independent of temperature over the range from 22° to 44°C but decreased sharply with increasing temperature from 44° to 47°C. The 'early critical load' reached a minimum between 47° and 51°C and increased with further increases in temperature up to 55°C. It was noted that the temperature range over which the 'early critical load' decreased sharply with increasing temperature corresponded closely to the 'glass-type' transition temperature which has been reported for collagen.

Dynamic stiffness and damping (hysteresis energy dissipation) measured under triangular wave loading with peak loads less than the early critical load, however, showed no marked changes at temperatures around the 'glass-type' transition temperature and appeared to be nearly independent of temperature over the entire range from 22° to 55°C. Possible trends toward a slight increase in damping and a slight decrease in dynamic stiffness with increasing temperature were noted, but these were not statistically significant in the results from this study.

Attempts to develop a mathematical model for the mechanical behavior of tendon were unsuccessful but did show that the damping displayed by cyclicly loaded tendon could not be adequately described by a linear differential equation in load and elongation or by simple Coulomb friction as the major dissipative process.

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1. INTRODUCTION

The mammalian organism normally regulates its body temperature within relatively narrow limits except during hibernation. Even in non-hibernaters such as man, however, significant shifts in body temperature do occur. DuBois (1948) has noted that rectal temperatures in normal human individuals may range from near 36° C in the early morning or in cold weather up to between 38.5° and 40° C with strenuous exercise. Also, rises of 3° C or more are commonly associated with a variety of diseases. In clinically induced "moderate" hypothermia, with the regulatory mechanisms temporarily suppressed as in hibernation, the core temperature is lowered to 23° C or below (Smith and Stetson, 1965). Relatively large changes also occur locally with external heating or cooling.

There is at least one instance in which the therapeutic application of heat is presumed to induce medically important changes in the mechanical properties of fibrous connective tissue. In the treatment of contractures resulting from fibrosis or scarring of tissues around joints, therapeutic heat (as applied by ultrasonic heating) has proved to be of value. It is presumed that the extensibility of the fibrous structures increases with increasing temperature, facilitating subsequent therapy through range of motion exercises (Lehmann, 1965).

Some previous experimental work, most notably that of Rigby et al (1959) on rat tail tendon, has indeed suggested that marked changes in the mechanical properties of collagen tissue are associated with temperature changes within the usual range of tissue temperatures. Among the changes which have been reported to occur with increasing temperature are an increase in the amount of stress relaxation and a decrease in the elongation at rupture. Some data also suggest a possible decrease in stiffness with increasing temperature (cf. LaBan, 1962). Rigby et al (1959) reported that some of these changes occurred in "an abrupt and irreversible manner" at about 40° C, and went so far as to speculate that fever-induced changes of this kind in human collagen might be severe enough to impair the function of the heart valves.

Another abrupt change in the properties of collagen, the hydrothermal shortening which typically occurs at around 60° C, has been studied extensively for some time. However, the changes occurring near body temperature have received only limited attention in the ten years since Rigby et al published their study. Since such changes could be of medical significance, the study reported here was undertaken in the hope of gaining further information which might be useful in anticipating and analyzing the effects of connective tissue changes associated with those shifts in tissue temperature which result from physiological and environmental factors. It was assumed that the changes in mechanical properties having the most physiological significance would be those changes in extensibility which would influence joint stiffness. Therefore, the study was designed to emphasize these characteristics -- dynamic stiffness, plastic-like elongation, and damping (hysteresis) -- while omitting traditional determinations of tensile strength. Failure of tendons following plastic-like elongation, however, was included in the study.

2. REVIEW OF THE LITERATURE

2.1 GENERAL

Most or all of the soft tissues, such as tendon, which appear to function primarily as passive mechanical elements in the animal body are normally referred to as "connective tissues". They are characterized by a relatively sparse population of cells associated with a matrix of extracellular protein fibers and surrounded by a viscous fluid or gel called the ground substance. Like the synovial fluid found within joint capsules, the ground substance contains significant concentrations of mucopolysaccharides, especially hyaluronic acid and chondroitin sulfate (see: Wells, 1954; Barnett et al, 1961). The fiber matrix is made up primarily of two types of protein fibers, collagen and elastin. Elastin fibers appear yellowish in bulk and are seen under the light microscope as singly occurring fibers which branch and anastamose frequently (cf. Clark, 1965). Electron microscope (Greenlee et al, 1966) and x-ray diffraction (Kolpak, 1935) studies suggest that the material of these fibers is largely amorphous, although possibly containing very fine (approximately 100 Å) fibrils. In contrast, the white collagen fibers occur in bundles[1], are unbranched, and have been found to be highly ordered molecular arrays (cf. Felsher, 1954). A third type of fiber, the reticular fiber, has been described in connective tissue, particularly in young animals, but there is evidence that reticular fibers are

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[1]Since collagen occurs in fiber bundles the term "fiber" has been used to describe both the bundle and the individual element. The terms fassiculi, fibers, sub-fibers, and fibrils are sometimes used to indicate decreasing orders of aggregation.

"immature" collagen. (A succinct review of this topic is presented by Clark, 1965.)

The tensile mechanical properties of different forms of connective tissue are determined largely by the composition and arrangement of the fiber matrix. Tissues which contain a high proportion of elastin fibers, such as the ligamentum nuchae (Wood, 1954) and the ligamentum flavum (Nunley, 1958; Nachemson and Evans, 1968), appear to be far more extensible than those which are composed largely of collagen. For example, Nachemson and Evans (1968) obtained full recovery from strains of 30 to 60 percent in various specimens of ligamentum flavum while Gratz (1931) found a typical strain at rupture of less than 9 percent for human fascia lata, and Rigby et al (1959) obtained full recovery from strains of no more than around 4 percent in rat tail tendon. In skin, as in tendon and fascia, collagen fibers predominate; however, skin can typically be extended by more than 30 percent. This is accounted for by the different arrangement of fiber bundles which are nearly parallel in tendon and fascia but in skin are arranged as a "feltwork" (cf. Rothman, 1954; Daly, 1966).

Seemingly all types of connective tissue behave as viscoelastic materials, showing both time-dependent and time-independent components in their mechanical behavior. The time-dependent behavior, as pointed out by Nachemson and Evans (1968), tends to be less significant in tissues where elastin fibers predominate than in those where collagen predominates. For the 'elastic tissues' in which elastin predominates, descriptions of mechanical behavior based on rubber-like thermoelasticity have been employed with some success (cf. Hoeve and Flory, 1958). Attempts to apply such a description to 'collagen tissues', however, have been quite unsuccessful (Hall, 1951b, 1952); and an adequate theoretical frame of reference for dealing with the properties of these tissues seems far from being realized.

2.2 MECHANICAL PROPERTIES OF COLLAGEN TISSUES IN UNIAXIAL TENSION

2.2.1 STRESS-STRAIN RELATIONS

Investigators who have dealt with the mechanical behavior of collagen tissues have generally found the load-extension characteristics of these materials to be markedly nonlinear, their stiffness increasing with increasing load (or elongation) except at quite large loads where yield may be presumed to occur with a resultant decrease in stiffness with increasing load (cf. Rigby et al, 1959). This behavior is illustrated in Figures 2.1 and 2.2. Figure 2.1 shows stress-strain curves obtained by Wright and Rennels (1964) from tests on several specimens of human plantar fascia. Figure 2.2 shows a stress-strain curve for human Achilles tendon as obtained by Abrahams (1967) and illustrates yield behavior. Specimens retested after yield occurred have been found to be less stiff and to yield again at smaller loads than in the initial test (Rigby et al, 1959).

Exceptions to the observation of increasing stiffness with increasing stress, however, are found in the works of Walker et al (1963, 1964) and Benedict et al (1966, 1968). These studies, all performed by the same group of investigators, yielded stress-strain curves of highly variable character for embalmed specimens (Walker et al, 1963, 1964) and unembalmed specimens (Benedict et al, 1966, 1968) of human tendon. In these studies, tests were performed in unsaturated air, and strains were determined with reference to gage lengths established with specimens under "impending tension" (Walker et al 1964) or under some preload (Benedict et al, 1968).

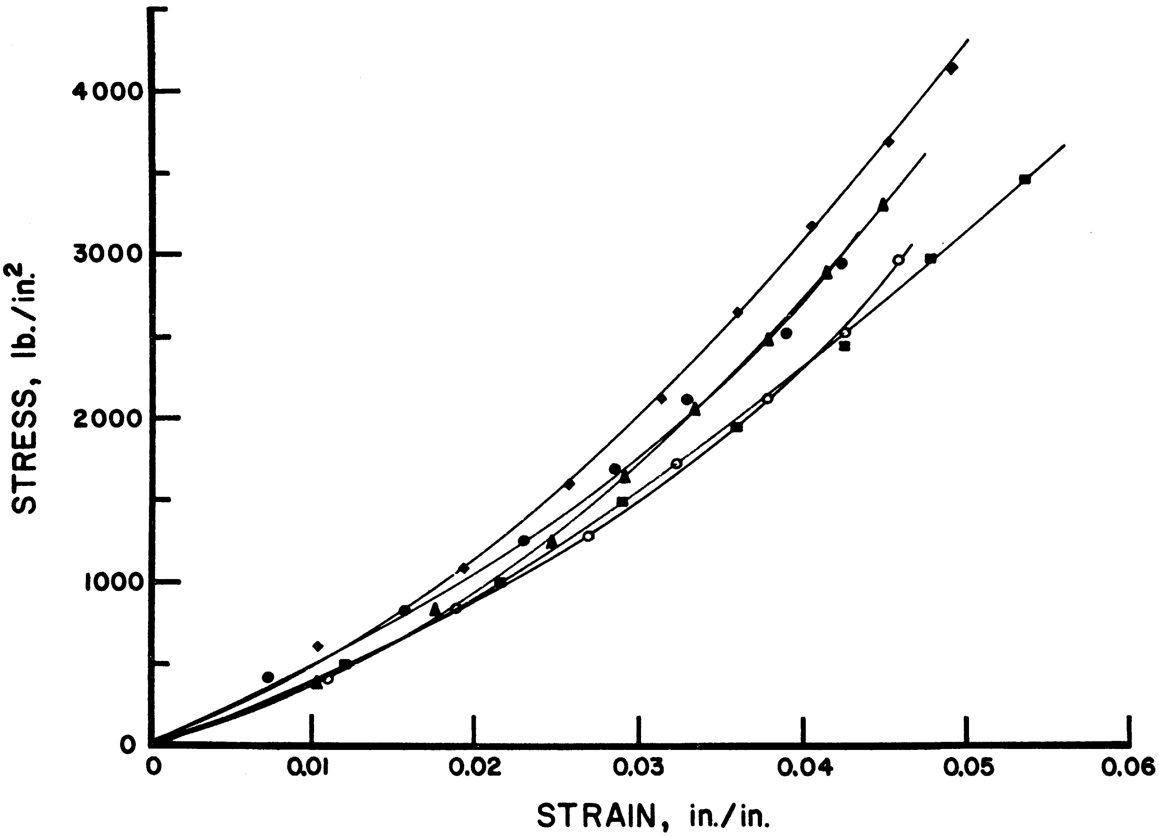

FIGURE 2.1 STRESS-STRAIN CURVES FROM TESTS ON HUMAN PLANTAR FASCIA. REDRAWN FROM WRIGHT AND RENNELS (1964).

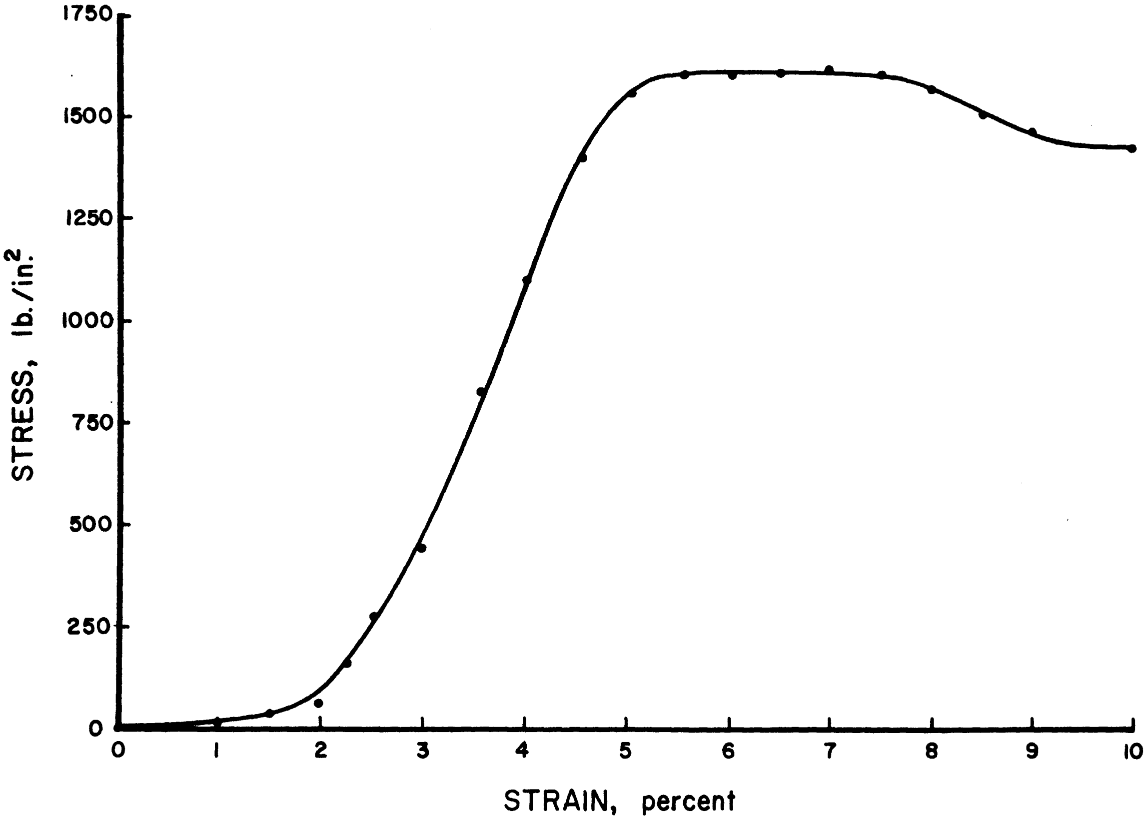

FIGURE 2.2 STRESS-STRAIN CURVE FOR HUMAN ACHILLES TENDON (NOTE YIELD BEHAVIOR). REDRAWN FROM ABRAHAMS (1967).

Wertheim (1847) noted the forms of stress-strain relations for several tissues including tendon and described them by a quadratic expression having two constants a and b:

$$(\text{strain})^2 = a(\text{stress})^2 + b(\text{stress}).$$

In more recent reports, a variety of descriptions for the stress-strain relations of collagen tissues have been used.

Several authors including Reuterwall (1921), Rigby et al (1959), Partington (1963), and Viidik (1966) have described stress-strain curves for collagen tissues below the yield region as being composed of two parts, a "toe" portion at low stresses, in which stiffness increases markedly with increasing stress, and an approximately linear region in which the proportionality between changes in stress and changes in strain is nearly constant.

Hall (1951b) attempted to apply equations derived from the theory of rubber-like thermoelasticity to the stress-strain characteristics of ox hide collagen, but found that while an empirical fit could be obtained, the constant relating length of the stretched molecular chain was negative and, hence, application of the theory was not physically meaningful. Noting that rubber-like thermoelasticity is mainly dependent on a significant decrease in entropy with stretching, Hall (1952) used the Wiegand-Snyder equation to determine the relative magnitudes of the entropy and internal energy changes associated with stretching and found that both quantities increased as the fibers were stretched at pH 7 although the entropy decreased with stretching at low pH (1.75).

Morgan (1960) analyzed 395 stress-strain curves for ox hide collagen and obtained an empirical stress-strain relation of the form $\epsilon = a\sigma^n$, where ϵ and σ represent strain and stress, respectively, and a and n are empirical constants. Hall (1951b) had reported that an equivalent equation obtained earlier by Mitton for leather fibers did not fit his data for untanned ox hide collagen. However, Kenedi et al (1964) found that a power function $\sigma = K\epsilon^d$, with empirical constants K

and d fitted their stress-strain data for human skin better than either an exponential function or the equation for a theoretical elastomer. Ridge and Wright (1964) also found that the stress-strain curve for human skin at intermediate stresses could be fitted by a power function of the form $\epsilon = c + K\sigma^b$, where b, c, and K were constants. For small stresses (the 'toe' region), however, they obtained better agreement using a logarithmic equation of the form $\epsilon = x + y \log \sigma$ with constants x and y. In other work (Wright and Ridge, 1964; Ridge and Wright, 1966) they found that the same formulations could be used to describe the stress-strain curve for rat tail tendon. Fung (1967) found that the stress-strain curve for rabbit mesentery could be represented by an exponential equation, corresponding to the logarithmic equation suggested by Ridge and Wright, over much of the range of stresses to which it was subjected. He also asserted the existence of a less extensive region at higher stresses in which a power function fitted the stress-strain relation for this material.

Elden has proposed a parabolic stress-strain relation for tendon (see: Elden, 1968). However, this represents only a special case of the power function formulation.

The two regions of the stress-strain curve for tendon below the yield region, i.e. the 'toe' region in which the stiffness increases markedly with increasing stress and the region of intermediate stresses over which the stiffness increases only slightly with increasing stress, were suggested by Reuterwall (1921) to correspond, respectively, to straightening of the initially wavy collagen fibers and to stretching of the straightened fibers. This has been demonstrated by Rigby et al (1959) who photographed unstretched and stretched rat tail tendons under a polarizing microscope and later by Abrahams (1967a,b) who employed formalin fixation of unstretched and stretched specimens. Both Rigby et al and Abrahams found that the wave pattern was restored after the release of stretch in fresh specimens which were not stretched beyond a "safe limit" of approximately 4 percent strain.

Similar fiber straightening and fiber stretching phenomena in skin were noted by Daly (1966). To describe the consequent mechanical behavior he developed a "semiempirical" model which was composed of four elastic (or viscoelastic) elements connected as a parallelogram with a fifth element forming a diagonal of the parallelogram.

2.2.2 DAMPING AND TRANSIENT PHENOMENA

Transient phenomena, creep and stress relaxation, have been observed and studied in collagen tissues by a number of investigators.

Reuterwall (1921) observed that specimens of human plantaris and extensor digitorum tendons subjected to a constant load showed time-dependent extension (creep). His data tend to suggest that a limiting value of extension was approached after some time.

Hall (1951a) performed creep experiments on ox hide collagen. From these experiments he identified a portion of the creep as unrecoverable and found that the amount of unrecoverable creep for each test decreased with repeated testing when specimens were subjected to load for 200 minutes and were then permitted to recover in the unloaded condition for 21 hours between tests.

Smith (1954) reported that specimens of rabbit anterior cruciate ligament, with portions of the tibia and femur atteched, which were subjected to a "submaximal" load for 5 minutes showed continued elongation throughout this period and failed to recover their ogiginal lengths thereafter, whereas specimens subjected to brief, periodic "submaximal" loads did recover their original lengths. Fry and her coworkers (Fry et al, 1962; Fry et al 1964) have reported that skin under a fixed load was observed to elongate at a constant rate. LaBan (1962) failed to obtain complete recovery in creep experiments in which various loads were applied to canine calcaneal tendon specimens for 15 minutes. But, since he measured elongations outside of the wedge-action clamps which held the specimens, it is possible that his observations reflect slippage at the clamps in addition to the behavior of the tissue. Jamison et al (1967) also reported large portions of the strain to have been unrecoverable in creep tests on guinea pig skin; but they, too, used the movement of their specimen clamps for measuring strain.

Stress relaxation in specimens of rat tail tendon was investigated in some detail by Rigby et al (1959). They found that the load in specimens held at a fixed extension decreased markedly with time but eventually approached a limiting value. In semilogarithmic plots of percent stress relaxation against the logarithm of time, they noted two approximately linear regions, the first extending for roughly 60 minutes. The amount of stress relaxation observed in their experiments was found to increase with increasing strain and with increasing rate of initial strain application. For initial strains of less than 4 percent (i.e., below the yield region) and stress relaxation times of "considerably less than 60 minutes" they found the stress relaxation behavior to be "entirely reproducible".

Haut and Little (1969) investigated stress relaxation in canine anterior cruciate ligament over 10 minute relaxation periods and found the percentage relaxation to increase with increasing initial stress. They also found that, at each strain, differences in stress which were associated with different rates of strain during the initial stretching persisted after 10 minutes of stress relaxation.

Both creep and stress relaxation in human skin were investigated by Daly (1966). His results also suggest that a limiting value of stress is approached in stress relaxation experiments and, further, tend to suggest approach to a limiting value of strain in creep experiments, contrary to the observations of Smith and of Fry and her coworkers. Daly undertook to apply several different approaches in analyzing his stress relaxation data, as will be discussed later.

Coupled with transient behavior like that which has been described for collagen tissues, one would expect to find hysteresis or damping when specimens of these materials are subjected to cyclic loading.

In studies of the tensile properties of human fascia lata by Gratz (1931) and of Achilles tendon by Stuke (1950) the stress-strain curves for loading and unloading did not coincide, the curve for unloading showing slightly greater stiffness. Thus, in one stress cycle, energy was dissipated; and the material might be regarded as having displayed some form of damping (cf. Lazan, 1964).

Over the past six years the behavior of collagen tissues under cyclic loading or extension has been studied by several investigators.

Rigby (1964) studied rat tail tendon subjected to cyclic extension for over 1000 cycles. He found that with repeated cycling the portion of the stress-strain curve above the 'toe' region became steeper (increasing stiffness) but that the overall length of the specimen increased. He also noted that the hysteresis energy dissipation in each cycle decreased with repeated cycling. These changes continued to occur, although rather gradually, even after several hundred extension cycles. An improvement in the definition of x-ray diffraction patterns and an increase in shortening temperature (see subsequent section) with cycling in these experiments suggested that cycling increased the crystallinity of the fibers.

Other investigators have reported that behavior in the first cycle or first few cycles differed noticeably from that in subsequent cycles but that after the first few cycles the stress-strain hysteresis loop became repeatable (VanBrocklin and Ellis, 1965; Daly, 1966; Viidik, 1968). Such a phenomenon had apparently also been considered by Rigby et al (1959) who applied a "conditioning stretch" to their specimens before initiating their testing procedures. Considering Rigby's findings

(Rigby, 1964) it seems probable that these latter observations do not represent the establishment of a true steady state response but rather of a condition in which the change with continued cycling is slow enough not to be detected in a few cycles.

Abrahams (1967a) noted the development of a permanent "residual" strain in specimens of horse extensor tendon which were subjected to strain cycles of large amplitude. His observation of this behavior agrees with much earlier findings of Gratz (1931) for human fascia lata.

The effective stiffness, taken from stress-strain tracings obtained under cyclic loading or with a ramp loading or strain function, has been observed to increase with increases in some rate parameter (strain rate, cycle frequency, etc.) for human toe extensor tendon (VanBrocklin and Ellis, 1965), human skin (Daly, 1966), rabbit mesentery (Fung, 1967), and canine anterior cruciate ligament (Haut and Little, 1969). However, the energy dissipated in each cycle in a cyclicly loaded specimen has been reported to be nearly independent of rate parameters (VanBrocklin and Ellis, 1965; Fung, 1967).

Fung (1967) has, in addition, pointed out that the dynamic stiffness of rabbit mesentery subjected to small amplitude stress cycles was greater than that with large amplitude stress cycles at the same rates of stress application and removal.

There have been several attempts to develop mathematical analogs for the dynamic and transient behavior of collagen tissues in simple tension. Based on LaBan's creep experiments and on measurements of finger-joint torque-displacement characteristics, a rather simple rheological model consisting of a Kelvin body in series with an elastic element having an extension limit was suggested (Final Report, The University of Michigan Orthetics Research Project, 1964). This model is diagrammed in Figure 2.3. An analog "displaying the creep phenomenon with an element of plasticity" was mentioned by Viidik (1966).

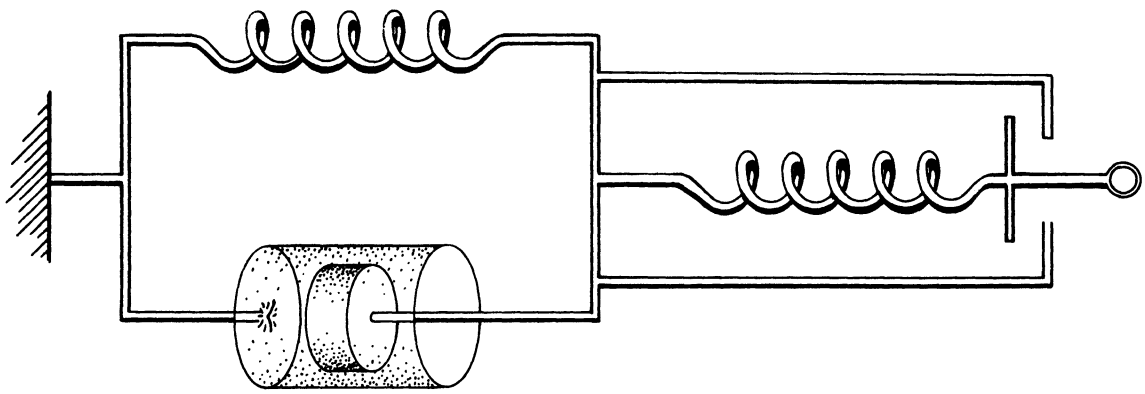

FIGURE 2.3 SIMPLE RHEOLOGICAL MODEL SUGGESTED AS AN ANALOG TO THE MECHANICAL BEHAVIOR OF COLLAGEN TISSUE. FINAL REPORT, THE UNIVERSITY OF MICHIGAN ORTHETICS RESEARCH PROJECT (1964).

Jamison et al (1967) applied a linear model consisting of a viscous element in series with a Kelvin body (Figure 2.4) in characterizing creep in guinea pig skin, and obtained several sets of model parameters. They pointed out, however, that the creep response for real skin depended on stress magnitude as well as time, and that this was not accounted for in their linear model. For ligament, Viidik and Mägi

(1967) suggested a two segment piecewise linear model in which a piecewise linear elastic element was placed in parallel with the viscous element as shown in Figure 2.5. Later, however, Viidik chose

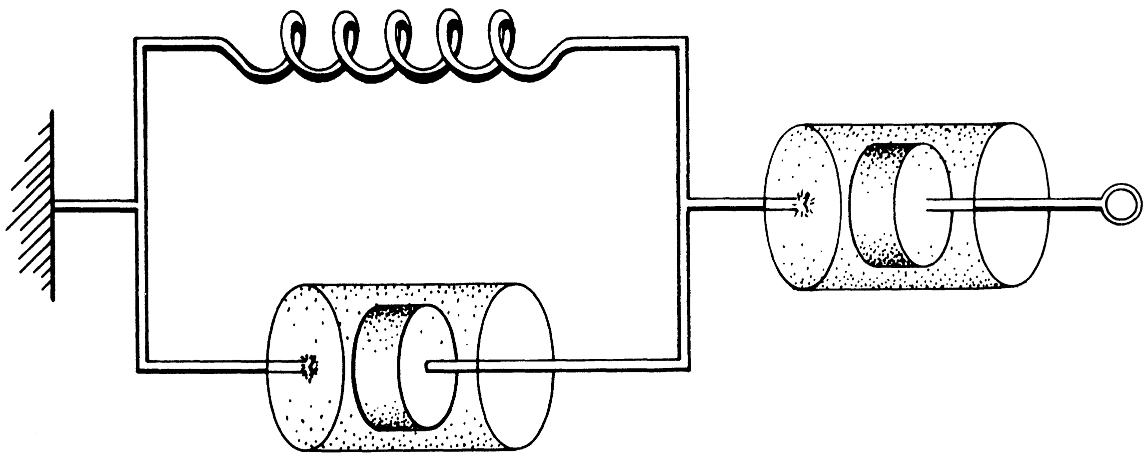

FIGURE 2.4 MODEL EMPLOYED BY JAMISON et al (1967) AS AN ANALOG TO THE MECHANICAL BEHAVIOR OF GUINEA PIG SKIN.

to employ a ligament model in which the piecewise linear elastic element was placed in series with the viscous element (Viidik, 1968a,b) and in which there were a number of additional elements. The form of this model is indicated in Figure 2.6. Here, the loosely linked friction elements and their associated viscous element were used to account for the increase in strain during the first few cycles for specimens subjected to cyclic loading; and the single friction element, at the right in the illustration, was supposed to account for a small region in which there was seen negligible change in strain with decreasing stress just after the maximum point in the stress cycle. It seems, however, that the

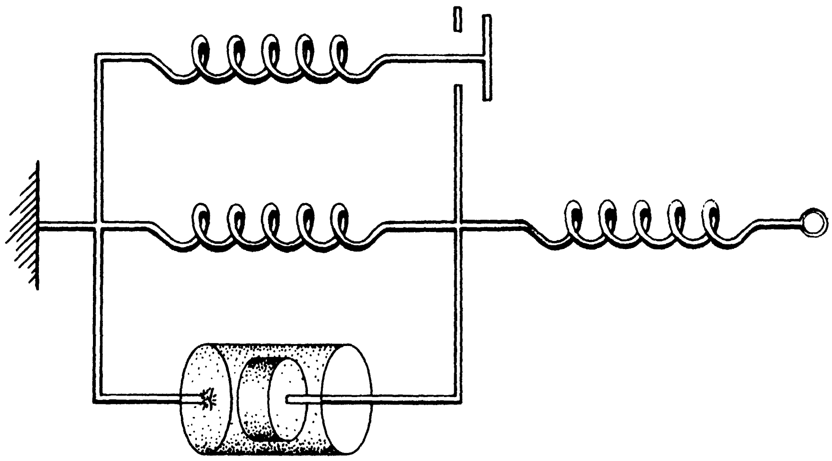

FIGURE 2.5 MODEL PROPOSED BY VIIDIK AND MÄGI (1967) AS AN ANALOG TO THE MECHANICAL BEHAVIOR OF LIGAMENT.

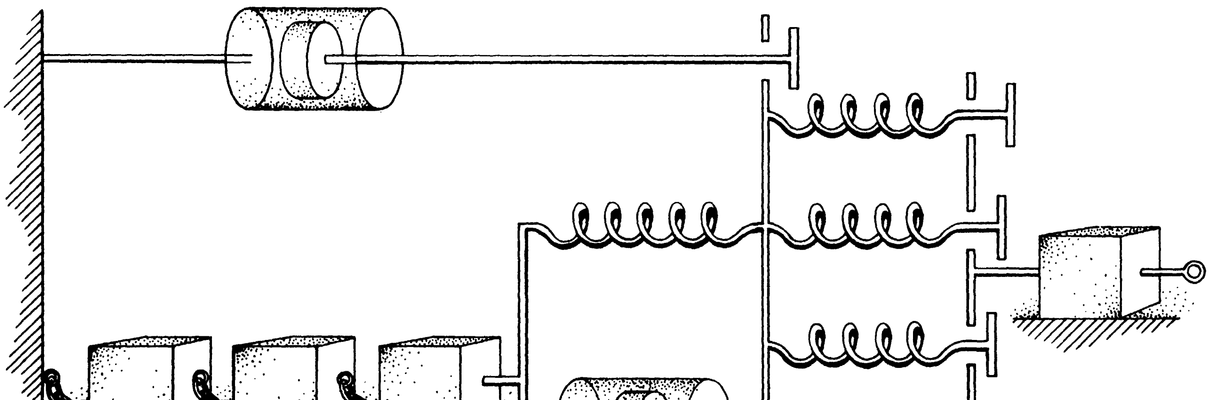

FIGURE 2.6 VIIDIK'S MODEL FOR THE MECHANICAL BEHAVIOR OF LIGAMENT (VIIDIK, 1968a,b).

absence of a corresponding region of increasing stress with negligible change in strain just after the minimum of the stress cycle would have precluded the use of a friction element in this position.

Frisén et al (1969) have reported applying Viidik's model in treating data from tests on rabbit anterior cruciate ligament. But they apparently neglected the problem of this friction element and took the model in the "stationary phase" as a series combination of a Kelvin body and a nonlinear elastic element. In discussing their results, they considered extensions of the model in which the Kelvin body contained a nonlinear viscous element or was replaced by a series of Kelvin bodies; but they concluded that "there are no evident reasons to choose any of these alternatives".

Fung (1968), however, stated that his observation that damping in specimens of rabbit mesentery was nearly independent of frequency "dispels at once the belief popular in the literature that the irreversible portion of the stress-strain law of biological materials is linear viscoelastic"; and he has suggested that either a nonlinear viscoelastic model or a model with a continuous relaxation spectrum might be appropriate (Fung, 1967;1968). He has not yet developed and reported such models because of the analytical difficulties involved.

Somewhat earlier, Daly (1966) attempted to apply a general theory of nonlinear viscoelasticity based on the assumptions that the material was isotropic and that the stress at a point depended only on the time history of the displacement of that point. Using an analysis employing three terms,

$$\sigma(t) = c_i J(t) + c_i^2 K(t,t) + c_i^3 L(t,t,t),$$

he concluded that "the presently available concepts of non-linear viscoelastic theory are not applicable to skin in the form considered." However, he was able to fit the stress relaxation curve for skin by an empirical equation of the form

$$\sigma(t) = a_0 + a_1 e^{-t/\tau_1} + a_2 e^{-t/\tau_2} + \ldots + a_5 e^{-t/\tau_5}$$

Also, he was reasonably succesful in applying a "semiempirical" model consisting of a parallelogram of linear viscoelastic elements having a viscoelastic diagonal member.

2.2.3 TENSILE STRENGTH

Numerous determinations of tensile strength have been made for collagen tissues. Since this report is not primarily concerned with strength characteristics, an extensive review of published work will not be presented here. Elliott (1965) has reviewed published information on the tensile strength of tendon, noting that strengths reported for fresh material have typically been in the range 5 - 10 kg/mm^2, although much lower values have been reported, for example 63.1×10^6 $dynes/cm^2$ (0.62 kg/mm^2) by LaBan (1962). The tendency of specimens to break where they are attached to the testing apparatus, as pointed out by Elliott, the occasional practice of disrupting the fiber array in order to obtain a test section of reduced cross-sectional area (see: Elliott, 1965), and the large differences among cross-sectional area determinations (Ellis,

1969) have probably all been major contributors to the considerable variability in the published data.

Tensile failure of tendon is typically of a "slipping" type (cf. Takigawa, 1953). In tests on muscle-tendon-bone preparations, though, failure has rarely been induced in the body of the tendon, but has typically occurred at the muscle-tendon junction or, more frequently, by a fracture of the bone around the insertion (McMaster, 1933; Stuke, 1951).

2.3 TEMPERATURE EFFECTS IN COLLAGEN TISSUES

2.3.1 HYDROTHERMAL SHORTENING

Certainly the most prominent effect of temperature on collagen tissues is the 'hydrothermal' shortening observed at a temperature which is usually well above the range of physiological temperatures and above the range of temperatures considered in the study reported here[1]. When an unloaded specimen of collagen tissue is heated to this temperature it is seen to shorten rather abruptly to roughly a quarter of its original length and to take on a slightly yellow, translucent appearance in contrast to the reflective white appearance of the original material. Its consistency changes to that of a rubber-like mass which can easily be stretched or compressed.

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[1] Studies of hydrothermal shortening will be reviewed here only briefly. For somewhat more detailed reviews of selected aspects of this work see Veis (1964), Chapter 1, and Elden (1968).

This profound change has probably been observed since the time man first began cooking meat and has attracted significant attention from biophysical chemists during the past several decades. It has been interpreted as a rate limited chemical reaction (Weir, 1949b), as the result of rupturing of inter-chain bonds (Thies and Steinhardt, 1950), and as a phase change like crystal melting (Wöhlisch and deRochemont, 1927; Garrett and Flory, 1956; and others).

In x-ray diffraction studies, both the high angle pattern, which shows prominent reflections corresponding to periodicities of 2.86 Å in the axial direction and approximately 10 Å in the transverse direction, and the low angle pattern which shows reflections corresponding to an axial periodicity of around 640 Å have been found to disappear upon shortening. The high angle pattern (which is the same for collagen and gelatin) has been found to reappear with cooling, drying, or stretching, but, for some time it was thought that the low angle pattern was destroyed irreversibly (cf. Bear, 1944; Wright and Wiederhorn, 1951). Recovery of the low angle pattern was demonstrated, however, by Frenkel et al (1965) who restretched shortened specimens to their original lengths and then cooled these stretched specimens.

The absorption of heat, analogous to latent heat of fusion, has been found to accompany shortening (Wöhlisch and deRochemont, 1927) as has a small increase in volume (Weir, 1949a). The volume increase has been used to identify the transition point in collagen-diluent systems; and a depression in the transition temperature with the addition of diluent, analogous to that generally found for crystal melting, has been observed (see: Garrett and Flory, 1956; Flory and Garrett, 1958).

It has been pointed out that the temperature at which shortening occurrs, T_s, is typically not the equilibrium melting temperature, T_m, but is higher than the equilibrium melting temperature unless the material has been partially premelted so that there is a fresh interface between crystalline and amorphous phases (Oth et al, 1957).

Tensile loads have been found to stabilize the native crystalline state and to retard the shortening transition, so that a specimen under load will attain a higher temperature before shortening than an unloaded specimen. This phenomenon has been described in terms of a one-dimensional analog to the Clausius-Clapeyron equation (Wöhlisch, 1940); and a refinement of the description has been obtained using the equilibrium melting temperature rather than the shortening temperature (Oth et al, 1957). Considering the transition as a helix-coil transformation, Frenkel et al (1965) predicted that there would be a critical stress above which the transition temperature would decrease with increasing load and were able to obtain experimental evidence for the existence of such a critical stress.

2.3.2 THE 'GLASS TYPE' TRANSITION AND LOWER TEMPERATURE EFFECTS

In their investigations on the "melting" of collagen, Flory and Garrett (1958) noted discontinuities in the slopes of volume-temperature curves for beef Achilles tendon specimens. These discontinuities, which were found at 95° C for dried specimens but between 40° and 45° C (only a few degrees above body temperature) for fully hydrated specimens, indicated a second order, "glass type", transition. The occurrence of such a transition in largely crystalline collagen was rather unexpected and was tentatively interpreted in terms of side chain mobility.

Somewhat earlier, Weiderhorn and Reardon (1952) had studied the properties of thermally contracted collagen and had obtained data indicating that the volume fraction of collagen in specimens which were

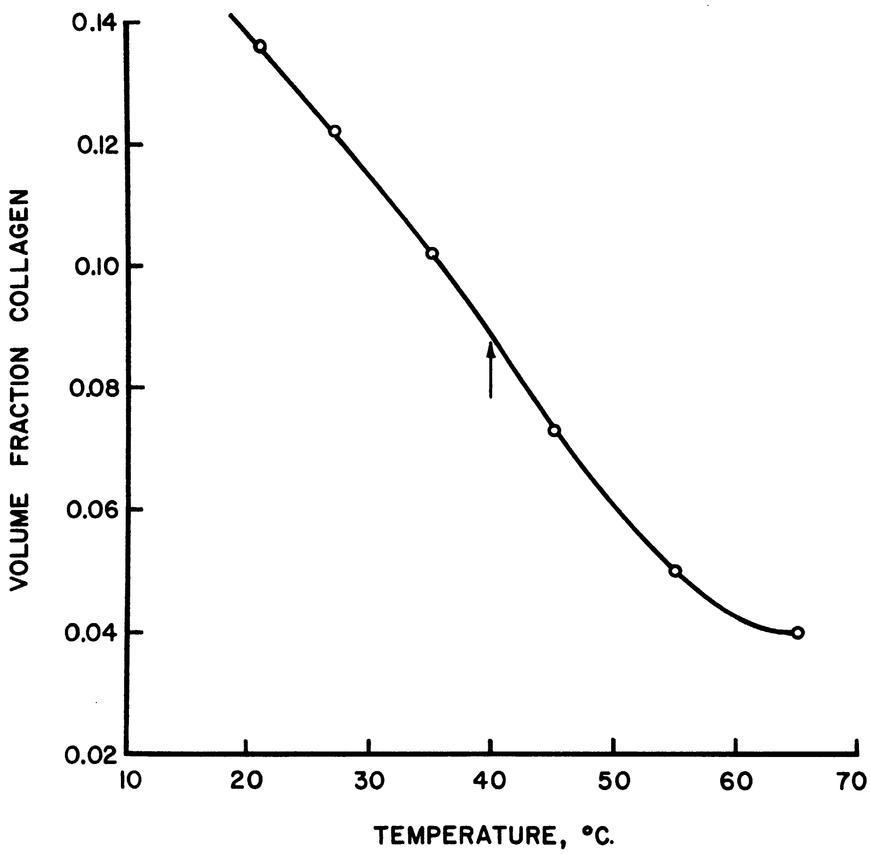

FIGURE 2.7 VOLUME FRACTION COLLAGEN AS A FUNCTION OF TEMPERATURE FOR THERMALLY CONTRACTED COLLAGEN SWOLLEN IN WATER. (ARROW INDICATES INFLECTION POINT) DATA FROM WEIDERHORN AND REARDON (1952).

swollen in a solvent (water, 1 molal LiCl in 1:1 water-ethanol, or formamine), and then blotted, decreased with increasing temperature over the range 21° to 65° C. Their data indicate, however, a possible inflection point in the volume fraction-temperature relation around 40° C. This is shown in Figure 2.7 in which Weiderhorn and Reardon's volume fraction data for thermally contracted collagen swollen in water have been plotted as a function of temperature.

Much earlier, Kitamura (1923) had investigated the effect of temperature on the hardness of tendon and of ligamentum nuchae, as measured by the penetration of an indenter perpendicular to the fiber axis. Starting at 20° C, he found that hardness decreased with increasing temperature until it reached a minimum around 50° C, and then increased with further increases in temperature. Since his data in the region of the minimum were taken at 10 ° C intervals, the location of the minimum is not known with sufficient accuracy to assess whether it was associated with shortening or occurred before the shortening temperature was reached.

Marked changes in the tensile mechanical properties of rat tail tendon which have been assumed to coincide with the 'glass type' transition (see: Elden, 1968) have been reported by Rigby et al (1959). They found that above approximately 37° - 40° C specimens frequently ruptured at strains of only 3 to 4 percent, while below 37° C specimens could be repeatedly strained to 4 percent and showed temperature independent load-strain behavior. In addition, they noted that stress relaxation behavior at strains below the "safe limit" of approximately

4 percent strain was nearly independent of temperature between 0° and 37° C but that as the temperature was increased above approximately 37°-40° C the rate and amount of stress relaxation became substantially greater with increasing temperature. This effect is illustrated in Figure 2.8 which shows stress relaxation curves at 3 percent strain obtained at several temperatures by Rigby et al. These changes were

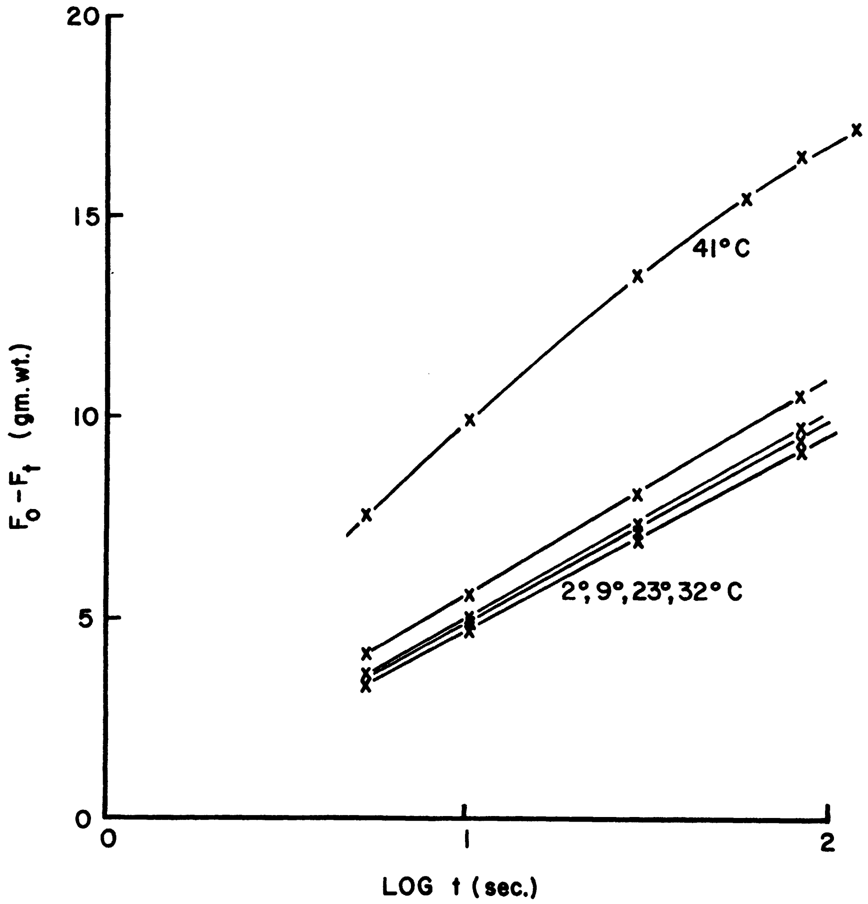

FIGURE 2.8 STRESS RELAXATION CURVES (DECREASE, F_o-F_t, IN LOAD PLOTTED AS A FUNCTION OF THE LOGARITHM OF TIME) FOR RAT TAIL TENDON AT 3 PERCENT STRAIN, AT SEVERAL TEMPERATURES. REDRAWN FROM RIGBY et al (1959).

described by Rigby et al as "abrupt and irreversible" and were interpreted as possibly reflecting the breakage of some hydrogen bonds in the collagen structure, a process analogous to that which occurs in melting.

Although Rigby et al found load-strain and stress relaxation behavior to be nearly temperature independent below 37° C, they found that the limiting stress in fully stress relaxed specimens changed in a reversible manner as the temperature was varied in the range below 37° C (Figure 2.9) so long as the specimen had not been strained beyond the

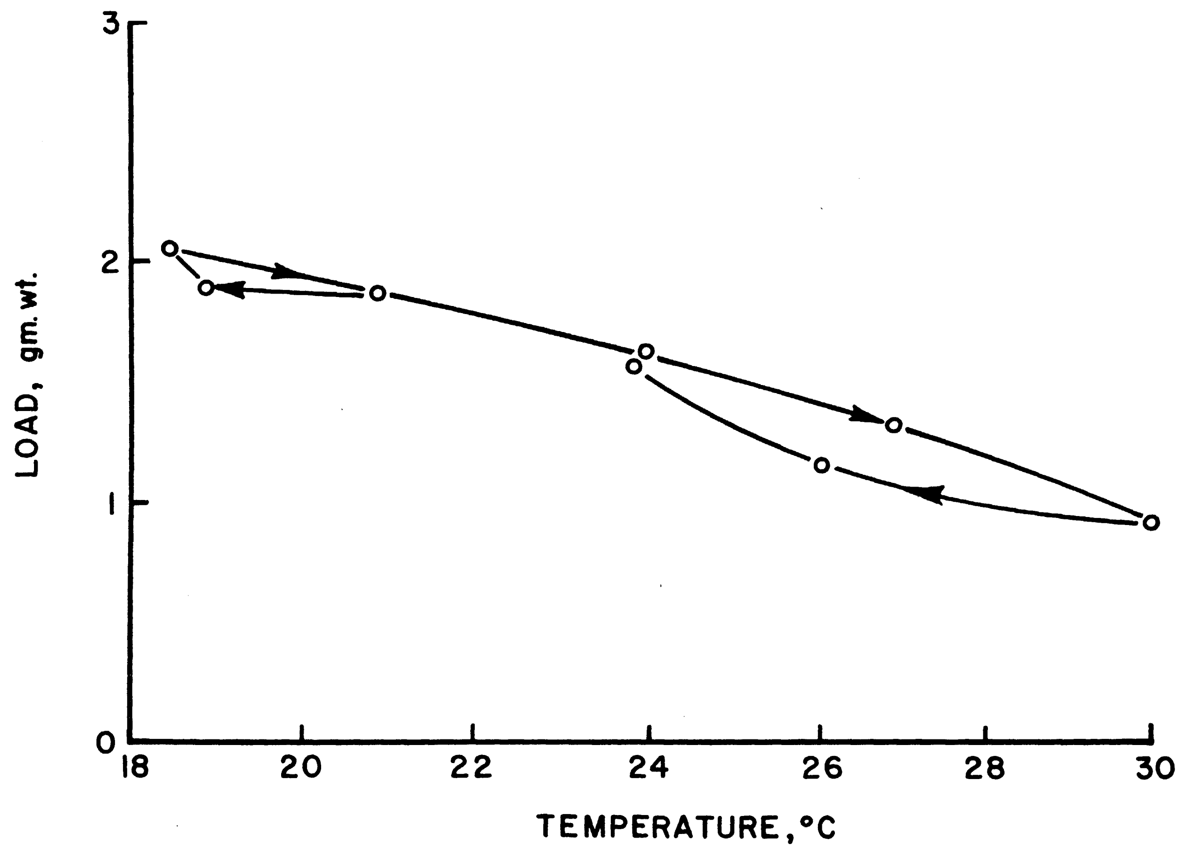

FIGURE 2.9 ISOMETRIC FORCE AS A FUNCTION OF TEMPERATURE IN FULLY STRESS RELAXED RAT TAIL TENDON. REDRAWN FROM RIGBY et al (1959).

"safe limit" of approximately 4 percent. For specimens strained beyond this limit, stress relaxation was almost complete; and the decrease in load associated with an increase in temperature was found to be irreversible.

A creep effect which possibly corresponds to this stress relaxation effect, but which more closely resembles the increase in the rate and amount of stress relaxation with increasing temperature which Rigby et al found at higher temperatures, had been noted earlier by Hall (1951a) in

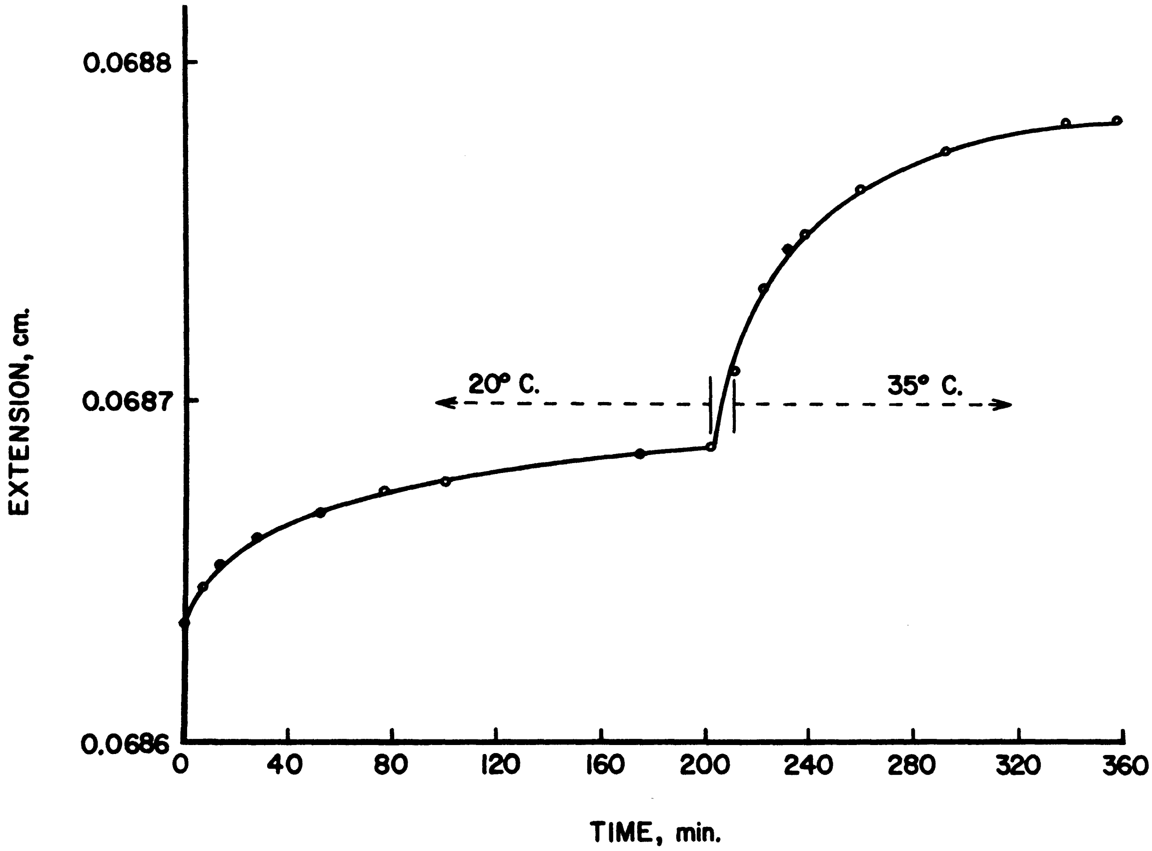

FIGURE 2.10 EXTENSION AS A FUNCTION OF TIME FOR A COLLAGEN SPECIMEN PERMITTED TO CREEP AT 20° C AND THEN HEATED TO 35° C. REDRAWN FROM HALL (1951a).

a single experiment in which he allowed a specimen to creep for 200 minutes at 20° C and then heated it to 35° C. Upon heating, a very pronounced increase in creep was seen, as shown in Figure 2.10.

Two additional studies have included measurements of tensile mechanical properties at more than one temperature and have involved temperatures near 40° C. LaBan (1962) obtained creep and recovery curves for canine calcaneal tendon at 37°, 39.8°, and 42.5° C. His data indicate that an increase in temperature was accompanied by an increase in initial extension which was not subsequently recovered. As has already been noted, however, his extension measurements may have been significantly in error, since he measured extensions outside the clamps which applied force to his specimens. Haut and Little (1969) obtained stress-strain curves and stress relaxation data for canine anterior cruciate ligaments moistened with saline, immersed in saline at 72° F (22.2° C) and immersed in saline at 101° F (38.4° C). Their stress-strain data were obtained at a number of strain rates and show a more pronounced increase in stiffness with increasing strain rate at the higher temperature than at the lower temperature. Their stress relaxation data, which were taken only over 10 minute periods, however, show little difference between the two temperatures. In reporting their results, these authors expressed some reservations about their results at the higher temperature.

Mason and Rigby (1963) obtained both specific volume and isometric tension (in specimens stretched to a fixed extension) as functions of temperature for tendon specimens and found that the 'glass transition' which Flory and Garrett (1958) had identified as a discontinuity in the slope of the volume-temperature curve could also be identified by a change in the slope of the isometric tension-temperature curve. This is indicated in Figure 2.11 which shows the isometric tension-temperature curves obtained by Mason and Rigby, one for native beef tendon and four for partially premelted beef tendon (see below). Similar changes in the

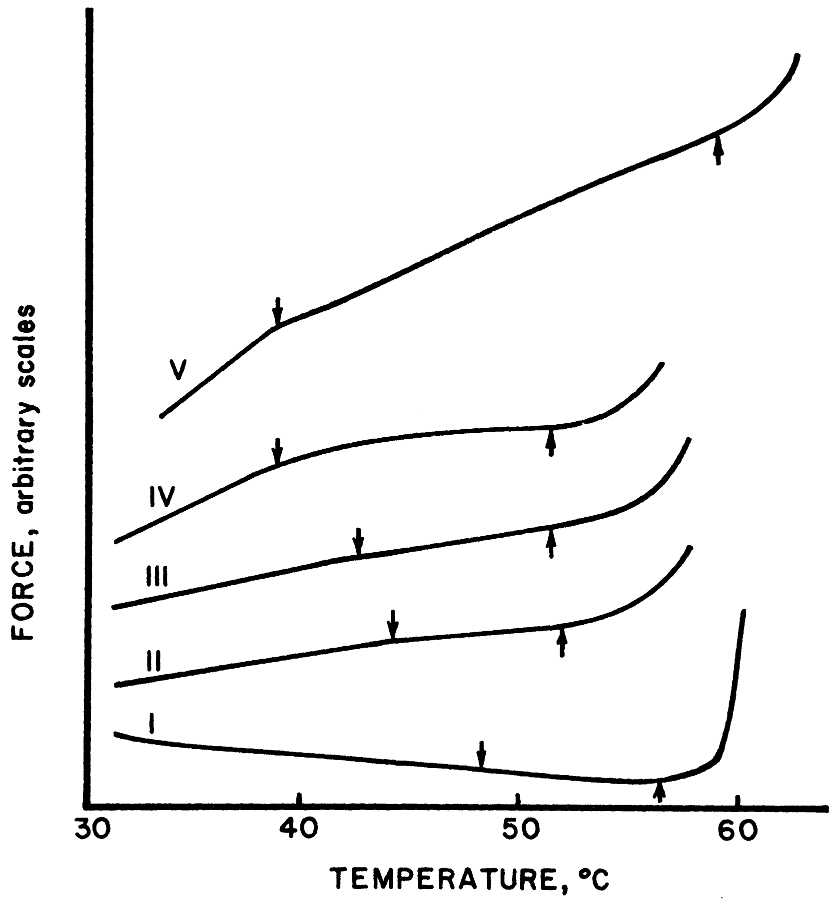

FIGURE 2.11 ISOMETRIC TENSION-TEMPERATURE CURVES FOR BEEF TENDON. (I) NATIVE, (II,III,IV) SUCCESSIVE TESTS, ALL PERFORMED THE SAME DAY ON A PARTIALLY PREMELTED SPECIMEN, (V) TEST ON THE SAME SPECIMEN AS IV, PERFORMED THE FOLLOWING DAY. ↓ T_G, ↑ T_S. (MASON AND RIGBY, 1963).

slopes of isometric tension-temperature curves have also been reported more recently by Elden (1964a,b;1965). Using both specific volume- and isometric tension-temperature curves for determining transition temperatures Mason and Rigby (1963) examined the effect of partial premelting and found partial premelting to decrease both the shortening temperature, T_S, (in accord with the work of Oth et al, 1957) and the 'glass transition' temperature, T_G. They reported the change in T_G to have been irreversible. Their data also show that partial premelting reverses the slope of the isometric tension-temperature curve below the shortening temperature (see Figure 2.11). In the native material the isometric tension decreases with increasing temperature below the shortening temperature, but according to the data of Mason and Rigby, isometric tension increases with increasing temperature in partially premelted specimens. If partial premelting was presumed to create amorphous regions displaying rubber-like thermoelasticity, this result would be anticipated in light of the discussion of partially melted proteins presented by Flory (1956).

Rigby (1964b) determined transition temperatures from isometric tension-temperature curves in a series of experiments in which he examined the effect of strain history on the transition temperatures of rat tail tendon in 0.9 percent saline. The curves which he obtained in these experiments (Figure 2.12) were significantly different from those obtained by Mason and Rigby (1963) and differed from one another depending on the strain history. For native specimens, Rigby found only one transition, 'shortening' at 60° C (curve I). For specimens strained for long periods at moderate strains (curve II) or for short periods at large strains (curve III), he found an additional transition, a 'partial shortening' at around 51° C. This effect was reported to have been reversible when small strains were employed, but not when large strains were used. After long periods at large strains he no longer observed this 51° transition but found, instead, a transition like that seen by Mason and Rigby (1963), occurring at around 35° C (curves IV and V). 'Shortening' in these specimens occurred at a lower temperature than

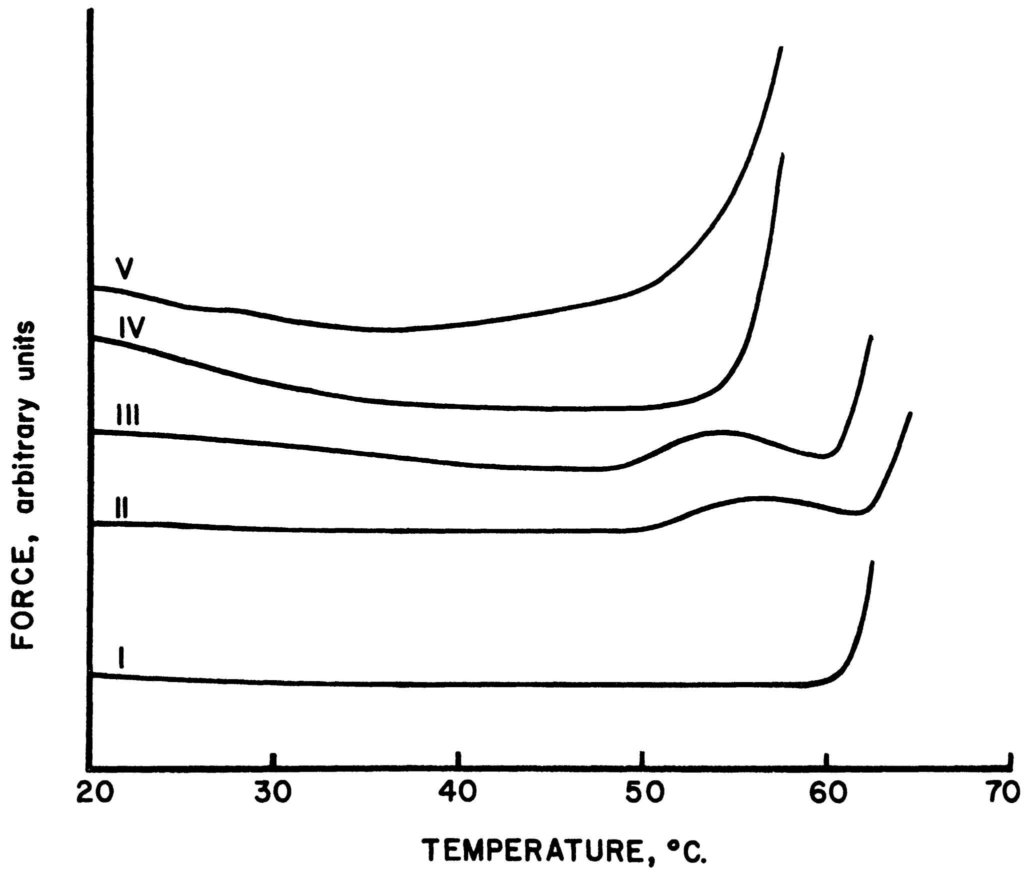

FIGURE 2.12 ISOMETRIC TENSION-TEMPERATURE CURVES FOR RAT TAIL TENDON IN 0.9 PERCENT SALINE, OBTAINED AFTER VARIOUS STRAIN HISTORIES. (I) UNSTRETCHED, (II) 1.0 PERCENT STRAIN FOR 22 HOURS, (III-V) 10 PERCENT STRAIN FOR 4, 17, AND 69 HOURS, RESPECTIVELY. (RIGBY, 1964b).

in unstrained specimens, around 50° - 55° C. Rigby noted correspondences between the transition temperature of 51° C and the equilibrium melting temperature (Oth et al, 1957), and between the 35° C transition temperature and the melting temperature of tropocollagen[1] in solution. He suggested that the melting processes corresponding to these two temperatures might occur in the mechanically disrupted material, resulting in the two transitions which he observed.

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[1]Tropocollagen is a soluable form of collagen with a "molecule" which is believed to consist of 3 polypeptide chains arranged as a triple helix. Collagen fibrils are believed to consist of parallel arrays of chains formed from tropocollagen units joined end-to-end.

3. OBJECTIVES AND SCOPE OF THE INVESTIGATION

Initially, the objective of this investigation was defined only in very general terms: to obtain a description of the way in which the tensile mechanical properties of tendon, exclusive of tensile strength, change with temperature. The description was viewed at first as a constitutive equation having coefficients which might be functions of temperature. The early work consisted of a loosely structured series of experiments, analyses, and computer simulations directed toward the development of such a constitutive equation but without temperature as a parameter (that is, with all experimental work performed at a single test temperature).

Since this work failed to yield a satisfactory mathematical analog to the mechanical behavior of tendon (see section 5.1) this approach was abandoned, and the subsequent work was directed toward the determination of values for parameters associated with prominent features of the mechanical behavior of tendon at a sequence of temperatures. The features included were damping (hysteresis energy loss), dynamic stiffness, and plastic flow. These were investigated over a temperature range from approximately 20° to 55° C which included the usual range of physiological temperatures and extended nearly to the shortening temperature.

4. MATERIALS AND METHODS

4.1 GENERAL CONSIDERATIONS:

Since the primary reason for undertaking this study of tendon mechanics was to obtain information which could be used in analyzing the effect of temperature on the functional characteristics of connective tissue structures in the living animal, it was desirable for the test situation to approximate the physiological condition of the tendons studied. The physiological environment of tendon fibrils is the ground substance. It might therefore have been desirable to test the specimens while they were surrounded by this substance, or by synovial fluid which has a similar composition. The large volume of bathing medium required in the test apparatus, however, made this impractical. The use of an artificial medium with similar ionic and specific mucopolysaccharide compositions would also have been impractical because of the difficulty of obtaining sufficiently large quantities of the mucopolysaccharides.

Therefore, the test environment was made to approximate the physiological environment only in inorganic ion composition (Ringer's solution at a neutral pH).

Although the effect of this deviation cannot be assessed quantitatively on the basis of the available information, two changes may be anticipated, at least qualitatively: (1) The tendons would be expected to take up water and swell slightly in the Ringer's solution (Viidik and Lewin, 1966). This is probably due to the absence of osmotically effective large molecules in the solution. (2) The tendons would probably lose some polysaccharides to the bathing medium. This would be expected to result in slight increases in the strength and the stiffness (elastic modulus) of the specimens, as suggested by the work of Milch (1966 a,b) which has indicated that polysaccharides act as plasticizers for collagen. Changes of this type have, to some extent, been demonstrated by Viidik and Lewin (1966), who found that the failure energy, failure load, and elongation at rupture for rabbit ligaments were increased by 5 hours' storage in 0.9 percent saline at 20°C although the stiffness was decreased by 24 hours storage in 0.9 percent saline at 4°C.

To assess the extent to which these and related changes were occurring in the experiments in this study, test-retest features were incorporated into the experimental protocol wherever practical.

As a matter of convenience, both fresh specimens and specimens which had been preserved by freezing were used in the experiments in this study. However, preserved specimens were used primarily in exploratory experiments such as those associated with attempts to develop mathematical models. In the temperature studies, results obtained from tests on preserved specimens were considered to be quantitatively valid only after they had been verified by subsequent tests on fresh material.

Earlier work has suggested that preservation by freezing may have some effect on the mechanical properties of connective tissue although the effect is probably small. Thomas and Gresham (1963) found no significant differences in tensile strength among fresh, frozen, and freeze dried specimens of human fascia lata. Viidik and Lewin (1966) also failed to find a significant difference between the strengths of fresh rabbit anterior cruciate ligaments and those which had been preserved by freezing, but they did find that specimens preserved by freezing exhibited higher failure energy than fresh specimens, even though stiffness, gross shape of the stress-strain curve, and elongation at rupture were not found to be altered. Matthews and Ellis (1968), however, did find a statistically significant decrease in stiffness (around 10%) associated with preservation by freezing in their experiments on extensor digitorum communis and extensor lateralis tendons from the cat.

4.2 EXPERIMENTAL PROTOCOL

Tendons used in this study were extensor digitorum communis tendons from adult cats (*Felis catus, L*). Each cat was sacrificed using an intraperitoneal injection containing 750 mg. sodium pentobarbital, and after dissection was completed the chest cavity was opened by an incission to ensure that the animal would not revive.

As soon as the cat became totally unresponsive to stimuli, the extensor digitorum communis tendons were dissected from its front paws, four from each paw, and were immediately placed in a tray of slightly chilled Ringer's solution. One of these eight tendons was cut to length and tested at once. The remaining seven were wrapped in paper towelling moistened with Ringer's solution and then in aluminum foil, and were stored in a freezer at approximately -20°C for subsequent testing. Just prior to the time of testing these specimens were thawed in Ringer's solution at room temperature (approximately 20°C) before being cut to the test length.

Most specimens as they were dissected from the animal were over 5 cm. long. These were cut to a standard length of 5.00 cm. immediately before they were tested using the cutting machine employed by Ellis (1969). In this machine, which is illustrated in Figure 4.1, the specimen is straightened by a pair of rubber pads sliding along the 'chopping block'; and is then cut by a pair of blades which come down onto the 'chopping block' simultaneously. Those few specimens less than 5 cm. long were trimmed with a scalpel so that their ends were cut straight across; their lengths were then measured with a rule having 0.05 cm. graduations, and the measurements were recorded on the test record.

After a specimen was cut to length, it was clamped in the testing machine; the strain-measuring vanes were clipped to it; and the Ringer's solution bath was raised so that the specimen was immersed. Then, a sequence of tests having the following general form was begun:

First, the specimen was subjected to a triangular wave loading function at roughly 2 cycles per minute until a steady state response was approached (that is, until a closed loop such as that shown in Figure 5.22A was obtained in the

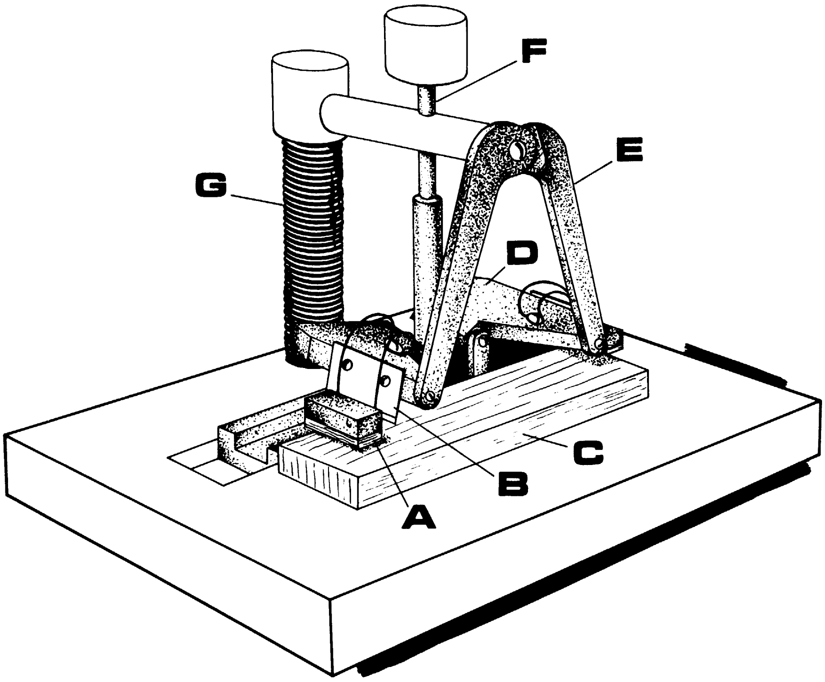


FIGURE 4.1 TENDON CUTTING MACHINE; (A) TENDON STRAIGHTENING PAD, (B) BLADE, (C) CHOPPING BLOCK, (D) ARM CARRYING PAD AND BLADE, (E) LINKAGE TO ARM, (F) GUIDE PIN, (G) SPRING-LOADED SLIDING POST. [DRAWING BY G.N.TAN, FROM ELLIS, 1969].

load-strain record). Then, the specimen was held at a constant small load (either 0.1 or 0.05 Megadyne) for approximately one hour. At the end of this time it was subjected to a triangular wave loading function of the same amplitude and frequency as the one used initially, again until a steady state response was approached. Following this, it was subjected to a sequence of triangular wave loading functions comprising the first main test sequence. All tests up to this point were performed at a low temperature, in the range 22° - 25° C. At the end of the first main test sequence, the specimen was completely unloaded; the strain-measuring vanes were removed; and the bath temperature was raised to a second test temperature. Although the second test temperature was attained within one hour, two hours were allowed before the specimen was subjected to further tests. The strain sensor was then re-standardized, using the output obtained with the unobstructed light beam as a reference for adjusting the sensitivity (see Section 4.4.4); the vanes were clipped to the specimen; and a second main test sequence was begun. The second main test sequence consisted of a sequence of triangular wave loading functions the same as that in the first main test sequence. At the end of the second main test sequence, the specimen was removed from the testing machine and hung on a stainless steel hook to dry.

There were two deviations from this general form: (1) In a few experiments, three test temperatures were used. In these, the specimen was unloaded and the vanes removed at the end of the second main test sequence; then a two hour interval was allowed during which the bath attained the third test temperature. At the end of this interval, the strain sensor was re-standardized; the vanes were reattached; and a third main test sequence, like the first and second main test sequences, was performed. (2) In experiments on plastic flow, the first main test sequence was omitted, and the second main test sequence consisted of a triangular wave loading function followed by a series of steady loads each held for several minutes and each attained from the preceding one by ramp loading.

After the tests in an experiment were completed, the specimen was dried in room air (temperature 20° - 25°C, relative humidity 50 - 80 percent) for at least 24 hours and was then weighed using a Mettler M5 microchemical balance having an accuracy better than ± 2 μg. over the range of specimen weights.

4.3 PRESENTATION OF DATA (UNITS):

The usual practice in reporting results of tests such as those described here is to express the applied force in terms of force per unit of cross-sectional area of the test specimen, or stress, (lb_f/in^2, $Mdyne/cm^2$, etc.). Various methods which have been used to measure cross-sectional areas of tendon specimen, however, have been found to give widely differing results and to show relatively poor repeatibility (Ellis, 1969). Of the measurements examined by Ellis, those based on the initial, unstressed, length and the weight of dry material comprising the specimen were found to be the most repeatable.

For this reason, and because it is reasonable to assume that tensile properties of tendon would correlate well with the mass of solid material per unit length, dry weight per unit length is adopted here as a measure of cross-sectional area. This practice follows that used by Elden in most of his work. When the quantity *dry weight per unit length* is used in defining a 'stress', the resulting expression,

$$\frac{\text{Force}}{\text{dry weight/unit length,}}$$

does not have the usual units of stress. To avoid confusion, a different name, 'specific load', is applied to the quantity defined by this expression. The unit of specific load in this report is Mdyne-cm./gm. (i.e. $\frac{\text{Mdyne}}{\text{gm./cm.}}$).

Stiffness or elastic modulus is normally defined as the ratio of stress to strain. Since strain is dimensionless, the units are the same as the units for stress. In reporting results in this study stiffness has been defined using specific load rather than stress and has the same units as specific load.

In analyzing data from experiments on damping, areas of hysteresis loops in the coordinates 'specific load' and 'strain' were used as measures of the energy dissipated by the specimen in each load cycle. Since strain is dimensionless, these areas also have units of Mdyne cm/gm. In an analogy to the term 'specific load' such areas are here referred to by the term 'specific damping'.

4.4 APPARATUS

4.4.1 TESTING MACHINE:

The testing machine used for these studies was a modification of the low capacity dead weight testing machine described by VanBrocklin and Ellis (1965). This machine, which is shown in Figure 4.2, operates as a balance in which load applied by filling a weight bucket (K) is balanced by tension in the test specimen. A counterweight (F) which is sufficiently massive to counterbalance a partially filled load bucket allows the machine to be balanced (specimen unloaded) by suitably adjusting the level of water in the weight bucket. A guide and pin assembly (H) prevents the specimen from twisting by preventing rotation of the upper, movable, specimen clamp while an adjustable limit stop (G) can be used to limit the downward excursion of this clamp.

The load which is applied to the specimen is measured by a strain gage load cell (E) in series with the movable specimen clamp; and elongation of the specimen is measured by an optical strain sensor (C) which detects the separation of a pair of vanes clipped to the specimen. Load and strain are recorded as functions of time on a Servoriter II recording potentiometer (Texas Instruments Co.); and, when desired, load is plotted as a function of strain on a Plotamatic 600 X-Y recorder (Data Equipment Co.).

The test section, including the specimen, specimen clamps (A), and strain sensor (C), is immersed in a controlled temperature bath

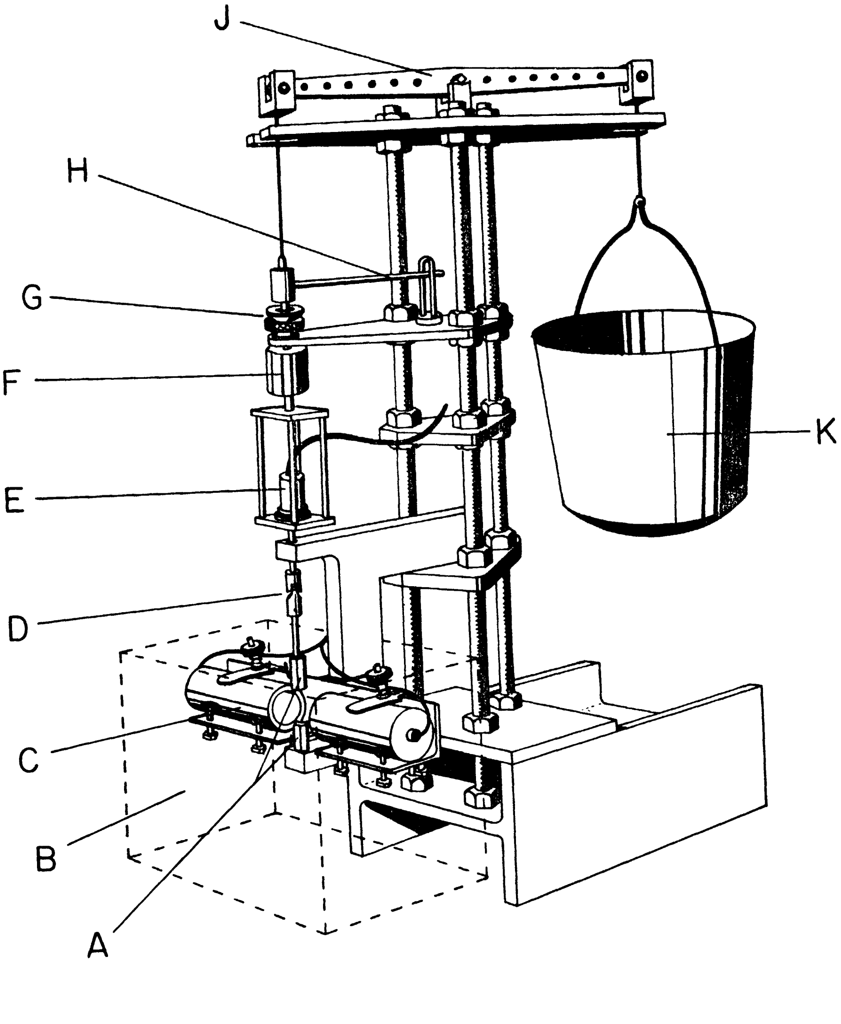


FIGURE 4.2 TESTING MACHINE; (A) SPECIMEN CLAMPS, (B) CONTROLLED TEMPERATURE BATH, (C) STRAIN SENSOR OPTICS, (D) UNIVERSAL JOINT [NOT USED], (E) LOAD CELL, (F) COUNTERWEIGHT, (G) ADJUSTABLE LIMIT STOP, (H) GUIDE AND PIN ASSEMBLY, (J) BALANCE BEAM, (K) WEIGHT BUCKET.

(B) containing Ringer's solution. This bath is supported on a jack (Cenco-Lerner "Lab jack", no. 19089) so that it may be lowered to permit mounting of the specimen in the specimen clamps.

During the initial phases of the study (the experiments associated with the modelling work) force was transmitted from the weight bucket to the force transducer-clamp assembly by means of a steel cable which ran over a pair of aluminum pulleys each of which turned on a stainless steel shaft supported in "Teflon" bushings. For the temperature studies, this arrangement was replaced by an aluminum balance beam as shown in Figure 4.2 (J). This beam and each of the two stirrups which it carries (one for the weight bucket and one for the transducer-clamp assembly) are pivoted on stainless steel knife edges.

At the same time as the pulley system was replaced by the balance beam, a universal joint (D) was added in the link between the force transducer and the movable clamp. This, however, was soon replaced by a solid rod since changes in specimen orientation which occurred with the universal joint were found to produce significant errors in the strain measurement.

4.4.2 SPECIMEN CLAMPS:

The clamps which were used to hold specimens in the testing machine are made of stainless steel and are of a lever action type.

In each of these clamps (Figure 4.3) the specimen was gripped between a cam-operated clamping lever and a cylindrical post which projects from the body of the clamp. This post is semicircular in section and has serrations over somewhat more than one third of its curved surface. The clamping surface of the lever is also serrated and mates with the serrated portion of the post surface. Friction between the specimen and the remaining (smooth) portion of the post surface serves to reduce the pull which must be resisted by the clamped portion of the specimen.

To allow for small variations in the thickness of the clamped specimen, the cam which holds the clamping lever against the post is not solid but is formed by a loop of stiff stainless steel spring wire which protrudes from a 3-turn coil of wire wrapped about the cam pivot.

4.4.3 FORCE TRANSDUCER:

During the experiments associated with the modelling studies, the force applied to the specimen was measured by a 20 lb. capacity aluminum proving ring (Figure 4.4) which was instrumented with four foil strain gages (Micromeasurements Co. type MA-13-125AC-350). For the subsequent work this was replaced by a more sensitive transducer consisting of a Statham universal transducing cell (Model UC3) attached to a 10 lb. load adapter (Statham UL4-10) which forms the bottom member of a supporting cage. This is the transducer configuration shown in Figure 4.2 (E).

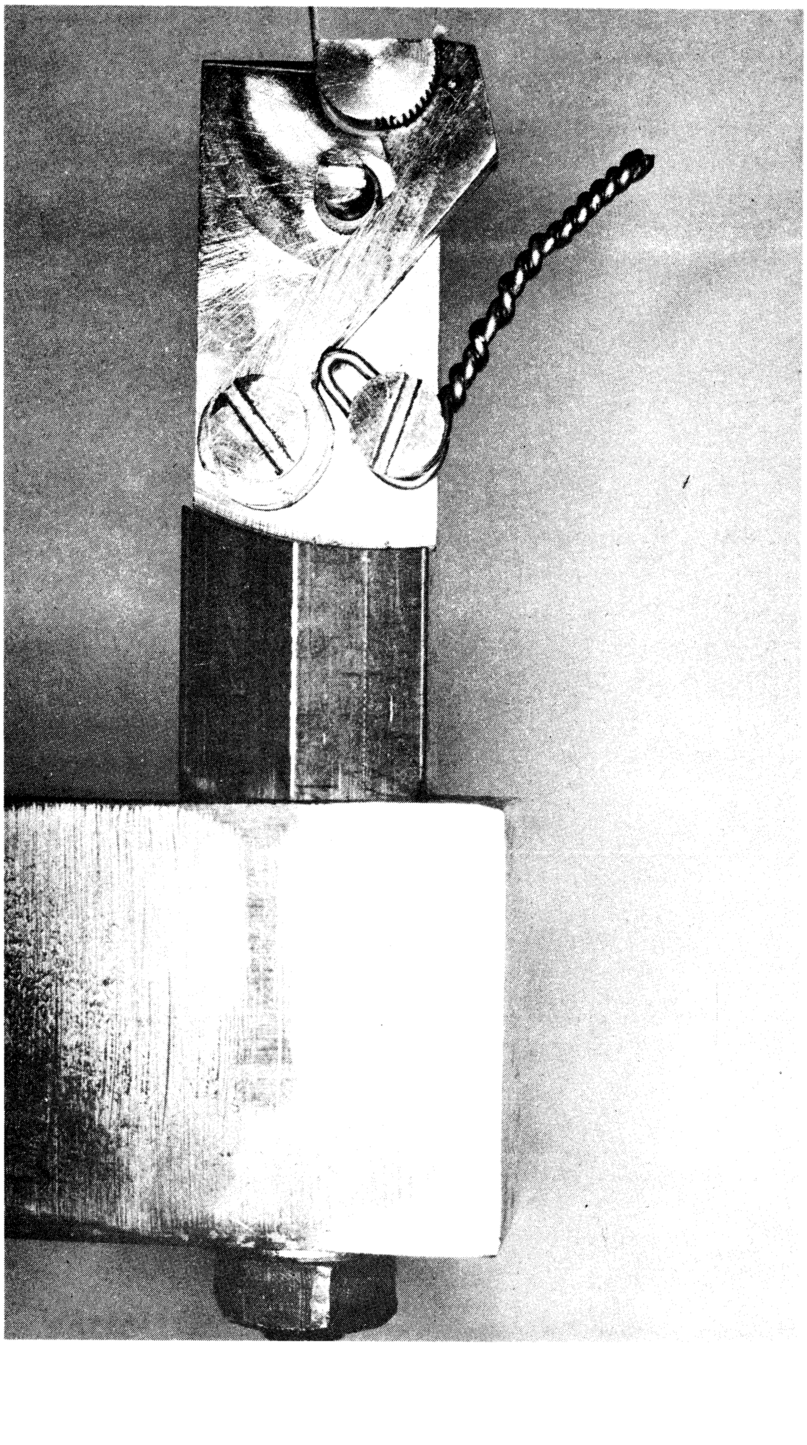


FIGURE 4.3 SPECIMEN CLAMP (APPROXIMATELY 2X ACTUAL SIZE).

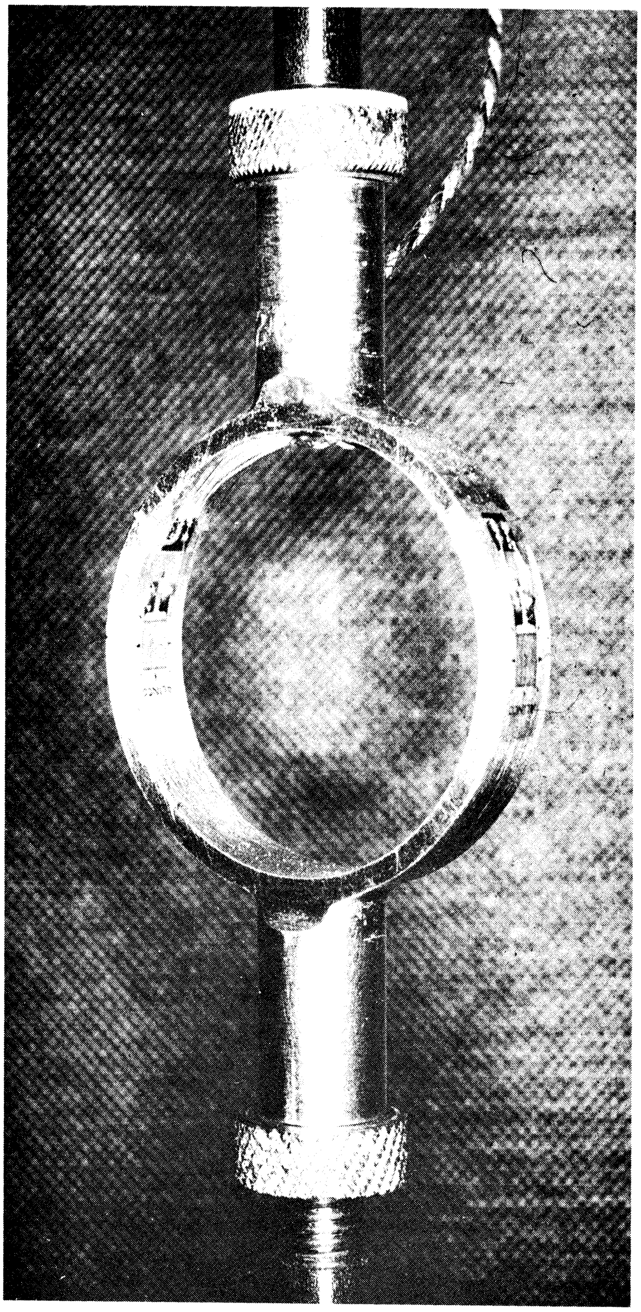


FIGURE 4.4 PROVING RING LOAD CELL USED DURING MODELLING STUDIES (APPROXIMATELY 2X ACTUAL SIZE).

An external resistive network connected to the transducer strain gage bridge was used to balance the bridge and to generate calibration signals. This network, which is diagrammed in Figure 4.5, was the same for the proving ring as for the Statham load cell except for resistance values as noted in the diagram.

When the proving ring was used, it was necessary to amplify the transducer output to a level suitable for recording on the X-Y recorder. This was accomplished by means of a Sanborn 8875A differential amplifier. Adjustment of the amplifier gain permitted the over all sensitivity of the load sensor to be set as required.

With the Statham transducer, the output of the transducer bridge is sufficiently large to be recorded without amplification, and sensitivity adjustment is accomplished by varying the bridge supply voltage.

Calibration of both force transducers was carried out as follows: The movable specimen clamp of the testing machine was removed and replaced by a weight pan. The transducer bridge was balanced to zero output. Then, calibration weights were placed on the weight pan; and the bridge output was recorded on the recording potentiometer. Calibration curves obtained in this way for the two transducers are presented in Appendix 1,A.

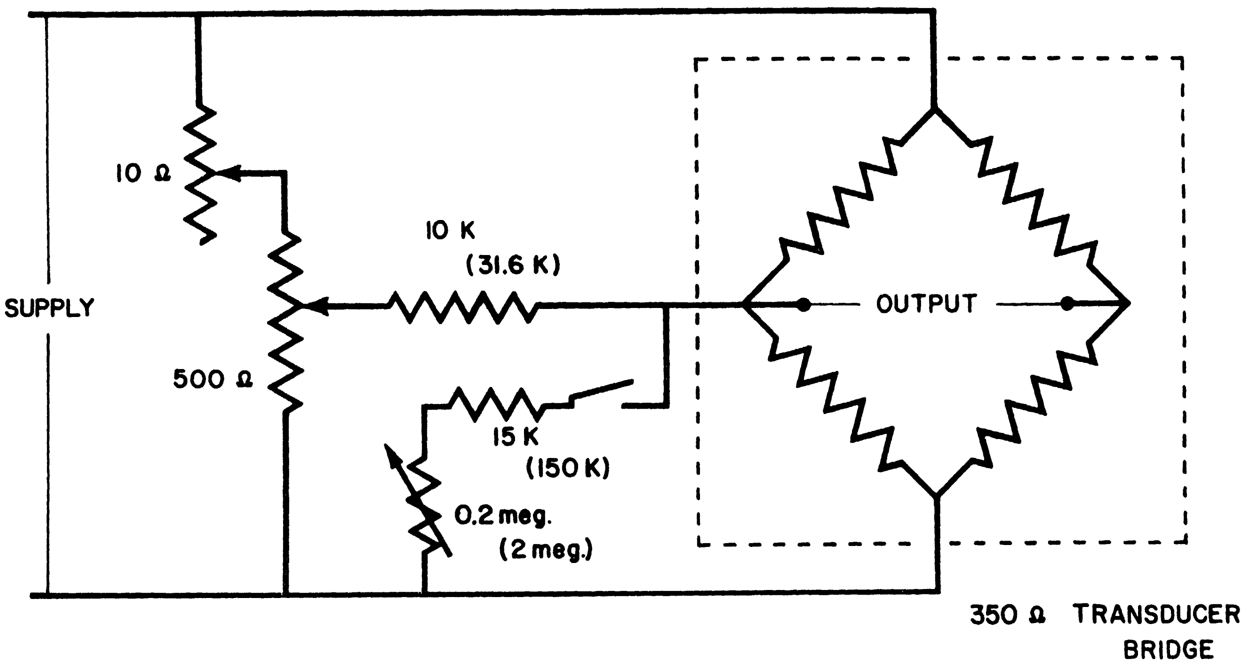


FIGURE 4.5 LOAD CELL BRIDGE CIRCUIT. VALUES IN PARENTHESES FOR PROVING RING LOAD CELL.

4.4.4 STRAIN SENSOR

Specimen strain was measured by measuring the separation of a pair of parallel vanes clipped to the specimen. This was done by measuring the light passing between the vanes when a rectangular, collimated light beam was directed so that it was intercepted by the vanes as shown in Figure 4.6. The instrument used for this measurement has been described by Ellis (1967). It consists of a light source and a detector each housed in an O-ring sealed stainless steel canister having an optically flat glass window at one end.

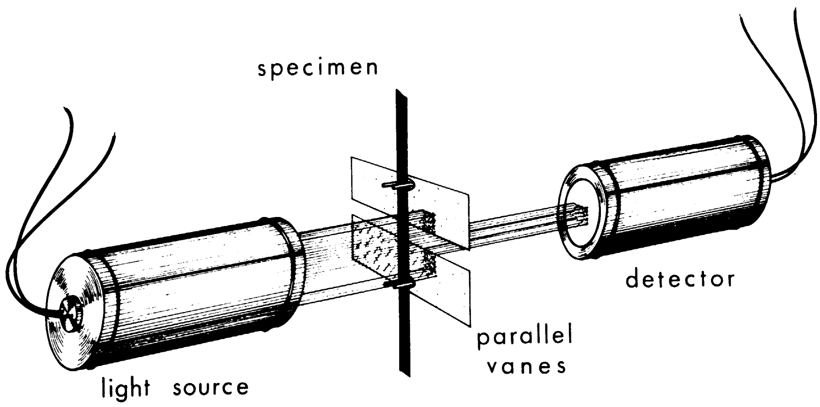


FIGURE 4.6 BASIC ELEMENTS OF THE STRAIN MEASURING SYSTEM (FROM ELLIS, 1967).

In the light source canister, light from a small indicator bulb (Sylvania 6ES) having a planar, V-shaped filament is collimated by a 55 mm. focal length achromatic lens. The collimated beam is then limited by a vertical slit 1.4 mm. wide before passing through the window. A 5 mm. horizontal slit in a stainless steel end cap which is fitted over the source canister window limits the vertical height of the beam.

The transducer element in the detector canister is a phototransistor (General Sensors GS600) upon which the light from the source canister is focused by a pair of achromatic lenses (55 mm. and 20.4 mm. focal lengths).

This phototransistor is connected into an external circuit so as to yield a voltage output which is proportional to the irradiance at the phototransistor surface plus an adjustable constant. (Note that since light from the source canister is focused onto the phototransistor the irradiance at the phototransistor surface is proportional to the total light from the source which is received by the detector.) The sensor circuit is shown in Figure 4.7. In this

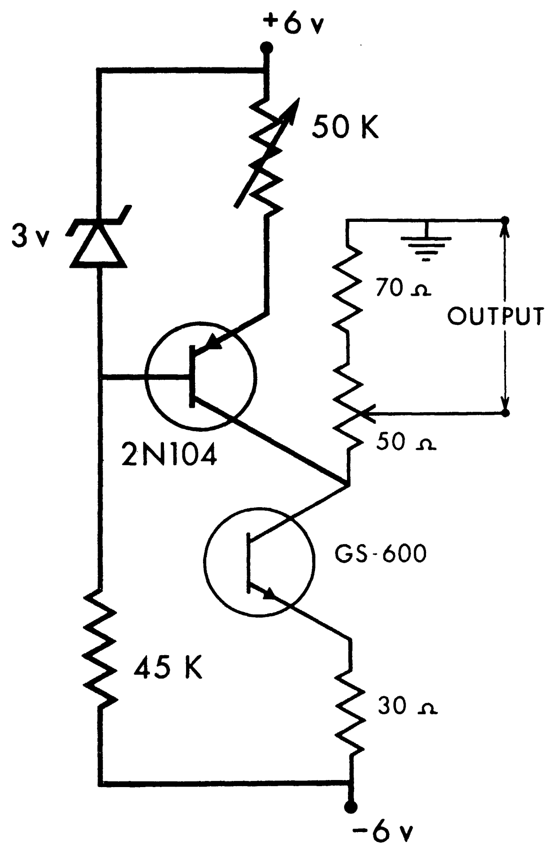


FIGURE 4.7 STRAIN SENSOR CIRCUIT.

circuit the phototransistor, as seen from the collector terminal, acts as a current source with current proportional to the irradiance at the phototransistor surface. Current from an adjustable current source is added to the phototransistor collector current to give a current proportional to irradiance plus a constant. This current then flows through a resistor to ground; and the voltage across this output resistor is the output voltage for the sensor. Making the output resistor a potentiometer, as shown in Figure 4.7, provides a means of adjusting the sensitivity of the device.

An additional sensitivity adjustment is provided by a rheostat in series with the source lamp. In operation, the two sensitivity adjustments are used to compensate for differences, from one test to another, in the transmittance of the bathing medium (by adjusting to a pre-established voltage the output excursion in going from "light off" to illumination of the detector by the unobstructed source beam) while the offset obtained with the adjustable current source is used to cancel the output due to light passing between the vanes when the test specimen is unstrained.

Both the source canister and the detector canister are mounted on a bracket made of 2 inch stainless steel angle. On this bracket, each canister is supported on 4 pads each of which rests on a ball-end adjusting screw. A clamping bar holds each canister against its support pads. This arrangement permits optical allignment of the instrument.

The most important criterion in alignment of the optical system is uniformity of the light beam as seen by the detector. Alignment was considered satisfactory when the detector output varied by less than 1.5 percent of its peak value as a 0.025 in. (0.64 mm.) slit was passed through the light beam. The arrangement used for alignment and calibration of the instrument is shown in Figure 4.8. Here, the slit for alignment and calibration is provided by the two blades of a vernier caliper (Brown and Sharpe "Tri-cal") which is positioned by means of a Boley compound slide mounted on the testing machine lower clamp support. Alignment of the instrument was performed at frequent intervals during the investigation. Each time the device was aligned, a calibration curve was obtained by setting zero output at a slit width of 0.017 in. (.43 mm.) and then determining the slit widths required to give output voltages at fixed intervals. Calibration data and alignment scan records are presented in Appendix I,B.

The bracket which carries the two canisters is mounted so that it can slide on the vertical portion of the testing machine lower clamp support. This allows the strain sensor optics to be moved away from the test section to facilitate setting up of the specimen in the clamps and permits vertical positioning of the light beam. An adjustable stop for the lower limit of the bracket excursion prevents movement of the bracket during a test.

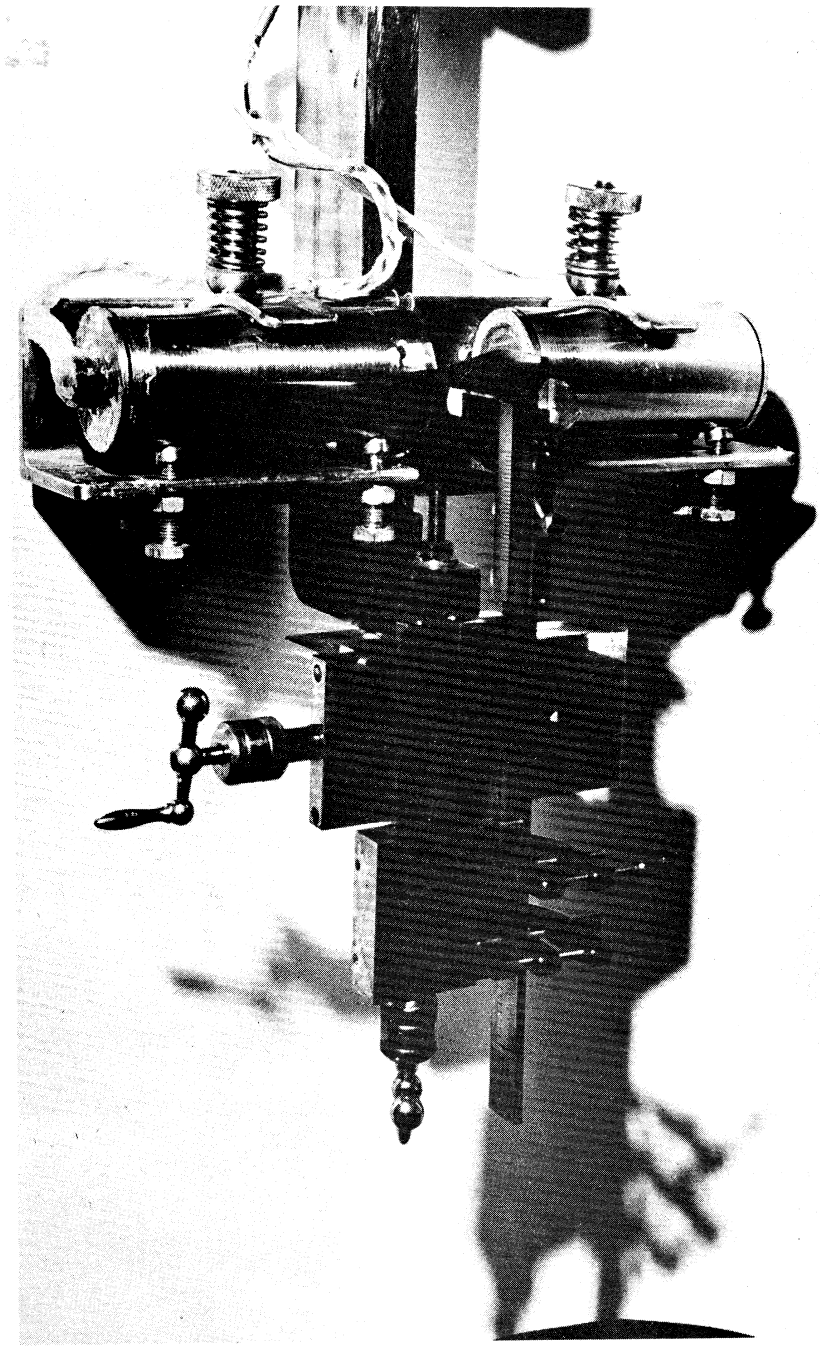


FIGURE 4.8 ARRANGEMENT OF EQUIPMENT USED FOR ALIGNMENT AND CALIBRATION OF THE STRAIN SENSOR.

The vanes used for strain measurement are of stainless steel 0.005 in. thick with their edges ground flat to a deviation of less than 0.0005 in. (0.013 mm.). Each vane is attached by a pair of .80 UNM stainless steel screws to a stainless steel support member which also carries a clip for attaching the vane to the specimen and forms part of a linkage for maintaining vane parallelism.

The linkage which maintains vane parallelism is based on Sarrut's mechanism for linear motion (cf. Hinkle, 1960) which is illustrated in Figure 4.9. In the vane linkage one redundant group

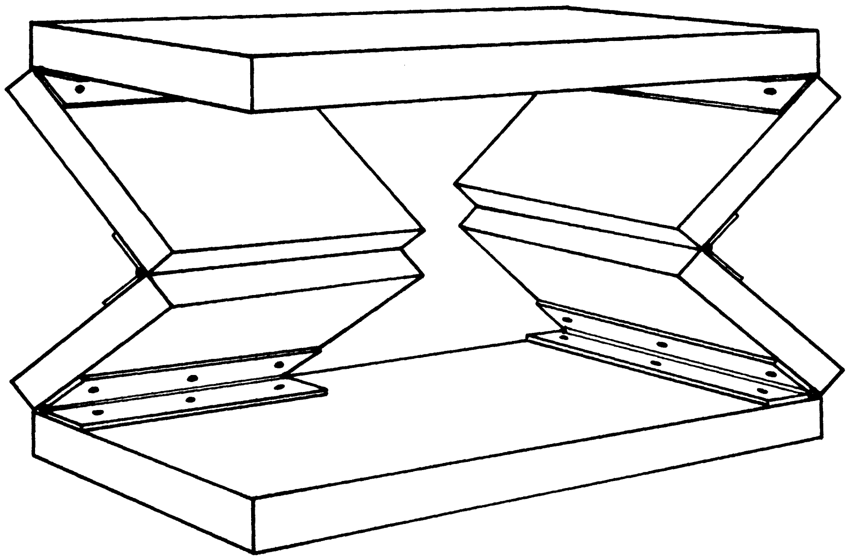


FIGURE 4.9 SARRUT'S MECHANISM FOR LINEAR MOTION.

of 3 hinges has been added to this basic mechanism to provide balance so that the center of mass of the assembly does not translate in a direction perpendicular to the direction of vane motion. Each of the 9 hinges in the mechanism consists of a stainless steel staff having at each end a small, tapered point which is inserted into a "Teflon" plug to form a pivot bearing. This point is in the form of a cone with a 5° half-angle and a diameter at its base of approximately 0.2 mm. Several circumferential grooves are cut into the cone surface to prevent relative axial movement between the point and the fluorocarbon plug into which it is inserted.

The clip which attaches each vane to the tendon specimen has three clamping surfaces which bear against the specimen, one on the supporting member which carries the vane and one on each of two levers which are attached to the supporting member and are pivoted about axes parallel to the specimen axis. Each of the clamping surfaces is radiused to 0.012 in. (0.31 mm.) radius and is polished. Each lever is held against the supporting member by a spring which serves to prevent relative movement of the two pieces in the direction of the lever axis. Clamping force is supplied by an additional spring which acts on both of the levers.

Four counter weights, two attached to each support member; serve to balance the vane assembly so that its center of mass lies approximately on the specimen axis. These counterweights also serve as stops which prevent the vane edges from coming into contact and establish the initial gage length of 11 mm. for the vane assembly.

The complete vane assembly is shown in Figure 4.10. The weight of this assembly is 7.45 gm.

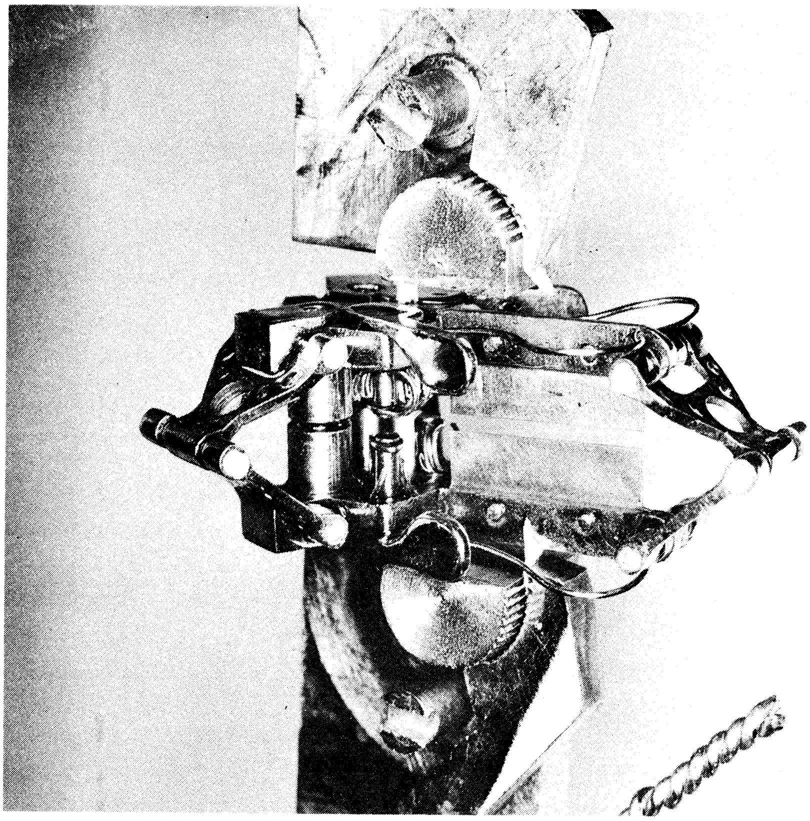


FIGURE 4.10 VANE ASSEMBLY ATTACHED TO SPECIMEN (APPROXIMATELY 2X ACTUAL SIZE).

4.4.5 CONTROLLED TEMPERATURE BATH:

The controlled temperature bath in which tests were performed is a somewhat modified "Sero-utility" bath (Precision Scientific Co., No. 66643) having a capacity of approximately 6 liters. This bath is constructed of stainless steel with a nickel-plated heating element in place of the original copper-sheathed element. A variable autotransformer supplies the heater current and limits the rate of heating. A circulation and filtering system, as diagrammed in Figure 4.11, serves to minimize temperature gradients and suspended

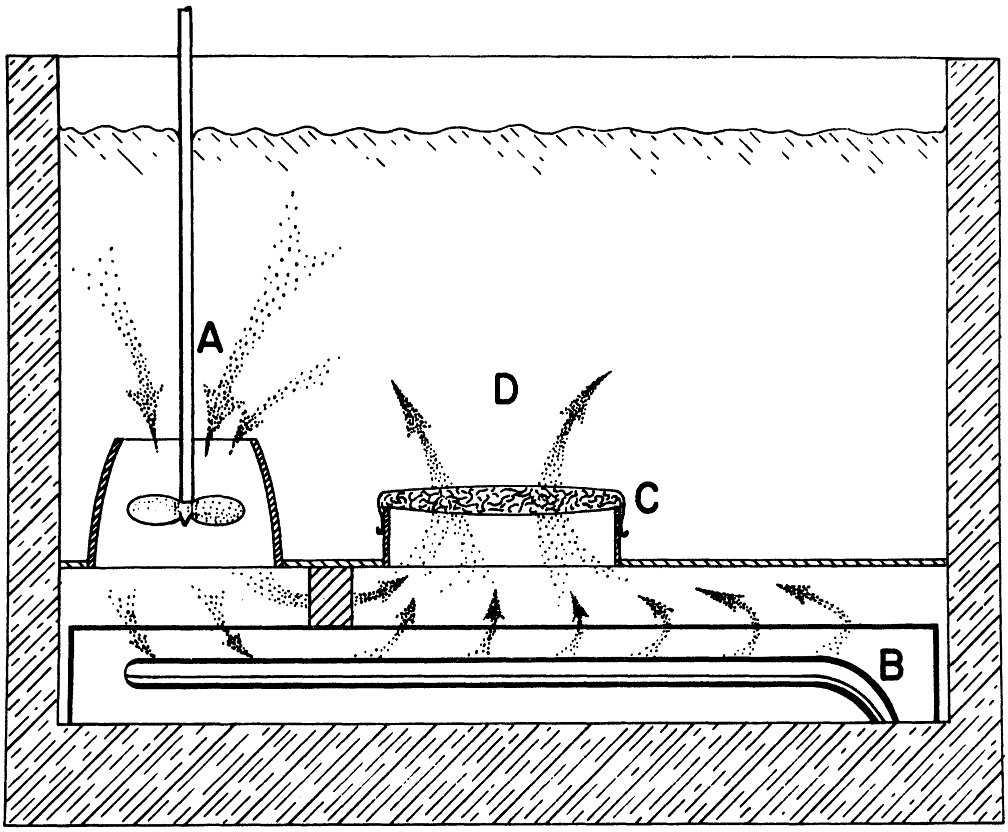


FIGURE 4.11 CIRCULATION AND FILTERING SYSTEM OF THE CONTROLLED TEMPERATURE BATH; (A) SHROUDED IMPELLER, (B) HEATING ELEMENT, (C) FILTER, (D) REGION OCCUPIED BY TEST SECTION.

dust particles which are seen as perturbations in the strain measurement. In this system a shrouded nylon impeller forces solution to flow under a baffle plate which covers the heating element. Solution emerges from below the baffle plate through a filter consisting of a bonded mat of Kodak "Kodel" polyester fibers supported between two layers of woven nylon mesh. It then flows upward around the test specimen.

Bath temperature is measured by a precision mercury in glass thermometer (Cenco 1933-A, NBS 54; or Princo 76-1, NBS 67341). In operation, fluctuations in bath temperature about its set point were less than $\pm$ .5°C.

Ringer's solution was used to fill the bath. Its pH was checked periodically and was maintained at 7.0 $\pm$ 0.3.

4.4.6 LOADING SYSTEM:

Water for filling the weight bucket of the testing machine is supplied from a constant head reservoir; and the weight bucket is drained to a second constant head reservoir. The supply reservoir level is maintained by cold tap water supplied at a rate greater than the rate of flow from the reservoir to the weight bucket, the excess being drained from the reservoir through a pair of large capacity overflow tubes. The drain reservoir consists of a tray placed in the laboratory sink. A 3-way valve (Figure 4.12) permits flow in a weight bucket siphon to be switched either to a supply

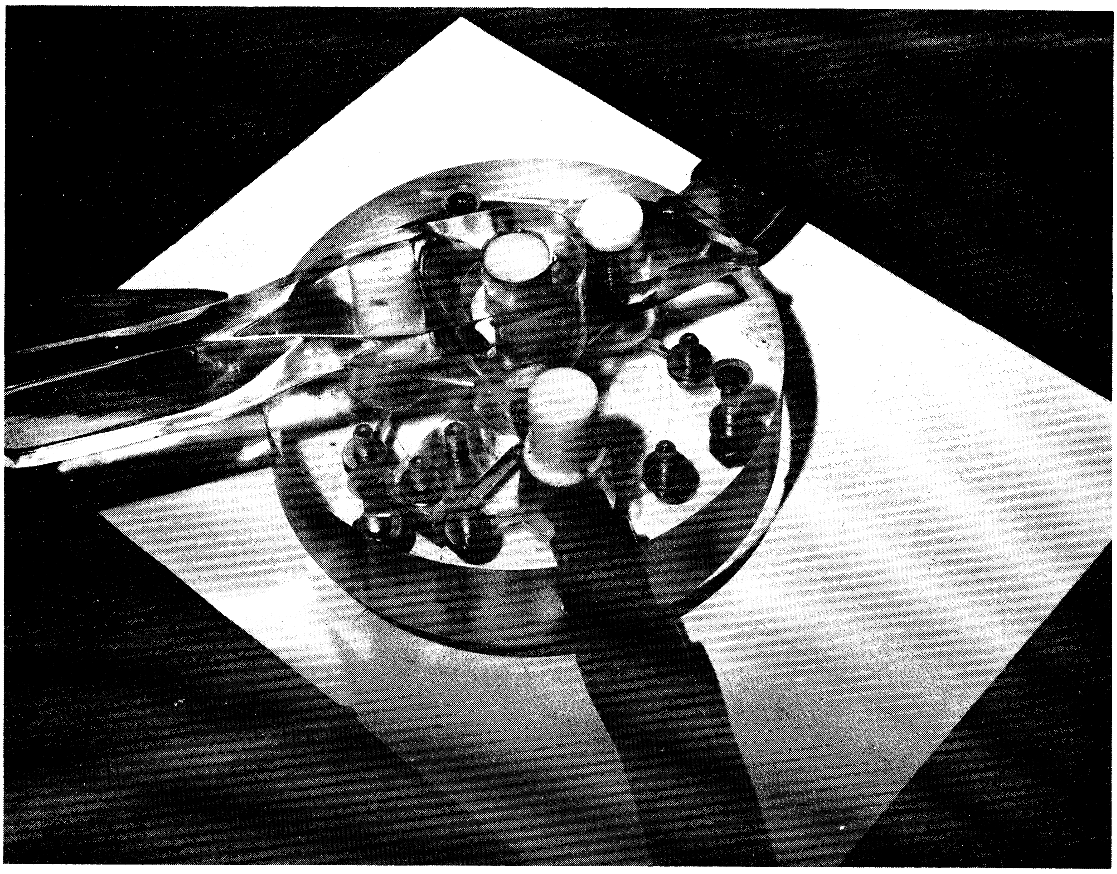


FIGURE 4.12 3-WAY VALVE USED TO CONTROL LOADING AND UNLOADING OF THE WEIGHT BUCKET.

siphon or to a drain tube. All three of these tubes, the weight bucket siphon, the supply siphon, and the drain tube, are of 5/8 in. I.D. "Tygon" tubing. As flow from each tube enters the valve assembly, it passes through an adjustable flow restrictor. These three restrictors permit adjustment of the loading rate (by the supply siphon restrictor), the unloading rate (by the drain tube restrictor), or both rates together (by the weight bucket siphon restrictor).

That portion of the weight bucket siphon which dips into the bucket is of rigid tubing and has at its bottom end an additional adjustable restrictor and a plunger-operated shut-off valve which permits flow to or from the bucket to be stopped so that the specimen can be held at a constant load. In addition, this rigid tubing segment has a vent at the upper end which may be opened to permit air to escape from the siphon. Figure 4.13 illustrates this dipping tube segment.

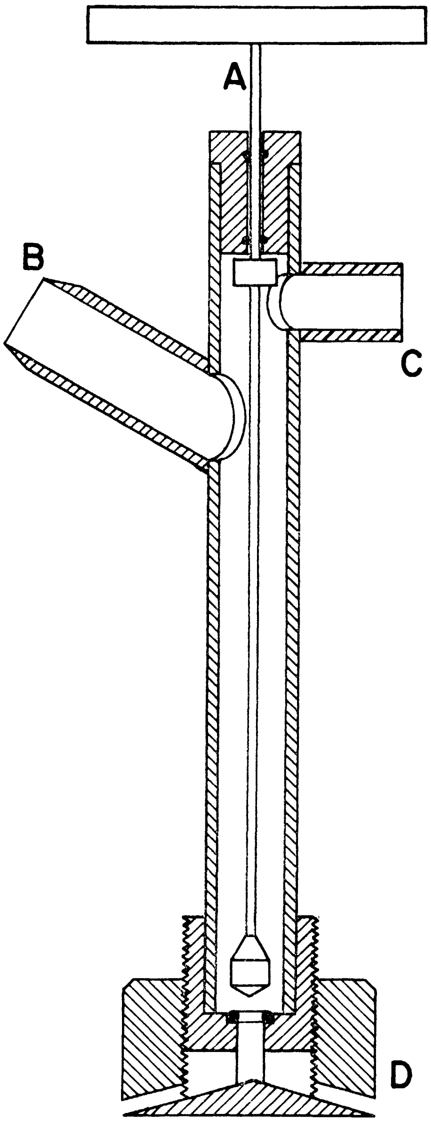


FIGURE 4.13 FINAL PORTION OF THE WEIGHT BUCKET SIPHON; (A) SHUT-OFF VALVE PLUNGER, (B) CONNECTION TO SIPHON TUBE, (C) VENT TUBE, (D) ADJUSTABLE RESTRICTOR.

5. RESULTS AND THEIR DISCUSSION

5.1 MODELLING STUDIES

Since the mathematical analogs which have been suggested as models for the mechanical behavior of collagen tissues seemed to lack either sufficient detail or adequate experimental verification, a series of experimental and model studies was undertaken in the hope of establishing an analog whose parameters could be used for reasonably accurate quantification of the behavior of tendon in the temperature experiments. As neither the final form of the analog nor the steps leading to it were anticipated at the outset, there was no attempt to establish a preformed protocol for this series. Rather, each experiment was designed either to test or to obtain parameter values for the particular model being considered at the time; and, at various stages, the model was altered in an attempt to obtain better agreement with the experimental findings.

The description of this phase of the investigation will trace the evolution of the mathematical analog and will emphasize those experiments which were important in its development. A complete list of the experiments performed in this series, indicating test sequence and conditions, is presented in Appendix III,A.

Two well established features of the mechanical behavior of collagen tissue served as the starting point for development of the analog. These were: (1) the nonlinear character of the load-elongation relation, and (2) the existence of history dependent behavior, as seen in creep and stress relaxation phenomena and in the dissipation of energy under cyclic loading.

The form of the load-elongation relation for rat tail tendon has been analyzed by Ridge and Wright (1966) who found that this relation could be described by the same equations that they used for describing skin (see also Ridge and Wright, 1964). These equations have the form:

$$E = a + c \ln P \qquad \text{...........at low loads}$$

$$E = (KP)^b \qquad \text{...........at intermediate loads}$$

where P and E represent load and elongation, respectively, and the other symbols represent constants.

Previous experience in dealing with the data obtained by VanBrocklin and Ellis (1965) had also suggested that the stress-strain relation for tendon at low stress levels might be represented by a logarithmic function. This may be seen from Figure 5.1 which shows the "mean low rate elastic modulus" (tangent modulus) reported by VanBrocklin and Ellis plotted as a function of stress in semilogarithmic coordinates. The applicability of the power function formulation at intermediate stresses for the material used in the present study is indicated in Figure 5.2, which shows a load-elongation hysteresis loop obtained under triangular wave loading at 3.8 cycles per minute, replotted in logarithmic coordinates.

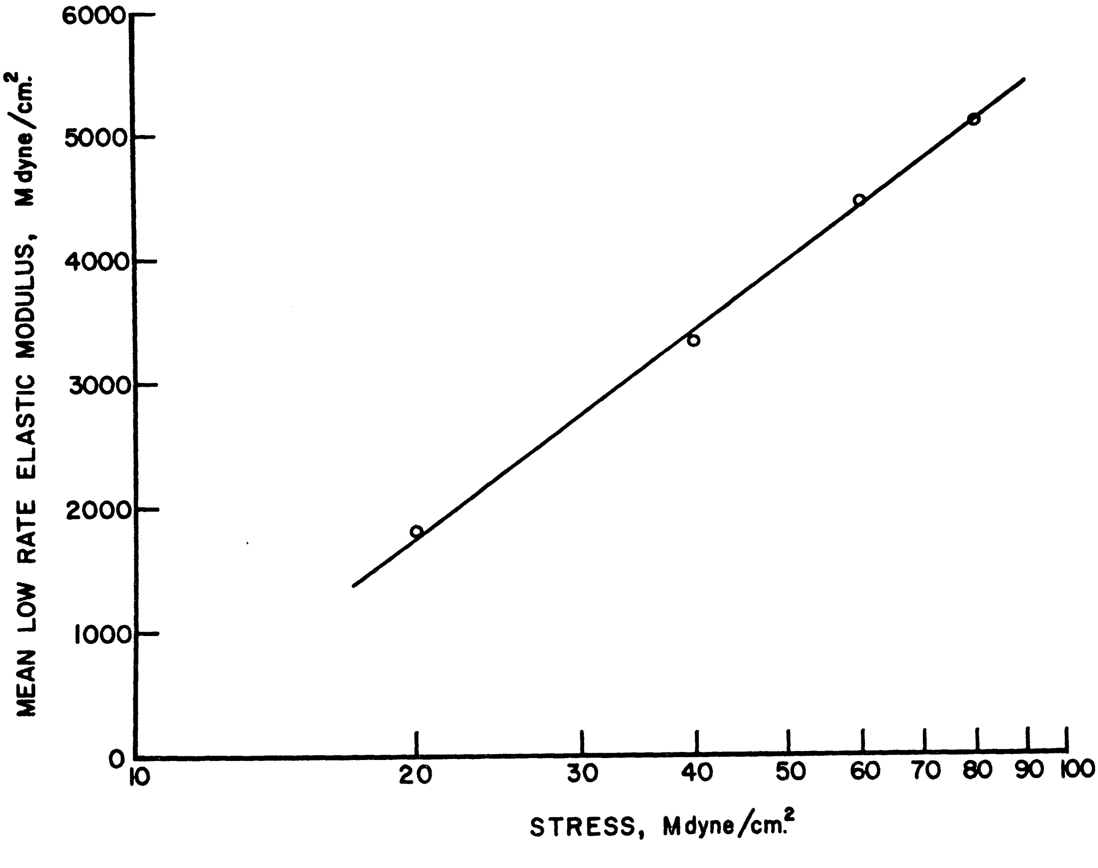


FIGURE 5.1 "MEAN LOW RATE ELASTIC MODULUS" AS A FUNCTION OF STRESS FOR HUMAN EXTENSOR DIGITORUM TENDONS, PLOTTED IN SEMI-LOGARITHMIC COORDINATES. DATA FROM VANBROCKLIN AND ELLIS (1965).

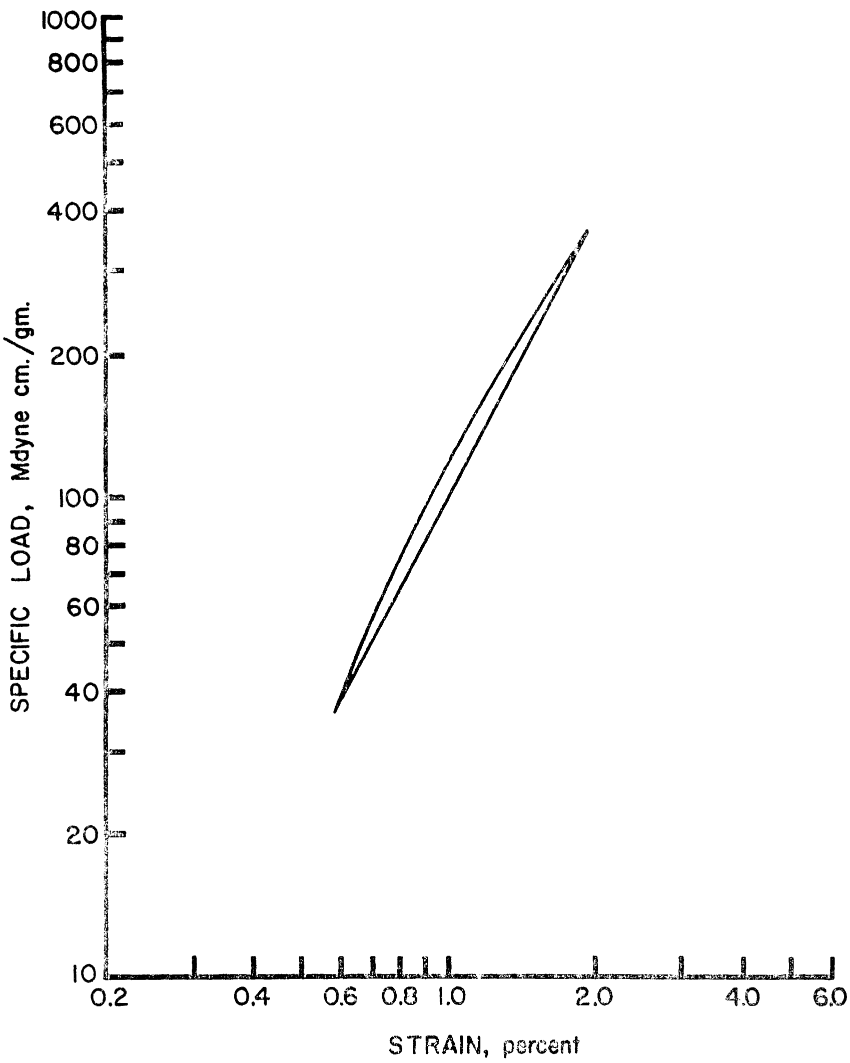


FIGURE 5.2 LOAD-ELONGATION HYSTERESIS LOOP FOR CAT EXTENSOR DIGITORUM COMMUNIS TENDON (SPECIMEN 30C5) CYCLED AT 3.8 CYCLES PER MINUTE, LOGARITHMIC COORDINATES. (SPECIMEN PRESERVED BY FREEZING)

A simple mechanical analog which would display creep, relaxation, and energy dissipation properties somewhat like those which have been reported for collagen tissue is the model for the 'standard anelastic solid' which is shown in Figure 5.3 (cf. Norwick, 1965). This model consists of a Kelvin body in series with an elastic element. As noted elsewhere, models based on this arrangement of elements have been prominent in the literature. The load-elongation characteristics of this model might be brought into correspondence with those for tendon by the use of a nonlinear elastic element.

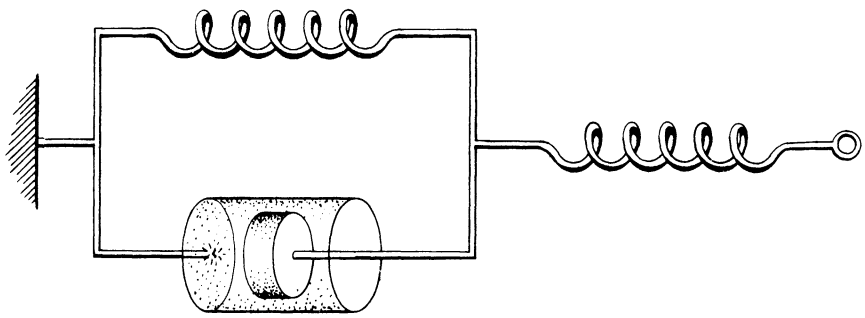


FIGURE 5.3 MECHANICAL ANALOG FOR THE STANDARD ANELASTIC SOLID.

Before doing this in the model studies, however, the damping characteristics of the linear model were compared with those measured for cat extensor digitorum communis tendon under triangular wave loading.

Since, in the model, energy would be dissipated only in the Kelvin body, only its behavior was considered. Taking the differential equation of the Kelvin body,

$$\dot{E} + \left(\frac{k}{c}\right)E = \frac{P(t)}{c}$$

where P represents force, E represents elongation, and where k and c represent the spring constant and the damping coefficient, respectively, the steady state response to a sinusoidal input, $P(t) = B \sin \omega t$, is:

$$E(t) = \frac{B}{c} \cdot \frac{1}{\omega^2 + \left(\frac{k}{c}\right)^2} \left[\left(\frac{k}{c}\right)\sin \omega t - \omega \cos \omega t\right].$$

A symmetrical triangular wave of peak-to-peak amplitude A and period $2\pi/\omega$ can be expanded into the Fourier series

$$P(t) = \sum_{n=1}^{\infty} \frac{4A}{\pi^2 (2n-1)^2} \cos[(2n-1)\,\omega t].$$

The steady state response to this input is:

$$E(t) = \sum_{n=1}^{\infty} \frac{4A}{(2n-1)^2 \pi^2} \cdot \frac{1}{c} \cdot \frac{1}{[(2n-1)\omega]^2 - \left(\frac{k}{c}\right)^2}$$

$$\left\{(2n-1)\,\omega \sin[(2n-1)\,\omega t] + \left(\frac{k}{c}\right)\cos[(2n-1)\,\omega t]\right\}.$$

The damping, or energy dissipation per cycle, in general, is:

$$D = \oint E dP$$

Using the above expressions for P and E, this yields

$$D = \sum_{n=1}^{\infty} \frac{16A^2\omega}{c\,\pi^3(2n-1)^2 \left\{[(2n-1)\,\omega]^2 + (\frac{k}{c})^2\right\}}$$

An approximation obtained from the first five terms of this expression for the damping is plotted as a function of frequency, ω , in Figure 5.4 for a Kelvin body having unit parameters.

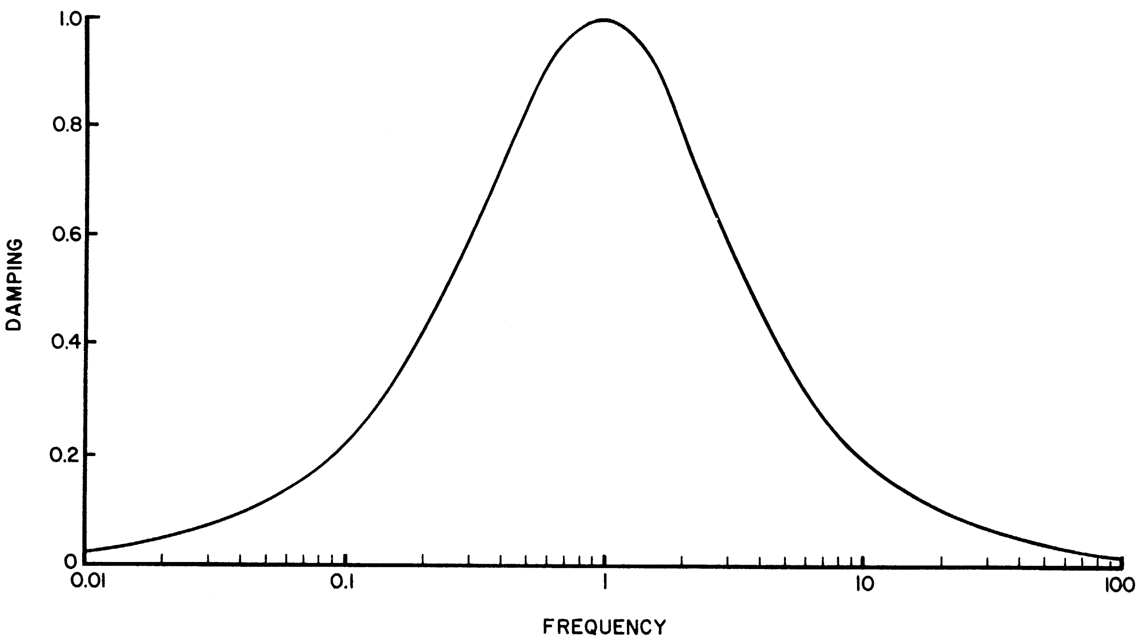


FIGURE 5.4 DAMPING SPECTRUM FOR A KELVIN BODY WITH UNIT SPRING CONSTANT AND DAMPING COEFFICIENT SUBJECTED TO A TRIANGULAR WAVE FORCE (5 TERM APPROXIMATION), NORMALIZED.

Figure 5.5 shows a corresponding plot of specific damping data obtained from a cat extensor digitorum communis tendon subjected to triangular wave loading of constant amplitude at a number of frequencies. These data are in substantial agreement with the observations by VanBrocklin and Ellis (1965) for human toe extensor tendons and by Fung (1967, 1968) for rabbit mesentery that the energy dissipation per cycle is relatively insensitive to frequency. As pointed out by Fung (1967, 1968) this situation is not compatible with a simple, linear viscoelastic model.

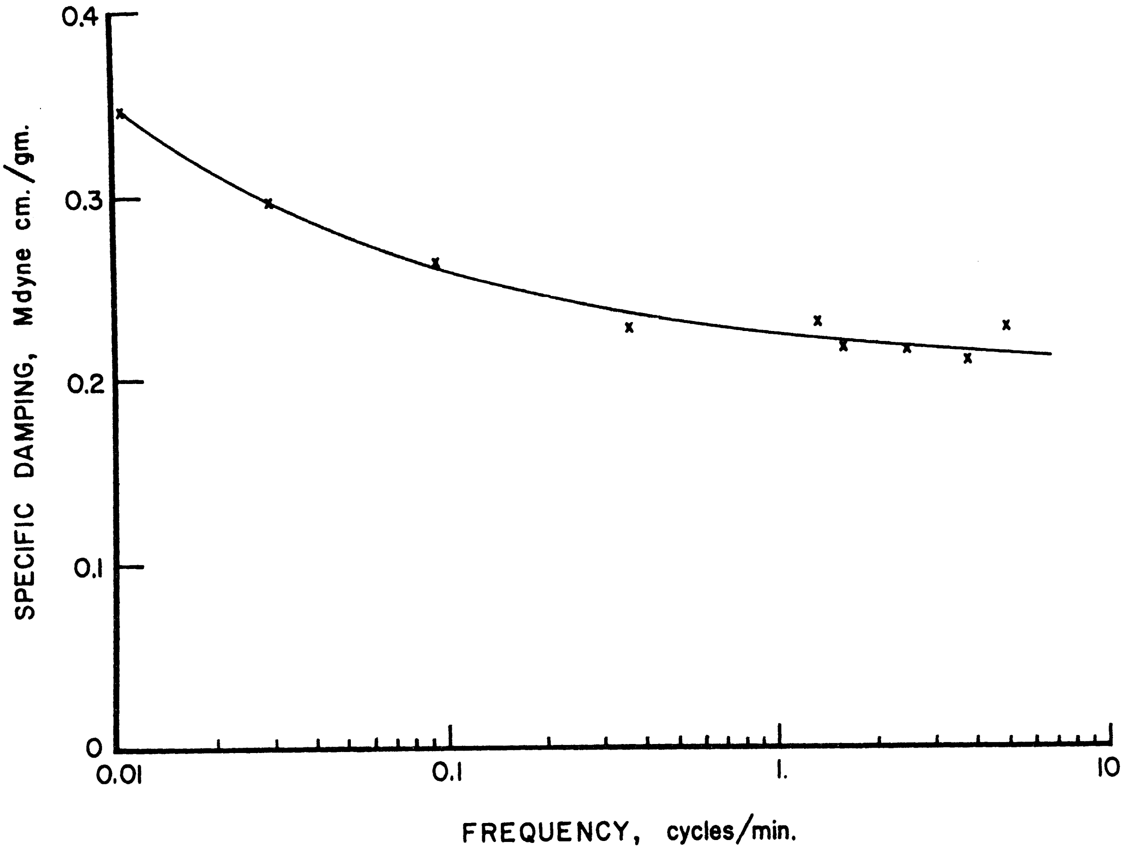


FIGURE 5.5 DAMPING SPECTRUM FOR CAT EXTENSOR DIGITORUM COMMUNIS TENDON (SPECIMEN NO. 30C5) CYCLED BETWEEN LIMITS OF 71 and 356 MEGADYNE CENTIMETERS/GRAM. (SPECIMEN PRESERVED BY FREEZING)

A number of approaches to modeling the behavior indicated in Figure 5.5 are possible. Among these are the use of several Kelvin bodies with different time constants, (corresponding to one of the approaches which was employed by Daly (1966) to describe stress relaxation in skin), the use of a continuous relaxation spectrum as suggested by Fung (1967, 1968), and the use of Coulomb damping to account for a major portion of the energy dissipation.

Initially, Coulomb damping was considered appealing because of the possibility of obtaining a model having only a small number of parameters to be determined experimentally. Such a model is represented in Figure 5.6. In this model a parallel combination of an elastic element and a Saint-Venant body was added to the model of Figure 5.3. The Kelvin body, now assumed to be quite stiff and to have a large time constant, was retained to account for the slight frequency dependence of the damping and for the transient phenomena.

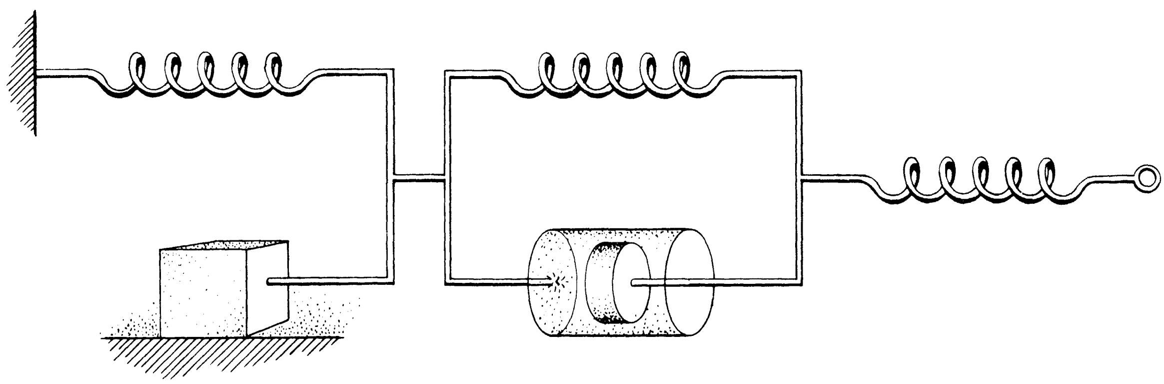


FIGURE 5.6 RHEOLOGICAL MODEL WITH BOTH VISCOUS AND COULOMB DAMPING, CONSIDERED AS AN ANALOG TO THE MECHANICAL BEHAVIOR OF TENDON.

From the data previously obtained (Figure 5.5) it appeared that at a frequency of 2 cycles per minute nearly all of the damping should be accounted for by the Saint-Venant body if this model were applicable. Hence, several tests were run at this frequency in an attempt to estimate model parameters.

In such "high frequency" tests performed on a material describable by this model, the energy dissipation per cycle at a fixed amplitude would be expected to be constant, independent of the steady load about which the periodic fluctuations were centered, unless the elastic element associated with the Saint-Venant body was nonlinear, or the critical load for incipient movement of the Saint-Venant body was not constant, or both. Figure 5.7 shows the energy dissipation per cycle as a function of steady specific load for a cat extensor digitorum communis tendon subjected to triangular wave loading at 2 cycles per minute at an amplitude of 138 Mdyne cm/gm. A decrease in energy dissipation with increasing steady load is clearly indicated.

Analytically, the simplest way to treat this phenomenon in the model was to assume that the elastic element associated with the Saint-Venant body was nonlinear. An elastic element whose stiffness increased with increasing strain would agree with the overall form of the load-elongation relation and could account for the dependence of energy dissipation on steady load. Such dependence can be demonstrated with reference to a hypothetical hysteresis loop such as the one shown in Figure 5.8. This is the loop which would be obtained when the parallel combination of a nonlinear elastic element described by the load-

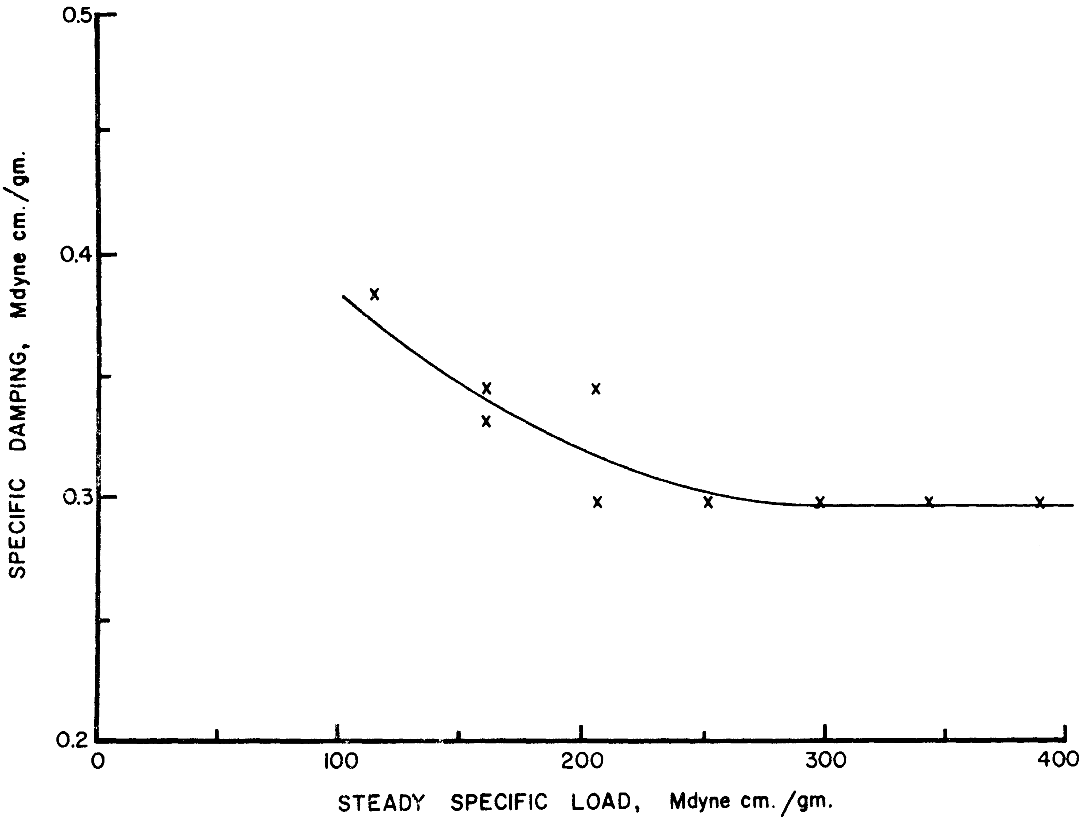


FIGURE 5.7 DAMPING AS A FUNCTION OF STEADY LOAD FOR CAT EXTENSOR DIGITORUM COMMUNIS TENDON (SPECIMEN 33C7) CYCLED AT 2 CYCLES/MIN AT AN AMPLITUDE OF 138 MDYNE CM/GM. (SPECIMEN PRESERVED BY FREEZING)

elongation relation $E = f(P)$ and a Saint-Venant body having a critical load P_c for incipient movement was cyclically loaded. The energy dissipation for each cycle is equal to the area within this loop, or

$$D = \int_{P_1}^{P_2-2P_c} f(P+P_c)\,dP - \int_{P_1+2P_c}^{P_2} f(P-P_c)\,dP + 2P_c f(P_2-P_c) - 2P_c f(P_1+P_c) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{for } P_2-P_1 > 2P_c$$

$$D = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{for } 0 < P_2-P_1 \leq 2P_c.$$

Since the magnitudes of the integral expressions are equal,

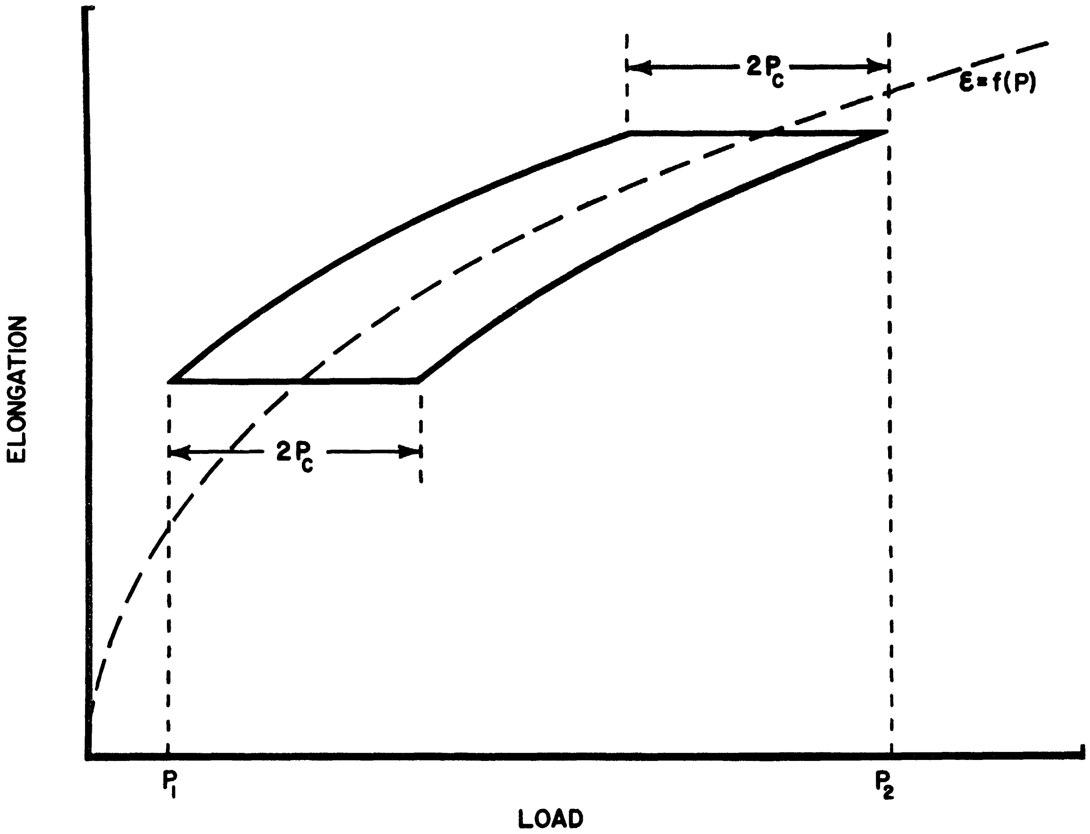


FIGURE 5.8 HYPOTHETICAL HYSTERESIS LOOP FOR A PARALLEL COMBINATION OF A SAINT-VENANT BODY WITH A CRITICAL LOAD P_c AND A NONLINEAR ELASTIC ELEMENT DESCRIBED BY $\varepsilon = f(P)$.

$$D = 2P_c \left\{ f(P_2-P_c) - f(P_1+P_c) \right\} \quad \text{............for } P_2-P_1 > 2P_c$$

$$D = 0 \quad \text{............for } 0 < P_2-P_1 \leq 2P_c$$

Here it may be noted that the damping decreases as $f(P_2-P_c)$ becomes closer to $f(P_1+P_c)$. For a fixed load amplitude, P_2-P_1, $f(P_2-P_c)$ will become closer to $f(P_1+P_c)$ as the stiffness of the elastic element is increased. Hence, if the stiffness of the elastic element increased with increasing strain the damping would decrease with increasing steady load for a fixed load amplitude.

Using the above equations, it should be possible to obtain both $f(P)$ and P_c for such a combination from experimental measurements. One way of doing this would be to hold the lower cycle limit, P_1, fixed while varying the upper cycle limit, P_2. With P_1 fixed, $f(P_1+P_c) = H$, a constant; and

$$\frac{D}{2P_c} = f(P_2-P_c) - H \quad , \quad \text{or....} \quad f(P_2-P_c) = \frac{D}{2P_c} + H.$$

Thus, if D were determined experimentally as a function of P_2, $f(P)$ with $P = P_2-P_c$ could be determined if P_c and H were known. Since D becomes zero for $0 < P_2-P_1 \leq 2P_c$ but $D \neq 0$ for $P_2-P_1 > 2P_c$, P_c is one half of the separation between the fixed limit P_1 and the $D = 0$ intercept in a graph of D as a function of P_2. Knowing this, H can be found by imposing the requirement $f(P) = 0$ for $P = 0$ (i.e. the elastic element is unstrained when it is under zero load).

Similarly, when the upper cycle limit is held constant,

$$f(P_1+P_c) = -\frac{D}{2P_c} - H^*$$

with H^* a constant; and $f(P)$ could as easily be found from an experiment in which the upper cycle limit was fixed while the lower cycle limit was varied.

In order to obtain $f(P)$ over a wide range of P values, it was desirable to combine these two procedures and to permit the variable cycle limit to range both above and below the fixed limit. The data from an experiment in which this was done could be combined into a single $f(P)$ curve if D was plotted as a function of the variable limit for the tests in which the variable limit was the higher one and -D was plotted as a function of the variable limit in the same coordinates for the tests in which the variable limit was the lower one. Figure 5.9 illustrates the type of curve one would expect to obtain by this procedure and indicates a sequence of translations which would bring the two branches of the curve into correspondence with a curve having the same form as f(P) except for the scale factor $2P_c$ and the location of the origin along the D-axis.

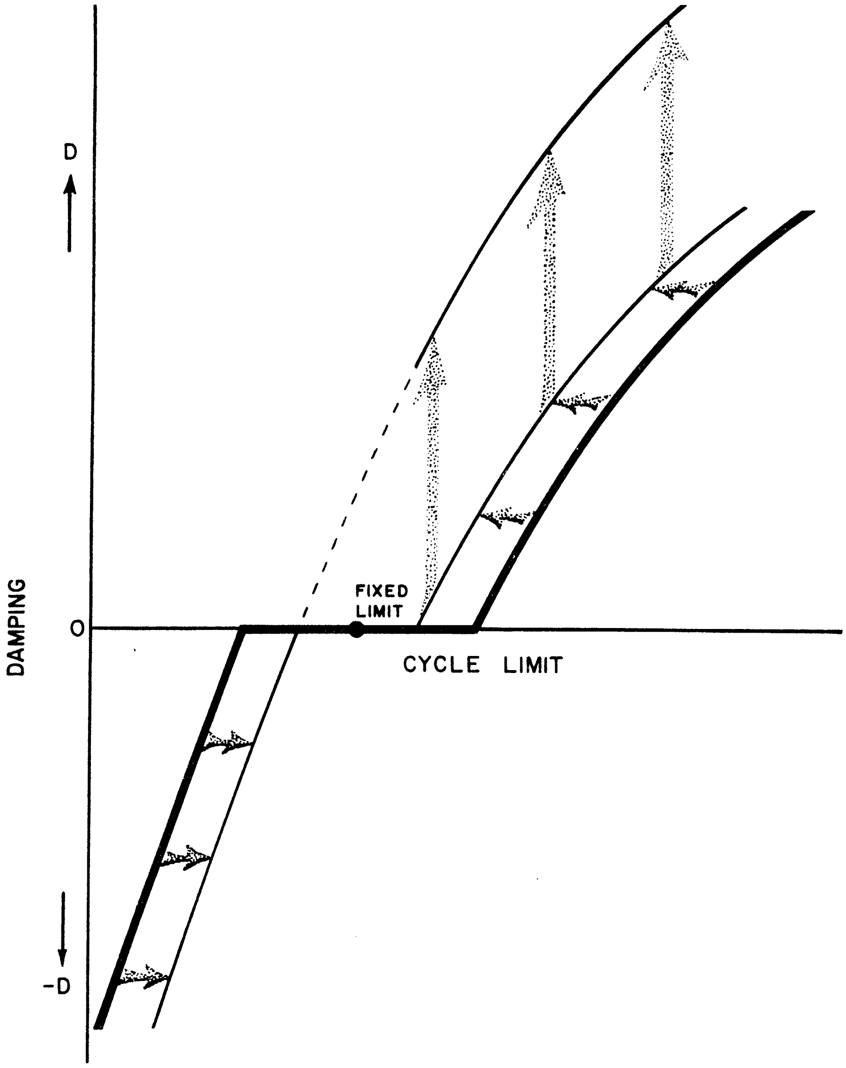


FIGURE 5.9 HYPOTHETICAL DAMPING-CYCLE LIMIT CURVE AND THE SERIES OF TRANSLATIONS WHICH YIELD THE FORM OF THE LOAD-ELONGATION CURVE FOR THE ELASTIC ELEMENT ASSOCIATED WITH THE SAINT-VENANT BODY (SEE TEXT).

Experimental curves obtained for one tendon cycled at 2 cycles per minute with fixed cycle limits at 2 different load levels are presented in Figure 5.10. Not only would it have been difficult to obtain a value for P_c from either curve because of the absence of the anticipated intercepts, but the load-elongation curves for the elastic element associated with the Saint-Venant body would have been different for the two experimental curves. Specifically, each experimental curve shows a change in the sign of its curvature at or near the fixed cycle limit. This corresponds to an inflection point in the load-elongation curve. But since the inflection points correspond to the fixed cycle limits, which were different, they could not coincide. In addition, the regions of negative curvature in the load-elongation curves which would be derived from these plots would have been expected to coincide with regions in which the damping would increase with increasing steady stress. This expectation, however, does not agree with the previous observation of damping decreasing somewhat with increasing steady load.

Hence, the Coulomb plus viscous damping model of Figure 5.6 was abandoned. Indeed, the form of each branch of the curves of Figure 5.10 approximates the parabolic damping-amplitude relation characteristic of viscous damping (equation: page 69). Noting this, it seemed appropriate to explore further the possibilities for an analog employing only viscous damping.

The previously noted observation that energy dissipation was dependent on steady load at once precluded the use of any linear model

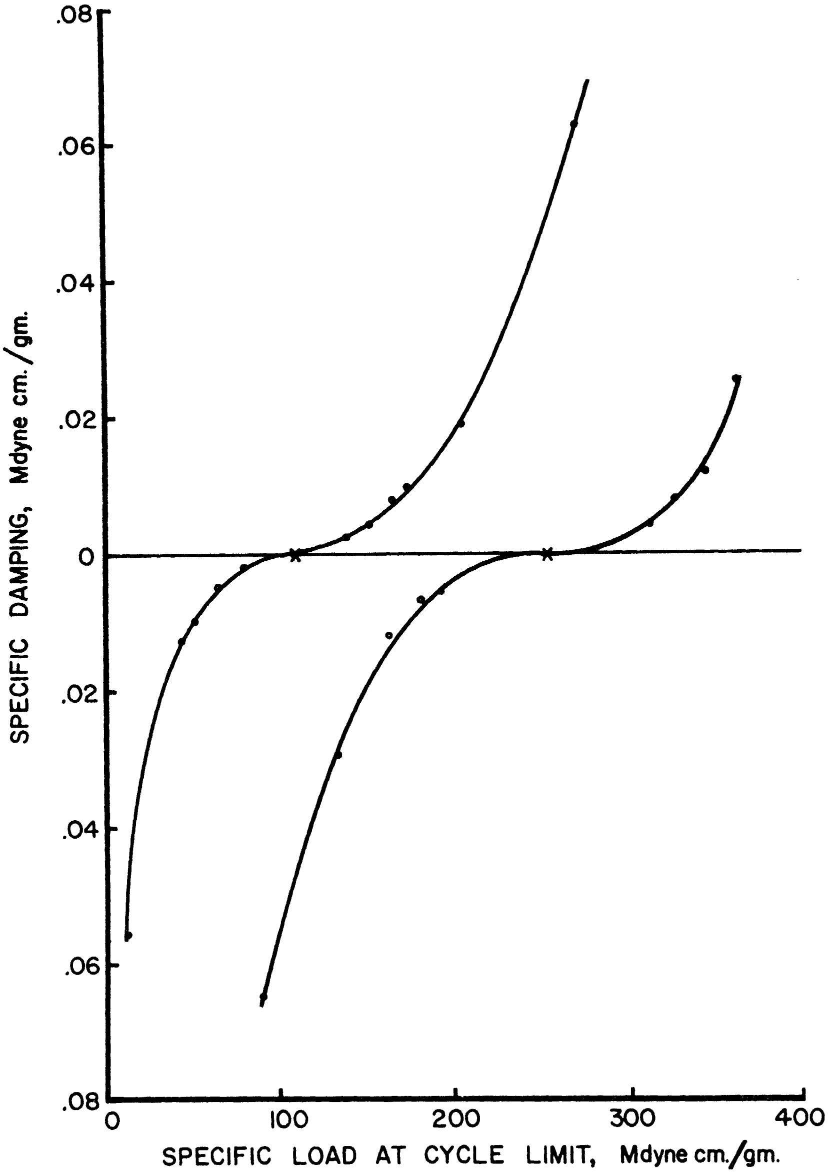


FIGURE 5.10 DAMPING AS A FUNCTION OF VARIABLE CYCLE LIMIT (SEE TEXT) FOR TWO VALUES OF THE FIXED CYCLE LIMIT FOR CAT EXTENSOR DIGITORUM COMMUNIS TENDON (SPECIMEN NO. 33C2) CYCLED AT 2 CYCLES/MIN. (SPECIMEN PRESERVED BY FREEZING)

to account for the damping characteristics. An additional observation which also provided evidence against the applicability of a linear model was the dependence of dynamic stiffness on cycle amplitude at a constant frequency for loading cycles centered about a fixed steady load. Figure 5.11 illustrates this effect in a plot of tangent modulus at the (fixed) steady load level as a function of cycle amplitude for one specimen. Here, a tendency for this tangent modulus to increase with decreasing amplitude is shown, whereas if energy dissipation occurred only in an array of elements which could be described by a linear differential equation, this modulus should be independent of amplitude, even if this array was in series with a nonlinear elastic element.

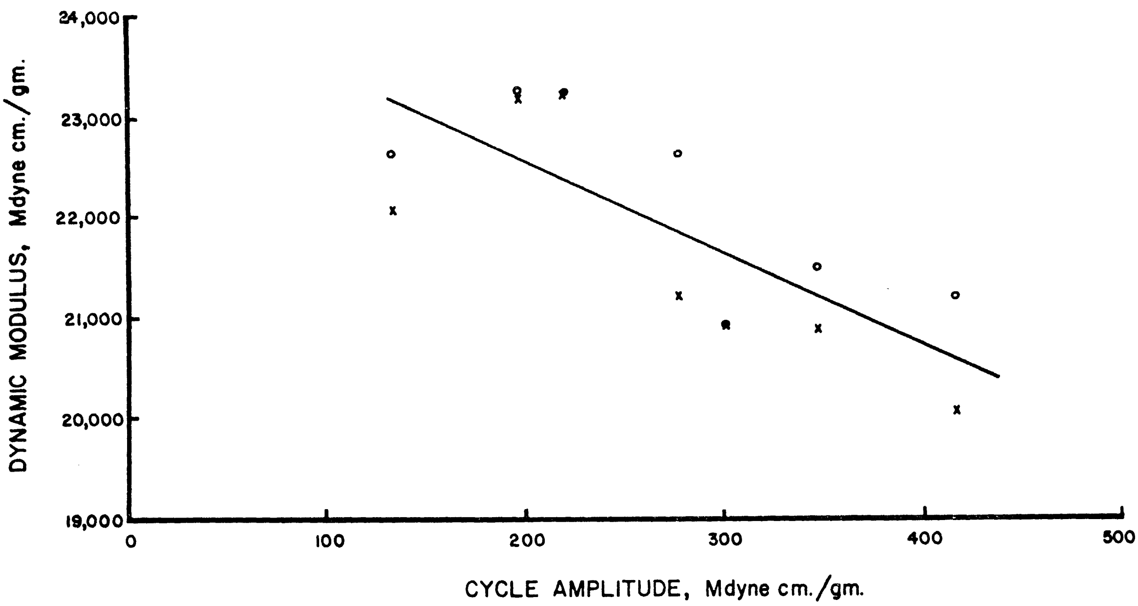


FIGURE 5.11 DYNAMIC TANGENT MODULUS AT 290 MEGADYNE CM./GM. OBTAINED UNDER CYCLIC LOADING AT 2 CYCLES/MIN. WITH STRESS CYCLES CENTERED AT 290 MEGADYNE CM./GM. AS A FUNCTION OF CYCLE AMPLITUDE FOR CAT EXTENSOR DIGITORUM COMMUNIS TENDON (SPECIMEN 31C7, PRESERVED BY FREEZING)

o LOADING
x UNLOADING

The nonlinear body which was considered as a possible analog to the damping behavior was a body with the configuration of a Kelvin body but having nonlinear viscous and elastic elements. In this body, the elastic element was described by a power function, $E = (KP_e)^b$, and the viscous element was pseudoplastic; i.e. the ratio of viscous stress to strain rate decreased with increasing strain rate. Two forms of pseudoplastic dashpot were tested in this partial model: (1) an element described by the equation for a "generalized Newtonian liquid", as presented by Reiner (1949),

$$\frac{1}{c} = \Phi_\infty - (\Phi_\infty - \Phi_0)e^{-P_v^2/x},$$

where P_v is the viscous load and c is the damping coefficient in the equation $P_v = c\dot{E}$, and where the other symbols represent constants. (2) an element displaying power function damping such that

$$\dot{E} = (GP_v)^a$$

where G and a are constants, with a required to be an odd positive integer.

The response of this nonlinear body to a triangular wave input was obtained, approximately, using digital simulations which were run on a Linc-8 computer. (Listings of the simulation programs, in the Focal language, are presented in Appendix II.) In general, results obtained from these simulations for both forms of the viscous element showed some broadening and skewing of the damping peak, as anticipated. With the power function viscous element, considerable broadening could be obtained by the choice of a sufficiently large exponent. As shown in Figure 5.12, an exponent of 17 yielded a peak which was sufficiently broad to have a region as large as the range of experimental measurements over which the frequency dependence of damping was comparable in magnitude to that found in the experimental measurements. This region, however, was near the peak and had a curvature opposite that of the experimental curve. In addition, a plot of dissipation as a function of variable cycle limit (as employed in assessing the hypothesized Coulomb damping) in this region more nearly approximated the theoretical anticipation for Coulomb damping than it did the experimentally obtained curve for tendon. Such a curve obtained from the simulation, with an exponent of 15, is shown in Figure 5.13.

From these observations, it appears that a model in the form of a Kelvin body might require a viscous element approximately as nonlinear as power function damping with an exponent of 17 to account for the frequency dependence of damping, but at the same time might require a more nearly linear viscous element to account for the amplitude dependence of damping. Thus, a single body of this type might be inadequate to account for the damping properties of tendon.

To provide a satisfactory analog to the behavior of tendon, a model consisting of viscous and elastic elements would be likely to contain a distribution of nonlinear elements. It is probable that the analytical difficulties associated with the development and use of such a model would outweigh its utility as a means of dealing with the data in this study.

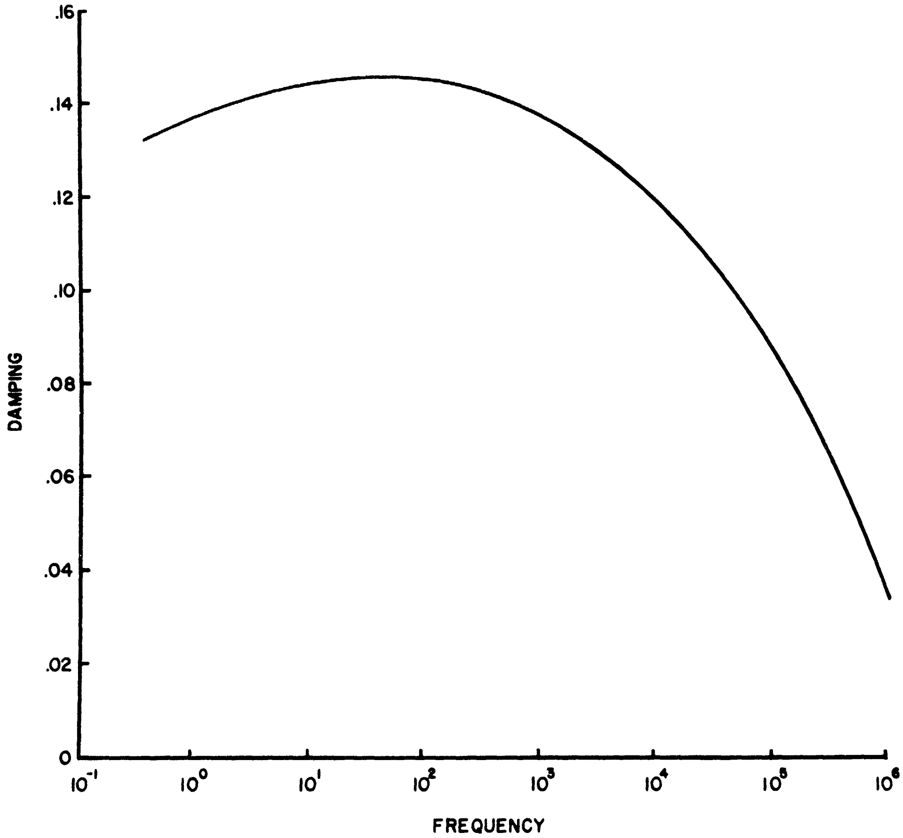


FIGURE 5.12 DAMPING SPECTRUM FOR KELVIN-TYPE BODY WITH POWER FUNCTION ELASTIC AND VISCOUS ELEMENTS, FROM DIGITAL SIMULATION ($G = 5$, $a = 17$, $K = 1$, $b = .5$, CYCLE LIMITS: 0.1 AND 1.0)

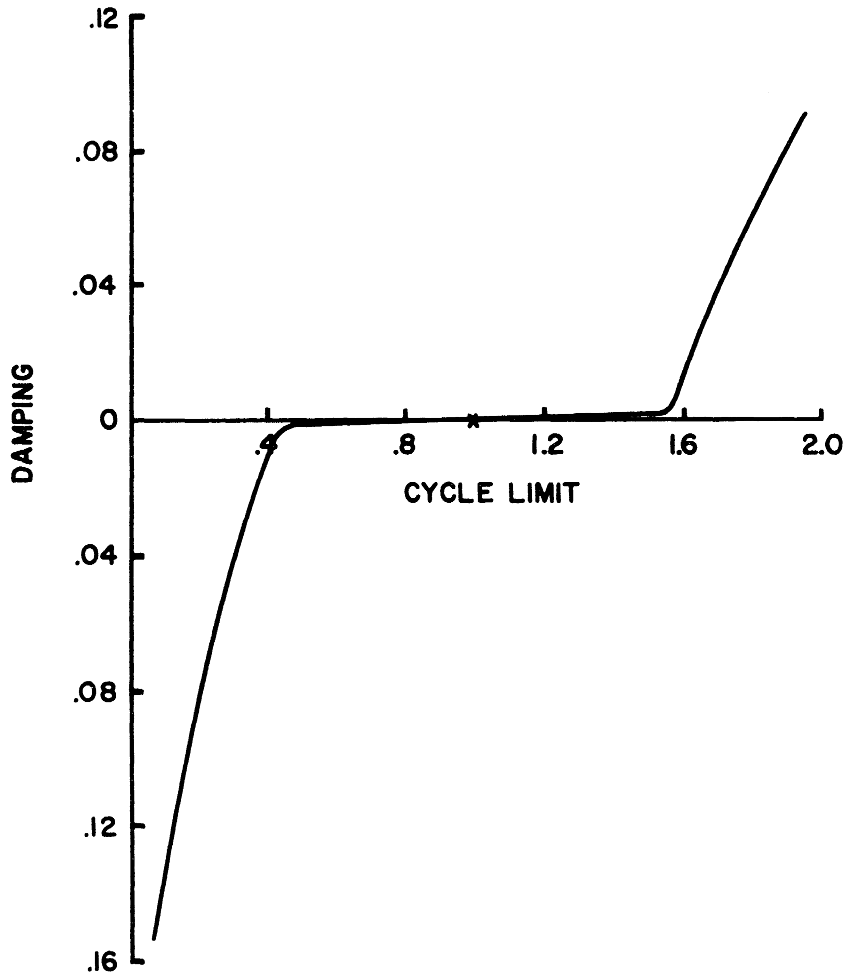


FIGURE 5.13 DAMPING AS A FUNCTION OF CYCLE LIMIT (SEE TEXT) WITH FIXED LIMIT AT 1.0, FOR A KELVIN-TYPE BODY WITH POWER FUNCTION VISCOUS AND ELASTIC ELEMENTS, FROM DIGITAL SIMULATION ($G = 5$, $a = 15$, $K = 1$, $b = .5$, FREQUENCY: 1000).

A different type of model for mechanical behavior, based on a chemical kinetics approach, was applied some time ago in describing muscle (Polissar, 1952; Ramsey, 1955) and was adapted to the description of stress relaxation in arterial segments by Stacy (1957). This type of model could be particularly useful in the present work because the chemical kinetics approach would provide a strong theoretical frame of reference for dealing with the temperature dependence of properties.

For the development of this model, the material was presumed to be composed of a series array of elements each of which could exist in either a long state or a short state. Transition from long to short and from short to long were assumed to proceed as first order reactions; that is, the rates of the two reactions are given by

$$R_{\ell-s} = k_{\ell}^{*} N_{\ell} \quad \text{for the reaction long} \longrightarrow \text{short}$$

$$R_{s-\ell} = k_{s}^{*} N_{s} \quad \text{for the reaction short} \longrightarrow \text{long}$$

where N_{ℓ} and N_{s} are the number of elements in the long state and the number of elements in the short state, respectively, and where k_{ℓ}^{*} and k_{ℓ}^{*} are the rate constants for the two reactions. Polissar (1952) has argued that the rates of the two reactions should depend on tension, P, in such a way that the rate of the reaction

long $\longrightarrow$ short

is reduced by a factor $e^{-C_{\ell}P}$ while the rate of the reaction

short $\longrightarrow$ long

is increased by a factor $e^{C_s P}$, where C_ℓ and C_s are constants. Thus, the rates of the two reactions could be written as:

$$R_{\ell - s} = k_\ell e^{-C_\ell P} N_\ell$$

and

$$R_{s - \ell} = k_s e^{C_s P} N_s$$

The rate of change in the number of elements in the short state, then, would be

$$\frac{dN_s}{dt} = k_\ell e^{-C_\ell P} N_\ell - k_s e^{C_s P} N_s ,$$

or

$$\frac{dN_s}{dt} = (N_t - N_s) k_\ell e^{-C_\ell P} - k_s N_s e^{C_s P}$$

where N_t is the total number of elements in the array.

If the elements are assumed to be inextensible in each of the two states and if the length of an element in the short state is a while its length in the long state is b, the length of the array will be:

$$L = aN_s + b(N_t - N_s) .$$

The strain can then be expressed as

$$E = \frac{L - L_o}{L_o}$$

where L_o is the length of the array having an initial or reference composition. If the reference composition is taken as the composition in the unstressed (P = 0) equilibrium ($\frac{d}{dt}(\frac{N_s}{N_t})$ = 0) condition,

$$L_o = \left\{ (a-b)\frac{1}{\frac{k_s}{k_\ell} + 1} + b \right\} N_t$$

and

$$E = \frac{(b-a)[1 - (\frac{N_s}{N_t})(\frac{k_s}{k_\ell} + 1)]}{(b\ (\frac{k_s}{k_\ell}) + a\ )}$$

For the equilibrium condition ($\frac{d}{dt}(\frac{N_s}{N_t})$ = 0) the load-strain relation can be evaluated. This was done for several sets of parameters; and it was found that qualitative agreement with the form of the experimental stress-strain curves for tendon could readily be obtained. Several equilibrium load-strain curves computed from this model are shown in Figure 5.14. (The computer program for generation of these curves is listed in Appendix II[1].) It is interesting to note that reasonable qualitative agreement between the form of the computed load-strain curve and the form of experimental curves was obtained for $\frac{k_s}{k_\ell}$ = 100 which was also the value which Stacy (1957) reported to yield agreement with his results on the stress relaxation of arterial segments.

---

[1]The program for computing equilibrium load-strain curves was written to take account of the compliances G_s and G_ℓ of the short and long states as they would appear in the equation for L. Unless these compliances are presumed to be sufficiently small or to be related in such a way that the difference in length between the long and the short states was always constant, C_ℓ and C_s should not be regarded as constants, but should become functions of the tension, P. To avoid this complication, the compliances were taken to be zero, even though this limited the generality of the model.

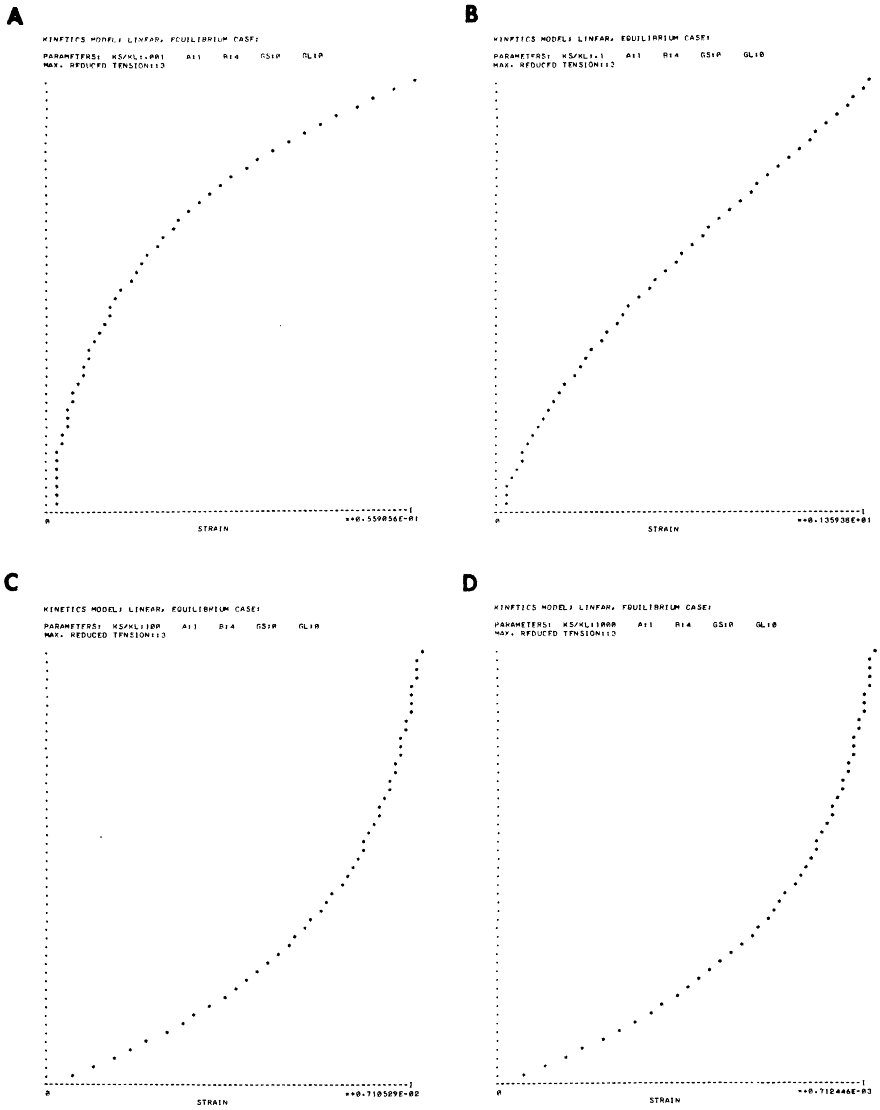


FIGURE 5.14 EQUILIBRIUM LOAD-STRAIN CURVES COMPUTED FROM THE KINETICS MODEL (SEE TEXT) WITH RELATIVE LENGTHS a = 1 and b = 4, ZERO COMPLIANCES, AND VARIOUS RATIOS OF RATE CONSTANTS: (A) K_s/K_ℓ = 0.001, (B) K_s/K_ℓ = 0.1, (C) K_s/K_ℓ = 100, (D) K_s/K_ℓ = 1000.

To evaluate the general form of the damping spectrum for this model, the integral

$$\oint \frac{N_s}{N_t}\, dP$$

was evaluated in an approximate digital simulation of the model (Appendix II). As with the viscoelastic models examined, this model yielded a pronounced damping peak and nowhere approached the frequency insensitivity of the experimental data. Various choices of parameters shifted the position of the peak and altered its height, but no set of parameters yielded a significant change in its form. Figure 5.15 illustrates the damping peaks for three of the sets of parameters tested.

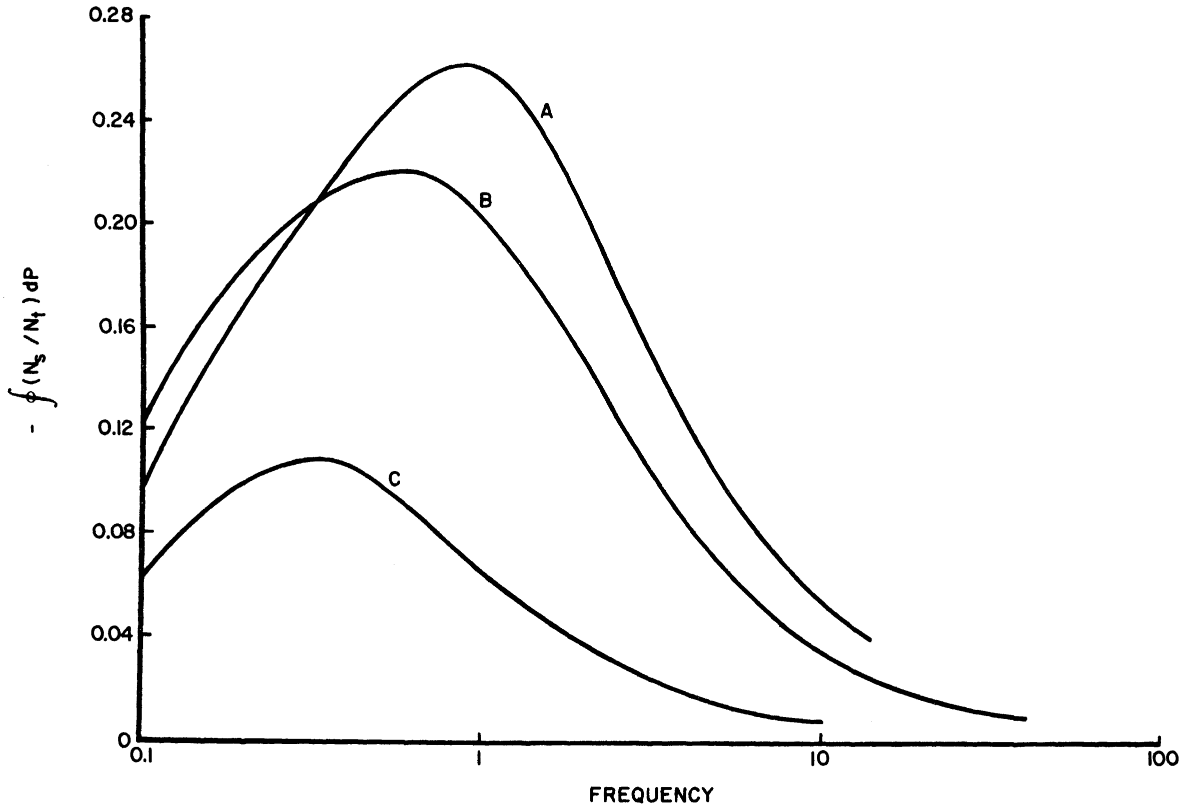


FIGURE 5.15 $-\oint \frac{N_s}{N_t}\, dP$ [which is proportional to damping] AS A FUNCTION OF FREQUENCY, FROM AN APPROXIMATE DIGITAL SIMULATION OF THE KINETICS MODEL, WITH $K_s = K_\ell = 1$, AND (A) $C_s = 1$, $C_\ell = 0.1$, (B) $C_s = C_\ell = 1$, (C) $C_s = C_\ell = 0.1$.

Although the work reported here did not fully explore the possibility of constructing either a conventional rheological model or a chemical kinetics model for the mechanical behavior of tendon, it has suggested some restrictions on the class of models which might be suitable. Most notably, it has indicated that the damping behavior cannot be adequately described by a linear differential equation in load and elongation or by taking simple Coulomb friction as the major dissipative process.

5.2 PLASTIC-LIKE ELONGATION

Relatively early in the experiments on temperature effects it was noted that some specimens, especially at temperatures high in the temperature range studied, failed to approach a steady state response with cyclic loading. Instead, they demonstrated progressive elongation such that each loop in the load-elongation record was displaced toward somewhat greater strains than the previous loop (Figure 5.16). Typically, the displacement between successive loops was nearly constant for several loops and then increased progressively until the specimen broke. This behavior appeared similar to the constant rate creep which had previously been described (c.f. Fry et al, 1964).

To further investigate this phenomenon, a sequence of experiments was performed according to the protocol presented earlier. Each specimen was subjected to triangular wave loading, first at a low temperature and subsequently at the test temperature; then it was subjected to a sequence of steady loads, each obtained by ramp loading from the preceeding load level and each held for several minutes. (The sequences of tests for these experiments are listed in Appendix III, B).

At each load level, the elongation showed an initial (viscoelastic) transient response. At small loads, the elongation seemed to approach a constant value after 5 to 10 minutes (Figure 5.17, A). At large loads, however, the elongation did not appear to

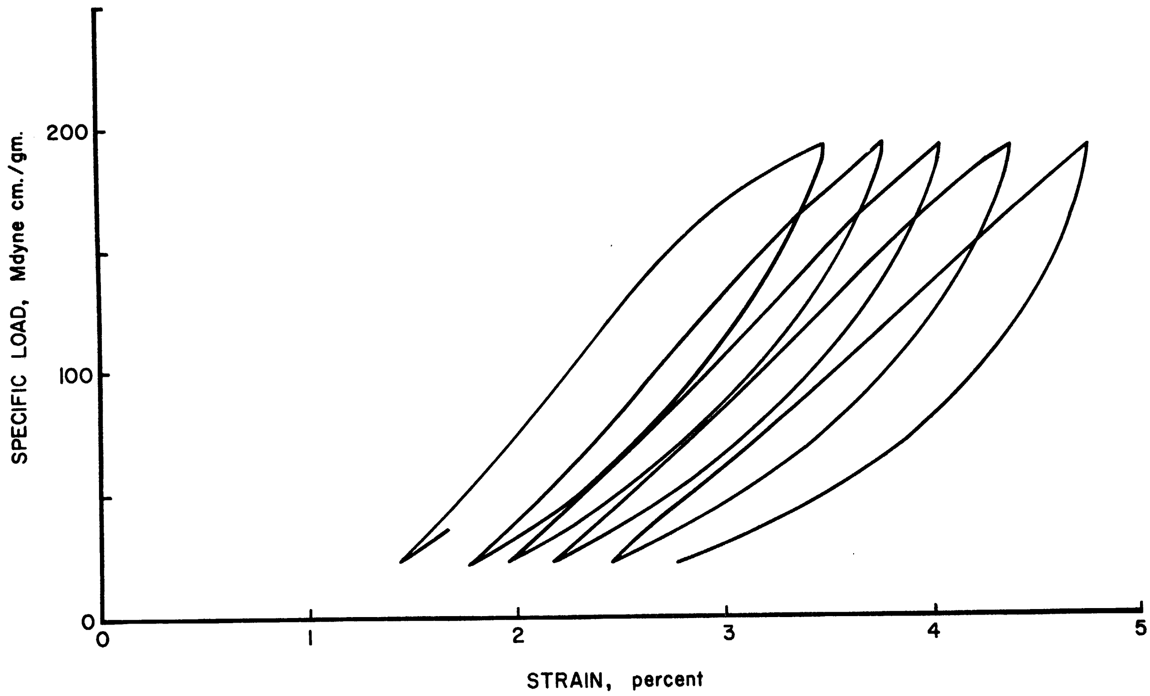


FIGURE 5.16 LOAD-ELONGATION RECORD FOR A TENDON SHOWING PROGRESSIVE ELONGATION UNDER CYCLIC LOADING (TEST AT 40°C, 0.072 CYCLES PER MINUTE: SPECIMEN 34C4, PRESERVED BY FREEZING).

approach a limiting value, but proceeded to increase at a rate which became nearly constant (in most instances) after a period of 3 to 6 minutes during which the viscoelastic transient appeared to be significant (Figure 5.17, B). The distinction between the viscoelastic transient and the constant rate (plastic-like) elongation was most clearly demonstrated when a specimen was unloaded from a large load to significantly smaller load which was

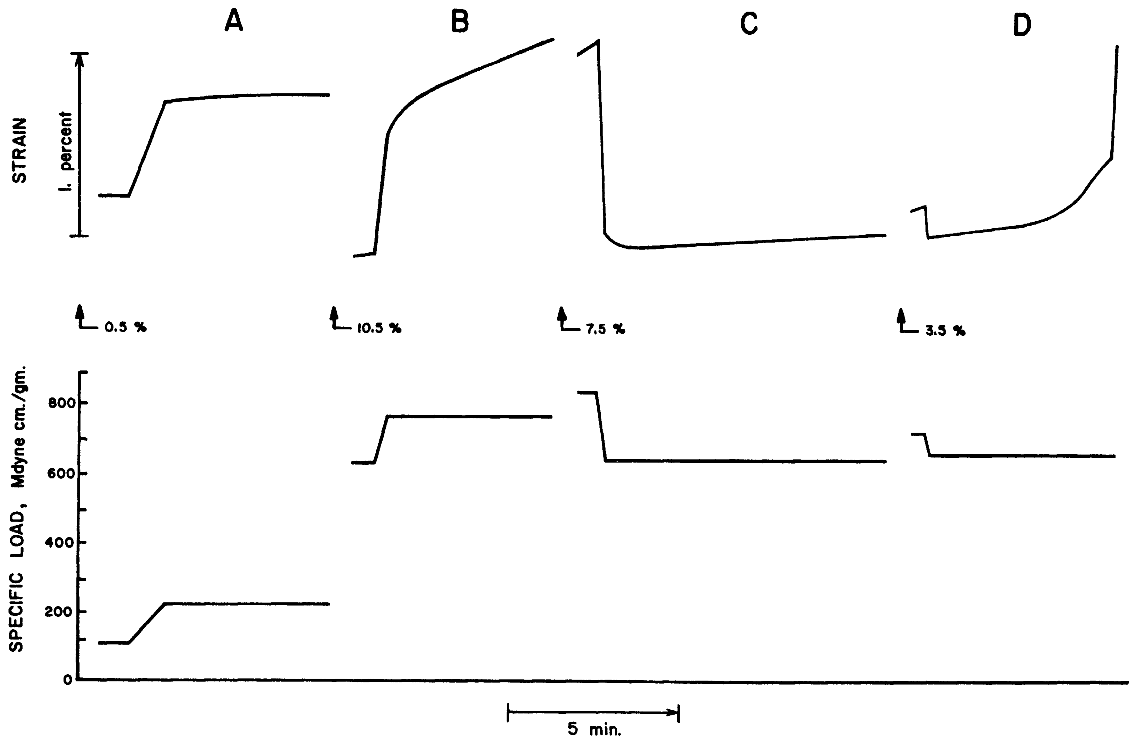


FIGURE 5.17 ELONGATION RESPONSES OF TENDONS HELD AT CONSTANT LOADS AFTER RAMP LOADING: (A) VISCOELASTIC EXTENSION. (TEST AT 46.6°C; SPECIMEN 36C6, PRESERVED BY FREEZING), (B) VISCOELASTIC AND PLASTIC-LIKE EXTENSION FOLLOWING AN INCREASE IN LOAD (TEST AT 25°C; SPECIMEN 43C1), (C) VISCOELASTIC SHORTENING AND PLASTIC-LIKE ELONGATION FOLLOWING A DECREASE IN LOAD (TEST AT 25°C; SPECIMEN 43C1), (D) PLASTIC-LIKE ELONGATION JUST PRIOR TO RUPTURE; NOTE INCREASING RATE OF ELONGATION (TEST AT 40.3°C; SPECIMEN 44C1).

still great enough for the plastic-like elongation to be seen. When this was done the viscoelastic transient was observed as an initial shortening of the specimen while the plastic-like elongation yielded a subsequent lengthening (Figure 5.17, C). In those instances when a specimen was held for a sufficiently long time at a load at which it appeared to be elongating plastically, the elongation proceeded until the specimen broke. Shortly before the failure occurred, the rate of elongation began to increase rapidly (Figure 5.17, D).[1]

To evaluate the plastic-like component of elongation, the rate of elongation (taken after the viscoelastic component appeared small) was plotted as a function of load for each specimen tested. The resulting graphs were either linear, as would have been expected for ideal plastic flow, (Figure 5.18) or concave upward (Figure 5.19), but never concave downward. The intercept of each such curve with the load-axis indicates a critical load above which plastic-like elongation was observed but below which such elongation was not to be expected. Such a critical load might be considered similar to a yield point, the load at which plastic flow commences. However, it differs from a yield point because it was determined only by the behavior of the material after plastic-like elongation had been initiated. The

[1] Frequently, specimens failed at the testing machine clamps before the rate of elongation within the gage length began to increase dramatically. The pattern described was typical of those specimens which did not fail at the testing machine clamps.

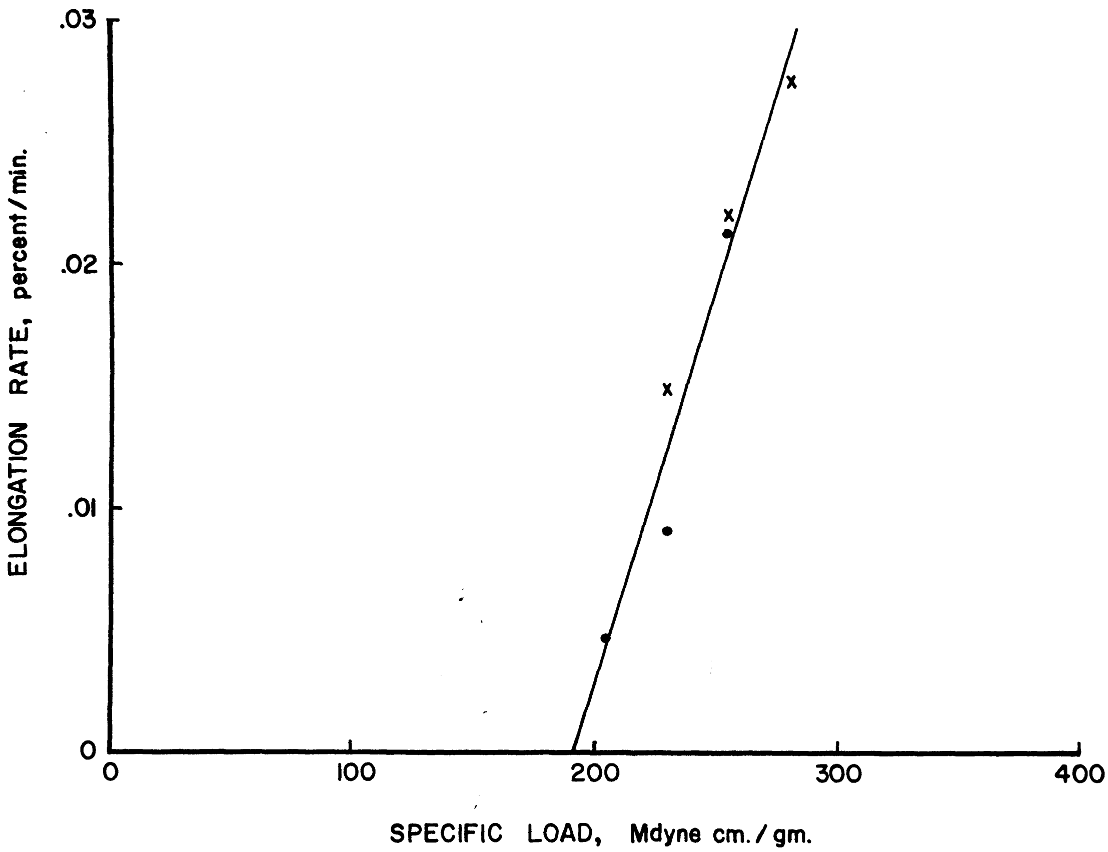


FIGURE 5.18 RATE OF PLASTIC-LIKE ELONGATION AS A FUNCTION OF SPECIFIC LOAD (TESTS AT 47.2°C; SPECIMEN 35C6, PRESERVED BY FREEZING).

X FOLLOWING AN INCREASE IN LOAD
• FOLLOWING A DECREASE IN LOAD

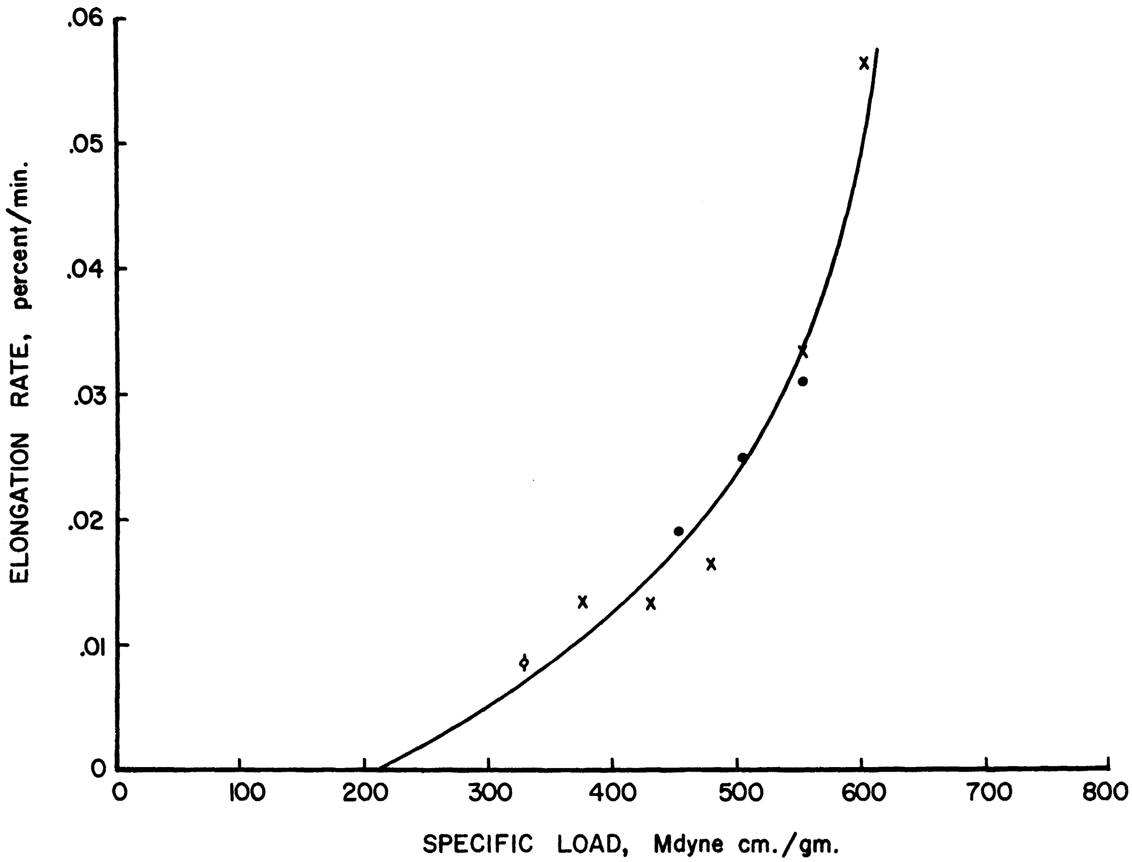


FIGURE 5.19 RATE OF PLASTIC-LIKE ELONGATION AS A FUNCTION OF SPECIFIC LOAD (TESTS AT 49.4°C; SPECIMEN 41C1).

× FOLLOWING AN INCREASE IN LOAD.
ϕ FOLLOWING AN INCREASE IN LOAD, WITH OVERSHOOT.
● FOLLOWING A DECREASE IN LOAD.

critical load should therefore be regarded as the load at which plastic-like elongation might have been expected to stop as the material was unloaded, rather than the load at which it commenced as the material was loaded.

While the rate of elongation, as determined after several minutes at a fixed load, depended on the magnitude of the load, it appeared not to depend on whether that magnitude was attained by loading from a smaller load or by unloading from a larger load (see: Figures 5.18 and 5.19). However, it did appear that the rate of plastic-like elongation increased with the amount of previous plastic-like elongation. Although this increase in elongation rate was most prominent in the period of rapidly increasing strain rates immediately prior to failure, it was also observed much earlier, especially in those few experiments in which plastic-like elongation proceeded for relatively long times, on the order of 2 hours or more. Figure 5.20 shows the graph of elongation rate as a function of load for one such experiment and indicates the sequence in which the elongation rates were measured. Although the trend toward increasing elongation rates has resulted in considerable scatter of the points in this figure, most moderate-sized sub-sequences from the sequence of measurements show substantially less scatter and could be fitted reasonably well by curves such as those of Figures 5.18 and 5.19. The slopes of such curves, of course, would vary considerably; but their intercepts would tend to cluster around 620 Mdyne cm./gm. In this experiment, then, it appears that the dependence of elongation rate on load changed considerably as plastic-like elongation proceeded

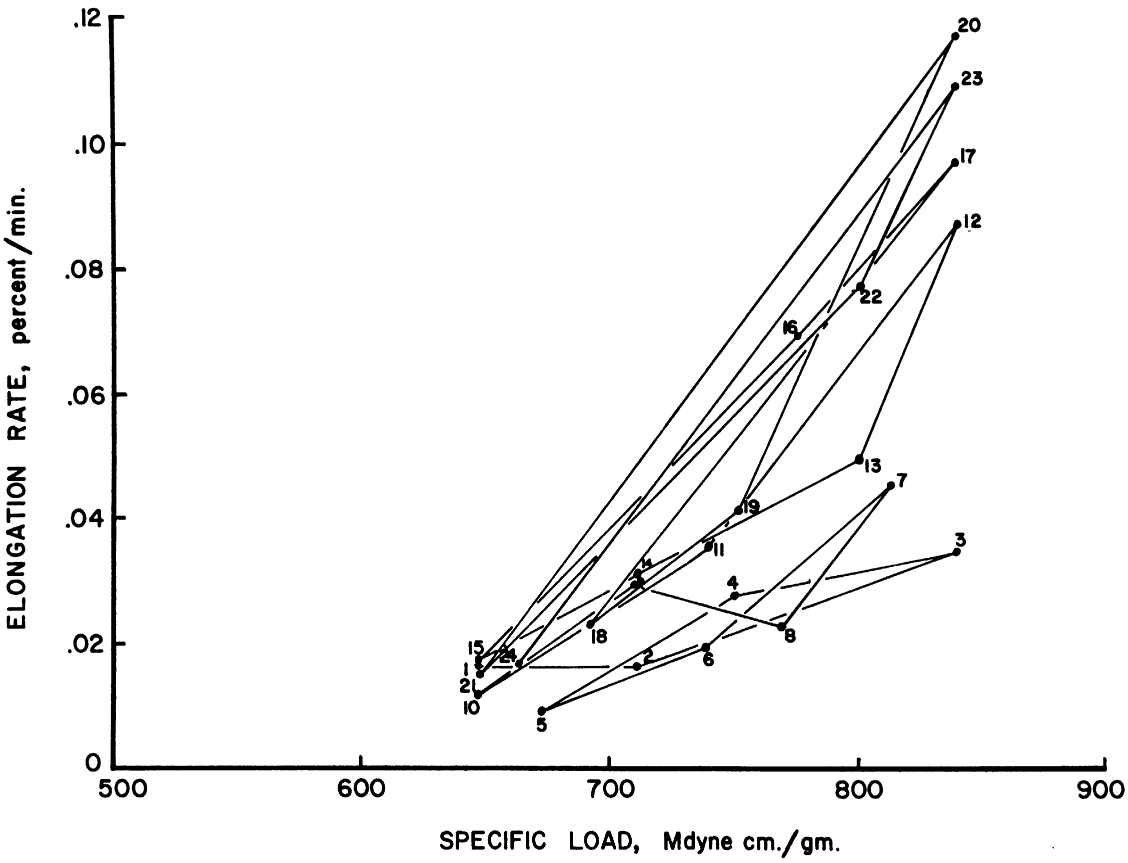


FIGURE 5.20 RATE OF PLASTIC-LIKE ELONGATION AS A FUNCTION OF SPECIFIC LOAD FROM AN EXPERIMENT IN WHICH PLASTIC-LIKE ELONGATION PROCEEDED FOR SEVERAL HOURS. NUMBERS INDICATE SEQUENCE IN WHICH TESTS WERE PERFORMED. (TESTS AT 25.0°C; SPECIMEN 43C1).

but that the critical load changed little. However, in another experiment, in which rupture of the specimen was prevented by the selection of smaller loads whenever rupture seemed imminent there appeared to be a sizable decrease in the critical load as plastic-like elongation proceeded. This may be inferred from the elongation rate trajectory for part of this experiment which is shown in Figure 5.21. At rupture, in this experiment, the specimen

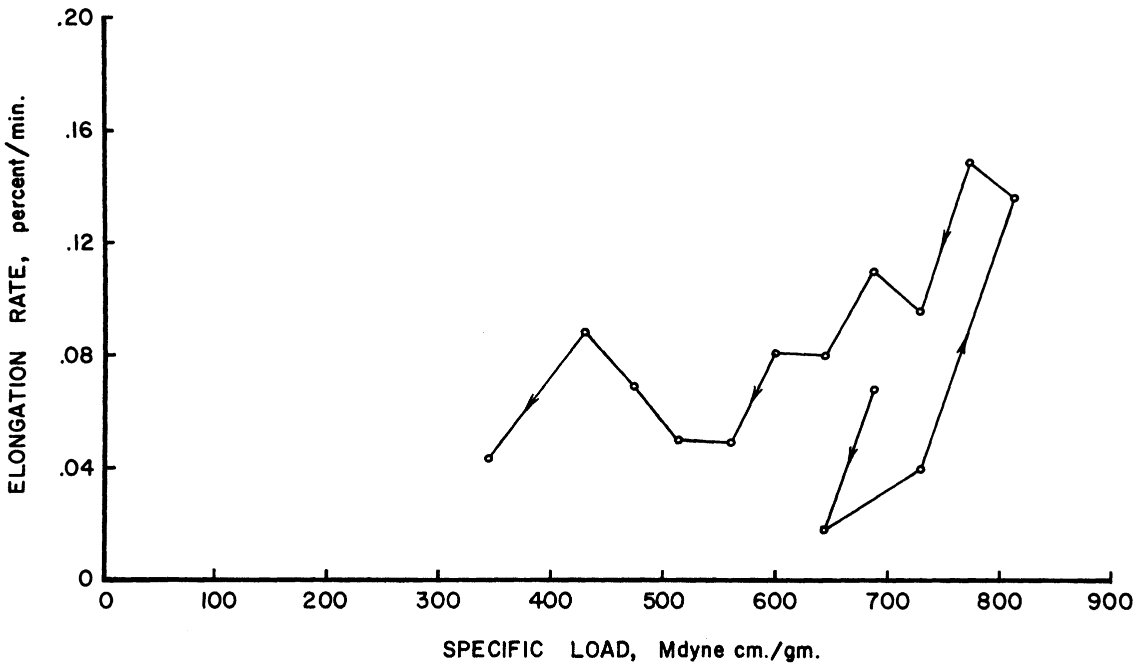


FIGURE 5.21 RATE OF PLASTIC-LIKE ELONGATION AS A FUNCTION OF SPECIFIC LOAD FROM AN EXPERIMENT IN WHICH RUPTURE WAS PREVENTED BY THE CHOICE OF PROGRESSIVELY DECREASING LOADS. ARROWS INDICATE THE SEQUENCE OF TESTS. (TESTS AT 32.4°C; SPECIMEN 50C1).

was elongating in a plastic-like manner at a specific load of less than 25 Mdyne cm./gm. and had attained an elongation of somewhat more than 22 percent. This may be contrasted with an elongation of less than 2.3 percent at a specific load of 430 Mdyne cm./gm. measured before plastic-like elongation was initiated. Shortly before failure occurred, this specimen had been unloaded and was seen to recover its original length to within 1.5 percent. This observation indicates that the progressive elongation which appeared to be plastic flow in tests at constant load levels was, instead, the result of a progressive decrease in stiffness. This conclusion was also supported by an earlier test in the experiment just discussed. The tendon was subjected to cyclic loading of the same amplitude and frequency both before plastic-like elongation was initiated and after plastic-like elongation had proceeded for some time. Load-elongation records obtained initially and after plastic-like elongation had occurred in this experiment are shown in Figure 5.22. These indicate that the specimen was much less stiff and displayed considerably more hysteresis after plastic-like elongation had occurred than before. They also tend to indicate a lowering of the critical load since the initial test yielded a closed load-elongation loop while in the later test there was a progressive displacement of the loops in the load-elongation record indicating that plastic-like elongation was occurring during this test. One other test of this type was performed on a different specimen and yielded similar results. The observation that the apparent constant rate or plastic-like elongation is the result of decreasing stiffness agrees well with earlier findings

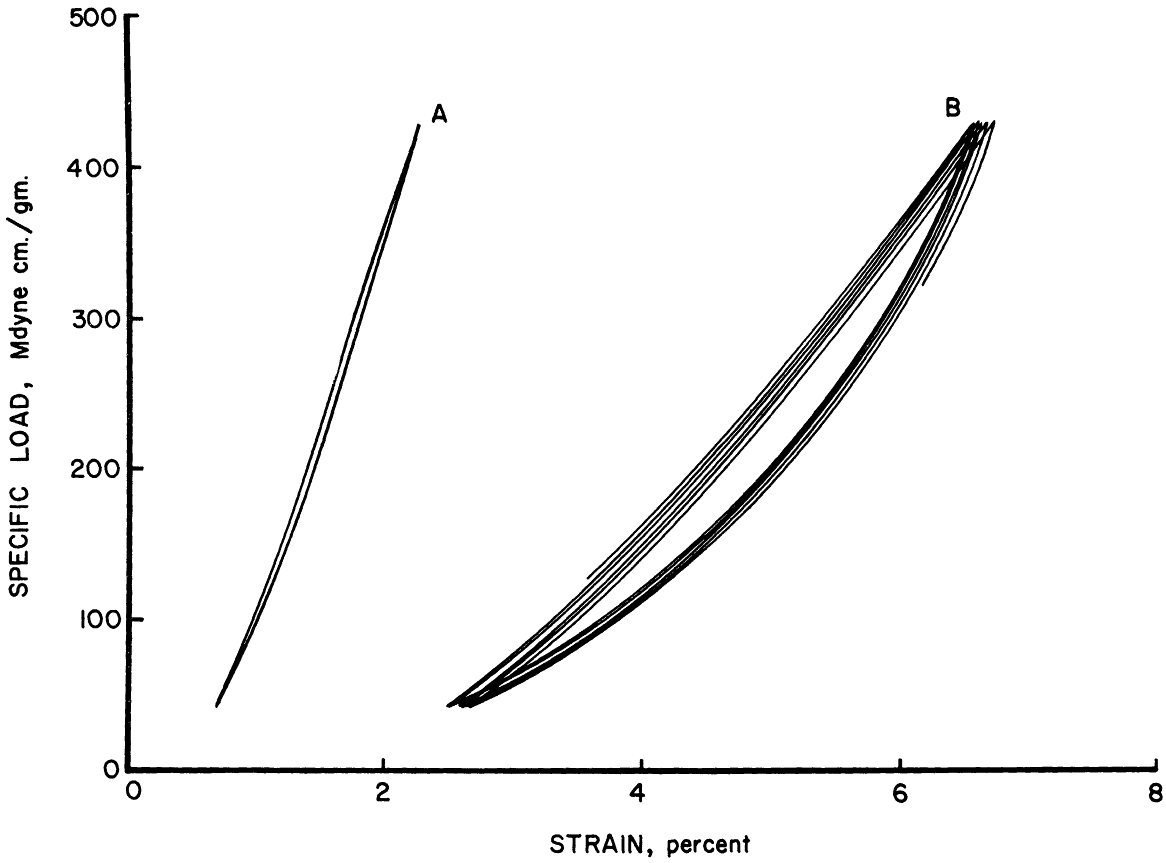


FIGURE 5.22 LOAD-ELONGATION RECORDS FOR A TENDON SUBJECTED TO CYCLIC LOADING AT 1 CYCLE PER MINUTE; (A) BEFORE PLASTIC-LIKE ELONGATION WAS INITIATED, (B) AFTER PLASTIC-LIKE ELONGATION HAD PROCEEDED FOR SOME TIME. (TESTS AT 32.4°C; SPECIMEN 50C1).

that "yielding" of tendon specimens resulted in a decrease in stiffness (Rigby et al, 1959; Graves, 1966).

To summarize the observations presented thus far in this section: (1) The time-dependent elongation in tendon appears to be composed of two components, a viscoelastic component and a component which appears plastic-like in creep experiments. (2) The plastic-like component is the result of a progressive decrease in stiffness rather than an increase in unstressed length as in classical plastic flow. (3) There appears to be a critical load at which plastic-like elongation would be expected to stop as a specimen undergoing plastic-like elongation was unloaded. (4) This critical load apparently decreases as plastic-like elongation progresses.

By using those elongation rates obtained early in each experiment (and in experiments in which plastic-like elongation did not proceed to a great extent), it was possible to obtain 'early critical loads' by plotting graphs such as those of Figures 5.18 and 5.19. Such early critical loads might be considered as approximations to a hypothetical 'initial critical load'. Values of the early critical load obtained in different experiments performed at temperatures below 44°C were remarkably consistent when compared, for example, with published values for tensile strength. Thus, the 'initial critical load' seems to be a relatively consistent property of the material.

Creep experiments were performed at various temperatures over the range 22° - 56°C, and the early critical load was determined from a plot of elongation rate as a function of load for each experiment that provided sufficient data. The values of early critical load from these experiments are plotted as a function of temperature in Figure 5.23. These data indicate that the early critical load remained nearly constant or decreased slightly with

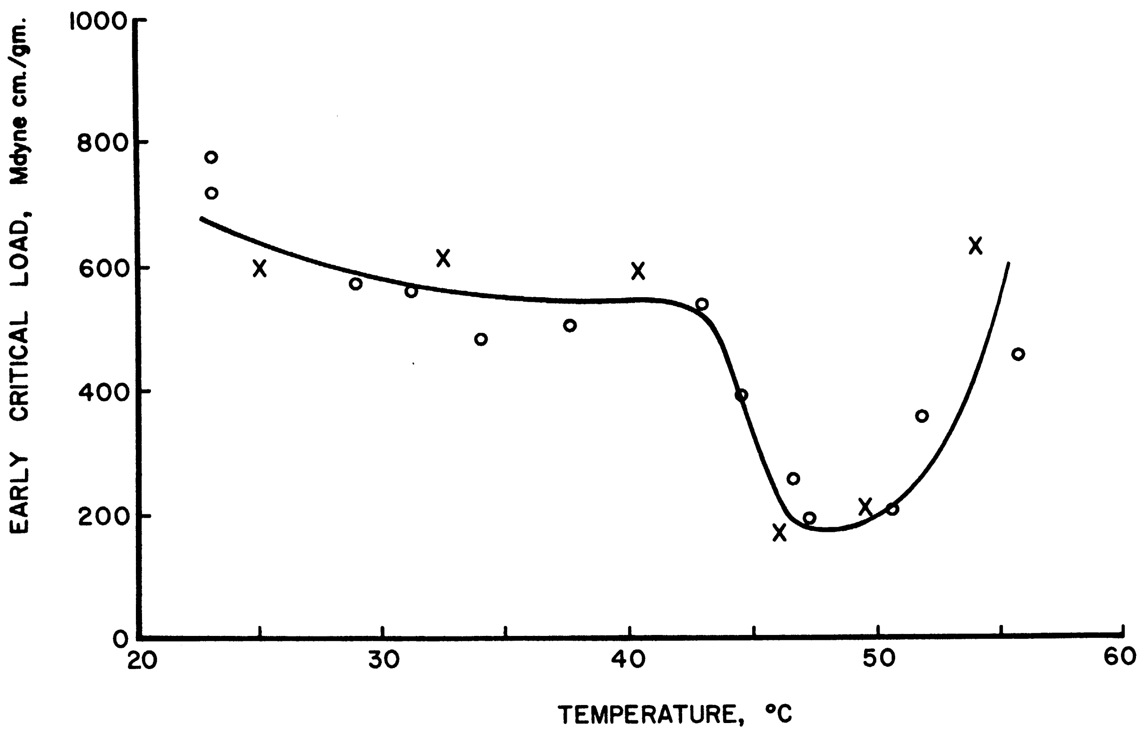


FIGURE 5.23 TEMPERATURE DEPENDENCE OF THE EARLY CRITICAL LOAD FOR PLASTIC-LIKE ELONGATION (SEE TEXT).

x EXPERIMENTS ON FRESH SPECIMENS
o EXPERIMENTS ON SPECIMENS PRESERVED BY FREEZING

increasing temperature over the range 22° - 44°C. Over the range 44° - 47°C the early critical load decreased sharply with increasing temperature, dropping from nearly 600 Mdyne cm./gm. to about 200 Mdyne cm./gm. Above approximately 50°C the early critical load increased with increasing temperature. There was little difference between early critical loads obtained from experiments on fresh tendons and those obtained from experiments on tendons which had been preserved by freezing; however, there appeared to be some tendency for values obtained from fresh material to be slightly higher (Figure 5.23).

To assess the reversibility of the transition responsible for the decrease in early critical load which was noted as the temperature increased above 44°C, several experiments were performed in which unloaded specimens were placed in 50°C Ringer's solution for 2 hours and were subsequently tested at temperatures around 24°C. In these experiments the specimens were permitted to remain in room temperature Ringer's solution (approximately 20°C) for various periods of time between their exposure to 50°C Ringer's solution and the time they were tested. The creep tests in these experiments constituted the first main test sequence in the protocol given in Section 4.2. Data from these experiments are presented in Table 5.1. A comparison of these data with the data from low temperature (< 44°C) tests on specimen which had not been preconditioned at 50°C (see: Figure 5.23) indicates that even after 10 or more hours at room temperature, specimens which had been immersed in 50°C Ringer's solution for 2 hours had early critical loads which were significantly lower than those of specimens which had never been subjected to a temperature above 44°C. Partial recovery, however, is indicated by the difference between values of early critical load obtained in these experiments and those obtained in experiments in which tests were performed at temperatures near 50°C. Also, a trend toward increasing early critical load with increasing time at room temperature suggests progressive recovery.

These data do not indicate whether the transition is fully reversible with respect to its effect on early critical load. But they do show that recovery occurs very gradually and that, for most purposes, the transition could be regarded as nearly irreversible.

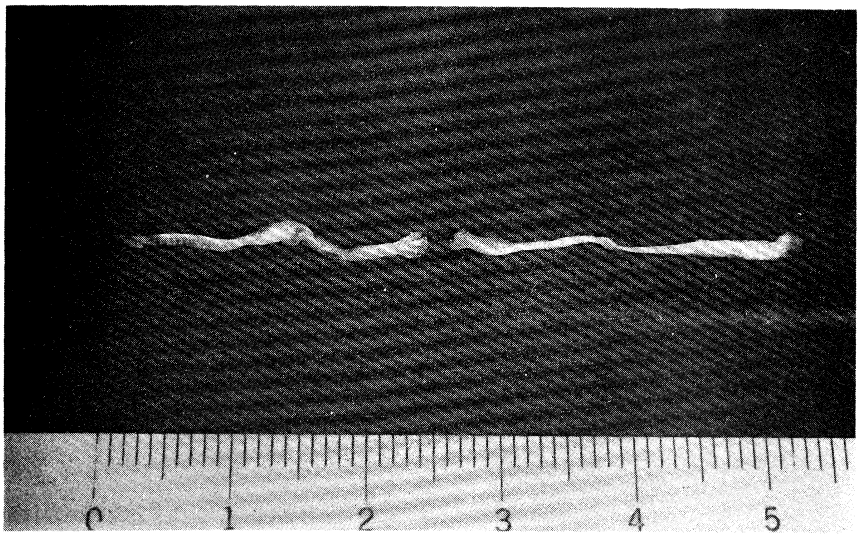
SPECIMEN NUMBER	57C1	54C7	57C3	53C2	51C3
SPECIMEN CONDITION	FRESH	FROZEN	FRESH	FROZEN	FROZEN
TIME AT ROOM TEMPERATURE, HOURS	1	1	4	10.5	17
EARLY CRITICAL LOAD, Mdyne cm./gm.	331	316	221	398	414

TABLE 5.1 RESULTS FROM EARLY CRITICAL LOAD DETERMINATIONS FOR SPECIMENS PRECONDITIONED IN 50°C RINGER'S SOLUTION FOR 2 HOURS, THEN IN ROOM TEMPERATURE RINGER'S SOLUTION FOR VARIOUS TIMES (TIMES REPORTED INCLUDE 1 HOUR IN THE TESTING MACHINE PRIOR TO THE CREEP TEST).

It is quite possible that the sharp decrease in early critical load with increasing temperature, beginning around 44°C, coincides with the phenomenon reported by Rigby et al (1959). If their stress relaxation experiments were begun at loads which would have been below the early critical load at lower temperatures, but above the early critical load at the temperatures of their higher temperature tests, the addition of a progressive decrease in stiffness of the type described above to the viscoelastic relaxation could account for the large increase in the rate and amount of stress relaxation which they found as the temperature was raised above 40°C. The effect of this progressive decrease in stiffness would have been an irreversible change such as they reported. The temperature range over which the early critical load was seen to decrease sharply with increasing temperature (44° - 47°C) corresponds closely to the second order transition temperature of 45°C reported by Flory and Garrett (1958) for hydrated beef Achilles tendon. The hypothesis that the decrease in early critical load is the result of a second order transition like that described by Flory and Garrett is an attractive one. Applying this hypothesis, one might conjecture that the progressive decrease in stiffness results from the disruption of regions which are glass-like in structure.

Similarly, the temperature above which the early critical load begins to increase with increasing temperature (around 50°C) is close to 51°C, the equilibrium melting temperature of collagen (c.f. Oth et al, 1957; Rigby, 1964b); and one might guess the apparent increase in early critical load to be associated with melting in disrupted regions of the collagen structure. The appearance of broken ends of specimens which did not fail at the machine clamps seems to partially support wuch a hypothesis. In those specimens which failed after plastic-like elongation at high temperatures the broken ends appeared somewhat rounded and rubber-like (Figure 5.24,A) while failures at low temperatures gave stringy or

A.



B.

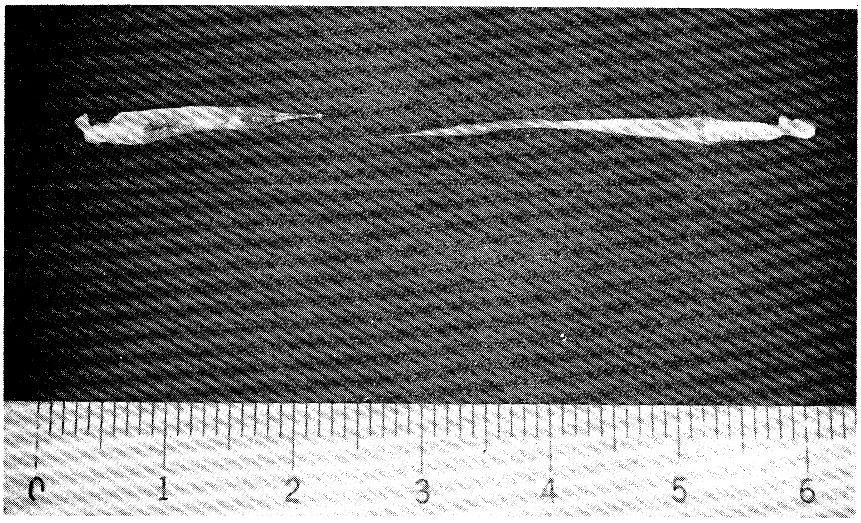


FIGURE 5.24 APPEARANCE OF SPECIMENS WHICH BROKE AFTER PLASTIC-LIKE ELONGATION; (A) AT A HIGH TEMPERATURE (49.4°C; SPECIMEN 41C1), (B) AT A LOW TEMPERATURE (32.4°C; SPECIMEN 43C2, PRESERVED BY FREEZING). [METRIC SCALES.]

shredded ends (Figure 5.24, B). However, the number of specimens which failed away from the clamps was small, 5 in all. One of these, which had been tested at 46.0°C, had broken ends which appeared intermediate between the two types. This could be taken as an indication that the change in failure type, if it is a common characteristic, is associated with the glass-type transition rather than with local melting at the equilibrium melting temperature.

It might be of interest to compare the early critical load below 44°C with published values for the tensile strength of tendon. For the 10 determinations of early critical load below 44°C the average value was 559 Mdyne cm./gm. and the standard deviation 46 Mdyne cm./gm. To compare this with published tensile strength data it will be necessary to estimate the stress corresponding to this critical specific load, since most published data have been presented in terms of stress rather than specific load. Ellis (1969) has given approximate factors for relating various cross-sectional area measurements to the dry weight per unit length for cat extensor digitorum communis tendon. These can be used for the required estimate. For areas based on moist specimen displacement volume per unit length, the appropriate factor is 2,259 mm^3/gm. or 2.259 cm^3/gm. Using this factor, the stress corresponding to 559 Mdyne cm./gm. would be 243 Mdyne/cm^2 (2.53 Kg/mm^2). This estimate is probably rather low, since moist specimen displacement volume tends to give high values for cross-sectional area. The area measuring device used by Walker et al (1964) tends to give low values for cross-sectional area. The factor for relating measurements with such an instrument to dry weight per unit length is 990.7 mm^3/gm. (Ellis, 1969), or .9907 cm^3/gm. Dividing 559 by this, the stress would be 561 $Mdyne/cm^2$ (5.84 Kg/mm^2). Comparing the range of stresses defined by these two estimates with the usual range of tensile strengths of 5-10 Kg/mm^2 given by Elliott (1965), it appears that the early critical load was somewhat more than half the tensile strength commonly measured. Since the progressive decrease in stiffness occurring above the critical load is probably a progressive failure it might be possible to obtain as a value of tensile strength almost any value between the critical load and perhaps twice the critical load, depending on the way in which the tensile strength was measured. (Even lower values, of course, might be obtained by initiating plastic-like elongation and then lowering the load progressively as was done in one experiment described above.)

5.3 DYNAMIC STIFFNESS AND DAMPING

A series of exploratory experiments was performed in an attempt to identify the effects of temperature on dynamic stiffness and damping in tendon in the load range above the 'toe' portion of the load-strain curve and below the region of plastic-like elongation.

When a tendon is subjected to cyclic loading with loads in this range, the strain response approaches a nearly steady state (see also: Sections 2.22 and 5.1). In this nearly steady state load-elongation records from successive load cycles coincide, and the load-elongation record for each cycle is a closed loop. The area within such a loop represents the energy dissipated in one load cycle (hysteresis energy loss, or damping). This energy dissipation increases roughly as the square of the amplitude, but decreases slightly as the mean load is increased. It also decreases slightly with increasing frequency, at least over the range of approximately 0.05 to 5 cycles/minute which has been investigated in this study. However, the frequency dependence of damping accounts for a change of only about 30 percent in the energy dissipation over this range, much less than would be found for a simple viscoelastic material (Section 5.1).

Dynamic stiffnesses (or moduli) can be found from load-elongation loops in two ways, by taking the slopes of tangents to the loops at defined locations or by taking the ratios of stress (or specific load) amplitudes to strain amplitudes. Dynamic stiffnesses found in either way tend to increase with increasing frequency and with increasing mean load, so long as the amplitude of the load cycle is held roughly constant. The increase in dynamic stiffness with increasing mean load might be thought of as resulting largely from nonlinearity of the elastic portion of the load-elongation characteristic. From such nonlinear elastic behavior one would expect that the dynamic stiffness taken as an amplitude ratio would increase with decreasing amplitude for load cycles with fixed mean loads and frequencies. The observation that the dynamic stiffnesses defined by tangents at the mean load also increase with decreasing load amplitude (Section 5.1) suggests a secondary effect, in addition to that which would result from nonlinear elastic behavior, because the slope of a tangent to the load-elongation curve for an elastic specimen would depend only on the value of load at the point of tangency.

These two effects of amplitude on dynamic stiffness were further demonstrated in a series of three experiments designed to assess the temperature dependence of the nonlinearity of the relation between (steady) load and dynamic stiffness. In these, each specimen was subjected to load cycles of various amplitudes centered at each of 5 steady loads. Two cycle frequencies, 2 and 0.25 cycles/minute, were used in each experiment; and the sequence of tests was performed twice for each specimen, once at a temperature between 22° and 25°C and once at a higher temperature.

To analyze the data from these experiments, it was assumed that the load-strain relation of a hypothetical elastic element approximating the tendon could be represented by a power function such as that proposed by Ridge and Wright (1964). This relation was taken to have the form

$$E = c + (KP)^b$$

with E and P representing strain and specific load, respectively, and the other symbols representing empirical constants. For load cycles with load limits P_1 and P_2, application of this equation gives extension limits E_1 and E_2 such that

$$E_2 - E_1 = K^b(P_2^b - P_1^b).$$

For each set consisting of all data from one specimen at a single frequency and temperature, values of K and b for this relation were obtained by a least squares fit of an equivalent form of this second equation to the set of data triplets $[P_1, P_2, (P_2 - P_1)/(E_2 - E_1)]$ using the scheme which is developed in Appendix IV. Then the equation with these two coefficients was used to obtain the dynamic stiffness, $(P_2 - P_1)/(E_2 - E_1)$, of the hypothetical elastic component as a function of load amplitude for each of the steady loads used in the experiment. Figure 5.25 shows one family of dynamic stiffness curves calculated in this way, along with the experimentally determined points from the

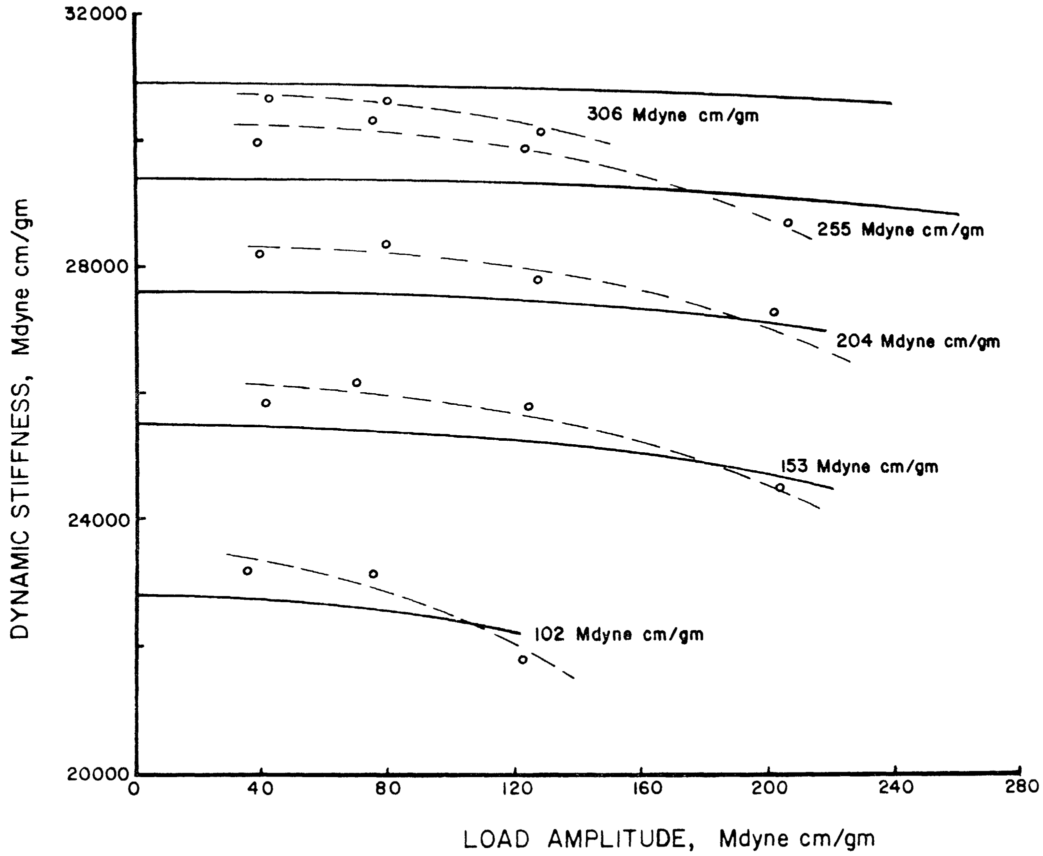


FIGURE 5.25 DYNAMIC STIFFNESS (AMPLITUDE RATIO) AS A FUNCTION OF CYCLE AMPLITUDE AT SEVERAL MEAN LOADS, AS INDICATED. (TESTS ON SPECIMEN 52C1 AT 2 CYCLE/MIN. AND 23.6°C) SOLID CURVES FROM THE EQUATION $(E_2 - E_1) = K^b(P_2^b - P_1^b)$ FITTED TO THE DATA; b = .723, $K = 8.66 \times 10^{-6}$ gm./Mdyne cm.

corresponding data set. At each steady load shown here, the measured dynamic stiffness increased more sharply with decreasing amplitude than did that calculated from the equation of the hypothetical elastic component. Such deviations, corresponding to the observation noted in Section 5.1, were found in all of the 12 data sets at all steady load levels except the highest. However, the magnitude of these systematic deviations seemed to be quite variable.

Despite these deviations, the hypothetical elastic components seemed to provide moderately good descriptions of dynamic stiffness at a fixed frequency. For example, only 3 of the 181 measured stiffness values in the 12 data sets differed by more than 10 percent from the corresponding calculated values.

The exponent b provides an indication of the degree of nonlinearity of the hypothetical elastic component and hence of the relationship between dynamic stiffness and load (most easily conceived of as steady load at zero amplitude). To assess the temperature dependence of this indicator of nonlinearity, results from analyses of the three experiments described above were taken together with results from similar analyses of two additional experiments in which tests were made at a frequency of 1 cycle/minute and in which the lower cycle limit was held constant. Values for b obtained from the 17 data sets in these 5 experiments[1] ranged from 0.589 to 0.880. However,

---

[1]One experiment at 1 cycle/minute included tests yielding data sets at each of 3 temperatures. Of the 5 specimens, 2 were fresh and 3 preserved by freezing. Test temperatures for second (or third) main test sequences were 33.8°, 36.4°, 39.2°, 43.1°, 49.5°, and 51.6°C.

b values from data sets for each specimen were relatively close together, with the ratio of b from a data set in the second (or third) main test sequence to b from the corresponding data set in the first main test sequence ranging only from 0.895 to 1.125. This ratio showed no obvious pattern of temperature dependence, and a linear regression analysis for the relation of this ratio to second test temperature (9 points) yielded a temperature coefficient of +0.0028/°C for the ratio b_2/b_1 (a trend toward decreasing nonlinearity with increasing temperature) but with a correlation coefficient of only .235. From the regression line, the value of b_2/b_1 within the range of initial test temperatures was 0.96.

Thus, these results fail to demonstrate a significant change in nonlinearity either with temperature over the range 22° - 52°C or between the first and second main test sequences, although they do suggest the possibility that nonlinearity decreases slightly with increasing temperature.

To explore the effect of temperature on the magnitude of dynamic stiffness, another set of experiments was performed in which the load limits were the same for all tests in each experiment. For the sake of resolution in these experiments the load limits were chosen so that the load excursion covered a large part of the range between the 'toe' region and the critical load.

For each main test sequence, which in these experiments consisted of tests at a series of frequencies, the dynamic stiffness (amplitude ratio) was plotted as a function of frequency as indicated in Figure 5.26. From such curves, values of stiffness were taken at three frequencies, 0.3, 1.0, and 3.0 cycles/minute, and the ratio *stiffness from the second test sequence/stiffness from the first test sequence* was found for each of the three frequencies for each specimen. Figure 5.27 shows this ratio for a frequency of 1 cycle/minute plotted as a function of second test temperature. Here, and for the other two frequencies, there is little indication of a systematic change in the magnitude of the dynamic stiffness with temperature. Linear regression analyses for the relation of this stiffness ratio to second test temperature for each frequency yielded the results given in Table 5.2. While these may suggest possible trends toward decreasing stiffness with increasing temperature and toward an increase in stiffness between first and second main test sequences, they fail to demonstrate a statistically significant correlation between stiffness and second test temperature.

	0.3 cycle/min.	1 cycle/min.	3 cycle/min.
Temperature coefficient for the stiffness ratio	-.00303/°C	-.00298/°C	+.00163/°C
Correlation coefficient	-.153	-.318	+.314
Values of the stiffness ratio within the range of first test temperature (22° - 25°C), from the regression line	1.032 - 1.041	1.033 - 1.042	.993 - .998

TABLE 5.2 RESULTS FROM LINEAR REGRESSION ANALYSES FOR THE RELATION OF THE STIFFNESS RATIO *STIFFNESS FROM THE SECOND MAIN TEST SEQUENCE/ STIFFNESS FROM THE FIRST MAIN TEST SEQUENCE* TO SECOND TEST TEMPERATURE.

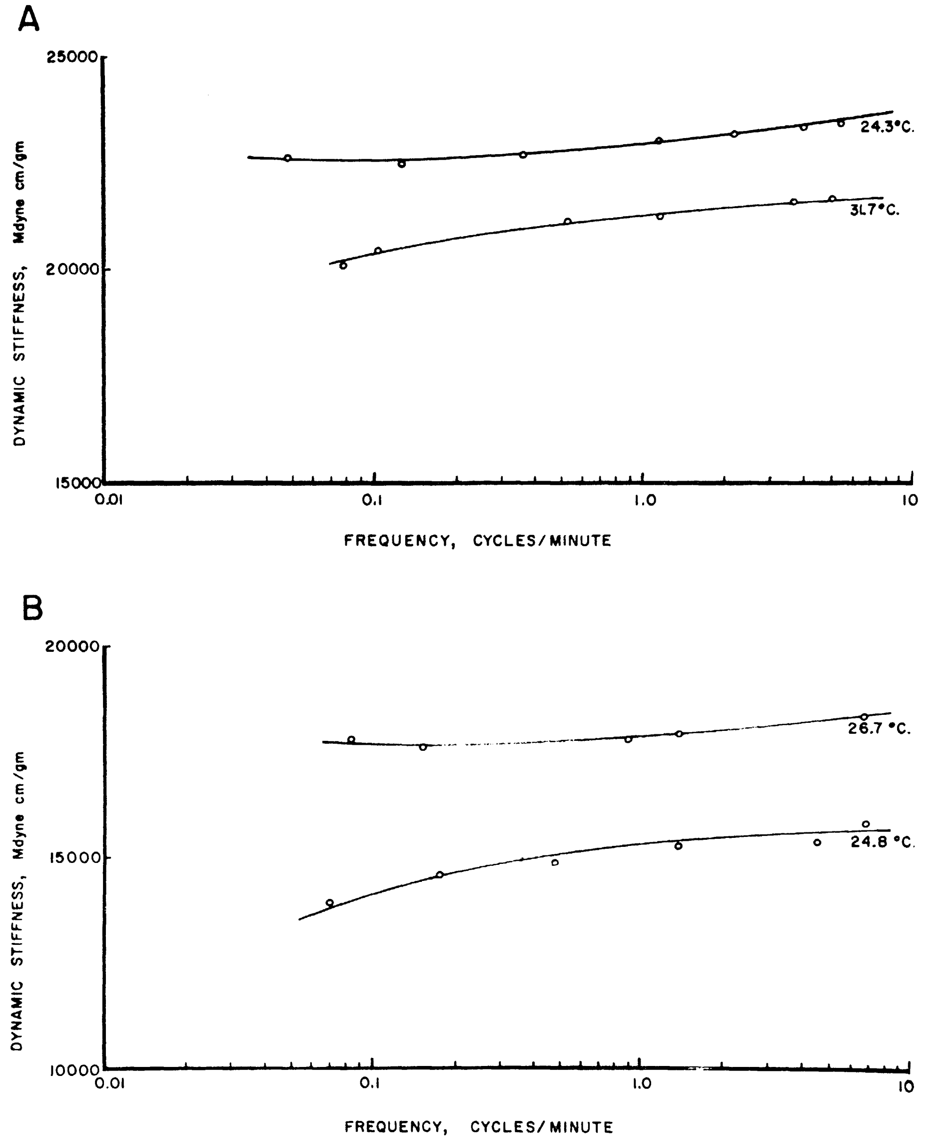


FIGURE 5.26 DYNAMIC STIFFNESS (AMPLITUDE RATIO) AS A FUNCTION OF FREQUENCY FOR EACH OF 2 SPECIMENS EACH TESTED AT 2 TEMPERATURES. (A) SPECIMEN 54C4 (PRESERVED BY FREEZING), CYCLE LIMITS 35 AND 353 MDYNE CM/GM. (B) SPECIMEN 61C1, CYCLE LIMITS 29 AND 232 MDYNE CM/GM.

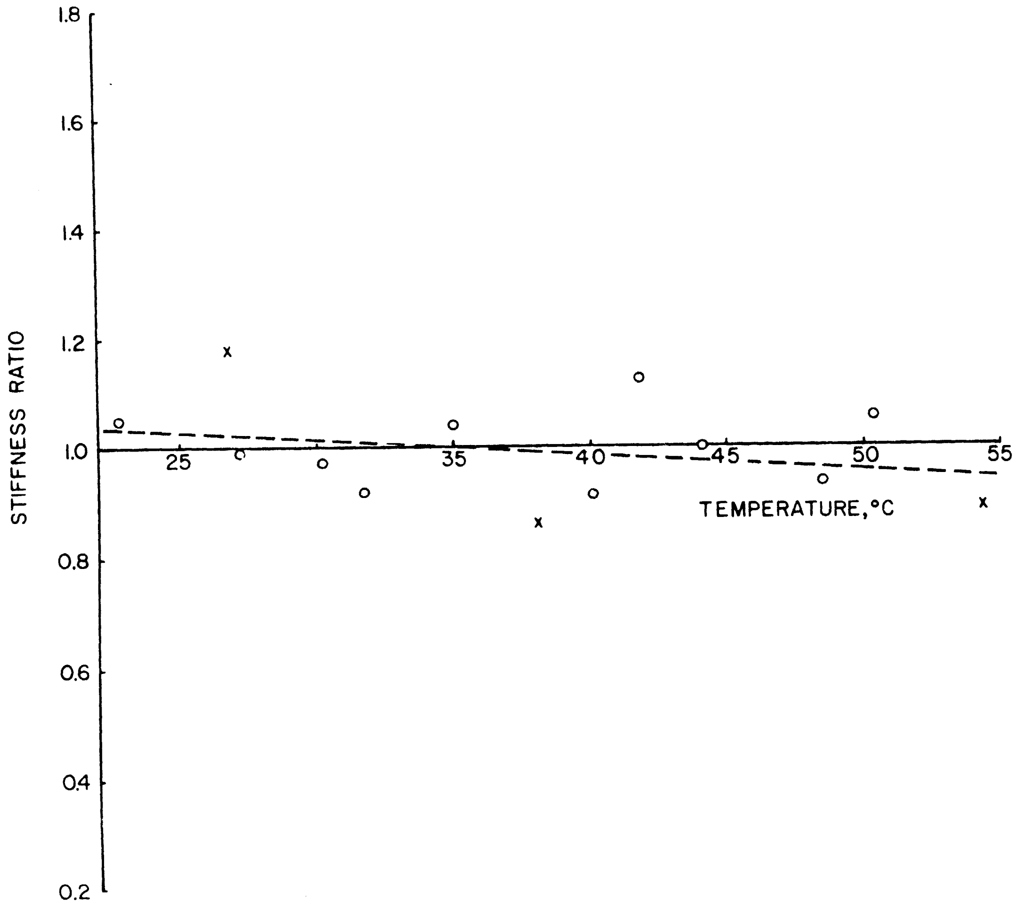
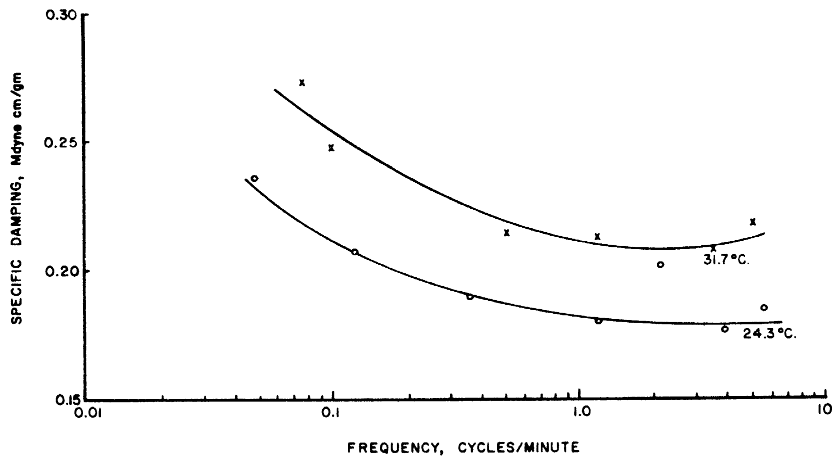


FIGURE 5.27 RATIO OF STIFFNESS FROM THE SECOND MAIN TEST SEQUENCE TO STIFFNESS FROM THE FIRST MAIN TEST SEQUENCE AS A FUNCTION OF SECOND TEST TEMPERATURE, FOR A FREQUENCY OF 1 CYCLE/MINUTE.

The effect of temperature on damping was explored in substantially the same way as the effect of temperature on the magnitude of the dynamic stiffness. Areas from the load-elongation hysteresis loops obtained in the series of experiments just described were measured with a polar planimeter (K & E 620022) and were plotted as functions of frequency for each test sequence (Figure 5.28). From the curves obtained in this way, values of damping at frequencies of 0.3, 1.0



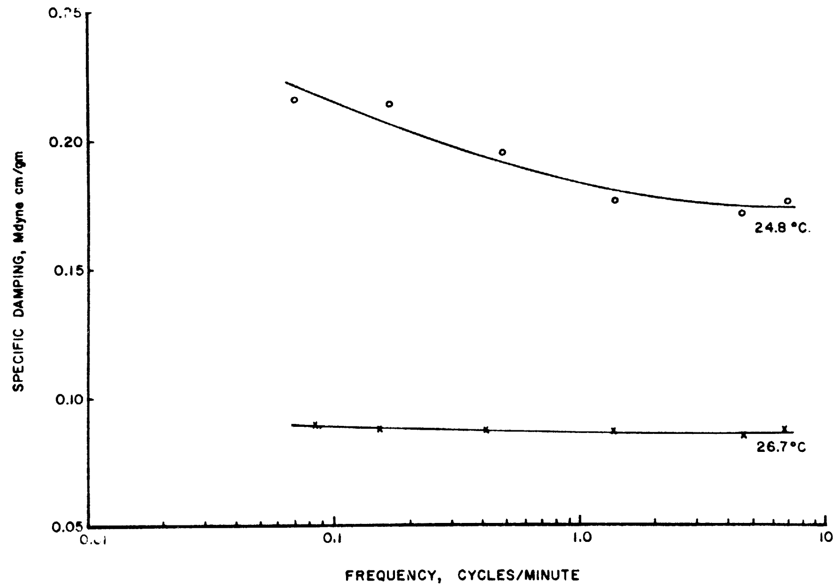


FIGURE 5.28 DAMPING AS A FUNCTION OF FREQUENCY FOR EACH OF 2 SPECIMENS EACH TESTED AT 2 TEMPERATURES. (A) SPECIMEN 54C4 (PRESERVED BY FREEZING), CYCLE LIMITS 35 AND 353 MDYNE CM/GM. (B) SPECIMEN 61C1, CYCLE LIMITS 29 AND 232 MDYNE CM/GM.

and 3.0 cycles/minute were taken; and the ratios *damping from the second main test sequence/damping from the first main test sequence* were computed for each of the three frequencies for each specimen. Figure 5.29 shows this ratio plotted as a function of second test temperature for a frequency of 1 cycle/minute. Again the data provide little indication of a systematic change with temperature. Results from linear regression analyses for the relation of this ratio to second test temperature for each of the three frequencies are given in Table 5.3. While these might suggest possible trends toward increasing damping with increasing temperature and toward a

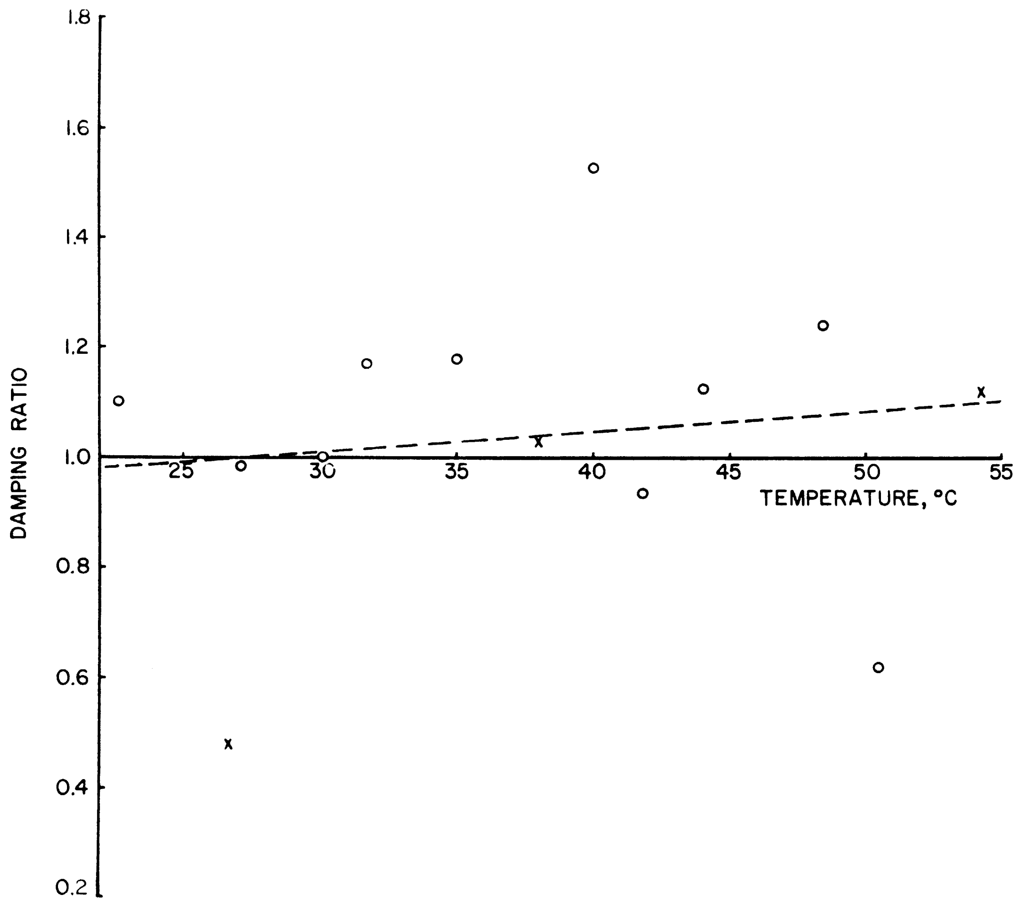


FIGURE 5.29 RATIO OF DAMPING FROM THE SECOND MAIN TEST SEQUENCE TO DAMPING FROM THE FIRST MAIN TEST SEQUENCE AS A FUNCTION OF TEMPERATURE, FOR A FREQUENCY OF 1 CYCLE/MINUTE.

decrease in damping between first and second main test sequences, they fail to demonstrate correlation between damping and temperature which is significant at any reasonable confidence level ($P < 0.8$).

There is some theoretical basis for expecting that damping would increase with increasing temperature. If the dissipative processes require activation energies, increasing the temperature is expected to influence damping in the same way as decreasing the frequency (Norwick, 1964). Since damping in tendon specimens was found to increase slightly with decreasing frequency, it might also have been expected

	0.3 cycle/min.	1 cycle/min.	3 cycle/min.
Temperature coefficient for the damping ratio	+.0172/°C	+.00373/°C	+.000876/°C
Correlation coefficient	+.478	+.139	+.031
Values of the damping ratio within the range of first test temperatures (22° - 25°C), from the regression line	.888 - .939	.981 - .992	.982 - .984
Number of ratios	10	13	11

TABLE 5.3 RESULTS FROM LINEAR REGRESSION ANALYSES FOR THE RELATION OF THE DAMPING RATIO *DAMPING FROM THE SECOND MAIN TEST SEQUENCE/DAMPING FROM THE FIRST MAIN TEST SEQUENCE* TO SECOND TEST TEMPERATURE.

to increase with increasing temperature. However, since the increase in damping with increasing frequency which was found over the frequency range covered by these experiments was relatively small, the increase in damping with increasing temperature would also be expected to be small, unless the activation energies for the dissipative processes were exceedingly large. Thus, the failure to clearly demonstrate temperature dependence for damping is not surprising.

Comparing the results of these experiments with those in Section 5.2, it is interesting to note that, unlike the early critical load, the dynamic stiffness and damping characteristics did not show large changes in the region of the 'glass type' transition temperature. This could be taken as evidence that those portions of the structure which are involved in the 'glass type' transition do not contribute significantly to the viscoelastic dissipative processes or the overall compliance of the material at loads below the early critical load.

5.4 CHANGES DUE TO STORAGE AND HANDLING

Throughout this investigation, two rather consistent differences were noted between the initial test, immediately after a specimen was mounted in the testing machine, and the second test, 1 hour later. These reflected (1) an increase in dynamic stiffness and (2) a decrease in damping between the first and second tests. Figure 5.30 presents a histogram indicating the magnitude of the increase in stiffness for

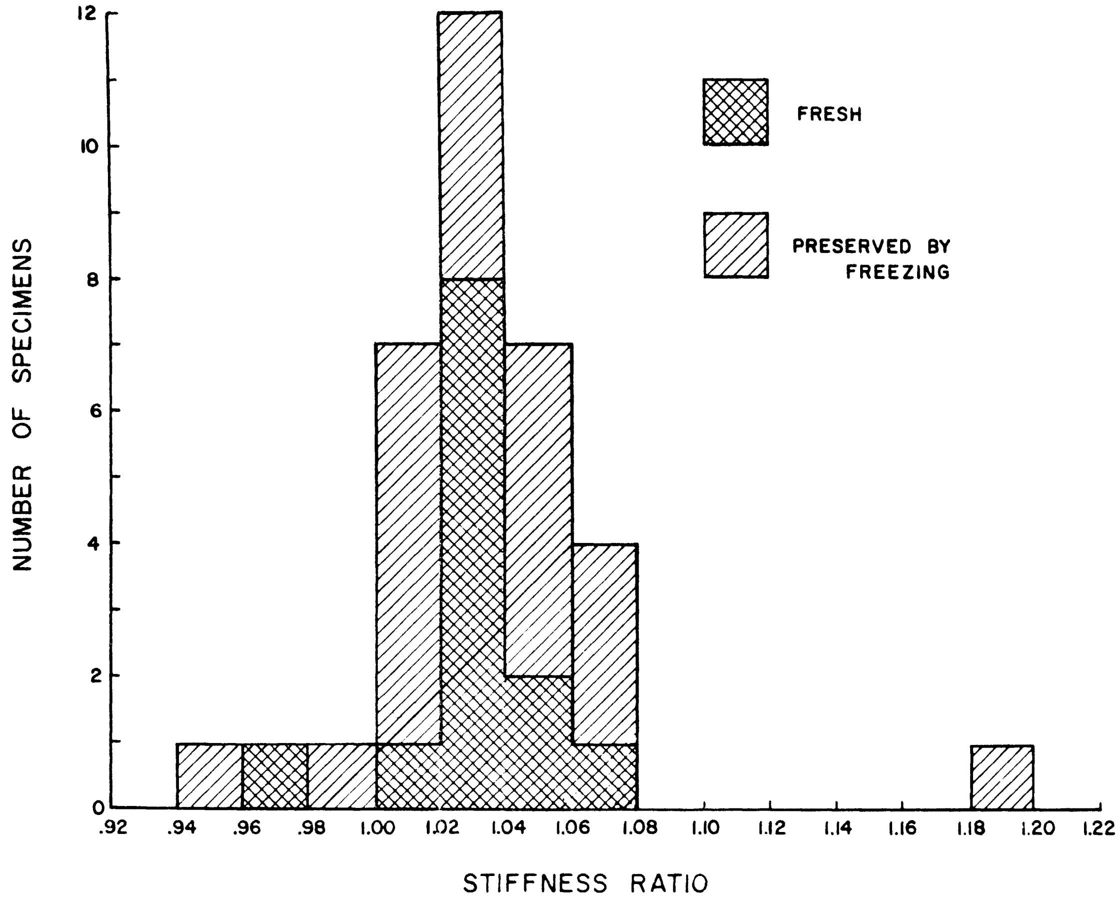


FIGURE 5.30 HISTOGRAM SHOWING THE DISTRIBUTION OF THE RATIO OF STIFFNESS AFTER 1 HOUR IN THE TESTING MACHINE TO INITIAL STIFFNESS FOR SPECIMENS USED IN THE TEMPERATURE STUDIES.

those specimens used in the temperature studies of Sections 5.2 and 5.3[1]. These data show that an increase in dynamic stiffness of

[1]Tests involving specimens preconditioned at 50°C have been excluded as have tests for which the load-strain record from the first test was not clear enough for both dynamic stiffness and damping to be evaluated.

roughly 3 percent (at approximately 2 cycles/minute) was typical for both fresh specimens and specimens which had been preserved by freezing. Figure 5.31 shows histograms for the ratio *damping from the second test/damping from the first test* for the same specimens. From these it appears that the decrease in damping was less for fresh specimens than for those preserved by freezing. A t-test for significance in the difference between the means of the damping ratios for fresh specimens (0.939) and for specimens preserved by freezing (0.758) indicated that this difference was significant at the 99 percent confidence level. Another possible difference between fresh and preserved specimens, a slightly lower early critical load for specimens preserved by freezing, was noted in Section 5.2.

The observed increase in stiffness and decrease damping in this initial interval might be accounted for by the loss of polysaccharide plasticizers (Section 4.1). There is an indication from the analyses of Section 5.3 that these changes may have continued to occur after the first main test sequence also (i.e. during the second and third hours of the experiment). However, such changes during this interval were not positively identified.

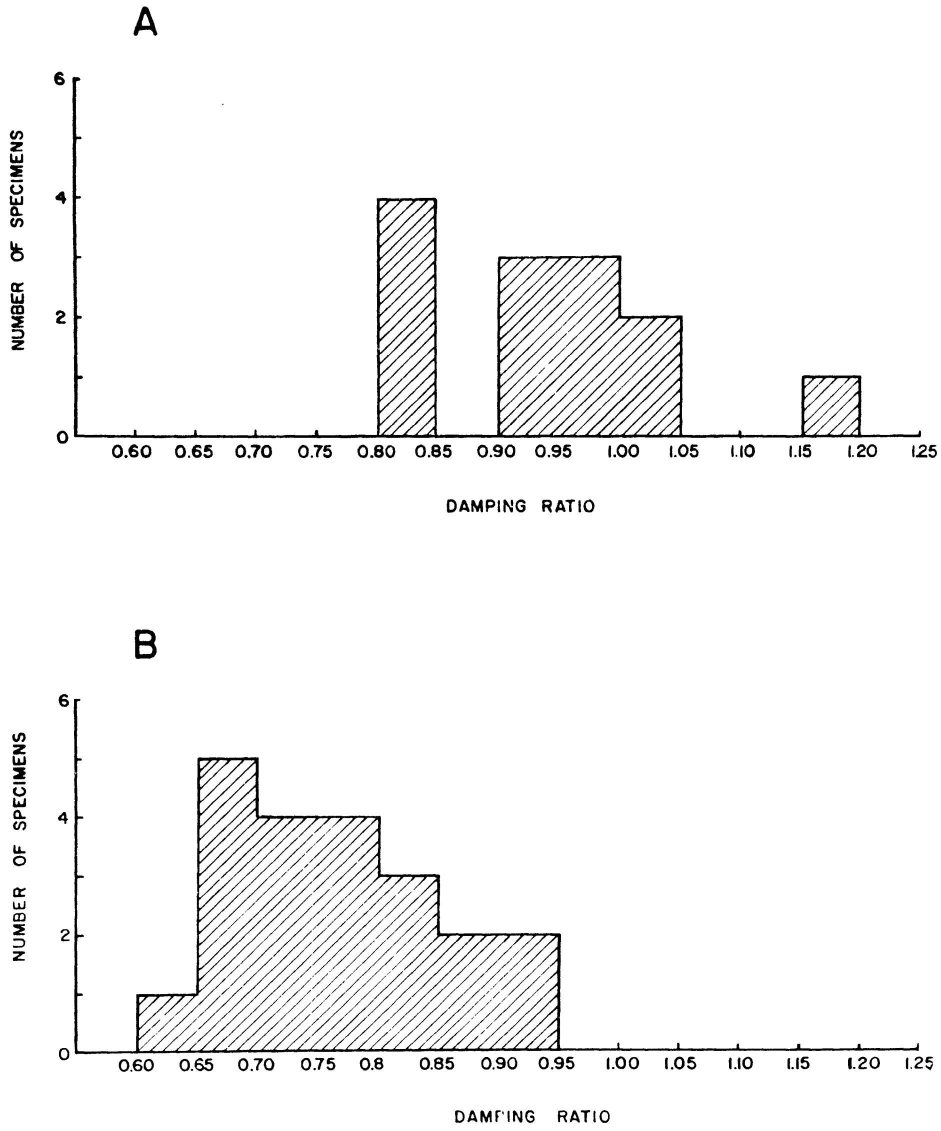


FIGURE 5.31 HISTOGRAMS SHOWING THE DISTRIBUTIONS OF THE RATIO OF DAMPING AFTER 1 HOUR IN THE TESTING MACHINE TO INITIAL DAMPING FOR SPECIMENS USED IN THE TEMPERATURE STUDIES (A) FRESH SPECIMENS, (B) SPECIMENS PRESERVED BY FREEZING.

APPENDIX I. TRANSDUCER CALIBRATIONS

I.A FORCE TRANSDUCERS

I.A.1 PROVING RING

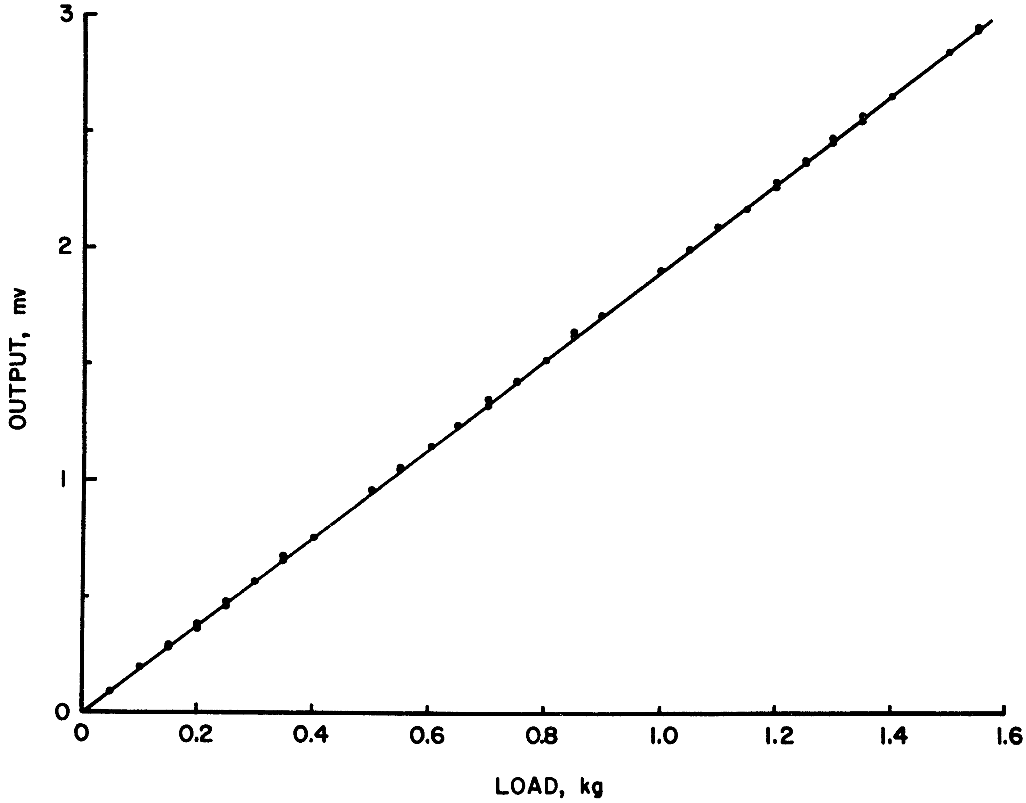


FIGURE I.1 CALIBRATION CURVE FOR ALUMINUM PROVING RING (6 v. excitation). DEAD WEIGHT CALIBRATION 27 NOV. 1968.

I.A.2 STATHAM LOAD CELL

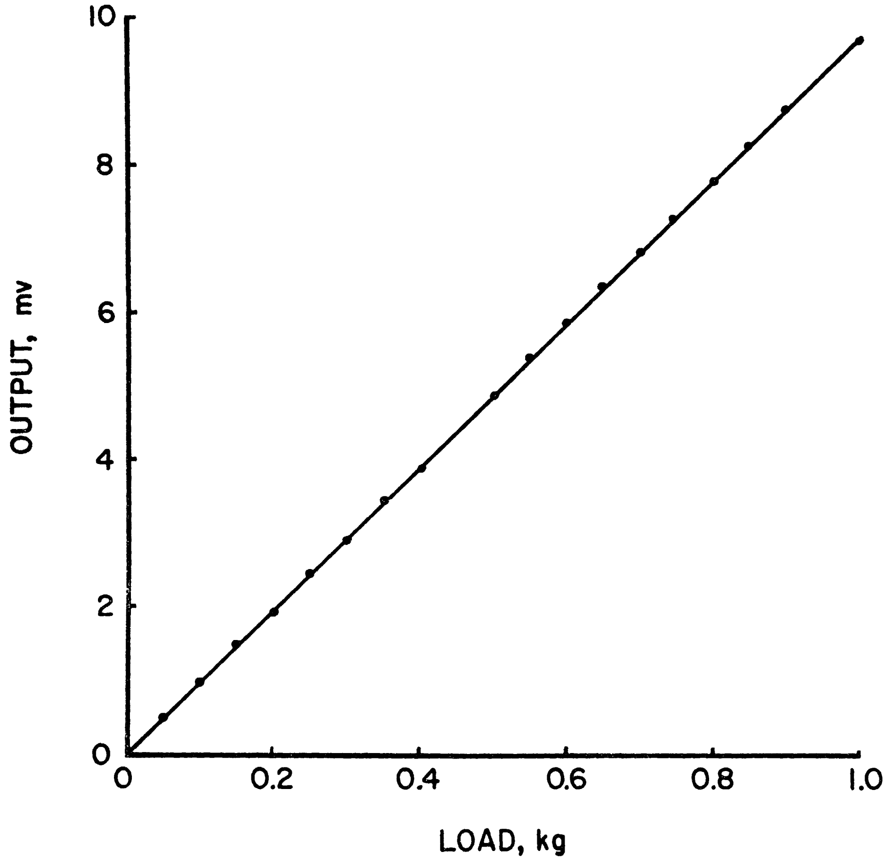


FIGURE I.2 CALIBRATION CURVE FOR STATHAM LOAD CELL (UC3) WITH 10 LB. ADAPTER (UL4-10) MOUNTED IN TESTING MACHINE (Excitation adjusted for 10 mv./Mdyne). DEAD WEIGHT CALIBRATION 14 MAY 1969.

I.B STRAIN SENSOR

Calibration of the strain sensor involved determining both the uniformity of the light beam as seen by the photo transistor and the overall sensitivity (output as a function of interposed slit width). Figure 1.3 presents a sensitivity calibration curve obtained with a slit of adjustable width (Brown and Sharpe 'Tri-cal' vernier calipers) at a fixed position in the light beam. Figures 1.4 through 1.6 present output records obtained when a slit of constant width (.025") was moved through the light beam as a check for uniformity. In each of these three figures the successive scans, from left to right, were obtained after successive slit displacements of 2 mm. toward the light source. Such uniformity tests were performed (and the instrument aligned if necessary) on the following dates during the course of the study:

28-29	Nov.	1968	10	Sept.	1969
24	Dec.	1968	27-8	Sept.	1969
16	May	1969	8	Oct.	1969
27	June	1969	10	Oct.	1969
22	July	1969	23	Oct.	1969
27	July	1969			
9	Aug.	1969			
17	Aug.	1969			

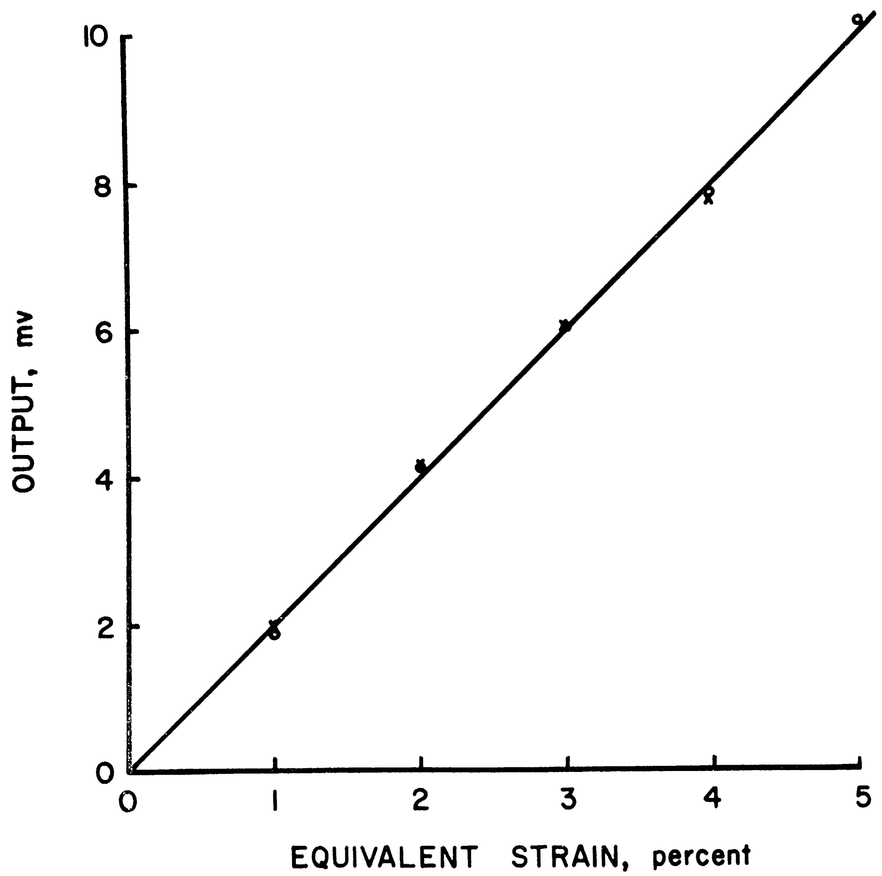


FIGURE 1.3 STRAIN SENSOR SENSITIVITY CALIBRATION (OUTPUT AS A FUNCTION OF INTERPOSED SLIT WIDTH). CALIBRATION 27 JUNE 1969 WITH EQUIVALENT STRAIN OF ZERO TAKEN AT A SLIT WIDTH OF 0.017" AND EQUIVALENT STRAIN AT 0.0043"/PERCENT. OUTPUT WITH UNOBSTRUCTED BEAM ADJUSTED TO 104 mv.

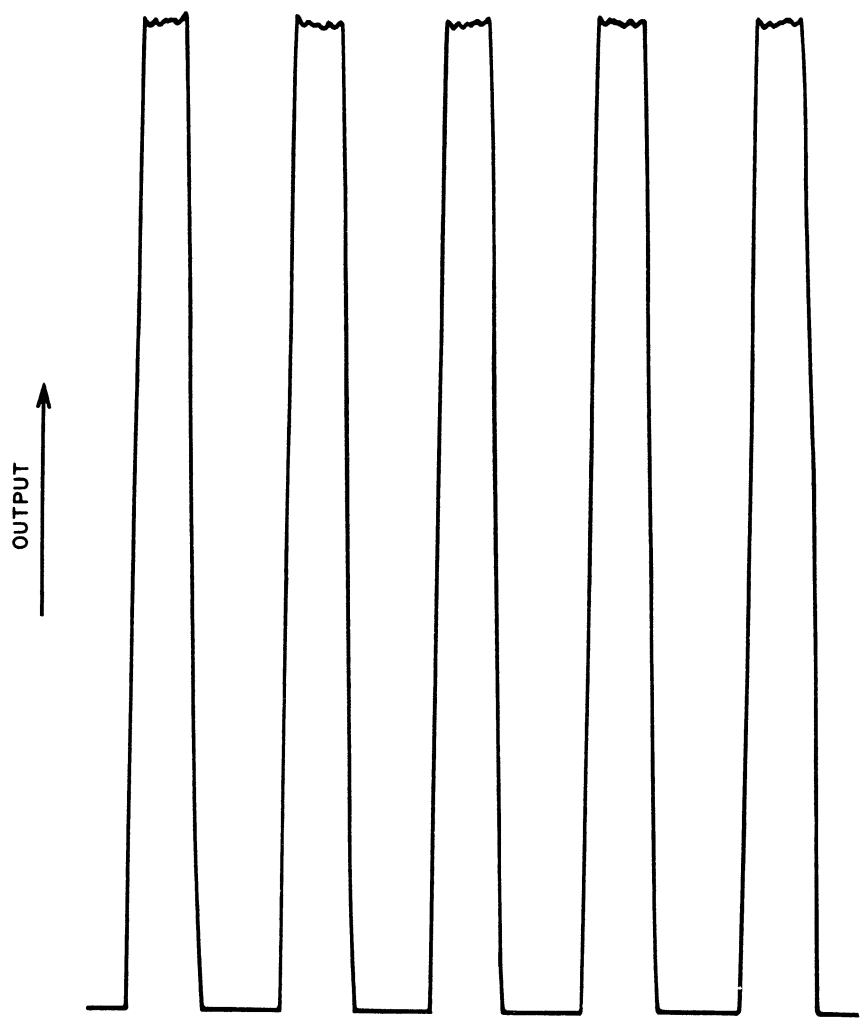


FIGURE I.4 STRAIN SENSOR OUTPUTS RECORDED AS A 0.025'' SLIT WAS PASSED THROUGH THE LIGHT BEAM. TEST 17 AUGUST 1969.

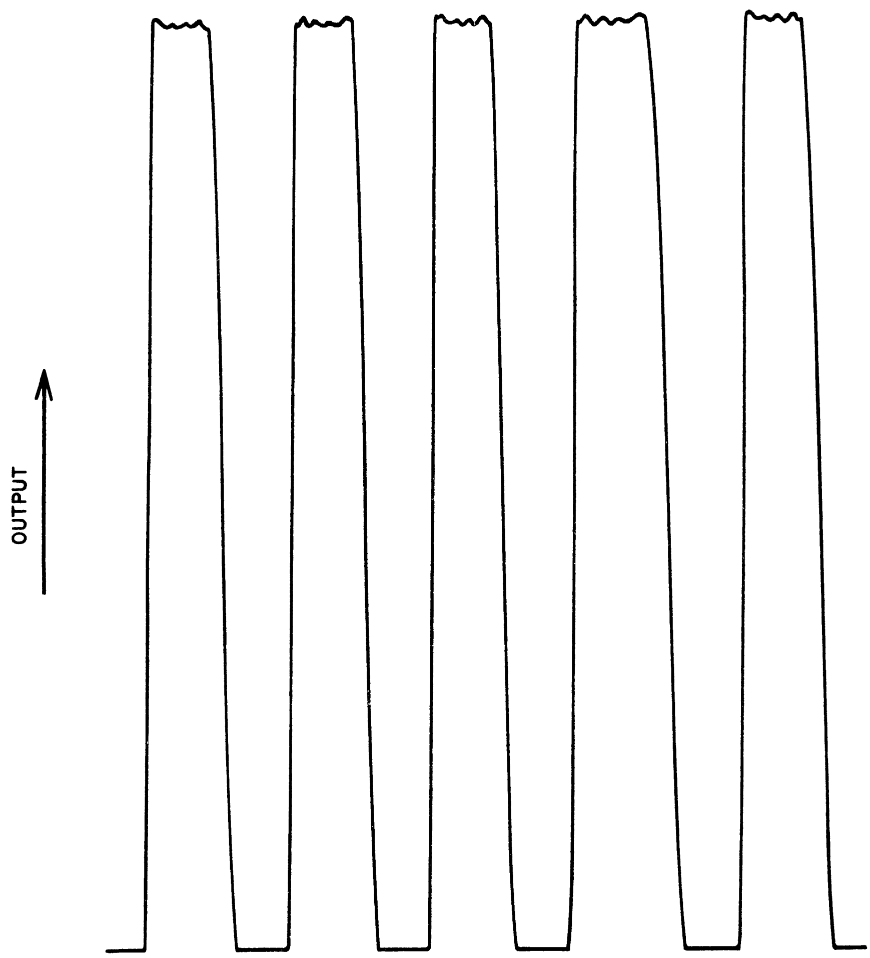


FIGURE I.5 STRAIN SENSOR OUTPUTS RECORDED AS A 0.025'' SLIT WAS PASSED THROUGH THE LIGHT BEAM. TEST 10 SEPTEMBER 1969.

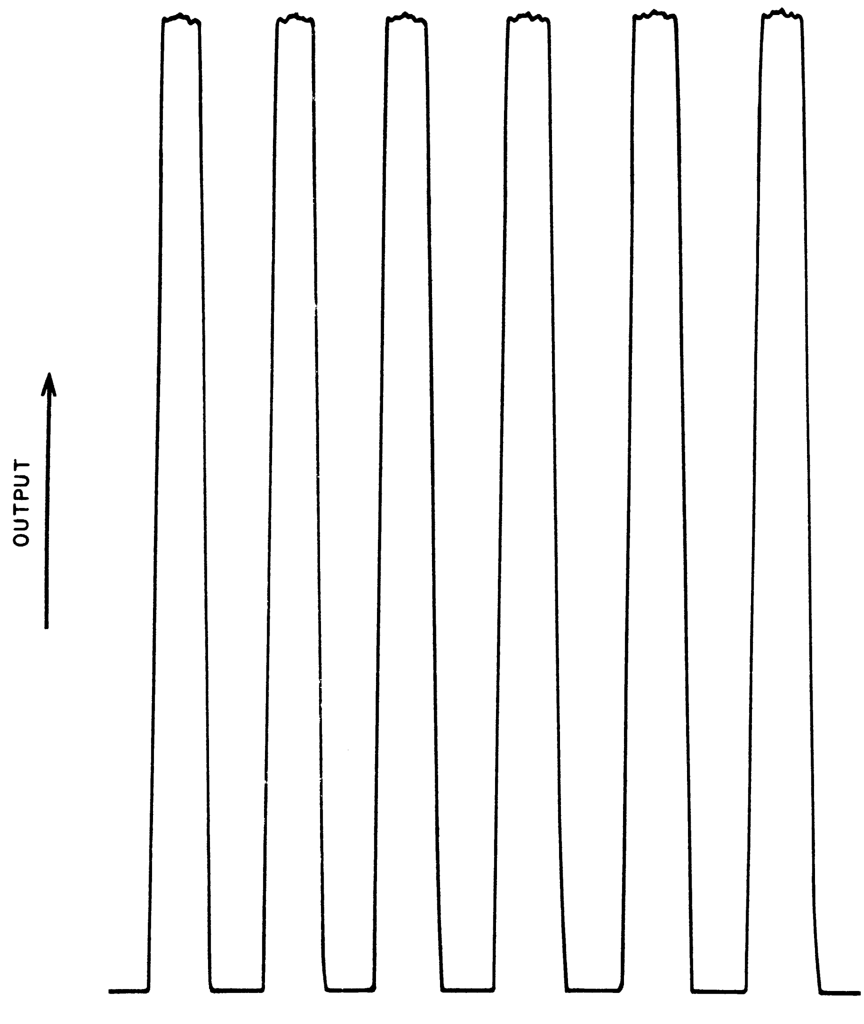


FIGURE 1.6 STRAIN SENSOR OUTPUTS RECORDED AS A 0.025" SLIT WAS PASSED THROUGH THE LIGHT BEAM. TEST 23 OCTOBER 1969.

APPENDIX II. SIMULATION PROGRAMS

Listings, in the FOCAL language, of computer programs used in the modelling studies.

II.A GENERALIZED KELVIN BODY

Simulates a Kelvin-type body with power function elastic element and "generalized Newtonian" dashpot subjected to triangular wave loading.

```
C-FOCAL., 1968

01.10 T !!!!!!"GENERALIZED KELVIN BODY:"!
01.15 T "DE/DT=<GI-(GI-GO)*EXP(<1/X>*<<<1/K>*↑<1/B>>-S>↑2)>"!
01.17 T "                                    *(S-<1/K>*E↑<1/B>)"!!
01.20 T "STRESS: S       STRAIN: E"!!!
01.22 A "GI",GI,"     GO",GO,"     K",K,"     B",B,"     X",X,!!!
01.30 A "MAX STRESS",ST,"  MIN STRESS",SB,"   STRESS INCREMENT",DS,!!
01.35 A "FREQUENCY",DE,!!; S DT=DS/<2*DE*<ST-SB>>; S S=0; S SE=0
01.40 S E=0.01; S EN=0; S DQ=DS; S PE=0
01.45 A "OUTPUT EACH ? STEPS",TO,!"CYCLES?",CY,!!; D 3.8
01.47 F CO=1,1,CY*2*<ST-SB>/<DS*TO>; D 2.0
01.50 A !!!!!!!"-1 NEW PARAMETERS, 0 NEW CYCLES, 1 CONTINUE",DI,!!!!!!!
01.60 I (DI)1.1,1.3,1.45

02.10 F CS=1,1,TO; D 3.0
02.20 T %,"     ",S,"     ",E,!

03.10 S S=S+DO
03.20 S VS=S-<<1/K>*FEXP(<1/B>*FLOG(E))>
03.30 S E=E+<DT*<GI-<GI-GO>*FEXP(-<VS*VS>/X)>*VS>
03.40 S EN=EN+<S-<DQ/2>>*<E-PE>; S PE=E
03.50 I (SB-S)3.7;S DQ=DS
03.60 R
03.70 I (S-ST)3.6; S DQ=-DS
03.75 T %,!!!!"CYCLE ENERGY",EN,!!!!; S EN=0
03.80 T !"         STRESS                  STRAIN"!!
*
```

II.B POWER FUNCTION KELVIN BODY

Simulates a Kelvin-type body with power function elastic and viscous elements subjected to triangular wave loading.

```
C-FOCAL., 1968

01.10 T !!!!"POWER FUNCTION KELVIN BODY"!
01.15 T "DE/DT=(G<S-<1/K>*<E↑(1/B)>>)↑A"!!
01.20 T "STRESS: S      STRAIN: E"!!!
01.22 A "G",G,"    A",A,"    K",K,"    B",B,!!
01.30 A "MAX STRESS",ST,"  MIN STRESS",SB,"   STRESS INCREMENT",DS,!!
01.35 A "FREQUENCY",DE,!!; S DT=DS/<2*DE*<ST-SB>>; S S=0.1; S SE=0
01.40 S E=0.1; S EN=0; S DQ=DS; S PE=0
01.45 A "OUTPUT EACH ? STEPS",TO,!"CYCLES?",CY,!!; D 3.8
01.47 F CO=1,1,CY*2*<ST-SB>/<DS*TO>; D 2.0
01.50 A !!!!!!!"-1 NEW PARAMETERS, 0 NEW CYCLES, 1 CONTINUE",DI,!!!!!!!
01.60 I (DI)1.1,1.3,1.45

02.10 F CS=1,1,TO; D 3.0
02.20 T %,"     ",S,"     ",E,!

03.10 S S=S+DQ
03.30 S E=E+DT*<(G*<S-<1/K>*FEXP(<1/B>*FLOG(E))>)↑A>
03.40 S EN=EN+<S-<DQ/2>>*<E-PE>; S PE=E
03.50 I (SB-S)3.7;S DQ=DS
03.60 R
03.70 I (S-ST)3.6; S DQ=-DS
03.75 T %,!!"CYCLE ENERGY",EN,!!; S EN=0
03.80 T !"         STRESS              STRAIN"!!
*
```

II.C DAMPING SPECTRUM

Computes damping at frequency intervals over a specified frequency range for a (linear) Kelvin body with triangular wave force input (5-term approximation).

```
C-FOCAL., 1968

01.20 T !!!"KELVIN BODY DAMPING SPECTRUM"!!
01.30 A "MAX. FREQ.",T,"   MIN. FREQ.",B,"   INCR. RATIO",R,!!
01.40 A "C",C,"   K",K,!
01.45 T !!"          FREQUENCY                D/A↑2"!!
01.50 S Z=<16*B>/<<3.14159↑3>*C>; S Q=0
01.60 F N=1,1,5; D 2.0
01.70 S DA=Q*Z
01.80 T %,"          ",B,"     ",DA,!
01.90 S B=B*R
01.95 I (B-T)1.5; Q

02.10 S Q=Q+1/<<<2*N-1>↑2>*<<<<2*N-1>*B>↑2>+<K/C>↑2>>
*
```

II.D KINETICS MODEL - EQUILIBRIUM CASE

Computes the equilibrium load-strain curve for the first order kinetics model.

```
C-FOCAL., 1968

01.10 T !!"-"!!!!"KINETICS MODEL; LINEAR, EQUILIBRIUM CASE:"!!
01.20 A "PARAMETERS:  KS/KL",K,"   A",A,"   B",B,"   GS",GS,"   GL",GL,!
01.30 A "MAX. REDUCED TENSION:",T,!!
01.40 D 5.0
01.50 S EM=E; S DT=T/50
01.60 F C=1,1,50; D 2.0
01.65 F C=1,1,69; T "-"
01.70 T "I"!
01.75 T "0"
01.77 F C=1,1,56; T " "
01.80 T %,EM,!
01.90 T "                                    STRAIN"!!!
01.95 G 1.1

02.10 T "."
02.20 F D=1,1,70*E/EM; T " "
02.30 T "*"!
02.40 S T=T-DT
02.50 D 5.0

05.10 S J=FEXP(T)
05.20 S E=<<B-A>*K*<J-1>+T*<K+1>*<A*GS+B*GL*K*J>>/<<K*J+1>*<A+B*K>>
*
```

II.E PARTIAL KINETICS MODEL

A partial simulation of the first order kinetics model, yielding $\oint \frac{N_s}{N_t}\, dP$ for triangular wave loading.

```
C-FOCAL., 1968

01.10 T !!!!!"PARTIAL KINETICS MODEL"!!
01.22 A "   KS",KS,"   KL",KL,"   CS",CS,"   CL",CL,!!
01.30 A " STRESS:  MAX.",ST,"  MIN.",SB,"   INCR.",DS,!!
01.35 A "FREQUENCY",DE,!!; S DT=DS/<2*DE*<ST-SB>>; S S=0; S SE=0
01.40 S N=1/<1+<KS/KL>>; S E=0; S EN=0; S DQ=DS; S PE=0
01.45 A "OUTPUT EACH ? STEPS",TO,!"CYCLES?",CY,!!; D 3.8
01.47 F CO=1,1,CY*2*<ST-SB>/<DS*TO>; D 2.0
01.50 A !!!"-1  PARAMETERS, 0  CYCLES, 1 CONTINUE",DI,!!!
01.60 I (DI)1.1,1.3,1.45

02.10 F HS=1,1,TO; D 3.0
02.20 T %,"    ",S,"    ",N,!

03.10 S S=S+DQ
03.20 S N=N+DT*<<<1-N>*KL*FEXP(-CL*S)>-<N*KS*FEXP(CS*S)>>
03.25 I (N-1)3.3; Q
03.30 S E=N
03.40 S EN=EN+<S-<DQ/2>>*<E-PE>; S PE=E
03.50 I (SB-S)3.7;S DQ=DS
03.60 R
03.70 I (S-ST)3.6; S DQ=-DS
03.75 T %,!!"CYCLE INTEGRAL",EN,!!; S EN=0
03.80 T !"       STRESS               NS/NT"!!
```

APPENDIX III. TEST SEQUENCES

III.A. EXPERIMENTS ASSOCIATED WITH MODELLING STUDIES (SECTION 5.1)

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
18 Dec. 1968	28C8	545	21.4	1.00	.10	2.34	8	Stored 24 hrs. in cold Ringer's solution after thawing.
				.97	.09	1.11	7	
				1.00	.10	.29	9	
				1.00	.10	1.74	5	
				1.00	.10	2.43	4	
				1.00	.10	.74	7	
25 Dec. 1968	29C4	555	20.9	1.00	.10	1.9	9	Stored 5 days in cold Ringer's solution after thawing.
					.60	held	1 hr.	
				1.00	.10	1.9	9	
					.60	held	1 hr.	
				1.00	.10	1.9	8	
					.60	held	1 hr.	
				1.00	.10	1.9	9	
30 Dec. 1968	29C8	673	21.8	1.00	.10	1.9	320	
31 Dec. 1968	31C7	807	21.7	1.00	.10	2.0	6.5	
5 Jan. 1969	30C4	616	20.7	1.00	.10	1.94	10	Stored 2 days in cold Ringer's solution after thawing.
				1.00		held	1 hr.	
				1.00	.10	1.9	4	
				1.00		held	1 hr. 35 min. rupture	
6 Jan. 1969	30C5	713	21.1	.50	.05	4.0	12	
					.05	held	1 hr.	
				.50	.05	4.0	4	
					.05	held	1 hr.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
6 Jan. 1969	30C5 (cont'd)		21.1	.50	.05	4.0	5	
					.05	held	1 hr.	
			21.15	.50	.05	4.0	4	
				.50	.05	3.8	4	
				.50	.05	2.54	3	
				.50	.05	1.6	4	
				.50	.05	.89	4	
				.50	.05	.34	4	
				.50	.05	.092	3	
				.50	.05	.03	2	
				.50	.05	.001	1.5	
				.50	.05	5.0	2	
				.50	.05	5.0	4	
				.015				
				.020				
				.040				
				.080				
				.125				Held 7.5 min. at each load
				.175				
				.220				
				.265				
				.450				
				.550				
				.430				
				.340				
				.245				
				.050				
				.05	0	15	6	
				.10	.075	3	4	
9 Jan. 1969	30C6	614	21.25	.50	.05	.77	6	
					.05	held	1 hr.	
				.50	.05	.77	4	
				.10	.05	5.6	4.5	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
9 Jan. 1969	30C6 (cont'd)			.15	.10	5.6	4.5	
				.20	.15	5.6	4.5	
				.25	.20	5.6	4.5	
				.30	.25	5.6	4.5	
				.35	.30	5.6	4.5	
				.40	.35	5.6	4.5	
				.45	.40	5.6	4.5	
				.50	.45	5.6	8	
				.45	.40	5.6	3.5	
				.40	.35	5.6	3.5	
				.35	.30	5.6	4.5	
				.30	.25	5.6	3.5	
				.25	.20	5.6	3.5	
				.20	.15	5.6	3.5	
				.15	.10	5.6	3.5	
				.10	.05	5.6	3.5	
					.05	held	2 hr. 20min.	
			44.4	.50	.05	.77	15	Much noise on strain tracing due to convection currents.
16 Jan. 1969	31C5	665	20.8	.11				
				.21				
				.32			held	1/2 hr. at each load
				.61				
				.81				
				1.02				
				.20				

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
16 Jan. 1969	31C5 (cont'd)			.815				
				.60				
				1.00	.20	.0137	1.5	
				1.00	.20	.156	2	
				1.00	.20	.0444	2	
				1.00	.20	.83	3	
				1.00	.20	5.2	5	
9 Feb. 1969	32C4	531	22.9	.50	.05	2.56	8	
					.05	held	1 hr.	
				.50	.05	2.6	6	
				.50	.05	1.0	5	
				.44	.08	0.5	3	
				.12	.103	.05	2	
				.30	.16	.50	2	
				.465	.311	.50	3	
				.33	.12	.50	3	
				.445	.235	.50	2	
				.377	.085	.50	3	
			22.6	.45	.041	.50	3	
				.50	.09	.50	3	
				.50	.04	.50	2	
				.37	.054	.50	3	
			22.6	.50	.10	.87	4	
				.50	.10	.00066	2	
				.50	.10	4.35	5	
			22.0	.50	.10	1.33	4	
				.50	.10	.15	3	
				.50	.10	4.9	6	
				.50	.10	.00029	1.5	
13 Feb. 1969	32C7	524	22.1	.50	.10	1.4	5	
					.10	held	1 hr.	
				.50	.10	1.4	4	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
13 Feb. 1969	32C7 (cont'd)			.50	.10	.181	3	
				.50	.10	3.0	5	
				.50	.10	.015	3	
				-	-	-	-	procedure test for stress relaxation.
17 Feb. 1969	31C7	580	23.2	.50	.10	1.91	8	some noise and instrument drift.
					.10	held	1 hr.	
				.50	.10	1.91	5	
				.66	.28	2.0	3	
				.69	.31	2.0	4	
				.87	.13	2.0	9	
				.81	.19	2.0	6.5	
				.67	.33	2.0	4	
				.76	.24	2.0	7	
				.61	.38	2.0	3	
				.74	.26	2.0	4....	specimen slipped from clamp.
19 Feb. 1969	33C1	471	26.4	.50	.10	2.4	6	
					.10	held	1 hr.	
				.50	.10	2.4	4	
				.57	.11	2.13	1.5	
				.63	.15	2.0	4	
				.48	.333	2.0	2	
				.525	.275	2.0	4	
				.59	.21	2.0	3	
				.66	.15	2.0	3.5	
				.585	.215	2.0	3.5	
				.56	.24	2.0	3	
				.62	.18	2.0	4	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
19 Feb. 1969	33C1 (cont'd)			.69	.11	2.0	5	
				.68	.12	.25	4	
				.49	.30	.25	2	
				.625	.17	.25	2	
				.55	.15	.25	2	
				.63	.17	.25	2.5	
				.76	.04	.25	3	
				.70	.10	.25	2.5	
				-	-	-	-	stress relaxation; 2 hr. at 2.1% strain
				.70	.10	.44	3	
				.60	.10	-	-	square wave 2 min. at .6, 4 min. at .1 Mdyne
7 March 1969	33C7	460	26.3	.50	.10	1.54	9	
					.10	held	1 hr.	
				.40	.10	4.4	4	
				.50	.20	4.3	4	
				.70	.40	4.35	5	
				.60	.30	4.44	6	
				.90	.60	4.4	4	
				1.00	.70	4.35	5	
				1.00	.70	4.4	4	
				.60	.30	4.4	6	
				.60	.30	4.4	3.5	
				.50	.20	4.27	4	
				.80	.50	4.1	5	
				.56	.44	2.0	2	
				.656	.35	2.0	3	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
7 March 1969	33C7 (cont'd)			.60	.405	2.0	2.5	
				.73	.27	2.0	3.5	
				.89	.11	2.0	4	
				.50	.10	5.0	5.5	
				.50	.10	.072	5	
				.50	.10	4.0	5	
				.50	.10	.33	2	
				.50	.10	1.5	4	
				.50	.10	.13	2	
18 March 1969	33C6	493	26.3	.50	.10	3.5	6	
					.10	held	1 hr.	
				.50	.10	3.5	5	
				.26	.10	2.0	5	
				.39	.10	2.0	5	
				.19	.10	2.0	4	
				.525	.10	2.0	5	
				.315	.10	2.0	4	
				.78	.10	1.88	6	
				.64	.10	2.0	1.....	specimen slipped from clamp
21 March 1969	33C8	468	26.2	.50	.10	2.9	8	
					.10	held	1 hr.	
				.50	.10	2.9	4	
				.56	.30	2.0	11	
				.30	.04	2.0	4	
				.363	.30	2.0	3	
				.30	.233	2.0	3	
				.30	.15	2.0	4	
				.44	.30	2.0	3	
				.30	.10	2.0	4	
				.50	.30	2.0	4	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
21 March 1969	33C8 (cont'd)			.68	.30	2.0	6	
				.85	.30	2.0	1	
				.87	.30	2.0	1	
				.87	.30	2.0	4	
22 March 1969	33C8			.50	.10	2.77	9	Specimen retested after 24 hours in cold Ringer's solution.
				.50	.10	2.8	4	
				.74	.30	2.54	1	
				.62	.30	2.05	4	
				.37	.18	2.0	<1	
				.408	.30	2.0	4	
				.30	.19	1.9	3	
				.55	.30	2.0	5	
				.30	.05	1.94	4	
				.30	.17	2.0	4	
				.43	.30	2.0	4	
				.30	.09	2.0	4	
				.52	.30	2.0	8	
				.30	.12	1.97	4	
				.48	.30	2.0	7	
				.77	.30	2.0	16	
24 March 1969	33C2	364	26.6	.50	.10	1.98	13	
					.10	held	1 hr.	
				.50	.10	2.0	4	
				.57	.30	2.0	4	
				.30	.035	2.0	5	
				.385	.30	2.0	3	
				.30	.22	2.0	4	
				.48	.30	2.0	4	
				.30	.12	2.0	9	
				.42	.30	2.0	4	
				.30	.18	2.0	7	
				.46	.30	2.0	4	
				.30	.14	2.0	7	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
24 March 1969	33C2 (cont'd)			.75	.30	2.0	8	
				.70	.25	2.0	4	
				.70	.37	2.0	4	
				1.00	.70	2.0	7	
				.80	.70	2.0	3.5	
				.70	.60	2.0	3	
				.90	.70	2.0	5	
				.70	.50	2.0	6	
				.95	.70	2.0	5	
				.70	.45	2.0	10	
				.86	.70	2.0	6	
				.70	.53	2.0	5	
17 April 1969	33C3	382	48.9	.50	.05	2.0	9	Some convection current noise in strain tracing.
					.10	held	1 hr.	
				.40	.10	1.0	3	
				.24	.10	1.0	3	
				.385	.10	.50	4	
				.53	.10	.33	4	
				.26	.10	.152	2	
				.29	.10	.125	2	
23 April 1969	33C4	431	32.2	.50	.05	2.01	8	
					.10	held	1 hr.	
				.50	.05	2.01	3	
				.45	.05	3.33	4	
				.35	.15	6.61	4.5	
				.40	.10	4.47	4	
				.3	.2	12.0	8	Some instrument draft.
			32.1	.45	.05	.0117	2	
			32.0	.43	.07	.54	4	
				.40	.10	.64	4	
				.35	.15	.97	4	
				.30	.20	1.9	5	
				.40	.10	.0337	3	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
23 April 1969	33C4 (cont'd)		32.0	.35	.15	.049	4	
				.40	.10	2.26	4	
				.35	.15	3.34	4	
				.30	.20	6.6	8	
				.30	.20	1.0	3	
				.35	.15	.50	3	
				.40	.10	.33	2	
			32.0	.35	.15	.313	5	
				.30	.20	.62	3	
				.35	.15	1.81	3	
				.30	.20	3.4	4	
				.35	.15	.264	2	
				.30	.20	.53	3	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

III.B EXPERIMENTS ON PLASTIC-LIKE ELONGATION (SECTION 5.2).

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT , cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
11 Aug. 1969	35C5	710	23.9	.5	.05	1.70	10	
				.05		held	1 hr.	
				.5	.05	1.72	4	
				.44	.05	.167	6	
				.63	.05	.172	7	
				.95	.05	.169	7	
				.43	.05	.167	5	
				0			2 hrs.	heating
			43.0	.43	.05	.167	6	
				.68	.05	.161	3	
				.99	.05	.166	9.5	
				.905		held	6 min.	
				.80		held	5.6min	
				.899		held	6.7min	
13 Aug. 1969	35C6	510	24.2	.5	.05	2.02	11	
				.05		held	1 hr.	
				.5	.05	2.02	10	
				.46	.05	.169	3	
				.65	.05	.167	3	
				.88	.05	.167	3	
				1.01	.05	.168	5	
				0			2 hrs.	heating
			47.2	.40	.05	.167	7	
				.68	.05	.166	4	
				.40	.05	.167	5	
				.60	.05	.167	3	
				.97	.05	.167	3.5	
				.80		held	3 min.	
				.90		held	4 min.	
				1.00		held	3 min.	
				.90		held	5 min.	
				1.10		held	4 min.	
				1.00		held	4 min.	
				1.20		held	.5min	.failure

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
19 Aug. 1969	36C2	749	23.2	.5	.05	2.16	7	
				.05		held	1 hr.	
				.5	.05	2.16	6	
				.32	.05	.167	5	
				.6	.05	.167	3	
				.905	.05	.167	4	
				0			2 hrs.	heating
			37.6	.37	.05	.167	10	
				.66	.05	.169	7	
				.99	.05	.168	2.5	
				.70		held	4.7min.	
				.90		held	3 min.	
				.75		held	2 min.	failure
22 Aug. 1969	36C3	481	23.8	.5	.05	2.12	8	
				.05		held	1 hr.	
				.5	.05	1.68	5	
				.38	.05	.166	6	
				.618	.05	.169	4	
				.94	.05	.166	4	
				.37	.05	.167	2	
				0			2 hrs.	heating
			50.6	.36	.05	.166	4	
				.63	.05	.166	2.5	
				.495		held	5.6min.	
				.60		held	3.5min.	
				.55		held	5 min.	
				.65		held	1.2min.	failure
23 Aug. 1969	36C4	593	23.3	.5	.05	2.14	7	
				.05		held	1 hr.	
				.5	.05	2.14	4	
				.47	.05	.167	4	
				.705	.05	.167	4	
				1.01	.05	.167	4	
				0			2 hrs.	heating

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
23 Aug. 1969	36C4 (cont'd)		34.0	.47	.05	.167	4	
				.755	.05	.167	4	
				.95	.05	.167	3.5	
				.80		held	3 min.	
				1.00		held	4 min.	
				.90		held	6.7min.	
				1.20		held	4 min.	
				1.095		held	6 min.	
				1.40		held	2 min..	.failure
25 Aug. 1969	36C5	614	24.1	.5	.05	2.15	6	
				.05		held	1 hr.	
				.5	.05	2.18	4	
				.535	.05	.167	7	
				.75	.05	.167	4	
				.98	.05	.167	4	
				0			2 hrs.	heating
			44.6	.43	.05	.167	3	
				.66	.05	.167	3.5	
				.60		held	2.5min.	
				.70		held	4 min.	
				.80		held	4.2min.	
				.75		held	4.7min.	
				.90		held	4.5min.	
				.85		held	3.7min.	
				1.00		held	2 min..	.failure
26 Aug. 1969	36C6	568	23.9	.5	.05	2.15	5	
				.05		held	1 hr.	
				.5	.05	2.10	4	
				0			2 hrs.	heating
			46.6	.2		held	6 min.	
				.4		held	6 min.	
				.5		held	6.8min.	
				.6		held	5.7min.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
26 Aug.	36C6		46.6	.7		held	4.7min.	
1969	(cont'd)			.65		held	5.8min.	
				.75		held	4.6min.	
				.8		held	3 min.	.failure
2 Sept.	37C3	625	23.9	.5	.05	2.20	7	
1969				.05		held	1 hr.	
				.5	.05	2.22	5	
				0			2 hrs.	heating
			32.0	.5	.05	2.22	4	
				.4		held	7.9min.	
				.6		held	6.6min.	
				.8		held	7.7min.	
				1.1		held	7 min.	
				1.2		held	9.4min.	
				1.4		held	2 min.	
				1.3		held	3 min.	.failure
3 Sept.	34C7	428	31.2	.5	.05	2.26	7	
1969				.05		held	2.5hr.	
				.5	.05	2.12	8	
				.54	.05	.08	5	
				.93	.05	.08	3	
				1.2		held	11 min.	
				1.4		held	10 min.	
				1.6		held	7.5min.	
				1.8		held	8 min.	
				2.0		held	1.5min.	
				1.83		held	1.2min.	
				1.7		held	6.2min.	
				1.6		held	4.8min.	
				1.5		held	3.5min.	
				1.4		held	7.5min.	
				1.5		held	4.7min.	.failure

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
4 Sept. 1969	36C7	698	23.4	.5	.05	2.05	6	
				.05		held	1 hr.	
				.5	.05	2.05	4	
				.75	.05	.222	3.5	
				1.1		held	5 min.	
				1.2		held	4 min.	
				1.1		held	3.5min	
				1.0		held	2 min.	
				1.1		held	3.9min	
				1.3		held	5.6min	
				1.4		held	4.4min	
				1.3		held	6.8min	
				1.5		held	3.3min	
				1.4		held	3.6min	
				1.3		held	3.9min	
				1.2		held	.4min	.failure
16 Sept. 1969	36C8	620	23.4	.5	.05	2.12	7	
				.05		held	1 hr.	
				.5	.05	2.13	8	
				0			2 hrs.	heating
			28.9	.5	.05	.515	4.5	
				1.0		held	5 min.	
				1.2		held	4.4min	
				1.1		held	6.3min	
				1.3		held	6.6min	
				1.4		held	4 min.	
				1.3		held	7.6min	
				1.5		held	3.6min	
				1.4		held	6.1min	
				1.3		held	4.8min	
				1.46		held	3 min.	
				1.36		held	6.4min	
				1.56		held	7 min.	
				1.44		held	7 min.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
16 Sept. 1969	36C8 (cont'd)		28.9	1.60		held	6.7min	
				1.66		held	3.9min	
				1.54		held	5.8min	
				1.7		held	.8min	.failure
19 Sept. 1969	40C2	449	23.4	.5	.05	2.03	8	
				.05		held	1 hr.	
				.5	.05	2.02	6	
				0			2 hrs.	heating
			51.8	.5	.05	.51	6.5	
				.65		held	1.6min	
				.80		held	5.5min	
				1.10		held	4.1min	
				1.20		held	5.6min	
				1.30		held	4 min.	
				1.40		held	3.9min	
				1.30		held	4 min.	
				1.20		held	4 min.	
				1.30		held	2.9min	.failure
21 Sept. 1969	37C6	445	24.0	.5	.05	2.06	9	
				.05+		held	1 hr.	
				.5	.05	2.04	5	
				0			2 hrs.	heating
			55.7	.5	.05	.436	8.5	
				.6		held	3.9min	
				.8		held	5.5min	
				1.0		held	6.2min	
				1.2		held	9.7min	
				1.4		held	6.8min	
				1.6	--	--	--	.failure
23 Sept. 1969	41C1	505	24.6	.5	.05	2.26	7	
				.05		held	1 hr.	
				.5	.05	2.22	6	
				0			2 hrs.	heating

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
23 Sept. 1969	41C1 (cont'd)		49.4	.45	.05	.526	5.5	
				.65		held	4.4min.	
				.75		held	4.7min.	
				.85		held	4.8min.	
				.95		held	4.7min.	
				1.10		held	6.1min.	
				1.20		held	4.8min.	
				1.10		held	5 min.	
				1.00		held	5.3min.	
				.90		held	7.2min.	
				1.04		held	2.2min.	.failure
30 Sept. 1969	43C1	647	23.0	.5	.05	2.03	9	
				.05		held	1 hr.	
				.5	.05	2.04	6	
				0			2 hrs.	heating
			25.0	.5	.05	.86	6.5	
				.80		held	7.8min.	
				.90		held	6 min.	
				1.00		held	4.7min.	
				1.10		held	5.8min.	
				1.20		held	6 min.	
				1.16		held	6.1min.	
				1.04		held	5.2min.	
				1.14		held	5.9min.	
				1.26		held	4.6min.	
				1.20		held	4.7min.	
				1.10		held	6.7min.	
				.995		held	6.8min.	
				1.14		held	6.9min.	
				1.32		held	6 min.	
				1.24		held	6.5min.	
				1.10		held	26min.	
				1.00		held	9.2min.	
				1.20		held	6 min.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
30 Sept. 1969	43C1 (cont'd)		25.0	1.30		held	5.1min.	
				1.06		held	6.5min.	
				1.16		held	5 min.	
				1.30		held	2.2min.	
				1.00		held	16min.	
				1.24		held	4.2min.	
				1.30		held	2.7min.	
				1.025		held	37min.	
1 Oct. 1969	44C1	708	23.8	.5	.05	2.10	8	
				.05		held	1 hr.	
				.5	.05	2.10	5	
				0			2 hr.	heating
			40.3	.5	.05	.875	20	
				.85		held	8 min.	
				1.0		held	7.9min	
				1.1		held	5 min.	
				1.04		held	6.3min	
				.96		held	5.5min.	failure
5 Oct. 1969	43C2	630	23.0	.5	.05	2.19	11	
				.05		held	1 hr.	
				.5	.05	2.20	7	
				0			2 hrs.	
				.5	.05	1.23	9.5	
				.9		held	8.2min.	
				1.2		held	9.4min.	
				1.3		held	7 min.	
				1.5		held	6.4min.	
				1.6		held	6.3min.	
				1.5		held	6.1min.	
				1.4		held	7.2min.	
				1.3		held	5 min.	
				1.4		held	6.2min.	
				1.3		held	6.4min.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
5 Oct. 1969	43C2 (cont'd)		23.0	1.5		held	6 min.	
				1.7		held	3.5min.	
				1.6		held	5 min.	
				1.3		held	6 min.	
				1.5		held	5.3min.	
				1.4		held	10min.	
				1.6		held	6.6min.	
				1.5		held	5.7min.	
				1.7		held	7 min.	
				1.6		held	7.1min.	.failure
9 Oct. 1969	48C1	411	24.4	.5	.05	2.10	9	
				.04		held	1 hr.	
				.5	.05	2.10	6	
				0			2 hrs.	heating
			42.5	.5	.05	1.03	14	
				.9		held	7.7min.	
				1.1		held	4 min.	
				1.3		held	8.2min.	
				1.5		held	12min.	
				1.7		held	9 min.	
				1.9		held	9 min.	
				0			2 hrs.	heating
			54.4	.5	.05	1.05	11.5	
				.7		held	4.5min.	
				.85		held	4 min.	
				.8		held	8.5min.	
				.9		held	6.5min.	.failure
13 Oct. 1969	49C1	615	23.3	.5	.05	2.11	8	
				.05		held	1 hr.	
				.5	.05	2.11	6	
				0			2 hrs.	heating
			46.0	.5	.05	.28	19	
				.75		held	3.3min.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
13 Oct. 1969	49C1 (cont'd)		46.0	.9		held	2.5min.	
				.85		held	3.3min.	
				.8		held	.5min.	
				.5	.05	1.09	10	
				.6		held	5 min.	
				.55		held	2.7min.	
				.5		held	5.2min.	
				.45		held	6 min.	
				.4		held	6.6min.	
				.35		held	12min.	
				.4		held	7.2min.	.failure
14 Oct. 1969	50C1	859	23.3	.5	.05	1.83	9	
				.04		held	1 hr.	
				.5	.05	1.83	7	
				0			2 hrs.	heating
			32.4	.5	.05	1.02	9.5	
				.8		held	5.9min.	
				.75		held	7.9min.	
				.85		held	7.7min.	
				.95		held	5.4min.	
				.9		held	8.6min.	
				.85		held	4.4min.	
				.8		held	4.6min.	
				.75		held	5.8min.	
				.7		held	5.5min.	
				.65		held	5 min.	
				.55		held	4.5min.	
				.5	.05	1.00	6	
				.2		held	4.2min.	
				.5		held	4.5min.	
				.4		held	3.7min.	
				.35		held	5.5min.	
				.45		held	1.7min.	
				.35		held	1.7min.	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
14 Oct. 1969	50C1 (cont'd)		32.4	.25		held	2 min.	
				.3		held	1.3min.	
				.25		held	7.5min.	
				.1		held	1.8min.	
				.18		held	1 min.	
				.15			2.7min.	
				.195		held	1.3min.	
				.15	.005		35min.	decreasing irregularly
				.02	--	--	--	..failure
6 Dec. 1969	54C7	644	50.0	--	--	--	2 hrs.	
			23.6	.4	.05	2.75	10	
				.05		held	1 hr.	
				.4	.05	2.76	8	
				.5		held	4.7min.	
				.6		held	2.7min.	
				.55		held	4.1min.	
				.65		held	3 min.	.failure
7 Dec. 1969	51C3	499	50.0	--	--	--	2 hrs.	
			room	--	--	--	16 hrs.	
			24.0	.4	.05	2.40	8	
				.05		held	1 hr.	
				.4	.05	2.67	7	
				.55		held	4 min.	
				.65		held	3.9min.	
				.755		held	4.2min.	
				.85		held	5.8min.	
				.95		held	4.3min.	
				1.1		held	4.7min.	
				1.2		held	4.2min.	
				1.1		held	5 min.	
				1.3		held	2 min.	.failure

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
8 Dec. 1969	53C2	639	50.0	--	--	--	2 hrs.	
			room	--	--	--	9.5hr.	
			23.8	.4	.05	2.88	9	
				.05		held	1 hr.	
				.4	.05	2.87	8	
				.7		held	5.9min.	
				.9		held	6.7min.	
				1.1		held	7.1min.	
				1.0		held	3.7min.	failure
8 Dec. 1969	57C1	779	50.0	--	--	--	2 hrs.	
			24.0	.4	.05	2.76	8	
				.05		held	1 hr.	
				.4	.05	2.74	10	
				.6		held	6.2min.	
				.7		held	6 min.	failure
8 Dec. 1969	57C3*	564	50.0	--	--	--	2 hrs.	
			room	--	--	--	3 hrs.	
			24.3	.4	.05	2.31	10	
				.05		held	1 hr.	
				.4	.05	2.38	6	
				.6		held	4 min.	
				.65		held	3.7min.	
				.7		held	5.5min.	failure

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

*FRESH

III.C DYNAMIC STIFFNESS AND DAMPING EXPERIMENTS (SECTION 5.3).

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
2 July 1969	34C4	429	24.4	.5	.05	2.52	8	
				.05		held	1 hr.	
				.5	.05	2.53	6	Possible plotter problems.
				.5	.05	.124	5	
				.5	.05	5.71	6	
				.5	.05	.0214	4	
3 July 1969	34C4	429	24.7	.45	.05	2.40	6	Rerun after storing overnight in cold Ringer's solution.
				.5	.05	2.18	13	
				.05		held	1 hr.	
				.5	.05	2.22	6	
				.45	.05	2.47	5	
				.45	.05	6.11	6	
				.45	.05	.167	3	
				.45	.05	.650	3	
				.45	.05	.0605	3	
				.45	.05	3.86	4	
				.45	.05	.0623	2	
				0			2 hr.	heating
			40.0	.45	.05	2.38	9	
				.45	.05	6.11	7	
			40.0	.45	.05	.19	12	
				.45	.05	.695	4	
				.45	.05	.0745	8	
7 July 1969	34C8	458	25.0	.5	.05	2.20	6	
				.06		held	1 hr.	
				.5	.05	2.23	4	
				.45	.05	2.50	4	
				.45	.05	6.13	5	
				.45	.05	.939	4	
				.45	.05	.368	7	
				.45	.05	.167	3	
				0			2 hr.	heating

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
7 July 1969	34C8 (cont'd)		34.7	.45	.05	2.45	10	
				.45	.05	6.23	5	
				.45	.05	.90	4	
				.45	.05	.417	7	
				.45	.05	.178	5	
				0			2 hr.	heating
			44.0	.45	.05	2.37	16	
				.45	.05	5.95	5	
				.45	.05	.90	4	
				.45	.05	.369	9	
				.45	.05	.150	3	
				0			2 hr.	heating
			53.6	.45	.05	2.52	4	
				.45	.05	2.48	8	vanes
				.45	.05	5.94	7	damaged
				.45	.05	.940	4	in mounting
				.45	.05	.330	3	
				.45	.05	.189	4	
				.5	.10	1.25	4...	failure
30 July 1969	35C1	514	24.4	.5	.05	2.03	6	
				.05		held	1 hr.	
				.5	.05	2.06	8	
				.082	.05	1.0	2	
				.164	.05	1.0	4	
				.334	.05	1.0	4	
				.590	.05	1.0	4	
				.92	.05	1.0	5	
				0			2 hr.	heating
			33.8	.085	.05	1.0	2	
				.165	.05	1.0	4	
				.350	.05	1.0	5	
				.610	.05	1.0	7	
				.917	.05	1.0	7	
				0			2 hr.	heating

SPECIMENS WITH A NUMBER ENDING IN Ɨ WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
30 July 1969	35C1 (cont'd)		46.1	.094	.05	1.0	5	
				.160	.05	1.0	4	
				.338	.05	1.0	4	
				.592	.05	1.0	7	
				.917	.05	1.0	5	
				0			2 hrs.	heating
			51.4	.172	.05	1.0	4	
				.335	.05	1.0	9	
				.615	.05	1.0	6	
				.890	.05	1.0	11	plastic-like elongation noted
2 Aug. 1969	35C3	502	24.1	.5	.05	1.58	6	
				.05		held	1 hr.	
				.5	.05	1.58	4	
				.137	.05	1.0	3	
				.214	.05	1.0	3	
				.316	.05	1.0	6	
				.630	.10	1.0	6	
				.850	.10	1.0	5	
				1.00	.10	1.0	6	
				0			2 hrs.	heating
			39.2	.139	.05	1.0	14	
				.192	.05	1.0	4	
				.35	.05	1.0	8	
				.60	.10	1.0	6	
				.56	.05	1.0	3	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
2 Aug. 1969	35C3 (cont'd)		39.2	.777	.05	1.0	7	
				.84	.10	1.0	5	
				.977	.10	1.0	4	
				.92	.05	1.0	6	
				0			2 hrs.	heating
			51.0	.15	.05	1.0	7	
				.204	.05	1.0	8	irregular
				.31	.05	1.0	13	strain
				.61	.10	1.0	12	record
24 Sept. 1969	37C8	439	22.7	.5	.05	2.49	10	
				.05		held	1 hr.	
				.5	.05	2.49	5	
				.4	.05	.167	3	
				.4	.05	.48	5	
				.4	.05	5.6	6	
				.4	.05	2.31	5	
				.4	.05	1.16	5	
				.4	.05	.0733	4	
				0			4 hrs.	
				.4	.05	.202	4	
				.4	.05	.54	5	
				.4	.05	5.55	5	
				.4	.05	2.08	5	
				.4	.05	1.14	4	
				.4	.05	.0471	1.5	
29 Sept. 1969	38C5	495	24.0	.5	.05	2.28	9	
				.05		held	1 hr.	
				.5	.05	2.29	6	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
29 Sept. 1969	38C5 (cont'd)			.4	.05	.942	4	
				.4	.05	4.76	6	
				.4	.05	2.59	5	
				.4	.05	1.48	8	
				.4	.05	.209	3	
				.4	.05	.416	3	
				.4	.05	.088	4	
				0			2 hr.	heating
			50.3	.4	.05	1.03	10	
				.4	.05	4.71	6	
				.4	.05	2.60	5	
				.4	.05	1.49	6	
				.4	.05	.229	6	
				.4	.05	.416	5	
4 Oct. 1969	39C2	512	23.3	.5	.05	1.96	13	
				.05		held	1 hr.	
				.5	.05	1.96	6	
				.5	.05	.943	4	
				.5	.05	3.61	5	
				.5	.05	2.13	5	
				.5	.05	1.18	5	
				.5	.05	.180	3	
				.5	.05	.313	4	
				.5	.05	.074	3	
				.5	.05	5.10	6	
				0			2 hr.	heating
			30.1	.5	.05	.815	6	
				.5	.05	3.67	6	
				.5	.05	2.10	6	
				.5	.05	1.08	4	
				.5	.05	.178	6	
				.5	.05	.318	5	
			30.1	.5	.05	.074	4	
				.5	.05	5.17	8	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
6 Oct. 1969	38C6	539	23.1	.5	.05	2.24	17	
				.05		held	1 hr.	
				.5	.05	2.24	6	
				.5	.05	.75	7	
				.5	.05	3.61	7	
				.5	.05	2.03	5	
				.5	.05	1.14	5	
				.5	.05	.168	2	
				.5	.05	.313	3	
				.5	.05	.059	4	
				.5	.05	5.30	8	
				0			2 hr.	heating
			48.4	.5	.05	.705	8	
				.5	.05	3.64	10	
				.5	.05	2.01	10	
				.5	.05	1.12	6	
				.5	.05	.181	3	
				.5	.05	.317	3	
				.5	.05	.083	3	
				.5	.05	5.11	4.5.	failure
21 Oct. 1969	41C2	556	22.9	.5	.05	2.20	9	
				.05		held	1 hr.	
				.5	.05	2.23	7	
				.192	.128	2.0	4	
				.212	.104	2.0	3	
				.244	.074	2.0	4	
				.228	.098	.25	3	
				.184	.136	.25	2	
				.272	.212	2.0	3	
				.300	.180	2.0	3	
				.340	.140	2.0	5	
				.388	.092	2.0	4	
				.270	.208	.25	2	
				.328	.152	.25	2	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
21 Oct. 1969	41C2 (cont'd)			.426	.056	.25	3	
				.356	.288	2.0	3	
				.380	.264	2.0	4	
				.414	.226	2.0	4	
				.464	.176	2.0	5	
				.510	.130	2.0	4	
				.348	.292	.25	2	
				.400	.240	.25	4	
				.522	.118	.25	3	
				.432	.368	2.0	3	
				.460	.340	2.0	3	
				.504	.296	2.0	3	
				.550	.250	2.0	5	
				.498	.302	.25	3	
				.574	.226	.25	3	
				.420	.380	.25	2	
				.512	.450	2.0	3	
				.538	.424	2.0	7	
				.578	.384	2.0	5	
				.526	.434	.25	2	
				.572	.388	.25	3	
				0			2 hr.	heating
			36.4	.194	.126	2.0	4	
				.220	.102	2.0	3	
				.268	.050	2.0	6	
				.206	.114	.25	5	
				.278	.048	.25	2	
				.276	.206	2.0	4	
				.300	.180	2.0	4	
				.340	.138	2.0	4	
				.400	.080	2.0	9	
				.272	.202	.25	2	
				.330	.150	.25	3	
				.428	.052	.25	3	
				.354	.288	2.0	5	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
21 Oct. 1969	41C2 (cont'd)		36.4	.384	.258	2.0	4	
				.424	.216	2.0	3	
				.474	.164	2.0	5	
				.408	.230	.25	2	
				.502	.138	.25	2	
				.436	.364	2.0	2	
				.458	.342	2.0	4	
				.510	.290	2.0	3	
				.544	.254	2.0	3	
				.496	.302	.25	2	
				.600	.202	.25	3	
				.512	.448	2.0	4	
				.544	.416	2.0	3	
				.594	.388	2.0	4	
				.518	.440	.25	3	
				.594	.386	.25	2	
27 Oct. 1969	41C3	526	24.1	.50	.05	2.07	11	
				.05		held	1 hr.	
				.5	.05	2.08	12	
				.191	.130	2.0	3	
				.221	.099	2.0	3	
				.261	.058	2.0	4	
				.202	.115	.25	2	
				.250	.071	.25	2	
				.269	.206	2.0	3	
				.299	.180	2.0	4	
				.338	.142	2.0	5	
				.421	.079	2.0	5	
				.282	.197	.25	5	
				.324	.151	.25	2	
				.404	.075	.25	2	
				.352	.286	2.0	2	
				.380	.260	2.0	2	
				.413	.230	2.0	4	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
27 Oct. 1969	41C3 (cont'd)		24.1	.486	.151	2.0	4	
				.370	.270	.25	2	
				.400	.241	.25	2	
				.480	.160	.25	2	
				.428	.366	2.0	4	
				.460	.340	2.0	3	
				.500	.299	2.0	4	
				.555	.244	2.0	3	
				.441	.359	.25	3	
				.481	.316	.25	2	
				.579	.221	.25	3	
				.512	.449	2.0	5	
				.565	.436	2.0	4	
				.581	.379	2.0	6	
				.541	.415	.25	2	
				.580	.380	.25	2	
				0			2 hr.	heating
			49.4	.191	.129	2.0	3	
				.221	.099	2.0	4	
				.257	.061	2.0	6	
				.199	.119	.25	4	
				.242	.076	.25	3	
				.271	.210	2.0	2	
				.299	.181	2.0	4	
				.341	.139	2.0	9	
				.399	.078	2.0	6	
				.291	.189	.25	2	
				.339	.141	.25	2	
				.413	.067	.25	2	
				.352	.287	2.0	4	
				.420	.220	2.0	5	
				.451	.159	2.0	8	
				.367	.272	.25	2	
				.421	.218	.25	3	
				.500	.140	.25	3	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
27 Oct. 1969	41C3 (cont'd)		49.4	.431	.370	2.0	3	
				.469	.335	2.0	4	
				.508	.290	2.0	4	
				.569	.230	2.0	4	
				.451	.349	.25	3	
				.502	.297	.25	2	
				.591	.209	.25	3	
				.514	.445	2.0	3	
				.540	.420	2.0	3	
				.570	.390	2.0	2	
				.640	.321	2.0	3	
				.532	.427	.25	4	
				.595	.365	.25	3	
28 Oct. 1969	52C1	638	23.6	.5	.05	2.03	11	
				.04		held	1 hr.	
				.5	.05	2.10	6	
				.191	.129	2.0	5	
				.219	.100	2.0	5	
				.257	.063	2.0	6	
				.209	.111	.25	2	
				.253	.064	.25	2	
				.272	.208	2.0	4	
				.293	.185	2.0	3	
				.338	.143	2.0	3	
				.400	.080	2.0	3	
				.293	.186	.25	2	
				.335	.142	.25	2	
				.401	.077	.25	4	
				.351	.289	2.0	3	
				.382	.258	2.0	3	
				.420	.217	2.0	4	
				.479	.160	2.0	4	
				.369	.273	.25	4	
				.431	.209	.25	2	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
28 Oct. 1969	52C1 (cont'd)		23.6	.496	.143	.25	2	
				.431	.369	2.0	3	
				.459	.341	2.0	3	
				.494	.301	2.0	3	
				.560	.236	2.0	4	
				.454	.344	.25	2	
				.497	.301	.25	2	
				.579	.220	.25	3	
				.513	.447	2.0	3	
				.542	.417	2.0	3	
				.581	.378	2.0	4	
				.530	.428	.25	2	
				.580	.379	.25	3	
				0			2 hr.	heating
			43.1	.191	.129	2.0	5	
				.218	.100	2.0	3	
				.256	.063	2.0	4	
				.218	.101	.25	3	
				.276	.043	.25	4	
				.270	.211	2.0	5	
				.300	.179	2.0	3	
				.339	.140	2.0	6	
				.399	.082	2.0	4	
				.296	.185	.25	2	
				.349	.131	.25	2	
				.417	.062	.25	3	
				.350	.290	2.0	3	
				.375	.267	2.0	5	
				.424	.215	2.0	6	
				.480	.160	2.0	4	
				.370	.270	.25	8	
				.429	.209	.25	2	
				.490	.150	.25	2	
				.431	.369	2.0	2	
				.457	.342	2.0	3	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
28 Oct. 1969	52C1 (cont'd)		43.1	.499	.300	2.0	4	
				.564	.233	2.0	3	
				.437	.360	.25	4	
				.511	.289	.25	3	
				.584	.216	.25	3	
				.511	.448	2.0	2	
				.538	.420	2.0	6	
				.583	.373	2.0	3	
				.523	.436	.25	2	
				.577	.382	.25	2	
				.665	.297	.25	2	
17 Dec. 1969	59C1	678	24.4	.5	.05	2.49	16	
				.05		held	1 hr.	
				.5	.05	2.60	6	
				.5	.05	.120	2	
				.5	.05	.378	4	
				.5	.05	1.19	3	
				.5	.05	.119	2	
				.5	.05	5.41	5	
				.5	.05	3.60	6	
				.5	.05	.0705	2	
				0			2 hr.	heating
			38.0	.5	.05	.370	8	
				.5	.05	1.12	4	
				.5	.05	.369	12	
18 Dec. 1969	60C1	767	23.5	.5	.05	2.17	11	
				.05		held	1 hr.	
				.5	.05	2.27	8	
				.4	.05	.425	4	
				.4	.05	1.38	5	
				.4	.05	.161	2	
				.4	.05	6.80	8	
				.4	.05	4.68	7	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
18 Dec. 1969	60C1 (cont'd)		23.5	.4	.05	.122	3	
				0			2 hr.	heating
			54.2	.4	.05	.402	4	
				.4	.05	1.35	5	
				.4	.05	.17	1....	failure
19 Dec. 1969	61C1	579	24.8	.5	.05	2.41	14	
				.04		held	1 hr.	
				.5	.05	2.22	9	
				.4	.05	.469	5	
				.4	.05	1.36	5	
				.4	.05	.170	3	
				.4	.05	6.86	15	
				.4	.05	4.46	7	
				.4	.05	.068	3	
				0			2 hr.	heating
			26.7	.4	.05	.417	7	
				.4	.05	1.36	7	
				.4	.05	.153	2	
				.4	.05	6.67	7	
				.4	.05	4.52	6	
				.4	.05	.084	2	
20 Dec. 1969	54C4	707	24.3	.5	.05	2.11	6	
				.05		held	1 hr.	
				.5	.05	2.14	7	
				.5	.05	.357	5	
				.5	.05	1.17	4	
				.5	.05	.127	3	
				.5	.05	5.38	8	
				.5	.05	3.79	6	
				.5	.05	.0476	3	
				0			2 hr.	heating

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
20 Dec. 1969	54C4 (cont'd)		31.7	.5	.05	.524	9	
				.5	.05	1.16	3	
				.5	.05	.103	2	
				.5	.05	5.00	7	
				.5	.05	3.56	6	
				.5	.05	.0784	4	
21 Dec. 1969	52C2	594	24.6	.5	.05	2.14	4	
				.05		held	1 hr.	
				.5	.05	2.14	5	
				.5	.05	.347	6	
				.5	.05	1.14	4	
				.5	.05	.123	2	
				.5	.05	5.34	6	
				.5	.05	3.66	6	
				.5	.05	.093	1.5	
				0			2 hr.	heating
			27.1	.5	.05	.398	4	
				.5	.05	1.115	3	
				.5	.05	.129	2	
				.5	.05	5.29	5	
				.5	.05	3.73	11	
				.5	.05	.0833	2	
21 Dec. 1969	49C2	488	24.6	.5	.05	2.11	10	
				.04		held	1 hr.	
				.5	.05	2.15	5	
				.5	.05	.343	3	
				.5	.05	1.15	3	
				.5	.05	.134	2	
				.5	.05	5.11	5	
				.5	.05	3.72	6	
				.5	.05	.0949	3	
				0			2 hr.	heating

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

DATE OF TEST	SPECIMEN NUMBER #	INITIAL LENGTH / DRY WEIGHT, cm./gm.	TEMPERATURE, °C	MAXIMUM LOAD, Megadynes	MINIMUM LOAD, Megadynes	FREQUENCY, cycles per minute	NUMBER OF CYCLES	REMARKS
21 Dec. 1969	49C2 (cont'd)		41.8	.5	.05	.333	4	
				.5	.05	1.144	4	
				.5	.05	.102	5	
				.5	.05	5.13	5	
				.5	.05	3.53	5	
				.5	.05	.09	1	

SPECIMENS WITH A NUMBER ENDING IN 1 WERE TESTED FRESH; OTHERS PRESERVED BY FREEZING, EXCEPT AS NOTED.

APPENDIX IV. METHOD OF FITTING THE EQUATION $(E_2 - E_1) = K^b(P_2{}^b - P_1{}^b)$ TO DYNAMIC STIFFNESS DATA

If a non-linear elastic element can be described by the equation

1) $$E = c + (KP)^b,$$

where E and P are elongation and load, respectively and where K and c are constants, the behavior under cyclic loading between load limits P_1 and P_2 will be

2) $$(E_2 - E_1) = K^b(P_2{}^b - P^b{}_1).$$

Taking $G = K^b$, this becomes

3) $$(E_2 - E_1) = G(P_2{}^b - P_1{}^b).$$

We will consider the problem of fitting this equation to a collection of data points

$$Q_i = (E_{2_i} - E_{1_i}) \text{ measured for load limit pairs}$$

$$(P_{1_i}, P_{2_i}):$$

For the best fit in the sense of least squares, the sum of squared errors

4) $$Z = \sum_i \left\{ Q_i - G(P_{2_1}{}^b - P_{1_i}{}^b) \right\}^2$$

will have its minimum valve. At this minimum

$$\frac{\partial Z}{\partial b} = 0 \quad \text{and} \quad \frac{\partial Z}{\partial G} = 0.$$

5) $$\frac{\partial z}{\partial b} = -2G\left\{\sum_i Q_i(P_{2_i}{}^b \ln P_{2_i} - P_{1_i}{}^b \ln P_{1_i}{}^b)\right\}$$
$$+ 2G^2\left\{\sum_i (P_{2_i}{}^b - P_{1_i}{}^b)(P_{2_i}{}^b \ln P_{2_i} - P_{1_i}{}^b \ln P_{1_i}{}^b)\right\}$$

and

6) $$\frac{\partial z}{\partial G} = -2\left\{\sum_i Q_i(P_{2_i}{}^b - P_{1_i}{}^b)\right\} + 2G\left\{\sum_i (P_{2_i}{}^b - P_{1_i}{}^b)^2\right\}$$

Thus, at the minimum:

7) $$\sum_i \left\{(P_{2_i}{}^b \ln P_{2_i} - P_{1_i}{}^b \ln P_{1_i})(P_{2_i}{}^b - P_{1_i}{}^b - \frac{Q_i}{G})\right\} = 0$$

and

8) $$G = \frac{\sum_i Q_1(P_{2_1}{}^b - P_{1_i}{}^b)}{\sum_i (P_{2_i}{}^b - P_{1_i}{}^b)^2}$$

Equations 7 and 8 can be solved graphically by plotting the left side of equation 7 as a function of b and taking the intercept as the solution for b. Equivalently, they can be solved using a computer search routine in which b is varied systematically until a value is found for which the sum in equation 7 differs from 0 by less than a specified error. (NOTE: With this approach it is necessary to avoid a search pattern which would yield the trivial solution to equations 7 and 8, b=0, rather than the desired solution.)

A listing of a program, written in the FOCAL language, for performing this search with the experimental data follows. This program accepts as inputs the data triplets

$$[P_{1_i}, P_{2_i}, (P_{2_i} - P_{1_i})/(E_{2_i} - E_{1_i})]$$

with loads in Mdyne and stiffnesses in Mdyne/percent, along with the value for the ratio of initial length to dry weight, and converts the elements of the data triplet to units of Mdyne - cm./gm. before performing the calculations. Outputs from the program are values of b and K (not G).

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C-FOCAL., 1968

01.10 S B=.5; S F=.001
01.20 S R=10; S A=.10; S N=0; A !!!!"SIZE",S,!!!
01.30 A "U",U(N); I (U(N))1.4,2.1
01.40 S U(N)=S*U(N); A "    L",L(N),"     P",P(N),!!; S L(N)=S*L(N)
01.60 S P(N)=(U(N)-L(N))/(100*S*P(N)); S N=N+1; G 1.3

02.10 S B=B+A; S K=0; S D=0; F I=0,1,(N-1); D 8.0
02.30 S K=K/D; S S=0; F I=0,1,(N-1); D 7.0
02.50 S D=FABS(S)-FABS(R); S R=S; I (D)2.7; S A=-.5*A
02.70 I (FABS(R)-F)3.1; G 2.1

03.10 T %,!!!!"K",FEXP((1/B)*FLOG(K))," GM/MDYNE CM"!!"B",B,!!
03.20 G 1.2

07.10 S V=FLOG(U(I)); S D=FLOG(L(I)); S Q=FEXP(B*V); S W=FEXP(B*D)
07.30 S V=(Q*V)-(D*W); S D=Q-W-(P(I)/K); S S=S+(V*D)

08.10 S S=FEXP(B*FLOG(U(I)))-FEXP(B*FLOG(L(I))); S K=K+(P(I)*S)
08.20 S D=D+(S↑2)
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