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THE ROLE OF MODERN EXCITATION SYSTEMS IN RAPID DEMAGNETIZATION OF LARGE SYNCHRONOUS MACHINES

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NOMENCLATURE

$c_1, \ldots, c_{16}$	constants defined by Equations (2-46)
$c_7, \ldots, c_{23}$	constants defined by Equations (2-50)
$c_{24}, \ldots, c_{35}$	constants defined by Equations (3-55)
C	Equivalent capacitance determined by Equations (2-12)
e_a	open-circuit voltage in the armature circuit of the d-c generator
e_{a-d}	generated voltage in the direct-axis circuit of the amplidyne armature circuit
e_c	voltage applied to the control-field circuit of the amplidyne
e_{iqo}	quadrature-axis internal voltage
e_s	induced voltage in the short-circuited path of the amplidyne
E_a	Laplace transform of the open-circuit voltage in the armature circuit of the d-c generator
E_c	Laplace transform of the input excitation voltage to the B-winding of the amplidyne
$E_c(0)$	initial voltage contribution term in the control circuit of the amplidyne
$E_e(0)$	initial voltage contribution term in the exciter circuit
$E_f(0)$	Initial voltage contribution term in the field circuit of the synchronous machine
$E_f''(0)$	initial-value voltage term defined by Equation (3-72)
$E_{feq}(0)$	initial-value equivalent voltage defined by Equation (3-71)
$E_{i.c}$	initial value contribution term and is determined by Equations (2-8)
$E_s(0)$	initial voltage contribution term in the short-circuited path of the amplidyne and is given by Equation (2-26)

G	system function defined by Equation (3-34)
G'	product system function and is determined by Equation (2-5)
G_4''	system function defined by Equation (3-73)
H	feedback loop system function and is determined by Equation (2-53)
i	steady-state short-circuit current
i'	initial transient component of short-circuit current
i''	initial subtransient component of short-circuit current
$i_{a.c}$	maximum possible initial value of a-c component of short-circuit field current
$i_c(0)$	initial value of current in the control-field circuit of the amplidyne
i_d	direct-axis current for the synchronous machine
$i_{d.c}$	maximum possible initial value of d-c component of current
$i_e'(t)$	the component of transient exciter current as a result of de-energizing the A-winding of the amplidyne
$i_e''(t)$	the component of transient exciter current as a result of esciting the B-winding of the amplidyne
$i_f(0)$	initial-value of field current that is necessary to maintain rated terminal voltage at the synchronous machine terminals under rated speed
i_f'	initial transient component of short-circuit field current
i_f''	initial subtransient component of short-circuit field current
$i_f'(t)$	the component of transient field current as a result of de-energizing the A-winding of the amplidyne
$i_f''(t)$	the component of transient field current as a result of exciting the B-winding of the amplidyne

NOMENCLATURE (CONT'D)

i_{Kd} — hypothetical equivalent current in the equivalent direct-axis eddy currents and damper paths

$i_{\ell}(0)$ — initial value of current in the load circuit of the amplidyne

i_q — quadrature-axis current for the synchronous machine

$i_s(0)$ — initial value of current in the short-circuited path of the amplidyne

I_c — Laplace transform of the control field circuit current of the amplidyne

I_{dqf}^{KdKq} — current matrix defined by Equation (3-26)

I_{e1} — component of transient exciter current response due to $E_c(0)$ acting alone

I_{e2} — component of transient exciter current response due to $E_s(0)$ acting alone

I_{e3} — component of transient exciter current response due to $E_e(0)$ acting alone

I_{e4} — component of transient exciter current response due to $E_f(0)$ acting alone

I_{f1} — component of transient field current response due to $E_c(0)$ acting alone

I_{f2} — component of transient field current response due to $E_s(0)$ acting alone

I_{f3} — component of transient field current response due to $E_e(0)$ acting alone

I_{f4} — component of transient field current response due to $E_f(0)$ acting alone

I'_{f4} — component of closed loop field current with $E'_f(0)$ acting alone

I''_{f4} — component of closed loop field current with $E''_f(0)$ acting alone

I_{ℓ} — Laplace transform of the current in the load side of the amplidyne

NOMENCLATURE (CONT'D)

I_s	Laplace transform of the current in the short-circuited path of the amplidyne
K_2	coefficient defined by Equation (2-35)
K_2'	coefficient determined by Equation (2-23)
K_3	coefficient defined by Equation (2-36)
K_3'	coefficient determined by Equation (2-24)
K_4	coefficient defined by Equation (2-37)
K_{A1}	amplidyne constant for the A-winding control circuit and is defined by Equation (2-20)
K_{A2}	coefficient defined by Equation (2-21)
K_{A3}	coefficient defined by Equation (2-33)
K_{B1}	amplidyne constant for the B-winding control circuit and is defined by Equation (2-11)
K_{B2}	coefficient determined by Equations (2-22)
K_{B3}	coefficient defined by Equation (2-34)
K_c	control-field circuit constant that relates the induced voltage in the short-circuited path of the amplidyne with the control-field circuit current.
K_g	proportionality constant for the d-c generator and is determined by Equation (2-27)
K_ℓ	constant that relates the voltage induced in the short-circuited path of the amplidyne with the current in the load circuit
K_s	constant that relates the voltage induced in the direct-axis armature circuit of the amplidyne with the current in the short-circuited path
L	inductance parameter that can be identified by any of Equations (3-3) with different subscripts
L_a	direct-axis armature inductance of the amplidyne

NOMENCLATURE (CONT'D)

L_c	control-circuit inductance of the amplidyne
L_d	direct-axis synchronous inductance
L_{Kd}	equivalent direct-axis eddy currents and damper paths of the synchronous machine
L_{Kq}	equivalent quadrature-axis eddy currents and damper paths of the synchronous machine
L_o	zero-sequence inductance
L_q	quadrature-axis synchronous inductance
L_s	short-circuited path inductance of the amplidyne
M	mutual inductance between the equivalent eddy currents and damper paths, and the field circuit
N_f	number of turns of the field circuit
N_{Kd}	effective number of turns of the equivalent eddy currents and damper paths
O	system function defined by Equation (2-7)
r	synchronous machine armature resistance
r_a	direct-axis armature resistance of the amplidyne
r_c	control-circuit resistance of the amplidyne
r_e'	modified exciter field resistance and is determined by Equations (2-14)
r_{Kd}	equivalent direct-axis eddy currents and damper paths resistance of the synchronous machine
r_{Kq}	equivqlent quadrature-axis eddy currents and damper paths of the synchronous machine
r_s	short-circuited path resistance of the amplidyne
R	equivalent resistance determined by Equations (2-12)
R_{oe}	effective output resistance of the amplidyne and is determined by Equation (2-13)

R_s	resistance that may be inserted in series with the armature circuit of the d-c machine and synchronous machine field winding
s	generalized frequency $\sigma + j\omega$ that arises in Laplace transform
$s_{1,2}$	characteristic roots determined by Equations (A-8), (A-9) or (A-10)
T	instantaneous value of electromagnetic torque
v_a, v_b, v_c	source voltages of the synchronous machine for phases a, b and c
v_d	source voltage applied to the direct-axis
v_f	exciter field voltage of the d-c generator
v_o	zero-sequence voltage
v_q	source voltage applied to the quadrature-axis
v_t	synchronous machine terminal voltage
V_c	magnitude of negative step voltage applied to the B-winding of the amplidyne
V_{dqf}	voltage matrix defined by Equation (3-27)
V_ℓ	Laplace transform of the voltage at load side of the amplidyne
V_o	Open-circuit voltage of the amplidyne that is used in Thevenin's equivalent of the amplidyne
W_f	stored magnetic energy in the synchronous machine field winding
x_d''	direct-axis subtransient reactance
x_q	quadrature-axis synchronous reactance
x_q''	quadrature-axis subtransient reactnace
x_d	direct-axis synchronous reactance
x_d'	direct-axis transient reactance

X_d	direct-axis system function defined by Equation (3-38)
X_q	quadrature-axis system function defined by Equation (3-40)
Z	impedance matrix defined by Equation (2-5)
Z_{dqf}^{KdKq}	impedance matrix defined by Equation (3-25)
Z_o	output system function for the amplidyne and is determined by Equation (2-9)
Δ_1 , Δ_2 , Δ_3	determinants defined by Equations (A-3)
Δ	determinant defined by Equation (A-4)
θ	angle between phase a and the d-axis measured in the clockwise direction
$\lambda(0)$	initial value contribution flux linkage as defined by Equations (3-22) with different subscripts
λ_a , λ_b , λ_c	flux linkages of the synchronous machine for phases a, b, and c
λ_d	direct-axis flux linkage for the synchronous machine
λ_f	flux linkage of the synchronous machine field winding
$\lambda_{i.v}$	initial-value contribution flux linkage matrix defined by Equation (3-28)
λ_{Kd}	effective flux linkage in the equivalent direct-axis eddy currents and damper paths of the synchronous machine
λ_{Kq}	effective flux linkage in the equivalent quadrature-axis eddy currents and damper paths of the synchronous machine
λ_q	quadrature-axis flux linkage for the synchronous machine
Λ	Laplace transform of flux linkage λ
σ	leakage coefficient of the coupled circuits
τ_d'	direct-axis short-circuit time constant
τ_{do}'	open-circuit time constant of the synchronous machine

τ_d''	direct-axis subtransient short-circuit time constant
τ_{do}''	direct-axis subtransient open-circuit time constant
τ_e'	modified time constant for the exciter circuit including loading effects and is determined by Equation (2-18)
τ_q''	quadrature-axis subtransient short-circuit time constant
τ_{qo}''	quadrature-axis subtransient open-circuit time constant
τ_s	amplidyne short-circuited path time constant
ω	radian frequency

CHAPTER I

INTRODUCTION

The philosophy for the protection of synchronous machines is essentially based on preventing as far as possible failures of any type, or, if any do occur, to limit the damage to the absolute minimum. There has been much emphasis on the importance of providing for the best protective devices. The capital cost of these devices can be looked upon as insurance premium to guard against possible damages or probable troubles. Damage to a single coil may develop into a fire which may require the major replacement of stator windings and laminations. In addition to the cost of repair, there is also a loss of revenue during the time for which the unit is out of service.

With the rapid and continued growth of the demand for electricity, the size of generating units is progressively increased. Therefore, more attention needs to be directed towards the protection problems of these units. The heavy investment represented by large units justifies increased emphasis on methods of protecting against possible sources of danger. These sources of danger can be divided into two types:

a. External sources such as voltage surges due to atmospheric conditions, dangerous temperature rises due to overloads or systems short circuits and dangerous voltage rises due to sudden loss of load.

b. Internal faults such as inter-turn short circuits, phase short circuits, or ground faults of the stator windings.

When an internal fault occurs in an alternator, the main circuit breaker and the field circuit breaker should be actuated to open the respective circuits. In addition, the field of the machine should be suppressed as quickly and completely as possible. Thus, the e.m.f of the machine will have little possibility of maintaining the fault current. This process of suppressing the field of the synchronous machine is called "demagnetization" or sometimes "de-excitation".

There are various methods by which this process can be achieved depending on the size of the units in question. A brief discussion of the existing methods for demagnetization of synchronous machines will be given. For details the reader is referred to the literature.[1-7]

For small and medium size generators, an exciter-shunt field switch in addition to a field discharge resistance, as shown in Figure 1 may serve the purpose. This has been adopted expecially in North America particularly for hydro-electric alternators.

For larger size machines, a main-field circuit breaker as shown in Figure 2 in conjunction with a discharging resistance provide more rapid reduction in flux and consequently for the e.m.f . It is clear that the field suppression is higher in the second case but this will necessitate a higher interruption rating of the circuit breaker. In this case, the decay of field current will be hastened by the introduction of the field discharge resistance in the exciter field circuit. High values for the discharge resistors seem favorable, but high transient voltages across the field windings would result.. An optimum value exists for the field discharge resistance value; this value allows a safe margin for insulation breakdown from this transient voltage. This field discharge

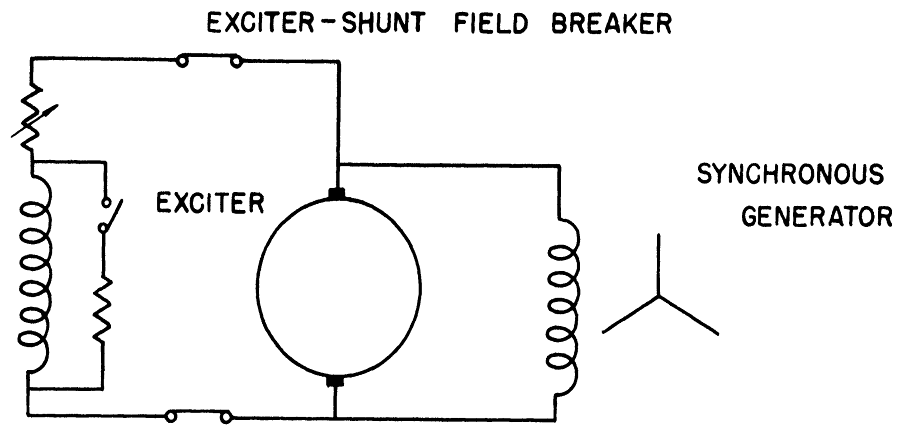


Figure 1. De-excitation system for small synchronous generators with exciter-shunt field circuit breaker.

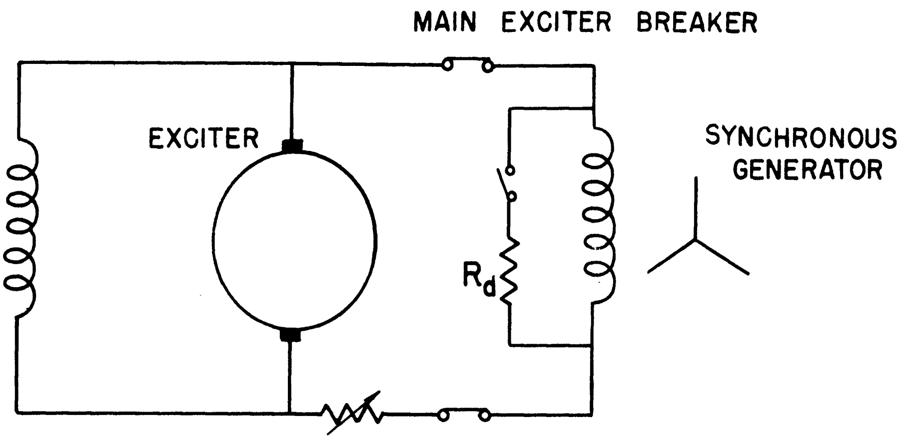


Figure 2. De-excitation system for larger size synchronous generators with field discharge resistance method.

resistance method is widely applied in the United Kingdom as well as in North America.

In Germany the oscillation resistance method suggested by Rudenberg was successfully applied;[4,6,7] this method utilizes a resistance that is inserted in series with the armature circuit of the d-c exciter. A reversal in polarity of the armature voltage would cause the exciter to temporarily run as a motor. Thus, part of the energy of the magnetic field sould be converted to mechanical energy in addition to that part of heat energy dissipated in the resistance.

In France and Switzerland, another method has been employed for large units in which a pilot exciter is usually arranged to provide the excitation for the main exciter. It is based on the idea of reversing the pilot exciter voltage[5] so that the full armature voltage of the main exciter is reversed. At a suitable instant this reversed voltage can be removed by means of a suitable relaying arrangement. Obviously, this would increase complexity of the circuit.

1.1 Statement of the Problem

In the last few years, new excitation systems have been applied in industry. These include ignitron exciters, rotating rectifier brushless exciters, harmonic exciters, inductor alternator exciters, amplidyne main exciters and a-c machine terminal voltage-current static exciters.[8]

Of special interest in this study, is the amplidyne application to large synchronous machines. One of the important characteristics of the amplidyne is its high speed of response which makes its application quite useful for power systems stability improvement. The elevation of the excitation voltage up to its ceiling voltage provides a useful means of improving power systems stability. The amplidyne plays a useful role is such field forcing because of its short time constant.[8,9]

It has been suggested[7] that the amplidyne could be utilized to force the excitation voltage down to a zero value in a demagnetization process. The generator field breaker, as well as, the main field discharge resistor could be eliminated. Thus, the high voltage impulse, which is applied across the machine field winding by using a high value of discharge resistor could be avoided. Then, much of the uncertainties and arguments about the optimum field discharge resistor value can be minimized.

The aim of this study is to analyze the amplidyne regulator excitation system application in order to effect a demagnetization process and to investigate the feasibility of adopting some particular schemes.

For this purpose, it is desired to establish mathematical models for the synchronous machine as well as the excitation system. A unified approach for studying the dynamic behavior of the total system is undertaken which adopts control systems techniques and especially transform methods and system functions.

The general differential equations of the synchronous machine, formulated by R.H. Park in 1929,[10] have provided an excellent means for studying any mode of machine operation. Full theoretical derivations based on these equations are presented for a three-phase short-circuit occurring from no-load conditions. There is much published material on this subject,[11-17] but all of the derivations neglect the winding resistances in the first analysis so that the principle of constant flux linkages, first advocated by R.E. Doherty in 1930, could be applied. The effect of winding resistance has been included in an approximate manner by assuming the components of the flux linkages of the winding to decrease at a rate directly proportional to the product of winding resistance and d-c component of current in the winding.[11,14]

In this work, the short circuit currents are derived without resort to the principle of constant flux linkage. It is then not necessary to neglect armature resistance at any stage or to adjust for the decay of short-circuit currents with the so-called "decrement factors". The conventional time constants that describe the machine behavior during the transient period are introduced in a systematic and natural manner. The solutions for the short circuit currents are arrived at using the Laplace transform method where initial conditions are continuously recognized. This has an advantage over the Heaviside operational method where it is usually necessary to use the form with zero initial values, and then to adjust the final solution in the last stages. Equivalent sources to represent initial value contribution terms are conveniently introduced throughout this treatment. A generalized solution for the three-phase short-circuit condition is established and the agreement of the solution utilizing certain assumptions, with the previously established expressions proves interesting. It is anticipated that for some applications where a certain degree of accuracy is desired, this generalized solution will be of value.

All the input-output relationsips in this thesis are described by "System Functions". This is believed to be more indicative of the actual situation than other nomenclature systems (e.g transfer functions, operational impedances or operational admittances).

1.2 Outline of this Study

This study is concerned with the analysis of an amplidyne regulator excitation system application for the demagnetization of large synchronous machines. Usually, the excitation system incorporates an amplidyne used as a pilot exciter to supply power to the field of the main d-c generator exciter. In this way, the d-c generator that supplies field excitation energy to the synchronous generator is controlled by the amplidyne control windings.

For very accurate representation of such a system, one has to consider nonlinearity and saturation effects of the individual components. However, this will complicate the analysis and a linearized model for the whole system permits much simplification.

Chapter II introduces the mathematical models to represent such a linearized system. A preliminary investigation is considered in which the synchronous machine is open-circuited at rated voltage. The speed and effectiveness for some suggested schemes utilizing the amplidyne excitation system for demagnetization purposes could then be investigated. Thus, an appraisal of the value of such schemes can be acquired in a rather simple manner.

In Chapter III, the analysis is extended to cover the actual and more important case when the unloaded synchronous machine is subjected to a three-phase short circuit directly at the end terminals of the machine winding. This case is more complicated because of the couplings between the different circuits involved. Reasonable assumptions are stated so that further simplifications may be carried out. The same physical operations suggested in Chapter II are performed and relations showing the speed of response of the decay of currents are obtained.

The experimental results are presented in Chapter IV and a comparison with the theoretical results is also given.

In Chapter V, the methods are applied to a large synchronous machine of typical parameters in order to evaluate such schemes suggested in this thesis. This machine has specifically been studied[6,7] for different existing methods of demagnetization.

A summary of the analytical and experimental work of the preceding Chapters is given in Chapter VI, together with the conclusions.

CHAPTER II

THE UNLOADED SYNCHRONOUS MACHINE

2.1 Introduction

In order to evaluate the effectiveness of different methods of demagnetization, the synchronous machine is first analyzed under an open-circuit condition. The exciting field current has a value just sufficient to generate rated voltage at rated speed. It should be realized that the transient behavior under fault conditions for the unloaded machine would be different from the open-circuit case.

For an excitation system utilizing the amplidyne as a pilot exciter, one of the control field windings, say A-winding, provides the necessary excitation to produce rated voltage at the synchronous machine terminals. Therefore, any desire to remove the excitation from the synchronous machine would naturally involve de-energizing the A-winding in the control fields. To accelerate this operation, a negative voltage may be impressed across one of the control fields at the same instant when the excitation is to be removed from the A-field winding. Thus, the stored magnetic energy in the field of the d-c machine can be dissipated in relatively shorter time. The magnitude of this negative voltage may be adjusted such that it would produce a zero terminal voltage if acting in conjunction with the original excitation. A reasonable procedure to accelerate the process would then be, to remove the original excitation from the A-winding and at the same time to excite another control field winding on the amplidyne, say B-winding, negatively. However, there will be some difficulty regarding the removal of the negative excitation before the terminal voltage starts building up after its zero value has been attained. From pure theoretical considerations and in order to gain insight into the performance under such a procedure, this case will be completely analyzed in Section 2.6.

Another arrangement that essentially depends upon feedback is next analyzed in Section 2.7. The negative voltage across the B-winding in this case will be proportional to the value of the decaying field current itself. Hence, as soon as the field current and consequently the terminal voltage are forced to zero, there will be no possiblity for the terminal voltage to build up.

Although it is not the purpose of this chapter to present an extensive discussion of network analysis or of Laplace transform methods, a short discussion will be carried out to introduce the approach that will be followed throughout this thesis. The initial value contribution terms due to initial conditions will be continuously identified as the necessary derivations are made.

Finally, in order to analyze such arrangements, it is necessary to represent each component of the total system by a methematical model. This is undertaken in the sections to follow.

2.2 The Amplidyne

The action of the amplidyne can be visualyzed as two cascaded d-c generators. It is therefore a two-stage generator. However, the two stages are combined in a single machine with a single armature winding. The construction, theory of operation, and applications of the amplidyne are widely covered in the literature.[17-23]

As mentioned before, the Laplace transform method will be followed. Since the Laplace transform is a linear transform, the following assumptions are made:

1. No consideration is given to hysteresis and saturation effects. The hysteresis may be considered as a cause for an additional phase lag between the input and output. Saturation due to the properties of the iron comprising the magnetic paths results in a nonlinearity between the generated voltage and excitation current. Its main effect is to reduce the gain, compared to that which would result if the nonlinearity did not exist.

2. The eddy currents generated in the field poles whenever the field current changes are assumed negligible. This assumption is reasonable since in most cases all magnetic circuits of an amplidyne are laminated. The effect of eddy currents can be accurately represented by a multiplicity of closed circuits in the iron which are mutually coupled to the field circuit as well as to each other. The effect of this multitude of circuits can be lumped in a single equivalent eddy current circuit which is coupled to the field circuit.

3. Brush-resistance drop is assumed to be a linear relation.

4. The brushes are located exactly on the neutral axis so that mutual coupling terms are zero.

5. Commutation is assumed to be perfect and to take place at one fixed point under the brush.

2.3 Circuits

In the amplidyne to be analyzed here, the principal circuits as shown in Figure 3 are:

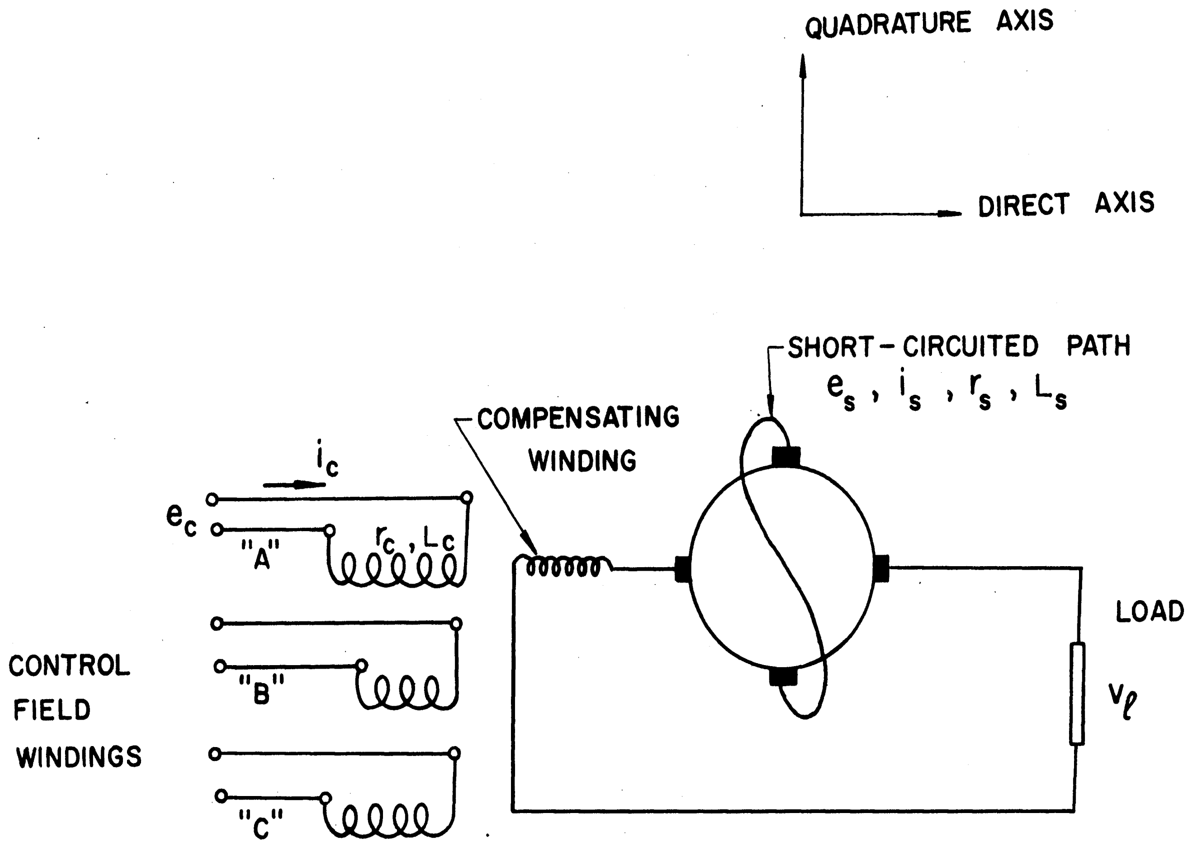


Figure 3. Schematic Diagram of the Amplidyne generator.

1. The control-field circuit, to which is applied the control excitation voltage e_c.

2. The quadrature armature circuit, a short-circuited path, in where there is an assumed driving voltage e_s (speed voltage) opposed by circuit impedance drop.

3. The direct armature circuit which there is an assumed generated voltage e_a that balances the sum of the impedance drop and the load terminal voltage v_ℓ. Included in this circuit is the compensating winding which is supposed to completely compensate for the load current.

The control-field circuit equation for the amplidyne generator of Figure 3 is:

$$e_c = r_c\, i_c + L_c \frac{di_c}{dt} \tag{2-1}$$

where the mutual coupling voltage terms have been dropped out according to the previous assumptions.

For the short-circuit path, the circuit equation is

$$e_s = K_c\, i_c = r_s\, i_s + L_s \frac{di_s}{dt} + K_\ell\, i_\ell \tag{2-2}$$

where e_s is proportional to i_c as a result of the assumed linearity of the magnetization curve. The voltage term $K_\ell\, i_\ell$ summarizes the effect of the load in the short-circuited path. Ideally, this term should be zero for a fully compensated amplidyne.

For the direct-axis circuit, one has

$$e_{a\text{-}d} = K_s\, i_s = r_a\, i_\ell + L_a \frac{di_\ell}{dt} + v_\ell \tag{2-3}$$

It is convenient in the Laplace transform notation to use capital letters to indicate the Laplace transform of a small-letter term. Capital letters are understood to be functions of the variable s where

$$s = \sigma + j\omega$$

is rather a generalized frequency.

2.4 Amplidyne System Function

By taking the Laplace transform of Equations (2-1) to (2-3) and rearranging them in such a way as to solve for V_ℓ as a function of E_c, I_ℓ and initial value contribution terms, one can write

$$(r_c+L_c s)I_c + (0)I_s + (0)\ V_\ell = E_c + L_c\ i_c(0) - (0)\ I_\ell \qquad (2\text{-}4a)$$

$$(-K_c)I_c + (r_s+L_s s)I_s + (0)\ V_\ell = L_s\ i_s(0) - (K_\ell)I_\ell \qquad (2\text{-}4b)$$

$$(0)I_c - (K_s)I_s + (1)V_\ell = L_a\ i_\ell(0) - (r_a+L_a s)I_\ell \qquad (2\text{-}4c)$$

In matrix notation, the above equations can be written as:

$$[Z]\begin{bmatrix} I_c \\ I_s \\ V_\ell \end{bmatrix} = \begin{bmatrix} E_c \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} L_c\ i_c(0) \\ L_s\ i_s(0) \\ L_a\ i_\ell(0) \end{bmatrix} - \begin{bmatrix} 0 \\ K_\ell \\ r_a+L_a s \end{bmatrix} I_\ell \qquad (2\text{-}4d)$$

where,

$$[Z] = \begin{bmatrix} r_c+L_c s & 0 & 0 \\ -K_c & r_s+L_s s & 0 \\ 0 & -K_s & 1 \end{bmatrix} \qquad (2\text{-}5)$$

It is interesting to note that the solution for V_ℓ can be written in the following form,

$$V_\ell = O.E_c + E_{i.c} - Z_o.I_\ell \tag{2-6}$$

where,

$$O = \begin{vmatrix} r_c+L_c s & 0 & 1 \\ -K_c & r_s+L_s s & 0 \\ 0 & -K_s & 0 \end{vmatrix} \Big/ |Z|$$

$$= \frac{K_c K_s}{r_c r_s} \frac{1}{(1+s\tau_c)(1+s\tau_s)} \tag{2-7}$$

is the open-circuit system function, and

$$E_{i.c} = \begin{vmatrix} r_c+L_c s & 0 & L_c i_c(0) \\ -K_c & r_s+L_s s & L_s i_s(0) \\ 0 & -K_s & L_a i_\ell(0) \end{vmatrix} \Big/ |Z|$$

$$= L_a i_\ell(0) + \frac{K_s/r_s}{1+s\tau_s} L_s i_s(0) + \frac{K_c K_s/r_c r_s}{(1+s\tau_c)(1+s\tau_s)} L_c i_c(0) \tag{2-8}$$

is the initial value contribution voltage term, and

$$|Z_o| = \begin{vmatrix} r_c+L_c s & 0 & 0 \\ -K_c & r_s+L_s s & K_\ell \\ 0 & -K_s & r_a+L_a s \end{vmatrix} \Big/ |Z|$$

$$= r_a + L_a s + \frac{K_\ell K_s/r_s}{1 + s\tau_s} \tag{2-9}$$

is the output system function for the amplidyne.

Now, the first term on the right hand side of Equation (2-6) determines the solution for the initially relaxed unloaded system. The second term $E_{i.c}$ in the same equation is an equivalent input excitation term to account for the initial values contribution terms. The last term $-Z_o I_\ell$ is to account for the drop in voltage of the loaded machine.

In the above form, one can easily determine the Thevenin's equivalent circuit of the amplidyne, thus

$$V_o = \frac{K_{B1}}{(1+s\tau_c)(1+s\tau_s)} E_c \qquad (2\text{-}10)$$

where,

$$K_{B1} = \frac{K_{cB} K_s}{r_{cB} r_s} \qquad (2\text{-}11)$$

It should be noted that the capital letters attached to the constant "K" refer to the particular winding under operation. In general, the value of these constants are determined by simple steady state tests for the amplidyne under investigation.

The Thevenin's impedance is given by Equation (2-9) above. Here, the first two terms represent the ordinary resistance and self-impedance of the direct-axis armature of the amplidyne. The third term represents the effect of improper compensation on the output impedance. It can be represented(19) by an R-C impedance circuit as in Figure 4 with the following values,

$$R = \frac{K_s K_\ell}{r_s} , \qquad (2\text{-}12a)$$

$$RC = \tau_s \qquad (2\text{-}12b)$$

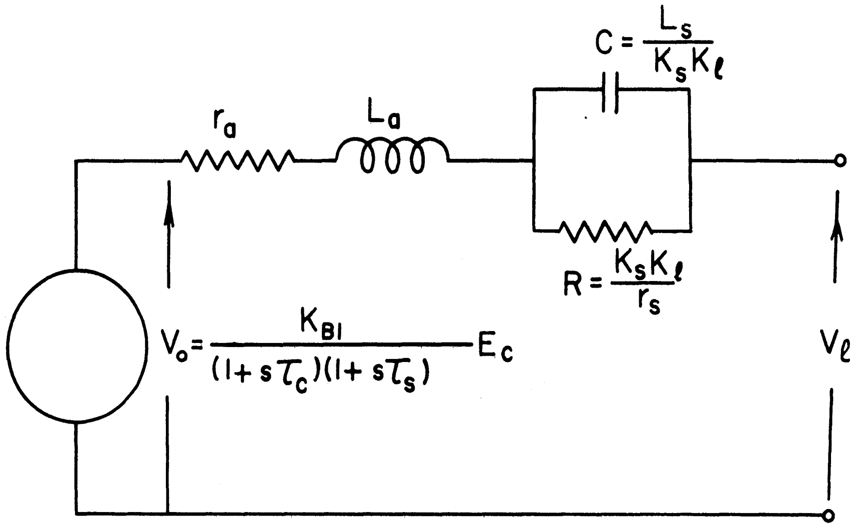


Figure 4. Equivalent circuit of the amplidyne.

A simplification for the output circuit of the amplidyne is shown in Figure 5. It is based on the fact that the load on the amplidyne is usually an inductive load of much higher inductance than L_a , so that the effect of inductance L_a as well as that of the capacitance C can be neglected. Hence Z_o is reduced to a pure resistance term R_{oe} given by:

$$R_{oe} = r_a + \frac{K_s\ K_\ell}{r_s} \tag{2-13}$$

This resistance can be interpreted as an "effective"resistance which takes account for an uncompensated amplidyne. The value of this resistance can be easily determined experimentally. For a fully compensated amplidyne, K_ℓ is zero and the effective output resistance would be that of the direct-axis armature circuit. This effective resistance can always be lumped together with that of the load resistance. In case the amplidyne is functioning as the main exciter, then the load will be the field excitation winding of the synchronous machine. It may be the field winding of a separately excited d-c generator that feeds the excitation energy for the synchronous machine, in which case the amplidyne is considered as a pilot exciter. For the amplidyne main exciter application the label ℓ attached to the output direct-axis circuit of the amplidyne can conveniently be changed to the label f to agree with the standard nomenclature in the synchronous machine literature for the field winding. Whereas, for the amplidyne pilot exciter application the label ℓ attached to the output direct-axis circuit of the amplidyne may conveniently be changed to the label e indicating the excitation for the d-c machine under consideration. The label f is then preserved for the output load of the d-c generator which is the field winding of the synchronous machine itself.

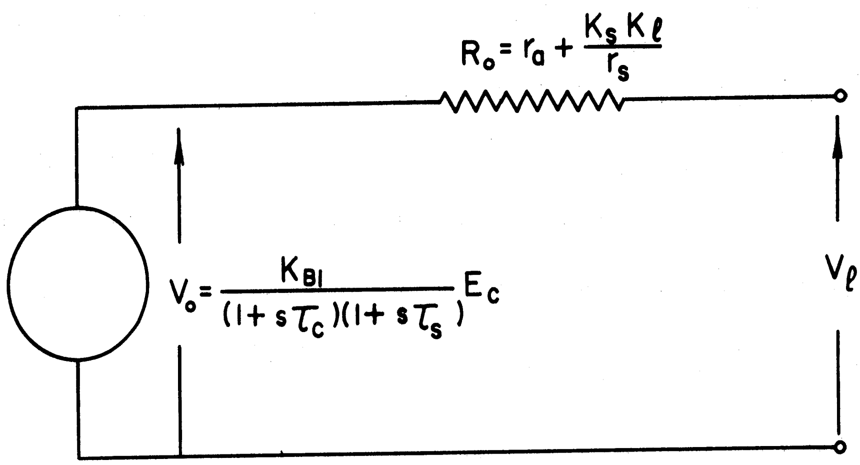


Figure 5. Simplified equivalent circuit of the amplidyne.

2.5 The Amplidyne-Exciter Combination

In the majority of power excitation systems, especially for larger units, the amplidyne is used as a pilot exciter for a d-c generator of considerable rating. In the following analysis the method will be considered in which the system function of the amplidyne will be extended to account for the cascaded d-c machine case. The addition of the d-c machine will undoubtedly introduce some difficulties in order to account for saturation. However, the d-c machine will be idealized to the extent that the linear theory will continue to hold.

Figure 6 represents the equivalent circuit of such a scheme where the open-circuit voltage of the amplidyne, $V_{o.c}$, can be interpreted as a voltage behind the effective output resistance R_{oe} of the amplidyne. Thus a modified exciter field resistance r_e' can be introduced such that

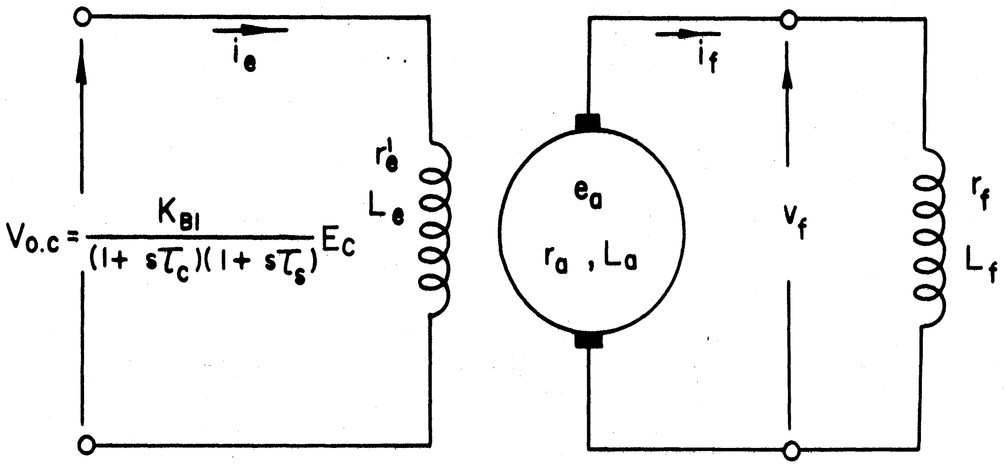
$$r_e' = r_e + R_{oe} \tag{2-14}$$

It is desired now to determine the system function which relates I_f, the transform of the output field or excitation current for the synchronous machine field, to the transform of excitation voltage E_c. But first an expression for the transform of exciter current I_e is derived. This is done by substituting R_{oe} for Z_o in Equation (2-6) so that,

$$V_\ell = 0 \cdot E_c + E_{i.c} - R_{oe} I_e \tag{2-15}$$

From Figure 3 another expression for V_ℓ can be written as

$$V_\ell = r_e(1+s\,\tau_e)I_e - L_e\, i_e(0) \tag{2-16}$$



r_e' = $r_e + R_{oe}$

= Equivalent resistance in the excitation circuit of the d-c machine

$V_{o.c}$ = Voltage behind the output resistance of the amplidyne

r_f' = $r_f + R_{of}$

= Equivalent resistance in the field circuit

e_a = $K_g i_e$

= Voltage behind the output resistance of the d-c machine.

Figure 6. Equivalent circuit of the d-c generator exciting the field winding of the synchronous machine.

Equations (2-15) and (2-16) can be equated to give an expression for I_e as:

$$I_e = \frac{0}{r_e'(1+s\tau_e')} E_c + \frac{1}{r_e'(1+s\tau_e')} E_{i.c} + \frac{L_e\ i_e(0)}{(r_e'(1+s\tau_e')} \qquad (2\text{-}17)$$

where,

$$\tau_e' = \frac{L_e + L_{ae}}{r_e'} \cong \frac{L_e}{r_e'} \qquad (2\text{-}18)$$

is a modified time constant for the exciter circuit including loading effects and assuming that the armature inductance in the direct-axis is much smaller than that of the excitation coil.

Now, if the initial value voltage term $L_a\ i_\ell(0)$ in Equation (2-8) is replaced by the term $L_e\ i_e(0)$, and if Equations (2-8) and (2-9) are substituted in Equation (2-17), one can write

$$\begin{aligned} I_e = {} & \frac{K_{A2}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')} E_c(0) \\ & + \frac{K_2'}{(1+s\tau_s)(1+s\tau_e')} E_s(0) \\ & + \frac{K_3'}{1+s\tau_e'} E_e(0) \\ & + \frac{K_{B2}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')} E_c \end{aligned} \qquad (2\text{-}19)$$

where,

$$K_{A1} = \frac{K_{cA}\ K_s}{r_{cA}\ r_s} \qquad (2\text{-}20)$$

$$K_{A2} = \frac{K_{A1}}{r_e'} \tag{2-21}$$

$$K_{B2} = \frac{K_{B1}}{r_e'} \tag{2-22}$$

$$K_2' = \frac{K_s}{r_s \; r_e'} \tag{2-23}$$

$$K_3' = \frac{1}{r_e'} \tag{2-24}$$

$$E_c(0) = L_c \; i_c(0) \tag{2-25}$$

$$E_s(0) = L_s \; i_s(0) \tag{2-26}$$

$$E_e(0) = L_e \; i_e(0) \tag{2-27}$$

Because of the assumed linearity of the magnetization curve of the d-c machine, the transorm of the generated voltage in the armature for rated speed is given by:

$$E_a = K_g \; I_e \tag{2-28}$$

where K_g is a proportionality factor for the d-c generator. In actual practice r_a , L_a the armature resistance and inductance of the d-c machine are usually much smaller than the resistance and the inductance of the load which will be the field winding of the synchronous machine. Furthermore, a modified time constant τ_f' for the field circuit can be introduced to account for the effective output resistance in the field circuit so that

$$r_f' = r_f + R_{of} \tag{2-29}$$

Hence, E_a as given by Equation (2-28) can be looked upon as a voltage behind the output resistance of the d-c generator and one can write,

$$K_g I_e = r_f'(1+s\tau_f')I_f - E_f(0) \qquad (2\text{-}30)$$

where,

$$E_f(0) = L_f \, i_f(0) \qquad (2\text{-}31)$$

Substituting Equation (2-30) into Equation (2-19) above, one has

$$\begin{aligned} I_f = & \frac{K_{A3}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_c(0) \\ & + \frac{K_2}{(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_s(0) \\ & + \frac{K_3}{(1+s\tau_e')(1+s\tau_f')} E_e(0) \\ & + \frac{K_4}{1+s\tau_f'} E_f(0) \\ & + \frac{K_{B3}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_c \end{aligned} \qquad (2\text{-}32)$$

where,

$$K_{A3} = \frac{K_g \; K_{A2}}{r_f'} \qquad (2\text{-}33)$$

$$K_{B3} = \frac{K_g \; K_{B2}}{r_f'} \qquad (2\text{-}34)$$

$$K_2 = \frac{K_g \; K_2'}{r_f'} \qquad (2\text{-}35)$$

$$K_3 = \frac{K_g \, K_3'}{r_f'} \tag{2-36}$$

$$K_4 = \frac{1}{r_f'} \tag{2-37}$$

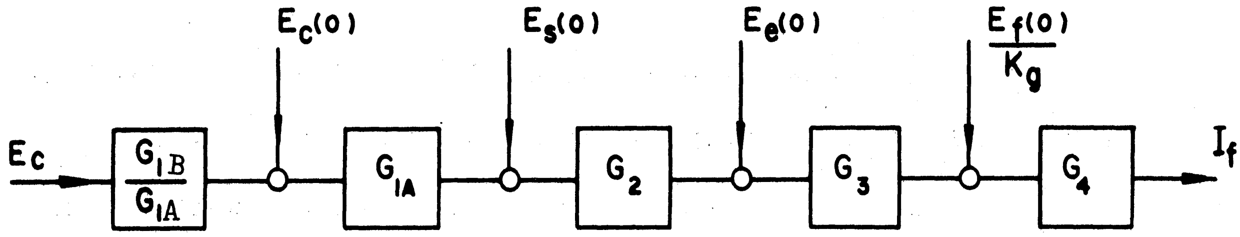
The block diagram representation of Figure 7 summarizes the above relationships and identifies clearly the different parameters involved.

Finally, it is desired to obtain time-domain solutions for the exciter and field currents. Before this is done, it is desirable to discuss the different factors upon which the solution depends. Equations (2-19) and (2-32) show that the nature of solution will depend to a large extent on the initial value contribution terms such as $E_c(0)$, $E_s(0)$, $E_e(0)$, and $E_f(0)$, as well as, on the form and magnitude of the excitation transform voltage E_c that may be applied to winding B of the amplidyne. Conveniently, the time domain solutions for both the exciter and the field current are best resolved into two distinct components. For the exciter current, one has

$$i_e(t) = i_e'(t) + i_e''(t) \tag{2-38}$$

where,

$$i_e'(t) = \mathcal{L}^{-1} \Big[\Big\{ \frac{K_{A2}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')} E_c(0)\Big\} + \Big\{ \frac{K_2'}{(1+s\tau_s)(1+s\tau_e')} E_s(0) \Big\} + \Big\{ \frac{K_3'}{1+s\tau_e'} E_e(0) \Big\}\Big] \tag{2-39}$$



$$G_{1A} = \frac{K_{cA}}{r_{cA}(1+s\tau_{cA})}$$

$$G_{1B} = \frac{K_{cB}}{r_{cB}(1+s\tau_{cB})}$$

$$G_2 = \frac{K_s}{r_s(1+s\tau_s)}$$

$$G_3 = \frac{K_g}{r_e'(1+s\tau_e')}$$

$$G_4 = \frac{K_g}{r_f'(1+s\tau_f')}$$

Figure 7. Block Diagram Representation of Equation (2-32).

and

$$i_e''(t) = \mathcal{L}^{-1}\left[\frac{K_{B2}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')} E_c\right] \qquad (2\text{-}40)$$

For the field current, one can similarly write

$$i_f(t) = i_f'(t) + i_f''(t) \qquad (2\text{-}41)$$

where

$$\begin{aligned} i_f'(t) = \mathcal{L}^{-1}\Big[&\Big\{\frac{K_{A3}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_c(0)\Big\} \\ &+\Big\{\frac{K_2}{(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_s(0)\Big\} \\ &+\Big\{\frac{K_3}{(1+s\tau_e')(1+s\tau_f')} E_e(0)\Big\} \\ &+\Big\{\frac{K_4}{1+s\tau_f'} E_f(0)\Big\}\Big] \end{aligned} \qquad (2\text{-}42)$$

and

$$i_f''(t) = \mathcal{L}^{-1}\left[\frac{K_{B3}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_c\right] \qquad (2\text{-}43)$$

It should be noted that $i_e'(t)$ and $i_f'(t)$ are to represent those parts of the solutions where a switching off operation is carried out from the A-winding. The other parts of solutions as given by i_e'' and $i_f''(t)$ are to represent in this case a switching on operation for another excitation winding say the B-winding. The final solutions for the exciter and field currents are partly determined by the type of negative forcing excitation voltage on this B-winding. Two different negative excitation functions are analyzed in the following two sections.

2.6 Exciter and Field Currents with Negative Forcing Step Voltage:

It can be shown by simple partial fraction expansion operations for Equations (2-39) and (2-42) that the time-domain solutions for the components of currents due to the switching off process for winding A are as follows:

$$i_e'(t) = K_{A2}\, E_c(0)\, [c_1\, \epsilon^{-t/\tau_c} + c_2\, \epsilon^{-t/\tau_s} + c_3\, \epsilon^{-t/\tau_e'}]$$
$$+ K_2'\, E_s(0)\, [c_4\, \epsilon^{-t/\tau_s} + c_5\, \epsilon^{-t/\tau_e'}]$$
$$+ K_3'\, E_e(0)\, [c_6\, \epsilon^{-t/\tau_e'}] \qquad (2\text{-}44)$$

and

$$i_f'(t) = K_{A3}\, E_c(0)\, [c_7\, \epsilon^{-t/\tau_c} + c_8\, \epsilon^{-t/\tau_s} + c_9\, \epsilon^{-t/\tau_e'} + c_{10}\epsilon^{-t/\tau_f'}]$$
$$+ K_2\, E_s(0)\, [c_{11}\, \epsilon^{-t/\tau_s} + c_{12}\, \epsilon^{-t/\tau_e'} + c_{13}\, \epsilon^{-t/\tau_f'}]$$
$$+ K_3\, E_e(0)\, [c_{14}\, \epsilon^{-t/\tau_e'} + c_{15}\, \epsilon^{-t/\tau_f'}]$$
$$+ K_4\, E_f(0)\, [c_{16}\, \epsilon^{-t/\tau_f'}] \qquad (2\text{-}45)$$

where,

$$c_1 = \frac{\tau_c}{(\tau_c-\tau_s)(\tau_c-\tau_e')} \qquad c_2 = \frac{\tau_s}{(\tau_s-\tau_c)(\tau_s-\tau_e')}$$

$$c_3 = \frac{\tau_e'}{(\tau_e'-\tau_c)(\tau_e'-\tau_s)}$$

$$c_4 = \frac{1}{\tau_s-\tau_e'} \qquad c_5 = \frac{1}{\tau_e'-\tau_s}$$

$$c_6 = \frac{1}{\tau_e'}$$

$$c_7 = \frac{\tau_c^2}{(\tau_c-\tau_s)(\tau_c-\tau_e')(\tau_c-\tau_f')}$$

$$c_8 = \frac{\tau_s^2}{(\tau_s-\tau_c)(\tau_s-\tau_e')(\tau_s-\tau_f')}$$

$$c_9 = \frac{\tau_e'^2}{(\tau_e'-\tau_c)(\tau_e'-\tau_s)(\tau_e'-\tau_f')}$$

$$c_{10} = \frac{\tau_f'^2}{(\tau_f'-\tau_c)(\tau_f'-\tau_s)(\tau_f'-\tau_e')}$$

$$c_{11} = \frac{\tau_s}{(\tau_s-\tau_e')(\tau_s-\tau_f')} \qquad (2\text{-}46)$$

$$c_{12} = \frac{\tau_e'}{(\tau_e'-\tau_s)(\tau_e'-\tau_f')}$$

$$c_{13} = \frac{\tau_f'}{(\tau_f'-\tau_s)(\tau_f'-\tau_e')}$$

$$c_{14} = \frac{1}{\tau_e'-\tau_f'}$$

$$c_{15} = \frac{1}{\tau_f'-\tau_e'}$$

$$c_{16} = \frac{1}{\tau_f'} \qquad (2\text{-}46)$$

A negative forcing step voltage is switched on to winding "B" at the same instant when the switching off process for winding "A" is effected. If the magnitude of this negative step voltage is V_c , then the Laplace transform of the input excitation to winding "B" is given by:

$$E_c = -\frac{V_c}{s} \qquad (2\text{-}47)$$

In a similar manner, other partial fraction expansions for Equations (2-40) and (2-43) after the value of E_c as in Equation (2-47) has been substituted may be carried out. The components of currents resulting from input excitation to winding "B" acting alone-initially relaxed system in this case - will be as follows:

$$i_e''(t) = -K_{B2} V_c [1 + c_{17} \epsilon^{-t/\tau_c} + c_{18} \epsilon^{-t/\tau_s} + c_{19} \epsilon^{-t/\tau_e'}] \qquad (2\text{-}48)$$

and

$$i_f''(t) = -K_{B3} V_c [1 + c_{20} \epsilon^{-t/\tau_c} + c_{21} \epsilon^{-t/\tau_s} + c_{22} \epsilon^{-t/\tau_e'} + c_{23} \epsilon^{-t/\tau_f'}] \qquad (2\text{-}49)$$

where,

$$
\begin{aligned}
c_{17} &= \frac{\tau_c^2}{(\tau_c-\tau_s)(\tau_c-\tau_e')} \\
c_{18} &= \frac{\tau_s^2}{(\tau_s-\tau_c)(\tau_s-\tau_e')} \\
c_{19} &= \frac{\tau_e'^2}{(\tau_e'-\tau_c)(\tau_e'-\tau_s)} \\
c_{20} &= \frac{-\tau_c^3}{(\tau_c-\tau_s)(\tau_c-\tau_e')(\tau_c-\tau_f')} \\
c_{21} &= \frac{-\tau_s^3}{(\tau_s-\tau_c)(\tau_s-\tau_e')(\tau_s-\tau_f')} \\
c_{22} &= \frac{-\tau_e'^3}{(\tau_e'-\tau_c)(\tau_e'-\tau_s)(\tau_e'-\tau_f')} \\
c_{23} &= \frac{-\tau_f^3}{(\tau_f'-\tau_c)(\tau_f'-\tau_e')(\tau_f'-\tau_e')}
\end{aligned}
\qquad (2\text{-}50)
$$

In the above, two switching processes in the input excitation circuits of the amplidyne have been analyzed. If the two processes are simultaneously initiated, then one can apply the principle of superposition to obtain the total solutions. It is clear that solutions for the total exciter and field currents will start from some particular initial values and will then decay towards some final steady state negative values. The zero values of currents will occur at some instants on the time-axis which will essentially depend upon the magnitude of the negative forcing voltage applied. Obviously, there will then exist a requirement to remove this negative voltage at some suitable instant of time before both the exciter and field currents start reversing directions.

2.7 Exciter and Field Currents, with Negative Forcing Feedback:

The requirement of removing the negative forcing voltage at some suitable instant of time as mentioned in the previous section poses some difficulties. Some auxiliary equipment including under-current or under-voltage relays in conjunction with some timing element may fulfill this requirement. However, the system will be complicated and some sort of automatic features may prove more favorable. A reasonable means is necessary to assure that the negative forcing voltage to the B-winding is related to the decaying field current itself. Figure 8(a) is a schematic diagram for such a provision and Figure 8(b) is a block diagram representation of it. This arrangement has the following advantages:

1. The negative signal is automatically removed from the "B" winding at the same instant that the field current reaches its zero value without the necessity of any auxiliary equipment.

2. The applied negative forcing voltage depends essentially on the behavior of the field current, thus providing more bucking voltage with increased field current values.

It will be evident that Figure 8(b) is very similar to that of Figure 7 except for the feedback block represented by H. The value of H depends upon the parameters of the B-winding as well as upon the value of resistance R_s in the field circuit across which the negative voltage is taken.

Therefore, by closing switch s_1, and opening switch s_2 simultaneously, a negative voltage signal from the decaying field current will be applied to the B-winding. This negative voltage should evidently decay as fast as the field current itself.

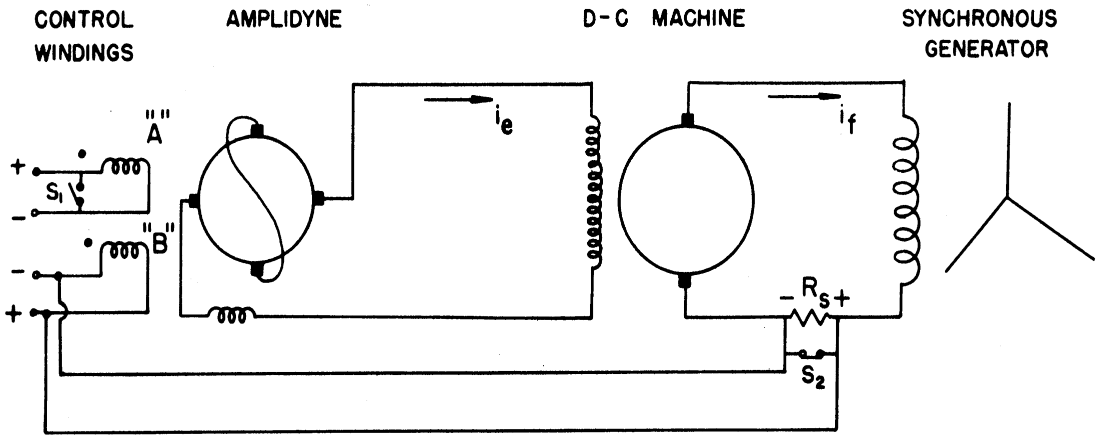


Figure 8a. Schematic Diagram for Negative Forcing Feedback Method.

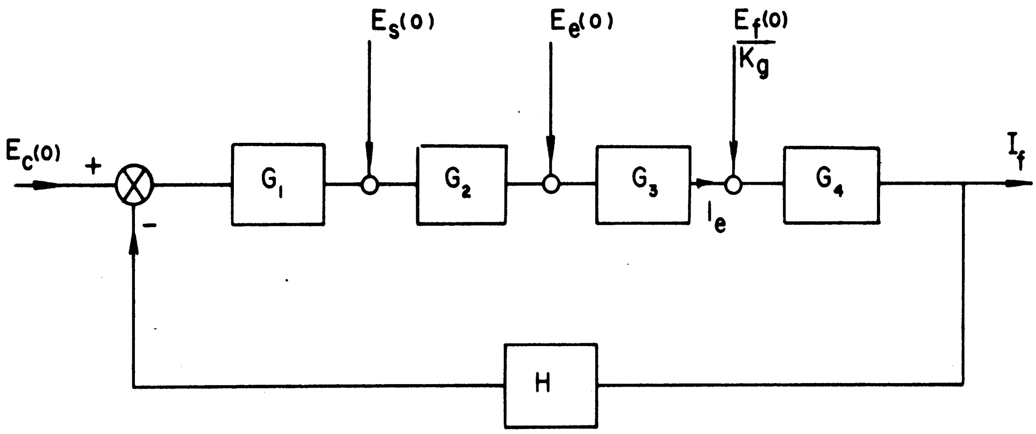


Figure 8b. Block Diagram Representation of Figure 8a.

Physically, this arrangement amounts to forcing the field current to its zero value by utilizing the decaying positive energy in the output field circuit. Thus, by reversing this positive energy, the decay of field current will be accelerated.

It should be mentioned at this occasion that the only restriction on such a procedure is to limit the value of R_s so that the system stability will be maintained.

The determination of exciter and field currents with such an arrangement will be somewhat more complicated because of the equivalent inputs injected between individual blocks. However, for the linearized system, one can apply superposition to determine the components of currents due to each input acting alone. Figure 9 shows the individual block diagram representations that will yield responses of different inputs acting alone. It will be seen that the total exciter and field currents are given by,

$$I_e = I_{e1} + I_{e2} + I_{e3} + I_{e4} \qquad (2\text{-}51)$$

$$I_f = I_{f1} + I_{f2} + I_{f3} + I_{f4} \qquad (2\text{-}52)$$

It is more convenient to solve for the field current components first. Next, the corresponding exciter current components can be easily determined. Before this is done, an expression for H the feedback loop system function can be shown to have the following relationship:

$$H = R_s \frac{K_{B3}(1+s\tau_{cA})}{K_{A3}(1+s\tau_{cB})} \qquad (2\text{-}53)$$

where it is assumed that the B-winding has no shunting effect on R_s since Z_B is much greater than R_s . For the case where the time constants of

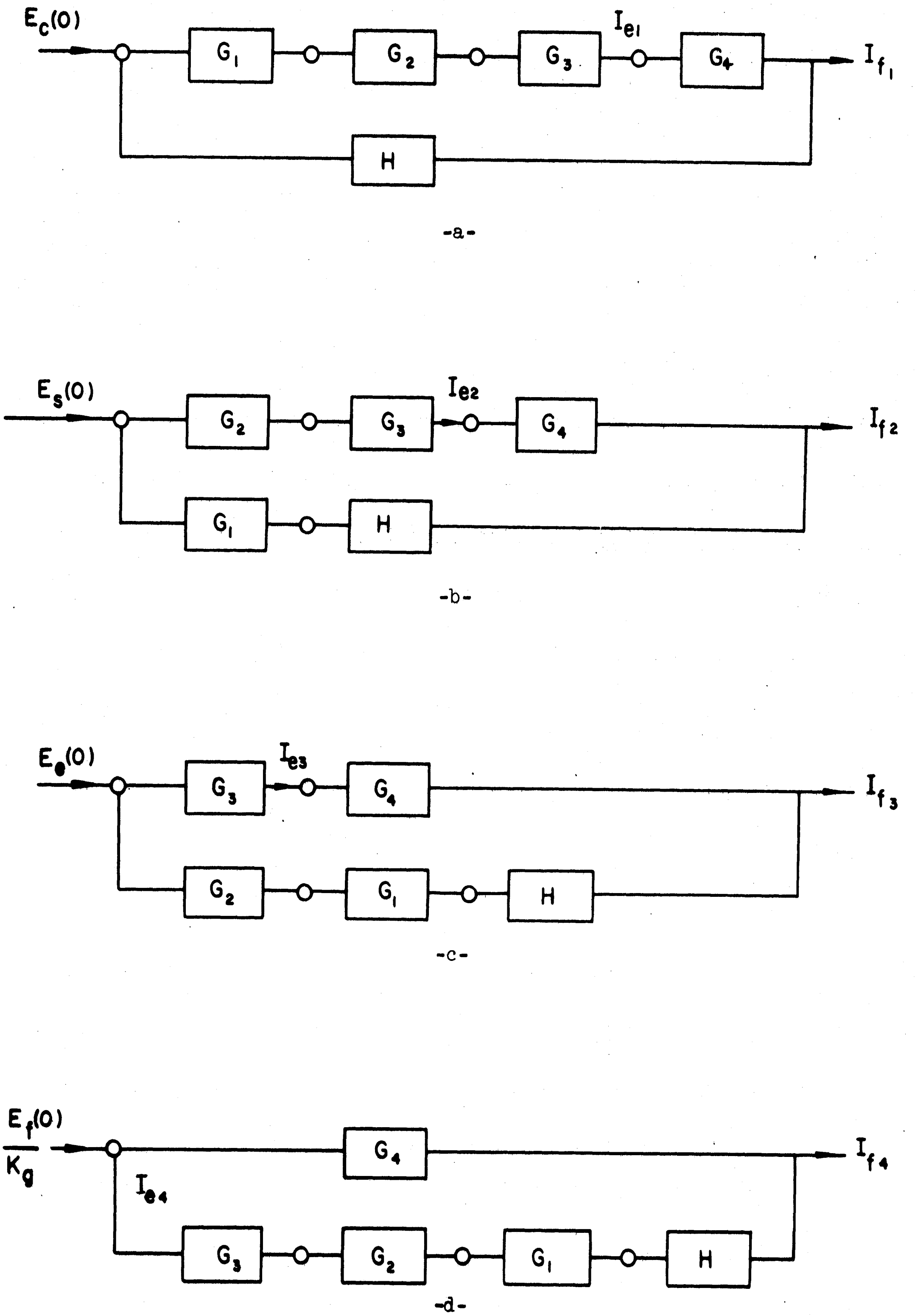


Figure 9. Block Diagram Representation for exciter and field current components.

both windings are equal, Equation (2-53) for H reduces to the simpler relationship

$$H = R_s \frac{K_{B3}}{K_{A3}} \tag{2-54}$$

Referring back to Figure 9, it is clear that I_{f1} will be given by the following relationship,

$$I_{f1} = \frac{G'}{1+G'H} E_c(0) \tag{2-55}$$

where,

$$\begin{aligned} G' &= G_1 \, G_2 \, G_3 \, G_4 \\ &= \frac{K_{A3}}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} \end{aligned} \tag{2-56}$$

By substituting the value of G' and H as given by Equations (2-56) and (2-54) respectively, one has

$$I_{f1} = \frac{K_{A3}}{K_{B3} \, R_s + (1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_c(0) \tag{2-57}$$

In a similar fashion and by referring to Figure 9 again for the other field current components, one can show that

$$I_{f2} = \frac{K_2(1+s\tau_c)}{K_{B3} \, R_s + (1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_s(0) \tag{2-58}$$

$$I_{f3} = \frac{K_3(1+s\tau_c)(1+s\tau_s)}{K_{B3} \, R_s + (1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_e(0) \tag{2-59}$$

$$I_{f4} = \frac{K_4(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')}{K_{B3} \, R_s + (1+s\tau_c)(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')} E_f(0) \tag{2-60}$$

Now, the polynomial of the denominator for all the above expressions as given by Equations (2-57) to (2-60) are the same. All of the terms agree with that of the opened-loop system except for the addition of $K_{B3} R_s$ to the absolute term. However, the numerator in each of the above expressions is different from the other. This difference will change the values of the coefficients in the partial fraction expansions, but the form of solution will be the same.

Evidently, the increased value of the absolute term in the denominator will move the location of the roots from the real axis. It will be shown later for a specific practical application that the resulting quartic equation for the denominator will have two pairs of conjugate roots. As a result, the time-domain solution for the field current components will be composed of damped sinusoidal and cosinusoidal terms. The rate of decay of the total field current will be governed by two effective time constants one of which is shorter than the field circuit time constant.

Finally, by referring to Figure 9, one can show that the exciter current components are given by:

$$I_{e1} = \frac{K_{A2}}{K_{A3}} (1+s\tau_f') I_{f1} \qquad (2\text{-}61)$$

$$I_{e2} = \frac{K_2'}{K_2} (1+s\tau_f') I_{f2} \qquad (2\text{-}62)$$

$$I_{e3} = \frac{K_3'}{K_3} (1+s\tau_f') I_{f3} \qquad (2\text{-}63)$$

$$I_{e4} = -\frac{R_s K_{A2} K_{B3}}{K_{A3}} \frac{1}{(1+s\tau_c)(1+s\tau_s)(1+s\tau_e')} I_{f4} \qquad (2\text{-}64)$$

where it can be seen that the time-domain solution for the exciter current component terms has the same form of solution as the field current terms.

Later in Chapter IV, where numerical values will be assigned for a specefic application, more insight will be gained for such a procedure.

2.8 The Total System - Synchronous Machine - D.C Generator-Amplidyne Combination.

As a final step, it is beneficial to include the a-c terminal voltage of the synchronous machine in the analysis. For the unloaded synchronous machine, the terminal voltage is more indicative of the potentiality of such a method than are the field or exciter currents. There exists however, a slight difficulty due to the induction of secondaty currents in the field poles and damper paths as a result of changes in field current. For accurate calculations, one has to account for a large number of closed circuits mutually coupled to the field circuit as well as to each other. However, a single closed equivalent circuit of resistance r_{Kd}, and inductance L_{Kd} has been found to be a satisfactory representation for these circuits. The situation is depicted in Figure 10 where M is to represent the mutual inductance coefficient between the equivalent eddy currents and damper paths, and the field circuit.

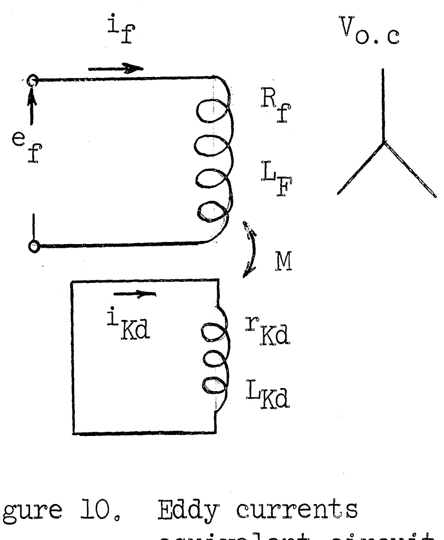


Figure 10. Eddy currents equivalent circuit.

It should be realized that the induced secondary currents will tend to sustain the flux at its initial value for some time. The terminal voltage being proportional to the air gap flux will accordingly lag in time. It is shown in Appendix A that the effect of an equivalent "eddy-current circuit" is to increase the time constant of both the field circuit and the terminal voltage. Thus, at least to the accuracy of the assumptions made in this development, an effective time constant τ'_{do} would be more indicative of the actual behavior than simply τ_f where,

$$\tau'_{do} = \tau_f + \tau_{Kd} \tag{2-65}$$

In other words, the open circuit time constant τ'_{do} for the synchronous machine may be looked upon as the equivalent field circuit time constant including eddy-current delay effects. According to the recommended IEEE (AIEE) Test Code for Synchronous Machines,(24) τ'_{do} is defined in Article 1.910 as follows:

> "The direct-axis transient open-circuit time constant is the time in seconds required for the rms a-c value of the slowly decreasing component present in the direct-axis component of symmetrical armature voltage on open-circuit to decrease to $1/\epsilon$ or 0.368 of its initial value when the field winding is suddenly short-circuited with the machine running at rated speed."

The same reference under Article 1.911 describes how to determine this time constant by test. Appendix A is actually a theoretical derivation for the above described test with the major assumption of representing the eddy-currents and damper effects by a single closed circuit. It is convenient to re-write the final expressions for $i_f(t)$, $i_{Kd}(t)$ and $v_t(t)$ as derived in Appendix A in terms of τ'_{do} so that,

$$i_f(t) = \frac{\tau_{Kd}}{\tau_f} i_f(0) \left[\frac{\tau_f}{\tau_{Kd}} \epsilon^{-t/\tau'_{do}} + \epsilon^{-\frac{t\,\tau'_{do}}{\sigma\tau_f\tau_{Kd}}}\right] \tag{2-66}$$

$$i_{Kd}(t) = -\sqrt{\frac{L_f}{L_{Kd}}\frac{\tau_{Kd}}{\tau_f}}\; i_f(0)\; [\epsilon^{-t/\tau'_{do}} - \epsilon^{-\frac{t\,\tau'_{do}}{\sigma\tau_f\tau_{Kd}}}] \qquad (2\text{-}67)$$

$$v_t(t) = K\, N_f\, i_f(0)\, \epsilon^{-t/\tau'_{do}} \qquad (2\text{-}68)$$

It is desired to examine the above expressions more carefully. Equation (2-66) for the transient field current is described by two components governed by two different time constants. The first component is decaying at a slower rate and it has a more pronounced effect particularly a few moments after the transient current starts decaying. In some applications, the second component may be completely neglected without affecting the accuracy of the results very much. If this assumption is carried out, then Equation (2-62) for the field current reduces to the simpler form:

$$i_f(t) = i_f(0)\, \epsilon^{-t/\tau'_{do}} \qquad (2\text{-}69)$$

The expressions already derived in the last two sections for field current would then still hold except that τ'_{do} the effective time constant should replace τ_f . As a result of this consideration and having determined the field current response, the terminal voltage response could easily be determined by referring to the open-circuit magnetization curve for the synchronous machine.

It is instructive at this stage to draw some interesting conclusions for the experimental plot of the decaying field current that may be obtained during the same test carried out to determine τ'_{do}.

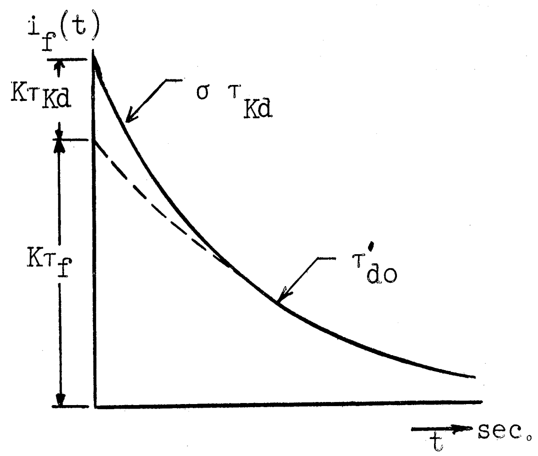


Figure 11. Field Current Decay.

Figure 11 is a rough representation of the decay of field current as given by Equation (2-66). The first component of the field current decays with a time constant given by:

$$\tau'_{do} = \tau_f + \tau_{Kd}$$

whereas the second component is decaying with a time constant given by:

$$\frac{\sigma \tau_f \tau_{Kd}}{\tau'_{do}} \cong \sigma \tau_{Kd} \qquad (2\text{-}70)$$

The relative magnitudes of the second to the first components for the initial values of field current as given by Equation (2-66) is seen to be in the ratio of $\tau_f : \tau_{Kd}$. This suggests an elegant method to determine τ_{Kd} from the experimental plot of the decaying field current. The importance of the parameter τ_{Kd} will be realized later in Chapter III when the occasion arises to determine the transient short circuit field current.

Moreover, the leakage coefficient represented by σ can be determined from a semilogarithmic plot of the first component of the decaying field current.

CHAPTER III

THE SYNCHRONOUS MACHINE UNDER THREE-PHASE SHORT-CIRCUIT

3.1 Introduction

In the previous chapter an analysis for demagnetization of the unloaded synchronous machine was carried out. The response in which the currents is forced to zero value, was derived for two cases. In both cases, the normal input excitation winding of the amplidyne was de-energized and at the same instant, a negative forcing voltage was applied across another winding of the amplidyne. In one case, the forcing voltage was a negative step while in the other case, it was a negative voltage proportional to the decaying field current of the generator.

In order to account for the actual performance, it is necessary to extend the analysis to include the synchronous machine under fault conditions. The field current will be shown to be the most important quantity for studying the effectiveness of demagnetization operations for different schemes. Hence, the field current will be used to formulate the equations under fault conditions. The results can thus be expressed in terms of the field current when a three-phase fault occurs at the machine terminals. Moreover, expressions will be given for the field current during the demagnetization period. It should be pointed out that the three-phase case is considered because it represents the most severe condition as far as the field current response is concerned.[1,2]

3.2 The Ideal Synchronous Machine

The machine will be idealized to the extent that the transient phenomena associated with fault conditions can be emphasized. The coupled circuit approach is followed where the machine can be looked upon as sets of linear circuits in relative motion. The stator consists of three identical circuits symmetrically distributed and the field winding is on the rotor. Each winding of the actual machine is represented by a single coil. Thus, there are three distributed armature coils a, b, and c displaced 120 electrical degrees from each other as indicated in Figure 12. The magnetic axes of the phase windings are assumed to coincide with the coil axes. The field winding is assumed to be symmetrical about the direct axis. Generally, a synchronous machine has additional circuits called damper or amortisseur circuits. There are also eddy-current paths in the iron, whether the rotor is solid or laminated. In order to represent the effects of these circuits in detail, a very large number of equations would have to be included. However, the effects of these circuits are usually lumped in one equivalent damper coil Kd on the direct axis and one equivalent damper coil Kq on the quadrature axis.[10-15] It should be realized that the damper circuits in turbo-altermators consist of eddy-current paths in the solid rotor body. In such cases, the damper circuits are not well defined and the problem becomes more involved. The derived equations can be applied to both nonsalient-pole as well as salient-pole machines provided suitable parameters are used in the approximate equivalent circuits.

The rotor is assumed to rotate in the clockwise direction, and θ is the angle from the axis of phase a to the d-axis. The corresponding angle from phase b to the d-axis is θ-120° while the angle from phase c is θ+120°.

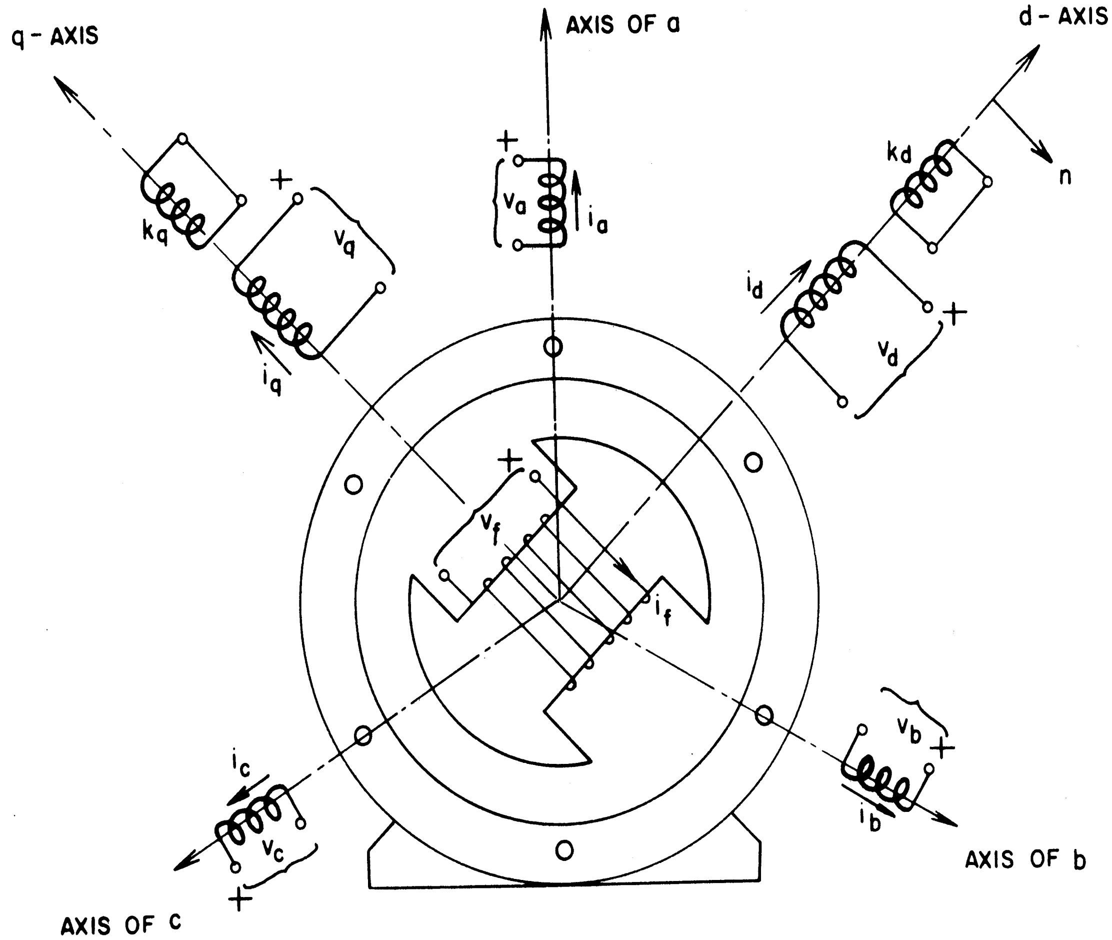


Figure 12. Equivalent Circuit Diagram of Synchronous Generator.

Each of the circuits a , b , c , Kd , Kq and f has its own resistance, self-inductance, and mutual inductance with respect to every other circuit. However, the circuits f and Kd are assumed to have no inductive coupling with the circuit Kq.

Saturation is neglected although it can be accounted for in an approximate manner by assigning different inductance values according to the degree of saturation. Since the coefficients of the differential equations describing the system are periodic functions of the rotor angle θ ,their solution will be rather involved and difficult. To simplify the analysis, use will be made of Park's dqo transformation of variables such that with reasonable assumptions the resulting differential equations are linear with constant coefficients. These assumptions are[10-17]

1. The armature windings are sinusoidally distributed to the extent that in practical machines the harmonics are usually minimized.

2. The effect of stator slots is neglected so that the self-inductances of the rotor circuits are assumed constant.

3. The space M.M.F. and flux waves are sinusoidally distributed.

Using the sign conventions proposed by W.A. Lewis[13] in which the stator circuits are considered as sources and the field circuits as loads, one can write the following set of equations:

$$v_a = - \frac{d\lambda_a}{dt} - r\, i_a \qquad (3\text{-}1a)$$

$$v_b = - \frac{d\lambda_b}{dt} - r\, i_b \qquad (3\text{-}1b)$$

$$v_c = -\frac{d\lambda_c}{dt} - r\, i_c \qquad (3\text{-}1c)$$

$$v_f = \frac{d\lambda_f}{dt} + r_f\, i_f \qquad (3\text{-}1d)$$

$$0 = \frac{d\lambda_{Kd}}{dt} + r_{Kd}\, i_{Kd} \qquad (3\text{-}1e)$$

$$0 = \frac{d\lambda_{Kq}}{dt} + r_{Kq}\, i_{Kq} \qquad (3\text{-}1f)$$

For the flux-linkage relations, with the assumption that the flux linkages of a circuit due to a positive current in it are also positive, one can write

$$\lambda_a = L'_{aa}\, i_a + L'_{ab}\, i_b + L'_{ac}\, i_c + L'_{af}\, i_f + L'_{aKd}\, i_{Kd} + L'_{aKq}\, i_{Kq} \qquad (3\text{-}2a)$$

$$\lambda_b = L'_{ba}\, i_a + L'_{bb}\, i_b + L'_{bc}\, i_c + L'_{bf}\, i_f + L'_{bKd}\, i_{Kd} + L'_{bKq}\, i_{Kq} \qquad (3\text{-}2b)$$

$$\lambda_c = L'_{ca}\, i_a + L'_{cb}\, i_b + L'_{cc}\, i_c + L'_{cf}\, i_f + L'_{cKd}\, i_{Kd} + L'_{cKq}\, i_{Kq} \qquad (3\text{-}2c)$$

$$\lambda_f = L'_{fa}\, i_a + L'_{fb}\, i_b + L'_{fc}\, i_c + L'_{ff}\, i_f + L'_{fKd}\, i_{Kd} + 0\, i_{Kq} \qquad (3\text{-}2d)$$

$$\lambda_{Kd} = L'_{Kda}\, i_a + L'_{Kdb}\, i_b + L'_{Kdc}\, i_c + L'_{Kdf}\, i_f + L'_{KKd}\, i_{Kd} + 0\, i_{Kq} \qquad (3\text{-}2e)$$

$$\lambda_{Kq} = L'_{Kqa}\, i_a + L'_{Kqb}\, i_b + L'_{Kqc}\, i_c + 0\, i_f + 0\, i_{Kd} + L'_{KKq}\, i_{Kq} \qquad (3\text{-}2f)$$

According to the previous idealizing assumptions relative to symmetry of the rotor and the sinusoidal winding distribution, the stator self-and mutual inductances depend upon rotor position. The self-inductance of any stator phase will consist of a positive fixed component plus a second-harmonic variation effect because of the different permeance values in the d- and q-axes. The mutual inductance between stator phases will also have a second-harmonic term whose coefficient is the same as that of the self-inductance, but with different phase shift angle. The mutual-inductances between stator and rotor circuits are seen to vary in a sinusoidal fashion with the angle θ, attaining a maximum value when the axis of the rotor winding is lined up with the axis of any coil in the stator.(15,16) This again is based on the classic assumption of considering only the space-fundamental component of flux that links the sinusoidally assumed distributed stator winding.

In accordance with the above considerations, one can write the following expressions for the inductance relations:

$$L'_{aa} = L_{aao} + L_{aav} \cos 2\theta \quad (3\text{-}3a)$$

$$L'_{bb} = L_{aao} + L_{aav} \cos (2\theta + 120°) \quad (3\text{-}3b)$$

$$L'_{cc} = L_{aao} + L_{aav} \cos (2\theta - 120°) \quad (3\text{-}3c)$$

$$L'_{ab} = L'_{ba} = - L_{abo} + L_{aav} \cos (2\theta - 120°) \quad (3\text{-}3d)$$

$$L'_{bc} = L'_{cb} = - L_{bco} + L_{aav} \cos 2\theta \quad (3\text{-}3e)$$

$$L'_{ca} = L'_{ac} = - L_{cao} + L_{aav} \cos (2\theta + 120°) \quad (3\text{-}3f)$$

$$L'_{af} = L'_{fa} = L_{af} \cos \theta \quad (3\text{-}3g)$$

$$L'_{bf} = L'_{fb} = L_{af} \cos (\theta - 120°) \quad (3\text{-}3h)$$

$$L'_{cf} = L'_{fc} = L_{af} \cos (\theta + 120°) \quad (3\text{-}3i)$$

$$L'_{aKd} = L'_{Kda} = L_{aKd} \cos \theta \quad (3\text{-}3j)$$

$$L'_{bKd} = L'_{Kdb} = L_{aKd} \cos(\Theta - 120°) \qquad (3\text{-}3k)$$

$$L'_{cKd} = L'_{Kdc} = L_{aKd} \cos(\Theta + 120°) \qquad (3\text{-}3l)$$

$$L'_{aKq} = L'_{Kqa} = -L_{aKq} \sin\Theta \qquad (3\text{-}3m)$$

$$L'_{bKq} = L'_{Kqb} = -L_{aKq} \sin(\Theta - 120°) \qquad (3\text{-}3n)$$

$$L'_{cKq} = L'_{Kqc} = -L_{aKq} \sin(\Theta + 120°) \qquad (3\text{-}3o)$$

Substituting the inductance values as given by Equations (3-3g) to (3-3o) in the rotor flux-linkage expressions as given by Equations (3-2d) to (3-2f) results in the following equations:

$$\lambda_f = L_{af}\,[i_a \cos\Theta + i_b \cos(\Theta-120°) + i_c \cos(\Theta+120°)] + L_{ff}\, i_f + L_{fKd}\, i_{Kd} \qquad (3\text{-}4a)$$

$$\lambda_{Kd} = L_{aKd}[i_a \cos\Theta + i_b \cos(\Theta-120°) + i_c \cos(\Theta+120°)] + L_{Kdf} i_f + L_{KKd}\, i_{Kd} \qquad (3\text{-}4b)$$

$$\lambda_{Kq} = -L_{aKq}[i_a \sin\Theta + i_b \sin(\Theta-120°) + i_c \sin(\Theta+120°)] + L_{KKq}\, i_{Kq} \qquad (3\text{-}4c)$$

Equations (3-4) lead to the familiar Park's transformation of the form:

$$i_d = K_d\,[i_a \cos\Theta + i_b \cos(\Theta-120°) + i_c \cos(\Theta+120°)] \qquad (3\text{-}5a)$$

$$i_q = -K_q[i_a \sin\Theta + i_b \sin(\Theta-120°) + i_c \sin(\Theta+120°)] \qquad (3\text{-}5b)$$

$$i_o = K_o\,[i_a + i_b + i_c] \qquad (3\text{-}5c)$$

where K_d and K_q have the value $\sqrt{\frac{2}{3}}$ in order to maintain reciprocity for the mutual inductances between the field and armature circuits as suggested by W.A. Lewis.[13] K_o is assigned a value of $\sqrt{\frac{1}{3}}$.

It should be realized that the newly introduced variables i_d, i_q and i_o transform the Θ-dependent terms as given by Equations (3-4) to new variables that are independent of Θ. The relation between the old and new variables is determined by the following equations written in matrix form:

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\Theta & \cos(\Theta-120°) & \cos(\Theta+120°) \\ -\sin\Theta & -\sin(\Theta-120°) & -\sin(\Theta+120°) \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{3-6}$$

or

$$i'] = [c][i \tag{3-7}$$

The inverse relationship is given by:

$$i] = [c]^{-1}[i' \tag{3-8}$$

where,

$$[c]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\Theta & -\sin\Theta & \sqrt{\frac{1}{2}} \\ \cos(\Theta-120°) & -\sin(\Theta-120°) & \sqrt{\frac{1}{2}} \\ \cos(\Theta+120°) & -\sin(\Theta+120°) & \sqrt{\frac{1}{2}} \end{bmatrix} \tag{3-9}$$

By substituting Equations (3-5) in the flux linkage equations as given by Equations (3-4), one obtains,

$$\lambda_f = \sqrt{\frac{3}{2}}\, L_{af}\, i_d + L_{ff}\, i_f + L_{fKd}\, i_{Kd} \tag{3-10a}$$

$$\lambda_{Kd} = \sqrt{\frac{3}{2}}\, L_{aKd}\, i_d + L_{Kdf}\, i_f + L_{KKd}\, i_{Kd} \tag{3-10b}$$

$$\lambda_{Kq} = \sqrt{\frac{3}{2}}\, L_{aKq}\, i_q + L_{KKq}\, i_{Kq} \tag{3-10c}$$

By analogy to Equations (3-5), the dqo flux linkage variables can be defined as,

$$\lambda_d = \sqrt{\frac{2}{3}}\, [\lambda_a \cos \Theta + \lambda_b \cos(\Theta-120°) + \lambda_c \cos(\Theta+120°)] \tag{3-11a}$$

$$\lambda_q = -\sqrt{\frac{2}{3}}\, [\lambda_a \sin \Theta + \lambda_b \sin (\Theta-120°) + \lambda_c \sin(\Theta+120°)] \tag{3-11b}$$

$$\lambda_o = \sqrt{\frac{1}{3}}\, [\lambda_a + \lambda_b + \lambda_c] \tag{3-11c}$$

The flux-linkage relations as given by Equations (3-2a) to (3-2c) can be substituted in Equations (3-11), and by substituting the inductance parameter values as in Equations (3-3), one ends up with the simpler relationships given by:

$$\lambda_d = L_d\, i_d + \sqrt{\frac{3}{2}}\, L_{af}\, i_f + \sqrt{\frac{3}{2}}\, L_{aKd}\, i_{Kd} \tag{3-12a}$$

$$\lambda_q = L_q\, i_q + \sqrt{\frac{3}{2}}\, L_{aKq}\, i_{Kq} \tag{3-12b}$$

$$\lambda_o = L_o\, i_o \tag{3-12c}$$

where,

$$L_d = L_{aao} + \frac{3}{2} L_{abo} + \frac{3}{2} L_{aav}\ , \tag{3-13a}$$

$$L_q = L_{aao} + \frac{3}{2} L_{abo} - \frac{3}{2} L_{aav}\ , \tag{3-13b}$$

$$L_o = L_{aao} \tag{3-13c}$$

The quantities L_d , L_q and L_o are defined as the direct-axis synchronous inductance, the quadrature-axis synchronous inductance, and the zero-sequence inductance(16) respectively, and can be easily determined by test.

3.3 The General Voltage Equations.

It can be proved that the choice of the transformation matirx [c] is such that it defines a Hermitian orthogonal one such that

$$[c]^{-1} = [c]_t^{*} \tag{3-14}$$

In this case the voltages transform by the same law as the currents so that

$$v_d = \sqrt{\frac{2}{3}}\,[v_a \cos\Theta + v_b \cos(\Theta-120°) + v_c \cos(\Theta+120°)] \tag{3-15a}$$

$$v_q = -\sqrt{\frac{2}{3}}\,[v_a \sin\Theta + v_b \sin(\Theta-120°) + v_c \sin(\Theta+120°)] \tag{3-15b}$$

$$v_o = \sqrt{\frac{1}{3}}\,[v_a + v_b + v_c] \tag{3-15c}$$

By substituting the voltage expressions as given by Equations (3-1a) to (3-1c) into Equations (3-15), one has the following set of equations:

$$v_d = -\sqrt{\frac{2}{3}}\,[\cos\Theta \frac{d\lambda_a}{dt} + \cos(\Theta-120°) \frac{d\lambda_b}{dt} + \cos(\Theta+120°) \frac{d\lambda_c}{dt}] - ri_d \tag{3-16a}$$

$$v_q = -\sqrt{\frac{2}{3}}\,[\sin\Theta \frac{d\lambda_a}{dt} + \sin(\Theta-120°) \frac{d\lambda_b}{dt} + \sin(\Theta+120°) \frac{d\lambda_c}{dt}] - ri_q \tag{3-16b}$$

$$v_o = -\frac{d\lambda_o}{dt} - ri_o \tag{3-16c}$$

Differentiating both sides of Equations (3-11a) and (3-11b) gives

$$\frac{d\lambda_d}{dt} = \sqrt{\frac{2}{3}}\left[\cos\theta\,\frac{d\lambda_a}{dt} + \cos(\theta-120°)\,\frac{d\lambda_b}{dt} + \cos(\theta+120°)\,\frac{d\lambda_c}{dt}\right]$$

$$- \sqrt{\frac{2}{3}}\left[\lambda_a \sin\theta\,\frac{d\theta}{dt} + \lambda_b \sin(\theta-120°)\,\frac{d\theta}{dt} + \lambda_c \sin(\theta+120°)\,\frac{d\theta}{dt}\right]$$

Substituting the value of λ_q as given by Equation (3-11b), the above expression reduces to:

$$\frac{d\lambda_d}{dt} = \sqrt{\frac{2}{3}}\left[\cos\theta\,\frac{d\lambda_a}{dt} + \cos(\theta-120°)\frac{d\lambda_b}{dt} + \cos(\theta+120°)\,\frac{d\lambda_c}{dt}\right] - \lambda_q\,\frac{d\theta}{dt} \quad (3\text{-}17)$$

Similarly, one can write for the quadrature-axis the following equation:

$$\frac{d\lambda_q}{dt} = \sqrt{\frac{2}{3}}\left[\sin\theta\,\frac{d\lambda_a}{dt} + \sin(\theta-120°)\frac{d\lambda_b}{dt} + \sin(\theta+120°)\,\frac{d\lambda_c}{dt}\right] + \lambda_d\,\frac{d\theta}{dt} \quad (3\text{-}18)$$

Equations (3-17) and (3-18), can now be used in conjunction with Equations (3-16) to write the following simplified forms for the direct and quadrature-axes voltage equations as:

$$v_d = -\frac{d\lambda_d}{dt} - \lambda_q\,\omega - r\,i_d \quad (3\text{-}19a)$$

$$v_q = -\frac{d\lambda_q}{dt} + \lambda_d\,\omega - r\,i_q \quad (3\text{-}19b)$$

3.4 The Synchronous Machine System Functions.

In order to adopt the same approach followed in the previous chapter, one should perform the Laplace transform operations on Equations (3-10a) to (3-10c), (3-12a), (3-12b) and (3-19a) to (3-19e), and by identifying the initial value contribution terms, one has the following set of equations:

$$\Lambda_d = \sqrt{\frac{3}{2}}\,L_{af}\,I_f + L_d\,I_d + \sqrt{\frac{3}{2}}\,L_{aKd}\,I_{Kd} \quad (3\text{-}20a)$$

$$\Lambda_q = L_q I_q + \sqrt{\tfrac{3}{2}}\, L_{aKq} I_{Kq} \qquad (3\text{-}20b)$$

$$\Lambda_f = L_{ff} I_f + \sqrt{\tfrac{2}{3}}\, L_{af} I_d + L_{fKd} I_{Kd} \qquad (3\text{-}20c)$$

$$\Lambda_{Kd} = L_{Kdf} I_f + \sqrt{\tfrac{3}{2}}\, L_{aKd} I_d + L_{KKd} I_{Kd} \qquad (3\text{-}20d)$$

$$\Lambda_{Kq} = L_{KKq} I_q + \sqrt{\tfrac{3}{2}}\, L_{aKq} I_q \qquad (3\text{-}20e)$$

and

$$s\Lambda_d + \omega\Lambda_q + r\, I_d = -V_d + \lambda_d(0) \qquad (3\text{-}21a)$$

$$s\Lambda_q - \omega\Lambda_d + r\, I_q = -V_q + \lambda_q(0) \qquad (3\text{-}21b)$$

$$s\Lambda_f + r_f I_f = V_f + \lambda_f(0) \qquad (3\text{-}21c)$$

$$s\Lambda_{Kd} + r_{Kd} I_{Kd} = \lambda_{Kd}(0) \qquad (3\text{-}21d)$$

$$s\Lambda_{Kq} + r_{Kq} I_{Kq} = \lambda_{Kq}(0) \qquad (3\text{-}21e)$$

Equations (3-20) and (3-21) constitute the starting point for any general transient analysis of the synchronous machine under different types of faults. For any mode of machine operation, different initial value contribution terms can be assigned from which the solutions may be established. For a three-phase short circuit, the terminal voltages v_a , v_b and v_c are all zero, and consequently V_d and V_q can be substituted by zero in Equations (3-21a) and (3-21b) for this type of fault. Furthermore, the initial value contribution terms in Equations (3-21) will assume the following values:

$$\lambda_d(0) = \sqrt{\tfrac{3}{2}}\, L_{af}\, i_f(0) + L_d\, i_d(0) \qquad (3\text{-}22a)$$

$$\lambda_q(0) = L_q\, i_q(0) \tag{3-22b}$$

$$\lambda_f(0) = L_{ff}\, i_f(0) + \sqrt{\tfrac{3}{2}}\, L_{af}\, i_d(0) \tag{3-22c}$$

$$\lambda_{Kd}(0) = L_{Kdf}\, i_f(0) + \sqrt{\tfrac{3}{2}}\, L_{aKd}\, i_d(0) \tag{3-22d}$$

$$\lambda_{Kq}(0) = \sqrt{\tfrac{3}{2}}\, L_{aKq}\, i_q(0) \tag{3-22e}$$

where all the currents in the damper winding assume zero values at the instant of short-circuit.

Substituting the values of flux linkages as given by Equations (3-20) into Equations (3-21) yield the following set of equations:

$$(sL_d+r)I_d + (\omega L_q)I_q + \left(\sqrt{\tfrac{3}{2}}\, sL_{af}\right)I_f + \left(\sqrt{\tfrac{3}{2}}\, sL_{aKd}\right)I_{Kd} + \left(\sqrt{\tfrac{3}{2}}\, \omega L_{KKq}\right)I_{Kq} = \lambda_d(0) \tag{3-23a}$$

$$(-\omega L_d)I_d + (sL_q+r)I_q + \left(-\sqrt{\tfrac{3}{2}}\, \omega L_{af}\right)I_f + \left(-\sqrt{\tfrac{3}{2}}\, \omega L_{aKd}\right)I_{Kd} + \left(\sqrt{\tfrac{3}{2}}\, sL_{aKq}\right)I_{Kq} = \lambda_q(0) \tag{3-23b}$$

$$\left(\sqrt{\tfrac{3}{2}}\, sL_{af}\right)I_d + (0)I_q + (sL_{ff}+r_f)I_f + (sL_{fKd})I_{Kd} + (0)I_{Kq} = V_f + \lambda_f(0) \tag{3-23c}$$

$$\left(\sqrt{\tfrac{3}{2}}\, sL_{aKd}\right)I_d + (0)I_q + (sL_{Kdf})I_f + (sL_{KKd}+r_{Kd})I_{Kd} + (0)I_{Kq} = \lambda_{Kd}(0) \tag{3-23d}$$

$$(0)I_d + \left(\sqrt{\tfrac{3}{2}}\, sL_{KKq}\right)I_q + (0)I_f + (0)I_{Kd} + (sL_{KKq}+r_{Kq})I_{Kq} = \lambda_{Kq}(0) \tag{3-23e}$$

The above relationships between currents and initial value contribution terms can be expressed in matrix form as:

$$Z_{dqf}^{KdKq}\, I_{dqf}^{KdKq} = V_{dqf} + \lambda_{i.v} \tag{3-24}$$

where,

$$Z_{dqf}^{KdKq} = \begin{bmatrix} s\,L_d + r & \omega L_q & \sqrt{\frac{3}{2}}\,s\,L_{af} & \sqrt{\frac{3}{2}}\,s\,L_{aKd} & \sqrt{\frac{3}{2}}\,\omega\,L_{aKq} \\ -\,\omega L_d & s\,L_q + r & -\sqrt{\frac{3}{2}}\,\omega L_{af} & -\sqrt{\frac{3}{2}}\,\omega\,L_{aKd} & \sqrt{\frac{3}{2}}\,s\,L_{aKq} \\ \sqrt{\frac{3}{2}}\,s\,L_{af} & 0 & s\,L_{ff} + r_f & s\,L_{fKd} & 0 \\ \sqrt{\frac{3}{2}}\,s\,L_{aKd} & 0 & s\,L_{Kdf} & s\,L_{KKd} + r_{Kd} & 0 \\ 0 & \sqrt{\frac{3}{2}}\,s\,L_{aKq} & 0 & 0 & sL_{KKq}+r_{Kq} \end{bmatrix} \qquad (3\text{-}25)$$

$$I_{dqf}^{KdKq} = \begin{bmatrix} I_d \\ I_q \\ I_f \\ I_{Kd} \\ I_{Kq} \end{bmatrix} \qquad (3\text{-}26) \qquad\qquad V_{dqf} = \begin{bmatrix} 0 \\ 0 \\ V_f \\ 0 \\ 0 \end{bmatrix} \qquad (3\text{-}27)$$

$$\lambda_{i.f} = \begin{bmatrix} \lambda_d(0) \\ \lambda_q(0) \\ \lambda_f(0) \\ \lambda_{Kd}(0) \\ \lambda_{Kq}(0) \end{bmatrix} \qquad (3\text{-}28)$$

Equation (3-24) is a rather general representation for the transient behavior of the synchronous machine. For the particualr case of a three-phase short-circuit at the machine terminals, V_{dqf} and $\lambda_{i.v}$ assume the values given by Equations (3-27) and (3-28) respectively. Thus, by some matrix manipulations, expressions for I_d , I_q and I_f can be easily obtained. However, it is desirable to identify some particular system functions commonly called operational impedances.[11,12] This can be accomplished by resolving Z_{dqf}^{KdKq} in a manner that will yield these functions. The main purpose of this development is the simplicity of expressing these system functions in terms of well-established and familiar machine constants that can be experimentally determined by means of standard tests.

Before carrying this out, the initial value flux linkage terms as given by Equations (3-22) are simplified for the particular application of the unloaded synchronous machine subjected to a three-phase short-circuit at its terminals. In such a case, $i_d(0)$ and $i_q(0)$ assume zero values and Equations (3-22) reduce to the following:

$$\lambda_d(0) = \sqrt{\frac{3}{2}}\, L_{af}\, i_f(0) \qquad (3\text{-}29a)$$

$$\lambda_q(0) = 0 \qquad (3\text{-}29b)$$

$$\lambda_f(0) = L_{ff}\, i_f(0) \qquad (3\text{-}29c)$$

$$\lambda_{Kd}(0) = L_{Kdf}\, i_f(0) \qquad (3\text{-}29d)$$

$$\lambda_{Kq}(0) = 0 \qquad (3\text{-}29e)$$

Moreover, assuming no voltage regulator action for the few moments after short-circuit is effected, then

$$V_f = r_f \frac{i_f(0)}{s} \qquad (3\text{-}30)$$

In order to express I_{Kq}, I_{Kd} and I_f as functions of I_d and I_q, one substitutes Equation (3-29e) into Equation (3-23e) so that

$$I_{Kq} = - \frac{\sqrt{\frac{3}{2}}\, s\, L_{KKq}}{s\, L_{KKq}+r_{Kq}}\, I_q \tag{3-31}$$

Substituting Equation (3-29d) into Equation (3-23d), one can write:

$$I_{Kd} = - \frac{\sqrt{\frac{3}{2}}\, s\, L_{aKd}}{sL_{KKd}+r_{Kd}}\, I_d - \frac{s\, L_{Kdf}}{sL_{KKd}+r_{Kd}}\, I_f + \frac{L_{Kdf}}{sL_{KKd}+r_{Kd}}\, i_f(0) \tag{3-32}$$

Substituting Equation (3-32) into Equation (3-23c) and using Equations (3-29c) and (3-30), and arranging terms, one has,

$$I_f = - s\, G\, I_d + \frac{i_f(0)}{s} \tag{3-33}$$

where,

$$G = \sqrt{\frac{3}{2}}\; \frac{L_{af}(sL_{KKd}+r_{Kd}) - sL_{aKd}L_{fKd}}{(sL_{ff}+r_f)(sL_{KKd}+r_{Kd}) - s^2L_{fKd}^2} \tag{3-34}$$

is the first identified system function to express I_d as a function of I_f in operational notation and is known in the literature.(11,12)

Substituting the value of I_f as given by Equation (3-33) back into Equation (3-32), one has the following relationship,

$$I_{Kd} = \frac{s^2L_{Kdf}\, G - \sqrt{\frac{3}{2}}\, s\, L_{aKd}}{s\, L_{KKd}+ r_{Kd}}\, I_d \tag{3-35}$$

By substituting Equations (3-31), (3-33) and (3-35) for I_{Kq} , I_f and I_{Kd} respectively into Equations (3-23a) and (3-23b) and using Equations (3-29a) and (3-29b), one can arrive at the following two simultaneous transform equations in I_d and I_q as follows:

$$(r+sX_d)I_d + (\omega X_q)I_q = 0 \tag{3-36}$$

$$(-\omega X_d)I_d + (r+sX_q)I_q = \sqrt{3}\, e_{iqo} \tag{3-37}$$

where,

$$X_d = \frac{s^2[L_d''(L_{KKd}L_{ff}-L_{fKd}^2)]+s[r_f(L_dL_{KKd}-\frac{3}{2}L_{aKd}^2)+r_{Kd}(L_dL_{ff}-\frac{3}{2}L_{af}^2)]+L_dr_fr_{Kd}}{s^2[L_{KKd}L_{ff}-L_{fKd}^2] + s[r_fL_{KKd}+r_{Kd}L_{ff}] + r_fr_{Kd}} \tag{3-38}$$

is the direct-axis system function with the subtransient reactance defined as:

$$x_d'' = \omega L_d'' = \lim_{s\to\infty} \omega X_d$$

$$= \omega[L_d - \frac{3}{2}\frac{L_{af}^2L_{KKd} + L_{aKd}^2L_{ff} - 2L_{af}L_{fKd}L_{aKd}}{L_{KKd}L_{ff} - L_{fKd}^2}] \tag{3-39}$$

and

$$X_q = \frac{s(L_qL_{KKq} - \frac{3}{2}sL_{aKq}^2) + r_{Kq}L_k}{sL_{KKq} + r_{Kq}} \tag{3-40}$$

is the quadrature-axis system function, and

$$e_{iqo} = \frac{\omega L_{af}}{\sqrt{2}} i_f(0) \tag{3-41}$$

is the quadrature-axis internal voltage and is equal to the open-circuit voltage at normal speed operation.

It should be noted that capital letters are meant to indicate transform quantities so that X_d (or $X_d(s)$ rather) is understood to be a function of s as in the Laplace transform notation. It is to be noted also that the subtransient reactance x_d'' as given by Equation (3-39) is introduced by utilizing the initial value theorem, so that it would be the appropriate parameter to determine the initial value of current. Finally, by solving Equations (3-36) and (3-37) simultaneously, one has the following expressions for the direct and quadrature-axis currents:

$$I_d = - \frac{\frac{\omega}{s} X_q}{(r+sX_d)(r+sX_q) + \omega^2 X_d X_q} \sqrt{3}\, e_{iqo} , \qquad (3\text{-}42)$$

$$I_q = \frac{\frac{1}{s}(r+sX_d)}{(r+sX_d)(r+sX_q) + \omega^2 X_d X_q} \sqrt{3}\, e_{iqo} \qquad (3\text{-}43)$$

The field current may be evaluated by substituting Equation (3-42) for the direct-axis current back into Equation (3-33). This results in the following expression:

$$I_f = \frac{i_f(0)}{s} + \frac{G X_q}{(r+sX_d)(r+sX_q) + \omega^2 X_d X_q} \sqrt{\frac{3}{2}}\, \omega^2 L_{af} i_f(0) \qquad (3\text{-}44)$$

3.5 Time Domain Solutions for Short-Circuit Currents.

In the previous section the transform direct, quadrature-axis and field currents have been expressed in terms of recognized system functions and the initial value of field current. It is clear that the no-load internal voltage is itself a function of the initial value of field current. In order to get the time-domain solutions for the currents as expressed in Section 3.4 above, one needs to arrange these expressions in partial fraction form. This can be accomplished by factoring out a fifth order polynomial in the denominators. Although this can be carried out by a standard digital program for individual applications, yet some detailed specifications for self-inductances and mutual-inductances appearing in system function expressions are necessary. Some of these parameters such as L_{KKd} , r_{Kd} , L_{aKd} , ... etc. are difficult to obtain either by calculation or experiment. To overcome this difficulty, some simplifying assumptions[12] with simple manipulations as shown in Appendix B lead to the following simpler expressions for the system functions:

$$G = \frac{(1+\tau_{KKd}s)}{(1+\tau_f s)(1+\tau''_{do}s)} \sqrt{\frac{3}{2}} \frac{L_{af}}{r_f}, \qquad (3\text{-}45)$$

$$X_d = \frac{(1+\tau'_d s)(1+\tau''_d s)}{(1+\tau_f s)(1+\tau''_{do}s)} L_d, \qquad (3\text{-}46)$$

and

$$X_q = \frac{1+\tau''_q s}{1+\tau''_{qo}s} L_q \qquad (3\text{-}47)$$

It should be noted that the assumptions as stated in Appendix B are such as to identify certain parameters known as "machine constants" that can be experimentally determined by standard tests. The oscillographic plots of phase currents resulting from a three-phase short-circuit test usually provide enough information to determine these constants. This information is obtained from an investigation of defenite regions in oscillographic plot of the short-circuit current.

Upon substituting the above expressions for system functions as given by Equations (3-45) to (3-47) into the current expressions given by Equations (3-42) to (3-44) and utilizing the approximate expression for the common denominator "D" as given by Equation (C-6) in Appendix C, one can show that:

$$I_d = -\frac{\omega^2(1+\tau'_{do}s)(1+\tau''_{do}s)}{s(1+\tau'_d s)(1+\tau''_d s)[(s+\frac{1}{\tau_a})^2+\omega^2]} \sqrt{3}\,\frac{e_{iqo}}{x_d}, \qquad (3\text{-}48)$$

$$I_q = \frac{\omega(1+\tau''_{qo}s)}{(1+\tau''_q s)[(s+\frac{1}{\tau_a})^2+\omega^2]}\sqrt{3}\,\frac{e_{iqo}}{x_q}, \qquad (3\text{-}49)$$

$$I_f = \frac{i_f(0)}{s} + \frac{\omega^2(1+\tau_{KKd}\,s)}{(1+\tau'_d s)(1+\tau''_d s)[(s+\frac{1}{\tau_a})^2+\omega^2]}(\tau'_{do}-\tau_d)i_f(0) \qquad (3\text{-}50)$$

with,

$$\frac{3}{2}\,\frac{L_{af}^2}{r_f L_d} \cong \tau'_{do} - \tau'_d \qquad (3\text{-}51)$$

as can be seen from the defining Equation (B-26) and by assuming

$$\tau'_{do} \cong \tau_f .$$

Equations (3-48) to (3-50) may be expanded in partial fractions so that the respective currents assume the following form:

$$I_d = -\left[\frac{C_{24}}{s} + \frac{C_{25}}{(1+\tau'_d s)} + \frac{C_{26}}{(1+\tau''_d s)} + \frac{C_{27}\,s + C_{28}}{\{(s+\frac{1}{\tau_a})^2+\omega^2\}}\right]\sqrt{3}\,\frac{e_{iqo}}{x_d}, \qquad (3\text{-}52)$$

$$I_q = \left[\frac{C_{29}}{(1+\tau''_q s)} + \frac{C_{30}\,s + C_{31}}{\{(s+\frac{1}{\tau_a})^2+\omega^2\}}\right]\sqrt{3}\,\frac{e_{iqo}}{x_q}, \qquad (3\text{-}53)$$

$$I_f = \frac{i_f(0)}{s} + \left[\frac{C_{32}}{(1+\tau'_d s)} + \frac{C_{33}}{(1+\tau''_d s)} + \frac{C_{34}\,s + C_{35}}{\{(s+\frac{1}{\tau_a})^2+\omega^2\}}\right](\tau'_{do}-\tau_d)i_f(0) \qquad (3\text{-}54)$$

where

$$C_{24} = 1 ,$$

$$C_{25} = \tau'_{do} - \tau'_d ,$$

$$C_{26} = \frac{\tau'_{do}}{\tau'_d} (\tau''_{do} - \tau''_d) \quad ,$$

$$C_{27} = - \frac{\tau'_{do}\ \tau''_{do}}{\tau'_d\ \tau''_d} \quad ,$$

$$C_{28} = C_{29} = C_{30} \cong 0 \; ,$$

$$C_{31} = \omega \frac{\tau''_{qo}}{\tau''_q} \quad ,$$

$$C_{32} = 1 \; ,$$

$$C_{33} = - \frac{\tau''_d - \tau_{KKd}}{\tau'_d} \quad ,$$

$$C_{34} = - \frac{\tau_{KKd}}{\tau'_d\ \tau''_d} \quad ,$$

$$C_{35} \cong 0 \qquad (3\text{-}55)$$

It is to be noted that in the evaluation of the above constants it was assumed that the subtransient time constants could be neglected with respect to the transient time constants. Moreover, some constants were assumed to be zero especially when such constants will be divided by $\omega = 377$ rad/sec to get inverse Laplace transform of sine functions as will be done in the coming development. By substituting the above values for the constants as given by Equations (3-55) into Equations (3-52) to (3-54) and by performing inverse Laplace operations, one can get the familiar time-domain solutions for the currents as follows:

$$i_d = -[\sqrt{3}\,\frac{e_{iqo}}{x_d} + \sqrt{3}\,e_{iqo}\left(\frac{1}{x_d'} - \frac{1}{x_d}\right)\epsilon^{-t/\tau_d'}$$

$$+\sqrt{3}\,e_{iqo}\left(\frac{1}{x_d''} - \frac{1}{x_d'}\right)\epsilon^{-t/\tau_d''} - \sqrt{3}\,\frac{e_{iqo}}{x_d''}\,\epsilon^{-t/\tau_a}\cos\omega t] \quad , \tag{3-56}$$

$$i_q = \sqrt{3}\,\frac{e_{iqo}}{x_q''}\,\epsilon^{-t/\tau_a}\sin\omega t \quad , \tag{3-57}$$

$$i_f = i_f(0) + i_f(0)\left(\frac{\tau_{do}'-\tau_d}{\tau_d'}\right)[\epsilon^{-t/\tau_d'} - \left(1-\frac{\tau_{KKd}}{\tau_d''}\right)\epsilon^{-t/\tau_d''} - \frac{\tau_{KKd}}{\tau_d''}\,\epsilon^{-t/\tau_a}\cos\omega t] \tag{3-58a}$$

Equation (3-58a) is conveniently re-written in the following form:

$$i_f = i_f(0) + i_f'\,\epsilon^{-t/\tau_d'} - i_f''\,\epsilon^{-t/\tau_d''} - i_{a.c}\,\epsilon^{-t/\tau_a}\cos\omega t \tag{3-58b}$$

with,

$$i_f' = \frac{\tau_{do}' - \tau_d}{\tau_d'}\,i_f(0) \quad , \tag{3-59a}$$

$$i_f'' = \frac{\tau_{do}' - \tau_d}{\tau_d'}\left(1-\frac{\tau_{KKd}}{\tau_d''}\right)i_f(0) \quad , \tag{3-59b}$$

$$i_{a.c} = \frac{\tau_{do}' - \tau_d}{\tau_d'}\,\frac{\tau_{KKd}}{\tau_d''}\,i_f(0) \tag{3-59c}$$

Finally, by substituting the values of i_d and i_q as given by Equations (3-56) and (3-57) above in the inverse relationship as given by Equation (3-8), with $\Theta = \omega t + \Theta_o$, one has the following expression for i_a :

$$i_a = -\sqrt{2}\,e_{iqo}\left[\frac{1}{x_d} + \left(\frac{1}{x_d'} - \frac{1}{x_d}\right)\epsilon^{-t/\tau_d'} + \left(\frac{1}{x_d''} - \frac{1}{x_d'}\right)\epsilon^{-t/\tau_d''}\right]\cos(\omega t + \Theta_o)$$

$$+\sqrt{2}\,e_{iqo}\,\frac{x_d'' + x_q''}{2x_d''\,x_q''}\,\epsilon^{-t/\tau_a}\cos\Theta_o$$

$$+ \sqrt{2}\, e_{iqo} \frac{x''_d - x''_q}{2x''_d\, x''_q} \epsilon^{-t/\tau_a} \cos\,(2\omega t + \Theta_o) \qquad (3\text{-}60)$$

The expressions for i_b and i_c may be obtained by replacing ωt with ωt $-120°$ and ωt $-240°$ respectively in Equation (3-60). Equation (3-60) can be simplified by assuming $x''_d = x''_q$, so that with $\Theta_o = 0$, one has

$$i_a \;=\; -\sqrt{2}\, e_{iqo} \left[\frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x_d}\right) \epsilon^{-t/\tau'_d} + \left(\frac{1}{x''_d} - \frac{1}{x'_d}\right) \epsilon^{-t/\tau''_d}\right] \cos\,\omega t$$

$$+ \sqrt{2}\, e_{iqo} \frac{1}{x''_d} \epsilon^{-t/\tau_a} \qquad (3\text{-}61)$$

Equation (3-61) is conveniently re-written in the following form:

$$i_a \;=\; -\sqrt{2}(i + i'\, \epsilon^{-t/\tau'_d} + i''\, \epsilon^{-t/\tau''_d}) \cos\,\omega t + i_{d.c}\, \epsilon^{-t/\tau_a} \qquad (3\text{-}62)$$

$$i \;=\; \frac{e_{iqo}}{x_d} \qquad (3\text{-}63a)$$

= steady-state short circuit current,

$$i' \;=\; e_{iqo} \left(\frac{1}{x'_d} - \frac{1}{x_d}\right) \qquad (3\text{-}63b)$$

= initial transient component of short-circuit current,

$$i'' \;=\; e_{iqo} \left(\frac{1}{x''_d} - \frac{1}{x'_d}\right) \qquad (3\text{-}63c)$$

= initial subtransient component of short-circuit current,

$$i_{d.c} = \sqrt{2}\,(i + i' + i'')$$

$$= \sqrt{2}\,\frac{e_{iqo}}{x''_d} \qquad (3\text{-}63d)$$

= maximum possible initial value of d-c component of current.

3.6 Energy Considerations.

In the preceeding sections, short-circuit currents in the armature circuit and the field current have been derived. The derivations pertain to an idealized synchronous machine. A single effective time constant was introduced in both the direct and quadrature axes to account for the damper windings and eddy current paths. The effect of damper windings is seen to be accounted for by a subtransient term in both the d-c and a-c components of field and armature currents respectively. Also, the amplitude of the d-c component in the armature circuit and the a-c component of current in the field circuit are modified by subtransient terms.

It turns out that, even with this approximate representations of the eddy current and damper windings effect, it is difficult to apply the derived results for the machine under two distinct transient stages. The first stage starts from the instant of short-circuit and continues until the demagnetization process is initiated. This stage is assumed to end after a period of time $t_1 = 0.1$ second which is customarily considered[1,2] as a reasonable amount of time to actuate the circuit breaker trip coil and for the switches indicated in Figure 2.6a to function. During this stage, the leakage flux is continuously changing due to the almost entirely demagnetizing action of the armature current. The changing nature of leakage flux, which accounts for the change in the reactance is described in an approximate manner by introducing the system functions X_d , X_q and G . When the second stage starts after $t_1 = 0.1$ second, other switching operations are carried out to initiate the demagnetization process and the analysis tends to be more involved. It is not essential to obtain the short-circuit currents in the second stage with extreme accuracy. If it were attempted to obtain complete rigorous solutions, the methods would become exceedingly difficult and require an undue amount of time. Actually, the situation is such that it would be almost impossible to get these rigorous solutions. In the first stage, many assumptions and simplifications were carried out and experience has shown that these approximate methods are sufficiently accurate for engineering purposes. In other words, emphasis will be on a more or less rigorous mathematical development and on obtaining a fundamental physical understanding of the system so that the extension of the theory is reasonably manageable. In order to achieve this and gain more insight to the overall physical nature of the problem, a complete energy account of the system will be considered. Thus, based on this physical approach, the short-circuit current expressions may be modified and fitted to yield the desired solution.

Originally, for an open circuited stator but with rated voltage on the terminals, the stored magnetic energy in the field winding of the synchronous machine is given by:

$$W_f = \frac{1}{2} L_f \, i_f^2(0) \tag{3-64a}$$

$$= \frac{1}{2} \frac{1}{L_f} \lambda_f^2(0) \tag{3-64b}$$

where,

$i_f(0)$ = field current value necessary to generate rated voltage at the stator terminals, and

$\lambda_f(0)$ = initial flux linkage value of the field winding and is given by Equations (3-29c).

It is evident that the stored magnetic energy as given by Equations (3-64) undergoes continuous variation as soon as the machine is short circuited. The field self-inductance is continuously varying and the field current itself follows a pattern as described by Equations (3-58). At the beginning of the second transient stage, the subtransient components $i_f'' \epsilon^{-t/\tau_d''}$ and $i'' \epsilon^{-t/\tau_d''}$ in Equations (3-62) and (3-65) will have almost decayed to zero values. This means that the armature and field currents approach values as given for a machine without dampers.[11,14] Consequently, the subtransient terms may be neglected and the uncertainties about accurate representation of damper windings and eddy current paths will have been nearly eliminated. Also, the stored magnetic energy will be less than that given by Equations (3-64) at the beginning of the second transient stage. It should be pointed out that the second transient stage is more influential than the first one as far as the demagnetization process is concerned. During the first stage, there is no control whatsoever on the decay of short-circuit currents, but fortunately, the decay is naturally accelerated in the subtransient interval. The major effort should then be exercised at the beginning of the second transient stage where the decay is relatively slower. Thus, any method of demagnetization should logically aim at accelerating the decay of the stored magnetic energy after 0.1 second from the instant of short-circuit.

Before proceeding further with this energy consideration, it is worthwhile to consider the electromagnetic torque under three-phase short-circuit conditions. The instantaneous value is given by the following genral expression:[11,17]

$$T = \lambda_d i_q - \lambda_q i_d \tag{3-65}$$

An approximate expression for the torque during a three phase short-circuit is derived by Concordia[11] and it can be generally separated into four components:

1. A fundamental frequency component.

2. A unidirectional component of torque proportional to field $i^2_{a.c} r_f$ loss, where i_{a-c} is the a-c component of rotor current, or the d-c component of stator current.

3. A unidirectional component of torque proportional to the stator i^2r losses, where i is now the a-c fundamental frequendy component of armature current.

4. A double-frequency component of torque, which is due to saliency.

The energy required to compensate for the copper losses cited under the above second and third items must be supplied from some source. In the second item the torque corresponds to the a-c component of rotor current and flux linkages, or equivalently to d-c component of stator current and flux. Looking upon the stator as if it were supplied with d-c excitation, then due to the relative motion, an a-c component of current is induced in the rotor. Obviously, this component must be supplied by the prime mover and has nothing to do with the stored magnetic energy in the field. However, the stator torque cited under the third item, is entirely determined by the direct components of i_d , i_q , or the fundamental frequency component of stator current. Since in the present case, the latter component of the stator current depends only on the d-c field current $i_f(0)$ - which flows irrespective of any relative motion - then the stator i^2r losses must be supplied by the stored energy in the magnetic field of the machine. In other words, due to the fact that the rotor and stator windings are magnetically coupled, the stored magnetic energy is dissipated in the stator by transformer action and appears as heat energy. This is not desirable, and every successful method for rapid demagnetization should result in minimizing the time integral of the stator i^2r losses. In order to accomplish this, a considerable portion of the stored magnetic energy may be diverted by some means or other to prevent much damage to the armature windings. The field discharge resistance method, for example, introduces the discharge resistance across the field winding for this purpose. In the negative forcing voltage method, introduced in Chapter II, some of the stored magnetic energy is extracted by reversing the polarity of the armature of the d-c exciter. This results in the machine running as a motor so that the stored magnetic energy may not only be dissipated as heat energy, but also as mechanical energy returned to the shaft.

Since the stored magnetic energy is generally proportional to the square of the magnetic flux, an acceleration of the decay of flux is obviously necessary. It is only after the flux linkage value is reduced to zero, that the prime mover will stop to supply the torque as given by

Equation (3-65). Needless to say any effort to accelerate the decay of field current after the second transient stage starts would also apply to the flux except for an amount of time lag. Due to the above considerations, the field current will be considered as the basic quantity around which the results will be formulated.

3.7 Extension of the Results of Sections 2.6 and 2.7 to the Machine Under Three Phase Short-Circuit.

In the previous sections, sufficient of the theory of synchronous generators was given to make clear the part played by the dynamic characteristics of the machine in determining the short-circuit currents. An energy consideration has been accounted for in Section 3.6 and an effort was made to grasp the overall physical picture in terms of two distinct transient stages. At the beginning of the second transient stage, the a-c component of field current and the d-c component of short-circuit armature currents are considerably reduced since both are decaying with the armature time constant which is rather small. This is why all the theoretical methods of analysis of demagnetization have always considered the d-c component of transient field current or the related fundamental-frequency a-c current alone. Moreover, the subtransient terms are usually assumed to be negligible by the end of the first transient stage,(11,15) and the effective time constant for the decay of the d-c component of field current is given by:

$$\tau_d' = \tau_{do}' \frac{x_d'}{x_d} \qquad (3\text{-}66)$$

This means that for larger units where the transient reactance tends to be smaller, the chances for a quicker decay of field current are greater than for lower capacity units. Consequently, all the previous equations developed in Section 2.6 can be applied for this case except that the time constant for the field circuit should be modified from τ_f' to τ_d' as far as the decay of the d-c component of field current is concerned. It should also be realized that in applying Equation (2-41), the value of $E_f(0)$ in the last term will be given by:

$$E_f(0) \quad = \quad L_f' \, [i_f(0)]_{0.1 \text{ second}} \qquad (3\text{-}67)$$

where,

$$[i_f(0)]_{0.1 \text{ second}} \quad = \quad i_f(0) + i_f' \, \epsilon^{-(0.1)/\tau_d'} \qquad (3\text{-}68)$$

and, i_f' is as defined by Equation (3-59a).

The value of τ_d' should, obviously, replace that of τ_f' for determining the component of current resulting from the input excitation to winding B acting alone as given by Equation (2-45). The law for the decay of the a-c component of field current will remain the same as that of Equation (3-58b).

In order to extend the results of Section 2.7 for the machine under fault conditions, but with negative forcing feedback, it is convenient to consider the negative forcing voltage as composed of two distinct terms. Thus, at the beginning of the second transient stage, when the subtransient terms have almost decayed to zero values, two equivalent excitations are suddenly applied to the B winding of the amplidyne. These two excitations are proportional to the d-c and a-c components of the negative value of field current respectively. By using superposition for the linearized system, the closed loop response may be determined by adding two separate solutions each derived alone. However, there will be some difficulties in extending these results and the block diagrams of Figures 9 should be modified. The field circuit of the synchronous machine was represented in Chapter II by the G_4 block, in which the principle parameter was the time constant τ_f'. The situation is not exactly the same under short-circuit conditions because of the various couplings which tend to reduce the time constant of the field winding to an effective value of τ_d' as far as the d-c component of field current is concerned and to a value of τ_a as far as the a-c component is concerned. Not only this, but also the field current at the beginning of the second transient stage is constrained as given by the following equation;

$$i_f = i_f(0) + i_f' \epsilon^{-t/\tau_d'} - i_{a.c} \epsilon^{-t/\tau_a} \cos \omega t \qquad (3\text{-}69)$$

where, as mentioned before, the subtransient term is neglected. Equation (3-69) has the following transform value:

$$I_f = \left[\frac{i_f(0)}{s} + \frac{i_f'}{s + \frac{1}{\tau_d'}} \right] \epsilon^{-0.1s} - \frac{i_{a.c} \left(s + \frac{1}{\tau_a}\right)}{\left(s + \frac{1}{\tau_a}\right)^2 + \omega^2} \epsilon^{-0.1s} \qquad (3\text{-}70)$$

where the exponential terms $\epsilon^{-0.1s}$ in the Laplace notation indicates a delayed response of 0.1 second. It is clear that the system function G_4 of Chapter II should be modified to G_4' or G_4'' depending on whether the d-c or a-c component of field current transform value is considered. For any of these components of transform field current, the modification should be such that when the input voltage before G_4 is multiplied by G_4' and G_4'', then the proper transform field current should be as constrained by Equation (3-70) above.

It is thus convenient to substitute $E_f(0)$ in Figure 8b by $E_{feq}(0)$ where,

$$E_{feq}(0) = E_f'(0) + E_f''(0) \qquad (3\text{-}71)$$

keeping in mind that $E_f'(0)$ is the equivalent input voltage before G_4' to account for the d-c component of short-circuit current. Since the governing time constant is till τ_d', then G_4' is equal to G_4 and $E_f'(0)$ will be given by Equation (3-67). Thus, all the basic equations developed in Section 2.7 can be applied provided τ_d' replaces that of τ_f'.

On the other hand, the equivalent input voltage due to the a-c component of field current is given by

$$E_f''(0) = -\frac{i_{a.c}}{K_4} \qquad (3\text{-}72)$$

In order to have the transform field current value as constrained by Equation (3-70) above, G_4'' will have the following value

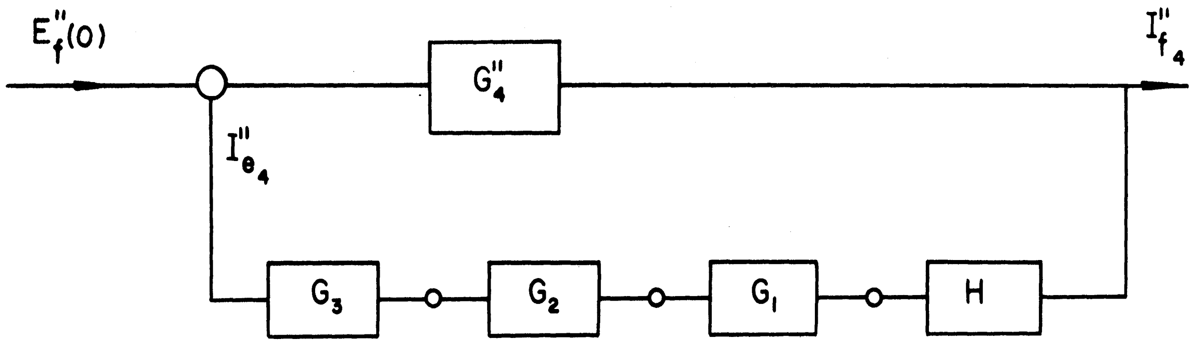
$$G_4'' = \frac{K_4(s+\frac{1}{\tau_a})}{(s+\frac{1}{\tau_a})^2 + \omega^2} \qquad (3\text{-}73)$$

Therefore, the response due to the total equivalent excitation is given by

$$I_{f4} = I_{f4}' + I_{f4}'' \qquad (3\text{-}74)$$

where, I'_{f4} and I''_{f4} are the respective closed loop responses with $E'_f(0)$ and $E''_f(0)$ acting alone. It should be noted that the block diagram representation of Figure 9 will apply also when $E'_f(0)$ is acting alone. As far as I''_{f4} is concerned, the block diagram representation of Figure 13 would be the proper one.

More insight will be gained when numerical values are substituted in Chapter V.



$$E_f''(0) = -\frac{i_{ac}}{K_4}$$

$$G_4'' = \frac{K_4(s+\frac{1}{\tau_a})}{(S+\frac{1}{\tau_a})^2 + \omega^2}$$

Figure 13. Block diagram representation for I_{f4}'' and I_{e4}'' with negative forcing feedback with the machine under fault conditions.

CHAPTER IV

EXPERIMENTAL RESULTS

4.1 Introduction

To verify the theoretical results of Chapter II, some experimental work has been carried out on a system involving an Amplidyne generator used as the pilot exciter for the separately excited d-c generator.

The theoretical results are obtained by substituting appropriate values of the different operating parameters in the derived expressions. To make a quantitative comparison between the theoretical predictions and the recorded results, all the parameters for the machines constituting the system must be known. Appendix D includes all the name plate data for these machines as well as the machine parameters used in the theoretical calculations. Brief descriptions of the necessary tests for determining these parameters are also presented in the same appendix.

The experimental verification of the derived results in Sections 2.6 and 2.7 was made on a scale model turbine generator in the Power Systems Laboratory. This is a specially designed machine that simulates a typical large one. It has approximately the per unit mechanical and electrical characteristics of a large cylindrical rotor power plant machine, except that the field resistance is too high and time constants correspondingly shorter.

The fact that the experimental machine has much faster response than conventional large synchronous machines should not present much difficulty as far as the verification of the theoretical results of Chapter II is concerned. However, with the machine under three-phase short-circuit, the governing time constant in the field circuit, τ_d' , replaces that of τ_{do}' and is even smaller. In such a case, the slightest error in measuring time quantities from the oscillographic plots would seriously impair the validity of the results. This difficutly may be overcome by employing a time-constant regulator,[25] which introduces a negative resistance into the field circuit. Thus, the effective field resistance can be reduced to any desired value in order to yield a reasonable simulation for large machines.

It should be pointed out, however, that the three-phase short-circuit expressions as presented in Chapter III, have been thoroughly discussed and subjected to experimental verification. Moreover, J.R. Hill et al.[2] have extended the experimental procedures to include the second transient stage under which the machine is subjected to the demagnetization process. In spite of the limitations of the theory that were brought out, particularly in regard to magnetic saturation and representation of all eddy-current paths by an effective time constant in each axis, these expressions were regarded as satisfactory. Therefore, it has been decided to assume reasonable validity of the mathematical model to represent the synchronous machine when subjected to three-phase short-circuit as given in Chapter III.

It is to be noted that the derived equations in Section 2.7 are not in a closed form so that it will be necessary to factor out the denominator for each individual case with different sets of parameters. This would then require much labor. Therefore, a standard digital computer program can be conveniently utilized to reduce the amount of labor. The CSAP[25] for example, is based upon a program obtained from the IBM and is placed as a subroutine on the library desk. This subroutine is a general computational aid and can perform many tasks. It has been used for the computation of the transient responses of Section 2.7. This program can also produce one page printer plot for each individual response so that the time responses can be directly obtained from the transform quantities.

The predicted transient currents according to the developments of Sections 2.6 and 2.7 together with the actual currents as obtained from the oscillograms are presented in the following two sections.

4.2 The System with Negative Forcing Step Voltage

Tests were performed on a system constituting the laboratory turbogenerator in order to compare the experimental results with the predicted ones. The schematic diagram of the system is shown in Figure 8a. The synchronous machine was driven by a d-c motor at synchronous speed during the tests. Oscillograms were taken for the exciter current of the d-c generator and the field current of the synchronous machine.

Excitation for the A-winding by means of one of the control field windings, $F_3 - F_4$, was first adjusted to maintain approximately the rated voltage at the synchronous machine terminals. The corresponding exciter and field currents and voltages were recorded. The different time constants in the system were modified as necessary to account for any resistances used to get signal voltages across the input terminals of the oscillograph.

The basic equations derived in Section 2.6 for the exciter and field currents after removing the excitation from the A-winding are

$$i_e' = K_{A2}\, E_c(0)\, [c_1\, \epsilon^{-t/\tau_c} + c_2\, \epsilon^{-t/\tau_s} + c_3\, \epsilon^{-t/\tau_e'}]$$

$$+ K_2'\; E_s(0)\, [c_4\, \epsilon^{-t/\tau_s} + c_5\, \epsilon^{-t/\tau_e'}]$$

$$+ K_3'\; E_e(0)\, [c_6\, \epsilon^{-t/\tau_e'}] \qquad (4\text{-}1)$$

and

$$i_f' = K_{A3}\, E_c(0)\, [c_7\, \epsilon^{-t/\tau_c} + c_8\, \epsilon^{-t/\tau_s} + c_9\, \epsilon^{-t/\tau_e'} + c_{10}\, \epsilon^{-t/\tau_f'}]$$

$$+ K_2\; E_s(0)\, [c_{11}\, \epsilon^{-t/\tau_s} + c_{12}\, \epsilon^{-t/\tau_e'} + c_{13}\, \epsilon^{-t/\tau_f'}]$$

$$+ K_3\; E_e(0)\, [c_{14}\, \epsilon^{-t/\tau_e'} + c_{15}\, \epsilon^{-t/\tau_f'}]$$

$$+ K_4\; E_f(0)\, [c_{16}\, \epsilon^{-t/\tau_f'}] \qquad (4\text{-}2)$$

Initially, the exciter and field currents have values of 0.335 and 1.725 amperes respectively and this gives a terminal voltage of 225 volts across the synchronous machine terminals. The corresponding control and quadrature-axis currents of the Amplidyne were 0.053 and 0.25 amperes respectively. When the above data for the system and the numerical values of the relevant constants are substituted in the above equations, the transient currents are

$$i_e' = -1.822\, \epsilon^{-18.2t} + 1.621\, \epsilon^{-11.8t} + 0.536\, \epsilon^{-27.8t} \qquad (4\text{-}3)$$

and

$$i_f' = 3.791\, \epsilon^{-4.15t} + 2.521\, \epsilon^{-18.2t} - 4.146\, \epsilon^{-11.8t} - 0.441\, \epsilon^{-27.8t} \qquad (4\text{-}4)$$

Figures 14a and 14b show plots of these calculated transient currents, together with the actual currents obtained from the oscillograms. For convenience, the amplitudes of currents are given in percentage of their normal rated values. This procedure will be followed for succeeding cases. The main difference between the predicted and the test results appears in delayed responses due to the effect of eddy current paths in the system and in the final values attained. Due to hystresis effects, the exciter and field currents attain nonzero values at the end of their decay.

With the A-winding unexcited, a negative forcing voltage of -4.9 volts was then applied to the B-winding. The corresponding exciter and field currents and voltages were recorded. The different time constants were also adjusted as in the previous case. The basic equations derived in Section 2.6 for the exciter and field currents after the negative excitation is applied to the B-winding through $F_7 - F_8$ are

$$i_e'' = - K_{B2} V_c [1 + c_{17} \epsilon^{-t/\tau_c} + c_{18} \epsilon^{-t/\tau_s} + c_{19} \epsilon^{-t/\tau_e'}] \tag{4-5}$$

and

$$i_f'' = - K_{B3} V_c [1 + c_{20} \epsilon^{-t/\tau_c} + c_{21} \epsilon^{-t/\tau_s} + c_{22} \epsilon^{-t/\tau_e'} + c_{23} \epsilon^{-t/\tau_f'}] \tag{4-6}$$

The final values of exciter and field currents are -0.35 and -1.74 amperes respectively which corresponds to a terminal voltage of 226 volts across the synchronous machine terminals. If the above data for the system and the numerical values of the constants are substituted in the above equations, the transient currents are

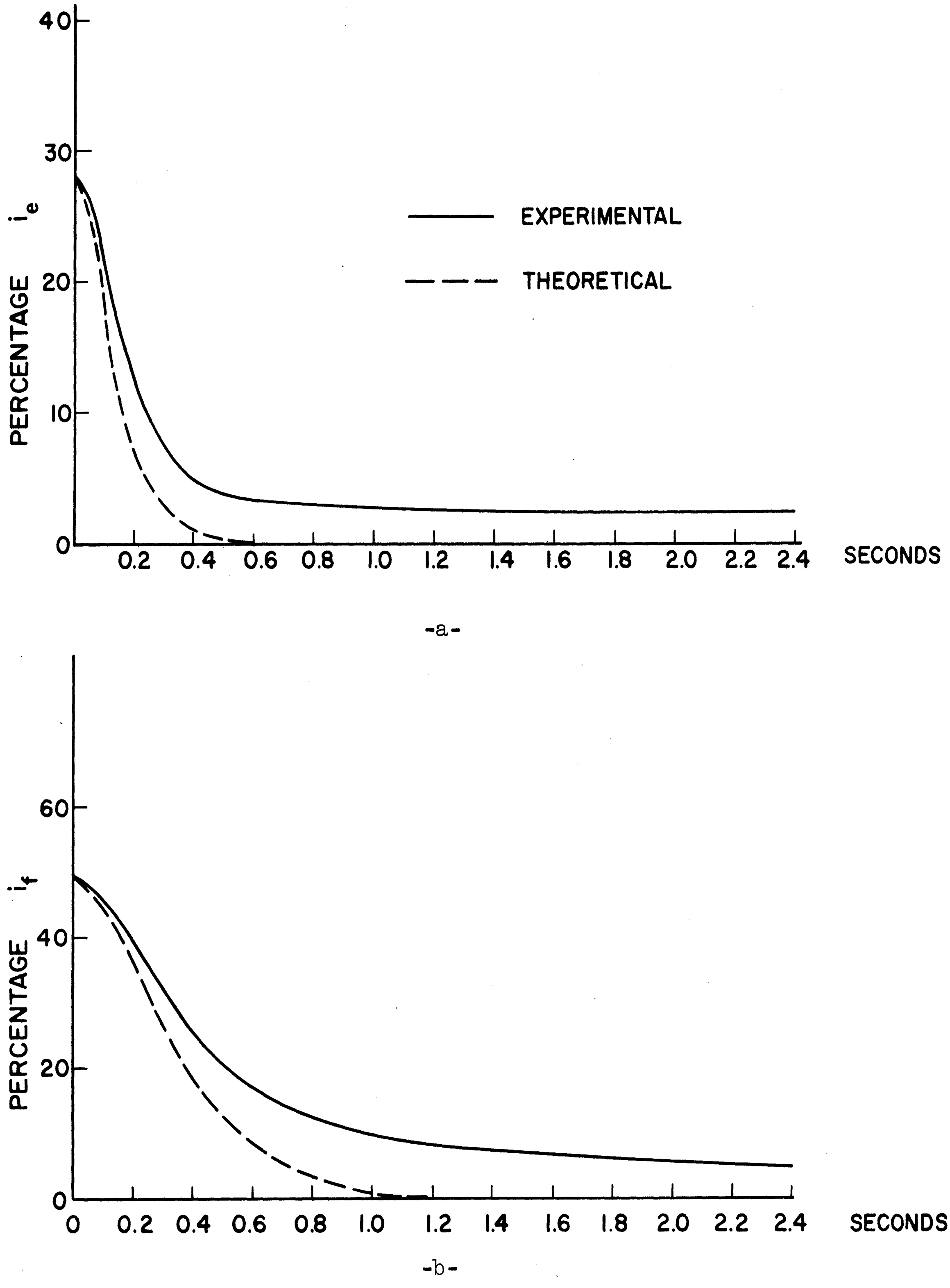


Figure 14. Transient currents due to removing excitation from F_3 - F_4 .

$$i_e'' = -0.35 - 1.858\,\epsilon^{-18.1t} + 1.722\,\epsilon^{-11.8t} + 0.486\,\epsilon^{-27.8t} \quad (4\text{-}7)$$

and

$$i_f'' = -1.74 + 4.11\,\epsilon^{-4.17t} + 2.75\,\epsilon^{-18.1t} - 4.7\,\epsilon^{-11.8t} - 0.42\,\epsilon^{-27.8t} \quad (4\text{-}8)$$

Figures 15a and 15b show plots of these calculated transient currents, together with the acutal currents obtained from the oscillograms. The main difference is also due to eddy currents and to hystresis effects. By constraining the final values of exciter and field currents to be -0.35 and -1.74 amperes respectively, then K_{B2} and K_{B3} Should be modified from 0.0944 and 0.44 to 0.0715 and 0.355 respectively.

Finally, by simultaneously removing the excitation from the A-winding and applying the negative forcing step voltage to the B-winding, the total exciter and field current responses are given by

$$i_e = i_e' + i_e'' \quad (4\text{-}9)$$

and

$$i_f = i_f' + i_f'' \quad (4\text{-}10)$$

where the individual component transient currents are given by Equations (4-1), (4-2), (4-5) and (4-6) above.

The exciter and field currents have initial values of 0.335 and 1.725 amperes respectively and this gives a terminal voltage of 225 volts across the synchronous machine terminals. The corresponding control and quadrature-axis currents of the Amplidyne were 0.053 and 0.25 amperes respectively. The final values of exciter and field currents have values of -0.335 and -1.665 amperes respectively.

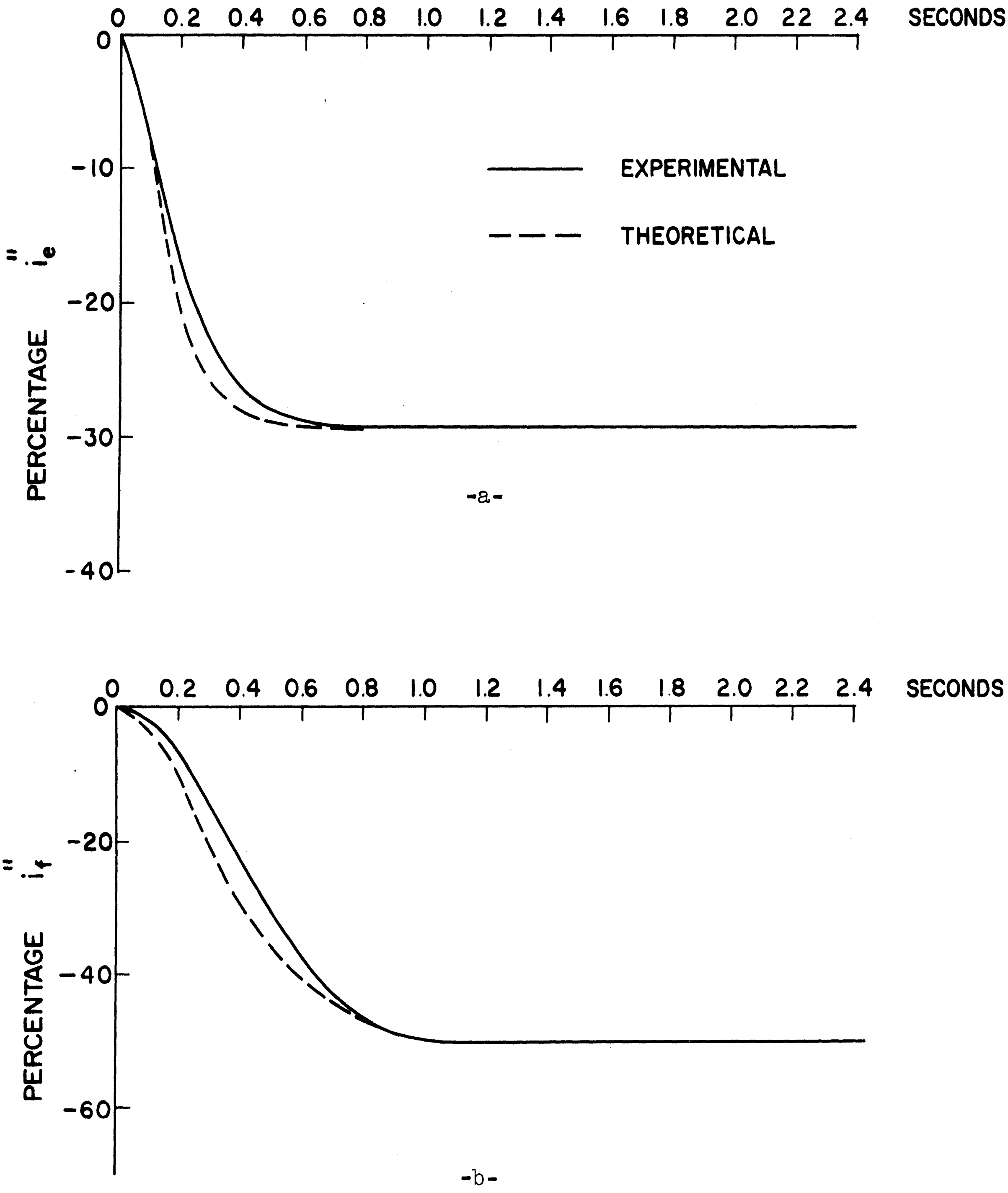


Figure 15. Transient currents due to the application of negative step voltage to $F_7 - F_8$.

If the above data for the system and the numerical values of the constants are substituted in the above equations, the transient currents are

$$i_e = -0.335 - 3.602\,\epsilon^{-18.15t} + 3.271\,\epsilon^{-11.8t} + 1.001\,\epsilon^{-27.8t} \qquad (4\text{-}11)$$

and

$$i_f = -1.625 + 7.631\,\epsilon^{-4.16t} + 5.089\,\epsilon^{-18.15t} - 8.537\,\epsilon^{-11.8t} - 0.833\,\epsilon^{-27.8t} \qquad (4\text{-}12)$$

Figures 16a and 16b show plots of these calculated transient currents, together with the actual currents obtained from the oscillograms. Similar results to those of the component currents response may be seen. The reasonable agreement between theory and experiment justifies the assumptions of Chapter II, particualrly that linear theory can be applied.

4.3 The System with Negative Forcing Feedback

The same type of experiments were carried out except that the negative forcing voltage is now proportional to the decaying field current. The schematic diagram of the system is shown in Figure 8a. Oscillograms of the exciter and field currents were taken for three different values of R_S. For each value of R_S, the effective time constant of the field circuit has to be ascertained. The experimental procedure was to remove the excitation from the A-winding and to simultaneously apply a feedback signal voltage with proper polarity to the B-winding. This was done by closing both switches s_1 and s_2 at the same instant. The oscillograms of the corresponding exciter and field currents were taken.

The basic equations derived in Section 2.7 for the transform of component exciter and field currents can be manipulated to show that

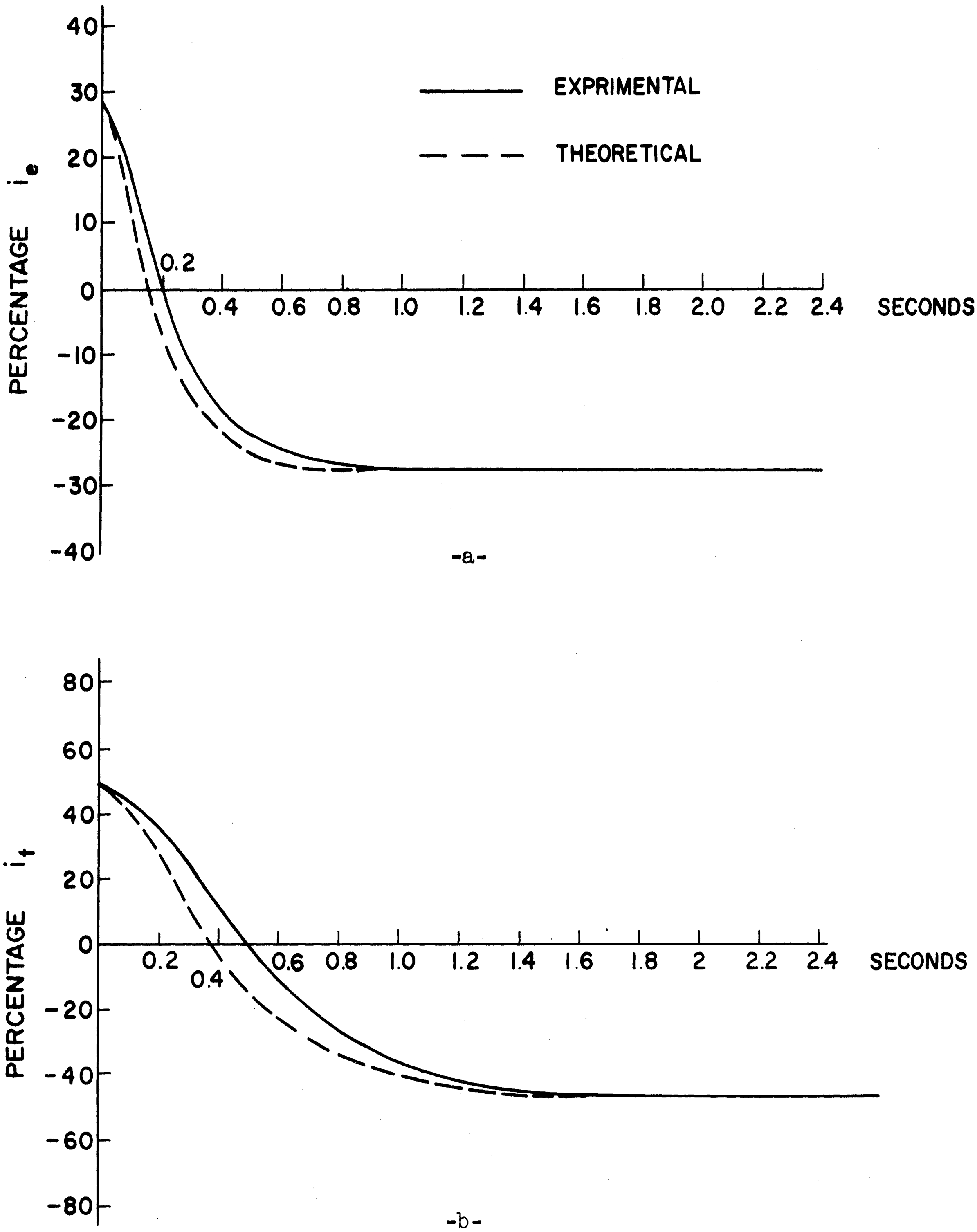


Figure 16. Transient currents due to simultaneously removing excitation from F_3 - F_4 and applying a negative step voltage to F_7 - F_8 .

the total transform exciter and field currents are given by

$$I_e = N_e \,/\, D_e \tag{4-13}$$

and

$$I_f = N_f \,/\, D_f \tag{4-14}$$

where,

$$N_e = \Big[\Big\{ - \frac{K_4\, K_{B2}\, R_s\, E_f(0)}{\tau_c \tau_s \tau_e' \tau_f'} \Big\} + \Big\{ \frac{K_{A2}\, E_c(0)}{\tau_c \tau_s \tau_e'} (s + \frac{1}{\tau_f'}) \Big\} + \Big\{ \frac{K_2'\, E_s(0)}{\tau_s \tau_e'} (s + \frac{1}{\tau_f'})(s + \frac{1}{\tau_c}) \Big\} + \Big\{ \frac{K_3'\, E_c(0)}{\tau_e'} (s + \frac{1}{\tau_f'})(s + \frac{1}{\tau_c})(s + \frac{1}{\tau_s}) \Big\} \Big] \tag{4-15}$$

$$N_f = \Big[\Big\{ \frac{K_{A3}\, E_c(0)}{\tau_c \tau_s \tau_e' \tau_f'} \Big\} + \Big\{ \frac{K_2\, E_s(0)}{\tau_s \tau_e' \tau_f'} (s + \frac{1}{\tau_c}) \Big\} + \Big\{ \frac{k_3\, E_e(0)}{\tau_e' \tau_f'} (s + \frac{1}{\tau_c})(s + \frac{1}{\tau_s}) \Big\} + \Big\{ \frac{K_4\, E_f(0)}{\tau_f'} (s + \frac{1}{\tau_c})(s + \frac{1}{\tau_s})(s + \frac{1}{\tau_e'}) \Big\} \Big] \tag{4-16}$$

and

$$D_e = D_f = s^4 + \Big[\frac{1}{\tau_c} + \frac{1}{\tau_s} + \frac{1}{\tau_e'} + \frac{1}{\tau_f'}\Big] s^3 + \Big[\frac{1}{\tau_c \tau_s} + \frac{1}{\tau_c \tau_e'} + \frac{1}{\tau_s \tau_e'} + \frac{1}{\tau_s \tau_f'} + \frac{1}{\tau_e' \tau_f'}\Big] s^2 + \Big[\frac{1}{\tau_c \tau_s \tau_e'} + \frac{1}{\tau_c \tau_s \tau_f'} + \frac{1}{\tau_c \tau_e' \tau_f'} + \frac{1}{\tau_s \tau_e' \tau_f'}\Big] s + \frac{1}{\tau_c \tau_s \tau_e' \tau_f'} \Big[1 + K_{B3}\, K_s\Big] \tag{4-17}$$

The CSAP[25] digital computer program was again used to compute the numerical values for the inverse functions of the transform exciter and field currents. Three different values of R_s, as in the experimental procedures, were used and for each value, different sets of data and adjusted values of constants were introduced. The corresponding transforms for these cases are as follows.

Case 1: $R_s = 2.16$ ohms

The exciter and field currents have initial values of 0.335 and 1.67 amperes and the corresponding control and quadrature-axis currents of the amplidyne were 0.05 and 0.25 amperes respectively. The transform exciter and field currents in this case are given by

$$I_e = [- 1620 + 78.7 (s+4.07) + 5.3 (s+4.07)(s+27.8) + 0.335 (s+4.07)(s+27.8)(s+11.8)] / [s^4 + 61.85 s^3 + 1283 s^2 + 10228 s + 43.400] \quad (4\text{-}18)$$

and

$$I_f = [1527 + 102.8 (s+27.8) + 6.51(s+27.8)(s+11.8) + 1.76 (s+27.8)(s+11.8)(s+18.18)] / [s^4 + 61.85 s^3 + 1283 s^2 + 10228 s + 43.400] \quad (4\text{-}19)$$

Case 2: $R_s = 3.09$ ohms

The exciter and field currents have initial values of 0.338 and 1.725 amperes and the corresponding control and quadrature-axis currents of the amplidyne were 0.05 and 0.2012 amperes respectively. The transform exciter and field currents in this case are given by

$$I_e = [- 2203 + 78.7 (s+4.19) + 5.3 (s+4.19)(s+27.8) + 0.338(s+4.19)(s+27.8)(s+11.8)] / [s^4 + 61.97 s^3 + 1289 s^2 + 10346 s + 52.250] \quad (4\text{-}20)$$

and

$$I_f = [1520 + 102.7 (s+27.8) + 6.56 (s+27.8)(s+11.8) + 1.725 (s+27.8)(s+11.8)(s+18.18)] / [s^4 + 61.97 s^3 + 1289 s^2 + 10.346 s + 52.250] \quad (4\text{-}21)$$

Case 3: $R_s = 6.34$ ohms

The exciter and field currents have initial values of 0.395 and 1.788 amperes and the corresponding control and quadrature-axis currents of the amplidyne were 0.06 and 0.30 amperes respectively. The transform exciter and field currents in this case are given by

$$I_e = [-4910 + 85.4\,(s+4.62) + 6.35\,(s+4.62)(s+27.8) + 0.396\,(s+4.62)(s+27.8)(s+11.8)] \,/\, [s^4 + 62.4\,s^3 + 1315\,s^2 + 10800\,s + 83.478] \tag{4-22}$$

and

$$I_f = [1832 + 123\,(s+27.8) + 7.67\,(s+27.8)(s+11.8) + 1.788\,(s+27.8)(s+11.8)(s+18.18)] \,/\, [s^4 + 62.4\,s^3 + 1315\,s^2 + 10800\,s + 83.478] \tag{4-23}$$

Figures 17 through 19 show plots of the transient currents as obtained from the output of the digital computer. On the same figures, the actual currents as obtained from the oscillograms are also plotted. It is seen that the agreement between theory and experiment is reasonably good. Although the theoretical curves are not exactly the same as the experimental ones, the comparison shows, however, the same type of variation for the theoretical and experimental results. The main discrepency is in the delay effect due to eddy current paths and hystresis effects. It can be seen that saturation has very slight effect due to the lower levels of voltages used.

It is to be noted that increasing the value of R_s tends to accelerate the decay of current but there is an upper bound for the value of R_s at which the system will start oscillation. The effects of varying R_s will be considered further in Chapter V.

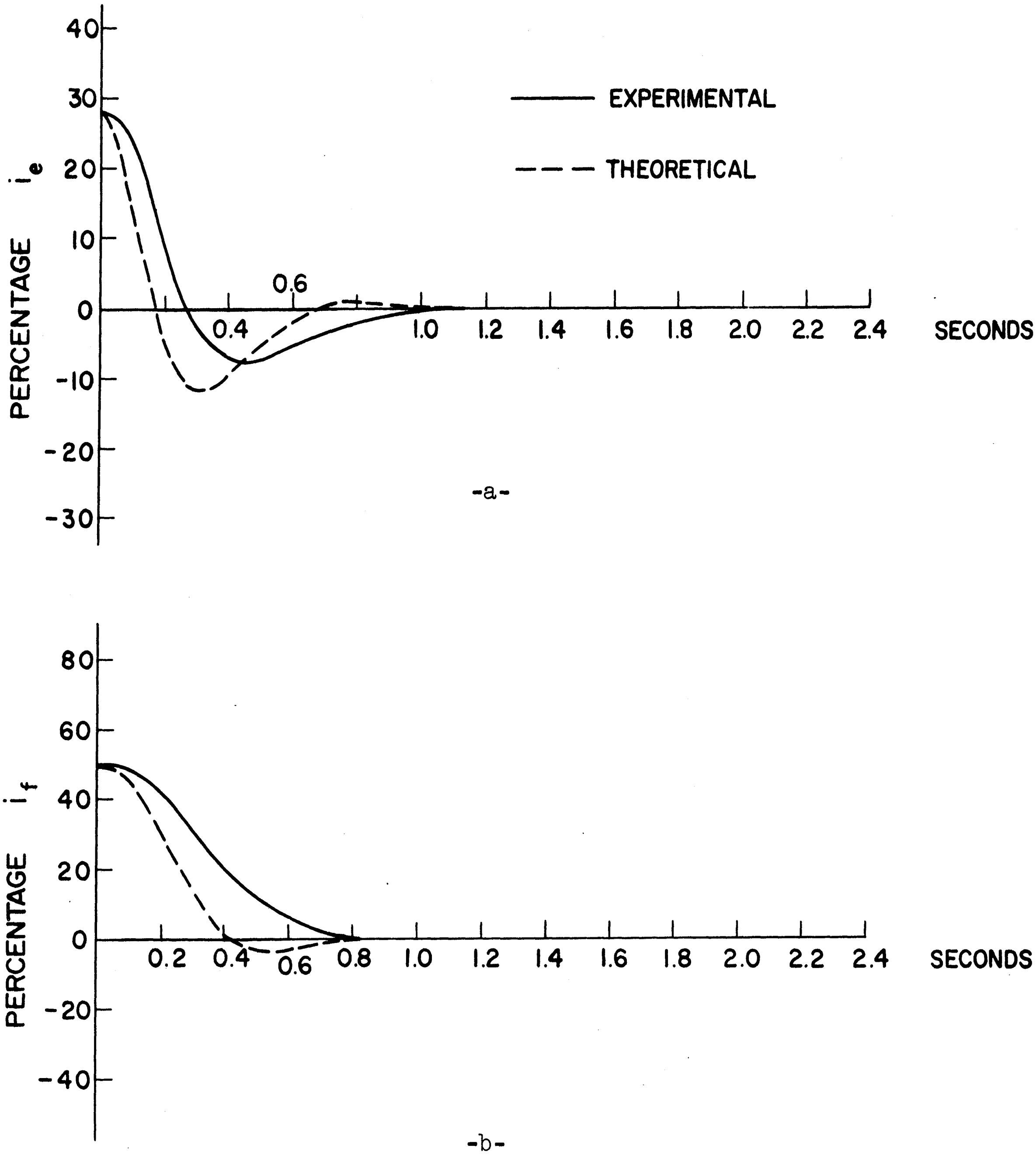
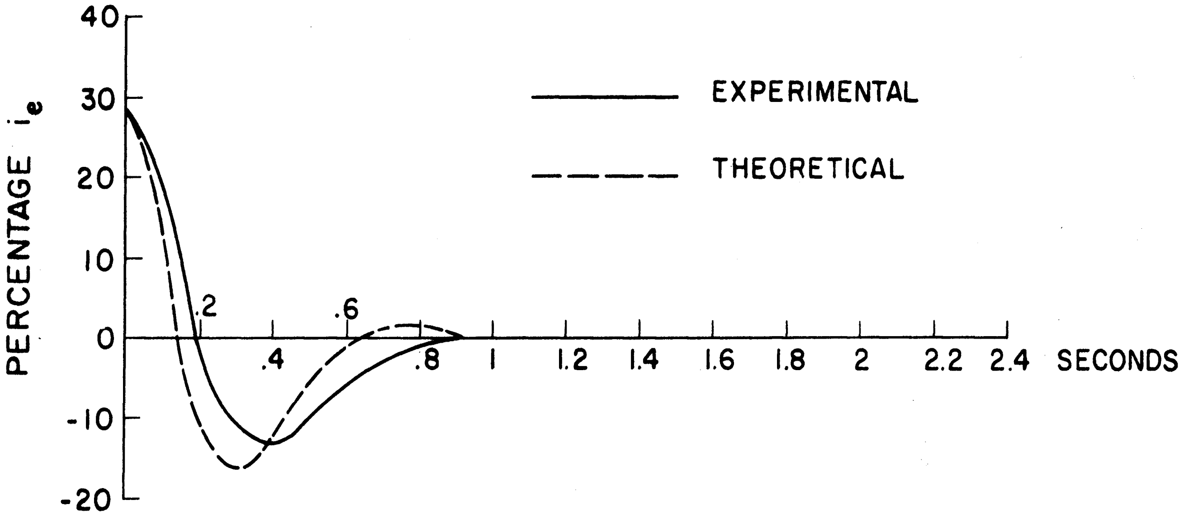


Figure 17. Transient currents with negative forcing feedback. Case 1. R_s = 2.16 ohms.



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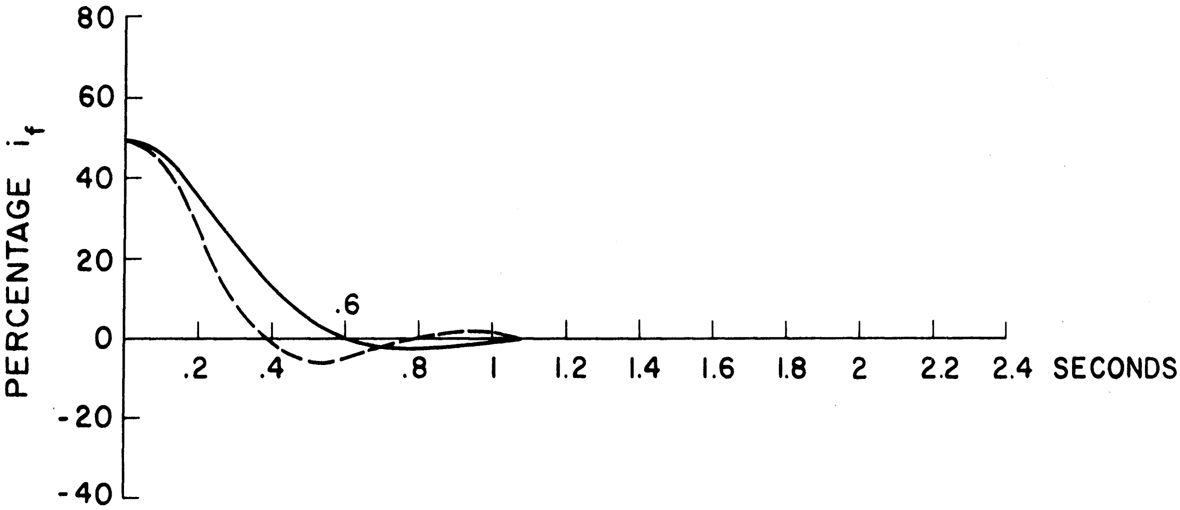

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Figure 18. Transient currents with negative forcing feed-back. Case 2. R_s = 3.09 ohms.

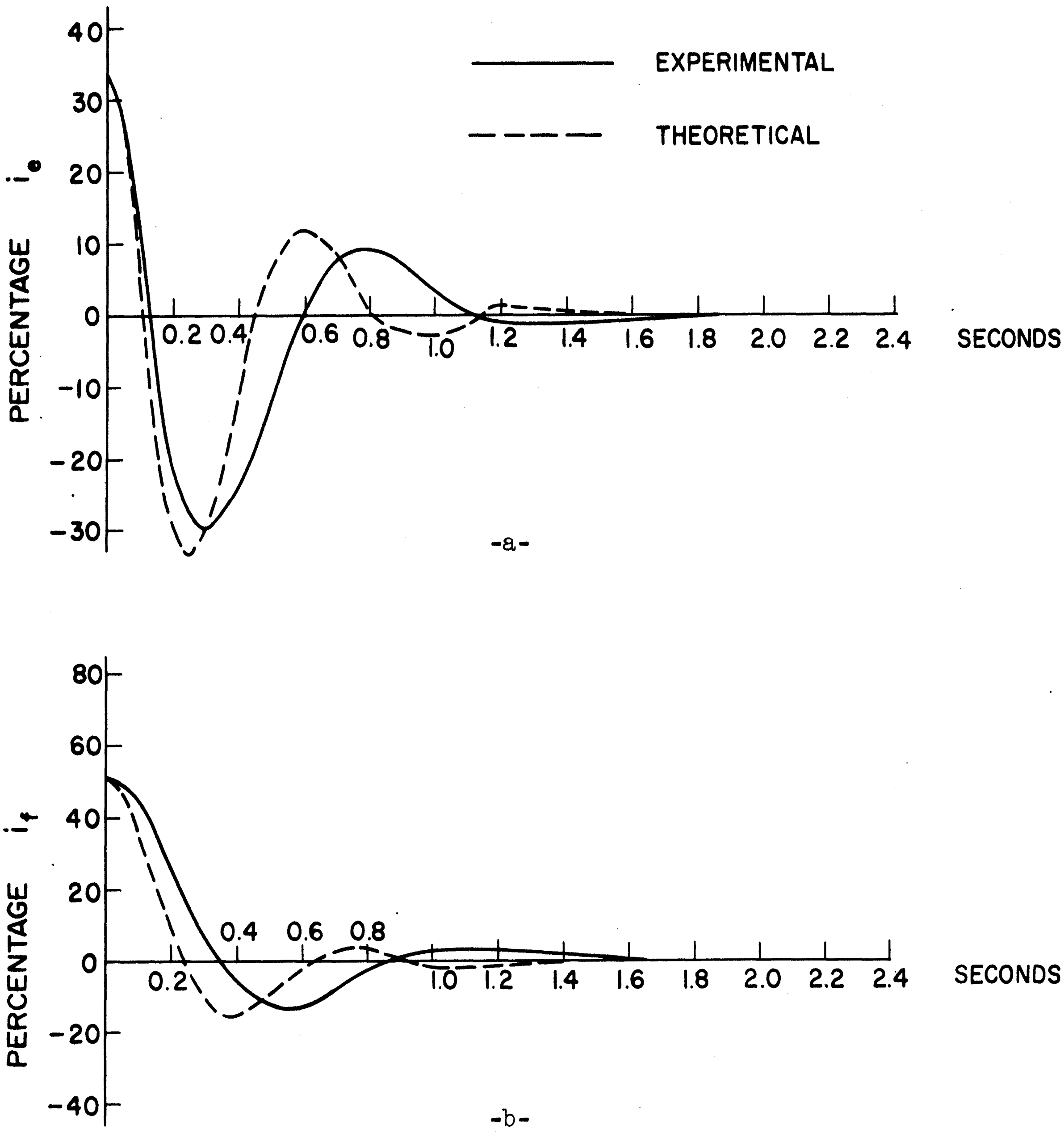


Figure 19. Transient currents with negative forcing feedback. Case 3. R_s = 6.34 ohms

CHAPTER V

APPLICATIONS AND RESULTS

5.1 Introduction

Having verified the adequately of the mathematical models to represent the system described in Chapter II, it is desired to apply the method to a typical power systems generator to test its potentialities.

This is achieved by applying the equations derived to the 265-mva generator used by K.S. Raman.(6) This method will also be compared with the oscillation resistance method, which is consedered as one of the best known methods of field suppression. For convenience, the constants and data for this machine and its excitation system are listed in Appendix E.

However, an amplidyne generator is used as a pilot exciter for the d-c main exciter generator unit. An AM-617, 7.5 kw, 250 volts amplidyne model has been chosen from technical information provided by the manufacturer.(27) The relevant constants and data are also listed in Appendix E. It has been assumed that the field windings of the main exciter unit can be redesigned so that the excitation flux and the time constant of the excitation winding will correspond to the original values in spite of the change of voltage level from 500 to 250 volts. This is necessary to provide a common basis for comparing results that essentially depend upon time.

5.2 Demagnetization Under Open-Circuit Conditions

5.2.1 The System with Negative Forcing Step Voltage

The main excitation, being supplied by the A-winding $(F_1\text{-}F_2)$, is adjusted to maintain rated voltage at the synchronous machine terminals.

The exciter and field currents have values of 5 and 935 amperes respectively and the corresponding control and quadrature axis currents of the amplidyne are 0.0341 and 0.413 amperes respectively. If the above data for the system and the numerical values of the relevant constants are substituted in Equations (4-1) and (4-2), the transient currents are

$$i'_e = 19.1\ \epsilon^{-1.183\ t} - 14.1\ \epsilon^{-1.667\ t} \qquad (5\text{-}1)$$

and

$$i'_f = 1197\ \epsilon^{-0.172\ t} + 264\ \epsilon^{-1.667\ t} - 526\ \epsilon^{-1.183\ t} \qquad (5\text{-}2)$$

It is to be noted that the exponential terms of the short-circuited path in the amplidyne have been dropped out. This can be justified since the associated time constant is much smaller than the other time constants.

A negative forcing step voltage of -3 volts, sufficient to maintain rated voltage at the machine terminals may be applied to the B-winding ($F_9 - F_{10}$). If this value and the numerical values of the constants are substituted in Equations (4-5) and (4-6), the transient currents are

$$i''_e = -5 + 8.9\ \epsilon^{-1.667\ t} - 3.9\ \epsilon^{-4.76\ t} \qquad (5\text{-}3)$$

and

$$i''_f = -813 + 950\ \epsilon^{-0.172\ t} - 164\ \epsilon^{-1.667\ t} + 27\ \epsilon^{-4.76\ t} \qquad (5\text{-}4)$$

Here, again the exponential terms of the short-circuited path have been neglected for the same reason as stated above.

By simultaneously removing the excitation from the A-winding and applying the negative forcing step voltage to the B-winding, the total exciter and field current responses are given by

$$i_e = -5 + 19.1\,\epsilon^{-1.183\,t} - 5.2\,\epsilon^{-1.667\,t} - 3.9\,\epsilon^{-4.76\,t} \qquad (5\text{-}5)'$$

and

$$i_f = -813 + 2147\,\epsilon^{-0.172\,t} + 100\,\epsilon^{-1.667\,t} - 526\,\epsilon^{-1.183\,t} + 27\,\epsilon^{-4.76\,t} \qquad (5\text{-}6)'$$

If the negative forcing step voltage is increased from -3 volts to -6 volts, then the total exciter and field current responses are given in this case by

$$i_e = -10 + 19.1\,\epsilon^{-1.183\,t} + 3.7\,\epsilon^{-1.667\,t} - 7.8\,\epsilon^{-4.76\,t} \qquad (5\text{-}5)''$$

and

$$i_f = -1626 + 3097\,\epsilon^{-0.172\,t} - 64\,\epsilon^{-1.667\,t} - 526\,\epsilon^{-1.183\,t} + 54\,\epsilon^{-4.76\,t} \qquad (5\text{-}6)''$$

Figures 20a and 20b are plots of these transient currents in which the amplitudes are given as percentages of their normal rated values.

5.2.2. The System with Negative Forcing Feedback

The procedure for this method is explained in Sections 2.7 and 4.3. However, Equations (4-13) to (4-17) should be slightly modified to account for this application where the system function H is as originally given in Section 2.7 by

$$H = R_s \frac{K_{B3}(1+s\tau_{cA})}{K_{A3}(1+s\tau_{cB})} \qquad (5\text{-}7)$$

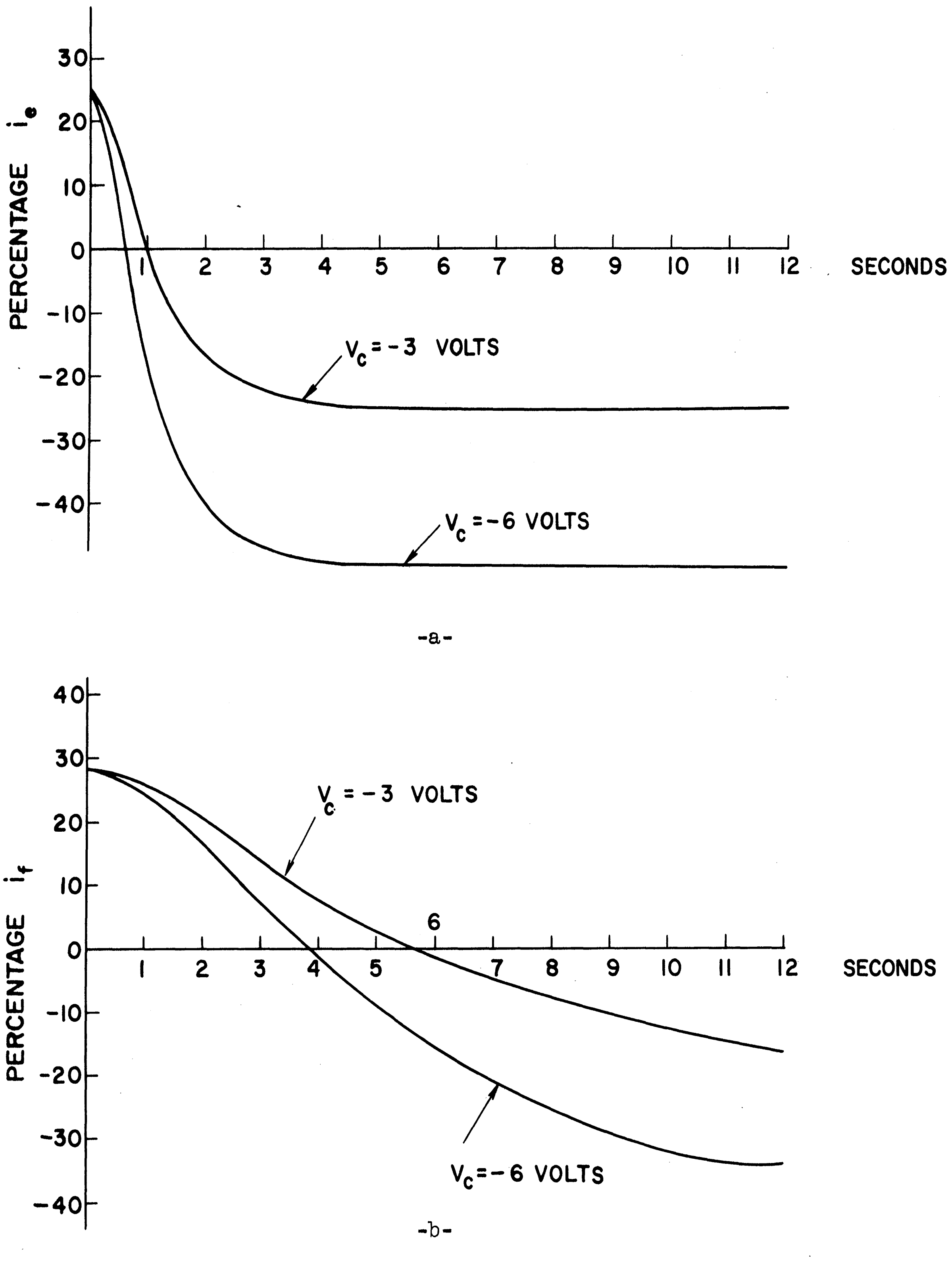


Figure 20. Transient currents due to simultaneously removing excitation from the A-winding and applying a negative step voltage to the B-winding.

The basic equations derived in Section 2.6 for the Laplace transform of component exciter and field currents can be manipulated to show that the total transform exciter and field currents in this case are given by

$$I_e = \frac{N_e}{D_e} \tag{5-8}$$

and

$$I_f = \frac{N_f}{D_f} \tag{5-9}$$

where,

$$N_e = [- K_4 K_{B2} R_s E_f(0) (1+s\tau_{cA}) + K_{A2} E_c(0) (1+s\tau_{cA})(1+s\tau_f')$$

$$+ K_2' E_s(0) (1+s\tau_{cA})(1+s\tau_{cB})(1+s\tau_f')$$

$$+ K_3' E_e(0) (1+s\tau_{cA})(1+s\tau_{cB})(1+s\tau_s)(1+s\tau_f')] , \tag{5-10}$$

$$N_f = [K_{A3} E_c(0) (1+s\tau_{cB}) + K_2 E_s(0) (1+s\tau_{cA})(1+s\tau_{cB})$$

$$+ K_3 E_e(0) (1+s\tau_{cA})(1+s\tau_{cB})(1+s\tau_s)$$

$$+ K_4 E_f(0) (1+s\tau_{cA})(1+s\tau_{cB})(1+s\tau_s)(1+s\tau_e')] \tag{5-11}$$

and

$$D_e = D_f = [(1+s\tau_{cA})(1+s\tau_{cB})(1+s\tau_s)(1+s\tau_e')(1+s\tau_f')+R_s K_{B3}(1+s\tau_{cA})] \tag{5-12}$$

It is to be noted that the transform voltage E_f across the field winding of the synchronous machine is given by

$$E_f = (R_f + s L_f)I_f - E_f(0) \tag{5-13}$$

Digital calculations were again carried out to determine the response of the exciter and field currents as well as of the voltage across the field winding. Different values of R_s were substituted and the constants were adjusted accordingly. Results of digital computer runs for $R_s = 0.074\ R_f$, $R_s = 0.1\ R_f$, $R_s = 0.11\ R_f$ and $R_s = 0.14\ R_f$ are reproduced in Figures 21 through 24. For convenience, the amplitudes of i_e , i_f and e_f are given as percentages of their normal rated values.

5.3 Demagnetization Under Short-Circuit Conditions with Negative Forcing Feedback

The discussions in Section 3.7 lead to substituting $E_f(0)$ in Figure 8b by $E_{feq}(0)$ where

$$E_{feq}(0) = E_f'(0) + E_f''(0) \qquad (5\text{-}14)$$

For d-c component of short-circuit current, $E_f'(0)$ is the equivalent input voltage before G_4' with τ_d' now as the governing time constant. Therefore, as far as the d-c component of short-circuit current is concerned, Equations (4-13) to (4-17) still hold except that τ_d' replaces τ_f' and L_f has a value of 0.141 H. instead of 0.93 H. In this case the d-c component of field current attains a value of 5610 amperes at the beginning of the second transient stage, i.e after 0.1 second from the occurrence of the fault. The corresponding exciter, control and quadrature axis currents have the same values as in Section 5.2.1. The first three values for R_s in Section 5.2.2 were used to calculate the corresponding responses as given by Equations (4-13) to (4-17), but with τ_d' replacing τ_f' . Results of the calculations are reproduced in Figures 25 through 27, where the amplitudes are given as percentages of their normal rated values.

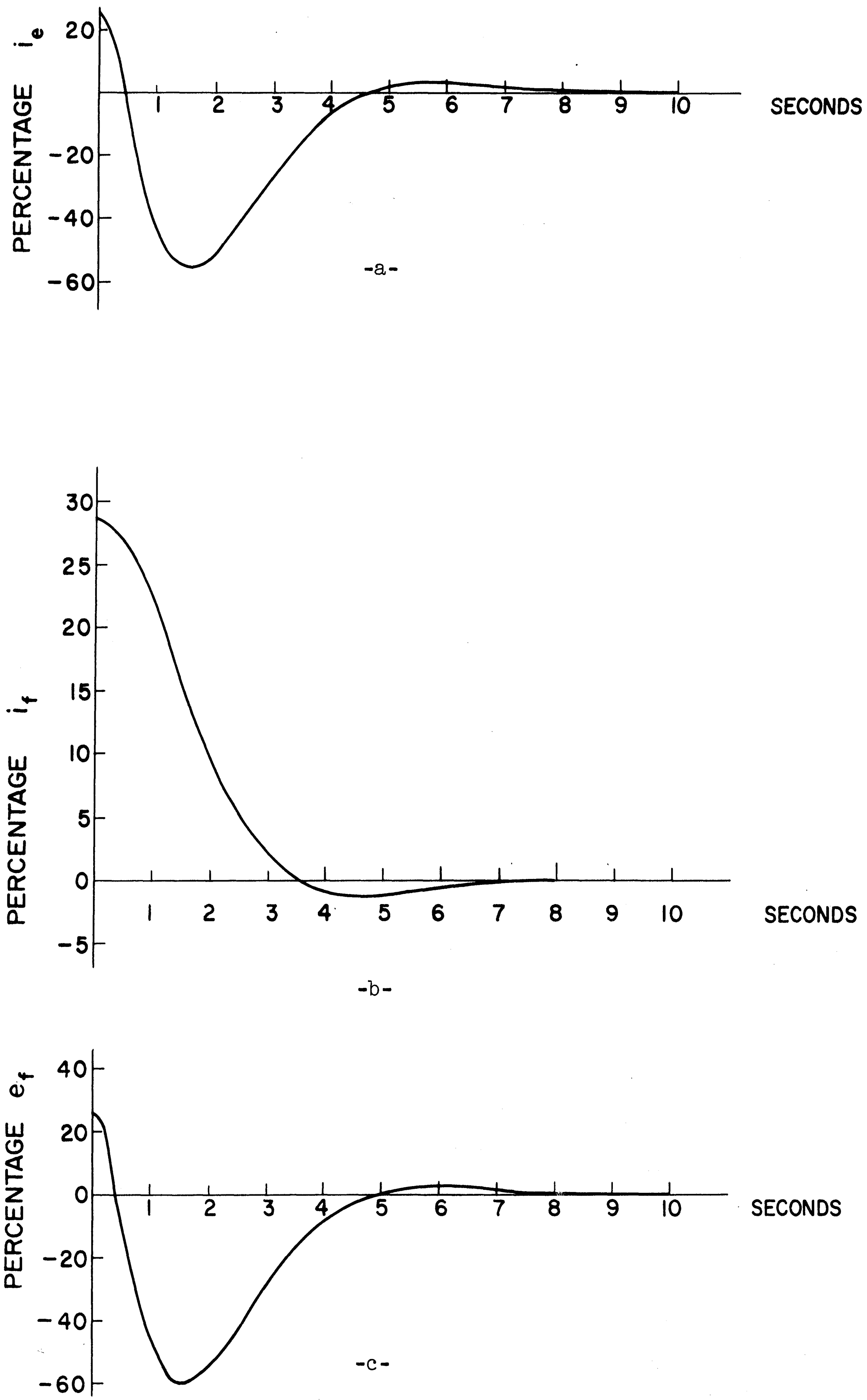


Figure 21. Responses with negative forcing feedback. $R_s = 0.074\ R_f$. Open-circuit conditions.

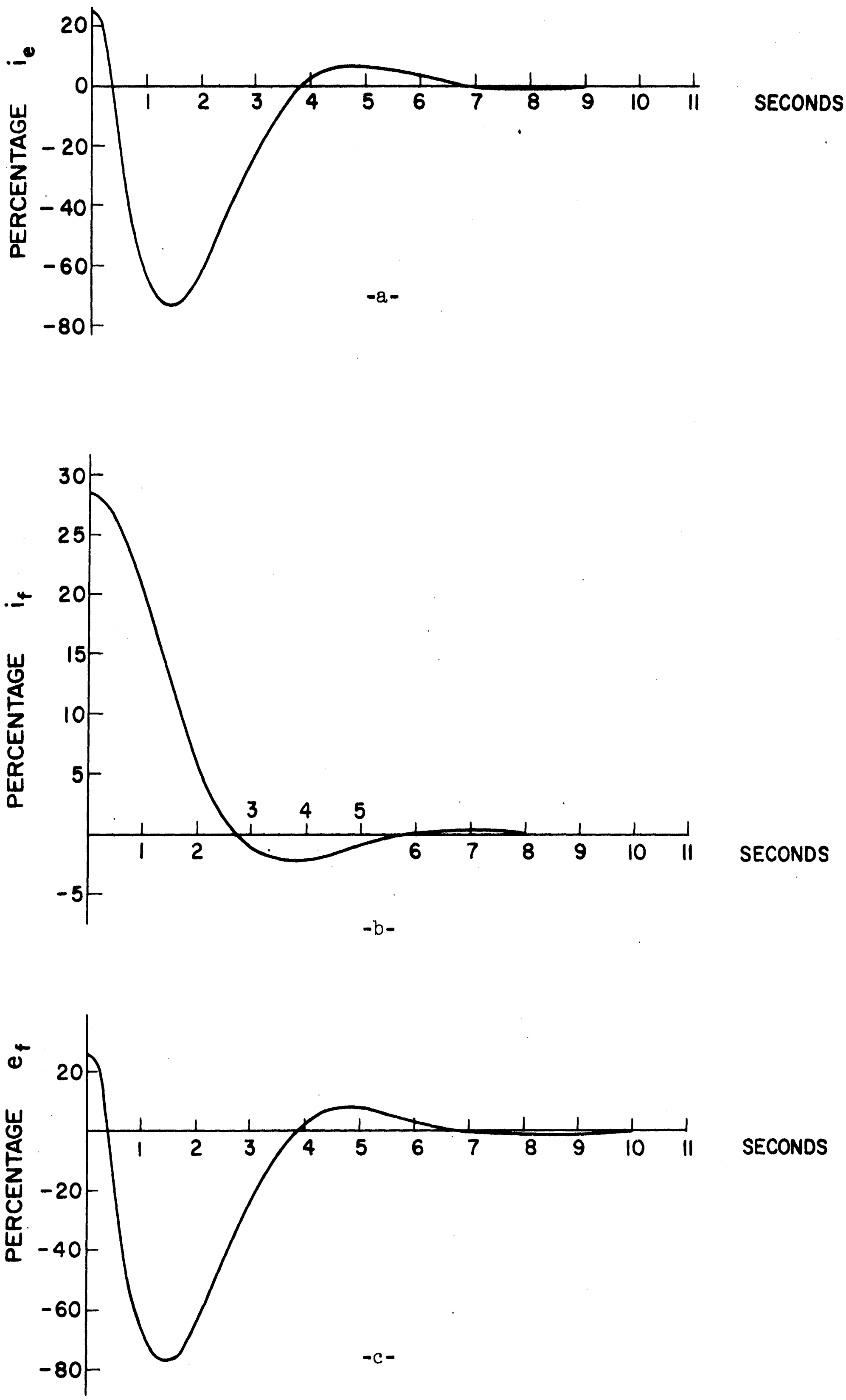


Figure 22. Responses with negative forcing feedback. $R_s = 0.1\ R_f$. Open-circuit conditions.

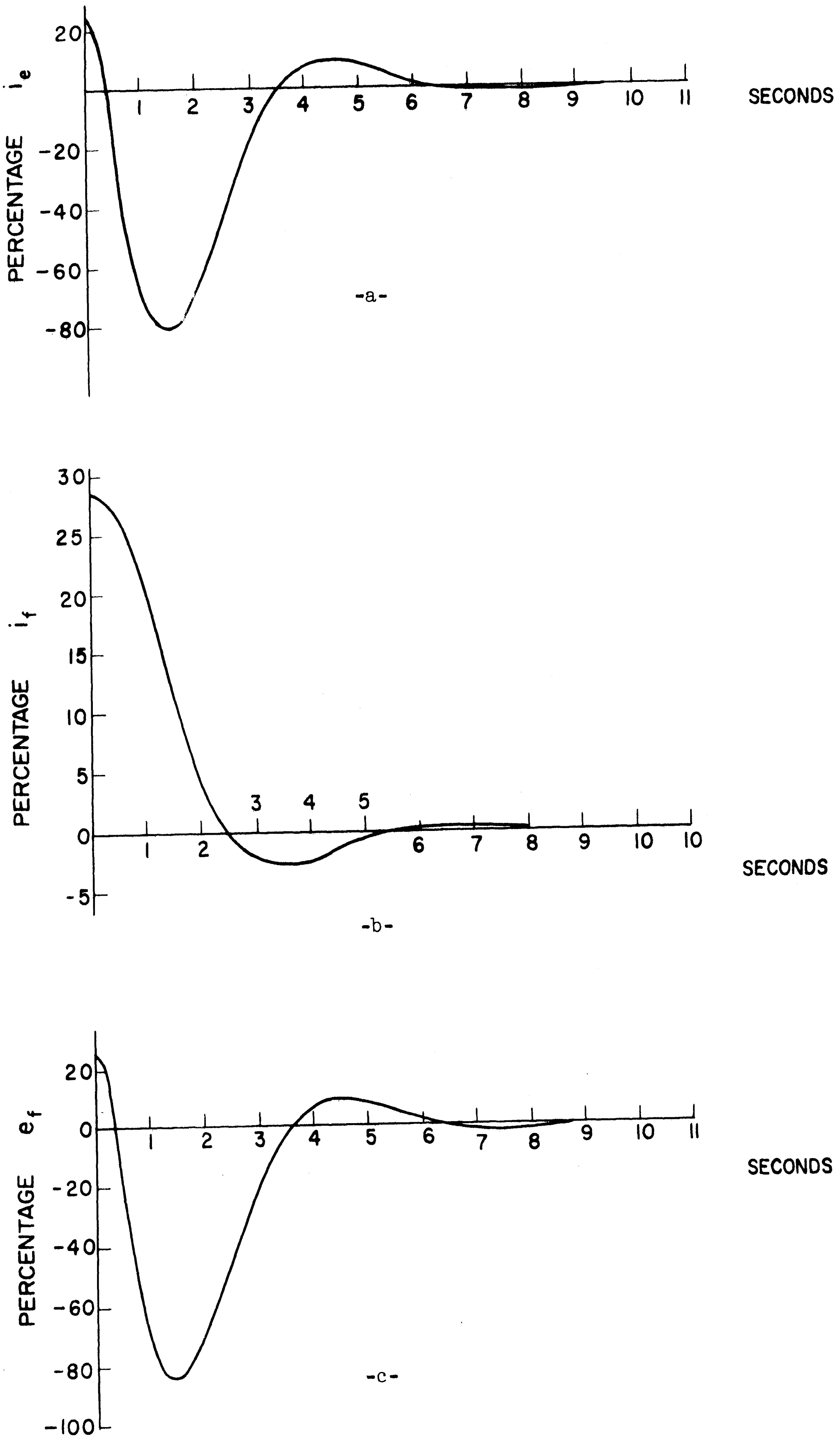


Figure 23. Responses with negative forcing feedback. $R_s = 0.11\ R_f$. Open-circuit conditions.

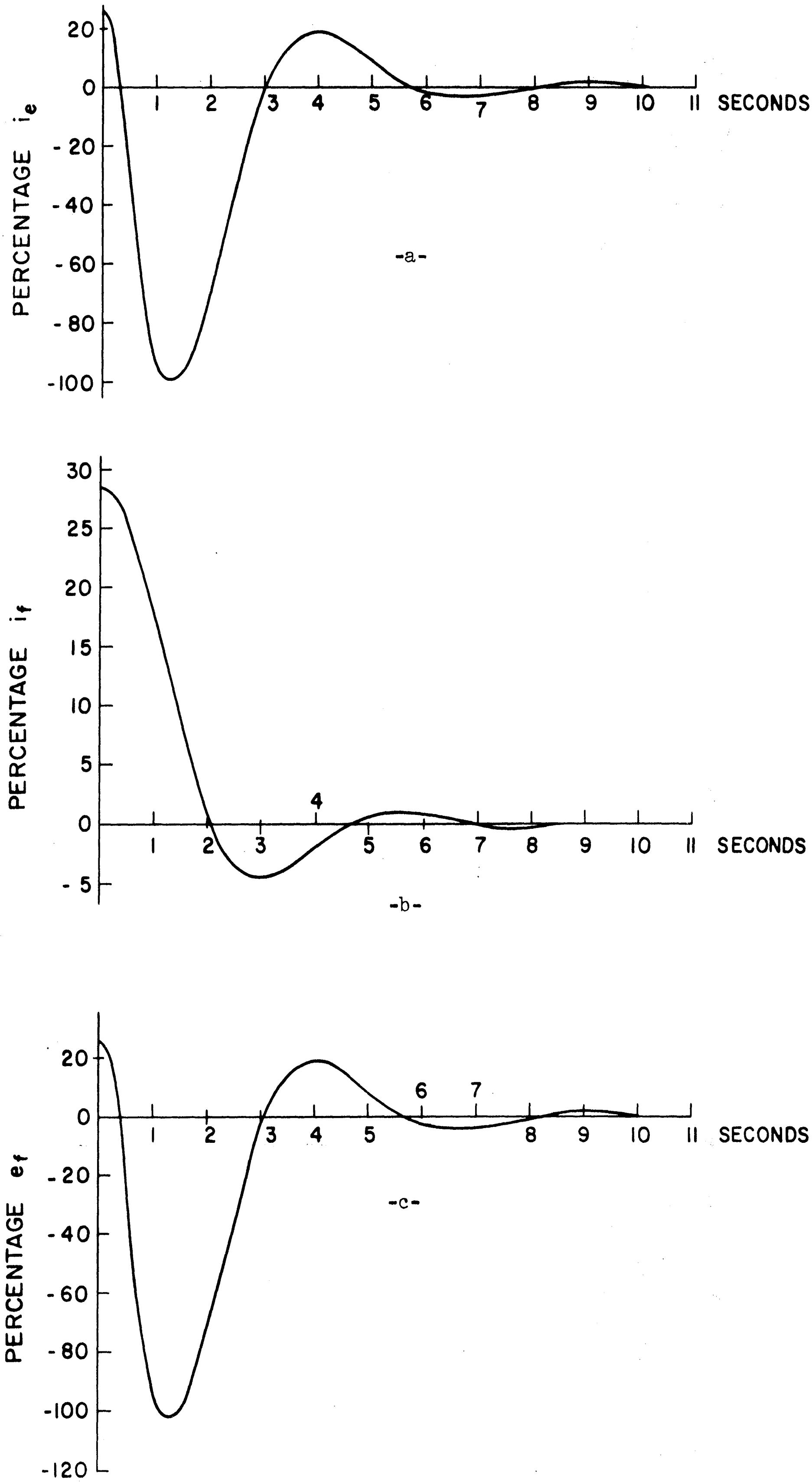
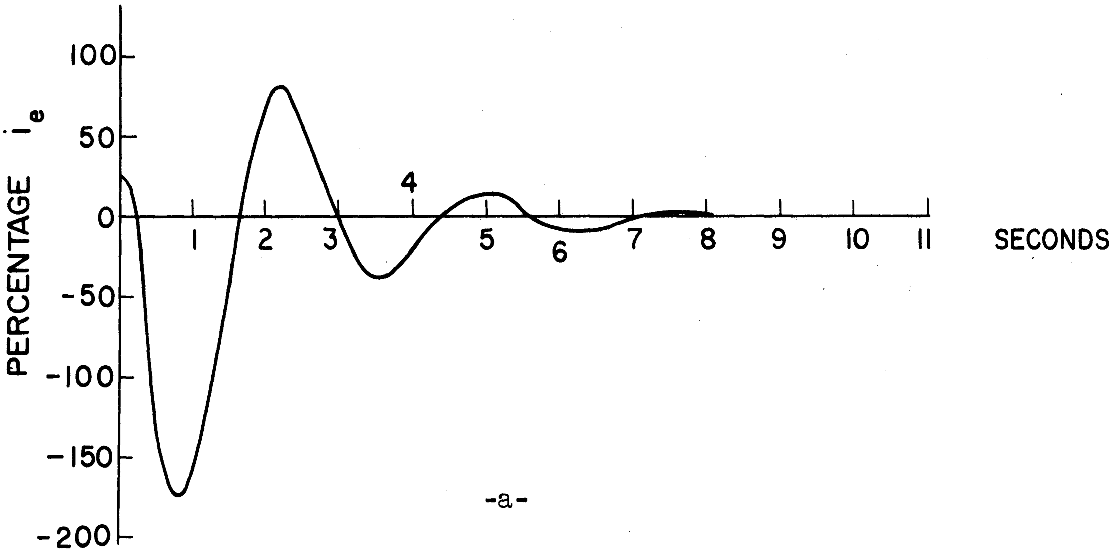


Figure 24. Responses with negative forcing feedback. $R_s = 0.14\ R_f$. Open-circuit conditions.



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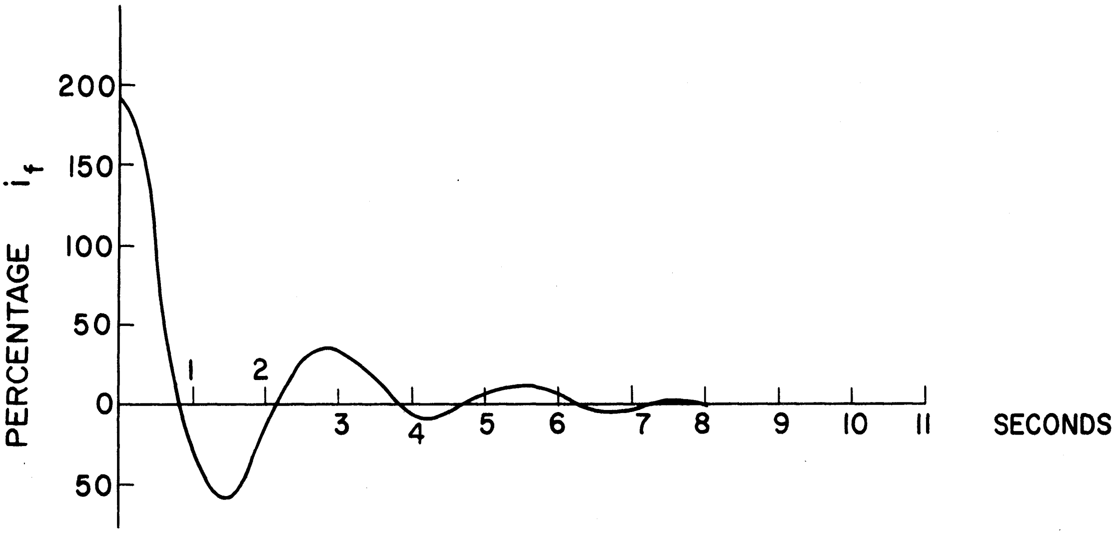

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Figure 25. Responses with negative forcing feedback. $R_s = 0.074R_f$. Short-circuit conditions.

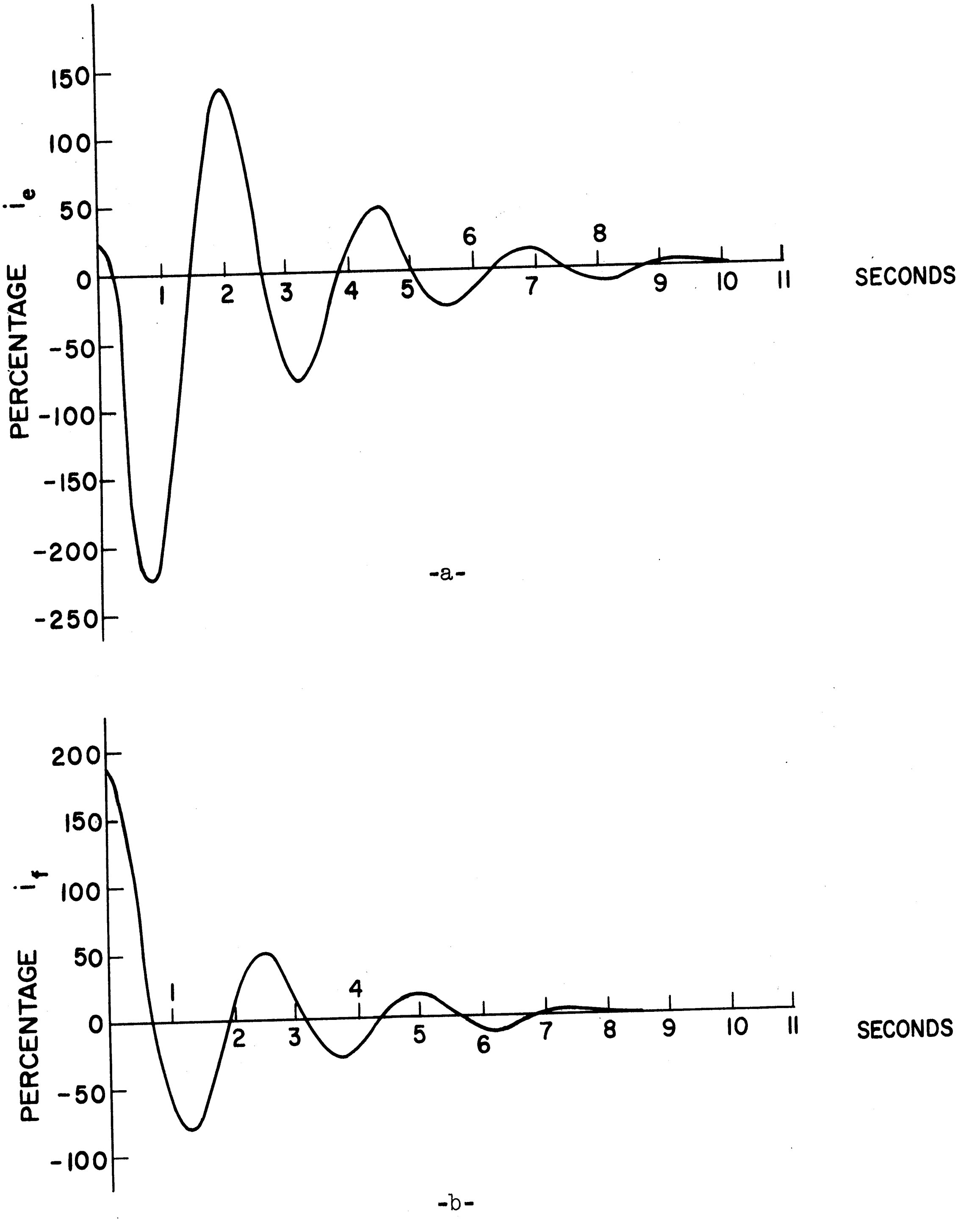
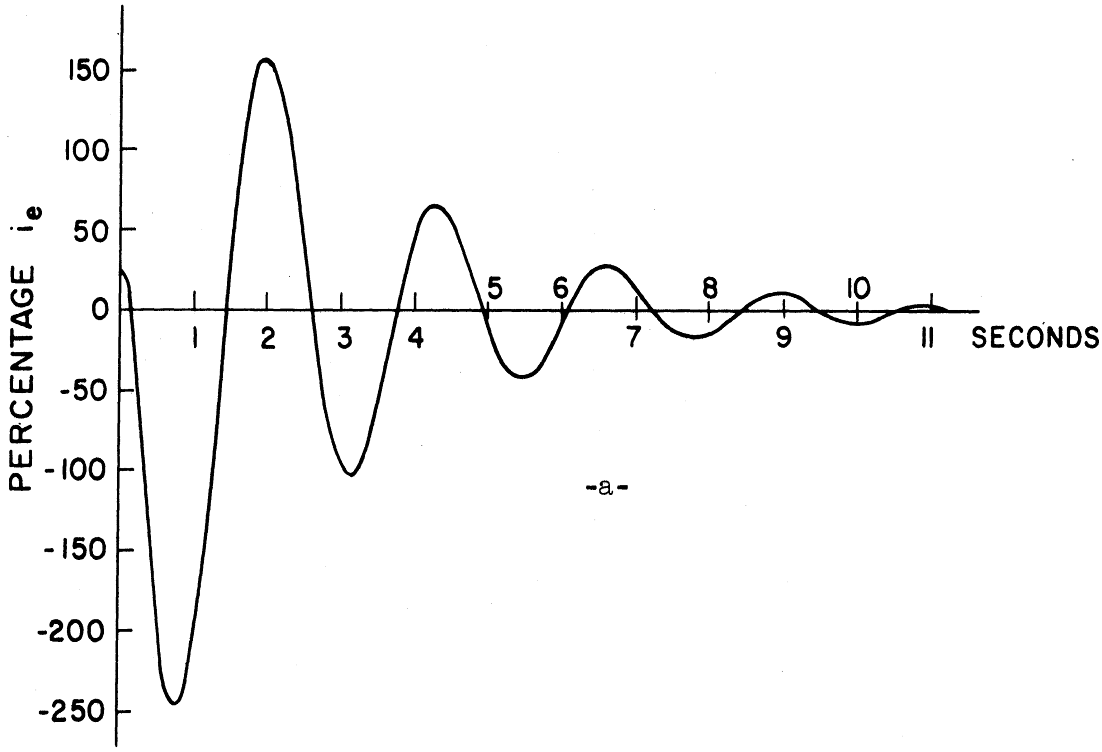


Figure 26. Responses with negative forcing feedback. $R_s = 0.1\ R_f$. Short-circuit conditions.



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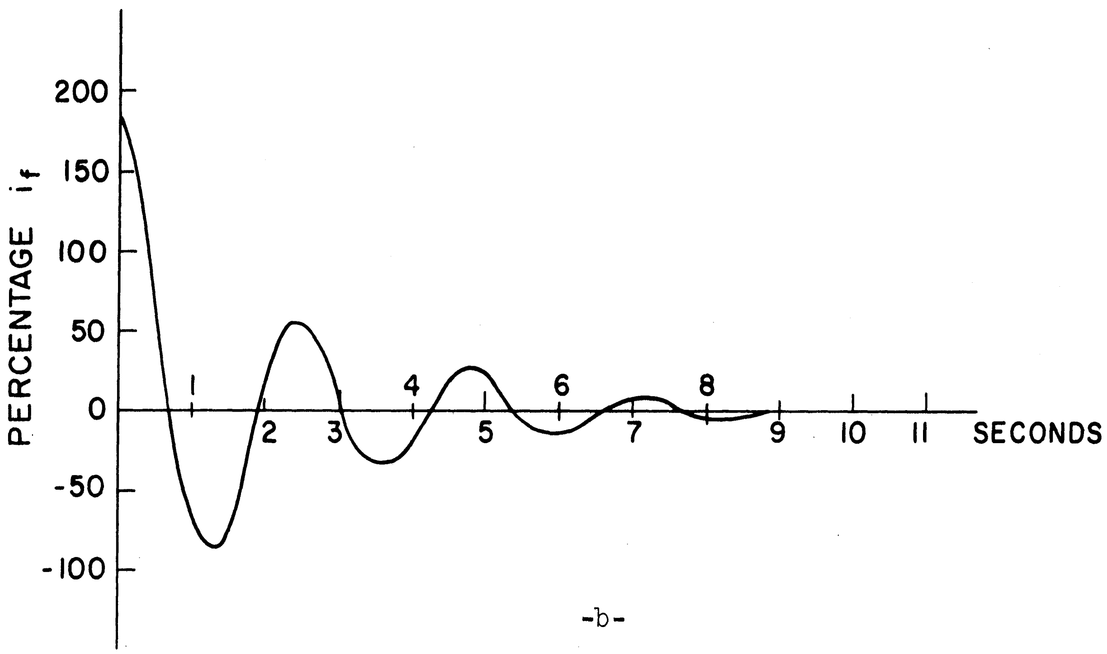

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Figure 27. Responses with negative forcing feedback. $R_s = 0.11\ R_f$. Short-circuit conditions.

On the other hand, $E_f''(0)$, the equivalent input voltage due to the a-c component of field current is shown in Section 3.7 to be given by

$$E_f''(0) = - \frac{i_{a.c}}{K_4} \tag{5-15}$$

Figure 13 is a block diagram representation for the resulting component transient currents for this case. The system function G_4'' is given by

$$G_4'' = \frac{K_4(s+ \frac{1}{\tau_a})}{(s+ \frac{1}{\tau_a})^2 + \omega^2} \tag{5-16}$$

It can be shown that with $\omega = 377$ radians/second, the closed loop system of Figure 13 can be approximated by an open loop system having a value of G_4'' . This means that, as far as the a-c component of short-circuit is concerned, feedback has no influence whatsoever on its response. Therefore, the a-c component of short-circuit current assumes its original value as dictated by Equations (3-58) and (3-59). For this reason, only the d-c component of short-circuit currents is being considered.

5.4 Discussion and Comparison of Results

Results of digital computer runs for the 265 mva machine, using the amplidyne method of demagnetization, have been reproduced in the figures of the previous sections. It is the purpose of this section to discuss these results and then ultimately to compare them with the oscillation resistance method.

Figures 20 through 24 show the results under open-circuit conditions. In particular, Figure 20 shows the results when a negative step voltage is applied to the B-winding at the same instant when the excitation to the A-winding is switched off. For a negative forcing step voltage of -3 volts, Figure 20a indicates the reversal of exciter current occurs after one second counted from the beginning of the switching operations. The generated field current decreases to about 53 percent of its original value at $t = 2.8$ seconds. However, for an increased negative step voltage of -6 volts, the field current decreases to about 30 percent of its original value at $t = 2.8$ seconds. Compared with a field discharge resistance of $R_{fd} = 1.5\ R_f$, the field current reaches about 32.9 percent of its original value at $t = 2.8$ seconds.[7] By applying higher values of negative step voltages, quicker responses can be achieved.

It should be realized, however, that the above method necessitates the adoption of some relaying equipment to cut off the negative step voltage at a suitable instant before the terminal a-c voltage starts building again. As mentioned before, the feedback method of negatively exciting the B-winding, does not require any such equipment. Results of the feedback methods are shown in Figures 21 through 24 for the switching operations when the machine is originally under open-circuit conditions. In Figure 21 with $R_s = 0.074\ R_f$, the exciter current passes through zero after 0.43 seconds from switching and the field current passes through zero after 3.65 seconds. The voltage across the field winding in this case attains a maximum negative value of about 59 percent in about 1.5 seconds. It then decreases very quickly and demagnetization is complete in about 10.5 seconds. If the rate of demagnetization is to be increased, a higher value of R_s must be used. In the last case shown in Figure 24 with $R_s = 0.14\ R_f$, the field current passes through zero after 2.07 seconds and a maximum negative value of about 104 percent across the field winding is reached in about 1.3 seconds. The field current becomes increasingly negative and this is favorable for eliminating the action of the damper windings that tend to resist flux changes. Consequently, quicker acceleration for the flux is achieved.

The above results are summarized in Table 5.1 together with the results already obtained by K.S. Raman(7) by means of the oscillations resistance method. For comparable amounts of time through which the field currents pass through zero, the maximum negative voltage across the field winding is somewhat higher by the Amplidyne method than with the oscillation resistance method. However, even in the last case, the maximum negative voltage has not exceeded 104 percent and the high negative values have lasted for no more than half a second.

Finally, a study of Figures 25 through 27 for the machine under short-circuit conditions occurring at no-load condition, shows that the exciter currents change polarity in a shorter time than under open-circuit conditions. Hence, demagnetization action is more intensive and quicker responses result. The currents are of a more oscillatory fashion and more forcing action is effected. Table 5-2 summarizes the results obtained from Figures 25 through 27, together with those obtained by the oscillation resistance method. In the amplidyne method of application, the passage of currents through zero occurs earlier but are somewhat oscillatory in nature. Also, the field currents attain higher negative values and this again would tend to eleminate the delaying action of damper circuits. Thus, more rapid reduction of the magnetic energy is achieved.

TABLE 5.1

SUMMARY OF RESULTS AND COMPARISON.
OPEN-CIRCUIT CONDITIONS

Method	Initial value of i_e in %	Initial value of i_f in %	Time in sec. at which i_f passes through zero	Maximum value of i_e in %	Maximum value of e_f in %
Amplidyne (feedback)					
$R_s = 0.074\ R_f$	25.0	28.5	3.65	-56.0	-60.0
$R_s = 0.1\ R_f$	25.0	28.5	2.78	-74.0	-77.4
$R_s = 0.11\ R_f$	25.0	28.5	2.50	-80.1	-84.7
$R_s = 0.14\ R_f$	25.0	28.5	2.07	-99.1	-103.8
Oscillation Resistance					
$R_a = R_f$	25.0	28.5	3.74	-27.5	-41.2
$R_a = 1.5\ R_f$	25.0	28.5	2.80	-37.0	-63.6
$R_a = 2\ R_f$	25.0	28.5	2.60	-45.0	-64.5
$R_a = 3\ R_f$	25.0	28.5	2.06	-55.0	-86.2

TABLE 5.2

SUMMARY OF RESULTS AND COMPARISON.
SHORT-CIRCUIT CONDITIONS

Method	Initial value of i_e in %	Initial value of i_f in %	Time in sec. at which i_f passes through zero	Maximum value of i_e in %
Amplidyne (feedback)				
$R_s = 0.074\ R_f$	25.0	184	0.82	-174.0
$R_s = 0.1\ R_f$	25.0	184	0.73	-230.0
$R_s = 0.11\ R_f$	25.0	184	0.71	-238.0
Oscillation resistance				
$R_a = R_f$	25.0	184	1.10	-55.0
$R_a = 1.5\ R_f$	25.0	184	0.86	-70.0
$R_a = 2\ R_f$	25.0	184	0.82	-80.0

CHAPTER VI

SUMMARY AND CONCLUSIONS

The main purpose of this study has been to develop a theory for the application of the amplidyne excitation system to effect a demagnetization process for a synchronous machine and then ultimately to determine the potentiality of such a method.

A unified approach using linear theory has been followed. The Laplace transform method has been shown to be a powerful tool for analyzing such a dynamic system. Equivalent sources to account for initial value contribution terms have been conveniently adopted. The application of control system techniques usch as system functions and block diagrams has facilitated the utilization of feedback concepts. One advantage of using feedback is to reduce the time response of the system under dynamic conditions and this is obviously, quite desirable.

In Chapter II, certain assumptions have led to the development of the mathematical model to represent the system under open-circuit conditions. Two methods for accelerating the decay of field current have been introduced. In the first method, a negative forcing step voltage is applied to one of the amplidyne auxiliary windings at the same instant as the excitation is switched off the main winding. In this method, quicker responses can be obtained by increasing the magnitude of the negative step voltage. In the second method, forcing action is obtained from the negative voltage across a resistance R_S in series with the field circuit. In this case, quicker responses can be obtained by increasing the value of the resistance R_S, but the field winding is subjected to higher voltages so that a compromise has to be achieved.

In Chapter III, the classic expressions for short-circuit currents have been developed using the same approach followed in Chapter II. Thus, there has been no need to resort to the constant flux linkage theorem with the usual assumption of zero resistance and then the modification of the expressions by decrement factors. This development has proved to be rather systematic and straightforward. An energy consideration has been also presented so that the physical situation may be grasped more readily. Based on this consideration, some assumptions have been stated in order to extend the theory of Chapter II to the important case where the synchronous machine is under fault conditions.

In Chapter IV, some experimental work has been carried out on a scale model synchronous machine in the Power Systems Laboratory. The agreement between the theory developed in Chapter II and experiment has proved to be reasonably good.

In Chapter V, the method is applied to a large synchronous machine having typical values of machine parameters. In particular, it has been compared with the results already obtained from the oscillation resistance method by another investigator. Compared with the oscillation resistance method, the feedback method produces quicker responses but the field winding of the synchronous machine is subjected to higher voltages. Since these transient voltages are of exponential character and very short duration, the insulation would not be stressed as severely as for a high-potential test of the same crest value. Moreover, it should be noted that the probability of developing the maximum three-phase short-circuits, during which the field current is highest, is very low. Less severe faults would result in decreased negative forcing voltages across the auxiliary control winding of the amplidyne. The demagnetization action is thus made to be graded in accordance with the severity of the fault. Indeed, this is another advantage of emplying feedback.

It should be noted that both the oscillation resistance method and the feedback method require the same auxiliary equipment such as a breaker and a resistor in the field circuit. With the negative forcing step voltage scheme, some relaying equipment is required to cut off the negative step voltage at the instant when the demagnetization is complete. This would complicate the circuitry but the breaker and resistor used in the other schemes could be dispensed with. It should also be noted that in both methods suggested, the switching operations at the amplidyne control fields are carried out easily since the power requirements there are small.

In conclusion, it can be said that the forcing action inherent in an amplidyne regulating system can provide an effective, powerful and flexible means for accelerating the decay of the stored magnetic energy in demagnetization of large synchronous machines. This forcing action decreases the time response according to the increased forcing voltage. However, the voltage across the field winding of the synchronous machine is somewhat higher. The analysis presented in this thesis provides a sound basis for the design of the optimum solution provided that the significant data are available. The formulation of the methods suggested is general and the computational requirements can be met by means of a standard digital computer program such as CSAP used in this study and cited under Reference 26.

APPENDIX A

EDDY CURRENTS AND DAMPERS EQUIVALENT CIRCUIT

In order to investigate the effect of eddy currents and damper circuits, it is desired to derive expressions for the transient field current $i_f(t)$ as well as for the hypothetical equivalent eddy currents and damper circuit current $i_{Kd}(t)$ when the IEEE(AIEE) tests as described in the code and quoted in Section 2.7, Chapter II is performed.

If the number of turns of the field circuit is N_f and if the effective number of turns of the equivalent eddy currents and dampers circuit is N_{Kd}, then one can write the following set of equations to describe the behavior as soon as the field winding is short circuited:

$$R_f\, i_f + L_f \frac{di_f}{dt} - M \frac{di_{Kd}}{dt} = 0$$

$$R_{Kd}\, i_{Kd} + L_{Kd} \frac{di_{Kd}}{dt} - M \frac{di_f}{dt} = 0 \qquad \text{(A-1)}$$

$$K(N_f\, i_f - N_{Kd}\, i_{Kd}) = v_t$$

where the sense of the equivalent circuit is assumed to give a negative sign for M which results in the opposite effect for i_{Kd} relative to that of i_f.

Taking the Laplace transform of the above set of equations and rearranging the terms, then in matrix notation, one can write:

$$\begin{bmatrix} R_f(1+s\tau_f) & -Ms & 0 \\ -Ms & R_{Kd}(1+s\tau_{Kd}) & 0 \\ -KN_f & KN_{Kd} & 1 \end{bmatrix} \begin{bmatrix} I_f \\ I_{Kd} \\ V_t \end{bmatrix} = \begin{bmatrix} L_f i_f(0) - M\, i_{Kd}(0) \\ -M\, i_f(0) + L_{Kd}\, i_{Kd}(0) \\ 0 \end{bmatrix} \qquad \text{(A-2)}$$

Thus, I_f , I_{Kd} , and V_t can be determined directly in the following fashion,

$$I_f = \frac{\Delta_1}{\Delta}, \quad I_{Kd} = \frac{\Delta_2}{\Delta} \quad \text{and} \quad V_t = \frac{\Delta_3}{\Delta}$$

where,

$$\Delta_1 = \begin{vmatrix} L_f\, i_f(0) - M\, i_{Kd}(0) & -Ms & 0 \\ -M\, i_f(0) + L_{Kd}\, i_{Kd}(0) & R_{Kd}(1+s\tau_{Kd}) & 0 \\ 0 & KN_{Kd} & 1 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} R_f(1+s\tau_f) & L_f\, i_f(0) - Mi_{Kd}(0) & 0 \\ -Ms & -M\, i_f(0) + L_{Kd}\, i_{Kd}(0) & 0 \\ -KN_f & 0 & 1 \end{vmatrix} \qquad \text{(A-3)}$$

$$\Delta_3 = \begin{vmatrix} R_f(1+s\tau_f) & -Ms & L_f\, i_f(0) - M\, i_{Kd}(0) \\ -Ms & R_{Kd}\,(1+s\tau_{Kd}) & -M\, i_f(0) + L_{Kd}\, i_{Kd}(0) \\ -KN_f & KN_{Kd} & 0 \end{vmatrix}$$

and

$$\Delta = \begin{vmatrix} R_f(1+s\tau_f) & -Ms & 0 \\ -Ms & R_{Kd}(1+s\tau_{Kd}) & 0 \\ -KN_f & KN_{Kd} & 1 \end{vmatrix} \qquad \text{(A-4)}$$

The denominator Δ , which determines the natural behavior of the system may be written as:

$$\Delta = R_f\, R_{Kd}[(\tau_f\, \tau_{Kd} - \frac{M^2}{R_f R_{Kd}})\, s^2 + (\tau_f + \tau_{Kd})s + 1] \qquad \text{(A-5)}$$

Considering the coefficient of s^2 , it may be written as:

$$(\tau_f \, \tau_{Kd} - \frac{M^2}{R_f \, R_{Kd}}) = \tau_f \, \tau_{Kd}(1 - \frac{M^2}{L_f \, L_{Kd}})$$

$$= \tau_f \, \tau_{Kd} \, \sigma$$

where,

$$\sigma = 1 - \frac{M^2}{L_f \, L_{Kd}} \qquad \text{(A-6)}$$

is the leakage coefficient of the coupled circuits.[4] Hence

$$\Delta = R_f \, R_{Kd}[\sigma \, \tau_f \, \tau_{Kd} \, s^2 + (\tau_f + \tau_{Kd})s + 1]$$

$$= \sigma \, L_f \, L_{Kd} \, (s - s_1)(s - s_2) \qquad \text{(A-7)}$$

where,

$$s_{1,2} = \frac{-(\tau_f+\tau_{Kd}) \pm \sqrt{(\tau_f+\tau_{Kd})^2 - 4\,\sigma\,\tau_f\tau_{Kd}}}{2\,\sigma\,\tau_f\tau_{Kd}} \qquad \text{(A-8)}$$

Usually, the two circuits are tightly coupled and the leakage is negligible so that in actual practice the roots may be distinguished as a small one and a large one. The small root is obtained by considering the positive sign in Equation (A-8) and by assuming

$$\sqrt{(\tau_f+ \tau_{Kd})^2 - 4\sigma \, \tau_f\tau_{Kd}} \cong (\tau_f+ \tau_{Kd}) \left\{1 - \frac{2\sigma \, \tau_f\tau_{Kd}}{(\tau_f+\tau_{Kd})^2}\right\} ,$$

then,

$$s_1 \cong - \frac{1}{\tau_f + \tau_{Kd}} \qquad \text{(A-9)}$$

The larger root is obtained by assuming $4\sigma\ \tau_f \tau_{Kd} \ll (\tau_f + \tau_{Kd})^2$ in Equation (A-8) so that

$$s_2 \cong -\frac{\tau_f + \tau_{Kd}}{\sigma\ \tau_f \tau_{Kd}} \qquad \text{(A-10)}$$

Expanding Δ_1 , Δ_2 and Δ_3 as given by Equations (A-3) and substituting the values of s_1 and s_2 as given by Equations (A-9) and (A-10) in Equation (A-7) for Δ , one can show that

$$I_f = \frac{i_f(0)\ [L_f\ L_{Kd}\ \sigma\ (s+ \frac{1}{\sigma\tau_{Kd}})] - i_{Kd}(0)\ R_{Kd}\ M}{\sigma\ L_f\ L_{Kd}\ (s+ \frac{1}{\tau_f+\tau_{Kd}})(s+ \frac{\tau_f + \tau_{Kd}}{\sigma\ \tau_f \tau_{Kd}})} , \qquad \text{(A-11)}$$

$$I_{Kd} = \frac{-i_f(0)\ [R_f M] + i_{Kd}(0)\ [R_f L_{Kd} + s(L_f L_{Kd} - M^2)]}{\sigma\ L_f\ L_{Kd}(s + \frac{1}{\tau_f+\tau_{Kd}})(s + \frac{\tau_f + \tau_{Kd}}{\sigma\ \tau_f\ \tau_{Kd}})} \qquad \text{(A-12)}$$

and,

$$V_t = K(N_f\ I_f - N_{Kd}\ I_{Kd}) \qquad \text{(A-13)}$$

Partial fraction expansions may be carried out and some of the resulting coefficients may be completely neglected if the following assumptions are made;

1. The terms involving $i_{Kd}(0)$ are much smaller than those multiplied by $i_f(0)$.

2. Perfect coupling exists between the two circuits implying the following relationship

$$\left(\frac{N_f}{N_{Kd}}\right)^2 = \frac{L_f}{L_{Kd}} \qquad \text{(A-14)}$$

Hence, by performing the inverse Laplace transform, one can show the time-domain solutions to be given by the following set of equations:

$$i_f(t) = \frac{\tau_{Kd}}{\tau_f} i_f(0) \left[\frac{\tau_f}{\tau_{Kd}} \epsilon^{-\frac{t}{\tau_f+\tau_{Kd}}} + \epsilon^{-\frac{t(\tau_f+\tau_{Kd})}{\sigma \tau_f \tau_{Kd}}}\right] \quad \text{(A-14)}$$

$$i_{Kd}(t) = -\sqrt{\frac{\tau_f}{L_{Kd}}}\, \frac{\tau_{Kd}}{\tau_f} i_f(0) \left[\epsilon^{-\frac{t}{\tau_f+\tau_{Kd}}} - \epsilon^{-\frac{t(\tau_f+\tau_{Kd})}{\sigma \tau_f \tau_{Kd}}}\right] \quad \text{(A-15)}$$

$$v_t(t) = K N_f i_f(0) \epsilon^{-\frac{t}{\tau_f+\tau_{Kd}}} \quad \text{(A-16)}$$

APPENDIX B

SYSTEM FUNCTIONS, TIME CONSTANTS AND REACTANCES OF THE IDEALIZED SYNCHRONOUS MACHINE

It is desired to simplify the expressions for the system functions X_d , G and X_q . Considering X_d first and dividing both the numerator and denominator of Equation (3-38) by $L_d r_{Kd} r_f$ and assuming that

$$\frac{3}{2}\frac{L_{aKd}^2}{r_{Kd}\, L_d} \ll \frac{L_{KKd}}{r_{Kd}}$$

in the numerator, one has

$$X_d = \frac{\left\{\frac{L_d''(L_{KKd}\, L_{ff} - L_{fKd}^2)}{L_d\, r_{Kd}\, r_f}\right\} s^2 + \left\{\left(\frac{L_{ff}}{r_f} - \frac{3}{2}\frac{L_{af}^2}{r_f\, L_d}\right) + \frac{L_{KKd}}{r_{Kd}}\right\} s + 1}{\frac{1}{L_d}\left[\left\{\frac{L_{KKd}\, L_{ff} - L_{fKd}^2}{r_{KKd}\, r_f}\right\} s^2 + \left\{\frac{L_{ff}}{r_f} + \frac{L_{KKd}}{r_{Kd}}\right\} s + 1\right]}$$

$$= \left[\frac{\tau_d'\, \tau_d''\, s^2 + (\tau_d' + \tau_{KKd})s + 1}{\tau_f\, \tau_{do}''\, s^2 + (\tau_f + \tau_{KKd})s + 1}\right] L_d \qquad \text{(B-1)}$$

where,

$$\tau_{do}'' = \tau_{KKd} - \frac{L_{fKd}^2}{r_{Kd}\, L_{ff}} \qquad \text{(B-2a)}$$

τ_{do}'' is the direct-axis subtransient open-circuit time constant,

$$\tau_d' = \tau_f - \frac{3}{2}\frac{L_{af}^2}{r_f\, L_d} \qquad \text{(B-2d)}$$

τ_d' is the direct-axis transient short-circuit time constant,

$$\tau_d'' = \frac{L_d''(L_{KKd}\, L_{ff} - L_{fKd}^2)}{r_{Kd}(L_d\, L_{ff} - \frac{3}{2} L_{af}^2)} \qquad \text{(B-2c)}$$

τ''_d is the direct-axis subtransient short-circuit time constant.

Equation (B-1) for X_d may be factored out by assuming that

$$\tau'_d + \tau_{Kd} \cong \tau'_d + \tau''_d ,$$

and

$$\tau_f + \tau_{Kd} \cong \tau_f + \tau''_{do}$$

since τ''_d and τ_{Kd} are small with respect to τ'_d , and τ_{Kd} and τ''_{do} are both small with respect to τ_f , so that

$$X_d = \frac{(1 + \tau'_d s)(1 + \tau''_d s)}{(1 + \tau_f s)(1 + \tau''_{do} s)} L_d \tag{B-3}$$

Considering G and dividing both the numerator and the denominator parts of Equation (3-34) by $r_{Kd}\, r_f$ and assuming that

$$\frac{L_{aKd}\, L_{fKd}}{r_{Kd}\, L_{af}} \ll \frac{L_{KKd}}{r_{Kd}}$$

in the nomerator, one has the following expression:

$$G = \left[\frac{\left\{ \frac{L_{KKd}}{r_{Kd}} s + 1 \right\}}{\left\{ \frac{L_{KKd}\, L_{ff} - L_{fKd}^2}{r_{Kd}\, r_f} \right\} s^2 + \left\{ \frac{L_{ff}}{r_f} + \frac{L_{KKd}}{r_{Kd}} \right\} s + 1} \right] \sqrt{\frac{3}{2}} \frac{L_{af}}{r_f}$$

$$= \left[\frac{(1 + \tau_{KKd}\, s)}{\tau_f\, \tau''_{do}\, s^2 + (\tau_f + \tau_{KKd})\, s + 1} \right] \sqrt{\frac{3}{2}} \frac{L_{af}}{r_f}$$

$$\cong \frac{(1 + \tau_{KKd}\, s)}{(1 + \tau_f\, s)(1 + \tau''_{do}\, s)} \sqrt{\frac{3}{2}} \frac{L_{af}}{r_f} \tag{B-4}$$

Similarly, by dividing both the numerator and denominator parts of Equation (3-40) by r_{KKq}, one has

$$X_q = \left[\frac{\left\{ \frac{L_{KKq}}{r_{KKq}} - \frac{3}{2} \frac{L_{aKq}^2}{r_{Kq} L_q} \right\} s + 1}{\frac{L_{Kq}}{r_{Kq}} s + 1} \right] L_q$$

$$= \frac{1 + \tau_q'' s}{1 + \tau_{qo}'' s} L_q \qquad \text{(B-5)}$$

where,

$$\tau_{qo}'' = \frac{L_{KKq}}{r_{Kq}} \qquad \text{(B-6a)}$$

τ_{qo}'' is the quadrature-axis subtransient open-circuit time constant,

$$\tau_q'' = \frac{L_{KKq}}{r_{Kq}} - \frac{3}{2} \frac{L_{aKq}^2}{r_{Kq} L_q} \qquad \text{(B-6b)}$$

τ_q'' is the quadrature-axis subtransient short-circuit time constant

It should be noted that the final expressions for the system functions as given by Equations (B-3) to (B-5) agree with those given by B. Adkins[12] except that in both the denominators of Equations (B-3) and (B-4), τ_f replaces τ_{do}'. It is believed that the difference between these time constants should be distinguished as was brought out in the previous appendix.

Finally, the reactance expressions for the idealized synchronous machine may be best obtained by applying the initial and final-value theorems as used in the Laplace transform theory for the respective system functions. It should be realized that the initial-value theorem is applied twice in order to identify the three regions of the short-circuit currents decay. In other words, after the subtransient currents have died out, the initial-value theorem is applied again for the transient currents as commonly done in synchronous machine literature. The reactance expressions will assume the following values:

$$x_d = \omega L_d$$

x_d is the direct-axis synchronous reactance.

$$x'_d \cong x_d \frac{\tau'_d}{\tau'_{do}}$$

$$= \omega\left(L_d - \frac{3}{2}\frac{L^2_{af}}{L_{ff}}\right)$$

$$x''_d = \omega L''_d$$

$$= x'_d \frac{\tau''_d}{\tau''_{do}}$$

$$= x'_d \frac{\tau'_d}{\tau'_{do}} \frac{\tau''_d}{\tau''_{do}}$$

x''_d is the direct-axis subtransient reactance.

$$x_q = \omega L_q$$

x_q is the quadrature-axis synchronous reactance.

$$x''_q = x_q \frac{\tau''_q}{\tau''_{qo}} = x_q\left(1 - \frac{3}{2}\frac{L^2_{aKq}}{L_q L_{KKq}}\right)$$

x''_q is the quadrature-axis subtransient reactance.

APPENDIX C

SIMPLIFICATION OF D

It is desired to simplify the denominator part of the transient currents solutions as given by Equations (3-42) to (3-44). For convenience, the denominator D is rewritten in the following form:

$$D = r^2 + s\, r\, X_d\, X_q \left(\frac{1}{X_d} + \frac{1}{X_q}\right) + X_d\, X_q\, (s^2 + \omega^2) \qquad \text{(C-1)}$$

In order to have D in a factored form, it is customary (Reference 11,12) to assume that the terms multiplied by s and the absolute terms can be neglected with respect to the terms multiplied by s^2 in Equations (3-46) and (3-47), so that

$$X_d \cong L_d''$$

and

$$X_q \cong L_q''$$

Another way to state this assumption is to consider the initial values of the above system functions so that the terms multiplied by s modify to:

$$s\, r\, X_d\, X_q \left(\frac{1}{L_d''} + \frac{1}{L_q''}\right)$$

Furthermore, neglecting the r^2 term in D while retaining the term multiplied by $(s^2 + \omega^2)$ as it is, Equation (C-1) may be rewritten as:

$$D = [s^2 + s\, r\, (\frac{1}{L_d''} + \frac{1}{L_q''}) + \omega^2]\, X_d\, X_q$$

$$= (s - s_1)(s - s_2)\, X_d\, X_q \qquad (C\text{-}2)$$

where,

$$s_{1,2} = -\frac{r}{2}(\frac{1}{L_d''} + \frac{1}{L_q''}) \pm j\sqrt{\omega^2 - \frac{r^2}{4}(\frac{1}{L_d''} + \frac{1}{L_q''})^2}$$

The imaginary part of these roots can be also approxiamted by neglecting the second term under the square root sign with respect to the ω^2 term, so that

$$s_{1,2} \cong -\frac{1}{\tau_a} \pm j\,\omega \qquad (C\text{-}3)$$

and

$$(s-s_1)(s-s_2) \cong (s+\frac{1}{\tau_a})^2 + \omega^2 \qquad (C\text{-}4)$$

where,

$$\tau_a = \frac{2\, L_d''\, L_q''}{r(L_d'' + L_q'')} \qquad (C\text{-}5)$$

τ_a approximates the average armature time constant considering the two axes. Substituting Equation (C-4) into Equation (C-2), one has

$$D = [(s + \frac{1}{\tau_a})^2 + \omega^2]\, X_d\, X_q \qquad (C\text{-}6)$$

APPENDIX D

EXPERIMENTAL MACHINES DATA

A commercial Amplidyne is used as the pilot exciter for exciting the separately excited d-c generator. Below are the name plate values and tabel D-1 gives its parameters as used in the calculations.

Machine rating	1.5	Kw
Voltage	250.0	Volts
Current	6.0	Amperes

Control Field	Ohms 50°C	Max. Amp.	Max. Volts
F_1-F_2	2.57	2.1	5.4
F_3-F_4 (A-winding)	11.07	1.0	11.07
F_5-F_6	50.5	0.25	12.62
F_7-F_8 (B-winding)	418.0	0.15	62.7

TABLE D-1

Parameter	Value		Determination
K_{A1}	124	Amp./Volt	Steady state test for e_a vs e_{cA}.
τ_{cA}	0.036	Sec.	Oscillographic plot of i_{cA} after application of a step voltage to the A-winding.
L_{cA}	0.4	H.	Calculated from r_{cA} and τ_{cA}.
K_{B1}	16	Amp./Volt	Steady state test for e_a vs e_{cB}.
τ_{cB}	0.036	Sec.	Oscillographic plot of i_{cB} after application of a step voltage to the B-winding.
L_{cB}	12.65	H.	Calculated from r_{cB} and τ_{cB}.

TABLE D-1 (Cont'd)

Parameter	Value		Determination
K_{cB}	460	Volts/Amp.	Steady state test for e_s vs i_{cB}.
R_{oe}	12.6	Ohms	Slope of e_a vs i_a with e_c fixed for different load resistances at the Amplidyne output circuit.
τ_s	0.085	Sec.	Oscillographic plot of e_a after application of a step voltage in series with 2 Mohms in the control circuit.
K_s	200	Volts/Amp.	Steady state test for e_a vs i_s.
r_s	13.6	Ohms	Calculated from Equation (2.11).
L_s	1.155	H.	Calculated from r_s and τ_s.

For the d-c main exciter generator, coupled to the synchronous machine shaft, the name plate values are as below. Table D-2 gives its parameters as used in the calculations. This table includes also the load on the d-c machine which is the field winding of the synchronous machine.

Machine rating			1	Kw
Voltage			125	Volts
Current			8	Amperes
Exciter current			1.2	Amperes
Excitation field resistance	$r_e(50°C)$	=	.146	Ohms
Alternator field resistance	$r_f(50°C)$	=	25.5	Ohms

TABLE D-2

Parameter	Value		Determination
τ_e	0.073	Sec.	Oscillographic plot of i_e after application of a step voltage to the excitation field winding.
L_e	9.35	H.	Calculated from $r_e(20°C)$ and τ_e .
K_g	144	Volts/Amp.	Steady state test for e_a vs i_e .
τ_f	0.33	Sec.	Oscillographic plot of i_f after the field winding has been suddenly short-circuited. The field current being already adjusted for rated terminal voltage at the synchronous machine terminals.
L_f	9.1	H.	Calculated from $r_f(20°C)$ and τ_f .
K_{A3}			Constant calculated from Equation (2-30a)
K_{B3}			Constant calculated from Equation (2-30b)

APPENDIX E

THE 265-MVA SYNCHRONOUS MACHINE DATA

In Chapter V, a 265 mva, 20 kv synchronous machine with a 1500 kw exciter, already chosen for different demagnetization studies, has been selected fro comparison with the amplidyne method as suggested in this thesis. The constants used have been based on actual designs and were considered to be typical of machines of these ratings. For amplidyne excitation system application, a 7.5-kw amplidyne used as a pilot exciter for the main exciter has been chosen from some technical information provided by the manufacturer.[27] The relevant constants and data for the synchronous machine and the whole excitation system are given below.

<u>Synchronous Machine</u>

265 mva, 20 Kv

$x_d = 1.69$ p.u.

$x_d' = 0.256$ p.u.

$x_d'' = 0.157$ p.u.

$x_q = 1.64$ p.u.

$x_q'' = 0.155$ p.u.

$\tau_{do}' = 5.82$ seconds

$\tau_d' = 0.88$ seconds

$\tau_{do}'' = 0.049$ seconds

$\tau_d'' = 0.03$ seconds

$\tau_a = 0.27$ seconds

τ'_{do} and τ_a are based on winding temperature of 75°C.

Open-circuit inductance L_f of generator field	= 0.93	henry
Generator field resistance r_f	= 0.16	ohms
Armature resistance R_a	= 0.00232	ohms.

Main Exciter

1500 kw   3000 Amperes   500 volts

Inductance of shunt field	=	9	h
Resistance of shunt field	=	15	ohms
Voltage constant K_e	=	26	volts/ampere
Residual voltage	=	20	volts

Number of turns in shunt field winding is to be adjusted in design so as to maintain the original flux as mentioned in Section 5.1.

Pilot Exciter

AM 617 7.5 Kw 2-pole Group 9 without quadrature series field.

Winding	Turns/pole	Ohms	Max. Continuous Amp.
F_1-F_2 (A-winding)	200	20	1
F_3-F_4	200	20	1
F_5-F_6	105	9.04	1.3
F_7-F_8	105	9.04	1.3
F_9-F_{10} (B-winding)	370	149	0.32

The total control field inductance in henries for 1000 turns per pole is 93 for the above type. From the typical saturation curves supplied by the manufacturer, K_{A1} and K_{B1} are found to have values of 100 and 25 respectively. Other constants are given below

Inductance (h.)	Resistance (ohms)	Time Constant (sec.)
L_{cA} = 18.6	21	τ_{cA} = 0.845
L_{cB} = 34.4	164	τ_{cB} = 0.21
L_s = 0.072	0.9	τ_s = 0.08

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