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# Evidence of Fraud, Audit Risk and Audit Liability Regimes

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**Abstract.** We investigate the effectiveness of proportionate liability in reducing the probability of fraud and audit risk relative to joint and several liability in two strategic audit settings: one that provides conclusive evidence of fraud and one that provides inconclusive evidence of fraud. In both settings the auditor makes an audit effort choice, but in the second setting the auditor also evaluates the audit evidence. Our results show that when the auditor chooses only effort, a proportionate liability rule with large marginal liability relief decreases audit risk. However, when the auditor also evaluates the audit evidence this result no longer holds.

**Keywords:** legal liability regimes, strategic auditing, proportionate liability, fraud

**JEL Classification:** D4, K13, M14

The audit profession contends that legal practices treat auditors unfairly. Until recently, they have been held jointly and severally liable for undetected material misstatements and have had to pay their own legal fees whether or not they prevail in court.[1] Under a joint and several liability regime the auditor pays damages that are often unrelated to his level of due care because other defendants are incapable of paying their share. For example, being 1% at fault for causing plaintiffs' damages is not necessarily related to paying 1% of the damages because the auditor is a "deep pocket." This issue is discussed by Arthur Andersen & Co., et al. (1992) who state that the joint and several liability system:

> . . . functions primarily as a risk transfer scheme in which marginally culpable or even innocent defendants too often must agree to coerced settlements in order to avoid the threat of even higher liability, pay judgments totally out of proportion to their degree of fault, and incur substantial legal expenses to defend against unwarranted lawsuits.

By contrast, a proportionate liability regime allows the court to determine the percent of auditor damages based on the percentage of fault in causing those damages irrespective of the other defendants' ability to pay. The Private Securities Litigation Reform Act of 1995 adopts a limited form of proportionate liability, and the Securities Litigation Uniform Reform Act of 1998 preempts state law for class action suits involving nationally traded securities. However, actions against auditors that are not class action suits may be filed under state common law and litigated under joint and several liability. In addition, controversy still exists

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over the effect of proportionate liability on audit failure rates and how best to implement a proportionate liability rule.

We examine the effect of a proportionate liability regime on audit failure rates in a strategic audit setting with various types of audit evidence. Our benchmark regime is joint and several liability within the context of a vague specification of due care. The term, "vague due care," coined by Schwartz (1997), reflects all uncertainty surrounding litigation outcomes for auditors resulting from imprecise interpretations of the auditor's due care standard. The auditor strategically formulates an audit plan giving consideration to how it would affect a court's judgment of negligence as well as the apportionment of damages according to liability regime; i.e., either joint and several or proportionate.

A primary concern about proportionate liability is its potential effects on stockholders.[2] Opponents claim that proportionate liability would decrease investor protection when firms are bankrupt due to the reduction in compensatory payments from auditors and the presumed increased likelihood of audit failures. Among others, Hanson and Gillett (1991) suggest that auditors' expected liability costs are inversely related to audit risk because larger liability costs motivate auditors to avoid audit failure.[3] However, unlike joint and several liability, proportionate liability includes other motivational factors that potentially decrease audit risk, offsetting the decrease in compensatory payments from bankruptcy and audit failure. Whether audit risk increases or decreases relative to joint and several liability depends on the audit setting and the form of the proportionate liability rule.

Previous research finds that proportionate liability decreases audit risk relative to joint and several liability in some audit and litigation settings. Narayanan (1994) shows that when the auditee does not behave strategically, a proportionate liability rule that provides large marginal liability relief for audit quality increases audit quality and decreases audit risk. Hillegeist (1999) examines a model that includes a strategic auditee and an apportionment rule that does not depend on audit quality. He finds that when the fraudulent-type auditee always misreports, proportionate liability increases audit risk. However, when the fraudulent-type auditee's misreporting behavior depends on the exogenously set apportionment rule, proportionate liability reduces audit risk. In this case, audit quality decreases but audit risk also decreases due to a decrease in the probability of misreporting.[4]

These previous studies examine different auditor and litigation settings but they share a common assumption about the auditor's detection capabilities: the probability of detecting a material misstatement depends only on the auditor's choice of effort. In our strategic model, this implies that conclusive evidence of fraud exists. The auditor detects fraud only when the conclusive evidence is found.[5] However, evidence of fraud is often inconclusive and inferential. For example, evidence that implies an inventory overstatement of 3% could be interpreted as "normal" due to typical inventory shrinkage while an inventory overstatement of 8% is more likely related to fraud. In this case, the auditor exercises his judgment in determining whether the evidence supports a finding of fraud. When the audit evidence is inconclusive, audit quality depends not only on audit effort but also evidence evaluation. Audit quality increases as effort increases and decreases as evidence evaluation becomes more liberal.[6]

We investigate whether or not the prescription of large marginal liability relief for audit quality can reduce audit risk relative to joint and several liability when the auditor faces a strategic auditee and when the audit evidence about fraud is inconclusive. Our model extends both the liability and strategic audit literature. Our audit setting is consistent with the prior strategic auditing research except that the auditor's anticipated damages, given undetected fraud, are not fixed, but depend on how the courts assess the quality of the audit.[7] Prior liability research either does not include a strategic auditee, audit damages that depend on audit quality, or an auditor that chooses both audit effort and evidence evaluation.[8]

Hillegeist (1999) includes a strategic auditee, but his study differs from ours in significant ways. For example, our apportionment rule depends on audit quality while Hillegeist's (1999) apportionment rule is fixed exogenously. Furthermore, the auditee in his model is deterred from fraud solely by the prospect of paying future damages rather than penalties incurred from audit detection. We assume that the auditee is penalized only when the auditor detects fraud. When the auditor does not detect fraud, the market eventually reveals that fraud has occurred. However, the undetected fraudulent auditee in our model is insolvent while Hillegeist's (1999) analysis of the auditee's strategic behavior crucially depends on a probability of bankruptcy less than one. The auditee in our model is never able to pay his share of investor damages. When the auditee commits fraud, he consumes the rewards from fraud, which is consistent with the setting discussed in Arthur Andersen & Co., et al. (1992).

Our results show that when conclusive evidence of fraud exists, Narayanan's (1994) policy prescription for a proportional liability rule extends to a setting where the manager (auditee) behaves strategically. However, when the evidence of fraud is inconclusive, we find that a proportionate liability rule with large marginal liability relief for audit quality can *increase* audit risk relative to joint and several liability. Large marginal liability relief that increases audit effort also reduces the auditor's expected liability cost, and in turn, provides less incentive for the auditee to reduce the fraud rate. Concurrently, the auditor's evidence evaluation becomes more liberal which increases the auditor's expected liability costs and provides more incentive for the auditee to reduce the fraud rate. In our equilibrium model of fraud, the net effect of these two forces can motivate the auditee to commit fraud more often and audit risk increases.

When the evidence of fraud is inconclusive, a proportionate liability rule that decreases the probability of fraud and audit risk relative to joint and several liability depends on characteristics other than large marginal liability relief based on court-assessed audit quality. These characteristics include the cost of effort, the auditee's motivation to commit fraud, and how the court assesses audit quality. Our results discuss how these factors affect a proportionate liability rule's ability to reduce audit risk.

The remainder of the paper is organized as follows. Section 1 investigates whether proportionate liability can reduce audit risk when conclusive evidence of fraud exists, while Section 2 examines a setting when evidence of fraud is inconclusive. Section 3 considers whether proportionate liability rules for various court-assessed quality measures can effectively reduce audit risk. Section 4 provides concluding remarks.

## 1. Audits that Provide Conclusive Evidence of Fraud

We compare audit risk under proportionate and joint and several liability regimes in a strategic audit setting. Similar to prior studies in strategic auditing, the auditee moves first by choosing the probability of fraud based on his motivation to commit fraud and taking into consideration the auditor's response in choosing an audit plan.[9] The auditor does not know whether or not fraud has occurred when he formulates his audit plan. The auditor formulates an audit plan based on the probability of fraud in addition to the liability cost of an undetected fraud and the cost of performing the audit. Unlike previous studies, we also incorporate the auditor's anticipated liability cost for an undetected fraud that depends directly on the court's quality assessment of the audit plan and the liability regime, either joint and several or proportionate.

When conclusive evidence of an existing fraud is available, the auditor formulates an audit plan by choosing audit effort, $n_j$, where $j = PL$ denotes proportionate liability and $j = JSL$ denotes joint and several liability. Evidence evaluation is fixed because once evidence of fraud is discovered the probability of fraud is one. For example, the discovery of falsified sales documents to nonexistent customers would substantiate the existence of fraudulent sales transactions and the likelihood of finding such documents increases in the auditor's search effort. We employ this audit technology by assuming that the probability of detecting an existing fraud is $1 - \text{Exp}(-n_j)$, while the probability of false fraud detection is zero.[10] In addition, we denote the probability that the auditee chooses to commit fraud as $t_j$, $j = PL$ or *JSL*.

In considering the effect of proportionate liability on audit risk, we first discuss how audit effort affects auditor liability for both proportionate and joint and several liability regimes. We then solve for the equilibrium strategies under proportionate liability, noting how joint and several liability affects these strategies. Finally, we compare audit risk under the two regimes.

### *1.1. Liability Rules When the Auditor Chooses Only Effort*

In a vague liability system (see Schwartz, 1997), the auditor is uncertain of whether the court would find audit quality to be sufficient to justify exonerating him from liability when financial statement users have been damaged. The uncertainty arises because standards for audit liability are not well defined.

Let $0 < \upsilon(n_j) \leq 1$, with $\upsilon'(n_j) \leq 0$, be the probability that the court finds the auditor liable for audit effort $n_j$. Let $L$ be the total liability cost that the auditor would pay under joint and several liability when the auditee is unable to pay his share of the damages. Thus, $L$ is the maximum amount of damages the auditor would pay under all liability regimes and the corresponding expected liability cost under joint and several liability is $\upsilon(n_{JSL})L$.[11]

For a proportionate liability regime, we define the proportionate liability rule as $0 < J(n_{PL}) \leq 1$, with $J'(n_{PL}) \leq 0$. The proportionate liability rule, $J$, determines the percentage of the total liability cost, $L$, that the auditor must pay, given his choice of effort, $n_{PL}$. Thus, under a proportionate liability regime, the auditor's expected liability cost is $\upsilon(n_{PL})J(n_{PL})L$. First the auditor must be found liable, which occurs with probability $\upsilon(n_{PL}) > 0$, and then the court determines an apportionment rule, $J(n_{PL})$.[12]

Next, we describe the strategic interaction between the auditor and auditee under vague proportionate liability. The case of joint and several liability is the same except that $J = 1$, with $J'(n_{JSL}) = 0$, for all effort levels.

### *1.2. Proportionate Liability*

*The auditee's expected payoff and equilibrium conditions.* We assume that the auditee's opportunity for fraud consists of fraudulent financial reporting or misappropriation of assets (also referred to as defalcation), but not both.[13] The auditee receives a zero payoff if he is honest and a payoff of $R$ if he commits fraud that goes undetected. However, if the auditor detects an existing fraud, the auditee is penalized $R$ and a punitive amount $P$ for a total penalty of $R + P$. Auditor detection of fraud allows for the recovery of $R$ and an imposition of the punitive penalty, $P$. We assume that if the auditor does not detect an existing fraud, the market eventually reveals that fraud has occurred. Sufficient time passes before a suit is brought against the auditor and auditee so that the auditee is unable to pay his share of damages. For example, the auditee might be insolvent or flee the country. Thus, the auditor's actions have no impact on the auditee's reward from fraud if fraud goes undetected.

Auditors have been most concerned about suits when auditees are unable to pay their share of damages. The national audit firms have argued for proportionate liability to reduce their payments of damages (see Arthur Andersen & Co., et al., 1992). The reduction in the amount of damages to be paid has caused resistance to the adoption of a proportionate liability rule. The amount the auditee is able to contribute to damages is expected to be limited (perhaps zero) and if the auditor does not pay the auditee's portion of damages, users are unable to recover their losses.

Based on the auditee's reward and penalty for fraud, the auditee chooses the probability of fraud, $t_{PL}$, that maximizes his expected payoff,

$$M(t_{PL}) = t_{PL}\{\text{Exp}(-n_{PL})R - (1 - \text{Exp}(-n_{PL}))P\}. \tag{1}$$

Because the auditee's expected payoff is linear in his strategy choice, he is either indifferent between honesty and fraud or strictly prefers one or the other where indifference corresponds to auditee randomization. Thus the auditee's equilibrium conditions are expressed as a combination of an indifferent condition and an inequality. The following are the auditee's equilibrium conditions:

$$\begin{aligned} &t_{PL} \in (0, 1) \quad \text{only if} \quad \text{Exp}(-n_{PL}) = \frac{P}{R + P} \quad \text{and} \\ &t_{PL} = 0(1) \quad \text{when} \quad \text{Exp}(-n_{PL}) < (>) \frac{P}{R + P}. \end{aligned} \tag{2}$$

The ratio of the auditee's payoffs, $\frac{P}{R+P}$, represents the portion of the total penalty that is punitive, which we call the auditee's effective penalty, $\rho$. A small effective penalty is due to either a small punitive penalty or a large reward from fraud. If the auditor chooses $\text{Exp}(-n_{PL}) = \rho$, then the auditee is indifferent between fraud and honesty. The higher the auditee's effective penalty, the less motivated he is to commit fraud and the auditor can choose a higher probability of non-detection (a lower probability of detection) in order to deter the auditee from committing fraud ($t_{PL} < 1$).

*The auditor's expected payoff and equilibrium condition.* The auditor formulates an audit plan that includes various procedures for detecting a (material) misstatement due to fraud. We simplify this as a choice of $n_{PL}$, which detects a misstatement due to fraud with probability $1 - \text{Exp}(-n_{PL})$. Thus, audit risk or the probability of audit failure is $t_{PL}\text{Exp}(-n_{PL})$ where effort, $n_{PL}$, increases the probability of detection, $1 - \text{Exp}(-n_{PL})$, and decreases audit risk for a given $t_{PL}$. The cost of audit effort is $k$ and the auditor's expected payoff is

$$A(n_{PL}) = -t_{PL}\,\text{Exp}(-n_{PL})\upsilon(n_{PL})J(n_{PL})L - k\,n_{PL}. \tag{3}$$

The auditor's action choice is continuous and in equilibrium his effort level satisfies the first order condition,[14]

$$A'(n_{PL}) = t_{PL}\,\text{Exp}(-n_{PL})LS(n_{PL}) - k = 0 \tag{4}$$

where

$$S(n_{PL}) = [\upsilon(n_{PL}) - \upsilon'(n_{PL})][J(n_{PL}) - J'(n_{PL})] - \upsilon'(n_{PL})J'(n_{PL}).$$

*Equilibrium strategies.* The solution concept for our setting is a Nash equilibrium of a simultaneous play game. The equilibrium to our game is given in Lemma 1.

**Lemma 1** *In a strategic audit setting that provides conclusive evidence of fraud, the auditor's and auditee's equilibrium strategies under a proportionate liability regime are:*

$$(1)\; n_{PL} = \text{Log}\left(\frac{1}{\rho}\right) \tag{5}$$

*and*

$$(2)\; t_{PL} = \frac{k}{L\rho}\frac{1}{S(\text{Log}(1/\rho))}. \tag{6}$$

(All proofs are in the Appendix.)

Consistent with prior studies, the auditor chooses an audit effort level based solely on the auditee's effective penalty, $\rho$. The auditee's choice of fraud rate, $t_{PL}$, depends on the cost of audit effort, $k$, the auditee's effective penalty, $\rho$, and the auditor's expected marginal liability cost for a given level of effort, $LS(\text{Log}(1/\rho))$. This is also similar to previous strategic auditing studies except that the auditor's liability cost is fixed at $L$ in those studies.[15]

### *1.3. Comparative Analysis of Liability Regimes*

A comparison of joint and several with proportionate liability shows that the auditee changes his rate of fraud while the auditor does not change his choice of audit effort, when the liability regime changes from joint and several to proportionate liability. Because the auditee anticipates that his penalties and rewards will be the same under the two regimes, there is no reason for the auditor to change the probability of detection. Instead, the auditor's inferred probability of fraud and audit risk changes. Audit risk under proportionate liability is

$$AR_{PL} = \frac{k}{L}\frac{1}{S(n)} \tag{7}$$

and under joint and several liability is

$$AR_{JSL} = \frac{k}{L}\frac{1}{\upsilon(n) - \upsilon'(n)} \tag{8}$$

where $n = n_{PL} = n_{JSL} = \mathrm{Log}(1/\rho)$. Proposition 1 provides the condition for audit risk to decrease under proportionate liability.

**Proposition 1** *The probability of detection is the same in both proportionate and joint and several liability regimes, while audit risk decreases in a proportionate liability regime, compared to a joint and several liability regime, if and only if*

$$J(n) - J'(n)\frac{\upsilon(n)}{\upsilon(n) - \upsilon'(n)} > 1 \tag{9}$$

*when* $n = \mathrm{Log}(1/\rho)$.

Previous studies that do not include a strategic auditee evaluate the relative benefits of proportionate and joint and several liability based on differences in audit effort.[16] By contrast, our strategic model shows that audit effort for joint and several and proportionate liability is *not* different. However, proportionate liability has the potential to reduce the probability of fraud, which in turn reduces audit risk, when marginal liability relief on audit effort is sufficiently large, as given in Proposition 1.[17] Because $0 < \frac{\upsilon(n)}{\upsilon(n)-\upsilon'(n)} < 1$ for a given vague liability probability that defines the collective beliefs of auditor negligence by the courts, a proportionate liability rule with sufficiently large liability relief, $-J'(n)$, results in a reduction in audit risk. Moreover, as the marginal probability of a non-negligence finding increases (i.e., $-\upsilon'(n)$ increases), the courts need to grant a larger amount of marginal liability relief under proportionate liability for audit risk to decrease because $\frac{\upsilon(n)}{\upsilon(n)-\upsilon'(n)}$ decreases in $-\upsilon'(n)$.

Our results for audits that provide conclusive evidence of fraud support Narayanan's (1994) policy prescription of large marginal liability relief on audit effort. Narayanan's (1994) Proposition 5 requirement for proportionate liability to decrease audit risk under a vague liability system is equivalent to expression (9) when the auditee firm pays zero damages.[18]

## 2. Audits that Provide Inconclusive Evidence of Fraud

In this section, we investigate whether audits that provide inconclusive evidence of fraud result in an increase or decrease in audit risk under proportionate liability relative to joint and several liability. To simplify and because the equilibrium characteristics of the joint and several liability game are similar to those of the proportionate liability game, we present and derive only the proportionate liability game's equilibrium. The equilibria of both games depend on the vague liability rule. The equilibrium strategies of the joint and several liability game are found by setting the proportionate liability rule $J = 1$.

In audits that provide inconclusive evidence of fraud, the elements of our strategic setting are the same except that the auditor evaluates audit evidence in addition to choosing audit effort. In audits that provide conclusive evidence of fraud, detection is certain when such evidence is uncovered during the audit. However, when evidence of fraud is inconclusive, the auditor must infer whether or not the evidence indicates that the manager has committed fraud.

Let $\tilde{x}$ be the audit evidence and $n_{PL}$ be the amount of audit effort where $\tilde{x}$ is distributed normally with mean zero, for no fraud, and mean $\omega$, for fraud, and with variance $\sigma^2/n_{PL}$.[19] The evidence, $\tilde{x}$, is a noisy estimate of the audit population's mean, (zero or $\omega$ for fraud). The auditor is uncertain about whether $\tilde{x}$ was drawn from an audit population with no fraud or one with fraud. This uncertainty is captured by the variance $\sigma^2$ but is reduced by increasing audit effort (i.e., gathering more evidence). The mean $\omega$ represents the measurable effects of fraud on the audit population.[20] For example, if the auditor tests inventory existence, $\omega$ may be a quantity measure or a dollar measure of fraudulently overstated inventory. The auditor observes evidence $x$ and then decides whether to accept or reject the financial statements as materially correct. The evidence distribution is the normal density $f(x \mid 0, n_{PL})$ when there is no fraud and $f(x \mid \omega, n_{PL})$ when fraud has been committed.

As discussed in Section 1, the proportionate liability rule, $J$, decreases in audit effort. Because audit effort is positively associated with the court-assessed audit quality we make no distinction between audit effort and the court-assessed audit quality. When the auditor evaluates evidence in addition to choosing effort, we define the court-assessed audit quality as $q(x, n_{PL})$. Audit quality increases in audit effort, $\partial q/\partial n_{PL} > 0$, but decreases in audit evidence $x$, $\partial q/\partial x < 0$, where $x$ is positively associated with fraud. Consequently, the proportionate liability rule is $J(q)$ with $J'(q) \leq 0$.[21]

When the auditor makes an accept/reject decision, he incurs two potential decision costs, one for an incorrect rejection, which we call $C_R$, and one for an incorrect acceptance, which is the auditor's expected liability cost, $\upsilon(q)J(q)L$. The auditor incurs a cost, $C_R$, from needlessly expanding the audit, the loss of clients, or the loss of reputation.[22] In effect the auditor chooses a cutoff value $c_{PL}$ where the auditor accepts when $x < c_{PL}$; otherwise, he rejects.[23] The audit plan consists of the auditor's evaluation rule, $c_{PL}$, and effort level, $n_{PL}$.

The auditor's expected payoff is

$$A(c_{PL}, n_{PL}) = -(1 - t_{PL})(1 - F(c_{PL} \mid 0, n_{PL}))C_R \\ - t_{PL} \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL})\upsilon(q(x, n_{PL}))J(q(x, n_{PL}))L\, dx - k\, n_{PL} \tag{10}$$

where $F$ is the normal distribution function corresponding to the density $f$.

The first term is the expected cost of an incorrect rejection where the realized cost, $C_R$, is constant and independent of the observed evidence. The second term is the expected cost of an incorrect acceptance where the realized liability cost, $J(q(x, n_{PL}))L$, and the probability that it occurs, $\upsilon(q(x, n_{PL}))$, depend on the evidence outcome, $x$.

In formulating the audit plan, $(c_{PL}, n_{PL})$, the auditor satisfies the following two first order conditions obtained from expression (10) with respect to $c_{PL}$ and $n_{PL}$:[24]

$$\begin{aligned}\frac{\partial A}{\partial c_{PL}} &= (1-t_{PL})f(c_{PL}\,|\,0,n_{PL})C_R - t_{PL}f(c_{PL}\,|\,\omega,n_{PL}))\upsilon(q(c_{PL},n_{PL}))\\ &\quad\times J(q(c_{PL},n_{PL}))L = 0 \qquad (11)\\ \frac{\partial A}{\partial n_{PL}} &= (1-t_{PL})f(c_{PL}\,|\,0,n_{PL})C_R\frac{c_{PL}}{2n_{PL}} - t_{PL}f(c_{PL}\,|\,\omega,n_{PL}))\upsilon(q(c_{PL},n_{PL}))\\ &\quad\times J(q(c_{PL},n_{PL}))L\frac{c_{PL}-\omega}{2n_{PL}} - t_{PL}L\int_{-\infty}^{c_{PL}} f(x\,|\,\omega,n_{PL})\\ &\quad\times\left\{\frac{(\omega-x)}{2n_{PL}}\frac{\partial q(x,n_{PL})}{\partial x} + \frac{\partial q(x,n_{PL})}{\partial n_{PL}}\right\}\frac{d(J\upsilon)}{dq}\,dx - k = 0. \qquad (12)\end{aligned}$$

Condition (11) shows how the auditor trades off type I and type II errors when choosing the evaluation rule, $c_{PL}$, for a fixed level of effort. The expected marginal cost of a type I error equals the expected marginal cost of a type II error. The first and second terms of condition (12) are the marginal costs of type I and type II errors with respect to effort without taking into consideration the marginal effects of the vague and proportionate liability rules. The marginal effects of a proportionate liability rule are found in the third term of condition (12). The marginal relief for court-assessed audit quality, $-d(J\upsilon/dq) \geq 0$, is averaged over all possible evidence outcomes, $x$, that are less than the auditor's evaluation rule, $c_{PL}$.

### *2.1. The Auditee's Expected Payoff and Equilibrium Conditions*

The auditee's expected payoff is essentially the same as the one found in an audit setting providing conclusive evidence of fraud. However, the probability of detection, given fraud, is defined as the probability of rejection given fraud or $1-F(c_{PL}\,|\,\omega,n_{PL})$. While the auditee could incur a penalty for an incorrect auditor rejection, we assume that it is zero to simplify the analysis and to be consistent with the auditee's payoffs in Section 1.[25] The auditee's expected payoff is

$$t_{PL}\{F(c_{PL}\,|\,\omega,n_{PL})R - (1-F(c_{PL}\,|\,\omega,n_{PL}))P\}, \qquad (13)$$

where $P$ is the auditee's punitive penalty for detected fraud and $R$ is the auditee's reward for undetected fraud. If we define the auditee's effective penalty as before, $\rho = \frac{P}{R+P}$, then the auditee's equilibrium conditions are

$$t_{PL} \in (0,1) \text{ only if } F(c_{PL}\,|\,\omega,n_{PL}) = \rho \quad \text{and} \quad t_{PL} = 0(1) \text{ when } F(c_{PL}\,|\,\omega,n_{PL}) < (>)\rho. \qquad (14)$$

Assuming audit cost, $k$, is small enough to allow the auditor to deter fraud ($t_{PL} < 1$), we have[26]

$$F(c_{PL}\,|\,\omega,n_{PL}) = \rho. \qquad (15)$$

In equilibrium, the auditor chooses the probability of non-detection, $F(c_{PL} \mid \omega, n_{PL})$, (called detection risk) equal to the auditee's effective penalty, $\rho$. Using a standard normal distribution function $\Phi(z)$ where $z = \frac{(c_{PL}-\omega)\sqrt{n_{PL}}}{\sigma}$, we rewrite (15) as

$$\Phi(z_\rho) = \rho. \tag{16}$$

Thus $z_\rho$ is fixed and defines the relation between $c_{PL}$ and $n_{PL}$. In equilibrium we have

$$c_{PL} = z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega. \tag{17}$$

The auditor's choice of evaluation rule, $c_{PL}$, depends on audit effort, $n_{PL}$, because $z_\rho$ is fixed based on the auditee's effective penalty, $\rho$. As the auditee becomes more motivated to commit fraud, $z_\rho$ decreases. Audit settings where detection risk is less than 50% results in $\Phi(z_\rho) < 0.5$ or $z_\rho < 0$, and the evaluation rule, $c_{PL}$, increases in audit effort, $n_{PL}$.

### 2.2. Equilibrium Strategies

Based on conditions (11), (12) and (17), we state the equilibrium strategies for the proportionate liability game in Lemma 2.

**Lemma 2** *In a strategic audit setting that provides inconclusive evidence of fraud and given a proportionate liability regime, the auditee's and auditor's equilibrium strategies satisfy:*[27]

$$(1)\ t_{PL} = \frac{\phi_0 C_R}{\phi_\rho \upsilon(q_\rho) J(q_\rho) L + \phi_0 C_R}, \tag{18}$$

$$(2)\ c_{PL} = z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega \quad \textit{and} \tag{19}$$

(3) $n_{PL}$ *that satisfies*

$$(1 - t_{PL}(n_{PL}))\phi_0 C_R \times \left( \frac{\omega}{2\sigma\sqrt{n_{PL}}} + \int_{-\infty}^{z_\rho \frac{\sigma}{\sqrt{n_{PL}}}+\omega} f(x \mid n_{PL}, \omega) M(x) \frac{(-J'(q)\upsilon(q) - J(q)\upsilon'(q))}{\phi_\rho J(q_\rho)\upsilon(q_\rho)}\, dx \right) - k = 0 \tag{20}$$

*where*

$$\phi_0 = \phi\left(z_\rho + \frac{\omega\sqrt{n}}{\sigma}\right), \phi_\rho = \phi(z_\rho), q_\rho = q(c, n)$$

*with* $c = z_\rho \frac{\sigma}{\sqrt{n}} + \omega$,

$$M(x) = \left\{ \frac{(\omega - x)}{2n} \frac{\partial q(x, n)}{\partial x} + \frac{\partial q(x, n)}{\partial n} \right\} \quad \textit{and} \quad n = n_{PL}.$$

As in the audit setting that provides conclusive evidence of fraud, the equilibrium probability of detection, $1 - \Phi(z_\rho)$, is constant across liability regimes and depends on the auditee's effective penalty $\rho$. Audit risk is the probability of fraud times the probability of non-detection and is equal to $t_{PL}\Phi(z_\rho) = t_{PL}\rho$. Comparing audit risk across liability regimes consists of comparing the fraud rates, $t_{PL}$ and $t_{JSL}$. When the probability of fraud decreases under proportionate liability, audit risk decreases.

When we compare the equilibria in the two audit settings that provide varying types of fraud evidence, there are two basic differences. First, the auditor has two strategy choices, $c_{PL}$ and $n_{PL}$ when audits provide inconclusive evidence of fraud, while he has only one strategy choice when the evidence is conclusive. Second, the court assesses audit quality based on the evidence outcome $x$ of random variable $\tilde{x}$, in addition to effort. For audits that provide conclusive evidence of fraud, the precise nature of the audit evidence results in standardized decisions about detection. There is no interpretation of audit evidence by the court. The auditor knows precisely how his effort choice affects the vague liability probability, $\upsilon$, and the amount of liability relief, $J$, where the marginal effects of $\upsilon$ and $J$ directly affect the probability of fraud. By contrast, for audits that do not provide conclusive evidence of fraud, the auditor does not know precisely how his audit plan affects the vague liability probability, $\upsilon$, or the amount of liability relief, $J$. Moreover, when the evidence of fraud is inconclusive, the marginal effects of both the vague liability probability and proportionate liability rule *indirectly* affect the probability of fraud while directly affecting audit effort (see $M(x)$ in Lemma 2, part (3)).[28]

### 2.3. *Comparison of Audit Risk under Proportionate and Joint and Several Liability Regimes*

In evaluating whether proportionate liability reduces audit risk, we simply consider the change in the equilibrium probabilities of fraud, $t_{PL}$ and $t_{JSL}$. If the probability of fraud under proportionate liability is smaller than the probability of fraud under joint and several liability, then proportionate liability reduces audit risk relative to joint and several liability.

Lemma 2 shows that for a fixed effort level, $n = n_{PL} = n_{JSL}$, the difference between the fraud rates, $t_{PL}$ and $t_{JSL}$, is due to the auditor's expected liability cost, $\upsilon JL$ and $\upsilon L$, respectively. For a given effort level, $n$, the probabilities of fraud under the two liability regimes are

$$t_{PL} = \frac{\phi_0 C_R}{\phi_\rho J \upsilon L + \phi_0 C_R} > \frac{\phi_0 C_R}{\phi_\rho \upsilon L + \phi_0 C_R} = t_{JSL}, \tag{21}$$

where $J < 1$. The probability of fraud and audit risk increase for the proportionate liability regime and a given amount of audit effort, $n$.

Whether or not the probability of fraud, $t_{PL}$, decreases in the proportionate liability equilibrium, relative to $t_{JSL}$, depends on how the rate of fraud changes with respect to audit effort in both liability regimes as well as the change in audit effort from one regime to the other. For example, in Figure 1, the joint and several liability fraud rate, $t_{JSL}$, decreases in audit effort, while the proportionate liability fraud rate, $t_{PL}$, first decreases and then increases in audit effort. In this case, a decrease in audit effort under proportionate liability from an initial

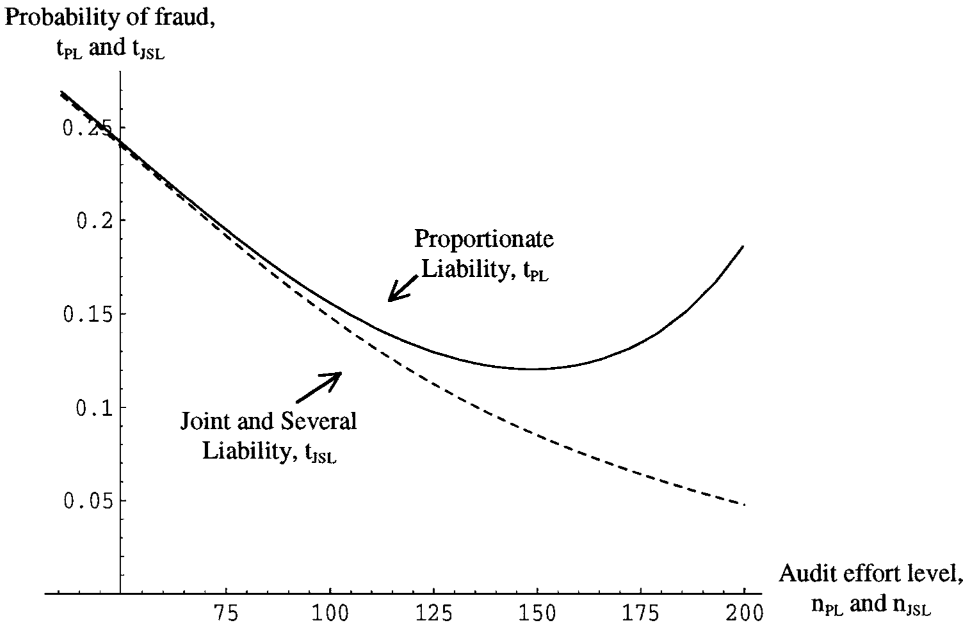


*Figure 1.* The relation between the probability of fraud and audit effort. (This graph is based on the auditee's equilibrium fraud condition (18) in Lemma 2.)

level, $\underline{n}_{JSL}$, would certainly increase the probability of fraud under proportionate liability. Thus, the fraud rate under proportionate liability can decrease only if audit effort increases.

However, an audit effort increase is not sufficient to decrease the fraud rate under proportionate liability. If audit effort starts out high under joint and several liability as in Figure 1 (say $n_{JSL} = 150$) due to a low cost of audit effort, $k$, then the proportionate liability rule cannot reduce the probability of fraud and audit risk. The proportionate liability fraud rate, $t_{PL}$, is greater than $t_{JSL}$ at effort 150 and $t_{PL}$ increases in effort for effort levels higher than 150. The marginal liability relief, $-J'(q)$, is too large to result in a reduction in the fraud rate for effort level 150. Arbitrarily large marginal liability relief excessively reduces the auditor's expected liability relief so that the auditee is no longer motivated to reduce the fraud rate when audit effort increases. Arbitrarily large marginal liability relief is not a good prescription for reducing audit risk because the same factors that cause higher levels of audit effort also cause the probability of fraud under proportionate liability to increase.

Based on the auditee's equilibrium strategy for the probability of fraud (expression (18)), $t_{PL}$ changes with respect to audit effort, $n_{PL}$, according to

$$\begin{aligned}\frac{dt_{PL}}{dn_{PL}} &= \frac{t_{PL}(1-t_{PL})}{J\upsilon}\left\{-\left(\frac{z_\rho}{2\sigma\sqrt{n_{PL}}}+\frac{\omega}{2\sigma^2}\right)\omega J\upsilon - \frac{d(\upsilon J)}{dq}\left(\frac{\partial q}{\partial n_{PL}}+\frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn_{PL}}\right)\right\}\\ &= \frac{t_{PL}(1-t_{PL})}{J\upsilon}\left\{-\left(\frac{z_\rho}{2\sigma\sqrt{n_{PL}}}+\frac{\omega}{2\sigma^2}\right)\omega J\upsilon - \frac{d(\upsilon J)}{dq}M(\mathrm{c}_{PL})\right\}. \qquad (22)\end{aligned}$$

The probability of fraud increases in effort when marginal liability relief, included in $-d(\upsilon J)/dq$, is large at $x = c_{PL}$ and the total marginal audit quality effect of audit effort at $(c_{PL}, n_{PL})$, $M(c_{PL})$, is positive.[29]

When $x = c_{PL}$, the effect of marginal liability relief on the auditor's equilibrium effort (see expression (20)) is equal to

$$-\frac{d(\upsilon J)}{dq} M(c_{PL}) = -\frac{d(\upsilon J)}{dq}\left(\frac{\partial q}{\partial n_{PL}} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn_{PL}}\right). \tag{23}$$

The most likely value for $x$ is $c_{PL}$ when the auditor's evaluation rule $c_{PL} < \omega$, a setting in which the auditee is relatively more motivated to commit fraud. Thus, a proportionate liability rule that includes large marginal liability relief to increase audit effort (expression (23) is large and positive) also increases the change in the probability of fraud with respect to audit effort, $n_{PL}$ (expression (22)).[30] These two conditions potentially work against each other to reduce audit risk. Large marginal liability relief that increases audit effort also increases the change in the probability of fraud with respect to audit effort, $n_{PL}$ and consequently audit risk can increase. This result is formally stated in Proposition 2.

**Proposition 2** *When the evidence of fraud is inconclusive, proportionate liability relates to joint and several liability as follows:*

*(1) Audit effort is higher under proportionate liability if and only if*

$$(1 - t_{PL})\phi_0 C_R \left(\frac{\omega}{2\sigma\sqrt{n}} + \int_{-\infty}^{z_\rho \frac{\sigma}{\sqrt{n}}+\omega} f(x \mid n, \omega) M(x) \frac{(-J'(q)\upsilon(q) - J(q)\upsilon'(q))}{\phi_\rho J(q_\rho)\upsilon(q_\rho)}\, dx\right)$$
$$> (1 - t_{JSL})\phi_0 C_R \left(\frac{\omega}{2\sigma\sqrt{n}} + \int_{-\infty}^{z_\rho \frac{\sigma}{\sqrt{n}}+\omega} f(x \mid n, \omega) M(x) \frac{(-\upsilon'(q))}{\phi_\rho \upsilon(q_\rho)}\, dx\right) \tag{24}$$

*at n equal to the equilibrium effort level for joint and several liability, $n_{JSL}$.*

*(2) When the fraud rate under joint and several liability decreases in audit effort ($dt_{JSL}/dn_{JSL} < 0$) for all positive effort levels ($n_{JSL} > 0$), a proportionate liability rule that increases audit effort is a necessary but not sufficient condition for audit risk to decrease.*

*(3) When the fraud rate under joint and several liability increases in audit effort ($dt_{JSL}/dn_{JSL} > 0$) over any interval of effort levels, the effect of a proportionate liability rule on audit risk is inconclusive.*

The design of a proportionate liability rule to lower audit risk relative to joint and several liability when the audit provides inconclusive evidence of fraud does not involve arbitrarily large marginal liability relief as suggested by the results in Section 1. The condition in Proposition 2 part (1) implies that audit effort increases only if marginal liability relief is positive ($-J'(q) > 0$) and sufficiently large over $x \in (-\infty, z_\rho \frac{\sigma}{\sqrt{n}} + \omega)$ for $n = n_{JSL}$. If proportionate liability relief is constant across all quality levels, audit effort decreases. Proposition 2 part (2) shows that audit effort must increase for audit risk to decrease when

$dt_{JSL}/dn_{JSL} < 0$. This is not a sufficient condition because large marginal liability relief that increases audit effort can concurrently influence the auditee to commit fraud more often. Unlike the setting with conclusive evidence of fraud, marginal liability relief indirectly affects the probability of fraud while directly affecting audit effort. The construction of a proportionate liability rule that reduces audit risk involves consideration of other factors, which we discuss in Section 3.

Our results are inconclusive when the probability of fraud under joint and several liability, $t_{JSL}$, *increases* in audit effort (Proposition 2 part (3)). This case occurs, for example, when the marginal probability of a non-negligence finding, $-\upsilon'(q)$, is large over a range of audit effort. As a result, the probability of a non-negligence finding becomes a primary motivational factor for the auditor. Alternatively, this situation occurs when audit effort is small and the auditee is extremely motivated to commit fraud (the first term in the curly brackets of expression (22) is positive).[31] For this case, an audit effort increase cannot reduce audit risk. Thus increasing marginal liability relief under proportionate liability plays no role in reducing audit risk relative to joint and several liability. Furthermore, we cannot rule out the possibility that an audit effort decrease could result in an audit risk decrease under proportionate liability relative to joint and several liability.[32]

The next section uses the results of Lemma 2 to examine the design of proportionate liability rules more closely, given two possible measures of audit quality.

## 3. Court-Assessed Quality Measures and Effective Proportionate Liability Rules

We assume that the quality measure, $q(x, n)$ (where $n$ is a generic level of audit effort for either the joint and several or proportionate liability regimes), reflects how the court would assess the audit quality of the pair $(x, n)$. The literature provides no consensus on court-assessed audit quality measures. The auditing standards consider audit failure rates (audit risk) related to audit quality while some research studies use the probability of detection as a measure of audit quality.[33] Users, on the other hand, may think of audit quality as the auditor's ability to predict bankruptcy. Of course there may be no one quality measure the courts use.

We consider two quality measures that exhibit reasonable properties. The first has a functional form that is similar to the probability of detection, which is one of the quality measures often referred to in the literature. The other is more generic and is simply designed to achieve a reduction in audit risk for a proportionate liability rule.

### *3.1. Court-Assessed Quality that Is Related to the Probability of Detection*

First, consider the quality measure

$$q_1(x, n) = 1 - F(x \mid n, \omega) \tag{25}$$

where $F$ is a normal distribution function with mean $\omega$ and variance $\sigma^2/n$. When $x = c_{PL}$ in a proportionate liability regime, this measure equals the probability of detection. In our strategic model, it also equals one minus the auditee's effective penalty, $\rho$. While this quality measure exhibits reasonable properties, a proportionate liability rule cannot reduce audit risk for typical audit settings when the court assesses quality based on $q_1$.

**Proposition 3** *When the court assesses quality as* $1 - F(x \mid n, \omega)$*:*

*(1) audit effort decreases under proportionate liability relative to joint and several liability, and*

*(2) no proportionate liability rule exists that decreases the probability of fraud and audit risk relative to joint and several liability when the cost of audit effort, k, is small or the auditee is not highly motivated to commit fraud ($\rho$ is not too small).*

For $q_1$ there are no marginal quality effects for changes in effort and evidence $x$ in expressions (22) and (20). We have,

$$\frac{\partial q_1}{\partial n_{PL}} + \frac{\partial q_1}{\partial c_{PL}} \frac{dc_{PL}}{dn_{PL}} = 0 \quad \text{and} \quad \left\{ \frac{(\omega - x)}{2n_{PL}} \frac{\partial q_1(x, n_{PL})}{\partial x} + \frac{\partial q_1(x, n_{PL})}{\partial n_{PL}} \right\} = 0. \quad (26)$$

Consequently, the *marginal* liability relief from a proportionate liability rule, $-J'(q)$, has no effect on the auditor's choice of audit effort and therefore the audit plan. In this case, equilibrium strategies under a proportionate liability regime result in lower audit effort when compared to a joint and several liability regime. Thus the only possibility for a proportionate liability rule to decrease audit risk is to find one that associates audit effort decreases with probability of fraud decreases. Our analysis shows that this is feasible only in unusual settings when the cost of audit effort, $k$, is large (audit effort small) or the auditee is highly motivated to commit fraud. Both of these settings correspond to part 3 of Proposition 2 where $dt_{JSL}/dn_{JSL} > 0$.

### *3.2. A Generic Court-Assessed Quality*

Next, consider a simple audit quality measure that is more generic. Let

$$q_2(x, n) = -x + \theta_0 + \theta_1 n \quad (27)$$

where $\theta_0$ and $\theta_1$ are constants with $\theta_1 > 0$. Quality measure, $q_2$, decreases in the evidence $x$ and increases in audit effort $n_{PL}$. The marginal quality effect of audit effort on changes in the probability of fraud (see expression (22)) is

$$\frac{\partial q_2}{\partial n_{PL}} + \frac{\partial q_2}{\partial c_{PL}} \frac{dc_{PL}}{dn_{PL}} = \theta_1 + \frac{\sigma z_\rho}{2n_{PL}\sqrt{n_{PL}}}. \quad (28)$$

When the marginal quality effect of audit effort on changes in the probability of fraud is positive, large marginal liability relief, $-J'(q)$, is more likely to increase the probability of fraud and audit risk under proportionate liability. Expression (28) is positive when audit effort, $n_{PL}$, is large (the cost of audit effort, $k$, is small), the auditee has relatively less motivation to commit fraud (represented by larger $z_\rho$ that corresponds to a larger effective penalty, $\rho$) or the marginal quality of effort, $\theta_1$ is high.

Because an analytical analysis cannot determine exactly how to construct a proportionate liability rule that reduces audit risk relative to joint and several liability, we use numerical analysis. Because $\upsilon$ and $J$ must lie between zero and one, we use a form of the normal distribution function for the proportionate liability rule $J$. Let $J(q) = \Phi((-q+\nu)/\gamma)$ where $\gamma$ and $\nu$ are positive constants that allow us to manipulate the slope (marginal liability relief) for various values of $q$. For convenience we use the vague liability probability, $\upsilon(q) = \delta\Phi(-q) + (1-\delta)$ where $0 < \delta < 1$.

Figure 2 shows three proportionate liability rules that change over audit effort, $n_{PL}$, when $x$ equals the equilibrium evaluation rule $c_{PL}$. The graphs differ due to different values of $\gamma$ and $\nu$. The large-dashed graph (2) uses a lower $\gamma$ than (1), resulting in a larger slope, $-J'(q)$, for the middle portion of the graph. The small-dashed graph (3) maintains the same shape as graph (1) but has been shifted to the right by a higher $\nu$. We adjust the amount of marginal liability relief for particular audit effort levels by appropriate choices of $\gamma$ and $\nu$. The form of the proportionate liability rule in Figure 2 is the one that affects the auditee's

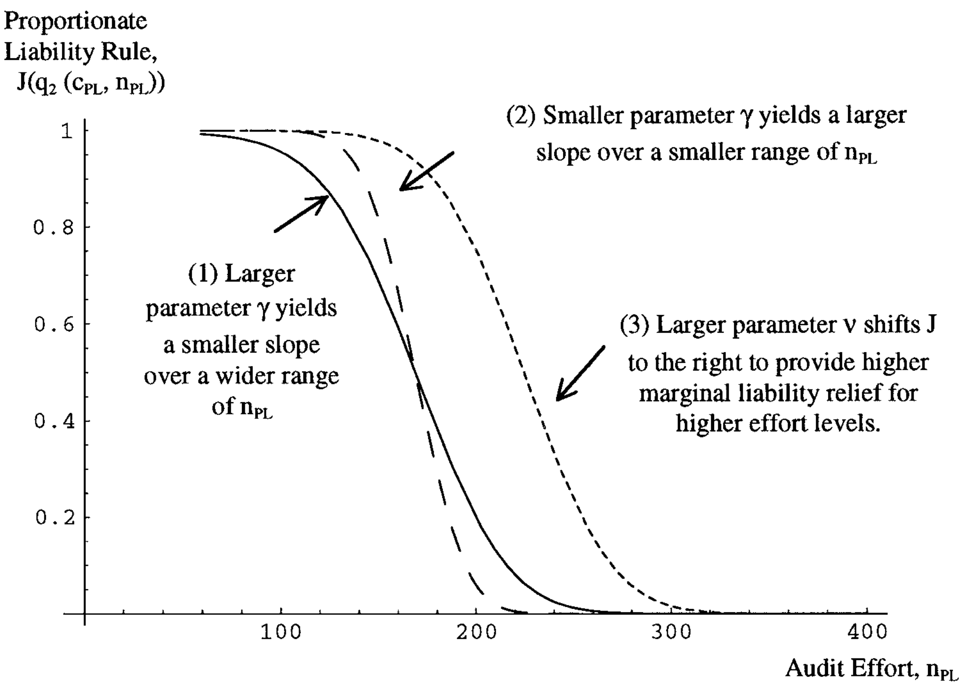


*Figure 2.* Three example proportionate liability rules. (This graph is based on the equilibrium $c_{PL}$ as shown in Lemma 2. Thus when $x = c_{PL}$, $J$ is a function of audit effort, $n_{PL}$. In this graph $J(q) = \Phi((-q_2+\nu)/\gamma)$ where $q_2(x, n) = -x + \theta_0 + \theta_1 n$ with $\theta_1 > 0$ and $\Phi$ is the standard normal distribution function.)

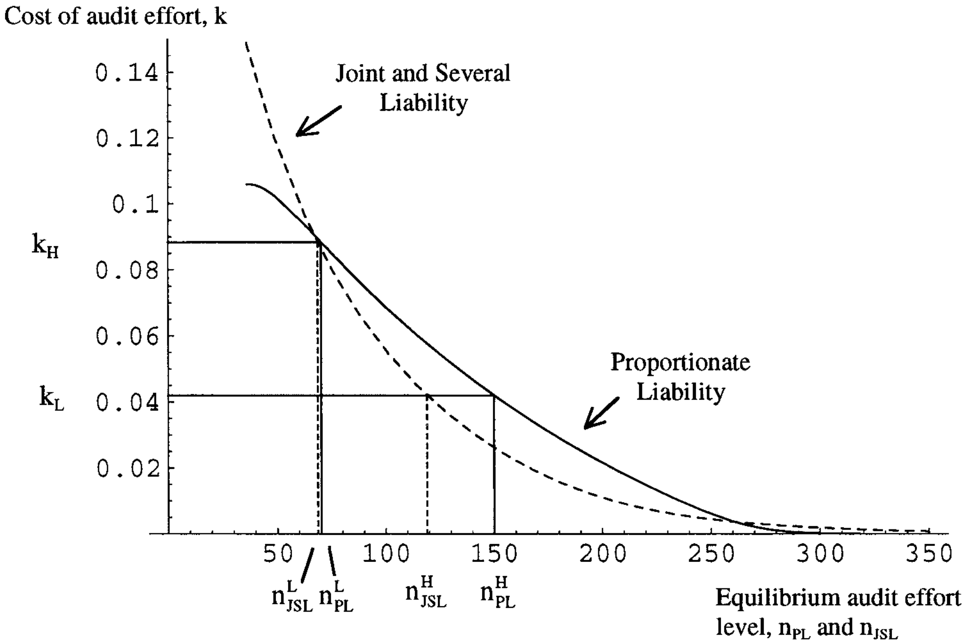


*Figure 3.* Equilibrium audit effort levels associated with the cost of audit effort, $k$, for joint and several and proportionate liability. (The graph is based on the auditor's equilibrium effort condition (20) in Lemma 2. Equilibrium audit effort levels for the costs of audit effort between $k_L$ and $k_H$ result in a smaller probability of fraud for proportionate liability. For $k_H = 0.0884, n^L_{PL} = 70, n^L_{JSL} = 68$, and $t_{PL} = t_{JSL} = 0.205$. For $k_L = 0.042, n^H_{PL} = 150, n^H_{JSL} = 119$, and $t_{PL} = t_{JSL} = 0.12$, where $t_{JSL}$ and $t_{PL}$ are the probabilities of fraud for joint and several and proportionate liability, respectively.)

fraud choice (expression (18)), while the auditor's choice of effort depends on the average marginal liability relief over evidence outcomes, $x$, that are less $c_{PL}$. Thus "large marginal relief" is not as easily defined compared to the audit setting where the auditor only chooses audit effort.

Figure 3 shows the relative equilibrium levels of audit effort for joint and several and proportionate liability for a given cost of audit effort, $k$. For the proportionate liability rule that corresponds to the solid graph (1) in Figure 2, proportionate liability provides lower audit risk for all audit effort costs between, $k_L = 0.042$ and $k_H = 0.0884$. Despite audit effort increases under proportionate liability for audit effort costs that are lower than $k_L = 0.042$, audit risk (or the probability of fraud) does not decrease at these lower audit cost levels. When audit effort exceeds 150, the probability of fraud increases in audit effort.

Figure 1 shows why this is true. For a given audit effort level, the probability of fraud is higher under proportionate liability. Thus, for audit effort levels that are greater than 150, an increase in audit effort cannot decrease the probability of fraud under this proportionate liability rule. As audit effort continues to increase, the probability of fraud eventually approaches one because the auditor's expected liability cost goes to zero.

In order to reduce audit risk under proportionate liability for effort equal to 150, marginal liability relief must influence the auditee to reduce the fraud rate (see expressions (18) and (22)) while maintaining the auditor's motivation to choose an appropriate increase in effort. For example, in Figure 2, the proportionate liability rule represented by graph (3) provides the correct incentives for both the auditor and auditee so that audit risk is less under proportionate liability. This liability rule provides a sufficient increase in audit effort without a corresponding decrease in the auditor's expected liability cost that results in a fraud rate increase. By contrast, the proportionate liability rules depicted in graphs (1) and (2) result in too high a decrease in the auditor's expected liability cost relative to effort so that the fraud rate and audit risk increase.

Figure 4 combines Figures 1 and 3. Audit effort is implicit in Figure 4 and shows how the *cost* of audit effort affects audit risk. Listed below each cost of audit effort is the equilibrium audit effort level under proportionate and joint and several liability, respectively. Figure 4 confirms our intuition about the maximum amount of audit effort, $n_{PL} = 150$, that can reduce audit risk. For small costs of audit effort, proportionate liability still provides higher

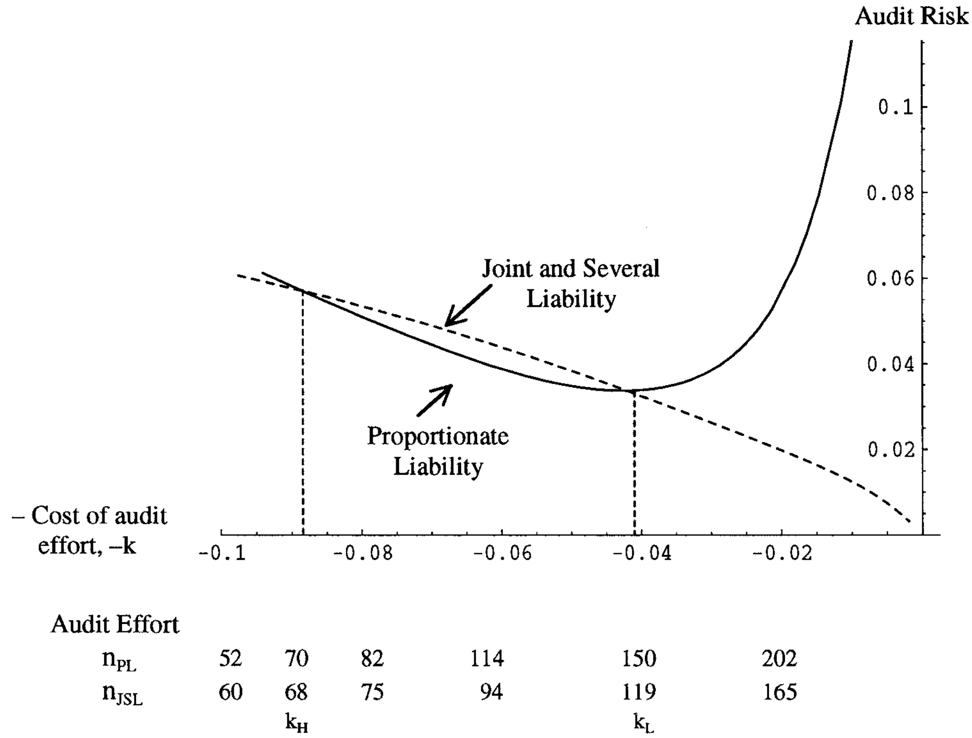


*Figure 4.* The change in audit risk with respect to changes in the cost of audit effort for proportionate and joint and several liability. (This graph combines the two graphs in Figures 1 and 3. We use "–$k$" on the horizontal axis because audit effort *decreases* in $k$. Audit risk is calculated as the equilibrium probability of fraud (from Figure 1) times the probability of non-detection, 0.28.)

effort levels, but they cannot influence the auditee to reduce the probability of fraud. For example when audit effort cost is equal to 0.02, proportionate liability increases audit effort by 37 (202 − 135) and yet audit risk is greater for proportionate liability.

Unlike audit settings that provide conclusive evidence of fraud, a policy prescription in an audit setting that provides inconclusive evidence of fraud should not emphasize arbitrarily large marginal liability relief. Whether a proportionate liability rule exists that decreases audit risk or whether large marginal liability relief decreases audit risk depends on the specific characteristics of the audit setting.

## 4. Conclusion

We consider how proportionate liability affects audit risk relative to joint and several liability when the auditee behaves strategically in two audit settings. One setting provides conclusive evidence of fraud, where the auditor chooses only audit effort, and the other setting provides inconclusive evidence of fraud, where the auditor evaluates the audit evidence in addition to choosing audit effort. This paper differs from prior research in strategic auditing because we allow the auditor's liability cost to change with respect to the audit plan. This paper differs from prior research on liability regimes because we allow the auditee to behave strategically and investigate a model in which the audit provides inconclusive evidence of fraud. In particular, we consider whether large marginal liability relief, as prescribed by Narayanan (1994), is effective in reducing audit risk in these various audit settings.

When the auditor chooses only audit effort, a proportionate liability rule with large marginal liability relief for audit effort (or audit quality) has no effect on audit effort and the probability of detection is constant across liability regimes. The probability of detection in a strategic audit setting depends on the auditee's rewards and penalties to commit fraud. Because the auditee has the opportunity to benefit from fraud when it goes undetected by consuming the rewards, the type of liability regime does not alter his payoffs. However, large marginal liability relief does motivate the auditee to reduce the probability of fraud and therefore, audit risk decreases.

By contrast, the general prescription of large marginal liability relief does not reduce audit risk in the audit setting that provides inconclusive evidence of fraud. Large marginal liability relief can increase audit effort but audit effort increases are not always associated with a decrease in the probability of fraud and audit risk. Changes in audit effort indirectly affect the probability of fraud. As audit effort increases, the auditor's expected liability costs decrease, and thus the auditee may choose a higher probability of fraud. A proportionate liability rule may exist that reduces the probability of fraud and audit risk, but it is not necessarily one that grants large marginal liability relief to the auditor.

## Appendix

### Proof of Lemma 1:

*Part (1).* We restrict the cost of audit effort to be small so that we consider only interior solutions. Thus, equilibrium audit effort is $n_{PL} = \mathrm{Log}(1/\rho)$.

*Part (2).* The solution to expressions (4) and (5) results in

$$t_{PL} = \frac{k}{L\rho} \times \frac{1}{\{[\upsilon(\text{Log}(1/\rho)) - \upsilon'(\text{Log}(1/\rho))][J(\text{Log}(1/\rho)) - J'(\text{Log}(1/\rho))] - \upsilon'(\text{Log}(1/\rho))J'(\text{Log}(1/\rho))\}}. \tag{A.1}$$

If we let $S(n_{PL}) = [\upsilon(n_{PL}) - \upsilon'(n_{PL})][J(n_{PL}) - J'(n_{PL})] - \upsilon'(n_{PL})J'(n_{PL})$, the equilibrium probability of fraud is $t_{PL} = \frac{k}{L\rho}\frac{1}{S(\text{Log}(1/\rho))}$. ■

**Proof of Proposition 1:** Effort for both joint and several and proportionate liability is $n = n_{PL} = n_{JSL} = \text{Log}(1/\rho)$. Let $\upsilon = \upsilon(\text{Log}(1/\rho))$, $J = J(\text{Log}(1/\rho))$, $\upsilon' = \upsilon'(n)$, and $J' = J'(n)$ evaluated at $n = \text{Log}(1/\rho)$, then

$$AR_{JSL} = \frac{k}{L}\frac{1}{(\upsilon - \upsilon')} > \frac{k}{L}\frac{1}{\{[\upsilon - \upsilon'][J - J'] - \upsilon'J'\}} = AR_{PL} \tag{A.2}$$

$$\Leftrightarrow \upsilon - \upsilon' < [\upsilon - \upsilon'][J - J'] - \upsilon'J' \Leftrightarrow (J - J') - \frac{\upsilon'J'}{\upsilon - \upsilon'} > 1 \tag{A.3}$$

and thus, $J - J'\frac{\upsilon}{\upsilon - \upsilon'} > 1$ is necessary and sufficient for audit risk to decrease under proportionate liability. ■

**Proof of Lemma 2:**

*Part (1).* The auditor's condition for choosing an evaluation rule $c_{PL}$ is

$$(1 - t_{PL})f(c_{PL} \,|\, 0, n_{PL})C_R - t_{PL}f(c_{PL} \,|\, \omega, n_{PL}))\upsilon(q(c_{PL}, n_{PL}))J(q(c_{PL}, n_{PL}))L = 0. \tag{A.4}$$

From the auditee's equilibrium condition we have

$$c_{PL} = z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega. \tag{A.5}$$

Recalling that $\phi$ is the standard normal density and substituting (A.5) into (A.4), we obtain,

$$(1 - t_{PL})\frac{\sqrt{n_{PL}}}{\sigma}\phi_0 C_R - \frac{\sqrt{n_{PL}}}{\sigma}t_{PL}\phi_\rho \upsilon(q_\rho)J(q_\rho)L = 0 \tag{A.6}$$

where

$$\phi_0 = \phi\left(z_\rho + \frac{\omega\sqrt{n}}{\sigma}\right), \quad \phi_\rho = \phi(z_\rho),\ q_\rho = q\left(z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega, n\right) \quad \text{and} \quad n = n_{PL}.$$

From (A.6), $\frac{\sqrt{n_{PL}}}{\sigma}\phi_0 C_R = \frac{\sqrt{n_{PL}}}{\sigma} t_{PL}(\phi_\rho \upsilon(q_\rho) J(q_\rho) L + \phi_0 C_R)$ and thus,

$$t_{PL}(n_{PL}) = \frac{\phi_0 C_R}{\phi_\rho \upsilon(q_\rho) J(q_\rho) L + \phi_0 C_R}. \tag{A.7}$$

■

*Part (2).* From the auditee's equilibrium condition (expression (16)) we know that $\Phi(z_\rho) = \rho \Rightarrow z_\rho = \Phi^{-1}(\rho)$ where $\rho$ is the auditee's effective penalty. Thus we have

$$z_\rho = \frac{(c_{PL} - \omega)\sqrt{n_{PL}}}{\sigma} \Rightarrow c_{PL} = z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega. \tag{A.8}$$

*Part (3).* Taking the first derivative of the auditor's expected payoff (expression (10)) with respect to $n_{PL}$ results in

$$\begin{aligned}\frac{\partial A}{\partial n_{PL}} &= (1 - t_{PL}) f(c_{PL} \mid 0, n_{PL}) C_R \frac{c_{PL}}{2n_{PL}} \\ &\quad + t_{PL} \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL}) \left( \frac{1}{2\sigma^2}(x - \omega)^2 - \frac{1}{2n} \right) \upsilon(q(x, n_{PL})) J(q(x, n_{PL})) L \, dx \\ &\quad - t_{PL} L \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL}) \frac{\partial q(x, n_{PL})}{\partial n_{PL}} \frac{d(J\upsilon)}{dq} \, dx - k = 0. \end{aligned} \tag{A.9}$$

To simplify the second term of (A.9), let

$$u = J(q(x, n_{PL}))\, \upsilon(q(x, n_{PL})) \quad \text{and} \quad dv = L\, f(x \mid \omega, n_{PL}) \left( \frac{1}{2\sigma^2}(x - \omega)^2 - \frac{1}{2n} \right) dx,$$

then

$$du = \frac{d(J\upsilon)}{dq} \frac{\partial q(x, n_{PL})}{\partial x} \, dx \quad \text{and} \quad v = L\, f(x \mid \omega, n_{PL}) \frac{\omega - x}{2n_{PL}}.$$

Using integration by parts where $\int_a^b u\, dv = uv]_a^b - \int_a^b v\, du$, we can write the second term of (A.9) as

$$t_{PL} \left\{ f(c_{PL} \mid \omega, n_{PL}) L J \upsilon \frac{\omega - c_{PL}}{2n_{PL}} - L \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL}) \frac{\omega - x}{2n_{PL}} \frac{\partial q(x, n_{PL})}{\partial x} \frac{d(J\upsilon)}{dq} \, dx \right\}.$$

Consequently,

$$\begin{aligned}\frac{\partial A}{\partial n_{PL}} &= (1 - t_{PL}) f(c_{PL} \mid 0, n_{PL}) C_R \frac{c_{PL}}{2n_{PL}} - t_{PL} f(c_{PL} \mid \omega, n_{PL}) L J \upsilon \frac{c_{PL} - \omega}{2n_{PL}} \\ &\quad - t_{PL} L \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL}) \frac{\omega - x}{2n_{PL}} \frac{\partial q(x, n_{PL})}{\partial x} \frac{d(J\upsilon)}{dq} \, dx \\ &\quad - t_{PL} L \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL}) \frac{\partial q(x, n_{PL})}{\partial n_{PL}} \frac{d(J\upsilon)}{dq} \, dx \\ &\quad - k = 0. \end{aligned} \tag{A.10}$$

Expression (A.10) results in expression (12) after collecting the last two terms. Next substitute for $t_{PL} f(c_{PL} \mid \omega, n_{PL}) L J \upsilon$ in (A.10) based on condition (11) and

$$\frac{\partial A}{\partial n_{PL}} = (1 - t_{PL}) f(c_{PL} \mid 0, n_{PL}) C_R \frac{\omega}{2n_{PL}} - t_{PL} L \times \int_{-\infty}^{c_{PL}} f(x \mid \omega, n_{PL}) \left\{ \frac{(\omega - x)}{2n_{PL}} \frac{\partial q(x, n_{PL})}{\partial x} + \frac{\partial q(x, n_{PL})}{\partial n_{PL}} \right\} \frac{d(J\upsilon)}{dq}\, dx - k = 0. \tag{A.11}$$

Finally we obtain $t_{PL}(n_{PL})$ from condition (A.7) and after using expression (A.4) again, the auditor's equilibrium choice of effort, $n_{PL}$, must satisfy

$$(1 - t_{PL}(n_{PL}))\phi_0 C_R \times \left( \frac{\omega}{2\sigma\sqrt{n_{PL}}} + \int_{-\infty}^{z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega} f(x \mid n_{PL}, \omega) M(x) \frac{(-J'(q)\upsilon(q) - J(q)\upsilon'(q))}{\phi_\rho J(q_\rho)\upsilon(q_\rho)}\, dx \right) = 0 \tag{A.12}$$

where

$$\phi_0 = \phi\left(z_\rho + \frac{\omega\sqrt{n}}{\sigma}\right), \phi_\rho = \phi(z_\rho),\ q_\rho = q\left(z_\rho \frac{\sigma}{\sqrt{n_{PL}}} + \omega, n\right),$$
$$M(x) = \left\{ \frac{(\omega - x)}{2n} \frac{\partial q(x, n)}{\partial x} + \frac{\partial q(x, n)}{\partial n} \right\} \quad \text{and} \quad n = n_{PL}. \qquad \blacksquare$$

**Proof of Proposition 2:** Audit risk under proportionate liability is $t_{PL}\rho$ and under joint and several liability is $t_{JSL}\rho$. Thus, audit risk decreases under proportionate liability when

$$t_{PL}(n_{PL}) = \frac{\phi_0^{PL} C_R}{\phi_\rho \upsilon\left(q_\rho^{PL}\right) J\left(q_\rho^{PL}\right) L + \phi_0^{PL} C_R} < \frac{\phi_0^{JSL} C_R}{\phi_\rho \upsilon\left(q_\rho^{JSL}\right) L + \phi_0^{JSL} C_R} = t_{JSL}(n_{JSL}) \tag{A.13}$$

where $\phi$ is the standard normal density with $\phi_0^j = \phi(z_\rho + \frac{\omega\sqrt{n_j}}{\sigma})$, $\phi_\rho = \phi(z_\rho)$ and $q_\rho^j = q(z_\rho \frac{\sigma}{\sqrt{n_j}} + \omega, n_j)$, $j = PL$ or $JSL$. The probabilities of fraud for $PL$ and $JSL$ differ only due to $n_j$ and $J$.

*Part (1).* From expression (20), audit effort increases in a proportionate liability relative to joint and several liability regime if and only if

$$(1 - t_{PL})\phi_0 C_R \left( \frac{\omega}{2\sigma\sqrt{n}} + \int_{-\infty}^{z_\rho \frac{\sigma}{\sqrt{n}} + \omega} f(x \mid n, \omega) M(x) \frac{(-J'(q)\upsilon(q) - J(q)\upsilon'(q))}{\phi_\rho J(q_\rho)\upsilon(q_\rho)}\, dx \right)$$
$$> (1 - t_{JSL})\phi_0 C_R \left( \frac{\omega}{2\sigma\sqrt{n}} + \int_{-\infty}^{z_\rho \frac{\sigma}{\sqrt{n}} + \omega} f(x \mid n, \omega) M(x) \frac{(-\upsilon'(q))}{\phi_\rho \upsilon(q_\rho)}\, dx \right) \tag{A.14}$$

for effort level $n = n_{JSL}$ because both the left-hand and right-hand side of (A.14) decrease in $n$ for any $n > \hat{n}$ where $\hat{n}$ is the smallest feasible solution for $n$.[34] ■

*Part (2).* $dt_{JSL}/dn_{JSL} < 0$. For each *fixed n* (including the equilibrium effort level for joint and several liability, $n_{JSL}$)

$$t_{PL}(n) = \frac{\phi_0 C_R}{\phi_\rho \upsilon(q_\rho) J(q_\rho) L + \phi_0 C_R} > \frac{\phi_0 C_R}{\phi_\rho \upsilon(q_\rho) L + \phi_0 C_R} = t_{JSL}(n) \tag{A.15}$$

where

$$\phi_0 = \phi\left(z_\rho + \frac{\omega\sqrt{n}}{\sigma}\right) \quad \text{and} \quad q_\rho = q\left(z_\rho \frac{\sigma}{\sqrt{n}} + \omega, n\right) \text{ because } J(q_\rho) < 1.$$

Therefore, if audit effort decreases from *JSL* to *PL*, the equilibrium probability of fraud under *PL* increases and an audit effort increase is a necessary condition for audit risk under *PL* to decrease. We also know that

$$\frac{dt_{PL}}{dn} = \frac{t_{PL}(1 - t_{PL})}{J\upsilon}\left\{-\left(\frac{z_\rho}{2\sigma\sqrt{n}} + \frac{\omega}{2\sigma^2}\right)\omega J\upsilon - \frac{d(\upsilon J)}{dq}\left(\frac{\partial q}{\partial n} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn}\right)\right\}. \tag{A.16}$$

In addition, $-\frac{d(\upsilon J)}{dq} > 0$ so that $-\frac{d(\upsilon J)}{dq}(\frac{\partial q}{\partial n} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn})$ is positive when $\frac{\partial q}{\partial n} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn} > 0$. This is true when $\frac{\partial q}{\partial n}$ is large relative to $-\frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn}$ given $\frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn} < 0$. If $-\frac{d(\upsilon J)}{dq}$ is large and $\frac{\partial q}{\partial n} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn}$ is positive then $\frac{dt_{PL}}{dn} > 0$. When $\frac{dt_{PL}}{dn} > 0$, an audit effort increase from a change in liability regime to proportionate liability cannot decrease $t_{PL}$ and therefore cannot decrease audit risk. ■

*Part (3).* $dt_{JSL}/dn_{JSL} > 0$ over some range of effort levels. This setting occurs when $-\upsilon'$ is large and $\frac{\partial q}{\partial n} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn} > 0$, or when $\frac{\partial q}{\partial n} + \frac{\partial q}{\partial c_{PL}}\frac{dc_{PL}}{dn} = 0$ and $-(\frac{z_\rho}{2\sigma\sqrt{n}} + \frac{\omega}{2\sigma^2})$ in expression (22) is positive. This second case corresponds to $c_{PL} < 0 < \omega$. In the first case, it is more difficult to manipulate the auditor's incentives with $-J'$ and our results are inconclusive. In both cases, an audit effort increase is no longer a necessary condition for the probability of fraud to decrease under *PL*. However, our numerical analysis is unable to produce a *PL* rule with either an effort increase or an effort decrease that reduces the probability of fraud. ■

**Proof of Proposition 3:** When $q_1(x, n) = 1 - F(x \mid \omega, n)$, $M(x) = 0$ in expression (20) and $M(c_{PL}) = 0$ in expression (22) because

$$\frac{\omega - x}{2n}\frac{\partial q_1}{\partial x} = \frac{\omega - x}{2n}\frac{\partial(1 - F(x \mid \omega, n))}{\partial x} = -\frac{\omega - x}{2n} f(x \mid \omega, n)$$

and

$$\frac{\partial q_1}{\partial n} = \frac{\partial(1 - F(x \mid \omega, n))}{\partial n} = \frac{\omega - x}{2n} f(x \mid \omega, n).$$

Thus, $M(x) = \frac{\omega - x}{2n}\frac{\partial q_1}{\partial x} + \frac{\partial q_1}{\partial n} = 0$ and $M(c_{PL}) = 0$.

As a result, the auditor's equilibrium condition for audit effort, $n$, and $\frac{dt_{PL}}{dn}$ become $(1 - t_{PL}(n))\frac{\sqrt{n}}{\sigma}\phi_0 C_R \frac{\omega}{2n} = k$ and $\frac{dt_{PL}}{dn} = t_{PL}(1 - t_{PL})\{-(\frac{z_\rho}{2\sigma\sqrt{n}} + \frac{\omega}{2\sigma^2})\omega J \upsilon\}$, respectively.

*Part (1).* We know $t_{PL}(n) > t_{JSL}(n)$ for each $n$ which implies that $(1 - t_{PL}(n))\frac{\sqrt{n}}{\sigma}\phi_0 C_R \frac{\omega}{2n} < (1 - t_{JSL}(n))\frac{\sqrt{n}}{\sigma}\phi_0 C_R \frac{\omega}{2n}$ for each $n$ and thus $n_{JSL} > n_{PL}$ (audit effort decreases). ■

*Part (2).* Assume for simplicity that there exists a lower positive bound on effort called $\underline{n}$. If $\underline{n} = 1$, for example, $\frac{dt_{PL}}{dn}$ and $\frac{dt_{JSL}}{dn}$ are less than zero for all $n > \underline{n}$ when $z_\rho > -\frac{\omega}{\sigma}$, resulting in no proportionate liability rule that reduces audit risk relative to joint and several liability. Furthermore, if $k$ is sufficiently small so that $t_{PL}(1) > t_{JSL}(n_{JSL})$ or $n_{PL} > (\frac{-z_\rho\sigma}{\omega})^2$ when $z_\rho < 0$, then again no proportionate liability rule can reduce audit risk relative to joint and several liability. When these conditions are not met, the effect on audit risk of a change to proportionate liability is inconclusive. ■

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## Notes

1. See Smith and Tidrick (1998) who investigate the issue of legal fee payments by defendants and plaintiffs.
2. See King and Schwartz (1997) for the policy debate related to the Private Securities Litigation Reform Act of 1995.
3. Prior research using a joint and several liability setting supports this assertion. See, for example, Balachandran and Nagarajan (1991) when the auditor does not behave strategically and Newman and Noel (1989) and Patterson and Noel (forthcoming) when the auditor behaves strategically.
4. Chan and Pae (1998) also investigate the benefits of proportionate liability relative to joint and several liability where investors can sue strategically and the apportionment rule is similar to Narayanan (1994). Proportionate liability in their model can also reduce audit risk by increasing audit quality. However, if audit quality decreases, they show that proportionate liability increases social welfare by decreasing audit fees and the frequency of suits against the auditor.
5. In terms of audit sampling models, discovery sampling corresponds to a setting with conclusive evidence of fraud. The auditor's evaluation rule is fixed because the auditor "discovers" fraud only if he finds definitive evidence of fraud. Acceptance sampling corresponds to a setting with inconclusive evidence of fraud where the auditor chooses evidence evaluation as well as sample size.
6. Evidence evaluation becomes more liberal when evidence that previously was considered indicative of fraud is interpreted as not indicative of fraud. This increases the probability of a type II error (concluding a fraud does not exist when it exists) and decreases the probability of a type I error (concluding a fraud exists when it does not exist).
7. See, for example, Newman and Noel (1989), Patterson (1993), Smith, Tiras and Vichitlekarn (2000), and Patterson and Noel (forthcoming) for strategic audit settings where the auditor's expected liability, given fraud, is fixed and does not depend on audit quality.
8. See for example, Narayanan (1994), Hillegeist (1999) and Schwartz (1997).
9. See, for example, Newman and Noel (1989), Newman, Park and Smith (1996) and Patterson and Noel (forthcoming).
10. This is consistent with discovery sampling in auditing. Studies that use this audit technology include Moore and Scott (1988), Patterson and Noel (forthcoming), Newman, Park and Smith (1996), Finley (1994), Narayanan (1994), Hillegeist (1999) and Schwartz (1997).

11. Our results would not change if the amount of damages depended on the equilibrium rate of fraud where damages increase in the rate of fraud. For simplicity we fix the amount of damages at $L$.
12. If the auditee were able to pay his proportionate share, then no difference would arise between proportionate and joint and several liability. In our model, $L$ is the maximum the auditor pays under joint and several liability and $J$ is the portion of $L$ paid under proportionate liability. Thus, if $J = 1$, the amount of damages paid is the same under the two regimes.
13. See Patterson and Noel (forthcoming) for a strategic setting that allows the auditee to choose both defalcation and fraudulent financial reporting.
14. We assume that the cost of audit effort is sufficiently small (or satisfies $k < L\rho S(\text{Log}(1/\rho))$) so that the auditor is motivated to deter the auditee from committing fraud ($t_{PL} < 1$). Otherwise, there is limited strategic interaction between the auditor and auditee.
15. See, for example, Patterson and Noel (forthcoming) and Newman, Park and Smith (1996).
16. Narayanan (1994) says on page 41, "... I compare deterrence across liability regimes by looking at the probability financial statements will be misstated. Since this probability is determined by audit quality, which in turn is determined by audit effort, I focus on audit effort when comparing liability regimes." In effect he ranks liability regimes according to audit risk, which happens to be the same as audit quality and audit effort in his setting.
17. See Smith, Tiras and Vichitlekarn (2000) for similar findings when comparing the benefits of control tests in a strategic setting.
18. However, our results do not support the specific proportionate liability rule that Narayanan (1994) identifies in his Proposition 4, part (ii) as the one that "always exists" for both the vague ($\upsilon < 1$) and non-vague ($\upsilon = 1$) negligence settings. In our notation, he defines the rule as $J(n_{PL}^{\text{Max}}) = 1$ where $J' < 0$ and $J'' > 0$. This policy prescription is unrelated to large liability relief and implies that $J > 1$ for all $n_{PL} < n_{PL}^{\text{Max}}$. An apportionment rule, $J$, greater than one would reduce the rate of fraud but it is not consistent with the concept of proportionate liability that allows for a liability apportionment that is less than joint and several liability. A similar finding arises in Narayanan's (1994) setting when non-zero liability costs are paid by auditees. Thus the policy prescription of interest for reducing audit risk depends on large marginal liability relief. Details of the comparison between our model and Narayanan (1994) are available from the authors upon request.
19. We assume that the audit evidence is distributed normally for analytic convenience. See Patterson (1993) and Patterson and Smith (2002) for similar assumptions.
20. For simplicity, our setting allows the auditee to choose either fraud or no fraud. Previous studies that allow the auditee to choose the level of fraud (see Newman, Park and Smith (1998), Patterson and Smith (2002) and Patterson (1995), for example) under strict liability result in varied equilibria, depending on the models' assumptions. We cannot predict how the results of our model would be affected by a continuous choice of fraud. However, we have examined a model in which the auditee can choose two levels of fraud as well as no fraud and found no difference in the results presented here. We leave further analysis of this question for future research.
21. Likewise, the vague liability probability is $\upsilon(q)$ with $\upsilon'(q) \leq 0$.
22. The possibility also exists that the auditor incurs a cost for a *correct* rejection. This would decrease the auditor's cost of an incorrect acceptance (type II cost). While this would affect the equilibrium strategies (e.g., the probability of fraud increases for each level of audit effort), this has no effect on our comparative analysis of liability regimes. Without loss of generality, we normalize this cost at zero.
23. We cannot assume that prior decision-theoretic results concerning a monotonic evaluation rule hold (See Berger, 1987, page 524 for a discussion of the Neyman-Pearson lemma) because the auditor's payoff for undetected fraud is not fixed. To determine the nature of the auditor's evaluation rule, we start from the end of the proportionate liability game in characterizing the auditor's expected payoff and show that in fact the specification of our game results in a monotonic decision rule for the auditor. The auditor chooses a cutoff value $c_{PL}$ where the auditor accepts when $x < c_{PL}$; otherwise, he rejects. To obtain the auditor's expected payoff for the audit plan ($c_{PL}, n_{PL}$), we calculate the ex-ante expected payoff for each possible audit effort, $n_{PL}$. which is expression (10). Details of this analysis are available from the authors upon request.
24. The conditions for formulating an audit plan under joint and several liability ($c_{JSL}, n_{JSL}$) are the same as expressions (11) and (12) with $J = 1$, $c_{JSL}$ substituted for $c_{PL}$ and $n_{JSL}$ substituted for $n_{PL}$.
25. Patterson (1993) shows that when the auditor's liability cost is fixed, a positive *auditee* type I penalty results in the probability of detection *decreasing* in audit effort. However, Patterson (1993) also shows that as audit effort increases the amount of decrease in the probability of detection becomes negligible. If the probability

of detection decreases in audit effort, the proportionate liability rule would require even greater reductions in the probability of fraud to reduce audit risk. To allow the greatest chance for a proportionate liability rule to decrease audit risk and to simplify the analysis, we set the auditee's type I penalty to zero.

26. If this were a defalcation game, then our equilibrium conditions require that $t_{PL} < 1$. However, if this represents a fraudulent financial reporting game then the updated probability of fraud, $t_{PL}$, is less than one only if the cost of audit effort is sufficiently small. Details of the restriction on $k$ are available from the authors upon request.
27. For our analysis, we require that $J$, $\upsilon$ and $q$ result in expression (20) decreasing in $n_{PL}$ and $n_{JSL}$ over the continuous range of equilibrium solutions (i.e., the equilibrium is unique). We can prove this is true only for specific $J$, $\upsilon$ and $q$ functions such as those discussed in Section 3.
28. We model court-assessed quality based on the observed evidence $x$ rather than decisions the auditor *might have made* for other outcomes of $x$, based on $c$. However, if the court were to assess quality based only on $q(c_{PL}, n_{PL})$, then the effects of a proportionate liability rule on the probability of fraud when $c$ is fixed would be similar to the audit setting of Section 1. However, when $c$ is not fixed, the marginal effects of proportionate liability still affect the probability of fraud indirectly and the results are similar to Proposition 2. Details of this analysis are available from the authors upon request.
29. This depends on how audit quality is defined. See Section 3 for a discussion of various quality measures.
30. We assume that the equilibrium is interior so that there is no auditor randomization between accepting (or rejecting) and collecting evidence. (See Patterson, 1993 for discussion.)
31. When the first term in the curly brackets of expression (22) is positive, $c < 0 < \omega$, which is contrary to the assumption of a standard statistical setting where $0 < c < \omega$.
32. Despite the inconclusive theoretical results, we also performed a numerical analysis for this case and were unable to find any proportionate liability rule that reduces audit risk.
33. See Konrath (1999), p. 29, for sources that define audit quality, which include the audit standards. The audit standards require the auditor to consider audit risk in both planning the audit and evaluating audit evidence (See AICPA, 1996). Narayanan (1994) considers audit quality in terms of audit effort, but he ranks the benefits of liability regimes based on audit risk.
34. Recall from Lemma 2 that we only consider $J$, $\upsilon$ and $q$ that result in a unique solution for $n$.

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