Journal of Risk and Uncertainty, 16:115–139 (1998)

Common Consequence Conditions in Decision Making under Risk

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Abstract

We generalize the Allais common consequence effect by describing three common consequence effect conditions and characterizing their implications for the probability weighting function in rank-dependent expected utility. The three conditions—horizontal, vertical, and diagonal shifts within the probability triangle—are necessary and sufficient for different curvature properties of the probability weighting function. The first two conditions, shifts in probability mass from the lowest to middle outcomes and middle to highest outcomes respectively, are alternative conditions for concavity and convexity of the weighting function. The third condition, decreasing Pratt-Arrow absolute concavity, is consistent with recently proposed weighting functions. The three conditions collectively characterize where indifference curves fan out and where they fan in. The common consequence conditions indicate that for nonlinear weighting functions in the context of rank-dependent expected utility, there must exist a region where indifference curves fan out in one direction and fan in the other direction.

Key words: Rank-dependent expected utility, prospect theory, common consequence effects, fanning out, fanning in, probability weighting function

1. Introduction

Maurice Allais presented the first serious challenge to the descriptive validity of expected utility (Allais, 1953, 1979). When faced with what has become known as the Allais Paradox, most individuals prefer a 10% chance at $5 million to an 11% chance at $1 million yet would rather have $1 million for sure than a lottery that gives a 10% chance at $5 million and an 89% chance at $1 million. Note that the gambles in the second pair are constructed by adding a common consequence, 89% chance at $1 million, to the gambles in the first pair. Put differently, an 89% chance is "shifted" from the lowest outcome ($0) to the middle outcome ($1 million). The choices described above are inconsistent with the independence axiom and hence with expected utility. Whereas the independence axiom requires that preferences be unaffected by changes in a common consequence, the Allais Paradox demonstrates that individuals are indeed sensitive to such shifts in probability mass. More generally, the *common consequence effect* defines a class of choice problems—of which the Allais paradox is one example—in which choices shift as probability is moved from one common consequence to another.

In this paper we explore variants of the common consequence effect. Empirical demonstrations by Kahneman and Tversky (1979) and MacCrimmon and Larsson (1979) indicate that the Allais Paradox is not an isolated example—individuals violate the independence axiom for small as well as large outcomes, for real as well as hypothetical payoffs, and for small as well as large probabilities. Recent research has shown that the common consequence effect is even more widespread: common consequence effect violations occur for lotteries that do not involve sure things as in Allais' original example (Prelec, 1990; Wu and Gonzalez, 1996), and in domains of uncertainty as well as risk (Tversky and Kahneman, 1992; Wu and Gonzalez, 1997). These studies, as well as others, suggest that expected utility is not a good descriptive model of choice under risk. The natural question to ask then is: What non-expected utility model can accommodate what is known about common consequence effect violations?

We address this question by cataloging three distinctly different common consequence effects. Consider the first example from Wu and Gonzalez (1996):

Example 1

	Question 1	
$R_H = (.05, \$240)$	vs.	$S_H = (.07, \$200)$
[38%]	$(n = 105)$	[62%]

	Question 1′	
$R_H' = (.05, \$240; .30, \$200)$	vs.	$S_H' = (.37, \$200)$
[65%]	$(n = 105)$	[35%]

	Question 1″	
$R_H'' = (.05, \$240; .90, \$200)$	vs.	$S_H'' = (.97, \$200)$
[39%]	$(n = 105)$	[61%]

As with the Allais example, the gambles in Question 1′ and 1″ are created from the gambles in Question 1 by shifting probability from the *lowest* or third highest outcome ($0) to the *middle* or second highest outcome ($200). Note that R_H'' (S_H'') first order stochastically dominates R_H' (S_H') which in turn first order stochastically dominates R_H (S_H). Also note that the "*R*" gambles are riskier than the "*S*" gambles in the sense that they have a higher probability of $0 (the worst outcome) and $240 (the best outcome). The pattern exhibited in Example 1 is inconsistent with expected utility: the number of subjects who chose the risky gamble increased from 38% (in Question 1) to 65% (in Question 1′) and then decreased to 39% (in Question 1″). A denser ladder of common consequence effects based on this example is given in Figure 1.

The results are also depicted in the probability triangle in Figure 2. The horizontal axis refers to the probability of $0, the worst outcome, while the vertical axis refers to the probability of $240, the best outcome. The middle outcome, $200, is implicit in the diagram. Note that gambles improve (in the sense of first-order stochastic dominance) with shifts in the west, north, or northwest direction. We refer to these as horizontal,

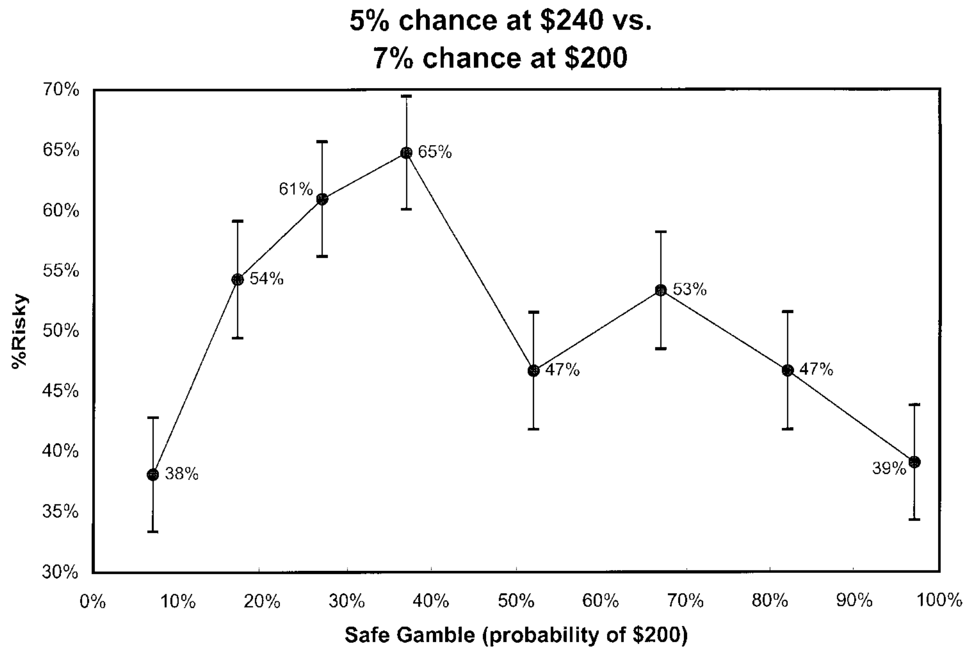

Figure 1. Example 1 (Horizontal common consequence shift)

vertical, and diagonal shifts respectively. The thick line bisecting the gambles is a sort of "quasi indifference curve" inferred from the choices above. The steeper the line, the greater the number of subjects choosing the safe gamble. If subjects were evenly divided between the risky and safe gamble, the thick line would connect the two gambles.

Next consider the following example created by shifting probability mass from the *middle* or second *highest* outcome ($100) to the *highest* outcome ($280)[1]:

Example 2

	Question 2	
$R_V = (.03, \$280; .90, \$100)$	vs.	$S_V = (.97, \$100)$
[51%]	(n = 59)	[49%]
	Question 2′	
$R_V' = (.33, \$280; .60, \$100)$	vs.	$S_V' = (.30, \$280; .67, \$100)$
[15%]	(n = 59)	[85%]
	Question 2″	
$R_V'' = (.93, \$280)$	vs.	$S_V'' = (.90, \$280; .07, \$100)$
[32%]	(n = 58)	[68%]

The pattern exhibited is the opposite of that exhibited in Example 1: the number of subjects choosing the risky option decreases from 51% (in Question 2) to 15% (in Ques-

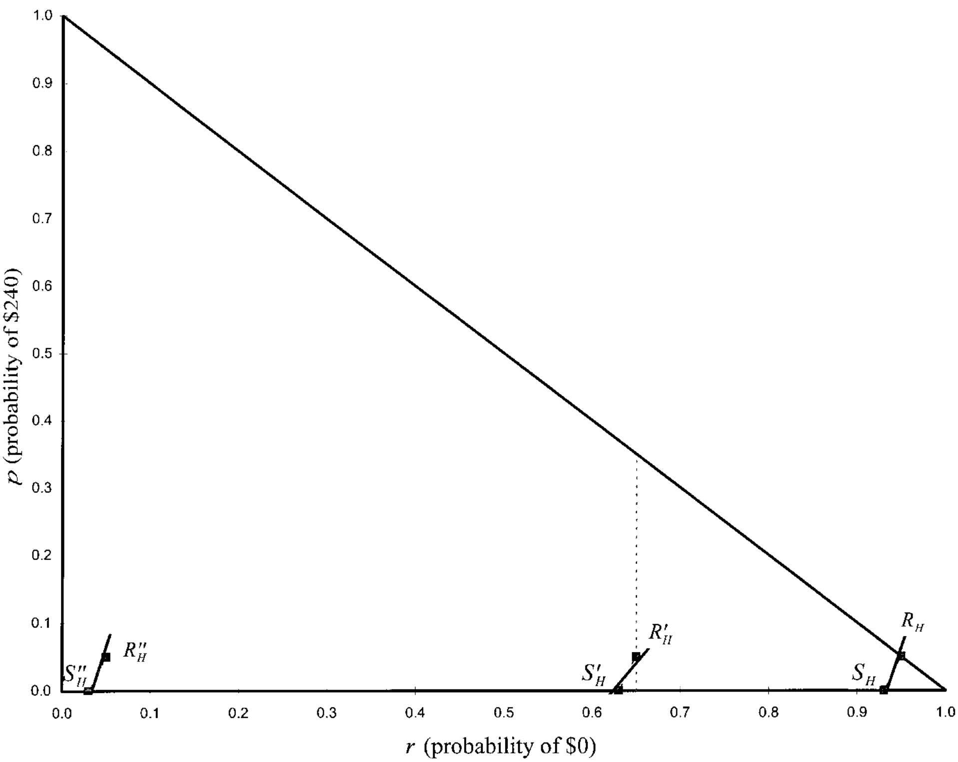

Figure 2. Example 1 in the probability triangle

tion 2′), and then increases to 32% (in Question 2″). Again, the questions in Example 2 are a subset of a more complete ladder found in Figure 3. The probability triangle illustrating these vertical shifts is found in Figure 4.

Finally, consider a third set of common consequence effects in which probability is shifted from the *lowest* or third highest outcome ($0) to the *highest* outcome ($25,000) (Camerer, 1989):

Example 3

	Question 3	
R'_d = (.10, $25000)	vs.	S_d = (.20, $10000)
[73%]	(n = 68)	[27%]
	Question 3′	
R'_d = (.30, $25000)	vs.	S'_d = (.20, $25000; .20, $1000)
[60%]	(n = 68)	[40%]
	Question 3″	
R''_d = (.90, $25000)	vs.	S''_d = (.80, $25000; .20, $10000)
[27%]	(n = 68)	[73%]

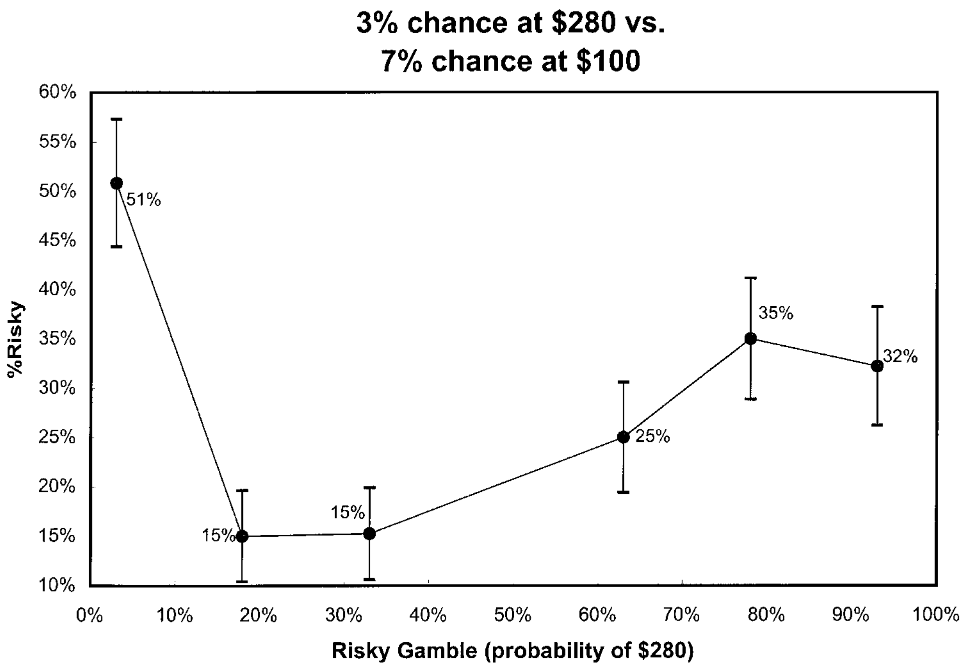

Figure 3. Example 2 (Vertical Common consequence shift)

In this example, the percentage of subjects who choose the risky option decreases as we shift probability mass from Question 3 to Question 3′ and decreases even more with a shift from Question 3′ to Question 3″. Figure 5 depicts a more complete set of common consequence shifts based on these gambles. Figure 6 shows the gambles in the probability triangle.

Put together, these three common consequence effects show the following characteristics:

1. The pattern observed (inverse U-shaped; U-shaped; or decreasing) depends on how common consequences are shifted (from the third highest to second highest; second highest to first highest; or third highest to highest).
2. The inflection point observed in Examples 1 and 2 seems to be roughly the same, between .25 and .40.
3. The patterns in the three examples are inconsistent with Machina's (1982) Hypothesis II that indifference curve "fan out" in the probability triangle, or put differently, that preferences become more locally risk averse as gambles get better in the sense of first order stochastic dominance. The shifts increase preference for the safe option ($S_H^{'}$ and $S_H^{''}$; S_V and $S_V^{'}$; S_D, $S_D^{'}$, and $S_D^{''}$), consistent with "fanning out", as well as the risky option (R_H and $R_H^{'}$; $R_V^{'}$ and $R_V^{''}$), consistent with "fanning in." This can be seen visually in

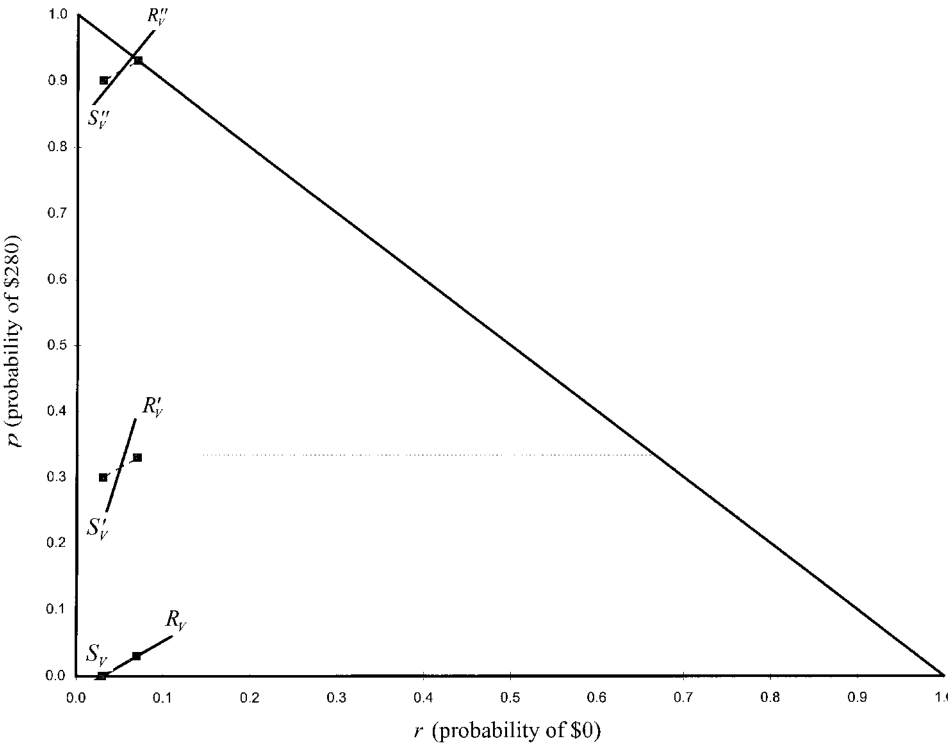

Figure 4. Example 2 in the probability triangle

Figures 2, 4, and 6. As a result of common consequence shifts, the quasi-indifference curves sometimes become steeper (i.e., they fan out), and sometimes become flatter (i.e., they fan in).

4. The effects do not involve sure things and thus do not rely on the certainty effect (Kahneman and Tversky, 1979).

These four observations are not entirely new. Several previous studies have demonstrated fanning in (Conlisk, 1989; Prelec, 1990; Camerer, 1992, Stylized Fact #3). Our paper makes two specific contributions. The first contribution is a more complete empirical characterization of the three natural types of common consequence effects for three-outcome gambles (Examples 1–3). The second contribution is a theoretical framework that accommodates the patterns of mixed fanning exhibited in the three examples.

The framework we use to explain the common consequence patterns is rank-dependent expected utility (RDEU; Quiggin, 1982; Yaari, 1987; Segal, 1989) with an inverse *S*-shaped probability weighting function. Many studies have suggested that RDEU best organizes the accumulated empirical evidence (see Camerer, 1989, 1992, 1995; Tversky

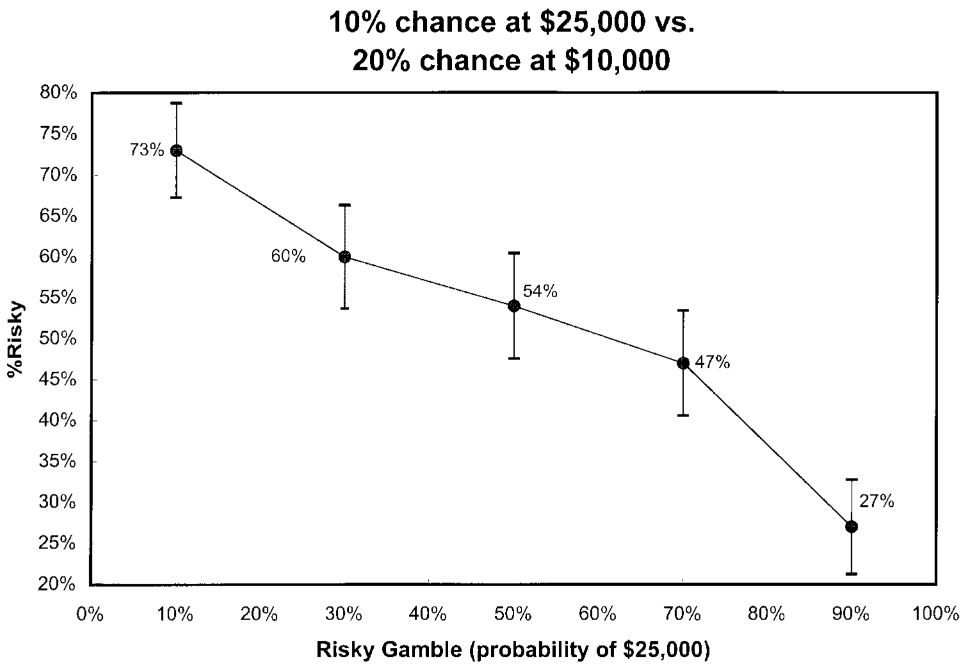

Figure 5. Example 3 (Diagonal common consequence shift)

and Kahneman, 1992).[2] Let $(p, x; q, y)$ refer to a three-outcome gamble which gives p chance at x, q chance at y, and chance at $r = 1 - p - q$ chance at $z = 0$ where $x > y > z = 0$.[3] RDEU represents choices between gambles as follows:

$$(p, x; q, y) \succ (p', x'; q', y') \Leftrightarrow \pi(p)v(x) + (\pi(p + q) - \pi(p))v(y) > \pi(p')v(x') + (\pi(p' + q') - \pi(p'))v(y'), \tag{1.1}$$

where $v(\cdot)$ is a value function and $\pi(\cdot)$ is a probability weighting function (also sometimes called a decision weighting function) such that $\pi(0) = 0$ and $\pi(1) = 1$. We also assume throughout that $v(\cdot)$ and $\pi(\cdot)$ are continuous and that $\pi(\cdot)$ is twice-differentiable.[4]

There is ample evidence that the probability weighting function, $\pi(\cdot)$, is inverse S-shaped, concave below approximately $p^* = .35$ and convex above (Tversky and Kahneman, 1992; Camerer and Ho, 1994; Tversky and Fox, 1995; Wu and Gonzalez, 1996). It turns out that this restriction on the weighting function is *almost* enough to characterize our three examples. Specifically, in the context of RDEU, the three natural manifestations of the common consequence effect (horizontal, vertical and diagonal shifts—Examples 1, 2, and 3 respectively) are each necessary and sufficient for a specific restriction on the probability weighting function. To be more precise, we start with the following two results. Within the context of RDEU:

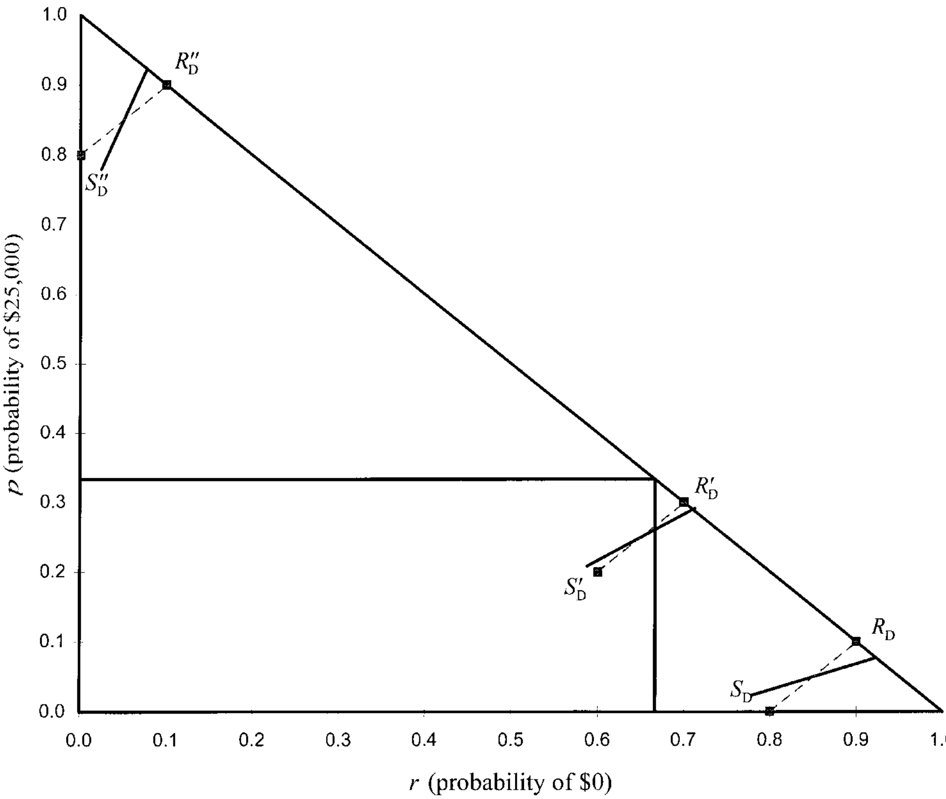

Figure 6. Example 3 in the probability triangle

(i) For shifts along the horizontal axis, concavity of $\pi(\cdot)$ implies horizontal fanning in (HFI), while convexity of $\pi(\cdot)$ implies horizontal fanning out (HFO).
(ii) For shifts along the vertical axis, concavity of $\pi(\cdot)$ is consistent with vertical fanning out (VFO), and convexity of $\pi(\cdot)$ consistent with vertical fanning in (VFI).

In Figure 7, we illustrate the global implications of (i) and (ii) for an *S*-shaped weighting function, concave below p^* and convex above p^*. In Region A, indifference curves fan out for both horizontal shifts and vertical shifts and thus fan out for diagonal shifts. However, in Regions B and C, the *S*-shape alone does not imply fanning in or fanning out for diagonal shifts parallel to the hypotenuse. Thus, that qualification alone falls short of completely characterizing fanning behavior within the triangle. We prove the following result concerning diagonal shifts:

(iii) For shifts parallel to the hypotenuse, decreasing absolute concavity of $\pi(\cdot)$, $\dfrac{-\pi''(p)}{\pi'(p)}$, implies diagonal fanning out (DFO).

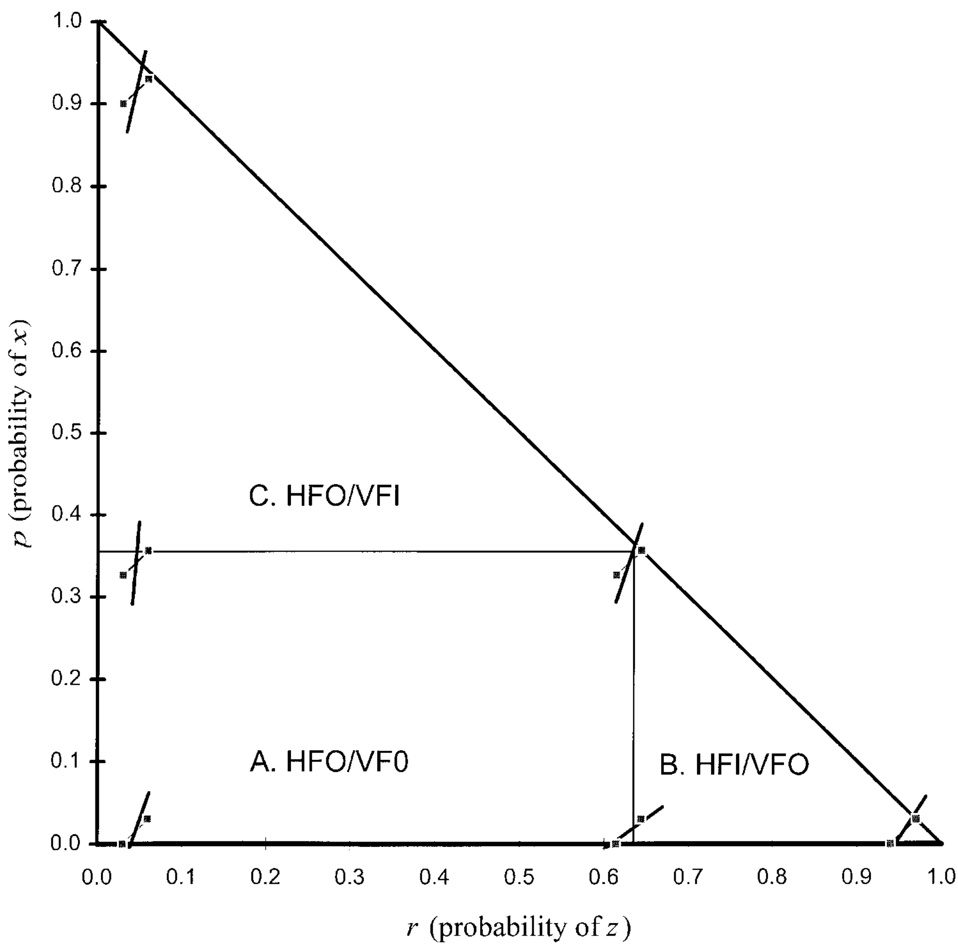

Figure 7. Fanning in and fanning out implications for *S*-shaped $\pi(\cdot)$

The measure of absolute concavity, $\frac{-\pi''(p)}{\pi'(p)}$, is the Pratt-Arrow measure for utility of wealth, $\frac{-u''(w)}{u'(w)}$, applied to the probability weighting function.

Thus, the three conditions together provide a formalism both for testing RDEU theory and for organizing past empirical studies. The paper proceeds as follows: In Section 2, we describe the three conditions and discuss empirical support for each condition, some previously reported and some new. The first condition, a shift along the horizontal axis, is a concavity/convexity condition discussed and tested empirically in Wu & Gonzalez (1996). That study provided a good deal of empirical support for the *S*-shaped weighting function. The second condition, a shift along the vertical axis is an alternative concavity/ convexity condition. The third condition, decreasing absolute concavity in the Pratt-Arrow sense, is a condition that is satisfied by most of the recently proposed weighting functions.

We review previous findings consistent with this property. In Section 3, we discuss the implications of these conditions in terms of global properties of indifference curves in the probability triangle. Finally, in Section 4, we discuss how this framework organizes much of the past empirical research on decision making under risk. Proofs are left for the Appendix, as well as the generalization of the conditions to gambles with more than 3 outcomes.

2. Common-consequence conditions

The conditions below have the same structure and hold outcomes x, y, and z fixed, and thus all such gambles can be depicted in the probability triangle. We start with two gambles, R and S, where R is mnemonic for Risky and S is mnemonic for Safe. R is riskier in the sense that it has a higher probability of x and z, the best and worst outcomes. R and S are transformed into R' and S' by shifting probability mass from one common consequence to another, i.e., from either z to y, y to x, or from z to x. Of course, the only preference patterns permitted by the independence axiom are R and R' or S and S'. Thus, a preference for R and S' or S and R' violates the independence axiom.

In all cases, transforming R to R' and S to S' yields strict improvements in the sense of first-order stochastic dominance. Thus, changes in preference from R to S' are consistent with fanning out of indifference curves (increased risk aversion) and changes in preference from S to R' are consistent with fanning in of indifference curves (decreased risk aversion).

We describe z to y shifts, y to x shifts, and z to x shifts in order. In the probability triangle, shifts from z to y involve *horizontal* (western) translations, shifts from y to x correspond to *vertical* (northern) translations, and shifts from z to x involve *diagonal* (northwest) translations parallel to the hypotenuse. The pairs of gambles used in the three conditions are depicted in Figure 8.

2.1. Horizontal translations: z-to-y probability shifts

We first consider shifts in probability mass from z to y. The following two conditions differ only in the direction of preference in the second pair.

Concavity condition I: Let $p > p'$, $q < q'$, $p + q < p' + q'$, $p' + q' + \varepsilon \leq 1$. Then if $R_H = (p, x; q, y) \sim (p', x; q', y) = S_H$, then $R_H' = (p, x; q + \varepsilon, y) \succeq (p', x; q' + \varepsilon, y) = S_H'$.

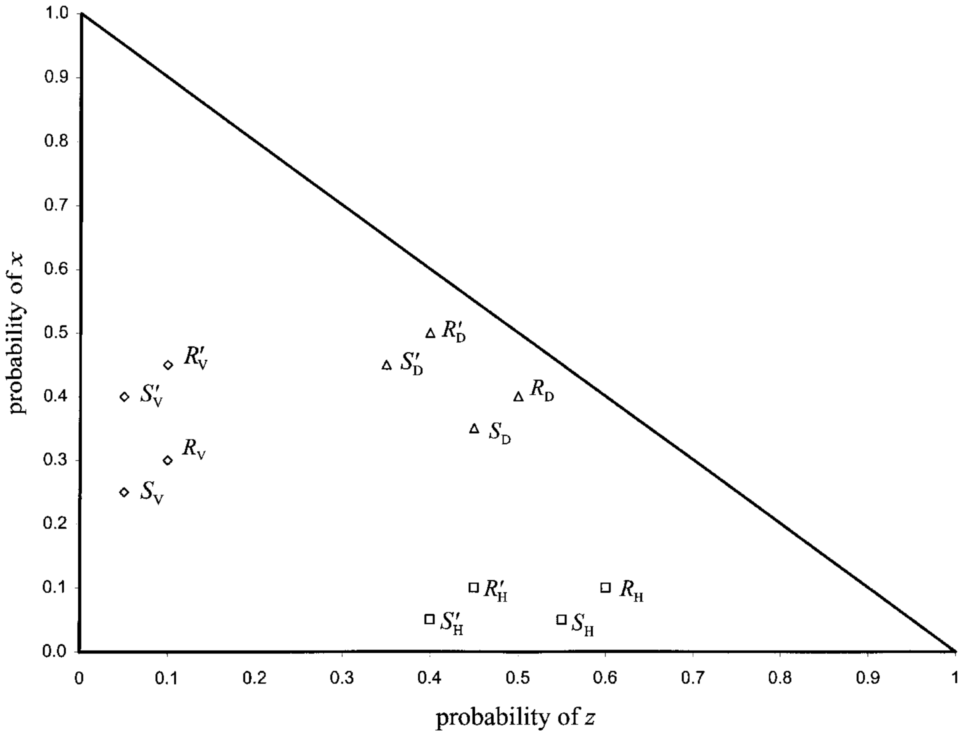

Figure 8. Common-consequence conditions depicted in the probability triangle

Convexity condition I: Let $p > p'$, $q < q'$, $p + q < p' + q'$, $p' + q' + \varepsilon \leq 1$. Then if $R_H = (p, x; q, y) \sim (p', x; q', y) = S_H$, then $R_H' = (p, x; q + \varepsilon, y) \preceq (p', x; q' + \varepsilon, y) = S_H'$.

Notice that R_H' and S_H' are merely R_H and S_H translated horizontally, a shift of ε probability from z to y (Figure 8). These conditions tighten Wu and Gonzalez (1996) in the following sense: if the Concavity/Convexity condition holds for $p' = 0$, it also holds for $p' > 0$. Convexity condition I was originally proposed by Segal (1987), who called it a "Generalized Allais Paradox."

Under RDEU, Concavity condition I (Convexity condition I) implies that $\pi(p + q + \varepsilon) - \pi(p + q) > (<)\ \pi(p' + q' + \varepsilon) - \pi(p' + q')$, which for ε small is approximately $\pi'(p + q) > (<)\ \pi'(p' + q')$. As $p' + q' \to p + q$, we approach a local condition: $\pi''(p + q) < (>)\ 0$. Note that both conditions, like all conditions in this paper, are "point conditions." Proposition 1 offers a precise generalization of the global implications, provided that the conditions hold for "all ε" within a range:

Proposition 1: In the context of CPT or RDEU, (i) and (ii) are equivalent:

(i) $\pi(\cdot)$ is strictly concave (convex) in the range $(\underline{r}, \bar{r})$;
(ii) Concavity (Convexity) condition I holds for all p, p', q, q', such that $p + q < p' + q'$, $\underline{r} \leq p + q$ and $p' + q' + \varepsilon \leq \bar{r}$.

Substantial empirical support for these both conditions (and hence $\pi(\cdot)$ S-shaped) was provided by Wu and Gonzalez (1996). Returning to Example 1, RDEU's explanation of the modal preferences, S_H and R_H' and S_H'', $\pi(.07) - \pi(.05) > \pi(.37) - \pi(.35)$ and $\pi(.97) - \pi(.95) > \pi(.37) - \pi(.35)$, is consistent with concavity of $\pi(\cdot)$ below (and convexity above) $p = .37$. The pattern also demonstrates the principle of diminishing sensitivity: additional probability has more impact at the boundaries (near 0 and near 1) than in the middle of the probability interval. Further examples of Concavity condition I in region B (Figure 2) are given by Prelec (1990) and Wu and Gonzalez (1996). For other illustrations of Convexity condition I (Region A), see Kahneman and Tversky (1979), Camerer (1989), and Wu and Gonzalez (1996).[5,6]

2.2. Vertical translations: y-to-x probability shifts

Next we consider shifts in probability mass from y to x. As before, the following two conditions differ only in the direction of preference in the second pair.

Concavity condition II: Let $p > p'$, $p + q < p' + q' + \varepsilon < 1$. Then if $R_V = (p, x; q + \varepsilon, y) \sim (p', x; q' + \varepsilon, y) = S_V$, then $R_V' = (p + \varepsilon, x; q, y) \preceq (p' + \varepsilon, x; q', y) = S_V'$.

Convexity condition II: Let $p > p'$, $p + q < p' + q' + \varepsilon < 1$. Then if $R_V = (p, x; q + \varepsilon, y) \sim (p', x; q' + \varepsilon, y) = S_V$, then $R_V' = (p + \varepsilon, x; q, y) \succeq (p' + \varepsilon, x; q', y) = S_V'$.

Both conditions involve an ε shift in probability mass from y to x. Recall that R_V' and S_V' represent a vertical translation of R_V and S_V (Figure 8).

Concavity (convexity) condition II implies that $\pi(p + \varepsilon) - \pi(p) < (>)\ \pi(p' + \varepsilon) - \pi(p')$, when $p > p'$. For ε small, this inequality is approximately $\pi'(p) < (>)\ \pi'(p')$, which approaches $\pi''(p) < (>)\ 0$ for $p - p'$ small. Even though these conditions only have point implications, as with the previous conditions, global curvature properties follow if the conditions hold for "all ε." The next proposition summarizes this result.

Proposition 2: In the context of CPT or RDEU, (i) and (ii) are equivalent:

(i) $\pi(\cdot)$ is strictly concave (convex) in the range $(\underline{r}, \bar{r})$;
(ii) Concavity (Convexity) condition II holds for all p, q, q', $\underline{r} \leq p$ and $p' \leq \bar{r}$.

Returning to Example 2, we found that preferences for the risky alternative decreased and then increased, consistent with an S-shaped probability weighting function. Preferences for R_V and $S_V^{'}$ in Example 2 imply that $\pi(.03) - \pi(.01) > \pi(.33) - \pi(.30)$, which is implied by concavity of $\pi(\cdot)$ below $p = .34$. Preferences for $S_V^{'}$ and $R_V^{''}$ are explained by $\pi(.93) - \pi(.90) > \pi(.33) - \pi(.30)$, an implication of convexity of $\pi(\cdot)$ above $p = .30$.[8]

It is noteworthy that Concavity condition I required a shift in preference from S to R', whereas Concavity condition II requires the opposite, a shift in preference from R to S'. This sign difference indicates that when $\pi(\cdot)$ is globally concave, it will exhibit horizontal fanning in, but vertical fanning out. In Section 3, we develop this observation and prove that RDEU with nonlinear $\pi(\cdot)$ implies regions in which there is both local fanning in *and* fanning out.[7] In Appendix B, we show that Concavity conditions I and II are special cases of the n-dimensional generalization of the concavity condition.

2.3. Diagonal translations: z-to-x probability shifts

Finally, we consider shifts in probability mass from z to x. The following condition describes such shifts:

Absolute Concavity condition: Let $p + q' < q$, and $x > y$. Then if $R_D = (p + p', x; q', y) \sim (p', x; q, y) = S_D$, then $R_D^{'} = (p + p' + \varepsilon, x; q', y) \preceq (p' + \varepsilon, x; q, y) = S_D^{'}$.

Note that $R_D^{'}$ and $S_D^{'}$ are constructed from R_D and S_D by moving ε probability from z to x (see Figure 8).

Preferences for R_D and $S_D^{'}$ imply that

$$\frac{\pi(p' + q + \varepsilon) - \pi(p' + p + q' + \varepsilon)}{\pi(p' + q) - \pi(p' + p + q')} \geq \frac{\pi(p' + p + \varepsilon) - \pi(p' + \varepsilon)}{\pi(p' + p) - \pi(p')}.$$

Re-writing this form as

$$\frac{-[(\pi(p' + q + \varepsilon) - \pi(p' + q)) - (\pi(p' + q) - \pi(p' + p + q' + \varepsilon))]}{\pi(p' + q) - \pi(p + p' + q')} \leq \frac{-[(\pi(p' + p + \varepsilon) - \pi(p' + p)) - (\pi(p' + \varepsilon) - \pi(p + \varepsilon))]}{\pi(p' + p) - \pi(p')}, \tag{2.1}$$

we have a difference equation analog to decreasing Pratt-Arrow absolute concavity. The numerator involves the difference of two differences (analog to a second derivative), while the denominator has a single difference (analog to a first derivative). As $q \to p + q'$ and $q \to p$ and (2.1) approaches a "local" condition, becoming

$$\frac{-\pi''(p' + q + \varepsilon)}{\pi'(p' + q + \varepsilon)} \leq \frac{-\pi''(p' + q)}{\pi'(p' + q)} \tag{2.2}$$

the Pratt-Arrow measure of absolute concavity applied to $\pi(\cdot)$, $\dfrac{-\pi''(p)}{\pi'(p)}$, decreasing in p (Pratt, 1964). Decreasing absolute concavity of the weighting function is satisfied for several proposed *S*-shaped weighting functions (Lattimore et al., 1992; Gonzalez and Wu, 1998; Prelec, 1998; Tversky and Kahneman, 1992). However, absolute concavity and *S*-shape are logically independent. *S*-shape implies that $-\pi''(p) / \pi'(p)$ must decrease at the inflection point, but not everywhere. For example, two functions proposed by Prelec (1998), $\pi(p) = [1 - \alpha \ln(p)]^{-\beta}$ and $\pi(p) = [\exp(p^{\alpha} - 1)]^{\beta}$, are *S*-shaped but do not satisfy decreasing $\dfrac{-\pi''(p)}{\pi'(p)}$ everywhere. Similarly, Segal's (1987) function, $\pi(p) = 1 - (1 - p)^{\alpha}$ has decreasing $\dfrac{-\pi''(p)}{\pi'(p)}$, but is convex everywhere.

The Pratt-Arrow measure also shows up prominently, but in a very different manner, in Prelec's (1998) work on the probability weighting function. Prelec characterizes $\pi(\cdot)$ using common-ratio effects, noting that the Pratt-Arrow ratio is a measure of subproportionality of $\pi(\cdot)$ transformed into log-log coordinates. The same measure is also found in Segal and Spivak (1988) who show that a "conditional risk premium" in RDEU is proportional to $\dfrac{\pi''(p)}{\pi'(p)}$.

The following proposition generalizes (2.2):

Proposition 3: In the context of CPT or RDEU, (i) and (ii) are equivalent:

(i) $\dfrac{-\pi''(p)}{\pi'(p)}$ decreases in p in the range $(\underline{r}, \bar{r})$;
(ii) the Absolute Concavity condition holds for all $p, p', q, \underline{r} < p + p', p + p' + q' < \bar{r}$.

To understand the condition, note that R_D and S_D each have a p' chance at x. Thus, p' chance at x might be regarded as the "endowment." Reframing, the choice between R and S can be seen as a choice between an incremental p chance at x or q chance at y. The absolute concavity condition requires that preferences become more risk averse as the endowment increases. Machina's (1987) intuitive justification for fanning out seems especially appropriate for this class of gambles: "Intuitively, if the distribution … involves very high outcomes, I may prefer not to bear further risk in the unlucky event that I don't receive it … But if (the distribution) … involves very low outcomes, I may be more willing to bear risk in the event that I don't receive it." (pp. 129–130)

Recall that Example 3 (Figures 5 and 6) shows a decreasing pattern of risky choices. Other tests consistent with the decreasing absolute concavity condition are reported in Camerer (1989, small gains and losses), Chew and Waller (1986, the "*H*" and "*L*" gambles in the "HILO" structure), Conlisk (1989), and Sopher and Gigliotti (1993). Several of Camerer's examples, as well as those of Chew and Waller and Conlisk are very close to the hypotenuse of the probability triangle. *Hypotenuse parallelism* (Camerer, 1989) suggests one explanation of the findings. Under RDEU, indifference curves along the hypotenuse are parallel, independent of the shape of $\pi(\cdot)$. The slope of the indifference curve through a gamble $(p, x; q, y)$ is $\dfrac{\pi'(p + q)v(y)}{\pi'(p)[v(x) - v(y)]}$. On the hypotenuse, $q = 0$, thus the slope, $\dfrac{v(y)}{v(x) - v(y)}$, does not depend on p or q. Most gambles that have tested absolute concavity have been near but not along the hypotenuse $-p$ varies while q is a constant measuring the distance from the hypotenuse. The indifference curves through $(p, x; q, y)$ and $(p', x; q, y)$ have nearly identical slopes as long as $\pi(\cdot)$ is linear in that range, which holds approximately except at the boundaries. When gambles move away from the hypotenuse, fanning out becomes more pronounced.

Buschena and Zilberman (1998), in an extensive test of 109 pairs of gambles in the probability triangle, provide a systematic test (292 subjects) of absolute concavity. Since their experiment was not designed specifically to test absolute concavity, only 70 pairs of choices constitute Absolute Concavity conditions. Many of these differences are small and statistically insignificant. Looking only at statistically significant shifts ($p < .05$), cases for decreasing absolute concavity ($\%R < \%R'$) outnumber cases for increasing absolute concavity ($\%R > \%R'$) 11 to 3 (binomial test, $p < .05$). Most of the statistically significant instances of decreasing absolute concavity are near the origin ($r < 1/3$, $p < 1/3$), where Convexity condition I and Concavity condition II both imply fanning out (and thus northwest shifts must also yield fanning out).[9]

3. Global implications

We can use Concavity/Convexity conditions I and II to provide a more precise characterization of the "ordinal" properties of indifference curves. Recall that a gamble $(p, x; q, y)$ is depicted in the triangle as $(1 - p - q, p)$ or (r, p), where r is the probability of z, the worst outcome. The slope of the indifference curves for a gamble $(p, x; q, y)$ is given by

$$f(p, r) = dp/dr = \frac{\pi'(1 - r)v(y)}{\pi'(p)[v(x) - v(y)]}.$$

Once x, y, and z are fixed, the slope of the indifference curves at (r, p) depends, of course, only on p and r. Indifference curves fan out everywhere if they become steeper for stochastically dominating improvements. Stated formally, $f(p + \varepsilon, r - \delta) > f(p, r)$ for $\varepsilon, \delta \geq 0$ and $\varepsilon + \delta > 0$, or, in differential form: for vertical fanning out (VFO), $\dfrac{\partial f}{\partial p} > 0$; and for horizontal fanning out (HFO), $\dfrac{\partial f}{\partial r} < 0$ (increasing r creates a worse gamble in the sense of stochastic dominance). We define the differential form of fanning in similarly: $\frac{\partial f}{\partial p} < 0$ (VFI) and $\frac{\partial f}{\partial r} > 0$ (HFI).

If $\pi(\cdot)$ is *S*-shaped, Propositions 1 and 2 together imply that there must exist a region in which indifference curves must fan in and a region where indifference curves fan out. In fact, this is true for $\pi(\cdot)$ globally concave or globally convex. In other words, for all nonlinear weighting functions, indifference curves cannot globally fan in or globally fan out. We formalize this last statement in the following Proposition:

Proposition 4: In the context of RDEU or CPT, if $\pi(\cdot)$ is nonlinear on $(p^*, p^* + \lambda)$, then:

$$\left.\frac{\partial f}{\partial r}\frac{\partial f}{\partial p}\right|_{p=\hat{p},\, r=\hat{r}} > 0, \text{ for } \hat{r} > 1 - p^* - \lambda, \hat{p} > p^*, \hat{r} + \hat{p} \leq 1.$$

Proposition 4 provides an alternative intuition for Roëll's (1987) result that Machina's Hypothesis II is compatible with RDEU only for $\pi(p) = p$. If $\pi(\cdot)$ is nonlinear, then there must exist a region in which indifference curves exhibit HFO and VFI, or HFI and VFO. Thus under RDEU, fanning out and fanning in cannot exist in different regions as in "mixed fanning models" (Gul, 1991; Neilson, 1992). There must exist indifference curves that exhibit a "saddle-point" quality, in which there is both local fanning in and local fanning out.

We prove this result graphically (the formal proof is given in Appendix A). Suppose that $\pi(\cdot)$ is concave on $(p^*, p^* + \lambda)$. Then, by Concavity condition I, there exists a region where HFI holds. The region is described in Figure 9. By Concavity condition II, VFO must hold within some portion of the triangle, also illustrated in Figure 9. When we combine these two results, we have a region in which both HFI and VFO hold.

4. Discussion

We presented three conditions that characterize the curvature of the probability weighting function. These conditions generalize Allais' common consequence effect by exploring three different types of "probability shifts" within the probability triangle. Within the context of rank-dependent expected utility theory, these common consequence conditions imply both fanning in and fanning out in the triangle, even when the weighting function is globally concave or globally convex. Our results both provide alternative intuition and extend Roëll's (1987) finding that Machina's Hypothesis II (global fanning out) is inconsistent with rank-dependent expected utility theory, except for the trivial case when expected utility holds. The Absolute Concavity condition is noteworthy because it extends the machinery of the Pratt-Arrow measure of risk preference to the probability weighting function. This opens up the possibility that utility theory results based on the Pratt-Arrow framework might yield insights for rank-dependent expected utility theory.[10]

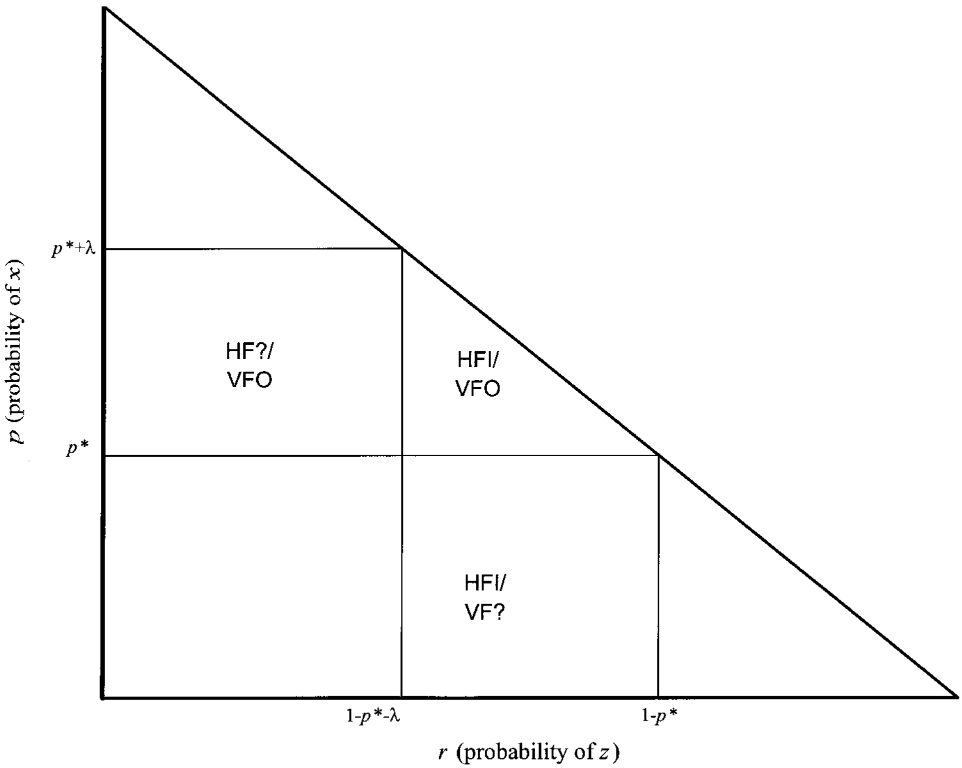

Figure 9. VFO and HFI are implied by concavity of $\pi(\cdot)$

Overall, the empirical evidence for these three conditions adds to the mounting support for an *S*-shape (first concave, then convex) probability weighting function. The psychological intuition is based on the principle of diminishing sensitivity (Tversky and Kahneman, 1992): individuals become less sensitive to probability changes away from the probability endpoints ($p = 0$, and $p = 1$). Thus diminishing sensitivity will produce a concave-convex shape in the probability weighting function.

The shape of the probability weighting function is currently a topic of both theoretical and empirical research. Many of the previous attempts to understand empirically the shape of the probability weighting function utilized goodness of fit tests on specific functional forms of both the value function and the weighting function. This paper diverges from that approach by offering qualitative conditions that are powerful enough to provide information about the curvature of the weighting function yet can also be used to test specific functional forms of $\pi(\cdot)$ (Wu and Gonzalez, 1996).

Finally, these conditions provide a framework for organizing much of the last 15 years of empirical risky choice research. With the exception of Starmer (1992), our common consequence framework is consistent with all the common consequence effect tests we could find (Buschena and Zilberman, 1995; Camerer, 1989; Chew and Waller, 1986; Conlisk, 1989; Harless, 1992; Kahneman and Tversky, 1979; Prelec, 1990; Sopher and

Gigliotti, 1993; Wu and Gonzalez, 1996). Machina's Hypothesis II has influenced how many researchers view empirical results: it has become standard to report not only the percentage of subjects who violate expected utility, but whether these violations are consistent with fanning in or fanning out. Researchers have debated what conclusions to draw from these such studies, the ambiguity attributed to weak or inappropriate statistical tests (e.g., Harless and Camerer, 1994) or incorrect specifications of a particular model (e.g., Quiggin, 1993). Like Quiggin, we believe that a consistent story of the empirical results emerges if these data are interpreted in terms of an *S*-shaped weighting function, characterized by the common consequence conditions presented in this paper. The framework explains fanning in/common consequence violations in the southeast corner (e.g., Prelec, 1990) and along the left edge (e.g., Conlisk, 1989). Of course, we do not wish to imply that RDEU (and its sign-dependent cousin, cumulative prospect theory) can explain all risky choice behavior. There are phenomena that RDEU and CPT cannot accommodate without resorting to editing operations or other explanations (e.g., Wu, 1994; Birnbaum and McIntosh, 1996; Birnbaum and Chavez, 1997), and surely researchers will find more in the future. Nevertheless, a rank-dependent model with an *S*-shaped weighting function offers a parsimonious and descriptively accurate picture of a large piece of what we currently know about decision making under risk. Our hope is that RDEU will continue to stimulate theoretical and empirical research in the area of risky choice.

Appendix A: Proofs

In all the proofs, we prove the result for either concavity or convexity. The "other" proof is identical with the inequalities reversed.

Proof of Proposition 1: Suppose that CPT holds and that $\pi(\cdot)$ is continuous. First, we prove that (i) implies (ii). If $(p, x; q, y) \sim (p', x; q', y)$, then

$$\pi(p)v(x) + [\pi(p + q) - \pi(p)]v(y) = \pi(p')v(x) + [\pi(p' + q') - \pi(p')]v(y). \quad \text{(A.1)}$$

Concavity of $\pi(\cdot)$ (and $p + q < p' + q'$) implies that $\pi(p' + q' + \varepsilon) - \pi(p' + q') \leq \pi(p' + q' + \varepsilon) - \pi(p + q)$ or

$$\pi(p' + q' + \varepsilon) - \pi(p + q + \varepsilon) + \pi(p + q) - \pi(p') \leq \pi(p' + q') - \pi(p'). \quad \text{(A.2)}$$

Substituting (A.2) into (A.1) and re-arranging, we get $\pi(p)v(x) + [\pi(p + q + \varepsilon) - \pi(p)]v(y) \geq \pi(p')v(x) + [\pi(p' + q' + \varepsilon) - \pi(p')]v(y)$. Thus, $(p, x; q + \varepsilon, y) \succeq (p', x; q'+\varepsilon, y)$.

Next, we prove that (ii) implies (i). Let $(p, x; q, y)^N (p', x; q', y)$. Then, by the concavity condition, $(p, x; q + \varepsilon, y) \succeq (p', x; q' + \varepsilon, y)$. Under CPT, the two conditions imply

$$\pi(p + q + \varepsilon) - \pi(p + q) \geq \pi(p' + q' + \varepsilon) - \pi(p' + q'). \tag{A.3}$$

Since the preceding preferences hold by assumption for all $p + q < p' + q'$ within the defined range, they also hold for all $p + q + \varepsilon = p' + q'$. Re-writing (A.3) for this case, we get $\pi(p + q + \varepsilon) \geq (\pi(p + q + 2\varepsilon) + \pi(p + q))/2$, which is midpoint concavity. Midpoint concavity is sufficient for concavity of $\pi(\cdot)$ on $(\underline{r}, \bar{r})$ for $\underline{r} < p + q, p' + q' < \bar{r}$ (Hardy, Littlewood, & Polya, 1952, Theorem 86) when $\pi(\cdot)$ is continuous. □

Proof of Proposition 2: Suppose that CPT holds and that $\pi(\cdot)$ is continuous. First, we prove that (i) implies (ii). If $(p, x; q + \varepsilon, y) \sim (p', x; q' + \varepsilon, y)$, then

$$\pi(p)[v(x) - v(y)] + \pi(p + q + \varepsilon)v(y) = \pi(p')[v(x) - v(y)] + \pi(p' + q' + \varepsilon)v(y). \tag{A.4}$$

Concavity of $\pi(\cdot)$ (and $p < p'$) implies that $\pi(p' + \varepsilon) - \pi(p') \leq \pi(p + \varepsilon) - \pi(p)$ or

$$\pi(p' + \varepsilon) - \pi(p + \varepsilon) + \pi(p) \leq \pi(p'). \tag{A.5}$$

Substituting (A.5) into (A.4) and re-arranging, we get $\pi(p + \varepsilon)[v(x) - v(y)] +$
$\pi(p + q + \varepsilon)v(y) \geq \pi(p' + \varepsilon)[v(x) - v(y)] + \pi(p' + q' + \varepsilon)v(y)$.
Thus,
$(p + \varepsilon, x; q, y) \succeq (p' + \varepsilon, x; q', y)$.

To prove that (ii) implies (i), we let $(p, x; q + \varepsilon, y) \sim (p', x; q' + \varepsilon, y)$. Then, by the concavity condition, $(p + \varepsilon, x; q, y) \succeq (p' + \varepsilon, x; q', y)$. The two conditions together imply $\pi(p + \varepsilon) - \pi(p) \geq \pi(p' + \varepsilon) - \pi(p')$. For the case of $p + \varepsilon = p'$, we have a midpoint concavity condition, $\pi(p + \varepsilon) \geq (\pi(p + 2\varepsilon) + \pi(p))/2$, which by Hardy, Littlewood, & Polya (1952, Theorem 86) establishes that $\pi(\cdot)$ is strictly concave on $(\underline{r}, \bar{r})$ for $\underline{r} < p, p' < \bar{r}$ if $\pi(\cdot)$ is continuous. □

Proof of Proposition 3: First, we establish that (i) implies (ii). Suppose that $-\pi''(p) / \pi'(p)$ is decreasing in p. If $(p + p', x; q', y) \sim (p', x; q, y)$, then

$$\frac{v(x) - v(y)}{v(y)} = \frac{\pi(p' + q) - \pi(p + p' + q')}{\pi(p' + p) - \pi(p')}. \tag{A.6}$$

Using Theorem 1 of Pratt (1964) (take $u_1(x) = \pi(x)$, $u_2(x) = \pi(x + \varepsilon)$ in Theorem 1e), $-\pi''(p) / \pi'(p)$ decreasing in p implies that

$$\frac{\pi(p' + q + \varepsilon) - \pi(p + p' + q' + \varepsilon)}{\pi(p' + p + \varepsilon) - \pi(p' + \varepsilon)} \geq \frac{\pi(p' + q) - \pi(p + p' + q')}{\pi(p' + p) - \pi(p')}. \tag{A.7}$$

Substituting (A.6) into (A.7) and re-arranging, we get

$$\pi(p + p' + \varepsilon)v(x) + [\pi(p + p' + q' + \varepsilon) - \pi(p + p' + \varepsilon)]v(y) \leq \pi(p' + \varepsilon)v(x) + [\pi(p' + \varepsilon + q) - \pi(p' + \varepsilon)]v(y),$$

which holds if and only if $(p + p' + \varepsilon, x; q', y) < (p' + \varepsilon, x; q', y)$.

The implication that (ii) implies (i) is similarly proved. The absolute concavity condition implies (A.7) for all $\varepsilon > 0$. Once again, using Theorem 1 of Pratt (1964), (A.7) implies that $-\pi''(p) / \pi'(p)$ decreases in p on $(p' + p, p + p' + q')$. □

Proof of Proposition 4: If $\pi(\cdot)$ is non-linear, then there exists a region, $(p^*, p^* + \lambda)$ where $\pi(\cdot)$ is concave or convex. If $\pi(\cdot)$ is convex in that region, $\pi'(p^* + \delta) > \pi'(p^*)$ for $\lambda > \delta > 0$, which implies

$$f(a, 1 - p^* - \delta) = \frac{\pi'(p^* + \delta)v(y)}{\pi'(a)[v(x) - v(y)]} < \frac{\pi'(p^*)v(y)}{\pi'(a)[v(x) - v(y)} = f(a, 1 - p^*), \quad \text{(A.8)}$$

for all $a < p^* + \delta$. $\pi(\cdot)$ convex also implies $\pi'(p^* + \varepsilon) > \pi'(p^*)$ for $\lambda > \varepsilon > 0$, which in turn implies

$$f(p^* + \varepsilon, b) = \frac{\pi'(1 - b)v(y)}{\pi'(p^* + \varepsilon)[v(x) - v(y)]} > \frac{\pi'(1 - b)v(y)}{\pi'(p^*)[v(x) - v(y)]} = f(p^*, b), \quad \text{(A.9)}$$

for all $b < 1 - p^* - \varepsilon$. For both (A.8) and (A.9) to hold, $1 - p^* - \lambda < b < 1 - p^*$ and $p^* < b < p^* + \lambda$. Since $f(a, 1 - p^* - \delta) > f(a, 1 - p^*)$ and $f(p^* + \varepsilon, b) < f(p^*, b)$ for all $\varepsilon, \delta < \lambda$, and a and b defined above, $\partial f/\partial p < 0$ and $\partial f/\partial r < 0$. Thus, $\partial f/\partial p\ \partial f/\partial r > 0$, therefore establishing the Proposition for $\pi(\cdot)$ convex. □

Appendix B: Generalization to gambles with more than 3 outcomes

Concavity conditions I and II, and Convexity conditions I and II readily generalize to more than three outcomes. An n-outcome prospect, $(p_1, x_1;\ldots; p_i, x_i;\ldots; p_n, x_n)$ offers p_i chance at x_i, where $x_i > x_{i+1}$ for all i. The general rank-dependent representation follows:

$$U(p_1, x_1;\ldots; p_i, x_i;\ldots; p_n, x_n) = \sum_{i=1}^{n} v(x_i)\,[\pi\,(\sum_{j=1}^{i} p_j) - \pi\,(\sum_{j=1}^{i-1} p_j)], \quad \text{(A.10)}$$

where $U(\cdot)$ represents preferences, i.e., $A > B \Leftrightarrow U(A) > U(B)$.

What follows is the n-outcome generalizations of the Concavity and Convexity conditions:

Generalized Concavity (convexity) condition: Let $\sum_{j=1}^{i} p_j < (>) \sum_{j=1}^{i} q_j$. Then if

$$A = (p_1, x_1;\ldots; p_i, x_i; p_{i+1}, x_{i+1};\ldots; p_n, x_n) \sim$$
$$(q_1, x_1;\ldots; q_i, x_i; q_{i+1}, x_{i+1};\ldots; q_n, x_n) = B,$$

then

$$A' = (p_1, x_1;\ldots; p_i + \varepsilon, x_i; p_{i+1} - \varepsilon, x_{i+1};\ldots; p_n, x_n) > (<)$$
$$(q_1, x_1;\ldots; q_i + \varepsilon, x_i; q_{i+1} - \varepsilon, x_{i+1};\ldots; q_n, x_n) = B'.$$

Note that Concavity conditions I and II are just special cases of the Generalized condition. In the Generalized condition, it is not necessary to distinguish between more and less risky. Since concavity and convexity are conditions about shifts in probability mass between two adjacent outcomes, x_i and x_{i+1}, what matters is the probability of receiving x_i or better, i.e., $\sum_{j=1}^{i} p_j$ and $\sum_{j=1}^{i} q_i$. To see that only adjacent outcomes matter, note that (A.10) implies

$$U(A') - U(A) = (v(x_i) - v(x_{i+1}))\,[\pi\,(\varepsilon + \sum_{j=1}^{i} p_j) - \pi\,(\sum_{j=1}^{i} p_j)] \tag{A.11}$$

and

$$U(B') - U(B) = (v(x_i) - v(x_{i+1}))\,[\pi\,(\sum_{j=1}^{i} q_j + \varepsilon) - \pi\,(\sum_{j=1}^{i} q_j)]. \tag{A.12}$$

Together, (A.11) and (A.12) lead to

$$A \sim B \text{ and } A' > B' \Leftrightarrow \pi\,(\varepsilon + \sum_{j=1}^{i} p_j) - \pi\,(\sum_{j=1}^{i} p_j) > \pi\,(\varepsilon + \sum_{j=1}^{i} q_j) - \pi\,(\sum_{j=1}^{i} q_j),$$

which concavity requires if $\sum_{j=1}^{i} p_j < \sum_{j=1}^{i} q_j$.

The Absolute Concavity condition is similarly extended to n-outcomes:

Absolute Concavity condition: Let $\sum_{j=1}^{i} p_j < \sum_{j=1}^{i} q_j$, $\sum_{j=1}^{i+1} p_j > \sum_{j=1}^{i+1} q_j$

If

$$A = (p_1, x_1;\ldots; p_i, x_i; p_{i+1}, x_{i+1}; p_{i+2}, x_{i+2};\ldots; p_n, x_n) \sim$$
$$(q_1, x_1;\ldots; q_i, x_i; q_{i+1}, x_{i+1}; q_{i+1}, x_{i+1};\ldots; q_n, x_n) = B,$$

then

$$A' = (p_1, x_1; \ldots; p_i + \varepsilon, x_i; p_{i+1}, x_{i+1}; p_{i+2} - \varepsilon, x_{i+2}; \ldots; p_n, x_n) \succeq$$
$$(q_1, x_1; \ldots; q_i + \varepsilon, x_i; q_{i+1}; x_{i+1}; q_{i+2} - \varepsilon, x_{i+2}; \ldots; q_n, x_n) = B'.$$

Thus, as the Generalized Concavity condition amounts to shifting probability to an adjacent outcome, the Absolute Concavity condition is characterized by shifting probability from outcomes spaced two apart as can be seen by examining the following two equalities:

$$U(A') - U(A) = (v(x_i) - v(x_{i+1}))\,[\pi\,(\varepsilon + \sum_{j=1}^{i} p_j) - \pi\,(\sum_{j=1}^{i} p_j)] +$$
$$(v(x_{i+1}) - v(x_{i+2}))\,[\pi\,(\varepsilon + \sum_{j=1}^{i+1} p_j) - \pi\,(\sum_{j=1}^{i+1} p_j)] \tag{A.13}$$

and

$$U(B') - U(B) = (v(x_i) - v(x_{i+1}))\,[\pi\,(\varepsilon + \sum_{j=1}^{i} q_j) - \pi\,(\sum_{j=1}^{i} q_j)] +$$
$$(v(x_{i+1}) - v(x_{i+2}))\,[\pi\,(\varepsilon + \sum_{j=1}^{i+1} q_j) - \pi\,(\sum_{j=1}^{i+1} q_j)] \tag{A.14}$$

Together, (A.13) and (A.14) lead to

$$A \sim B \text{ and } A' \prec B' \Leftrightarrow \frac{\pi\,(\varepsilon + \sum_{j=1}^{i+1} p_j) - \pi\,(\varepsilon + \sum_{j=1}^{i+1} q_j)}{\pi\,(\varepsilon + \sum_{j=1}^{i} q_j) - \pi\,(\varepsilon + \sum_{j=1}^{i} p_j)} \leq \frac{\pi\,(\sum_{j=1}^{i+1} p_j) - \pi\,(\sum_{j=1}^{i+1} q_j)}{\pi\,(\sum_{j=1}^{i} q_j) - \pi\,(\sum_{j=1}^{i} p_j)},$$

which is implied by decreasing absolute concavity (Pratt, 1964, Theorem 1).

Acknowledgements

We thank Drazen Prelec, Patrick Sileo, Amos Tversky, Peter Wakker for comments. We also thank Ben Sommers and Jason Brown for help collecting data. This work was supported by the Research Division of the Harvard Business School and Grant SES 91-10572 from the National Science Foundation.

Notes

1. The subjects were University of Chicago undergraduates. Surveys and instructions are available upon request.

2. In particular, Camerer and Ho (1994) found systematic violations of betweenness, both in the direction of quasi-concavity and quasi-convexity, suggesting the important role of nonlinearity of probability in describing risky choice behavior. See, however, Hey and Orme (1994) and Camerer and Harless (1994).
3. In Appendix B, we generalize these results to higher-dimensional gambles. There is no loss of generality in restricting attention to the case $z = 0$. Under these restrictions, however, RDEU coincides with Cumulative Prospect Theory (CPT; Starmer and Sugden, 1989; Luce and Fishburn, 1991; Tversky and Kahneman, 1992; Wakker and Tversky, 1993). Since CPT is basically two separate rank-dependent representations for gains and losses, CPT reduces to RDEU for gambles involving all gains or all losses.
4. In the context of RDEU, continuity of $v(\cdot)$ and $\pi(\cdot)$ follows from some version of the continuity axiom (e.g., Quiggin and Wakker, 1994, or Wakker and Tversky, 1993). Furthermore, since $v(\cdot)$ is unique up to a positive affine transformation, we have set $v(0) = 0$ for convenience, thus simplifying (1.1).
5. We are aware of only one counter-example, Starmer (1992), who found fanning in (Convexity) in Region A.
6. Convexity condition I is a special case of the upper subadditivity condition of Tversky and Wakker (1995) when $p' = 0$ and $p' + q' + \varepsilon = 1$. Our formulation of upper subadditivity tightens Tversky and Wakker, in which $q = 0$, in the following sense: if upper subadditivity holds for $q = 0$, it also holds for $q > 0$.
7. Concavity condition II is a special case of Tversky and Wakker's (1995) lower subadditivity condition when $p' = 0$ and $q = 0$. Since Tversky and Wakker require that $q' + \varepsilon = 1$, i.e., $(p, x; \varepsilon, y) \sim y \Rightarrow (p + \varepsilon, y) < (\varepsilon, x; 1 - \varepsilon, y)$, our formulation of lower subadditivity is slightly tighter. In other words, if lower subadditivity of $\pi(\cdot)$ holds for $q' + \varepsilon = 1$, it also holds for $q' + \varepsilon < 1$. Note that there are functions other than piecewise linear, $\pi(p) = \alpha + \beta p$, that are subadditive but not *S*-shaped. To demonstrate, take a function that satisfies subadditivity and is concave and then convex, such as Prelec (1998) or Karmarkar (1978). Replace a concave portion in a small interval with a slightly convex function. The resulting weighting function is no longer *S*-shaped but is still subadditive. Note also that Example 2 is evidence for concavity and convexity away from the boundaries, thus supporting the stronger hypothesis of concavity and convexity over lower and upper subadditivity.
8. We know of only one other test of Concavity and Convexity conditions II. In Region C, Camerer (1989) found non-significant fanning in (convexity) of $\pi(\cdot)$ in two gambles. In contrast, there are several tests of lower subadditivity, including Chew and Waller (1986) and Camerer (1989).
9. Interestingly, within the framework of Original Prospect Theory (Kahneman and Tversky, 1979), the absolute concavity condition is a concavity condition. According to OPT, $(p, x; q, y)$ is represented by $\pi(p)v(x) + \pi(q)v(y)$. For example, preferences for R_D and S_D' in Example 3 are accounted for by $\pi(.10) > \pi(.20) - \pi(.10)$, which follows from concavity of $\pi(\cdot)$ below $p = .20$. Thus, we can use the absolute concavity condition to test between OPT and CPT, a matter of some controversy (see, Camerer and Ho, 1994; Wu, 1994; Wu and Gonzalez, 1996). If CPT holds and $-\pi''(p)/\pi'(p)$ is decreasing in p from 0 to 1, then the percentage of subjects choosing R should decrease monotonically in the common consequence. However, if OPT holds and $\pi(\cdot)$ is *S*-shaped, then the percentage of subjects choosing R should decrease and then increase in the common consequence.
10. It is interesting to contrast our approach with that of Prelec (1998). Prelec has investigated the relationship between the functional form of $\pi(\cdot)$ and common ratio effects. Both common ratio effects (Prelec) and common consequence effect (our paper) are independently powerful enough for qualitatively similar implications on $\pi(\cdot)$, leading Prelec to conclude that the weighting function is in some sense over-determined.

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