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ANALYSIS OF FEASIBILITY OF USING CONVOLUTED DIAPHRAGMS IN A PARTICULAR DESIGN APPLICATION

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ABSTRACT

A preliminary examination has been made of the possibilities of using convoluted diaphragms as sealing and positioning devices in a valve unit subjected to exacting specifications.

No useful analytical design information was found to be available. Nor can such analysis be carried out within the scope of this project. Certain experimental data are available; these are not directly applicable to this problem, but are nonetheless useful in its examination.

On the basis of these data, nominal diaphragm dimensions necessary to achieve desired deflection characteristics are established. The resultant configurations are then examined from the strength standpoint. Results are pessimistic; it appears unlikely that diaphragms of sufficient strength can be achieved which will meet the required deflection specifications. A number of practical difficulties that might arise from the positioning requirements of the diaphragms are also examined. Again pessimism prevails.

While pessimistic, the results of the preliminary examinations are not absolutely conclusive and final. Experimentation will be necessary for positive information. This experimentation can begin in a relatively crude manner.

It is recommended strongly that alternate design approaches be explored concurrent with any possible further work on convoluted diaphragms, in view of the unpromising outlook. It is further recommended that the present specifications be subjected to rigorous re-examination, to note any and all possibilities of significant beneficial modification.

I. STATEMENT OF THE PROBLEM

A. OVERALL PROBLEM

The stated overall purpose of the subject study is to establish optimum designs for two particular convoluted diaphragms to be used in a proposed electro-hydraulic servo system.* The diaphragms have two main purposes:

1. They are to act as hydraulic insulators, or seals, of a valve unit.

2. They are to serve as positioning units for the valve actuator, in response to controllable forces from a torquemotor.

The study was to encompass theoretical design analysis and subsequent verification tests. The desired end results are design recommendations as to the necessary dimensions and materials for the two diaphragms.

Certain rigorous limits or specifications are imposed upon the size and the deflection characteristics of the diaphragms. It is recognized that the initial specifications are extremely exacting. To some extent they represent the most severe operating requirements anticipated. In the event the initial specifications prove infeasible, the extent to which they can be met was to be specified.

B. DETAILED SPECIFICATIONS

The general requirements on the size, mounting and mode of operation of the diaphragms are illustrated in the schematic diagram of Figure 1. It will be noted that the effective outside diameters of the two diaphragms are 0.442 inch and 0.625 inch, respectively. In the following, the smaller of the two will be denoted by "S" and the larger by "L."

*Refer to Vickers No. TO 12238-70-UM-2

Specifications on Sealing:

	Diaphragm "S" (0.442 inch diam)	Diaphragm "L" (0.625 inch diam)
Normal operating pressure:	300 psi	150 psi
Maximum operating pressure, short-term surge:	1500 psi	750 psi

It should be noted that the projected areas of the two diaphragms, and the operating pressures, are such that the total forces arising from these pressures are equal, or balanced.

Specifications on Positioning (both diaphragms):

Maximum central deflection: $\delta_S = \delta_L = \pm 0.008$ inch

Composite spring constant: $K = k_S + k_L \leq 100$ lb/inch

(where $k_S = P_S/\delta_S$, δ_S being central deflection obtained on diaphragm S under a central force P_S, etc.)

The combined spring constant K may be any value less than 100 lb/inch, but in the final application must not vary more than $\pm$ 10% from unit to unit.

Linearity: The load-deflection curve (central-load and central deflection) of each diaphragm is to be linear within the deflection range within $\pm$ 1/2% of 0.008 inch (i.e., within $\pm$ 0.00004 inch). (Later discussions have indicated that this requirement may be reduced somewhat at the extremes of the deflection range.)

Return to zero: Upon removal of load, the diaphragms shall return the actuator to within $\pm$ 1/2% of 0.008 inch ($\pm$ 0.00004 inch) of its original position.

In addition, the following general requirements relate to both the sealing and positioning aspects.

Ambient Temperature: 500°F

Frequency of Operation: 0-100 cycles per second. (At most, 100,000 cycles at full deflection may be expected.)

As noted earlier, these initial specifications are recognized as extremely rigid and as a measure of the ultimate performance desired. Modification may be possible, if necessary.

C. LIMITATIONS ON SCOPE OF THIS REPORT

This report does not profess to fulfill all of the overall objectives of the problem statement of section I-A. Indeed, the work to date demonstrates the need for a re-orientation of the entire project.

This report does the following:

1. Discusses the difficulty of obtaining reliable <u>analytical</u> solutions to the problem.

2. Emphasizes the need for an experimental development program.

3. Presents design recommendations for diaphragms that should have approximately the desired deflection characteristics, <u>if strength considerations can be met</u>.

4. Notes the likelihood that strength considerations will prevent achievement of the desired specifications.

5. Notes other areas of practical difficulty in the achievement of the desired specifications.

These aspects are summarized in the following Section II, and are detailed in the remainder of the report.

II. SUMMARY AND CONCLUSIONS

A preliminary examination of the problem, as represented by this report, creates decided pessimism as to the eventual possibilities of achieving the desired specifications or close approximations to them. To introduce convoluted diaphragms into the subject application would appear to require significant modification of the specifications. Discretion will require that alternate design approaches to the subject problem be explored concurrent with any further work on the diaphragms.

On the basis of the preliminary examination, the following conclusions can be drawn.

1. Analytical solution of the problem will be complex and time-consuming, beyond the scope of the currently-considered project in terms of both time and money. The literature of the field does not appear to contain any existing solutions suited to design use. Moreover, any solution obtained by analytical means will not have an accuracy commensurate with the present specifications.

2. Development of the ultimate diaphragm designs must be on an experimental basis. This will be no small program. Many variables are involved, and the rigidity of the specifications will require development of refined experimental techniques. Note the implication: if the experimental measurements are difficult to obtain due to the small quantities being measured, then the efficient use of the diaphragms in the ultimate installation will require equally-refined associated units.

3. At this stage it appears that ultimate achievement of the desired specifications is unlikely and significant modifications of the specifications will be necessary, in all probability.

   This conclusion is reached by an examination of available experimental data on convoluted diaphragms larger in diameter and subjected to much lower pressures than those we are concerned with. These data can be extrapolated to provide an estimate of diaphragm dimensions necessary to achieve approximately the desired deflection characteristics. Crude analogies and computations indicate that these diaphragms will have excessive stresses under the stipulated operating conditions.

   Even if the approximate deflection characteristics can be obtained without exceeding strength bounds, a number of additional considerations indicate the unlikelihood of achieving the desired accuracies in linearity and zero return.

4. The initial specifications are remarkably impressive to an observer unacquainted with the characteristics and operation of the valve proper, and the associated units of the complete servo system. To such an observer, it is impressive that the operation of the valve can be modified significantly by the stipulated small difference in actuator displacement, that the valve and housing can be manufactured within commensurate limits, that applied forces can be controlled within similar limits, and that the external units governed by the valve can respond with corresponding accuracy.

In view of the difficulties foreseen in achieving the desired diaphragm designs, it is recommended that the above considerations be re-examined. This would be with the view of loosening or down-grading the present specifications.

5. No reliable estimate can be made by analytical means as to the modification of the specifications that is likely to be necessary to permit introduction of convoluted diaphragms into the subject application. This must be derived from experiment.

As noted, then, the establishment of the desired designs must rely upon experimentation. Such experimentation may demonstrate that convoluted diaphragms are not practical for the proposed application. Since strength may prove to be the major obstacle, the initial experimentation should be directed against this aspect.

Certainly the above remarks warrant one summary conclusion: all of the design eggs should not be placed in the convoluted-diaphragm basket.

III. ANALYTICAL APPROACHES TO THE PROBLEM

A. INTRODUCTION

Clearly no designer would consider using convoluted (or "corrugated") diaphragms in a design application if the much simpler flat diaphragms would serve the purpose. As we shall see, it is the requirement of a linear load-deflection curve (or constant spring rate) that has led to consideration of convoluted diaphragms in this particular instance. It is worthwhile to re-examine this aspect.

Consider a thin flat circular plate with fixed edges, under the action of a uniform pressure.* The analytic solution of this problem for finite deflections and associated stresses is of itself no mean task. (This, incidentally, points up the difficulties to be encountered in solution of the much more complicated convoluted case.) The central deflection δ of such a plate can be approximated by**

*In the subject problem, the pertinent diaphragm loading associated with linear deflections is not a pressure, but a concentric loading applied by the actuator. However, the pressure case is somewhat simpler to discuss, and reveals the same type of necessary information and guidance.

**Theory of Plates and Shells, S. Timoshenko, McGraw-Hill Book Co., Inc., New York, 1940 (p. 335).

$$\delta = \frac{3}{16}\frac{pa^4(1 - \mu^2)}{t^3E}\left[\frac{1}{1 + .488(\frac{\delta}{t})^2}\right] \tag{1}$$

where

δ = central deflection (inches)

p = uniform pressure (psi)

a = plate radius (inches)

t = plate thickness (inches)

E = modulus of elasticity (psi)

μ = Poisson's ratio (dimensionless)

As will be noted, the deflection δ is not a linear function of the pressure p, as a result of the 0.488 $(\delta/t)^2$ term in the brackets. Thus the δ-p relation will be approximately linear only if $(\delta/t)^2$ is small. This is not true of the subject instances.

In the derivation of Equation (1), the deflection is due both to bending and to extensions of the central plane of the plate. It is this extension of the central plane that leads to the non-linearities. Thus, if the central plane is assumed unstrained, and the deflections due to bending alone, one obtains

$$\delta = \frac{3}{16}\frac{pa^4(1 - \mu^2)}{t^3E} \tag{2}$$

This is the well-known expression for infinitesimal bending. As noted, it applies only if $(\delta/t)^2$ is small.

Thus linearity is obtained only if the extensional strains are negligible, and bending predominates. This is the purpose of the convolutions: to eliminate extensions and make the action one of bending. To the extent that this can be done, linearity is obtained. The same general behavior is noted with concentric loadings as with the above pressure loadings.

B. REVIEW OF EXISTING LITERATURE

The useful published literature dealing directly with the analytic treatment of convoluted diaphragms is scanty indeed. The more pertinent items uncovered in our literature search are summarized briefly in the following:

1. The Theory of Pressure Capsules, A. A. Griffith, Aeronautical Research Committee, Air Ministry (Great Britain), Rand M 1136, August, 1927.

   This brief paper does not deal directly with convoluted diaphragms. Rather it "brackets" the convoluted case between the two extremes of the flat plate: one in which central plane extensions are considered, the other in which they are neglected. The ranges obtained are generally too large to be positively useful.

   The approach professes to bracket the strength considerations as well as the deflection aspects. No details are given. The extension to strength appears nebulous, and it is best not to rely upon it for our problem.

2. Der Spannungszustand einer Kreisringschale ("State of Stress in a Circular Shell"), K. Stange, Ingenieur-Archiv, March, 1931, Julius Springer, Berlin.

   This paper sets out to be an overall attack on the complete problem of convoluted diaphragms. It ends up as a method of solution for the deflections and stresses of a portion of one corrugation, per Figure 2. The analysis of such a portion is quite involved and tedious, although the method is a simplification of a more rigorous attack. Solution is by series.

   Application of the approach is beyond the scope of this study. Even were the method applied, it would not provide the requisite accuracy of this problem and experimentation would still be necessary. Since this is so, the computations can well be bypassed and the experimentation relied upon.

   While not convenient for this study, this reference would be useful in a long-term detailed analytical attack on the convoluted diaphragm problem.

3. Etude des Capsules Aneroides, F. Charron, Sci. at Tech. du Ministeres de L'Air, Pub. No. 160, Paris, 1940.

   This paper is concerned with effects of corrugation shape. Its material is not quantitatively useful to this problem. Since it cannot replace experimentation, it may as well be by-passed and experimentation relied upon.

4. A Note on the Theory of Corrugated Diaphragms for Pressure-Measuring Instruments, A. Pfeiffer, The Review of Scientific Instruments, Vol. 18, No. 9, September, 1947.

   This paper sets up a simple bending-tension model of convolution action. It then uses this model to correlate experimental data which will be discussed in the following section. The results have little usefulness in our particular instance.

As noted, none of the above items are particularly useful within the scope of the subject design problem. (Much more useful are certain papers containing experimental results; these are discussed in the following Section IV.)

C. SUMMARY OF ANALYTIC POSSIBILITIES

Reliable analytical treatment of the subject problem would be complex and time-consuming, beyond the scope of this project. Independent contemplation indicates this; the literature search verifies it. Even were such an analytical solution obtained, its accuracy would not be commensurate with the limits imposed by the specifications. Experimentation would be required.

Nonetheless, such a solution would be useful indeed, in providing guidance in experimentation and in indicating general feasibilities of convoluted diaphragms. Undoubtedly it could be arrived at in some form. It would not be of a "closed" nature; one could not use it directly to compute required dimensions for specified performance. Rather it would establish an analysis procedure, whereby a specified configuration could be analyzed to predict its approximate performance. This process would have to be repeated until the required configuration was obtained.

The process would undoubtedly be a step-by-step affair, in which various concentric zones of the diaphragm were considered in turn. However, with the likely use of computing machines, enough computations might be carried out and something like a set of curves obtained, useful to the design process.

As noted, this is far beyond the scope of the present project, in terms of both time and money. Primary recourse must be to experimentation.

IV. AVAILABLE EXPERIMENTAL DATA

Even though at this stage primary reliance must be placed upon experimentation, the published literature contains certain experimental data which will provide a guide to such a program, and-indeed-a guide to the possibilities of using convoluted diaphragms in our particular problem. The data are presented in the following:

1. Corrugated Metal Diaphragms for Aircraft Pressure-Measuring Instruments, W. A. Wildhack and V. H. Goerke (National Bureau of Standards), National Advisory Committee for Aeronautics, Technical Note 738, November, 1939.

2. The Limiting Useful Deflections of Corrugated Metal Diaphragms, W. A. Wildhack and V. H. Goerke (National Bureau of Standards), National Advisory Committee for Aeronautics, Technical Note 876, December, 1942.

These reports summarize and correlate experimental results obtained in tests of a great many convoluted diaphragms of various proportions. The primary variables are taken as thickness and diameter. Much less emphasis is placed on corrugation number and shape of the large majority of the tests incorporating the convolution pattern of Figure 3.

The tests were directed primarily at deflection characteristics, including such aspects as load-deflection curves and the effects of hysteresis, drift, aftereffect, recovery and creep. Little or no emphasis is on strength. Materials and methods of experimental manufacture and test are discussed.

The presented data include no tests on diaphragms as small as the ones we are considering, nor on test pressures even remotely as large as ours. Nor is our configuration (with its combined pressure and central loadings) considered. The smallest diameter tested was 1.5 inches, the largest pressures were apparently of the order of 35 psi; this compares to our 0.44 inch diameter and 300 psi. However, thickness-diameter and deflection-diameter ratios similar to ours were considered.

No attempt will be made herein to summarize or abstract these rather lengthy reports in a complete manner. The reports themselves should become part of the library of anyone intimately concerned with the convoluted diaphragm problem. Thus only that material directly used in the following treatment will be abstracted.

The basic load-deflection results are summarized in Figure 4, in terms of the dimensionless parameters $F\delta/pD$ and t/D. The symbols are defined on the figure.

Diaphragms of the configuration of Figure 3 were found to be nominally linear for deflections δ less than $0.02D$, where D is the diaphragm diameter.* Slight deviations from linearity were least in the range $2 < 1000t/D < 4$. These ranges are applicable to our problem, as we shall see.

The tests also demonstrate that diaphragms are subject to slight non-linearities, hysteresis, drift, and the like. While not particularly severe in the quoted tests, they become items of some concern under our specifications. Furthermore, the spring rates (under applied pressure) were slightly different for loadings on the concave and convex sides of the diaphragm. Again the effects are not particularly large, but are significant when viewed against our specifications.

These reports should be examined closely at the planning stage of any future experimental program.

V. DESIGN COMPUTATIONS-DEFLECTION CHARACTERISTICS

A. GENERAL APPROACH

In this and the following sections, the afore-mentioned experimental data will be used as a base in deriving preliminary designs for the subject problem. Because of the limitations on the available data, these proposed designs are likely to have the following characteristics:

1. The load-deflection curve will be nominally linear (within either the specified deflections range or within the range of elastic behavior of the material, whichever is the smallest).

2. Since little or no stress and strength data are available, there is no guarantee whatsoever that the designs will be strong enough to accomodate either the operating pressures or the operating deflections. (Indeed, crude computations of the following Section VI will indicate that it is quite unlikely that a satisfactory design strength-wise can be obtained within the subject specifications.) Experimentation is required in this respect.

*This is a somewhat conservative figure. A number of diaphragms were nominally linear for deflections δ as great as, or greater than $0.04D$.

3. The proposed designs are quite likely to have slight non-linearities or effects from drift, hysteresis and the like that are beyond the bounds of the specifications. This, too, requires experimentation.

4. The convolution pattern is to be as shown in Figure 3, since only this pattern is backed up by experimental data.

5. The material is to be beryllium copper. Again the majority of the data are for this case. This is quite proper, in that its relatively high strength-flexibility ratio makes it a generally accepted material for such usage. Nor will any direct account be taken of the pulse characteristics or vibratory characteristics of the system. To do so requires consideration and knowledge of the nature and operation of all units associated with the subject diaphragms.

B. ESTABLISHMENT OF DIMENSIONS ON THE BASIS OF DEFLECTION CHARACTERISTICS

The procedure for establishing the approximate dimensions will consist of evaluating the $F\delta/pD$ ordinate of Figure 4, and then determining the desired thickness t from the value of $1000t/D$ obtained from the curve of the figure. The diameter D is 0.442 inch and 0.625 inch, for diaphragms S and L respectively. F is the so-called plate modulus of the material, and is taken as 18.9×10^6 for beryllium copper; this is the value used in the subject reports. The remaining δ/p term is the inverse of a spring rate, based on pressure loading. Our requirements for spring rate are more nearly those of concentrated central loads. The report TN876 provides, with extrapolation, experimental factors relating the central loads and pressure loads causing the same central deflections. Thus, if Q is taken as a concentrated central load, we obtain

$$\frac{Q}{p\pi a^2} \simeq 0.35 \qquad (3)$$

Thus we replace p by $Q/.35\pi a^2$ in the ordinate of Figure 5. Then δ/p becomes $.35\pi a^2\delta/Q$, where δ/Q is the inverse of our desired spring rates. In this term, a = .22 inch for the small diaphragm, 0.31 inch for the large one.

On this basis, the $F\delta/pD$ values have been computed for each of the diaphragms, over a range of individual spring rates from 0 to 100 pounds per inch. The corresponding values of $1000t/D$ have been taken from Figure 4, and the thicknesses t required for the various spring rates have been determined.

The results (thickness vs spring rate, for the established diameters) are presented in Figure 5. For comparison purposes, the same relation is plotted for <u>small</u> deflections of a flat plate, the case in which extensional strains are insignificant. Thus the spring rates of the convoluted diaphragms are seen to be greater, for given thicknesses and diameters, than the <u>initial</u> (small deflection) rates of flat plates. This result is as it should be.

The design requirements are on composite spring rate, not on the individual rates. Thus any number of combinations of thicknesses of the two diaphragms can be selected which will fulfill the composite spring rate requirement. To be somewhat on the conservative side, let us aim at a composite spring rate of 80 pounds per inch ($K = k_S + k_L = 80$). Among the possibilities, then, are the following:

1. Configuration 1: equal spring constants. ($k_S = k_L = 40$)

   thickness: $t_S \simeq 0.00105$ inch

   $t_L \simeq 0.0011$ inch

2. Configuration 2: equal thicknesses.

   thickness: $t_S = t_L \simeq 0.00107$ inch

   $k_S \simeq 43$ lb/inch

   $k_S \simeq 37$ lb/inch

3. Configuration 3: unequal spring rates, unequal thicknesses. (an infinity of such possibilities exists; only two are noted.)

   (a) $k_S = 60$ lb/inch; $k_L = 20$ lb/inch

   $t_S \simeq 0.0014$ inch

   $t_L \simeq 0.0008$ inch

   (b) $k_S = 20$ lb/inch, $k_L = 60$ lb/inch

   $t_S \simeq 0.0007$ inch

   $t_L \simeq 0.0016$ inch

Note the very minor variations in thickness in cases (1) and (2).

The thickness values noted above are the effective thicknesses of the fabricated diaphragms. They are somewhat less than the thicknesses of the blanks from which the diaphragms are to be formed, because of the stretching inherent in the forming process. The presented data of the NACA reports indicate the blanks must be approximately 10-15% thicker than the eventual desired diaphragm thickness. (It should be noted that the thickness of the formed diaphragm will vary from point to point, as a result of the forming process.)

As noted earlier, Figure 4 is most valid in the ranges $\delta/D \leq 0.02$ and $2 \leq 1000t/D \leq 4$. Our results are either in these ranges or sufficiently close to them.

Clearly the eventual choice of the particular combination of individual spring rates to be employed will depend to a very considerable extent upon which combination presents the best strength characteristics. This can be answered only by experimentation.

C. LINEARITY CONSIDERATIONS

As noted earlier, the NACA experimental data show slight linearity deviations that can be serious in view of our rigid specifications. It was noted further that this aspect can be determined only by experimentation.

Still another linearity effect must be considered. The NACA data demonstrate that essentially linear spring rates can be obtained with pressure loadings on either the "concave" or "convex" (see Figure 3) sides of the diaphragms. However, the rates obtained with loadings on the two sides are not exactly the same. (The diaphragms appear slightly more flexible when loaded on the "concave" side.) The differences may be of the order of 2 or 3%, outside our specified bounds.

This effect can be compensated for in the final design, to some extent, by having the concave side of one diaphragm and the convex side of the other attached to the actuator.

VI. DESIGN CONSIDERATIONS—STRESS AND STRENGTH CHARACTERISTICS

A. GENERAL CONSIDERATIONS

No portion of the available literature on convoluted diaphragms lends itself to a reliable estimate of the stress and strength of the particular configurations evolved in the preceding section. Analytical solutions are nonexistent, and their development within the scope of this project is out of the question. The data of the NACA reports does not extend to our loading configuration or to the pressures we are considering.

Yet intuition indicates that the strength aspects effectively may prevent use of the convoluted diaphragms in the subject application. While no reliable estimates can be obtained without specific experimentation, several crude estimates of the general orders of magnitude of the stresses can be obtained. This is done in this section. As will be seen, the results warrant pessimism over the outlook for the introduction of convoluted diaphragms in the subject application. Nonetheless, they cannot be relied upon as being either conservative or non-conservative. Experimentation is required.

B. STRENGTH CRITERIA

The diaphragms cannot be presumed safe strength-wise if they successfully resist rupture. The stringent positioning and linearity specifications are such that no significant yielding of the material can be tolerated. (The only alternative to this might be an involved process of controlled yielding during the manufacturing process. Even this becomes invalid at our operating temperature of 500°F.)

Fatigue strength considerations will play a relatively minor role in the subject application. The total number of cycles accumulated should be less than 100,000. Thus stresses considerably above the infinite-life endurance limit can be successfully withstood. Indeed, in view of the crudity of the following stress checks, we can consider allowable stresses approaching the yield point.

If, in other applications of this type, the operating life is increased so that a significantly larger number of cycles is accumulated, fatigue must be considered with care. Then the strength problems will be multiplied, relative to the subject application.

In view of the crudity of the following stress checks, no particular attention is paid in the following to the strength degradations resulting from the 500°F operating temperature. These should not be excessive at this temperature; note, however, that we are approaching the threshold at which decided degradation of material strength will result. In a similar sense, strength degradations likely to arise from attachment of the diaphragms to the housing are neglected.

Clearly the most severe loading condition of the diaphragms is that of maximum actuator amplitude, coupled with the high operating pressures.

The following sections present some crude approximations to the stresses in the proposed diaphragms of the preceding Section V. Since the results obtained are subject to very large errors in any case, no refinements whatsoever are warranted or considered. The reader should not look for rigor.

C. AVERAGE SHEAR STRESS

It is intuitively obvious that strength failure of the proposed diaphragms will result from bending action, if at all. Nonetheless, a rough approximation of the shear stresses at the diaphragm peripheries will be worthwhile. If the average shear stresses at these regions are high, the results will be discouraging. If they are not, no conclusions can be reached.

Thus we compute

$$\text{average shear stress} = \frac{\text{pressure load}}{\text{peripheral shear area}}$$

Consider the small diaphragm. The pressure load is clearly $\pi p_S(a_S^2 - a_a^2)$, where a_a is the actuator diameter. The shear area at the peripheries is $2\pi t(a_S + a_a)$. Thus, for the smaller diaphragm

$$\tau_{avg} = \frac{\pi \cdot 300(.22^2 - .046^2)}{2\pi(.0011)(.22 + .046)} \simeq 50{,}000 \text{ psi} .$$

The actual maximum shear stress will be somewhat greater than this value.

Neglecting the degradations of strength due to temperature, one might expect to attain a tensile yield point of approximately 120,000 psi with beryllium copper. This would indicate a shear yield point of approximately 70,000 psi.

Clearly, then, the above computation indicates a highly critical strength situation, even if the thickness of the small diaphragm is increased. Note that attempts to go to alloy steels in the hopes of achieving a better strength situation are not particularly beneficial. The modulus of elasticity of steel is greater than that of beryllium copper; thus smaller thicknesses will be required for the same spring rates. Thus any increases in allowable stress are counterbalanced, at least in part, by smaller shear areas.

D. COMPARISON WITH FLAT PLATES

Attempts to "bracket" the convoluted diaphragm stress state by computation of the stresses in various flat-plate configurations appear useless. The ranges of values obtained are very wide, and reliable interpolation seems out of the question. The difficulty lies in drawing a reliable analogy between the convoluted diaphragm and some flat-plate configuration. As an example, attempts made in this direction have indicated bending stresses ranging from 40,000 to 300,000 psi, due to actuator deflection alone (i.e., exclusive of pressure considerations).

E. COMPARISON WITH CORRUGATED TUBE

What appears to be a reasonably good intuitive concept of the strength capabilities of corrugated diaphragms might be obtained by an examination of the strength capabilities of a corrugated tube. The analysis of such tubes is contained in the technical literature. Pertinent design information is presented in Figure 6.

Let us examine the stresses in such a corrugated tube, proportioned to fulfill the same deflection characteristics as the smaller diaphragm, and having the same thickness and diameter as the smaller diaphragm. Thus $p/\delta = 40$ pounds per inch, $a = 0.22$ inch, and $t = 0.0011$ inch. The dimension b is taken as 0.15 inch. Again consider the use of beryllium copper, with approximately 17×10^6 psi and μ approximately 0.33.

On this basis, the equations of Figure 6 indicate that approximately eleven semicircular corrugations will be required, leading to a tube length of approximately 3.3 inches.

Now consider the stresses in this tube <u>due to an 0.008 inch actuator deflection only</u>. (Stresses due to the internal pressure will be considered later.) This corresponds to an axial load of $p = (40)(0.008) = 0.32$ pounds. Under these conditions, the maximum meridional bending stresses are computed to be of the order of 30,000 pounds per square inch. Note that these stresses must be additive to the stresses due to internal pressure at one time or another, since the actuator deflects in both directions.

Intuition would indicate that the corresponding stresses in the associated convoluted diaphragm will be larger than this value. Note that one cannot take the view that the diaphragm stresses will be lower as a result of stiffening effects on the diaphragm from extensions of its central plane. This negates the necessary purpose of the convolutions, which is to make the action one of bending rather than extension.

The stresses in the corrugated tube <u>due to internal pressure</u> would seem to have much less meaning for comparative use. The two cases do not appear analogous. These stresses have been computed, however, with the results indicating maximum meridional bending stresses of the order of 270,000 psi at an operating pressure of 300 psi.

Again the results must be viewed more with pessimism than with optimism.

F. SUMMARY OF STRENGTH POSSIBILITIES

All of the foregoing indicates the likelihood that strength will prove a stumbling block in achievement of the desired specifications. The high operating pressures are likely to prove more detrimental than the actuator deflections. However, due to the crude nature of the computations, the results cannot be taken as entirely conclusive. (The average shear stress computation comes very close to being conclusive, however.)

Thus some experimentation will be required. This should probably be begun with relatively crude strength tests. Then, if the strength is obviously far too low, the time and expense of developing the refined methods of testing necessary to check the other variables will have been avoided.

VII. OTHER CONSIDERATIONS

A number of other significant considerations enter into the eventual problem of using convoluted diaphragms in the subject application. Some of these are noted in this section.

A. PULSE PRESSURES

The specifications of the problem note that pulse pressures of 1500 and 750 psi can be expected on the small and large diaphragms, respectively. No account of this has been made of these pulses in the foregoing computations. To do so would require knowledge of likely pulse durations as well as information on associated units of the complete system.

In any case, very short-time pulses will have little or no deleterious effects. However, if the duration became large enough so that the loading effectively could be considered as static, all hope of meeting the strength requirements would have to be abandoned.

B. VIBRATORY CHARACTERISTICS

The specifications call for actuator operation at frequencies up to 100 cycles per second. Clearly the possibilities of deleterious vibratory effects must be considered. These cannot be evaluated, however, without considerations or knowledge of the characteristics of attached units.

Consider, however, the unrealistic case in which the torquemotor exercised no restraint on the axial vibrations of the actuator and diaphragms. (This is tantamount to considering the system with no current in the torquemotor.) In this case, resonance of the system with the 100 cps excitation would occur if the total weight of the moving parts was 0.1 pounds or more. While this situation is unrealistic, it impresses the requirement of rigid attachment of the actuator and diaphragms to the torquemotor.

C. PERMISSIBLE VARIATIONS OF SPRING RATE

The specifications demand that the composite spring rate shall not vary more than ± 10% of its established value (< 100 pounds per inch) from unit to unit. This turns out to impose some rather rigid demands on the thickness and material properties of the diaphragm.

Let us assume no process of selective assembly is to be used with the diaphragms. Then the ± 10% limitation will apply to each individual diaphragm. From Figure 5 we see that the likely variation of spring rate with thickness of the small diaphragm is of the order of

$$\frac{\Delta k}{\Delta t} \simeq 6 \times 10^4$$

On this basis, an allowable variation of ± 4 pounds per inch in a nominal spring rate of 40 pounds per inch corresponds to an allowable thickness variation of ± 0.00007 inch in a diaphragm thickness of 0.0011 inch. This is, of course, far beyond the tolerances of commercially available sheet stock. Note further that this allowable variation is after forming, and that thickness decreases are incurred during forming.

In the same sense, the spring rate varies directly with the modulus of elasticity of the material. The modulus of drawn beryllium copper may be expected to show variations of the order of 3%.

D. IMPLICATIONS OF POSITIONING REQUIREMENT

The specifications impose very rigid positioning and return-to-zero requirements on the diaphragm. The return to zero is to be within ± 0.00004 inch of the initial position, and the load-deflection curve is not to vary more from linearity by more than this amount, at least over the central portion of its range.

These requirements are certainly impressive to an observer unacquainted with the characteristics and operation of the valve and the associated units of the complete system. To such an observer, it seems remarkable that:

1. Operation of the valve can be modified significantly by such small differences in actuator displacement.

2. A valve and housing can be manufactured within commensurate limits.

3. An applied force can be controlled within similar limits, and

4. External units governed by the valve can respond with corresponding accuracy.

In view of the obvious difficulties noted, and to be noted, in developing diaphragms capable of meeting the established specifications, it would appear worthwhile to re-examine the diaphragm specifications in light of the above.

It is granted that it will be desirable to have all units of the complete system as accurate as possible. However, it may prove profitable to tighten specifications on some other unit and thus obtain less strict requirements for the knotty diaphragm design.

The positioning requirements have ramifications in a number of areas. Some of these are outlined briefly in the following subsections.

E. HYSTERESIS, DRIFT, AFTEREFFECT, RECOVERY, AND ZERO SHIFT

All of the effects noted in the title, as well as the afore-mentioned differences in spring rate for convex-side and concave-side loadings, will work against achievement of the desired positioning specifications.

The experimental data of the NACA reports demonstrates that small deviations from desired performance will result for each of these effects. It would appear unlikely, then, that the sum of all of these individual errors could be held within the allowable limits of $\pm$ 0.00004 inch.

However, the available data must be extrapolated in order to obtain quantitative estimates. This cannot be done in a reliable and conclusive way. Thus experimentation is required. It should be noted that the variances under examination are so small that highly-refined measurement techniques will have to be employed.

F. EFFECT OF APPLIED PRESSURE

The proposed design calls for pressures of 300 and 150 psi to act upon the small and large diaphragms, respectively. The diaphragm areas are so proportioned that the total pressure force (pressure x area) is the same for each diaphragm. This is intended to result in a balanced system, with no resultant central deflection of either diaphragm.

The data of the NACA reports indicates that this will not be exactly so. Report TN738, for example, discusses the ratio of the concentrated central forces and the pressure forces necessary to provide the same central deflection. This ratio is found to vary slightly with diameter, for a given diaphragm thickness. (That is, the factor taken as 0.35 in Equation (3) is not actually a constant, but appears to increase slightly as the diameter increases.) For the diameters under consideration, the data indicate that as a result of this behavior the actuator will be shifted somewhat toward the larger diaphragm. This can be compensated for by decreasing slightly the diameter of the larger diaphragm in the final design.

A somewhat similar behavior will result if the two pressures (nominally 300 and 150 psi) are not maintained in an exact 2:1 ratio. Clearly the actuator would be displaced toward the smaller diaphragm if the 150 psi pressure on the large diaphragm was slightly less than 150 psi, while the 300 psi value was maintained as the smaller. A displacement δ of 0.00004 inch corresponds to a central force Q of $k\delta = (40)(0.00004) = .0016$ pound. This in turn corresponds to a pressure p which can be approximated from Equation (3) of Section V-B as

$$p \simeq \frac{Q}{0.35\pi a^2} \simeq \frac{0.0016}{(0.35)(\pi)(.31^2)} \simeq 0.015 \text{ psi} .$$

Note, then, that a change in pressure of 0.015 psi in the 150 psi value would result in excessive displacement of the actuator, if the 300 psi value remained constant.

It would be remarkable indeed if the 2:1 pressure ratio could be maintained with this accuracy. Indeed, a pressure drop of only 3 psi in the 150 psi side (the 300 psi remaining constant) would cause a deflection of the full 0.008 inch magnitude.

This aspect should be re-examined very carefully.

G. EFFECT OF ACCUMULATION OF TOLERANCES IN FABRICATION AND ASSEMBLY

Ideally the actuator, diaphragms and housing will be so manufactured and so assembled that the length L of the actuator is exactly the same as the distance between diaphragm centers before installation of the actuator, per Figure 7a. Obviously, however, these dimensions will not be exactly the same—some tolerances must be allowed for manufacture.

A number of variations can be obtained as a result of such tolerances; we shall examine only one of these. Assume, for example, that the actuator is manufactured to the length L, but the distance between the unattached diaphragm centers is $L + e$. Assume further that one diaphragm is so fabricated and installed that it is exactly in the position desired. These assumptions are pictured in Figure 7b. Now the unit is assembled. Clearly the actuator is displaced an amount $e/2$ from its desired position. Thus e cannot exceed 0.00008 inch unless special measures are taken during assembly.

Such measures must obviously be taken; otherwise the dimensions of the parts would have to be held within fantastic limits. Note that e can build up from variations in many sources: variations in the length of the actuator, variations in the length of the housing, "out-of-flatness" of the diaphragms, and the thicknesses of whatever brazing-type materials might be used to attach the diaphragms to the housing and the actuator.

H. SUMMARY

The above considerations point up some of the difficulties arising from attempts to achieve the high accuracies required of the specifications. They should be carefully examined.

The writers grant that certain of the above items may prove of no concern to the client, in the light of their detailed knowledge of the other units of the system. They appear pressing problems to the writers, however, in view of their limited knowledge of these other units.

VIII. RÉSUMÉ AND NEXT STEPS

Of necessity, the subject design has been approached in the following manner:

1. The approximate dimensions required for the desired deflection characteristics have been extrapolated from available experimental data.

2. The strength capabilities of the resultant configuration has been examined in a necessarily crude and highly approximate manner.

3. Other considerations affecting the possibilities of achieving the desired specifications have been examined.

The results of this procedure are pessimistic at best. Primary obstacles will be in the areas of strength and of high-accuracy positioning. The strength problems are noted in Section VI, the positioning problems in Section VII. Of the latter, the one relating to pressure differentials and discussed in subsection VII-F may prove the most severe.

However, none of the results of the above are absolutely conclusive and final. Experimentation will be necessary.

It is recommended that the experimental program be begun with relatively crude measurements of load-deflection characteristics and strength. The initial tests should be conducted on diaphragms approximating the proposed configurations (A) and (B) of Section V-B. Development of the highly-refined measurement techniques necessary to an evaluation of the secondary variables (such as hysteresis, zero return, and the like) can well be postponed until the above results are in. The initial crude tests may well indicate that the situation is hopeless.

It is further recommended that very careful attention be given to the possibilities of drastically downgrading the present specifications. The <u>absolute need</u> for the present requirements should be established, before extensive and expensive corollary experimentation is carried out.

As noted earlier, the primary obstacles are in the fields of strength and positioning. The outlook for the positioning aspects can be improved only through modification of the specifications.

In the area of strength, however, the possibility of design approaches utilizing spring arrangements other than the contemplated diaphragms should be explored. This will be difficult in view of the flexibilities required.

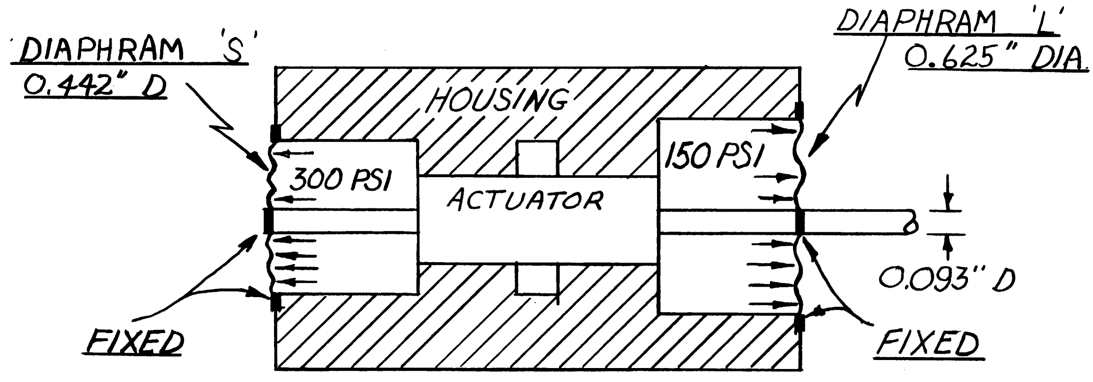
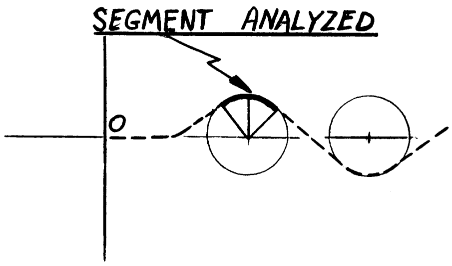
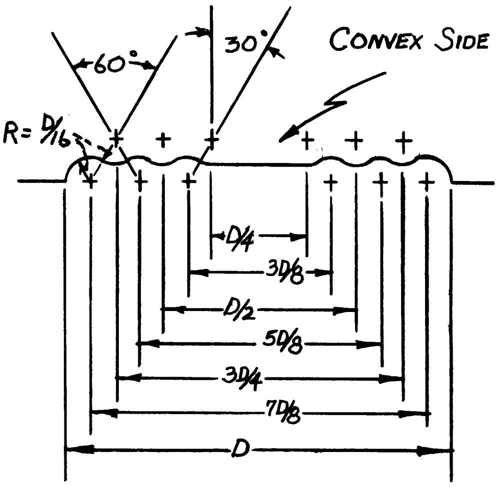


FIGURE 1



FROM 'DER SPANNUNGSZUSTAND EINER KREISRINGSCHALE' K. STANGE

FIGURE 2



FROM NACA TN 876

FIGURE 3

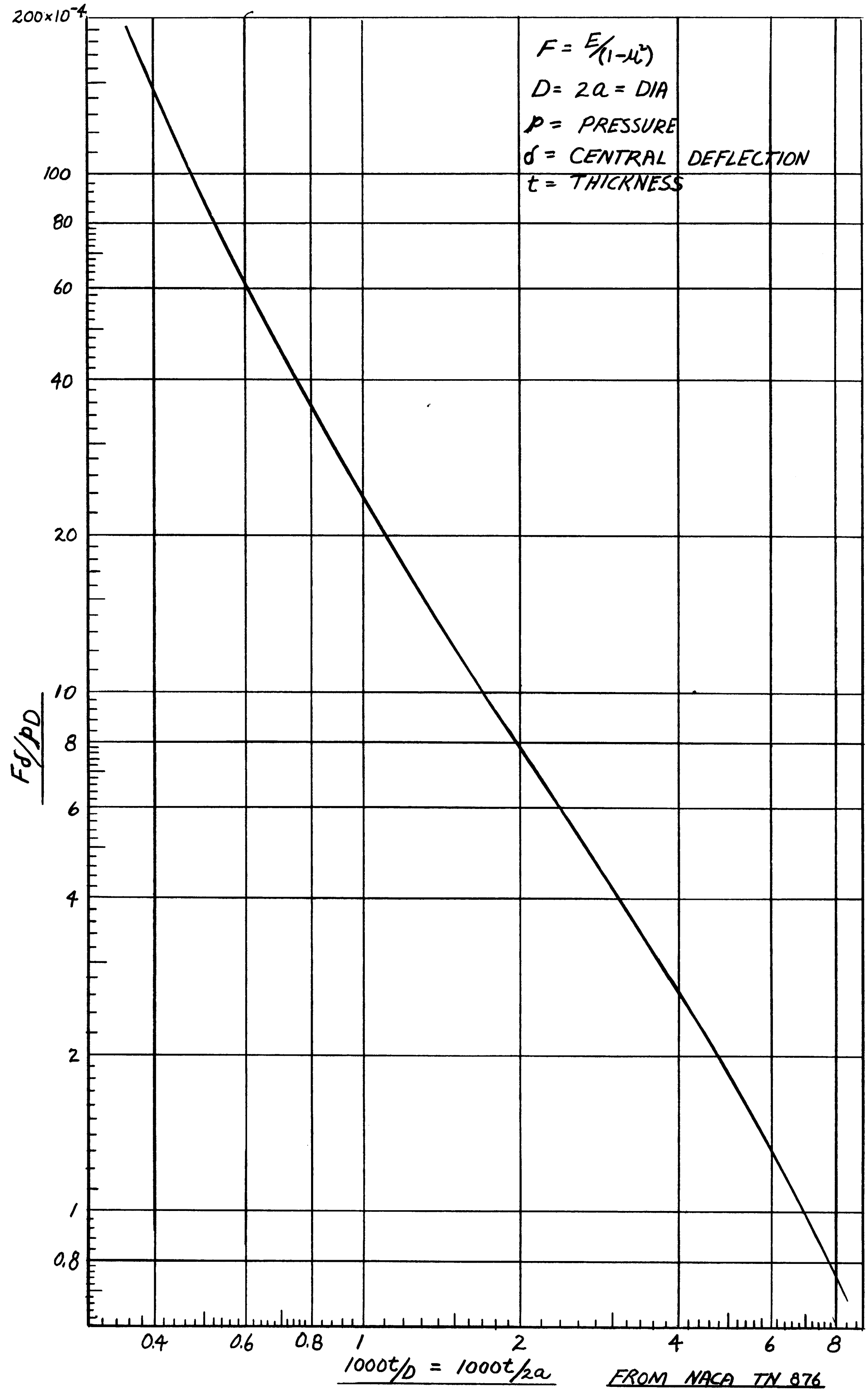

200×10⁻⁴
100
80
60
40
20
10
8
6
4
2
1
0.8
$F\delta/pD$
0.4
0.6
0.8
1
2
4
6
8
$1000t/D = 1000t/2a$
$F = E/(1-\mu^2)$
$D = 2a = DIA$
p = PRESSURE
δ = CENTRAL DEFLECTION
t = THICKNESS
FROM NACA TN 876

FIGURE 4

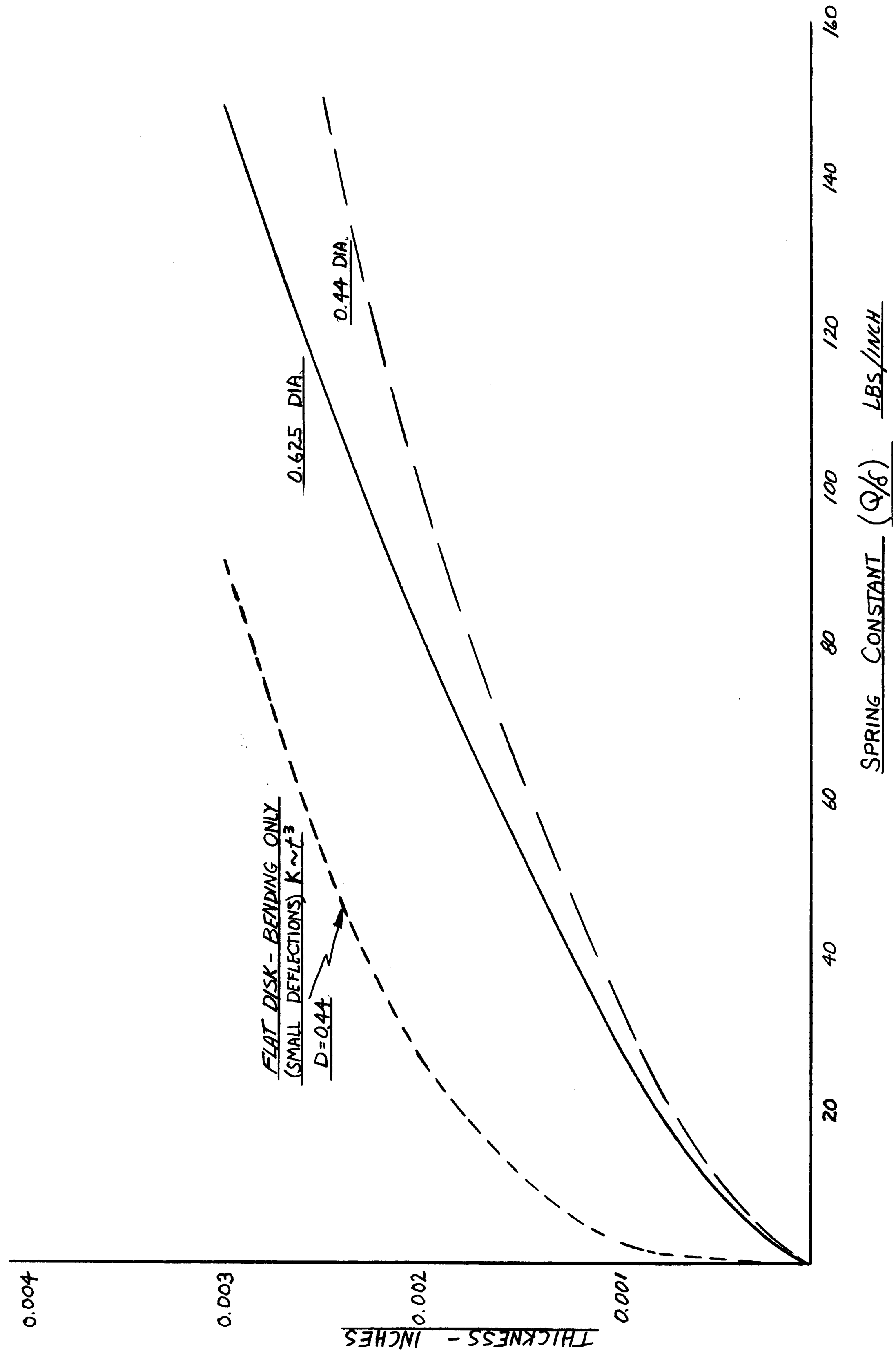
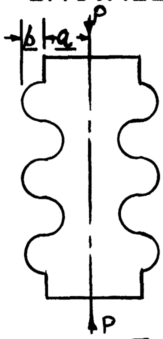

0.625 DIA.
0.44 DIA.
FLAT DISK - BENDING ONLY
(SMALL DEFLECTIONS) K~t³
D=0.44
THICKNESS - INCHES
0.004
0.003
0.002
0.001
20
40
60
80
100
120
140
160
SPRING CONSTANT (Q/δ) LBS/INCH

FIGURE 5



CORRUGATED TUBE UNDER AXIAL LOAD 'P'

$$\text{STRETCH} = \frac{mP}{\pi E t^2 \sqrt{12(1-\mu^2)}}\left(\frac{ab}{a-b}\right)$$

MAX MERID. BNDG. STRESS $S_1' = \frac{1.63\,P}{2\pi t(a-b)\sqrt[6]{1-\mu^2}}\sqrt[3]{\frac{ab}{t^2}}$

MAX CIRC. MEM. STRESS $S_2 = \frac{0.925\,P\sqrt[3]{1-\mu^2}}{2\pi t(a-b)}\sqrt[3]{\frac{ab}{t^2}}$ (COMPRESSIVE)

HERE m = NO. OF SEMICIRCULAR CORRUGATIONS (5 IN FIGURE)

CASE 1

FIGURE AS ABOVE BUT LOADED ONLY BY UNIFORM PRESSURE p, LBS. PER SQ. IN.

STRETCH = 0

MAX MERID. BNDG. STRESS $S_1' = \frac{0.955\,p}{\sqrt[3]{1-\mu^2}}\sqrt[3]{\left(\frac{ab}{t^2}\right)^2}$

MAX. CIRC. MEM. STRESS $S_2 = 0.955\,p\sqrt[6]{1-\mu^2}\sqrt[3]{\left(\frac{ab}{t^2}\right)^2}$

CASE 2

FROM 'FORMULAS FOR STRESS & STRAIN' ROARK

FIGURE 6

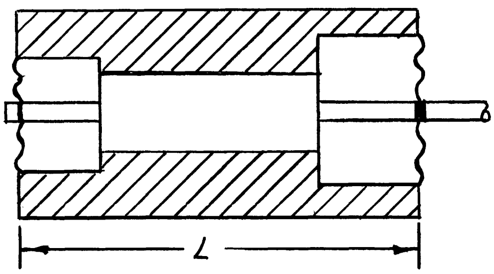


FIGURE 7a

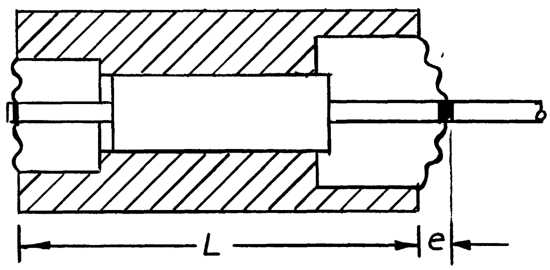


FIGURE 7b