## UNIVERSITY OF MICHIGAN

# A CHART USEFUL FOR STUDY OF OSTRACOD CARAPACES 

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## Volume XIV

1. An Upper Cretaceous Crab Avitelmessus grapsoideus Rathbun, by Robert V. Kesling and Irving G. Reimann. Pages $1-15$, with 4 plates.
2. A Chart Useful for Study of Ostracod Carapaces, by Robert V. Kesling. Pages 17-20, with 2 figures.

## A CHART USEFUL FOR STUDY OF OSTRACOD CARAPACES

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## INTRODUCTION

Studies of ostracod populations, both living and fossil, require consideration of several measured factors. Measurements of the carapace are the most important bases for separation of fossil ostracods into instars. Their variations through ontogeny seem to be specific characters, and as such deserve careful evaluations.

The quantity of data makes comparison of species, or of instars within a species, difficult. For example, how can one quickly compare two species, each represented by several hundred carapaces, for variation in length, height, width, height/length ratio, and volume? Tables are unsatisfactory; the larger the population, the more unwieldy they become. It seems that all significant trends cannot be solved by any one method. It is possible, however, to devise new ways of presenting the information which will make certain relationships more apparent.

The chart (Fig. 1) described here can be used to plot length vs. height, length vs. height/length ratio, length vs. volume, ${ }^{1}$ or length, height, and width. On it length, height, width, volume, and height/length ratio are shown simultaneously for all specimens being considered. Any combination is available for determining the limits of an instar. Furthermore, the chart solves the product of length, height, and width graphically.

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## CONSTRUCTION OF CHART

Two important relationships are graphically portrayed (Fig. 1) for the two variables plotted on double logarithm paper. Although mathematicians regard them as obvious, most students of micropaleontology are amazed that both the ratio and the product of the variables are represented by the simple plotted position on the paper. In the normal position of the graph, with the $y$-axis $(x=1)$ vertical at the left and the $x$-axis $(y=1)$ at the bottom, the 45 -degree line from the upper left through a point gives the product of the $x$ and $y$ values; the answer can be read at the intersection of the sloping line with either the $x$-axis or the $y$-axis. It can be shown that the values of such 45 -degree lines increase logarithmically to the upper right, on a scale equal to that of the regular co-ordinates times the sine of 45 degrees. The 45 -degree line from the upper right through a point gives the ratio of the $x$ and $y$ values; the ratio of $x / y$ can be read at the intersection of the sloping line with the $x$-axis, and the ratio of $y / x$ at the intersection with the $y$-axis.

As adapted for study of ostracods, the double logarithm scales (Fig. 1) are tilted 45 degrees so that the former $y$-axis slopes up toward the right, and the former $x$-axis slopes down in the same direction. The axis of the logarithm scale representing the products thus becomes horizontal, and the logarithm scale representing ratios becomes vertical.

## USE OF CHART

The chart can be used to study one specimen or several. It is particularly valuable in revealing ontogenetic relationships of many specimens of a species, from either a living population or a fossil collection. The uses are discussed singly.

1. To find the ratio of height/length.-If length is plotted as a co-ordinate of the axis sloping downward and height of the axis sloping upward, the ordinate of this point is the height/length ratio. The value of the ordinate is the same as that of the height at the intersection with the heightaxis.
2. To find the product of length $\times$ height.-If length and height are plotted as above, the abscissa of the point is the product of length $\times$ height. The value of the abscissa is the same as that of either length or height at the intersection with the length-axis or the height-axis. See Figure 1.
3. To find the product of length $\times$ height $\times$ width. -This product, comparable to the volume of the carapace, is derived with accuracy commensurate with that of the measurement of the three variables. Plot length
vs. height as above. Measure off a distance corresponding to the value of width on the horizontal scale, and draw a line this long extending to the right from the plotted point of length vs. height. The abscissa through the right end of the line is the product of length $\times$ height $\times$ width. See Figure 1 .
4. To compare two specimens not in the same instar.-The percentage increase in length, height, width, or volume can be readily measured with the slide rule described previously (Kesling, 1953) and used to assign an instar number to the unknown specimen; however, the increases can also be evaluated for data plotted on the chart. The percentage increase in length can be found by measuring off a distance between the plotted positions parallel to the length-axis and finding the value of this distance on the scale of length. Similarly, the percentage increase in height can be found by measuring the distance between the plotted positions parallel to the height-axis. The physical difference between the lines for width, evaluated on the width scale, gives the percentage increase in that dimension. The distance between the abscissas for volume, evaluated on the horizontal scale, gives the increase in volume.
5. To graphically show length, height, and width.-Plot length vs. height as a point for each specimen. Measure off a distance corresponding to the value of width on the horizontal scale, and draw a line this long extending upward from the plotted point. The uppermost ends of such lines do not represent volume, but their spacing shows how well the three variables agree for each instar. Figure 2 shows the data for a collection of Amphizona asceta Kesling and Copeland.
6. To separate specimens in a collection into instars according to length and height.-Plot length vs. height for each specimen. Draw a line parallel to the length-axis through each significant gap in distribution according to length. Similarly, draw a line parallel to the height-axis through each significant gap in distribution according to height. If growth has progressed in the ideal way suggested by Przibram, and if the ostracods have maintained the same shape in successive instars, and if the animals in each instar did not have large variations in each dimension, then the lines in each set should be spaced at intervals of the logarithm of 1.26 . A series of dashed squares in Figure 2, drawn with this spacing, fits around most of the points plotted for specimens in the collection.
7. To separate specimens in a collection into instars according to vol-ume.-Derive the product of length $\times$ height $\times$ width as explained in paragraph 3 above. Draw a vertical line through each significant gap in distribution of the products. If the volume has doubled from one instar of this
species to another, the lines should be spaced at intervals of the logarithm of 2. The dashed lines in the lower part of Figure 2, drawn with this spacing, separate all the specimens into instar groups.


Fig. 2. Plotted data for Amphizona asceta Kesling and Copeland taken from Kesling and Copeland, 1954, Tables I-VI. In the upper part of the figure length, height, and width are shown graphically. Length varies from .290 to 1.025 mm ., height from .215 to .605 mm ., and width from .080 to .320 mm . The series of dashed squares represents ideal limits of instars according to the concept of Przibram. In the lower part of the figure, the volume (product of length, height, and width) is plotted against the height/length ratio. Dashed lines represent ideal limits of instars. It can be seen that in this sample the specimens are grouped into instars better by volume than by one dimension.

## LITERATURE CITED

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Przibram, H. 1931. Connecting Laws in Animal Morphology. Four Lectures Held at the University of London, March, 1929. London: Univ. London Press. 62 pp.



[^0]:    1 "Volume" is used in this paper to mean the product of length, height, and width. Although this is actually the volume of a rectangular parallelopiped bounding the carapace, it is very useful in comparing one instar against another. Kesling (1952, pp. 772-80) has discussed the problem of volume in ostracod carapaces.

