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FALLING SPHERE METHOD FOR UPPER AIR DENSITY AND TEMPERATURE

UNIVERSITY OF MICHIGAN - SIGNAL CORPS

UPPER ATMOSPHERE RESEARCH PROJECT

OXFORD CONFERENCE

ON

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FALLING SPHERE METHOD FOR UPPER AIR DENSITY AND TEMPERATURE

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Abstract

A method of calculating upper atmosphere densities and temperatures from the measured velocities and accelerations of a falling sphere is described. The sphere apparatus and the DOVAP (Doppler radar) tracking system are described. The equations used in the calculations are given, and the sources and magnitudes of errors are discussed. The results from one Aerobee flight are given, and this flight and two others from which the data are being reduced are described.

1. Introduction

The velocity and acceleration trajectories of three spheres dropped from rockets on three flights at White Sands have been measured by Doppler radar. The spheres are approximately 4 feet in diameter when inflated, weigh approximately 50 pounds, and carry internal miniature Doppler receiver-transmitters. Ambient air density is calculated from the drag equation, and ambient temperature from the hydrostatic equation and the equation of state of a perfect gas. Results have been calculated from the first flight; the other two are in process.

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2. Other Methods for Pressure, Density, and Temperature

Before describing the details of the method, it may be of interest to mention briefly three methods which were used previously at Michigan to measure density, pressure, and temperature.[1] In each of these methods the idea was to measure the Mach number and velocity of the rocket from which the velocity of sound and ambient temperature may be directly calculated.

In one method the pressures at the tip and at four points on the side of a cone were measured. From these the Mach number may be calculated by a theory due to Taylor-Maccoll[2] and Stone.[3] In the second method the angle of the shock wave formed by a wedge at the front of the rocket was to have been measured by taking shadowgraph pictures with an optical apparatus. Mach number is known as a function of the shock wave angle. In the third method, the angle of a shock wave formed by a cone was measured by an array of moving probes containing pressure gages; these gages giving signals upon passing through the shock wave.

Of five flights in which the missile performance was satisfactory, temperature measurements of useful precision were obtained from one, the moving probes. An analysis of the difficulties encountered showed that equipment complexity was a major cause of failure and that errors in measuring the angle of attack and velocity of the missile were larger than anticipated.

3. Sphere Method

The falling sphere method was conceived as an answer to the above difficulties. The only flight instrumentation required, in addition to the sphere and its ejection mechanism, is the Doppler transceiver. This transceiver or a similar unit (for velocity measurement) must be used by the above methods in addition to the basic end-organs. The sphere itself presents no angle of attack problem although the spinning of the internal antenna is a source of some error.

The choice of the size and weight of the sphere is a compromise between engineering practicability and a wish to meet other conflicting criteria as ideally as possible. Factors which must be taken into account are:

1) Small weight-to-size ratio so that the drag acceleration is large enough to be measured.
2) Size small enough so that sphere can be fitted in rocket.
3) Size large enough to accomodate Doppler antennas.
4) Peak altitude so that Mach number and Reynolds number will lie in regions where the coefficient of drag is known or can be measured.
5) Manufacturing techniques, ejection, skin temperature rise, location of center of gravity, etc.

In the third and most recent trial of the method,the sphere was 4 feet in diameter and weighed 49 pounds. It was made of 16 nylon (nylon cloth impregnated with nylon) gores cut on longitudinal lines and heat sealed together. The inflating air was carried in an internal pressure cylinder made of woven Fiberglas, inside of which was mounted the single-loop Doppler antenna which served for both receiving and transmitting. On one end of the pressurizing cylinder was mounted an inflation valve for releasing the air into the sphere upon ejection; on the other end was mounted the miniaturized Doppler transceiver. Fig. 1 shows the Doppler unit being installed in the inflated sphere. In the rocket the sphere, packed tightly around its inner cylinder, was mounted in a cylinder 15 inches in diameter and 5 feet long surmounted by an ogive nose cone 3 feet long. The latter two comprised the outer forward end of the rocket. The sphere rested in a "sling shot" of stretched rubber bands. At the peak of the trajectory the sling shot was released by a command signal to the Doppler. The sphere, pushing the nose cone out of the way, was ejected from the frontend and, in 20 seconds, inflated. A schematic of the ejection is shown in Fig. 2.

Briefly, the Doppler system DOVAP[4] operates as follows: A continuous wave radio signal of 36.94 megacycles is sent from a ground transmitter to the sphere. The sphere transceiver doubles and retransmits the 73.88-megacycle signal to an array of ground receivers. At the receiver stations the received signal is heterodyned against the transmitter signal which has been received on another receiver directly from the transmitter and doubled. The difference frequency which is proportional to the velocity with which the transmitter-sphere-receiver path length is changing, is sent over wire lines to a central recording station. Each Doppler cycle represents about 13 feet of distance so that by counting cycles as a function of time, one can keep track of the instaneous transmitter-sphere-receiver distances (μ_i). The transmitter-launcher-receiver distances are fixed and known at any given time the sphere lies on the loci of points of the μ_i distances. These loci are ellipsoids of revolution, three of which determine uniquely the position of the sphere in space. In practice more than three stations are used to give a statistical and reliability advantage. Also, by using more than one receiver with differently polarized antennas at each station, much can be learned about the spin of the sphere. This is desirable because it is necessary to correct for false Doppler cycles introduced by the spinning of the antennas in the sphere. Fig. 3 illustrates the Doppler system.

4. Equations Used to Calculate Density and Temperature

The following equation may be used to calculate density for a nonvertical sphere trajectory with unknown winds:

$$\rho = \frac{-2m}{C_D A} \frac{\ddot{x}_3 + |g|}{(\dot{x}_3 - w_3)\sqrt{(\dot{x}_1 - w_3)^2 + (\dot{x}_2 - w_2)^2 + (\dot{x}_3 - w_3)^2}} \qquad (1)$$

where

ρ = density

m = mass of sphere

C_D = coefficient of drag of sphere

A = cross-sectional area of sphere

$\ddot{x}_3$ = vertical acceleration of sphere with respect to launcher

g = acceleration of gravity

$\dot{x}_1, \dot{x}_2, \dot{x}_3$ = east, north, and vertical velocity components of sphere with respect to launcher

w_1, w_2, w_3 = velocity components of wind with respect to launcher In the calculations these are neglected.

The temperature function derived from the hydrostatic equation and equation of state is:

$$T_s = \frac{1}{\rho_s R}\left[\int_0^s \rho g \, ds + P_o\right] = \frac{I_s}{\rho_s R} + \frac{\rho_o}{\rho_s} T_o \qquad (2)$$

where

P_o, ρ_o, T_o = ambient pressure, density, and temperature at the highest point of the trajectory where the drag acceleration may be measured with sufficient precision to calculate density (this will be referred to as the initial point)

P_s, ρ_s, T_s = ambient pressure, density, and temperature at a distance s below the initial point

$$I_s = \int_0^s \rho g \, ds$$

In equation (2), T_o is assumed. Temperatures obtained are valid starting approximately 50,000 feet below the initial point.

5.1 Sources of Error.

The precision of the densities and temperatures measured with this experiment changes with altitude for a given flight and depends on the peak altitude of that flight.

The sources of error in density are: possible winds, which are ignored in the calculations (vertical components have the greatest effect); errors in measurement of velocity and acceleration; error in the drag coefficients used; and errors in the cross-sectional area and mass of the sphere (with care, these last two should be negligibly small).

The sources of error in temperature are those listed above plus the error in T_o and the error in the integral I_s. These last two are appreciable only in the upper range of altitudes.

5.2 Error Due to Ignoring Possible Winds.

If a vertical wind component of 100 ft/sec is ignored, the density error obtained may be as large as 10 percent.

The neglect of a horizontal wind of the same magnitude will produce a maximum error of about 1 percent throughout the useful range of the experiment.

5.3 Effect of Errors in Velocity and Acceleration.

Experience has shown that the Doppler cycle counts per 1/2 second are accurate to within approximately $\pm$ 0.1 cycle (neglecting spin effects). The resulting error in velocity is approximately $\pm$ 1 ft/sec, which is negligible; the error in acceleration is $\pm$ 2 ft/sec^2. When $(\ddot{x}_3 + |g|)$ equals 20 ft/sec^2, we have a $\pm$ 10 percent error in this term.

At the start of the sphere drop $(\ddot{x}_3 + |g|)$ is small and the error relatively large. As the sphere falls $(\ddot{x}_3 + |g|)$ increases and may increase to a maximum of approximately 80-100 ft/sec^2 and then decrease again to 50 ft/sec^2. Thus the error is large at first, decreases gradually to 2 or 3 percent, and then increases to perhaps 5 percent.

The false Doppler cycles introduced by the spin of the sphere are only approximately corrected for, since they enter the data non-uniformly and are corrected for linearly. The "residual" errors due to spin may amount to $\pm$ .3 cycle in a given 1/2-second interval. These residual errors tend to be periodic and tend to stay well within the $\pm$ .3 cycle value. It is necessary to smooth the data to eliminate these periodic fluctuations.

5.4 Drag Coefficient Errors.

Experimental evidence[5] indicates that the sphere drag coefficient C_D is adequately represented as a function of Reynolds number and Mach number in the range of this experiment. Experimental values of C_D for most of the range of Reynolds numbers and Mach numbers required for this experiment were measured at the U. S. Naval Ordnance Laboratory.[6] A summary of the values obtained is shown in Fig. 4. It is estimated that the error of C_D values used will be less than 2 percent within a good deal of the lower altitude range of the experiment, and less than 5 percent in the higher altitude regions.

5.5 Errors in T_o and I_s.

In calculating temperatures a value of T_o (for the initial point of the calculation) is assumed. This value can be estimated to have less than 50 percent error. This term enters into the expression for T_s as a part of the sum $(I_s + \rho_o R T_o)$. As the calculation proceeds, the term I_s gradually becomes the dominant value in the sum. At 50,000 feet below the starting point, for example, I_s is 10 times as large as $\rho_o R T_o$. At this point the error in the sum due to a 50 percent error in T_o will only be 5 percent. From this point downward the error in the sum is determined primarily by the factor I_s. The error in I_s can be shown to be less than the error in ρ_s, the density at the point s.

5.6 Net Error.

All of these errors are assumed to be random in nature; they vary in an unknown fashion throughout the trajectory. When they are combined according to error theory, we obtain a predicted error band, which is a function of altitude.

Typical error bands in temperature for a 400,000-foot peak and a 244,000-foot peak are shown in Fig. 5. The latter trajectory corresponds to the sphere experiment of SC-23. Temperatures calculated for SC-23 are seen to lie within the predicted error band.

Two predicted error bands are shown for the case of a 244,000-foot peak. The narrow band is based on the assumption that no vertical winds exist. The wider band indicates the error if a 100 ft/sec vertical wind exists at all points of the trajectory.

6. Results

Three flights carrying the sphere experiment were carried out. All were partially successful, but none was completely successful. In the first two the spheres leaked. In the third the sphere was good, but the central recording at camp failed for all but two of the receiver stations during the most interesting portion of the trajectory.

Aerobee SC-23 was flown on May 14, 1952, at 18:16 MST. The sphere was ejected from the rocket at 244,000 feet above sea level, almost exactly the peak of the trajectory. The sphere's internal pressure record indicated that the sphere had a leak; however, the internal pressure was high enough to insure inflation during the initial portion of the fall. Excellent tracking telescope films obtained by the personnel of the Flight Determination Laboratory at WSPG showed erratic motion of the sphere beginning at approximately 165,000 feet above sea level. Presumably, the sphere began to collapse at this point. Densities and temperatures have been calculated for the portion of the trajectory between 215,000 to 165,000 feet above sea level.

Curves of the components of velocity, the resultant velocity, and $\ddot{x}_3$ are shown in Figs. 6 and 7. The maximum value of V through the range of data reduced was 1880 ft/sec. The maximum value of $(\ddot{x}_3 + |g|)$ was approximately 36 ft/sec^2. The densities and temperatures calculated from this data are shown in Figs. 8 and 9. A parabola fitted to the temperature data by the method of least squares is also shown in Fig. 9. The UARRP average density and temperature curves(7) are shown for reference.

Aerobee SC-29 was flown on December 11, 1952, at 16:47 MST. The sphere was ejected at approximately 341,000 feet above sea level, slightly before the peak of the trajectory--344,800 feet above sea level. This sphere also had a leak. The leak rate was about the same as that of the SC-23 sphere. Because of heavy cloud coverage and the high altitude, tracking telescope films were not available for this flight. The data has not been completely evaluated as of this date.

Aerobee SC-30 was flown on April 23, 1953, at 12:32 MST. The trajectory of this flight has not been computed as yet; however, a rough calculation indicates that the peak was approximately 395,000 feet above sea level with ejection of the sphere about 10,000 feet below peak on the upward portion of the flight. Pressure records indicate a good inflation of the sphere. DOVAP data on this flight are available up through peak and back down again to approximately 250,000 feet. At this point data from all except two stations were lost because of a power failure at the recorder. Power was restored when the sphere was at approximately 100,000 feet above sea level. An attempt will be made to reduce the data available on this flight.

7. Conclusion

On the basis of the initial partially successful flight, it is concluded that the method works as predicted. Data from an operationally successful flight are needed to fully evaluate the method.

A lighter sphere will extend the altitude range of the experiment. To this end and to provide more data for evaluating the method, the plan at present is to drop a 4-foot, 20-pound sphere at White Sands Proving Ground.

Several variations of the sphere experiment have been considered. An adaptation to the Deacon rocket-balloon combination for a latitude survey has been considered. The trajectory of a large, light-weight sphere for tracking winds was predicted. It appears that winds might be measured up to 250,000 feet. The use of two spheres or an up-and-down trajectory for one sphere would give some indication of the vertical wind error.

References

1. Reports of SCEL Upper Air Research Project, Engineering Research Institute, University of Michigan, Unpublished.

2. Taylor, G. I. and Maccoll, J. W., Proc. Roy. Soc., A139 (1933), pp. 278-311; Maccoll, J. W., Proc. Roy. Soc., A159 (1937), pp. 459-472.

3. Stone, A. H., "On Supersonic Flow Past a Slightly Yawing Cone," J. Math. Phys, 27 No. 1 (1948).

4. Hoffleit, D., "Dovap - A Method for Surveying High-Altitude Trajectories," Scientific Monthly, Vol. 68 (Mar. 1949), p. 172.

5. Charters, A. C., and Thomas, R. N., "The Aerodynamic Performance of Small Spheres from Subsonic to High Supersonic Velocities," J. Aero. Sci., Vol. 12 No. 4 (Oct. 1945), p. 468.

   University of California Technical Reports on Sphere Drag Tests:
   No. HE-150-65, by E. D. Kane.
   No. HE-150-71, by F. S. Sherman and E. D. Kane.
   No. HE-150-92, by N. A. Jensen.

6. Navord Report No. 2352, by May, A. and Witt, W. R., Jr., Free Flight Determinations of the Drag Coefficient of Spheres, U. S. Naval Ordnance Laboratory, White Oak, Maryland, Aug. 1952, Unclassified.

7. The Rocket Panel, "Pressures, Densities and Temperatures in the Upper Atmosphere," Phys. Rev., Vol. 88 (Dec. 1952), pp. 1027-1032.

Fig. 1

Installation of Doppler Receiver-Transmitter in Inflated Sphere

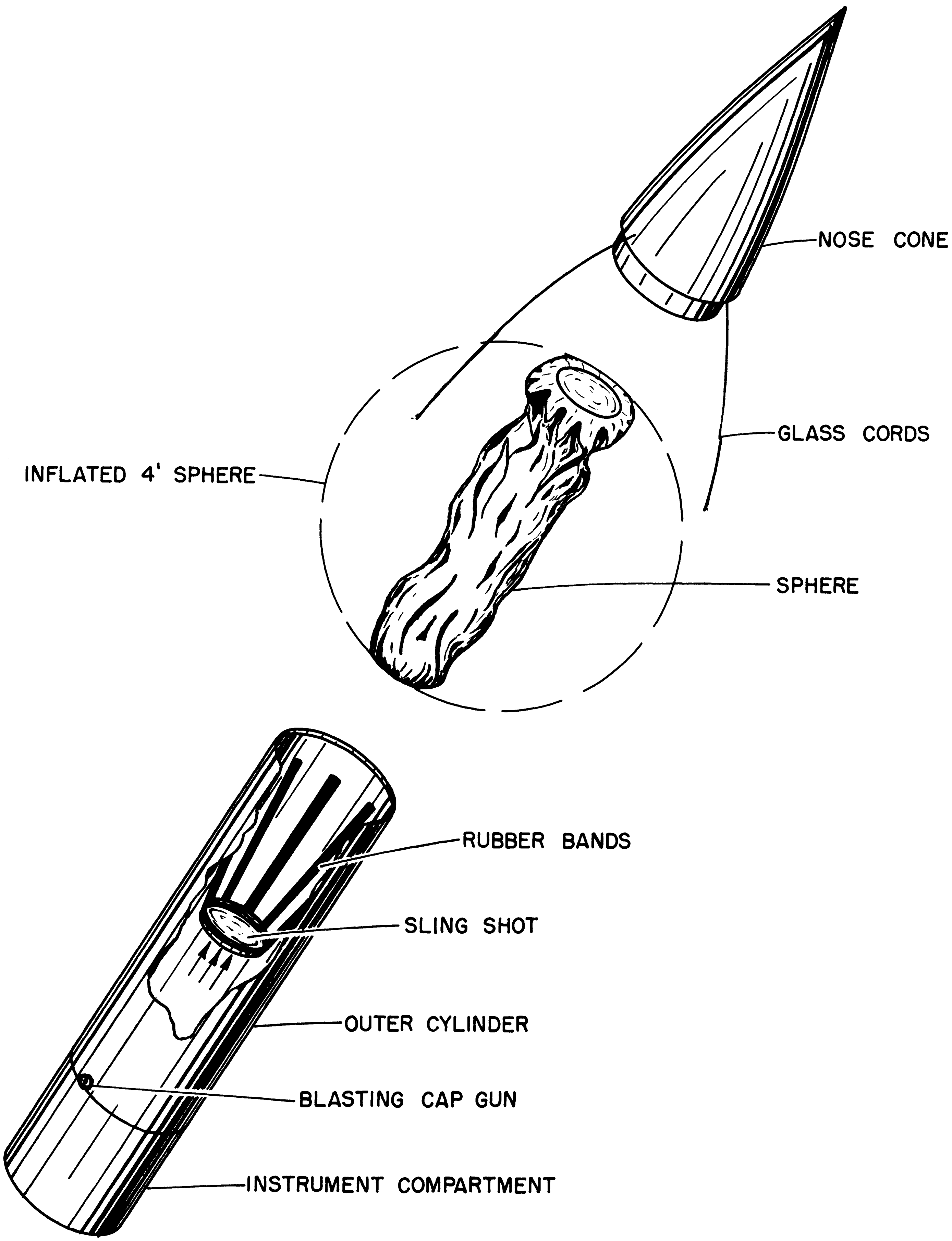


Fig. 2

Sphere Ejection System

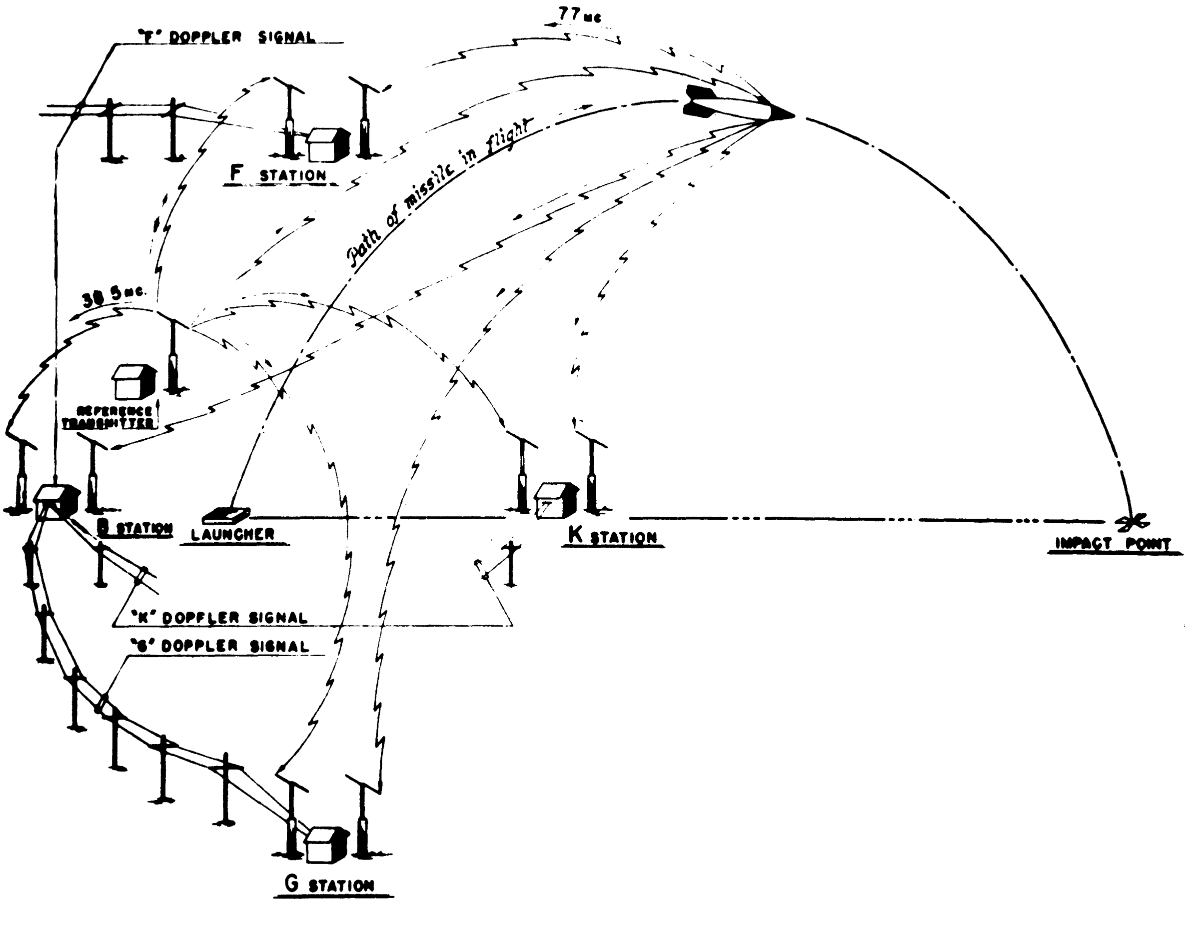


Fig. 3

DOVAP (Doppler) System
(Diagram Taken from Reference 4)

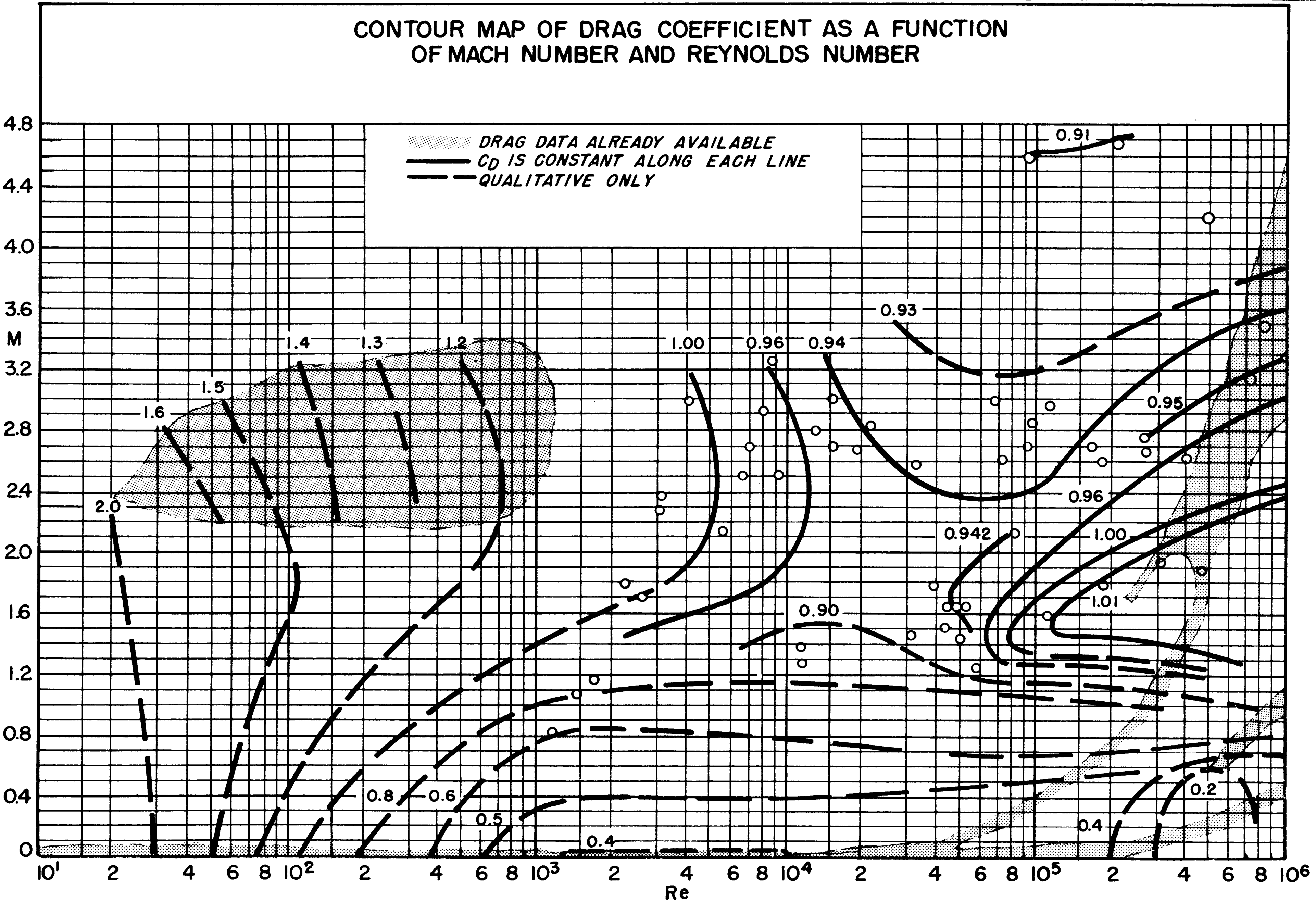
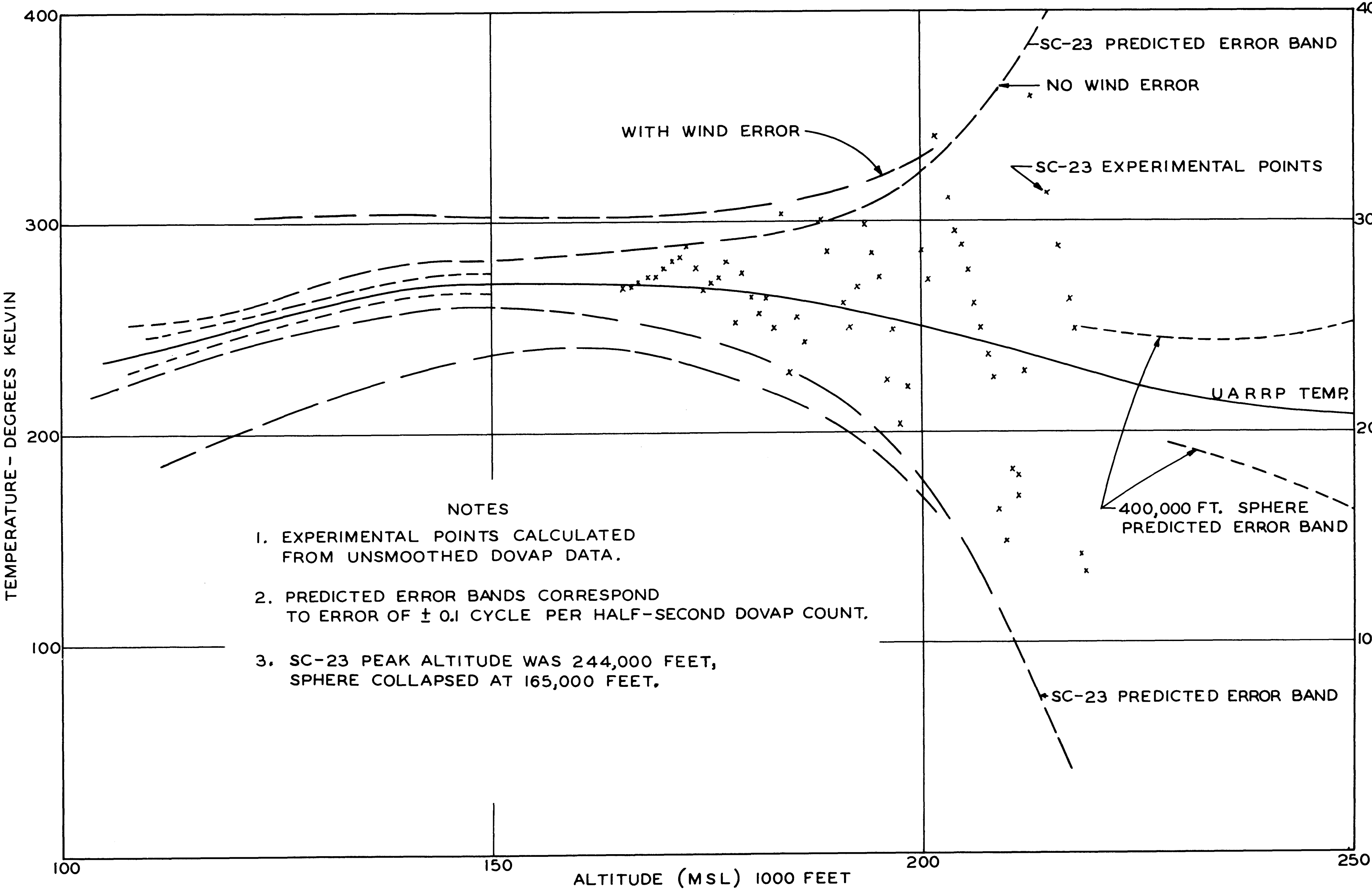
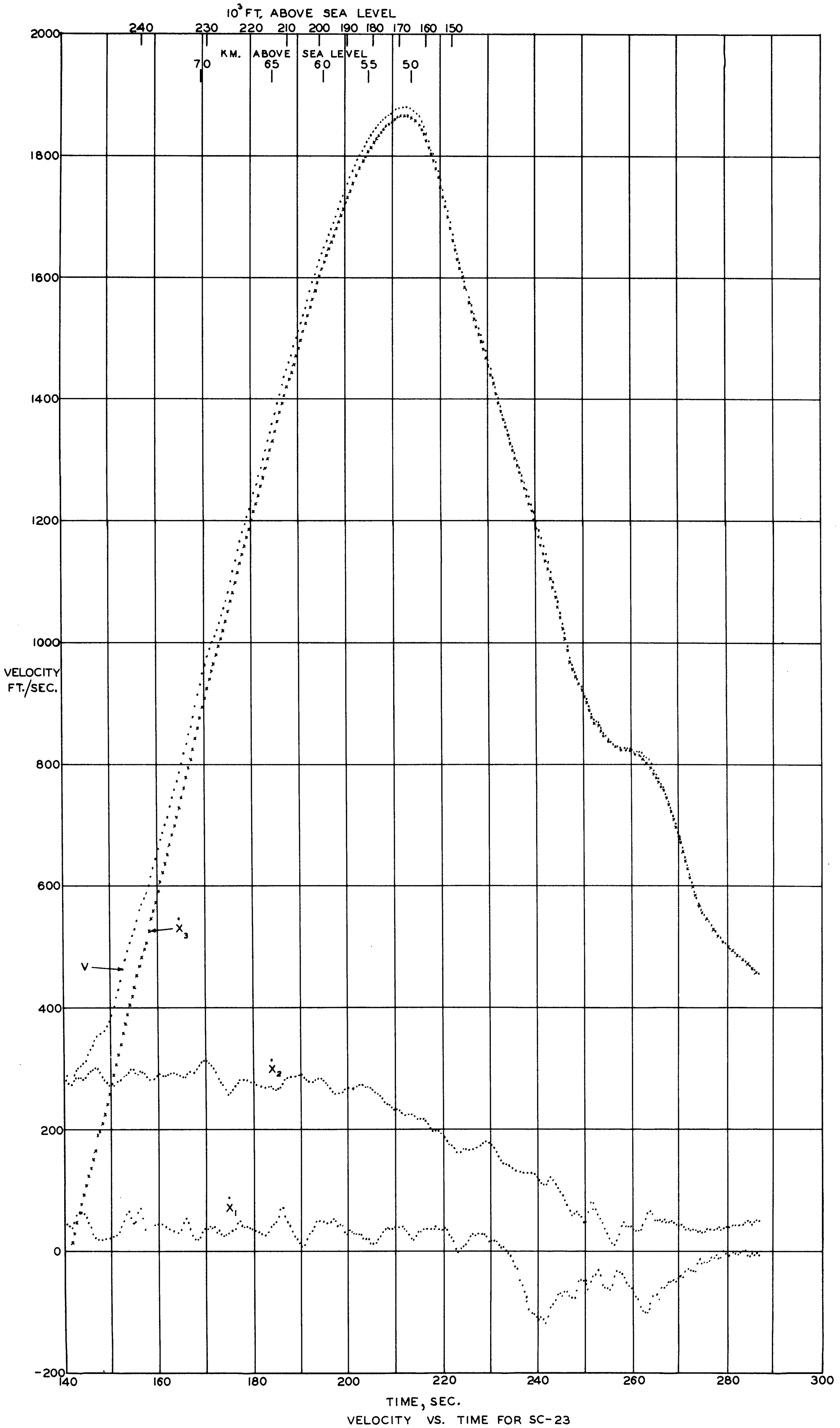

CONTOUR MAP OF DRAG COEFFICIENT AS A FUNCTION OF MACH NUMBER AND REYNOLDS NUMBER
DRAG DATA ALREADY AVAILABLE
C_D IS CONSTANT ALONG EACH LINE
QUALITATIVE ONLY
M
Re
4.8
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
10¹ 2 4 6 8 10² 2 4 6 8 10³ 2 4 6 8 10⁴ 2 4 6 8 10⁵ 2 4 6 8 10⁶
2.0
1.6
1.5
1.4
1.3
1.2
1.00
0.96
0.94
0.93
0.91
0.95
0.96
0.942
1.00
1.01
0.90
0.8
0.6
0.5
0.4
0.4
0.2

Fig. 4



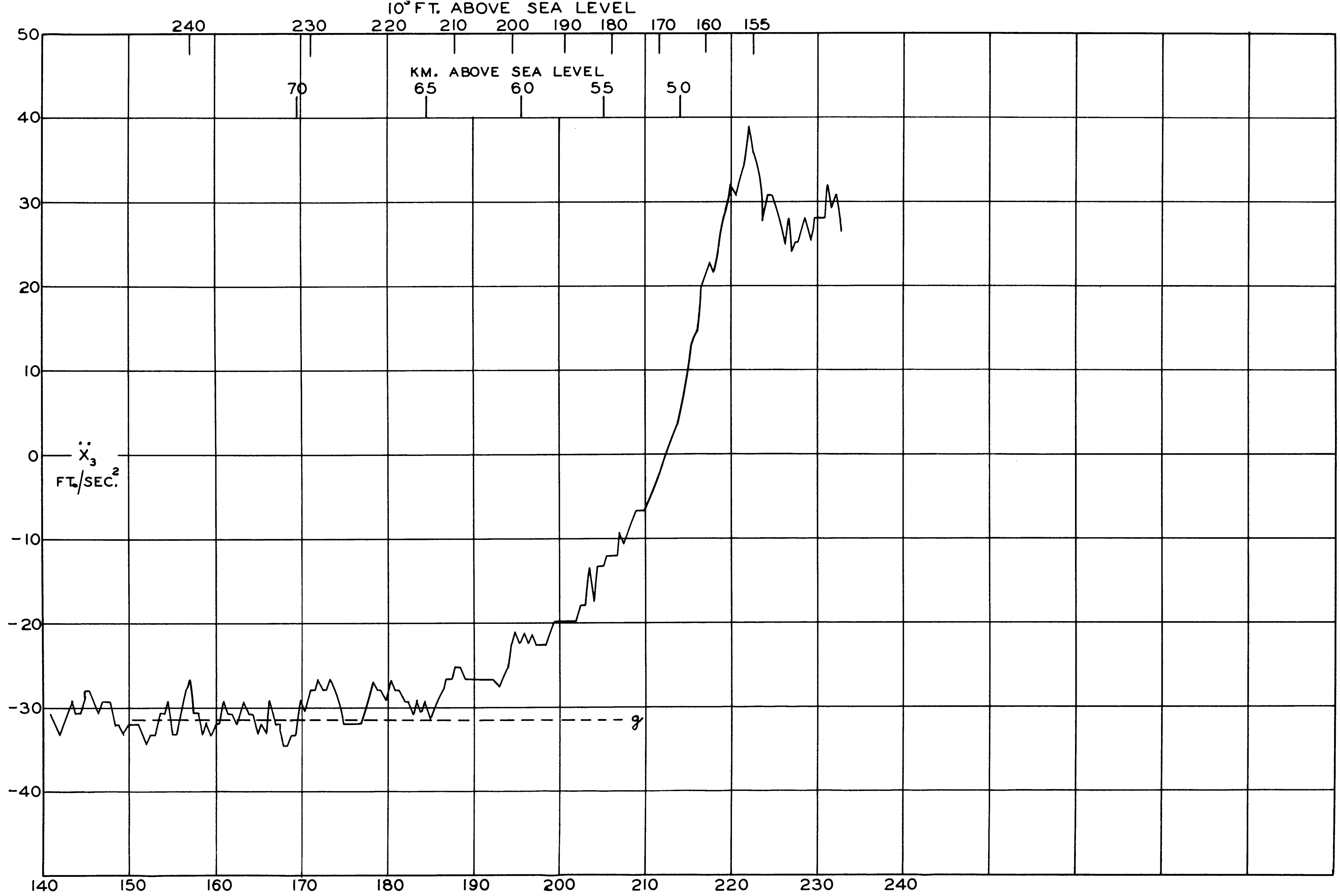
PREDICTED TEMPERATURE ERROR BANDS AND RESULTS OF SC-23

FIGURE - 5



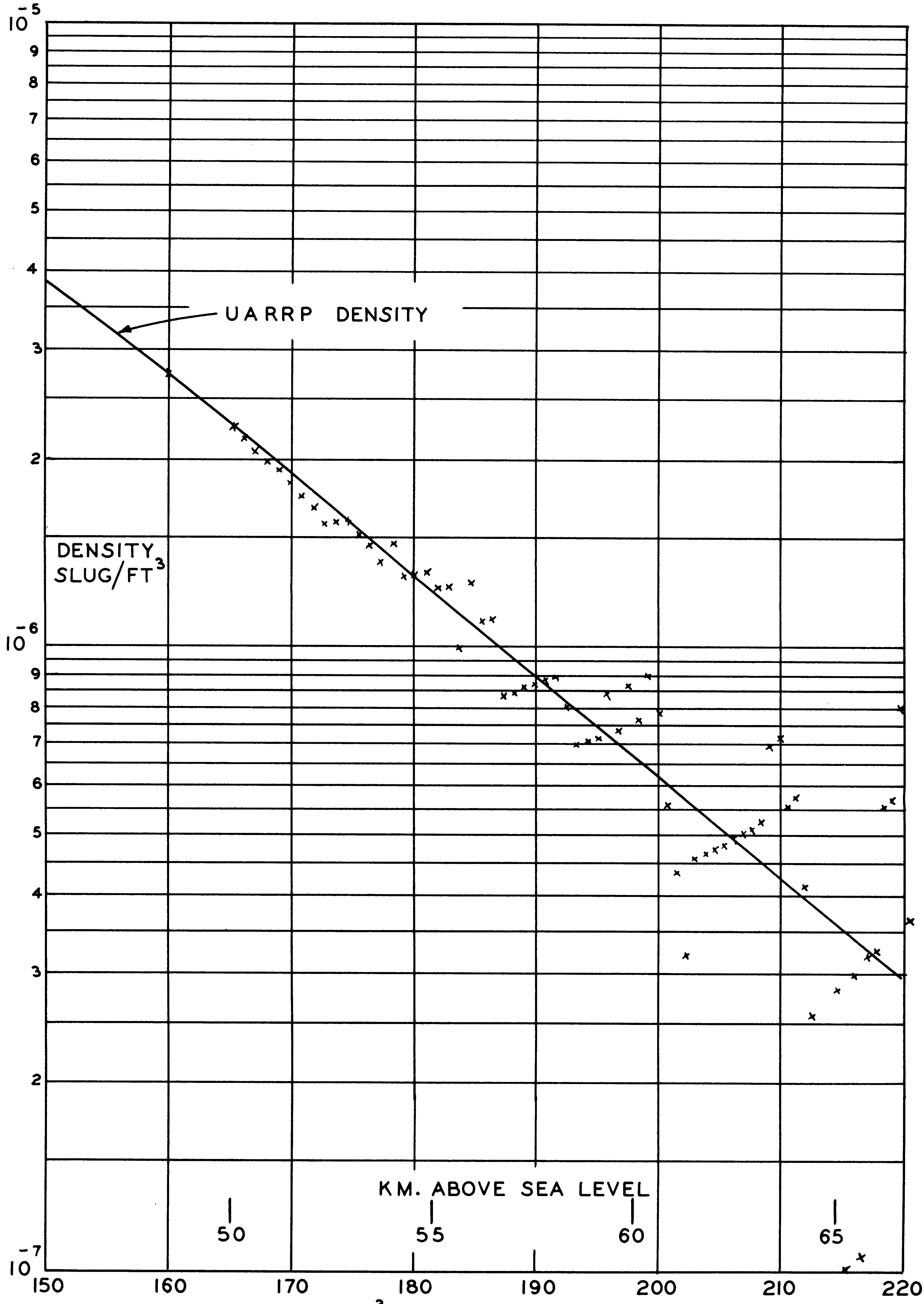
VELOCITY VS. TIME FOR SC-23

FIGURE - 6



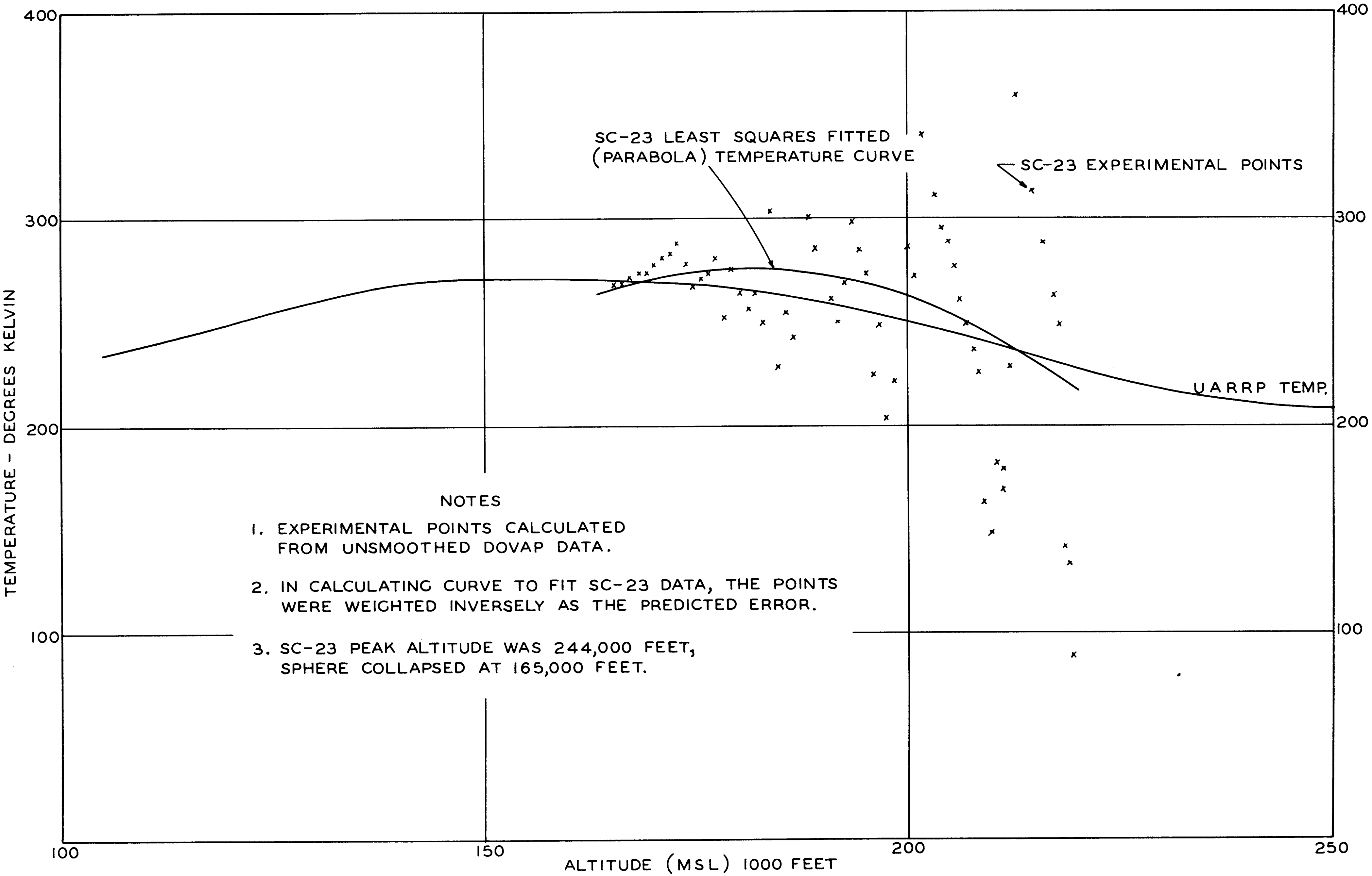
$\ddot{X}_3$ VS. TIME FOR SC-23

FIGURE - 7



DENSITY VS. ALTITUDE FROM SC-23

FIGURE-8



TEMPERATURE VS. ALTITUDE FROM SC-23

FIGURE - 9