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Experimental Evaluation of an Encoder Trailer for
Dead-reckoning in Tracked Mobile Robots

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Abstract

This paper describes a device developed for improved position and orientation estimation of tracked mobile robots. This device, called Encoder Trailer, consists of two incremental encoders, one absolute encoder, two "knife-edge" wheels, and a slip ring. It is a unique design in the sense that it can minimize the systematic errors and eliminate the most important nonsystematic errors such as uneven floors and unexpected obstacles. Both theoretical analysis and experimental results demonstrated that this encoder trailer has excellent robust property with respect to uneven floors and unexpected obstacles.
1. Introduction

Unmanned Ground Vehicles (UGVs) have received considerable attention recently because they can perform high risk missions in hazardous environments. In military applications, UGVs can be used for high risk missions, such as tactical and chemical reconnaissance and mine clearing. Other potential applications includes random patrols, responding to and assessing security and safety alarm actions, detecting and deterring intruders. So far, most of the UGV are wheeled vehicles. However, wheeled vehicles can not travel on rugged terrain or soft ground, climb stairs or travel over an obstacle. In contrast, tracked vehicles can perform all these tasks. We are interest in autonomous tracked vehicles that are capable of intelligent motion and action without requiring guide wires or teleoperator control. Even though autonomous wheeled robots have been around for quite a few years, tracked robots can only be tele-operated because it is much easier to automatically control wheeled robots than tracked robots. For a wheeled robot, we can easily predict its trajectory given driving wheel velocity due to the simplicity of its kinematics. However, the kinematics of a tracked robot is very complex due to the complex interaction between tracks and grounds. There are infinite contact points and infinite closed-link chains which make slippage inevitable. All those factors make the modeling of tracked robots a very difficult problem.

Currently, much research is concentrated on idealized mathematical models of military tank to have a general understanding of the interrelationship between the terrain factors (such as soil type, soil shear strength, and compressibility) and the vehicle characteristics (weight, track length and width, location of center of gravity, etc.) during steering [Wong, 1989]. To them, performance and stability are the primary task. In contrast, our Andros Mark V by REMOTEC is primarily used for non-military hazardous environments, many of which are indoor. To move the Andros along a specified trajectory, we need accurate knowledge of its position. Dead reckoning with a shaft encoder accumulates errors with time such that the estimated position of the vehicle becomes useless. In order to correct for those cumulative errors, we must use some kind of references. GPS is a good choice for outdoor vehicles. However, we can not use it for our tracked vehicle which will be used primarily indoor. Transmitters or Bar codes should work, however, they require costly installation and maintenance. For tracked vehicles, slippage is inevitable, so that regular shaft encoder used in wheeled robots are of little use. To overcome this problem, we have developed an encoder trailer for tracked mobile
robots that can provide us accurate position information in spite of floor roughness and obstacles.

This paper is organized as follows: the University of Michigan's tracked vehicle is described in Section 2. The Encoder Trailer is described in Section 3. Errors analysis is discussed in Section 4. The robust property of the Encoder Trailer is proven in Section 5. Experimental validation is described in Section 6. Conclusions are given in Section 7.

2. The University of Michigan's Tracked Vehicle

The Remotec Andros is a uniquely designed tracked vehicle for tele-operation on different terrain (Figure 2.1). The platform consists of a main chassis with two main tracks, a front auxiliary track, and a rear auxiliary track. The vehicle is 724 mm wide, 1092 mm high and 1574 mm long. The weight of the vehicle is 319 kg. The two auxiliary tracks can be raised or lowered individually to allow the vehicle to climb over large obstacles and move up and down stairs. It is under tele-operator control and the operator provides visual feedback to control the vehicle. Therefore the lack of dead-reckoning information is not a problem. However, this information is essential for the fully autonomous tracked vehicle.

![Figure 2.1 Remotec Andros](image)

In recent years there has been growing interest, especially in the DOE Robotics community, in converting tracked vehicles to fully autonomous operation. In order to
have accurate velocity control, two incremental encoders are mounted on the left and right main motors of the vehicle. For autonomous operation, the original onboard computer and motor controllers (for tele-operation by a human operator) were replaced by a 486-66 MHz PC-compatible single board computer and our own controllers which are based on HCTL-1100 motor controller chips. The new controller communicates with the host computer through parallel communication via a DG96 digital I/O board [Fan, et. al., 1994].

To assess the extend of dead-reckoning errors based on the two main track encoders, we performed University of Michigan Benchmark test (UMBmark) [Borenstein and Feng, 1994]. However, the nominal test-procedure had to be modified because of the dead-reckoning errors: The vehicle could not complete a square path. Typically, we encountered errors of 10 to 12 meters for a square path of nominally 16 meter total length, as shown in Figure 2.2. The orientation error was typically on the order of 120 – 130°! Therefore, the dead-reckoning information based on the main track encoders is useless due to the significant slip, which is inevitable because there are infinite contact points for the tracked vehicle. In order to have reliable position information, The University of Michigan has developed a unique attachment for the tracked vehicle (or any robot with poor dead-reckoning), called the "Encoder Trailer" (ET).

![Figure 2.2 Return position errors for the Remotec Andros with 4 × 4m bi-directional square path](image)
3. The Encoder Trailer

The Encoder Trailer (ET) is a small two-wheel trailer that drags behind the main vehicle, as shown in Figure 3.1. Mounted on each wheel is an optical incremental encoder. A rotary joint allows the trailer to rotate horizontally around a fixed point behind the pulling vehicle. The trailer can rotate around this joint without limitation due to a slip ring that provides the electrical connections to the wheel encoders. When the Andros travels backward, the ET is designed to swivel and lead due to a hinge which connects the absolute encoder with the vehicle. The ET is also designed to be raised off the floor when the tracked vehicle drives over the obstacles, although this function is not yet implemented on our system.

Figure 3.1 Remotec Andros with Encoder Trailer attached
The Encoder Trailer consists of two measurement wheels, two incremental encoders, one slip ring, one absolute encoder and connecting components as shown in Figures 3.2. The rim is made of aluminum, with an O-ring as a traction-providing tire. The incremental encoders have a resolution of 1200 pulses per revolution, and the absolute encoder has a resolution of 1024 pulses per revolution. Figure 3.3 is a schematic diagram of the encoder trailer.

Figure 3.2 Measurement wheel design

Figure 3.3 Location and velocity from Encoder Trailer
Using the same dead-reckoning algorithm as is used for differential-drive robots, one can easily compute the position and orientation of the center-point of the trailer. Combining this information with the angle measured by the absolute encoder and with the joint length, the position and orientation of the main vehicle can be computed.

Figure 3.3 defines the position and orientation of the encoder trailer \(x_k, y_k, \text{ and } \theta_k\) at time \(t_k\). \(X_k, Y_k, \text{ and } \beta_k\) are the position and orientation of the center of gravity for tracked vehicles. \(\phi_k\) is the orientation of the tracked vehicle with respect to the encoder trailer measured by the absolute shaft encoder. \(\Delta S_k\) is the distance traveled by the encoder trailer at the \(k\)th step. \(\Delta \theta_k\) is the change of orientation of the encoder trailer at the \(k\)th step. From Figure 3.3, we derive:

\[
x_k = x_{k-1} + \Delta S_k \cos(\theta_{k-1} + \Delta \theta_k / 2) \tag{3.1}
\]

\[
y_k = y_{k-1} + \Delta S_k \sin(\theta_{k-1} + \Delta \theta_k / 2) \tag{3.2}
\]

\[
\theta_k = \theta_{k-1} + \Delta \theta_k \tag{3.3}
\]

\[
\Delta S_k = \frac{N_R D_R + N_L D_L}{2} \tag{3.4}
\]

\[
\Delta \theta_k = \frac{N_R D_R - N_L D_L}{b} \tag{3.5}
\]

Where \(N_R\) and \(N_L\) are the right and left encoder reading respectively. \(D_R\) and \(D_L\) are the right and left wheel diameter.

The velocity of the encoder trailer can be expressed as

\[
\dot{x}_k = \frac{v_L + v_R}{2} \cos \theta_k \tag{3.6}
\]

\[
\dot{y}_k = \frac{v_L + v_R}{2} \sin \theta_k \tag{3.7}
\]

\[
\dot{\theta}_k = \frac{v_R - v_L}{b} \tag{3.8}
\]

Where \(v_R\) and \(v_L\) are the speed of the right and left measurement wheel at time \(t_k\). Correspondingly, we can express the position and orientation of the tracked vehicles as follows:

\[
X_k = x_k + l_1 \cos \theta_k + l_2 \cos \beta_k \tag{3.9}
\]
\[ Y_k = y_k + l_1 \sin \theta_k + l_2 \sin \beta_k \] (3.10)
\[ \beta_k = \theta_k + \phi_k \] (3.11)

Differentiating the above equations yields the velocity of the tracked vehicle:

\[ \dot{X}_k = \dot{x}_k - l_1 \dot{\theta}_k \sin \theta_k - l_2 \dot{\beta}_k \sin \beta_k \] (3.12)
\[ \dot{Y}_k = \dot{y}_k + l_1 \dot{\theta}_k \cos \theta_k + l_2 \dot{\beta}_k \cos \beta_k \] (3.13)
\[ \dot{\beta}_k = \dot{\theta}_k + \dot{\phi}_k \] (3.14)

Therefore, the robot's position is uniquely determined from the reading of the left and right incremental encoders, and of the absolute encoder. The rational for using a device like the ET, despite its obvious disadvantage, is the fact that many indoor applications might require a tracked vehicle because of occasional obstacles, but would offer relatively smooth concrete floors during most of the robot's travel. For autonomous operation, continuous absolute position updates are required. With conventional techniques, this would require the costly installation and maintenance of beacon systems in which several of the beacons must be seen at all times. We believe that the installation cost for such a system could be dramatically lower, if dead-reckoning information was available most of the time. The encoder trailer is economical, effective, and computationally cheap. In addition, it can provide fast position updates without modification of the environment. We will show in the following section that the Encoder Trailer can minimize the systematic errors due to unequal wheel diameters and uncertainty about the effective wheelbase.

4. Errors Analysis

According to Eqs. (3.1)-(3.5), the position and orientation of the center of the trailer can be easily computed based on the two incremental encoders, which are mounted on the trailer wheels. This computation is called dead-reckoning. However, dead-reckoning is based on the assumption that wheel revolutions can be translated into linear displacement relative to the floor. This assumption is only of limited validity. For example, a wheel might rotate without any linear displacement, a condition known as total slippage. In this case, the wheel encoder will register wheel revolutions and calculate the predicted linear displacement even though there is no actual linear displacement at all. Under normal conditions, dead-reckoning errors may fit into one of two categories: (1) systematic errors and (2) non-systematic errors [Borenstein, et. al., 1994].
1. Systematic errors
   a. Two wheel diameters are different
   b. Wheelbase is uncertain
   c. Wheels are misaligned
   d. The average of two wheel diameters is different from the nominal value
   e. Encoder resolution is finite
   f. Encoder sampling rate is finite

2. Non-systematic errors
   a. Travel over uneven floors
   b. Travel over unexpected objects on the floor
   c. Slip due to
      • slippery floors
      • over-acceleration
      • fast turning (skidding)
      • non-point wheel contact with the floor

4.1 Systematic Dead-reckoning Errors

Systematic errors are usually caused by imperfections in the design and mechanical implementation of a mobile robot. The two most notorious systematic error sources are unequal wheel diameters and the uncertainty about the effective wheelbase [Borenstein, et. al., 1994].

a) Unequal wheel diameters: Most mobile robots use rubber tires to improve traction. These tires are difficult to manufacture to exactly the same diameter. Furthermore, rubber tires compress differently under asymmetric load distribution. Either one of these effects can cause substantial dead-reckoning errors.

b) Uncertainty about the wheelbase: The wheelbase is defined as the distance between the contact points of the two drive wheels of a differential-drive robot and the floor. The wheelbase must be known in order to compute the amount of rotation of the vehicle. Uncertainty in the effective wheelbase is caused by the fact that rubber tires contact the floor not in one point, but rather in a contact area. The resulting uncertainty about the wheelbase can be on the order of 1% in some commercially available robots.
Systematic dead-reckoning errors can be reduced by adaptive calibration [Fan, et. al., 1994] or UMBark [Borenstein and Feng, 1994]. In this paper, the huge systematic errors of the tracked vehicle are reduced by the special design of Encoder Trailer. The wheels of the encoder trailer are "knife-edge" thin and not compressible because they are made of aluminum with a thin layer of rubber as a tire. Two wheels are easily manufactured to exactly the same diameter. Also, since the measurement wheels carry only a small well balanced load, asymmetric load distribution on the robot has no effect on measurement accuracy. Therefore, the problem of unequal wheel diameters is minimized. Furthermore, O-ring "tire" contacts the floor only one point, the uncertainty about the effective wheelbase is minimized, too. In addition, misalignment of wheels is also minimized by careful design and assembly of the trailer. Therefore, all important systematic errors sources are minimized except minor sources such as limited incremental encoder resolution, which is 1200 pulse/revolution, and limited encoder sampling rate, which is 16 ms/sample in our low level controller. Both of them are good enough in typical applications.

Systematic errors are particularly grave because they accumulate constantly. On most smooth indoor floor systematic errors are dominant. However, on rough floors with significant irregularities, non-systematic errors are dominant (Borenstein, et. al., 1994) One particularly grave problem with non-systematic errors is that they may appear unexpectedly (for example, when the robot traverses an unexpected objects such as cables), and they cause an orientation error, which, in turn, can cause unbounded position errors. Such disturbances can decrease the stability of the vehicle's trajectory controller.

4.2 Non-systematic Dead-reckoning Errors

Non-systematic dead-reckoning errors are those errors that are caused by the interaction of the robot with unpredictable features of the environment. For example, irregularities of the floor surface, such as bumps, cracks, or debris, will cause a wheel to rotate more than predicted, because the affected wheel travels up or down the irregularity, in addition to the-expected-horizontal amount of travel. Non-systematic errors are a great problem for actual applications, because it is impossible to predict an upper bound for the dead-reckoning error [Borenstein, 1995].

The typical dead-reckoning is very sensitive to typical floor irregularities such as cracks or bumps, because such disturbances cause orientation errors which induce
unbounded lateral errors. To overcome this problem, we have introduced a method called internal position error correction (IPEC) [Borenstein, 1995]. With this approach two mobile robots mutually correct their dead-reckoning errors. However, it requires that both robots can measure their relative distance and bearing continuously and accurately. Such redundancy may only have limited application in practical application. To overcome this problem, we introduced the ET, which is accurate and robust with respect to the typical floor irregularities.

4.3 The Most Significant Motion Error: the Orientation Error

Motion errors can be decomposed into orientation errors, contour errors, and tracking errors, as shown in Figure 4.1. The vehicle is given instructions to pass through point A, but it is actually at point B. The orientation error $E_o$ is defined as the angular difference between the actual orientation and the desired orientation. The contour error $E_c$ is the distance between the actual position B and the desired trajectory in the direction perpendicular to the direction of travel. The tracking error $E_t$ is the distance between the actual position and the desired position in the direction of travel. In mobile robot motion control, the orientation errors are the dominant errors since they cause unbounded growth of the contour errors. The tracking errors are usually of less concern (Fan, et. al., 1994, Feng, et. al., 1993).

Figure 4.1 Motion Error Diagram

As shown in the next section, an essential property of the Encoder Trailer is that it is robust with respect to the typical irregularities and unexpected obstacles. Due to this property, the vehicle orientation does not change after ET travels over bumps.

5. The Robust Property of the Encoder Trailer
Due to the design of the Encoder Trailer, it is shown to be robust with respect to typical floor irregularities and unexpected obstacles in the following propositions:

**Proposition 1.** The robot's orientation remains the same if one wheel of the encoder trailer travels over a bump.

Proof. To prove Proposition 1, it is sufficient to show that the robot's orientation remains the same while one wheel of the encoder trailer travels over a bump during the straight line motion. For straight line motion, the angular velocities are approximately the same for the left and right wheels of the encoder trailer while one wheel travels over a bump. Therefore, the total distance traveled by two wheels are approximately the same. However, the horizontal distance traveled by the left wheel (traveling over a bump) is ΔL less than that of the right wheel. This difference causes an orientation change of γ (positive in this example). As shown in Figure 5.1, The initial heading of the robot is 90 degrees or β = 90°. As AB̂LED and ĈD̂LAC, φ equals to γ in magnitude. As γ is defined to be positive if the rotation from AB to AC is counter-clockwise and φ is defined to be positive if the rotation from CD to DE is counter-clockwise, we can conclude:

\[ \phi = -\gamma \]  

(5-1)

From Figure 5.1, we can see that:

\[ \theta = 90^\circ + \gamma \]  

(5-2)

\[ \beta = \theta + \phi \]  

(5-3)

Substitute Eqs. (5-1) and (5-2) into Equ. (5-3) to get:

\[ \beta = 90^\circ + \gamma - \gamma = 90^\circ \]  

(5-4)
Figure 5.1 The change of orientation due to traversing a bump

Hence Proposition 1 is proved.

**Proposition 2.** The robot's orientation remains the same if both wheels of the encoder trailer travel over the same kind of bumps.

Proof. Proposition 2 is obvious.

**Proposition 3.** The robot's orientation remains the same if both wheels of the encoder trailer travel over two different bumps.

Proof. To prove Proposition 3, it is sufficient to show that robot's orientation remains the same during the straight line motion. If the bump traversed by left trailer wheel is larger than that of right wheel, the horizontal distance traveled by the left wheel is less than that of right wheel. This difference causes an orientation change of \( \gamma \) (positive in this case). Similarly, if the bump traversed by left trailer wheel is smaller than that of right wheel, there is a negative orientation change of \( \gamma \). Proposition 3 can be proved similarly as in the proof of Proposition 1.

According to those propositions, the dead-reckoning accuracy of the ET is not affected by the uneven floors or unexpected objects on the floor. Therefore, all non-systematic errors are compensated except wheel-slippage which has minor effect for the typical application, as shown in the experiments.
If the floor has irregularities or expected obstacles, which is the case in reality, the trailer's accuracy is not affected. However, the regular dead-reckoning encoder is much worse (orders of magnitude worse, as reported in Borenstein, 1995). This robustness with respect the typical irregularity is the essential feature of the Encoder Trailer.

6. Experimental Validation

Experiments are conducted to assess the dead-reckoning accuracy of the ET. The vehicle is driven to follow the $4 \times 4$ m path and compared its computed (i.e., using dead-reckoning) position with the vehicle's actually measured stopping position. The actual starting and stopping position is measured by 3 sonars installed on the vehicle and a corner bracket. The initial position of the ET is calculated based the measured starting position of the vehicle by 3 sonars. The experiments consist of the nominal $4 \times 4$ m bi-directional square path. The Figure 6.1 and 6.2 show position errors for the tracked vehicle with Basic Encoder Trailer, where CG represents the mean value of the 5 sets of data. In Figure 6.1, the concrete floor is smooth without large bumps. However, in Figure 6.2, 10 bumps, each 8-9 mm high, were introduced under the inside wheel of the ET. Compared Figure 6.1 with Figure 6.2, we can see that 10 bumps has no effects on the position errors of the ET.

![Figure 6.1 Basic Encoder Trailer without bumps under one encoder wheel](image)

Figure 6.1 Basic Encoder Trailer without bumps under one encoder wheel
Figure 6.2 Basic Encoder Trailer with 10 bumps under one encoder wheel

Each bump may cause an orientation error of 0.6 degree for the dead-reckoning of the typical mobile robot [Borenstein 1994], resulting in a 6 degree total orientation errors for 10 bumps. However, according to Table 1, the average orientation errors for the Encoder Trailer is approximately the same whether there are 10 bumps or not. From Table 2, we can observe that the average position errors for ET is approximately the same with or without bumps. This robust property with respect to the typical uneven floors and expected objects is the most essential feature of the ET.

<table>
<thead>
<tr>
<th>Average Orientation Errors</th>
<th>Without Bumps</th>
<th>10 Bumps</th>
</tr>
</thead>
<tbody>
<tr>
<td>CW</td>
<td>-0.32 deg.</td>
<td>0.57 deg.</td>
</tr>
<tr>
<td>CCW</td>
<td>0.076 deg.</td>
<td>-0.71</td>
</tr>
</tbody>
</table>

Table 1. The Average Orientation Errors for the ET

<table>
<thead>
<tr>
<th>Mean Position Errors</th>
<th>X (no bumps)</th>
<th>Y(no bumps)</th>
<th>X(10 bumps)</th>
<th>Y (10 bumps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCW</td>
<td>-9.27 cm</td>
<td>3.7 cm</td>
<td>-15.2 cm</td>
<td>11.8 cm</td>
</tr>
<tr>
<td>CW</td>
<td>-1.2 cm</td>
<td>-4.6 cm</td>
<td>-0.3 cm</td>
<td>-0.1 cm</td>
</tr>
</tbody>
</table>

Table 2. The Average Position Errors for the ET
According to Table 2, the average position errors are relatively small. However, there is a large spread of errors about its center of gravity for both CW and CCW experiments, as shown in Figure 6.1 and Figure 6.2. These may be larger than what could be explained as caused by the irregularities of the floor. For the regular mobile robot systems, the spread of errors may be due to the irregularities. For the encoder trailer, this is not the case for it is robust to the irregularities. This phenomena are due to slip of encoder trailer and low cost slip-ring.

During turning motion, especially 90° turns on the spot, the encoder trailer experiences much larger angular acceleration than the regular mobile robot systems (the main track body in our example), for the trailer has smaller moment of inertia and wheelbase than the regular mobile robot systems does. Experiments demonstrated that a minor change in the orientation of CG of main tracks caused a larger change in the orientation of the encoder trailer. We can also observe small oscillation of the ET during the turning. During the experiments, the tracked vehicle is controlled to follow a 4x4 m path via joystick. There is a fair amount of noise in the transmitting signals, which may cause sudden change in the vehicle's orientation and larger oscillation of the trailer's orientation during consecutive sampling times. Therefore, we can see that non-systematic errors due to the slippage of wheel is larger than that of regular mobile robot systems, even though the non-systematic errors due to uneven floors and unexpected objects are approximately eliminated due to the robust property of the encoder trailer. The electrical noise from the low-cost slip-ring was observed to cause erratic encoder readings sometimes. We observed that there is a sudden orientation changes up to 80 degree if two slip ring wires (under two adjacent slip ring grooves) are in contact accidentally due to the wear of the slip ring.

7. Conclusion

A basic encoder trailer is developed and discussed in detail in this paper. Its unique design minimized the systematic errors and compensated for the most important nonsystematic errors. Experiments verified that this encoder trailer can give excellent results in both clockwise direction and counterclockwise direction, while the main track encoders fail to provide any useful information on the vehicle's position and orientation due to slippage. Three propositions prove ET's robustness with respect to unexpected
obstacle. To verify those propositions, experiments are conducted with bumps and without bumps. The experimental results confirm those propositions.

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References


