COLLAPSE ANALYSIS
OF REINFORCED CONCRETE FRAMES

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COLLAPSE ANALYSIS OF REINFORCED CONCRETE FRAMES

ABSTRACT

Based on the energy principle and the finite element method, a mathematical criterion for predicting the ultimate state of reinforced concrete structures is formulated and discussed. The criterion may be used in member systems, two-dimensional and even three-dimensional structures. The criterion may be applied for structures composed of elastic-perfectly plastic or general elasto-plastic material. Because it is easy to calculate and has similar forms for different structures, it is well suited for computer-aided analysis of reinforced concrete structures.

A FORTRAN listing of the general subroutine for predicting the ultimate state of all kinds structures is suggested and input data is presented to illustrate the data preparation procedure and format. The subroutine can be easily integrated with any other program to provide the full-range nonlinear analysis of a reinforced concrete structure.

A extended Clough's element model and a piecewise restoring force model which simulate the main behavior of beam and column elements at all deformation stages including yield, maximum load and descending range (load decrease for corresponding deformation increase) are suggested. Based on these models and the mathematical criterion for predicting the
maximum load-carry capacity of structures, a computer program for the full-range nonlinear analysis of reinforced concrete framed structures is developed and a series of collapse analyses are carried out.

The calculated results show that when using a general tri-linear moment-rotation relationship instead of an elastic-perfectly plastic material, the maximum load-carry capacity of a reinforced concrete frame does not depend on the total number of the plastic hinges at the member ends in the structure. In this case, the "mechanism" stage is not a necessary condition for the maximum load state of the structure. After the structure reaches the maximum load-carry capacity, the internal forces at some member ends in the structure are still increasing until the structure reaches the "mechanism" stage.

The stiffness of the descending branch in the restoring force model has a great influence on structural behavior, such as the maximum load-carry capacity, deformability, distribution of the internal force and mechanism state, and thus, should receive considerable attention. Even though frame design typically is based the recommendation of strong column-weak beam, lower column strengths which still satisfy this general criteria will greatly affect the collapse behavior of reinforced concrete frames.

Key Words: Reinforced Concrete Structure, Frame, Collapse Analysis, Nonlinear Analysis, Variation, Potential, Limit State, Ultimate state, Ductility, Mechanism.
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CHAPTER 1
INTRODUCTION

During a strong earthquake, reinforced concrete structures are usually deformed into a state of severe material and geometrical nonlinearity. This has been verified by both post earthquake damage surveys of structures and dynamic analyses. In such cases, the material nonlinearity and the structural deformations are usually far beyond the scope of building codes for reinforced concrete structures. The earthquake resistant capacity of a structure mainly depends on its load-carry capacity, deformability, stiffness and energy dissipation after yielding. This important concept has been widely accepted by researchers and engineers. Thus, it is necessary to investigate the behavior of the structure during a large deformation including the yield, ultimate and even the decending range [Ref. 1]. Computer-aided full-range nonlinear analysis of a reinforced concrete structure is one of the most important methods for evaluating the seismic resistance of a structure. Such an analysis technique is also important because it reduces the need for laboratory experiments.

Consideration of the behavior of reinforced concrete frames at or near the maximum load-carry capacity and even in the descending range is necessary to determine the possible deformations and distributions of moments, shears, and axial forces that could be used or referenced in the
seismic design. During a strong earthquake, there exists a great difference between the real distribution of moments and forces and those given by an elastic structural analysis. Therefore, a nonlinear analysis of structures is necessary. Also, a determination of the available ductility or deformability of the structure when subjected to earthquake-type loading is important because the present seismic design philosophy relies on energy dissipation by inelastic deformation during a major earthquake.

By carrying out experiments on reinforced concrete frames under the static, monotonic or cyclic loading, structural behavior, such as maximum load-carry capacity, deformability and energy dissipation may be investigated [Ref. 2,3]. However, the testing of a real building is very expensive and involves a considerable time commitment. Also, due to practical restrictions during experiments it is difficult to measure certain important data, such as the bending moments, shear forces, axial forces and plastic hinge distribution in the structure tested. From this point of view, the use of computer-aided nonlinear analysis of reinforced concrete structure is an attractive alternative. It not only can be used in design, but it also partially replaces the need for experiments on real structures.

Generally speaking, if the material relationships of the structure are known, the bending moments, shear and axial forces, and deformations of reinforced concrete frames at any stage of loading from zero to maximum load can be determined analytically using the conditions of static equilibrium and geometric compatibility. However, difficulties are caused
by the inelastic behavior of reinforced concrete materials when considering the problem of the full-range nonlinear analysis, including the descending range. Most limit analysis methods for determining the limit or maximum load are based on an elastic-perfectly plastic model, and the model is not suitable for reinforced concrete members. Thus, a generalized method which can reproduce the elasto-plastic behavior of reinforced concrete should be developed. The element model must represent not only the behavior at and near the maximum load-carry capacity, but also in the descending range. Corresponding to the element model, full-range nonlinear analysis of frames requires simple and reliable material constitutive relationships, such as the moment-curvature of the cross section or moment-rotation at the element's ends. Therefore, predicting the maximum load-carry capacity of a structure and developing an element model which accurately reproduces inelastic behavior of reinforced concrete elements under large deformations are the two primary problems.

A mathematical method for solving the first problem has been suggested in Chapter 3. Based on Clough's two-component model, a extended Clough's model which includes a descending branch is developed in the report. Also, a moment-rotation relationship at the element's ends is suggested.

By using the mathematical criterion and the extended Clough's model, a computer program for the full-range nonlinear analysis of reinforced concrete frames is developed. A series of collapse analyses of frames are carried out and some significant results are discussed.
CHAPTER 2
ULTIMATE ANALYSIS OF STRUCTURE

Several valuable research efforts have been conducted all over the world [Ref. 4, 5, 6,7,8] to develop a full-range nonlinear analysis of reinforced concrete structures. However, there are some basic problems for which a satisfactory solution has not been obtained.

The primary problem in the full-range nonlinear analysis is to predict behavior beyond the ultimate state (maximum load-carry capacity) of a reinforced concrete structure. During a strong earthquake a structure may approach the ultimate state or even go into the descending strength range beyond the ultimate state. However, even when the deformations go beyond the ultimate state, a ductile structure will still have good earthquake resistant capacity. Therefore, a full-range nonlinear analysis should include the calculation of the ultimate state and the descending range (load decrease but deformation increase).

To date, the ultimate state of reinforced concrete structures cannot be simply and accurately predicated and the calculation of the descending range being beyond the ultimate state is impossible. As a result, current full-range nonlinear analyses of reinforced concrete structures are actually restricted to the stages before the ultimate state. Further progress seems to be difficult in dealing with the problem of calculating
the maximum load-carry capacity of a structure and its behavior beyond this ultimate state.

Ultimate state analysis or limit analysis of a structure is a classical mechanics problem in plasticity. A variety of ultimate analysis methods for structures from a single member to complex slabs and frames have been developed. Previous ultimate analysis methods have almost all been based on assumption of elastic-perfectly plastic material. This assumption is acceptable for steel structures. However, significant errors develop when using this for constitutive relationships of reinforced concrete structures [Ref. 9]. Also, when using the assumption of a elastic-perfectly plastic material the structure forms a mechanism at the ultimate state. Thus, it is impossible to calculate the deformation at the ultimate state and in the descending strength range of behavior for the structure. In summary, assuming an elastic-perfectly plastic material in the ultimate state analysis of reinforced concrete structures neither agrees with actual material relationships nor permits a complete full-range nonlinear analysis involving the ultimate state deformation and behavior in the descending range.

Although there are many different methods of ultimate state analysis, they can be divided into two main groups. The first group of methods concentrates on physical analysis [Ref. 10, 11]. In these methods, the ultimate or maximum load can be obtained from the loads carried by the structure in the certain state, for example, the mechanism method. The mechanism method, which is based on the lower-bound theorem and
uniqueness theorem for predicting the ultimate state, determines the minimum load among all the loads which simultaneously satisfy equilibrium and mechanism conditions. When using this method all possible mechanisms must be determined in order to obtain the real ultimate load. This method relies on trials and experience and therefore, when a structure is complex, this method is not suitable.

The second group of the methods employs mathematics. In these methods, the ultimate state can be predicted according to energy principles and structural stiffness [Ref. 12,13,14]. The finite element method and digital computers make it possible to predict the ultimate state of complex structures through the use of convenient and reliable mathematical methods. For instance, the secant stiffness method is a typical mathematical method and has a great advantage when employing computers. Its basic procedure can be briefly explained as follows:

At any time of the step-by-step calculation, the incremental stiffness equation for a structure is given by:

\[
[K]{d\Delta} = {dP}
\]  
(2.1)

where, \([ K ]\) is the incremental structure stiffness matrix which is updated at each step according to the stress state and material relations;

\({d\Delta}\), \({dP}\) are vectors of the deformation and load increments for the whole structure, respectively.
When the structure approaches the ultimate state, it becomes a mechanism and Eq. (2.1) has no unique solution because of the possible rigid body motion of the structure. At this time, the value of the determinant of Eq. (2.1) is equal to zero, i.e.

\[ \det[K] = 0 \quad (2.2) \]

Thus, the mathematical criterion for predicting the ultimate state of a structure can be described as follows: for an elastic-perfectly plastic structure, when the value of the determinant of its increment stiffness matrix is equal to zero, the structure is at the ultimate state.

This method is more direct than the mechanism method, but because the assumption of the elastic-perfectly plastic material results in an "ill condition" of the structural stiffness equation (2.1) at the ultimate state, it still cannot be used for a full-range nonlinear analysis including the ultimate deformation and the descending strength range.

To go beyond the present limitation for a full-range nonlinear analysis of reinforced concrete structures, a general multi-dimensional stress-strain relationship which describes the basic characteristics of reinforced concrete is used. Based on the relationships, a generalized mathematical method is developed in terms of the energy principle and the finite element method. This method can be used accurately and conveniently for predicting the maximum load-carry capacity of a structure.
CHAPTER 3
MATHEMATICAL CRITERION

3.1 Basic Concepts

When a reinforced concrete structure carries loads or surface traction, the general relationship between the load \( P \) and corresponding deformation \( U \) may be shown in Fig. 3.1. At the point \( B \) and in the descending range \( BC \), or the horizontal line \( BC' \) for elastic-perfectly plastic material, the structure presents a state of unstable equilibrium. In this case, the load \( P \) stays constant or even decreases, but the deformation \( U \) corresponding to the loading direction continues to increase. Therefore, when discussing the maximum load-carry capacity and descending range for a reinforced concrete structure, it is significant and acceptable to make use of the structure stability concept.

A theorem of the minimum total potential may be stated as follows [Ref. 15]: for every compatible perturbation of the deformation field, if the first variation of the total potential of a structure is equal to zero and the second variation is larger than zero, i.e.

\[
\delta \Pi_p = 0 \quad ; \quad \delta^2 \Pi_p > 0 \quad (3.1)
\]

then, the structure is in a state of stable equilibrium.

A corollary which follows from the theorem is: If \( \delta^2 \Pi_p \leq 0 \) for at
least a compatible perturbation, then, the equilibrium is unstable.

Hence, the value of the second variation of the total potential may be used to predict the ultimate state or maximum load-carry capacity of the structure. In summary, the first time the second variation of the total potential of a structure is equal to or less than zero, the structure approaches the ultimate state.

3.2 Expression of $\delta^2 \Pi_p$ for a Structure

In plasticity, there are two major theories: deformation theory and incremental strain theory or flow theory. The deformation theory is only a special case of the flow theory and has been found unsuitable for a complete description of the plastic behavior of a reinforced concrete structure. In this report, all discussions are based on flow theory.

For the flow theory, the plastic strains depend upon a combination of factors such as the increment of stresses and strains and state of stress. The strain vector $\{\varepsilon\}$ can be decomposed into its elastic $\{\varepsilon^e\}$ and plastic $\{\varepsilon^p\}$ components as follows:

$$\{\varepsilon\} = \{\varepsilon^e\} + \{\varepsilon^p\} \quad (3.2)$$

In flow theory, the plastic strain increment is function given by:

$$\{d\varepsilon^p\} = f (\{\sigma\}, \{d\varepsilon\}, \{d\sigma\}) \quad (3.3)$$
therefore, Eq. (3.2) may be written in the incremental form:

\[
\{d\epsilon\} = \{d\epsilon^e\} + \{d\epsilon^p\} = [D^e]^{-1}\{d\sigma\} + \{d\epsilon^p\} \quad (3.4)
\]

where, \([D^e]\) is the matrix of elastic behavior.

From Eq. (3.4), incremental stress can be taken as:

\[
\{d\sigma\} = ([D^e] - [D^p])\{d\epsilon\} = [D^{ep}]\{d\epsilon\} \quad (3.5)
\]

where, \([D^p]\) = \([D^e]\) \{d\epsilon^p\} / \{d\epsilon\}, is called the matrix of plastic behavior. It is usually symmetric matrix.

Formula (3.5) expresses the relations between incremental stress and incremental strain in the structure at any time.

As far as an elastic-plastic structure is concerned, at any time, its total potential functional is given by:

\[
\Pi_p = \iiint_V \int_{\epsilon} \sigma_{ij} \, d\epsilon_{ij} \, dv - \iiint_V T_i \, U_i \, dv - \iint_s P_i \, U_i \, ds \quad (3.6)
\]

where, term 1 is the internal work or strain energy due to internal stresses;

term 2 is the body force work;

term 3 is the surface traction work or load work.

\(dv = dx dy dz\) and \(ds\) are the elementry volume and the elementry area of the surface of the structure, respectively.
By using the finite element method and matrix formulation, all the stress, strain components and body forces, surface tractions and deformations may be expressed in terms of vectors of nodal deformations and nodal forces:

\[
\varepsilon_{ij} = \{ \varepsilon(q) \} = [B] \{q\}
\]

\[
\sigma_{ij} = \{ \sigma(q) \}
\]

\[
T_i = \{T\}; \quad P_i = \{P\}; \quad U_i = \{q\}
\]

where, \(\{q\}\) is the nodal deformation vector;

\(\{T\}, \{P\}\) are the equivalent nodal body force and surface traction, respectively;

\([B]\) is deformation function.

The expression of the total potential is taken as:

\[
\Pi_p = \sum \int_V \int_{\{q\}} (\sigma(q))^T d(\varepsilon(q)) \, dv - \sum \int_V \int_{\{T\}} (T)^T \{q\} \, dv - \int_{\partial V} \{P\}^T (q) ds
\]

(3.8)

where the notation \(\sum\) means summation over all the elements.

Assuming \(\{q\}\) is an independent function, then, the first variation of the total potential functional (2.8) about \(\{q\}\) can be indicated as follows:

\[
\delta \Pi_p = \sum \int_V (\sigma(q))^T [B] \delta q \, dv - \sum \int_V (T)^T dv (\delta q) - \int_{\partial V} \{P\} ds (\delta q)
\]

(3.9)
Because \( \{ \delta q \} \) may have any value, the structural equilibrium equation expressed by \( \delta \Pi_p = 0 \) is:

\[
\Sigma \int \int \int_V \{ B \}^T \{ \sigma(q) \} dv - \Sigma \int \int \int_V \{ T \} dv - \int_{s^e} \{ P \} ds = 0 \quad (3.10)
\]

From Eq. (3.10) and considering the second variation of the first order terms of \( \{ q \} \) are zero, the second variation of the total potential of the structure is equal to:

\[
\delta^2 \Pi_p = \delta \Sigma \left( \int \int \int_V \{ \sigma(q) \} \{ B \} dv \right) \{ \delta q \} \\
= \{ \delta q \}^T \left( \Sigma \int \int \int_V \{ \partial \sigma/\partial q \} \{ B \} dv \right) \{ \delta q \} \\
= \{ \delta q \}^T \left( \Sigma \int \int \int_V \{ \partial \sigma/\partial q \} \{ B \} dv \right) \{ \delta q \} \\
= \{ \delta q \}^T \left( \int \int \int_V \{ \partial \sigma/\partial q \} \{ B \} dv \right) \{ \delta q \} \quad (3.11)
\]

From Eq. (3.5) and Eq. (3.7), the following expression can be obtain:

\[
\{ \partial \sigma/\partial q \} = \left( \left[ D^{ep} \right] \{ \partial \epsilon/\partial q \} \right)^T = \left[ B \right]^T \left[ D^{ep} \right] \quad (3.12)
\]

By substituting Eq. (3.12) into Eq. (3.11), the second variation of the total potential are shown by:

\[
\delta^2 \Pi_p = \{ \delta q \}^T \left( \Sigma \int \int \int_V \{ B \}^T \left[ D^{ep} \right] \left[ B \right] dv \right) \{ \delta q \} \\
= \{ \delta q \}^T \left[ K \right] \{ \delta q \} \quad (3.13)
\]

where, \( \left[ K \right] \) is the elastic-plastic incremental stiffness matrix of the
structure at any time, it depends on the material stress-strain relation:

\[ \{d\sigma\} = [D\epsilon^p] \{d\epsilon\} \]

and geometrical relation:

\[ \{d\epsilon\} = [B] \{dq\} \]

Expression (3.13) may be used to calculate the value of the second variation of the total potential of structure. According to the corollary mentioned above, if \( \delta^2 \Pi_p \leq 0 \), then the structure has reached the unstable state. The matrices \([B]\) and \([D\epsilon^p]\) will be developed at the end of this Chapter.

### 3.3 Criterion to Predict Ultimate State

In order to discuss the criterion directly and conveniently, the vector \(\{\delta q\}\) and matrix \([K]\) are further separated into subvectors and submatrices as follows:

\[
\{\delta q\} = \begin{bmatrix} \delta U \\ \delta \theta \end{bmatrix} \quad [K] = \begin{bmatrix} K_{uu} & K_{u\theta} \\ K_{\theta u} & K_{\theta \theta} \end{bmatrix} \quad (3.14)
\]

where, \( U \) is the vector of nodal deformations corresponding to the loading directions;

\( \theta \) is the vector of other nodal deformations.

and, \( \theta = L U + C \); \( \delta \theta = L \delta U \) \quad (3.15)
where, \( C \) is a constant vector not relating to vector \( U \).

After substituting Eq. (3.14) and Eq. (3.15) into Eq. (3.13), the second variation of the total potential is equal to:

\[
\delta^2 \Pi_p = \{\delta U\}^T [K_{uu} + L^T K_{u\theta} L + L^T K_{\theta\theta} L] \{\delta U\} \quad (3.16)
\]

By using the incremental equilibrium equation, \( d(\delta \Pi_p) = 0 \), the vector \( L \) is expressed as:

\[
L = -[K_{\theta\theta}]^{-1} [K_{u\theta}]
\quad (3.17)
\]

The second variation value can be shown as Eq. (3.18) after substituting Eq. (3.16) into Eq. (3.17). Then, if the structure carrying loads is in a state of instability in the loading direction, its \( \delta^2 \Pi_p \) corresponding to the deformation component of the loading direction should be equal to or less than zero, i.e.

\[
\delta^2 \Pi_p = \{\delta U\}^T [K_{uu} - K_{u\theta} K_{\theta\theta}^{-1} K_{u\theta}] \{\delta U\}
\]

\[
= \{\delta U\}^T [K]^* \{\delta U\} \leq 0 \quad (3.18)
\]

The expression (3.18) is a real quadratic form. According to linear algebra, if for any \( \{\delta U\} \in \mathbb{R}^n \) (real solution space), the quadratic form, \( \delta^2 \Pi_p \),

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is larger than zero, then $\delta^2 \Pi_p$ is positive definite. The sufficient and necessary conditions for a positive definite quadratic form $\delta^2 \Pi_p$ are that its coefficient matrix $[K]^*$ is positive definite. This condition is equal to the requirement that all the eigenvalues of the matrix $[K]^*$ must be larger than zero. For the reason given above, it can be concluded the term is not positive definite, or $\delta^2 \Pi_p \leq 0$, as long as any eigenvalue of the matrix $[K]^*$ is equal to or less than zero. In this case, Eq. (3.18) can be obtained and the structure is in a state of instability in the loading direction.

Thus, a generalized mathematical criterion to predict the ultimate state or the maximum load-carry capacity of any structure can be indicated as follows:

For an elastic-plastic structure carrying loads, based on the finite element method, its current incremental elasto-plastic stiffness matrix corresponding to the loading direction is taken as:

$$[K]^* = [K_{uu} - K_{u\theta} K_{\theta \theta}^{-1} K_{\theta u}] \quad (3.19)$$

The first time any one of the eigenvalues of the $[K]^*$ is equal to or less than zero, the structure has reached the ultimate state, i.e. its maximum load-carry capacity.

For an elastic-perfectly plastic structure, some eigenvalue of the
matrix $[K]$ is equal to zero when the structure is at the ultimate state. This means that at the ultimate state the structure has a rigid body motion [Ref. 16]. Therefore, the prior criterion can also effectively be used for a structure composed of elastic-perfectly plastic material.

3.4 Deformation matrix $[B]$

In the finite element method, relationships between the element strains and element nodal deformations may be indicated by the deformation function $[B]$ as given in Eq. (3.7). According to element type, the matrix $[B]$ has different forms. In the proof of the mathematical criterion developed previously, the expression $(d\varepsilon) = [B](dq)$ is used, i.e. the matrix $[B]$ is constant in the elastic-plastic deformation history and is just related to the coordinate value of the strain.

Because loading history may be taken into account in the finite element method based on flow theory, the matrix $[B]$ may be changed during loading. In other words, when the matrix $[B]$ is changed into another form, e.g. a new matrix $[C]$, all the results mentioned above are still valid as long as the matrix $[C]$ replaces the matrix $[B]$.

3.5 Material Matrix $[D^{ep}]$

Although the criterion discussed above may be applied to the entire range of solid mechanics, only the material relations and matrix $[D^{ep}]$
corresponding to reinforced concrete structures will be discussed. Generally speaking, most mathematical models for concrete material which may be employed in a nonlinear analysis of reinforced concrete structures are either elasticity-based models or plasticity-based models. From a purely formal point of view, there is no basic difference between elasticity-based models and the plastic models. They all result in variable incremental material stiffness matrices or incremental "elasticity" matrices applicable for certain ranges of loading.

In nonlinear analysis, however, due to the approximate considerations that avoid the use of more sophisticated concepts, such as loading functions (surfaces) and flow rules, the elasticity-based models are more convenient and successful than plasticity-based models. Therefore, a brief discussion is given to illustrate the material behavior matrix \( [D^p] \) founded on the elasticity-based models.

The simplest elasticity-based models are uniaxial piecewise models. These may not only be relationships between the stress and strain of material, but also relationships between the force and deformation of the elements of a structure. For instance, when the full-range nonlinear analysis of a reinforced concrete frame is carried out, the uniaxial piecewise model represents either the moment versus curvature of element sections or moment versus rotation at the element ends [Ref. 17].
In this case, the linear incremental stiffness \([ K ]\) may be obtained from the model during the process of the step-by-step nonlinear analysis calculation.

For general conditions, such as biaxial or triaxial loading, the main assumptions of the elasticity-based models can be explained as follows:

(1) Concrete is regarded as an incrementally linear biaxial or triaxial orthotropic material;

(2) During the loading history, principal stress axes coincide with corresponding principle strain axes.

The incremental constitutive equation is taken as:

\[
\{d\sigma\} = [D^e_p]\{d\varepsilon\}
\]  \hspace{1cm} (3.20)

For biaxial conditions, a successful application of the elasticity-based model was worked out by Darwin and Pecknold [Ref. 18, 19]. The model is based on the concept of "equivalent uniaxial strain", whereby the effects of biaxial stress on internal damage in concrete are represented by the equivalent uniaxial stress-strain curves for each of the principle stress axes. In such a case, the material matrix \([D^e_p]\) has the form:
\[
[D^{ep}] = \begin{bmatrix}
E_1 & \frac{1}{2}(E_1 + E_2) & 0 \\
\frac{1}{(1-\mu)^2} & 1/(1-\mu^2) & E_2 \\
\text{symm.} & 0 & 1/4(E_1 + E_2 - 2\mu \sqrt{E_1 E_2})
\end{bmatrix}
\] (3.21)

where, \(\mu = \sqrt{\nu_1 \nu_2}\), "equivalent" Poisson's ratio;

\(E_i\) (\(i = 1, 2\)) are tangent moduli of the "equivalent uniaxial" concrete stress-strain relations.

For example, using the uniaxial relations provided by Saenz [Ref. 20]:

\[
\sigma_i = \frac{E_0 \varepsilon_{ie}}{1 + (E_0/E_{ci} - 2) \varepsilon_{ie}/\varepsilon_{ci} + (\varepsilon_{ie}/\varepsilon_{ci})^2} \quad (i = 1, 2, 3) \quad (3.22)
\]

Then, \(E_{ti}\) is equal to:

\[
E_{ti} = \frac{E_0 [1 - (\varepsilon_{ie}/\varepsilon_{ci})^2]}{[1 + (E_0/E_{ci} - 2) \varepsilon_{ie}/\varepsilon_{ci} + (\varepsilon_{ie}/\varepsilon_{ci})^2]^2} \quad (i = 1, 2, 3) \quad (3.23)
\]

Because of the difficulty of 3-dimensional finite element analysis, triaxial elasticity-based models have not been applied as often as the biaxial model. However, some valuable applications have been done. Elwi
and Murray extended the equivalent uniaxial strain approach of Dariwn and Peknold and represented the incremental material matrix \([D^{\text{ep}}]\) for the axisymmetric case [Ref. 21]. The general form of the material matrix \([D^{\text{ep}}]\) for triaxial loading was also given by Bathe and Ramaswamy [Ref. 22]. The expression is:

\[
[D^{\text{ep}}] = \beta \begin{bmatrix}
(1-\mu)E_{11} & \mu E_{21} & (1-\mu)E_{22} & 0 \\
\mu E_{31} & \mu E_{32} & (1-\mu)E_{33} & 0 \\
0 & (1/2-\mu)E_{12} & 0 & (1/2-\mu)E_{12} \\
(1/2-\mu)E_{31} & 0 & (1/2-\mu)E_{31} & 0
\end{bmatrix} \tag{3.24}
\]

where, \(\beta = 1 / (1 + \mu) (1 - \mu)\);

\[
E_{ij} = \frac{\sigma_i E_i + \sigma_j E_j}{\sigma_i + \sigma_j} \quad (i = 1,2,3; \ j = 1,2,3) \tag{3.25}
\]

Generally speaking, in the full-range nonlinear analysis of a reinforced concrete structure, the incremental material matrix \([D^{\text{ep}}]\) at each step of the calculation may be evaluated according to stress state and material relations. Thus, the calculation of the stiffness matrix \([K]\) in Eq. (3.13) at any time is quite easy.
CHAPTER 4
ELEMENT STIFFNESS MATRIX

4.1. Moment - Rotation Relationship

In this report the backbone portion of the moment - rotation relationship at the element end was assumed to be trilinear. It included the elastic, post-yielding and post-maximum (descending branch) stiffnesses. The stiffness value of the post-maximum portion is negative. The idealized moment - rotation relationship is shown in Fig. 4.1.

The following two-step procedure was taken to determine the values at the yield and maximum moment points:

1) Using the stress - strain relationships of the concrete and steel, a theoretical moment - curvature relationship for a reinforced concrete section under flexure and axial force can be derived on basis of standard assumptions.

2) From the moment - curvature relationship, taking stiffness changes along the element length and plastic hinge zone lengths into account, the moment - rotation relationship at the element end may be obtained.

In the calculation of the moment - curvature relationship, the following assumptions were used:
Plane sections before bending remain plane after bending;
Stress - strain relationship of the steel is elastic-perfectly plastic
as shown in Fig. 4.2 (a);
Stress - strain relationship of concrete is a parabola plus a linear
descending branch as shown in Fig. 4.2 (b) [Ref. 23];
Tensile strength of concrete may be neglected.

From these assumptions, equilibrium and deformation compatibility
conditions, the yield moment $M_y$ and corresponding curvature $\phi_y$, and the
maximum moment $M_u$ and corresponding curvature $\phi_u$ can be obtained.

The relationship of the moment, $M_{A,B}$ and the rotation, $\theta_{A,B}$ at the each
end, A and B, of the element may be generated after the moment -
curvature relationship of each section is known. In general, the
contra-flexure point of the element varies during the step-by-step
calculation procedure and results in the different rotation values at the
yield and maximum moment points. For simplicity, however, an
assumption that the contra-flexure point of the element is at midspan
was used. The idealized deformation shape and the curvature distribution
along the element length at the maximum moment state are shown in Fig.
4.3. The rotations at the element ends, A and B, are:

$$\theta_A = \Delta_{CA} / 0.5L = 2 \cdot MA_{AB} / L$$
$$\theta_B = \Delta_{CB} / 0.5L = 2 \cdot MA_{BC} / L$$

(4.1)
where, \( MA_{AB} \) is the moment of the area of curvature diagram between the points A and C about the point C;

\( MA_{BC} \) is the moment of the area of curvature diagram between the point B and C about point C.

Therefore, the yield rotations \( \theta_{yA} \) and \( \theta_{yB} \) are:

\[
\theta_{yA} = \varphi_{yA} \cdot \frac{L}{6}
\]

\[
\theta_{yB} = \varphi_{yB} \cdot \frac{L}{6}
\]

(4.2)

where, \( \varphi_{yA}, \varphi_{yB} \) are the yield curvatures of sections at the element ends A and B, respectively,

and the rotations, \( \theta_{UA} \) and \( \theta_{UB} \), corresponding to maximum moments are:

\[
\theta_{UA} = \varphi_{yA} L (\frac{2}{3} - \lambda_1 + \lambda_1^2) + \varphi_{UA} L_{PA} (1 - \lambda_1)
\]

\[
\theta_{UB} = \varphi_{yB} L (\frac{2}{3} - \lambda_2 + \lambda_2^2) + \varphi_{UB} L_{PB} (1 - \lambda_2)
\]

(4.3)

where, \( \varphi_{UA}, \varphi_{UB} \) are curvatures corresponding to the maximum moment of the section at the element ends, A and B, respectively;

and: \( \lambda_1 = L_{PA} / L; \lambda_2 = L_{PB} / L, \) where \( L_{PA} \) and \( L_{PB} \) are the lengths of the plastic hinge zones at the element ends, A and B, respectively. In maximum moment state, those are computed using the following
expression for equivalent plastic hinge length  [Ref. 24]:

\[ L_p = 0.8 \ k_1 \ k_2 \ (z/d) \ c \]  \hspace{1cm} (4.4) \]

where, \( k_1 = 0.7 \) and \( 0.9 \) for mild and cold-worked steel, respectively;
\( k_2 = 0.6 \) and \( 0.9 \) when \( f_{c'} = 5100 \) psi and \( f_{c'} = 1700 \) psi, respectively.

\( z \) is the distance from element end to contraflexure point;
\( d \) and \( c \) are effective depth and neutral axis depth at the maximum moment, respectively.

Also, according to the moment distribution, the plastic hinge length may be given by:

\[ L_p = \frac{(M_u - M_y)}{M_u} \times \left( \frac{L}{2} \right) \]  \hspace{1cm} (4.5) \]

In the collapse analysis of frames, the larger of the values obtained from Eqs. (4.4) and (4.5) was used.

After the yield and maximum moment points in the moment - rotation relationship have been obtained, the stiffnesses of each linear branch are determined as follows:

(1) before yielding:

\[ K_0 = \frac{M_y}{\theta_y} \]  \hspace{1cm} (4.6) \]
(2) After yielding, but before the maximum moment state:

\[ K_y = \frac{(M_u - M_y)}{(\theta_u - \theta_y)} \quad (4.7) \]

(3) Because it is difficult to determine the incremental stiffness value after the maximum moment point in the moment - rotation relationship, following expression is used in the report:

\[ K_u = \alpha \cdot K_y \quad (4.8) \]

where, \( \alpha \) is the assumed stiffness ratio and is equal to or less than zero.

### 4.2. Beam - Column Element Model

Clough's two-component element model has been widely used to simulate the elasto-plastic behavior of reinforced concrete beams and columns. The two-component element model consists of two parallel components: one is elastic and the other is elastic-perfectly plastic. This model has been applied extensively in seismic analysis of steel and reinforced concrete structures [Ref. 25, 26]. Another extended two-component element, which can represent the elastic, post-cracking and post-yielding stages of a reinforced concrete member, is developed [Ref. 27].
Because the descending branch of the moment - rotation relationship should be employed in the full-range nonlinear analysis or collapse analysis of reinforced concrete frames, Clough's two-component element had to be extended.

A three-component beam - column element model, which consists of an elastic and two elastic-perfectly plastic elements in parallel as shown in Fig. 4.4, was developed. The elastic element represents the strain softening in the descending branch and each of the elastic-perfectly plastic elements represents the yield and maximum strength of the element, respectively.

The stiffness coefficients should satisfy the following conditions:

\[ q_1 + q_2 + q_3 = 1 \]
\[ q_2 + q_3 = \rho \] \hspace{1cm} (4.9)
\[ q_3 = g \]

As for the extended Clough element, there are nine possible states at the element ends. The corresponding stiffness ratios for each state are indicated in Table 4.1. The new element model has more five states than original Clough model. The extra states are necessary for carrying out collapse analysis of reinforced concrete frames.

The incremental stiffness matrix of the element was taken as follows:
<table>
<thead>
<tr>
<th></th>
<th>$S_a$</th>
<th>$S_b$</th>
<th>$S_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>end A: E; end B: E</td>
<td>$k$</td>
<td>0.5$k$</td>
<td>$k$</td>
</tr>
<tr>
<td>end A: E; end B: Y</td>
<td>$(0.75+0.25p)k$</td>
<td>0.5$pk$</td>
<td>$pk$</td>
</tr>
<tr>
<td>end A: E; end B: U</td>
<td>$(0.75+0.25g)k$</td>
<td>0.5$gk$</td>
<td>$pk$</td>
</tr>
<tr>
<td>end A: Y; end B: E</td>
<td>$pk$</td>
<td>0.5$pk$</td>
<td>$(0.75+0.25p)k$</td>
</tr>
<tr>
<td>end A: Y; end B: Y</td>
<td>$pk$</td>
<td>0.5$pk$</td>
<td>$pk$</td>
</tr>
<tr>
<td>end A: Y; end B: U</td>
<td>$(0.75p+0.25g)k$</td>
<td>0.5$gk$</td>
<td>$gk$</td>
</tr>
<tr>
<td>end A: U; end B: E</td>
<td>$gk$</td>
<td>0.5$gk$</td>
<td>$(0.75+0.25g)k$</td>
</tr>
<tr>
<td>end A: U; end B: Y</td>
<td>$gk$</td>
<td>0.5$gk$</td>
<td>$(0.75p+0.25g)k$</td>
</tr>
<tr>
<td>end A: U; end B: U</td>
<td>$gk$</td>
<td>0.5$gk$</td>
<td>$gk$</td>
</tr>
</tbody>
</table>

where: E, Y, U represent elastic, yield and ultimate state, respectively.

Table 4.1 Stiffness Coefficients of Extended Clough Model
\[
[K] = \begin{pmatrix}
(S_a+S_b+2S_c) & -(S_a+S_c)L & - (S_a+S_b+2S_c) & (S_b+S_c)L \\
S_aL^2 & (S_a+S_c)L & S_cL^2 \\
\frac{1}{L^2} & \text{Symmetric} & (S_a+S_b+2S_c) & (S_b+S_c)L \\
S_bL^2 & \\
\end{pmatrix}
\] (4.10)

The element model may also have axial extension in addition to flexural rotations at the element ends.

Because both the yield state and maximum load state are considered in the theoretical analysis and the computer program, two types of plastic hinges were defined as follows:

**Definition 1.** When the moment at the critical section reaches the yield moment, \( M_y \), the section is called a yield plastic hinge.

**Definition 2.** When the moment at the critical section reaches the maximum moment, \( M_u \), the section is called an ultimate plastic hinge.

### 4.3. Computer Program For Collapse Analysis of Frames

Based on the extended Clough element model, corresponding moment - rotation relationship at the element ends and the previously developed criterion for predicting the maximum load-carry capacity of reinforced concrete structures, a computer program for the full-range nonlinear
collapse analysis of reinforced concrete frames subjected to monotonic loading was developed. A brief description of the operation of the program follows:

1. The program may be used to carry out an elasto-plastic static collapse analysis of frame structures, including the elastic, yield, maximum load, unstable and mechanism stages of the structure.

2. The reinforced concrete frame to be analyzed may have any shape. The frame is idealized as a number of nodes and assemblage of columns and beams. The cross sections of the members in the structure must be rectangular, but may be reinforced unsymmetrically.

3. During the step-by-step procedure, with the load increased increment by increment, nonlinearity of the moment - rotation relationship may occur. This means the moment - rotation relationship at the element ends is dependent not only on the element geometry and the material properties, but also on the level of axial force. In the program there is a parameter which can be selected by the user to control whether or not to recompute the moment - rotation relationship at each increment of loading.

4. When a member approaches its maximum moment capacity, the tension reinforcement at the critical section should yield, otherwise, the calculation will automatically stop and the following message will be printed: "TENSION REINFORCEMENT DID NOT YIELD, IMPROVE YOUR DESIGN OF ELEMENT NO = ".

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5. When the first ultimate plastic hinge occurs in some column of the structure, a special subroutine named PREUSS starts to predict whether the structure is approaching the maximum load-carry capacity. After the structure reaches the maximum load, the corresponding information is given and a negative value of load increment should be used.

6. When the maximum displacement of the structure is equal to or larger than some control value provided by the user and the structure has formed a "mechanism" consisting of ultimate plastic hinges, the calculation will automatically be terminated.
CHAPTER 5.
STUDY FRAMES

Three moment resisting building frames were designed according to the provisions of the Uniform Building Code, 1985 edition (UBC - 85) [Ref. 28] and the recommendations of ASCE - ACI Committee 352 [Ref. 29]. These frames are referred to as FRAME1, FRAME 2 and FRAME 3. The first example, FRAME 1, represents an interior reinforced concrete frame which is five stories by two bays and has 25 ft. spans. The second example, FRAME 2, also represents an interior reinforced concrete frame which is five stories by two bays and has 15 ft. spans. The third example, FRAME 3, was similar to FRAME 1 in dimensions, but the reinforcement amounts in the columns for the first three stories were different. Figure 5.1 shows the overall dimensions of the three study frames and the types of members in the structures.

The equivalent earthquake lateral loads for FRAME 1 (also FRAME 3) and FRAME 2 are given by UBC - 85 in accordance with the following formula:

\[ V = Z I K C S W \]  \hspace{1cm} (5.1)

where, \( V \) = total lateral force or the base shear;
\( Z = 1.0 \) for a building in "ZONE 4";
\( I = 1.0 \) for a non-essential facility;
\[ K = 0.67 \text{ for a building with a ductile moment-resistance frame;} \]
\[ C = 1.0/15 \sqrt{T}, \text{ where } T \text{ is the period of the first mode of vibration.} \]
For a building with ductile moment-resistance frame, \( T \) can be set equal to \( 0.1N \), where \( N \) is the total number of stories of the building. The value of \( C \) need not exceed 0.12.
\[ S = \text{numerical coefficient for site-structure resonance, assumed here to be 1.5. However, the product of } CS \text{ need not exceed 0.14.} \]
\[ W = \text{the total dead load of the building plus partition loads.} \]

According to Eq. \((5.1)\), the total lateral seismic forces for FRAME 1 and FRAME 2 were:

**FRAME 1:**
\[ V_1 = Z I K C S W \]
\[ = (1.0)(1.0)(0.67)(0.14)(1875) = 176 \text{ (Kip)} \quad (5.2) \]

**FRAME 2:**
\[ V_2 = (1.0)(1.0)(0.67)(0.14)(675) = 63.3 \text{ (Kip)} \quad (5.3) \]

The required strength of cross sections subjected to bending moment due to the equivalent earthquake lateral load shall be based on:

\[ \phi M_n > M_u = 1.4 \, M_E \]
\[ M_n \geq [1.56] \, M_E \quad (5.4) \]
where, $M_n$ - nominal moment strength;

$M_E$ - seismic bending moment.

All members sections were designed for a concrete compressive strength of 4,000 psi and steel yield strength of 60,000 psi. The assumed material properties are given in Table 5.1. Dimensions and reinforcement details for the member sections used in the frames FRAME 1, FRAME 2 and FRAME 3 are shown in Table 5.2 and Table 5.3.

In accordance with the ACI - ASCE Committee 352 recommendations, the joint flexural strength ratio ($M_R$) should be based on following formula:

$$M_R = \frac{\Sigma M_{\text{columns}}}{\Sigma M_{\text{beams}}} \geq 1.4 \quad (5.5)$$

where $\Sigma M_{\text{columns}}, \Sigma M_{\text{beams}}$ are the nominal moment capacities of columns and beams at a particular joint, respectively.

In order to reduce the number of different sections used in the frames, the beam and column section types are assumed constant over a few stories. The column to beam flexural strength ratios at external and internal joints of all stories of FRAME1, FRAME 2 and FRAME 3 are shown in Table 5.4.

Two types of loads were applied to the structures. One was the
Concrete: The stress-strain relationship was assumed in accordance with Hognestad's model having the following parameters:

\[ f_c' \] maximum compressive strength; \hspace{1cm} 4,000 psi
\[ \varepsilon_o \] strain at maximum stress; \hspace{1cm} 0.002
\[ \varepsilon_{\text{max}} \] maximum strain corresponding to maximum moment, \( M_u \); \hspace{1cm} 0.005
\[ E_c \] modulus of elasticity. \hspace{1cm} 3605 ksi
\[ Z \] descending slope of the linear branch of the stress-strain curve. \hspace{1cm} 100

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_c' )</td>
<td>4,000 psi</td>
</tr>
<tr>
<td>( \varepsilon_o )</td>
<td>0.002</td>
</tr>
<tr>
<td>( \varepsilon_{\text{max}} )</td>
<td>0.005</td>
</tr>
<tr>
<td>( E_c )</td>
<td>3605 ksi</td>
</tr>
<tr>
<td>( Z )</td>
<td>100</td>
</tr>
</tbody>
</table>

Steel: elastic-perfectly plastic model:

\[ f_y \] yield stress; \hspace{1cm} 60,000 psi
\[ E_s \] modulus of elasticity \hspace{1cm} 29,000 ksi

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_y )</td>
<td>60,000 psi</td>
</tr>
<tr>
<td>( E_s )</td>
<td>29,000 ksi</td>
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Table 5.1. Assumed Material Properties for Members
### BEAMS

<table>
<thead>
<tr>
<th>Beam Section Types</th>
<th>Cross Section (in. * in.)</th>
<th>Reinforcement (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bottom</td>
</tr>
<tr>
<td>SB1</td>
<td>22*30</td>
<td>5.94</td>
</tr>
<tr>
<td>SB2</td>
<td>22*25</td>
<td>5.53</td>
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</table>

### COLUMNS

<table>
<thead>
<tr>
<th>Column Section Types</th>
<th>Cross Section (in. * in.)</th>
<th>Reinforcement (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Left</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Right</td>
</tr>
<tr>
<td>SC1</td>
<td>22*22</td>
<td>7.62 (5.08)</td>
</tr>
<tr>
<td>SC2</td>
<td>22*22</td>
<td>6.00</td>
</tr>
<tr>
<td>SC3</td>
<td>24*24</td>
<td>9.36 (6.24)</td>
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<tr>
<td>SC4</td>
<td>24*24</td>
<td>7.62</td>
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</tbody>
</table>

Table 5.2. Types and Designs of Beam and Column

Cross Sections for FRAME 1 and FRAME 3

(numbers in round brackets are for FRAME 3)
### BEAMS

<table>
<thead>
<tr>
<th>Beam Section Types</th>
<th>Cross Section (in. * in.)</th>
<th>Reinforcement (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top</td>
</tr>
<tr>
<td>SB1</td>
<td>14 * 24</td>
<td>2.64</td>
</tr>
<tr>
<td>SB2</td>
<td>12 * 22</td>
<td>2.64</td>
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</tbody>
</table>

### COLUMNS

<table>
<thead>
<tr>
<th>Column Section Types</th>
<th>Cross Section (in. * in.)</th>
<th>Reinforcement (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Left</td>
</tr>
<tr>
<td>SC1</td>
<td>16 * 16</td>
<td>4.0</td>
</tr>
<tr>
<td>SC2</td>
<td>16 * 16</td>
<td>3.16</td>
</tr>
<tr>
<td>SC3</td>
<td>20 * 20</td>
<td>4.0</td>
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</table>

Table 5.3. Types and Designs of Beam and Column Cross Sections for FRAME 2
<table>
<thead>
<tr>
<th>Joint No.</th>
<th>FRAME 1</th>
<th>FRAME 2</th>
<th>FRAME 3</th>
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<tbody>
<tr>
<td>1.</td>
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<td>2.785</td>
<td>1.665</td>
</tr>
<tr>
<td>2.</td>
<td>2.058</td>
<td>2.447</td>
<td>1.548</td>
</tr>
<tr>
<td>3.</td>
<td>2.245</td>
<td>2.758</td>
<td>1.665</td>
</tr>
<tr>
<td>4.</td>
<td>2.127</td>
<td>2.601</td>
<td>1.547</td>
</tr>
<tr>
<td>5.</td>
<td>1.945</td>
<td>2.264</td>
<td>1.436</td>
</tr>
<tr>
<td>6.</td>
<td>2.127</td>
<td>2.601</td>
<td>1.547</td>
</tr>
<tr>
<td>7.</td>
<td>1.854</td>
<td>2.185</td>
<td>1.324</td>
</tr>
<tr>
<td>8.</td>
<td>1.687</td>
<td>1.815</td>
<td>1.226</td>
</tr>
<tr>
<td>9.</td>
<td>1.854</td>
<td>2.185</td>
<td>1.324</td>
</tr>
<tr>
<td>10.</td>
<td>2.092</td>
<td>1.957</td>
<td>2.092</td>
</tr>
<tr>
<td>11.</td>
<td>1.827</td>
<td>1.520</td>
<td>1.827</td>
</tr>
<tr>
<td>12.</td>
<td>2.092</td>
<td>1.957</td>
<td>2.092</td>
</tr>
<tr>
<td>13.</td>
<td>1.012</td>
<td>0.919</td>
<td>1.012</td>
</tr>
<tr>
<td>14.</td>
<td>0.874</td>
<td>0.706</td>
<td>0.874</td>
</tr>
<tr>
<td>15.</td>
<td>1.012</td>
<td>0.919</td>
<td>1.012</td>
</tr>
</tbody>
</table>

Table 5.4. Joint Flexural Strength Ratio $M_R$
gravity loads which were kept constant during the loading history. The other was the horizontal loads which were distributed along frame height in an inverted triangular shape and either increase or decrease during the step-by-step calculation procedure. Both of these loads were applied directly to the frame joints.
CHAPTER 6
ANALYTICAL RESULTS AND DISCUSSIONS

6.1. General

In this chapter the results of general collapse analyses conducted for the two standard design frames, FRAME 1 and FRAME 2, and the modified frame, FRAME 3, are discussed. The primary points to be discussed are the maximum load-carry capacity, deformability and mechanism state of the structures. In particular, the main difference between the mathematical criterion for predicting the maximum load-carry capacity of the structure and the conventional procedure is described. Comparisons of the collapse analyses and UBC - 85 results are also given.

In order to gain a better understanding of the influence of the stiffness parameters on the load-carry capacity and deformation of the structure after several sections have reached their maximum moment, two additional calculations were conducted for FRAME 1 using different stiffness ratios, \( \alpha \), (see Eq. (4.8) ) in the descending branches.

The influence of the joint flexural strength ratio \( (M_R) \) on structural behavior is discussed with the results for FRAME 3.

6.2. General Results of Collapse Analyses

From the collapse analyses of FRAME 1 and FRAME 2, the general
numerical results, such as the load-carry capacity, deformability and ductility of the structure, are given in Table 6.1. The full-range analytical curves for the load vs. displacement at the top of the structures and the shear vs. relative drift of the first story are shown in Fig. 6.1 and Fig. 6.2, respectively.

In Table 6.1, Fig. 6.1 and Fig. 6.2, three important structural states are indicated. The first one is that the first ultimate plastic hinge appears in the structure. From Fig. 6.1, the load vs. displacement relationship is clearly softened after this plastic hinge appears. The second state is when the structure approaches the maximum load-carry capacity and the load increments decrease after this state. The last one is when the structure becomes a mechanism composed of ultimate plastic hinges.

In this report, three different expressions for calculating ductilities of the frames are defined as follows:

\[ D_1 = U_u / U_y \]
\[ D_2 = U_u^* / U_y \]
\[ D_3 = U_u^* / U_u \]

(6.1)

where: \( U_y \), \( U_u \) and \( U_u^* \) are the displacements when; (1) the first ultimate plastic hinge appears, (2) the maximum load-carry capacity is reached and

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<table>
<thead>
<tr>
<th>FRAME No.</th>
<th>First Ult. Plastic Hinge</th>
<th>Maximum Load</th>
<th>Mechanism State</th>
<th>$\frac{P_y}{P_u}$</th>
<th>$\frac{U_y}{U_u}$</th>
<th>$\frac{P_u^*}{P_u}$</th>
<th>$\frac{U_u^*}{U_u}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRAME 1</td>
<td>91.8</td>
<td>4.63</td>
<td>101.</td>
<td>14.4</td>
<td>91.5</td>
<td>38.9</td>
<td>0.91</td>
</tr>
<tr>
<td>FRAME 2</td>
<td>33.8</td>
<td>3.91</td>
<td>40.4</td>
<td>11.7</td>
<td>38.3</td>
<td>23.0</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Comment: P, U -- Load applied at the top of the frames and corresponding displacement, respectively.

Stiffness Ratio, $\alpha = -0.5$ (see Eq. 4.8)

Dimensions: Kip, In.

Table 6.1 Numerical Results of FRAME 1 and FRAME 2
(3) the mechanism state is reached, respectively.

From Table 6.1 and Fig. 6.1, when the first ultimate plastic hinge appears in the structure the loads applied to the structures are about 90% of the maximum load-carry capacity of the structure. After this point, the displacements increase at a significantly faster rate for each load increment. Thus, this point may be regard as the yield point of whole frame.

When approaching the maximum load-carry capacities, the ductilities $D_1$ of FRAME 1 and FRAME 2 are 3.11 and 3.00, respectively. After the maximum load, the applied loads decrease, but the corresponding displacement still increases and the internal forces in some members of the frames also increase until the structure becomes a mechanism.

When the collapse mechanism has been formed, the load-carry capacities of the frames are still at least 90% of the maximum load. However, the corresponding displacements have increase greatly. The ductilities, $D_2$, of FRAME 1 and FRAME 2 are 8.40 and 5.89, respectively, approximately twice as much as $D_1$. The third measure of ductility, $D_3$, was 2.7 and 2.0 for FRAME 1 and FRAME 2, respectively. Because it is difficult to determine the yield point of a complex structure, using ductility value $D_3$ instead of $D_1$ and $D_2$ may be more direct for evaluating the ductile behavior of a structure when the maximum load-capacity of the structure can be easily and accurately obtained. Clearly, in the
descending range the load-carry capacity of the structure decreased slightly, but the energy dissipation of the structure increased rapidly. During a strong earthquake the mechanism state of a framed structure may be regarded as the ultimate state and both the load-carry capacity of members in the structure and the energy dissipation ability of structure under large deformations may be utilized.

If the maximum load-carry capacity for FRAME 1 and FRAME 2 are compared to results from Eq. (5.2) and (5.3), the ratios are as follows:

\[ \beta_1 = \frac{\sum P_i}{V_1} = \frac{303}{176} = 1.72 \]
\[ \beta_2 = \frac{\sum P_i}{V_2} = \frac{121}{63.3} = 1.92 \]  \hspace{2cm} (6.2)

where \( \beta_1, \beta_2 \) are the coefficients for FRAME 1 and FRAME 2, respectively.

\( P_i \) is the lateral force at the ith story.

According to Eq. 5.4, both \( \beta_1 \) and \( \beta_2 \) should be slightly larger than 1.56. Therefore, because the results in Eq. 6.2 are comparable to those from a UBC design approach, the computer program in this report could be used for seismic design.

6.3. Plastic Hinge Distribution

As mentioned previously, there are two types of the plastic hinges defined in this report; one is a yield plastic hinge and the other is an
ultimate plastic hinge. Figure 6.3 and 6.4 represent the plastic hinge distributions at the maximum load and mechanism states (or the ultimate state) for FRAME 1 and FRAME 2, respectively.

From Fig. 6.3 and Fig. 6.4 it can be seen that when the structure approaches the maximum load-carry capacity, the number of ultimate plastic hinges is not large enough to make the frame form a traditional "mechanism". This means that when using the general elasto-plastic material relationship, the maximum load-carry capacity of reinforced concrete frame does not depend on the number of yield or ultimate plastic hinges, but on the total resistant forces provided by the structure. In other words, the mechanism condition based on elastic-perfectly plastic material is not suitable for predicting the maximum load of a reinforced concrete structure composed of a general elasto-plastic material. This is the most important difference between the traditional principle of limit analysis and the mathematical criterion defined in Chapter 3.

Beyond the maximum load point or in the descending range, the applied loads and the internal forces at the ultimate plastic hinges "passively" decrease. However, the internal forces at the yield plastic hinges still increase. Some of yield plastic hinges become ultimate plastic hinges, one after another, until a mechanism consisting of the ultimate plastic hinges is formed. After the mechanism state is reached, as shown in Figs. 6.3 and 6.4, the internal forces at all the member ends decrease in accordance with either a stiffness value $K_0$ or a stiffness value $K_u$. 

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defined in Fig. 4.1, and the structural state would not change. This process is different from that of an elastic-perfectly plastic structure.

6.4. Stiffness Ratio: \( \alpha \)

For the full-range nonlinear analysis of a reinforced concrete structure, the incremental stiffness values of the descending branch of the moment - rotation relationship at the member ends may have a great influence on the structure behavior. Because it is difficult to determine the stiffness values of the descending branches, an assumed stiffness ratio, \( \alpha \), equal to -0.5 was used for all the members in the evaluations of FRAME 1 and FRAME 2.

In order to study the importance of the stiffness value of the descending branch, two more calculations for FRAME 1 were carried out. In these two extra examples, the stiffness ratios were -0.3 and -0.7, respectively.

The relationships of load vs. displacement at top of the structure and the story shear vs. relative drift curves in the first story for the frame with different stiffness ratios are shown in Fig. 6.5 and Fig. 6.6, respectively. The comparison of numerical results corresponding to maximum load-carry capacity and mechanism state are represented in Table 6.2.
### Data

<table>
<thead>
<tr>
<th>Stiffness Ratio</th>
<th>Maximum Load</th>
<th>Mechanism State</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 0.3 P₁</td>
<td>103.</td>
<td>100.</td>
</tr>
<tr>
<td>(1) U₁</td>
<td>17.9</td>
<td>29.7</td>
</tr>
<tr>
<td>- 0.5 P₂</td>
<td>101.</td>
<td>91.5</td>
</tr>
<tr>
<td>(2) U₂</td>
<td>14.4</td>
<td>38.9</td>
</tr>
<tr>
<td>- 0.7 P₃</td>
<td>99.8</td>
<td>70.0</td>
</tr>
<tr>
<td>(3) U₃</td>
<td>10.9</td>
<td>60.5</td>
</tr>
</tbody>
</table>

### Comprison

<table>
<thead>
<tr>
<th></th>
<th>Maximum load</th>
<th>Mechanism State</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁ / P₂</td>
<td>1.02</td>
<td>1.09</td>
</tr>
<tr>
<td>P₃ / P₂</td>
<td>0.989</td>
<td>0.766</td>
</tr>
<tr>
<td>U₁ / U₂</td>
<td>1.24</td>
<td>0.76</td>
</tr>
<tr>
<td>U₃ / U₂</td>
<td>0.76</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Table 6.2 Comprison of Results for Different Stiffness Ratios $\alpha$

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From these results it appears that the descending branch stiffness ratio had little influence on the maximum load-carry capacity. The maximum load increased slightly with an increase of the value of the stiffness ratio. However, the stiffness ratio did have a significant effect on the displacement corresponding to maximum load. These displacements rapidly increased with an increase of the value of the stiffness ratio.

Smaller values of the descending branch stiffness ratio had a negative influence on structural behavior. The frames with a larger value of the stiffness ratio had better deformability at the maximum load-carry capacity state. The larger value of the stiffness ratio for the members delayed the occurrence of the maximum load state and allowed more energy dissipation by the structure.

Displacements along the structure height are shown in Fig. 6.7. At the maximum load-carry capacity and mechanism state, the stiffness ratio had little influence on the horizontal deformation shape of the frames.

Because these results demonstrate that the seismic resistant behavior of reinforced concrete frames is sensitive to the stiffness ratio of the the descending branch of the moment-rotation relationship, it is necessary to determine a reasonable value for the descending branch stiffness ratio in order to obtain accurate results for the collapse analysis of the frame.

6.5. Joint Flexural Strength Ratio $M_R$

In the example frames, FRAME 3, the joint flexural strength ratio $M_R$
was reduced in the first three stories of the frame in order to study the influence of the ratio $M_R$ on the structure behavior.

Figures 6.8 and 6.9 show the load vs. displacement relationships at the top of the frames and the first and the fourth story shear vs. relative drift curves of the original design frame (FRAME 1) and the modified design frame (FRAME 3), respectively. The numerical results corresponding to the first ultimate plastic hinge state, the maximum load-carry capacity and mechanism state are given in Table 6.3.

In general, the comparison of the results reveals that decreasing the joint flexural strength ratio $M_R$ has a great influence on the load-carry capacity and deformability of the structure. The load value at the maximum load and the mechanism state decrease about 10% and the corresponding displacement decrease about 30%. It can be seen that before yielding, the ratio $M_R$ had little influence on the structure behavior, but the effect of $M_R$ increased rapidly after yielding.

Changing values of $M_R$ also changed the distribution of the plastic hinges in the frame. From Fig. 6.9(b), it appears that the main difference between the mechanisms of FRAME 1 and that of FRAME 3 was in the fourth story. Figures 6.10(a) and 6.10(b) show the displacements along the structure height at both the maximum load and mechanism states. Because $M_R$ was decreased in the first three stories of FRAME 3, the mechanism of
<table>
<thead>
<tr>
<th></th>
<th>FRAME 1</th>
<th>FRAME 3</th>
<th>FRAME 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>FRAME 1</td>
</tr>
<tr>
<td>First</td>
<td>$P_y$</td>
<td>91.8</td>
<td>85.7</td>
</tr>
<tr>
<td>U.P.H.</td>
<td>$U_y$</td>
<td>4.63</td>
<td>4.48</td>
</tr>
<tr>
<td>Maximum</td>
<td>$P_u$</td>
<td>101.</td>
<td>91.0</td>
</tr>
<tr>
<td>Load</td>
<td>$U_u$</td>
<td>4.11</td>
<td>9.67</td>
</tr>
<tr>
<td>Mechaism</td>
<td>$P_u^*$</td>
<td>91.5</td>
<td>81.7</td>
</tr>
<tr>
<td>State</td>
<td>$U_u^*$</td>
<td>38.9</td>
<td>29.0</td>
</tr>
</tbody>
</table>

Table 6.3 Comparison of Original and Modified Design
the structure has formed in the lower stories. After the structure reached the maximum load-carry capacity, the fourth story shear decreased with a stiffness $K_0$ instead of decreasing with a stiffness $K_u$ as in FRAME 1. In such a mechanism the higher stories are not fully utilized. Therefore, in order to provide more load-carry capacity and deformability the joint flexural strength ratios, $M_R$, in the lower stories should be increased so plastic hinges will occur in the higher stories of the structure.

In FRAME 1 and FRAME 3 the joint flexural strength ratios, $M_R$, in the top floor were all less than 1.4 and no ultimate plastic hinges formed in the columns at these locations. Thus, for practical design a lower value for the ratio $M_R$ may be utilized in the upper stories.
CHAPTER 7
CONCLUSIONS

Based on results given in the previous chapters, the following conclusions are made:

1. Using conventional procedures to predict the maximum load-carry capacity of a reinforced concrete structure is not only inaccurate, but also nearly impossible when the structure is complicated or when large deformations beyond the point of maximum load are to be calculated. Thus, the proposed mathematical method for predicting the ultimate state of a structure is significant and convenient for computer use. The criterion developed in this report can be employed for full-range nonlinear analysis of multi-dimensional reinforced concrete structures.

2. The mathematical criterion can be used for either elastic-perfectly plastic or general elastic-plastic material structures. The elastic-perfectly plastic material is only a special case in the application of the criterion.

3. According to the User's Guide (Appendix A) of the subroutine containing the mathematical criterion, the PREUSS subroutine may be easily intergrated with any other program to provide full-range nonlinear analysis of reinforced concrete structures. The PREUSS subroutine gives information about whether the structure has reached the maximum load state.

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4. General computer programs for the full-range nonlinear analysis, or collapse analysis, of reinforced concrete structure may be developed as long as the mathematical criterion for predicting the maximum load-carry capacity of the structure and element models which reasonably represent descending branch stiffness are used. The program developed in this report can be used to investigate the behavior of reinforced concrete frames at all loading stages, such as yielding, maximum load and the descending range.

5. Even if it goes into the descending strength range, a ductile moment-resisting frame still has good deformation and load-carry capacity until the mechanism state is reached. Therefore, for earthquake resistant design the mechanism state of a structure may be regarded as the ultimate state in order to fully utilize the energy dissipation of the structure.

6. When using a general elasto-plastic material relationship, the maximum load-carry capacity of a reinforced concrete frame does not depend on either the number of the yield plastic hinges or ultimate plastic hinges. In other words, the mechanism condition based on elastic-perfectly plastic material is unsuitable in this case.

7. Under large deformations structural behavior, such as the load-carry capacity, deformability, mechanism state and energy dissipation, is sensitive to the stiffness ratio, \( \alpha \), of the descending branch in the moment vs. rotation relationship. Thus, when a full-range nonlinear analysis or collapse analysis of reinforced concrete frame is to be carried
out, it is necessary to investigate the stiffness value of the descending branch in the member moment vs. relationship to correctly evaluate structure behavior during large deformations.

8. The joint flexural strength ratio \( M_R \) has a great influence on structure strength, deformability and even plastic hinge distribution. In order to increase the load-carry capacity and deformability of a structure, it is necessary to increase the value of ratio \( M_R \) in the lower stories and decrease the value of \( M_R \) in the upper stories.
REFERENCES

1. R. Park and T. Paulay, "Reinforced Concrete Structure." 1975


3. Okamoto, S., Wight, J. K., Nakata, S., Yoshimura, M. and Kaminosono, T., "Original testing; Repair and Strengthening; and Retesting of the Full Scale Structure" Earthquake Effect on Reinforced Concrete Structures, SP-84, American Concrete Institute, Detroit, MI, March, 1985.


5. Gervenka, V. and Gerstle, K. H., "Inelastic Analysis of Reinforced Concrete Panels." Proceedings of International Association for Bridge and Structure Engineering, Vol. 31-II, 1971 (1,2) and Vol. 32-II, 1972 (1,2).


22. Bathe, K. J. and Ramaswamy, S. "On Three-Dimensional Nonlinear
Analysis of Concrete Structures", Nuclear Engineering and Design. 52. 1979.


Appendix A

USER'S GUIDE FOR SUBROUTINE PREUSS

Writte in the FORTRAN language, PREUSS is a subroutine for predicting the maximum load-carry capacity of reinforced concrete structures. By using CALL statements, the subroutine may be conveniently connected with any computer program which is used to do full-range nonlinear analysis of reinforced concrete structures.

The contents and use of the subroutine PREUSS are explained as follows:

SUBROUTINE PREUSS

Purpose: Predict the maximum load-carry capacity of a structure. The subroutine is based on incremental theory in plasticity and the finite element method.

Format: CALL PREUSS (GSTIF,ASTIF,BSTIF,CSTIF,ESTIF,LNODS,EIGNV,
               *         NCFIX,HLOAD,BUNIT,CUNIT,LUNIT,SUNIT,MDISP,
               *         NBWID,NDISP,NLOAD,MVFIX,NELEN,NELEM,MDOFN,
               *         NNODE,MCORL)

Remark: 1. ARRAYS:

    GSTIF(MDISP,NBWID) -- Incremental stiffness matrix of the
structure. GSTIF is stored in half-band width form and it is not necessary to input data for GSTIF when calling PREUSS.

ASTIF(NDISP,NDISP), BSTIF(NLOAD,NDISP), CSTIF(NLOAD,NLOAD)
-- Work arrays. It is not necessary to input this data when calling PREUSS.

ESTIF(NELEN,MDOFN,MDOFN) -- Incremental stiffness matrix of an element. It is necessary to input data for ESTIF when calling PREUSS.

LNODS(NELEN,NNODE) -- Nodal index array of element. It is necessary to input this data when calling PREUSS.

EIGNV(NLOAD) -- Eigenvalue array. The calculated eigenvalues are returned to the MAIN program.

NCFIX(MVFIX) -- Index array of fixed deformation components of the structure. It is necessary to input this data when calling PREUSS.

HLOAD(NLOAD) -- Index array of deformation components corresponding to loading directions. It is necessary to input this data when calling PREUSS.

BUNIT(NDISP),CUNIT(NDISP), LUNIT(NDISP), SUNIT(NDISP) -- Work
arrays. It is not necessary to input this data when calling PREUSS.

2. VARIABLES:

MDISP -- Total number of deformation components of whole structure.

NBWID -- Half-band width of the structural stiffness matrix.

NDISP -- Total number of deformation components; no relationship with loading directions

NLOAD -- Total number of deformation components corresponding to loading directions (or total number of loads).

MVFIX -- Total number of the fixed deformation components.

NELEN -- The dimension of array ESTIF and LNODS. It must be equal to or larger than the total number of the elements in the structure.

NELEM -- Total number of the elements in the structure.

MDOFN -- Total number of deformation components of the element.
NNODE -- Total number of the element nodes.

MCORL -- Control parameter. It is returned to the MAIN program.

MCORL = 0  Normal Condition;
MCORL = 1  Structure has reached maximum load-carry capacity.

NOTE: All variables must be input data when calling PREUSS.

The FORTRAN listing of the subroutine PREUSS is presented in Appendix B. Two simple examples are given to illustrate calling the subroutine PREUSS.

Example 1. Figure A.1 shows a framed structure to be subjected to both lateral and gravity loads. During loading, the lateral loads gradually increase and the gravity loads remains constant.

**Input data preparation:**

(1) During loading, the incremental stiffness matrices of elements, ESTIF, are changed according to the internal forces at the element ends. After the sixth loading, the first ultimate plastic hinge appears in the structure and the stiffness matrices of the elements, ESTIF, are as follows:
### Element 1

\[
\begin{bmatrix}
0.33105 & 0.0 & 74.2857 & \text{Symmetric} \\
-4.07753 & 0.0 & -285.427 \\
-0.33105 & 0.0 & 4.07753 & 0.33105 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
50.4249 & 0.0 & -285.427 & -50.4249 & 0.0 & 7344.92
\end{bmatrix}
\]

### Element 2

\[
\begin{bmatrix}
0.443913 & 0.0 & 74.2857 & \text{Symmetric} \\
6.11629 & 0.0 & 570.854 \\
-0.443913 & 0.0 & -6.11629 & 0.443913 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
56.0315 & 0.0 & 285.427 & -56.0315 & 0.0 & 7558.99
\end{bmatrix}
\]

### Element 3

\[
\begin{bmatrix}
0.346655 & 0.0 & 74.2857 & \text{Symmetric} \\
43.3663 & 0.0 & 5830.23 \\
-0.346655 & 0.0 & -43.3663 & 0.346655 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
5.16546 & 0.0 & 241.055 & -5.16546 & 0.0 & 482.110
\end{bmatrix}
\]

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Element 4.

\[
\begin{pmatrix}
0.346655 \\
0.0 & 74.2857 & \text{Symmetric} \\
43.3663 & 0.0 & 5830.23 \\
-0.346655 & 0.0 & -43.3663 & 0.346655 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
5.16546 & 0.0 & 241.055 & -5.16546 & 0.0 & 482.110
\end{pmatrix}
\]

Element 5.

\[
\begin{pmatrix}
45.0 \\
0.0 & 0.01029 & \text{Symmetric} \\
0.0 & -1.33799 & 231.918 \\
-45.0 & 0.0 & 0.0 & 45.0 \\
0.0 & -0.01029 & 1.33799 & 0.0 & 0.01029 \\
0.0 & -1.33799 & 115.959 & 0.0 & 1.33799 & 231.918
\end{pmatrix}
\]

Element 6.

\[
\begin{pmatrix}
45.0 \\
0.0 & 0.401280 & \text{Symmetric} \\
0.0 & -52.1662 & 9042.13 \\
-45.0 & 0.0 & 0.0 & 45.0 \\
0.0 & -0.40128 & 52.1662 & 0.0 & 0.40128 \\
0.0 & -52.1662 & 4521.07 & 0.0 & 52.1662 & 9042.13
\end{pmatrix}
\]
(2) Nodal index array of the elements, LNODS:

\[
\begin{bmatrix}
1 & 3 \\
2 & 4 \\
3 & 5 \\
4 & 6 \\
3 & 4 \\
5 & 6 \\
\end{bmatrix}
--- element 1.
--- element 2.
--- element 3.
--- element 4.
--- element 5
--- element 6

(3) Index array of fixed deformation components of the structure, NCFIX:

\[ NCFIX = [ 1 \ 2 \ 3 \ 4 \ 5 \ 6 ]^T \]

(4) Index array of deformation components corresponding to loading directions, HLOAD:

\[ HLOAD = [ 7.0 \ 13.0 ]^T \]

(5) Variables:

\[
\begin{align*}
MDISP &= 18 \\
NBWID &= 9 \\
NDISP &= 10 \\
NLOAD &= 2 \\
MVFIX &= 6 \\
NELEN &= 10 \\
\end{align*}
\]
\[
\begin{align*}
\text{NELEM} & = 6 \\
\text{MDOFN} & = 6 \\
\text{NNODE} & = 2 \\
\text{MCORL} & = 0
\end{align*}
\]

By inputing all data presented above and calling the subroutine \text{PREUSS}, the eigenvalues of the structural stiffness matrix are calculated and returned to the \text{MAIN} program. The output results are written as follows:

\[
\begin{align*}
\text{ILOAD} & = 1 \quad \text{EIGENVALUE} = \quad 182418+E05 \\
\text{ILOAD} & = 2 \quad \text{EIGENVALUE} = \quad 185634+E03 \\
\text{MCORL} & = 0
\end{align*}
\]

Because the all eigenvalues are larger than zero, according to the criterion, the control parameter \text{MCORL} is equal to zero and the structure is still in a stable state.

Example 2. After the ninth loading, the incremental stiffness matrices of the elements in the structure from Example 1 become:

Element 1.

\[
\begin{pmatrix}
0.331050 \\
0.0 & 74.2857 & \text{Symmetric} \\
-4.07753 & 0.0 & -285.427 \\
-0.33105 & 0.0 & 4.07753 & 0.33105 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
50.4249 & 0.0 & -285.427 & -50.4249 & 0.0 & 7344.92
\end{pmatrix}
\]

65
Element 2.

\[
\begin{bmatrix}
0.331050 \\
0.0 & 74.2857 & \text{Symmetric} \\
-4.07753 & 0.0 & -285.427 \\
-0.33105 & 0.0 & 4.07753 & 0.33105 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2853 \\
50.4249 & 0.0 & -285.427 & -50.4249 & 0.0 & 7344.92
\end{bmatrix}
\]

Element 3.

\[
\begin{bmatrix}
0.263640 \\
0.0 & 74.2857 & \text{Symmetric} \\
39.4922 & 0.0 & 5649.44 \\
-0.26364 & 0.0 & -39.4922 & 0.26364 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
-2.58273 & 0.0 & -120.528 & 2.58273 & 0.0 & -241.055
\end{bmatrix}
\]

Element 4.

\[
\begin{bmatrix}
0.346660 \\
0.0 & 74.2857 & \text{Symmetric} \\
43.3663 & 0.0 & 5830.23 \\
-0.34666 & 0.0 & -43.3663 & 0.34666 \\
0.0 & -74.2857 & 0.0 & 0.0 & 74.2857 \\
5.16546 & 0.0 & 241.055 & -5.16546 & 0.0 & 482.110
\end{bmatrix}
\]

66
Element 5.

\[
\begin{array}{ccc}
45.0 & & \\
0.0 & -0.001287 & \text{Symmetric} \\
0.0 & 0.668993 & -115.959 \\
10^4 \cdot & -45.0 & 0.0 & 0.0 & 45.0 \\
0.0 & 0.001287 & -0.668993 & 0.0 & -0.001287 \\
0.0 & -0.33450 & -57.9794 & 0.0 & 0.33450 & 144.979
\end{array}
\]

Element 6.

\[
\begin{array}{ccc}
45.0 & & \\
0.0 & 0.401280 & \text{Symmetric} \\
0.0 & -52.1662 & 9042.13 \\
10^4 \cdot & -45.0 & 0.0 & 0.0 & 45.0 \\
0.0 & -0.40128 & 52.1662 & 0.0 & 0.40128 \\
0.0 & -52.1662 & 4521.07 & 0.0 & 52.1662 & 9042.13
\end{array}
\]

Inputing the new stiffness matrices of the elements and calling PREUSS, the output eigenvalues are given as follows:

\[
\begin{align*}
\text{ILOAD} & = 1 \quad \text{EIGENVALUE} = 0.158347 + 0.00E05 \\
\text{ILOAD} & = 2 \quad \text{EIGENVALUE} = -0.557231 + 0.00E02 \\
\text{MCORL} & = 1 \\
** \text{STRUCTURE HAS REACHED MAXIMUM LOAD} **
\end{align*}
\]
Because one of the eigenvalues is less than zero, the control parameter, MCORL, is equal to 1 and the structure has reached the maximum load-carry capacity. Therefore, in the full-range nonlinear analysis of the structure, a negative load increment should be applied to the structure at the tenth loading.

These examples show that the general subroutine PREUSS is easily called to predict the maximum load-carry capacity of a structure when doing a full-range nonlinear analysis. The MAIN program employed to call the subroutine PREUSS is presented in Appendix C.
APPENDIX B.

LISTING OF SUBROUTINE PREUSS

C ******************************************************************************
C * SUBROUTINE: PREUSS                                           *
C * PREDICT THE ULTIMATE STATE OF ELASTO-PLASTIC STRUCTURE.      *
C * ANALYSIS METHODS ARE REPORTED IN CHAPTER 3                   *
C ******************************************************************************

SUBROUTINE PREUSS (GSTIF,ASTIF,BSTIF,CSTIF,ESTIF,LNODS,EIGNV,
  *     NCFIX,HLOAD,BUNIT,CUNIT,LUNIT,SUNIT,MDISP,
  *     NBWID,NDISP,NLOAD,MVFIX,NELEN,NELEM,MDOFN,
  *     NNODE,MCORL)
DIMENSION GSTIF(MDISP,NBWID),ASTIF(NDISP,NDISP),BSTIF(NLOAD,
  *     NDISP),CSTIF(NLOAD,NLOAD),ESTIF(NELEN,MDOFN,MDOFN),
  *     LNODS(NELEN,NNODE),EIGNV(NLOAD),NCFIX(MVFIX),
  *     HLOAD(NLOAD)
DIMENSION BUNIT(NDISP),CUNIT(NDISP),LUNIT(NDISP),SUNIT(NLOAD,
  *     NLOAD)
DATA EPSSS/0.000001/

C ----------------------------------- ASSEMBLE STRUCTURE STIFFNESS MATRIX WITH HELLOW-BAND WIDTH -----------------------------------

DO 30 IDISP=1,MDISP
DO 30 JDISP=1,NBWID
30 GSTIF(IDISP,JDISP)=0.0
DO 40 IELEM=1,NELEM
DO 40 INODE=1,2
DO 40 IDOFN=1,3
KDOFE=3*(INODE-1)+IDOFN
KDOFS=3*(LNODS(IELEM,INODE)-1)+IDOFN
DO 40 JNODE=1,2
DO 40 JDOFN=1,3
MDOFE=3*(JNODE-1)+JDOFN
MDOFS=3*(LNODS(IELEM,JNODE)-1)+JDOFN
NEWDF=MDOFS-KDOFS+1
IF (NEWDF.LE.0) GOTO 40
GSTIF(KDOFS,NEWDF)=GSTIF(KDOFS,NEWDF)+ESTIF(IELEM,KDOFE,MDOFE)
40 CONTINUE
C --- DIVIDING STRUCTURE STIFFNESS MATRIX [GSTIF] INTO SUBMATRIX --
C --- [ASTIF], [BSTIF] AND [CSTIF] --
C

KASTF=0
KBSTF=0
IFIXA=1
ILOAD=1
DO 130 IDISP=1,MDISP
   IF(IDISP.EQ.NCFIX(IFIXA)) GOTO 120
   IF(IDISP.EQ.IFIXHLOAD(ILOAD)) GOTO 80
   KASTF=KASTF+1
   LASTF=0
   JFIXA=1
   JLOAD=1
   DO 70 JDISP=1,MDISP
      IF(JDISP.EQ.NCFIX(IFIXA)) GOTO 50
      IF(JDISP.EQ.IFIXHLOAD(JLOAD)) GOTO 60
      LASTF=LASTF+1
      KDISP=JDISP-IDISP+1
      IF(KDISP.LE.0) GOTO 70
      IF(KDISP.GT.NBWID) ASTIF(KASTF,LASTF)=0.0
      IF(KDISP.LE.NBWID) ASTIF(KASTF,LASTF)=GSTIF(IDISP,KDISP)
   GOTO 70
50   IF(JFIXA.NE.MVFIX) JFIXA=JFIXA+1
   GOTO 70
60   IF(JLOAD.NE.NLOAD) JLOAD=JLOAD+1
70   CONTINUE
    GOTO 130
80   IF(ILOAD.NE.NLOAD) ILOAD=ILOAD+1
       KBSTF=KBSTF+1
       LBSTF=0
       LCSTF=0
       IFIXB=1
       KLOAD=1
       DO 110 JDISP=1,MDISP
          IF(JDISP.EQ.NCFIX(IFIXB)) GOTO 100
          IF(JDISP.EQ.IFIXHLOAD(KLOAD)) GOTO 90
          LBSTF=LBSTF+1
          KDISP=JDISP-IDISP+1
          IF(KDISP.GT.0) GOTO 85
KDISP=IDISP-JDISP+1
BSTIF(KBSTF,LBSTF)=GSTIF(JDISP,KDISP)
GOTO 110

85 IF(KDISP.GT.NBWD) BSTIF(KBSTF,LBSTF)=0.0
IF(KDISP.LE.NBWD) BSTIF(KBSTF,LBSTF)=GSTIF(IDISP,KDISP)
GOTO 110

90 IF(KLOAD.NE.NLOAD) KLOAD=KLOAD+1
LCSTF=LCSTF+1
KDISP=JDISP-IDISP+1
IF(KDISP.LE.0) GOTO 110
IF(KDISP.GT.NBWD) CSTIF(KBSTF,LCSTF)=0.0
IF(KDISP.LE.NBWD) CSTIF(KBSTF,LCSTF)=GSTIF(IDISP,KDISP)
GOTO 110

100 IF(IFIXB.NE.MVFIX) IFIXB=IFIXB+1
110 CONTINUE
GOTO 130

120 IF(IFIXA.NE.MVFIX) IFIXA=IFIXA+1
130 CONTINUE
DO 140 IDISP=1,NDISP
DO 140 JDISP=IDISP,NDISP

140 ASTIF(JDISP,IDISP)=ASTIF(IDISP,JDISP)
DO 150 ILOAD=1,NLOAD
DO 150 JLOAD=ILOAD,NLOAD

150 CSTIF(JLOAD,ILOAD)=CSTIF(ILOAD,JLOAD)
C -------------------------------
C -- INVERSE OF MATRIX [ASTIF] --
C -------------------------------

160 DO 160 IDISP=1,NDISP
LUNIT(IDISP)=IDISP
DO 230 IDISP=1,NDISP
KA1=IDISP+1
Y=ASTIF(IDISP,IDISP)
J0=IDISP
IF(IDISP.EQ.NDISP) GOTO 180
DO 170 JDISP=KA1,NDISP
W=ASTIF(IDISP,JDISP)
IF(ABS(W).LE.ABS(Y)) GOTO 170
Y=W
J0=JDISP
170 CONTINUE
180 IF(ABS(Y).GE.EPSS) GOTO 200
WRITE(6,190) Y
FORMAT(4HSTOP,5X,40HINVERT MATRIX OF "ASTIF" IS ILL CONTION, * 3HY= ,E15.6)
STOP

Y=1.0/Y
DO 210 JDISP=1,NDISP
CUNIT(JDISP)=ASTIF(JDISP,J0)
ASTIF(JDISP,J0)=ASTIF(JDISP,IDISP)
ASTIF(JDISP,IDISP)=-CUNIT(JDISP)*Y
BUNIT(JDISP)=ASTIF(IDISP,JDISP)*Y
ASTIF(IDISP,JDISP)=BUNIT(JDISP)
ASTIF(IDISP,IDISP)=Y
I=LUNIT(IDISP)
LUNIT(IDISP)=LUNIT(J0)
LUNIT(J0)=1
DO 230 JDISP=1,NDISP
IF(JDISP.EQ.IDISP) GOTO 230
DO 220 LDISP=1,NDISP
IF(LDISP.EQ.IDISP) GOTO 220
ASTIF(JDISP,LDISP)=ASTIF(JDISP,LDISP)-CUNIT(JDISP)*BUNIT(LDISP)
CONTINUE

NS1=NDISP-1
DO 260 IDISP=1,NS1
IF(LUNIT(IDISP).EQ.IDISP) GOTO 260
K=IDISP+1
DO 250 JDISP=KA1,NDISP
IF(LUNIT(JDISP).NE.IDISP) GOTO 250
DO 240 LDISP=1,NDISP
W=ASTIF(JDISP,LDISP)
ASTIF(JDISP,LDISP)=ASTIF(IDISP,LDISP)
W=ASTIF(IDISP,LDISP)=W
LUNIT(JDISP)=LUNIT(IDISP)
GOTO 260
CONTINUE

C -----------------------------------------------
C -- [BSTIF] * INVERSE MATRIX OF [ASTIF] * --
C -- TRANSPOSE MATRIX OF [BSTIF] --
C -----------------------------------------------

DO 270 ILOAD=1,NLOAD
DO 270 JLOAD=1,NLOAD
SUNIT(ILOAD,JLOAD)=0.0
DO 270 IDISP=1,NDISP
DO 270 JDISP=1,NDISP
270 SUNIT(ILOAD,JLOAD)=SUNIT(ILOAD,JLOAD)+BSTIF(ILOAD,IDISP)*
   ASTIF(IDISP,JDISP)*BSTIF(JLOAD,JDISP)

C -------------------------------------
C     MATRIX [CSTIF] - MATRIX [SUNIT]  
C -------------------------------------
DO 300 ILOAD=1,NLOAD
DO 300 JLOAD=1,NLOAD
300 CSTIF(ILOAD, JLOAD)=CSTIF(ILOAD, JLOAD) - SUNIT(ILOAD, JLOAD)
   IF (NLOAD.EQ.1) EIGNV(NLOAD)=CSTIF(NLOAD, NLOAD)
   IF (NLOAD.EQ.1) GOTO 510

C -------------------------------------
C     CALCULATE ALL EIGENVALUES OF MATRIX [CSTIF] 
C -------------------------------------
DO 310 ILOAD=1,NLOAD
   EIGNV(ILOAD)=CSTIF(ILOAD, ILOAD)
   BUNIT(ILOAD)=EIGNV(ILOAD)
310 CUNIT(ILOAD)=0.0
   IROTH=0
   DO 500 I=1,50
      SM=0.0
      NM1=NLOAD-1
   DO 320 ILOAD=1,NM1
   DO 320 JLOAD=1,NLOAD
320 SM=SM+ABS(CSTIF(ILOAD, JLOAD))
   IF (SM) 330,510,330
330 IF (I-4) 340,350,350
340 TRESH=0.2*SM/(FLOAT(NLOAD)*FLOAT(NLOAD))
   GOTO 360
350 TRESH=0
350 DO 490 ILOAD=1,NM1
   IPP1=ILOAD+1
   DO 490 JLOAD=IPP1,NLOAD
   G=100.0*ABS(CSTIF(ILOAD, JLOAD))
   IF (I.GT.4 .AND. ABS(EIGNV(ILOAD))+G.EQ.ABS(EIGNV(ILOAD))) .AND.  
   ABS(EIGNV(JLOAD))+G.EQ.ABS(EIGNV(JLOAD)) .LE. TRESH) GOTO 480
   IF (ABS(CSTIF(ILOAD, JLOAD)) .LE. TRESH) GOTO 490
490
H=EIGNV(JLOAD)-EIGNV(ILOAD)
IF(ABS(H)+G.EQ.ABS(H)) GOTO 370
THETA=0.5*H/CSTIF(ILOAD,JLOAD)
T=1.0/(ABS(THETA)+SQRT(1.0+THETA*THETA))
IF(THETA.LT.0.0) T=-T
GOTO 380
370 T=CSTIF(ILOAD,JLOAD)/H
380 C=1.0/SQRT(1.0+T*T)
S=T*C
H=T*CSTIF(ILOAD,JLOAD)
CUNIT(ILOAD)=CUNIT(ILOAD)-H
CUNIT(JLOAD)=CUNIT(JLOAD)+H
EIGNV(ILOAD)=EIGNV(ILOAD)-H
EIGNV(JLOAD)=EIGNV(JLOAD)+H
CSTIF(ILOAD,JLOAD)=0.0
IPM1=ILOAD-1
IF(IPM1) 410,410,390
390 DO 400 KLOAD=1,IPM1
G=CSTIF(KLOAD,ILOAD)
H=CSTIF(KLOAD,JLOAD)
CSTIF(KLOAD,ILOAD)=C*G-S*H
400 CSTIF(KLOAD,JLOAD)=S*G+C*H
410 IQM1=JLOAD-1
IF(IQM1-IQP1) 440,420,420
420 DO 430 KLOAD=IQP1,IQM1
G=CSTIF(ILOAD,KLOAD)
H=CSTIF(KLOAD,JLOAD)
CSTIF(ILOAD,KLOAD)=C*G-S*H
430 CSTIF(KLOAD,JLOAD)=S*G+C*H
440 IQP1=JLOAD+1
IF(NLOAD-IQP1) 470,450,450
450 DO 460 KLOAD=IQP1,NLOAD
G=CSTIF(ILOAD,KLOAD)
H=CSTIF(JLOAD,KLOAD)
CSTIF(ILOAD,KLOAD)=C*G-S*H
460 CSTIF(JLOAD,KLOAD)=S*G+C*H
470 IROTH=IROTH+1
GOTO 490
480 CSTIF(ILOAD,JLOAD)=0.0
490 CONTINUE
DO 500 ILOAD=1,NLOAD
BUNIT(ILOAD) = BUNIT(ILOAD) + CUNIT(ILOAD)
EIGNV(ILOAD) = BUNIT(ILOAD)

500  CUNIT(ILOAD) = 0.0
510  DO 520 ILOAD = 1, NLOAD
     IF (EIGNV(ILOAD).GT.0.0) GOTO 520
     MCORL = 1
     GOTO 530
520  CONTINUE
530  RETURN

END
APPENDIX  C.

LISTING OF MAIN PROGRAM

C -----------------------
C   MAIN PROGRAM   
C -----------------------

DIMENSION Gstif(100,20),Astif(30,30),BSTIF(10,30),CSTIF(10,10)
  *   ESTIF(10,6,6),LNODS(10,2),EIGNV(10),NCFIX(20),
  *      HLOAD(10),BUNIT(30),CUNIT(30),LUNIT(30),SUNIT(10,10)

C -----------------------
READ(5,1) MDISP,NBWID,NDISP,NLOAD,MVFIX,NELEN,NELEM,MDOFN,
  *      NNODE,MCORL
1 FORMAT(10I5)
DO 11 IELEM=1,NELEM
READ(5,2)(ESTIF(IELEM,I,J),J=1,I),J=1,MDOFN
DO 10 I=1,MDOFN
DO 10 J=1,I
10 ESTIF(IELEM,J,I)=ESTIF(IELEM,I,J)
11 CONTINUE
2 FORMAT(3E15.6)
READ(5,3)((LNODS(IELEM,I),I=1,NNODE),IELEM=1,NELEM)
3 FORMAT(2I5)
READ(5,4)(NCFIX(IVFIX),IVFIX=1,MVFIX)
4 FORMAT(3I5)
READ(5,5)(HLOAD(ILOAD),ILOAD=1,NLOAD)
5 FORMAT(5F5.1)

C -----------------------
CALL PREUSS (GSTIF,ASTIF,BSTIF,CSTIF,ESTIF,LNODS,EIGNV,NCFIX,
  *          HLOAD,BUNIT,CUNIT,LUNIT,SUNIT,MDF,NDISP,
  *          NLOAD,MVFIX,NELEN,MELEM,MDOFN,NNODE,MCORL)

C -----------------------
WRITE(6,101)(ILOAD,EIGNV(ILOAD),ILOAD=1,NLOAD)
101 FORMAT(/10X,‘ILOAD =’,I5,3X,‘EIGENVALUE =’,E15.6)
WRITE(6,102) MCORL
102 FORMAT(/10X,‘MCORL =’,I5)
IF(MCORL.EQ.0) GOTO 100
WRITE(6,103)
103 FORMAT(/10X,‘** STRUCTURE HAS REACHED MAXIMUM LOAD **’)
100 STOP
END

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Fig. 3.1 P - U Relationship
Fig. 4.1 Moment - Rotation Relationship
Fig. 4.2 Idealized Stress – Strain Relationships for Steel and Concrete
Fig. 4.3 Deformation and Curvature Distribution of Beam - Column Element
Fig. 4.4  Extended Clough Beam - Column Element
Fig. 5.1 Study Frames
FLOOR 5. LOAD-DISPLACEMENT

Figure 6.1 (a) Load - Displacement of FRAME 1
FLOOR 5. LOAD-DISPLACEMENT

Figure 6.1 (b) Load - Displacement of FRAME 2
STORY 1. SHEAR-DRIFT

Figure 6.2 (a) Shear - Drift of FRAME 1
1. FIRST U.P.H.
2. MAXIMUM LOAD
3. MECHANISM STATE.

STORY 1. SHEAR-DRIFT

Figure 6.2 (b) Shear - Drift of FRAME 2
Fig. 6.3 Plastic Hinge Distribution of FRAME 1
Fig. 6.4 Plastic Hinge Distribution of FRAME 2
FLOOR 5. LOAD-DISPLACEMENT

Figure 6.5 Comparison of Load - Displacement (Ratio $\alpha$)
Figure 6.6 Comparison of Shear - Drift (Ratio $\alpha$)
1. STIFFNESS RATIO = -0.3
2. STIFFNESS RATIO = -0.5
3. STIFFNESS RATIO = -0.7

Figure 6.7 (a) Displacements along Structure Height at Maximum Load State (Ratio $\alpha$)
1. STIFFNESS RATIO = -0.3

2. STIFFNESS RATIO = -0.5

3. STIFFNESS RATIO = -0.7

STORY NO.

Figure 6.7 (b) Displacements along Structure Height at Mechanism State (Ratio $\alpha$)
FLOOR 5. LOAD-DISPLACEMENT

Figure 6.8 Comparison of Load - Displacement (Ratio $M_R$)
STORY 1. SHEAR-DRIFT

Figure 6.9 (a) Comparison of Shear - Drift of Story 1 (Ratio $M_R$)
STORY 4. SHEAR-DRIFT

Figure 6.9 (b) Comparison of Shear - Drift of Story 4 (Ratio $M_R$)
Figure 6.10 (a) Displacement along Structure Height at Maximum Load State (Ratio $M_R$)
Figure 6.10 (b) Displacement along Structure Height at Mechanism State (Ratio $M_R$)
Figure A.1  Example 1 and 2