Azimuth and Elevation Direction Finder Techniques

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ABSTRACT

A block diagram of the proposed DF azimuth - elevation direction finder is presented and discussed. A simple analysis is presented for a two-dimensional direction finder and is extended into a three-dimensional system. As for the three-dimensional azimuth - elevation direction finder analysis, a number of computer programs have been utilized to determine the accuracy of the system. In general it has been found that the system accuracy is strongly dependent upon the element pattern characteristics (i.e., it is important that they have broadband radiation pattern characteristics both as a function of element orientation and frequency). Further, it has been found that because of the elements being distributed over $2\pi$ steradians that the predicted elevation angles have associated with them a discrepancy from the true angle. This discrepancy has been found to be predictable and can easily be handled by the computer to provide the true angle. Discussions have been held with several computer representatives and it has been determined that it is feasible to employ a small computer to perform the necessary data processing to obtain the required azimuth - elevation data from the hemispherical antenna system.
FOREWORD

This report was prepared by The University of Michigan Radiation Laboratory of the Department of Electrical Engineering under United States Army Electronics Command Contract No. DAAB07–67–C0547. This contract was initiated under United States Army Project No. 5A6 79191 D902–05–11 "Azimuth and Elevation Direction Finder Techniques". The work is administered under the direction of the Electronics Warfare Division, Advanced Techniques Branch at Fort Monmouth, New Jersey. Mr. S. Stiber is the Project Manager and Mr. E. Ivone is the Contract Monitor.

The authors wish to express their thanks to Messrs. A. Loudon, D. Marble and E. Bublitz for their efforts in the experimental work that has been performed during this reporting period, and to Mr. P. Wilcox for preparing the computer program required for the three-dimensional analysis.

The material reported herein represents the results of the preliminary investigation into the study of techniques designing a broadband circularly polarized azimuth and elevation direction finder antenna.
TABLE OF CONTENTS

ABSTRACT ii
FOREWORD iii
LIST OF ILLUSTRATIONS v
I INTRODUCTION 1
II A SIMPLE METHOD FOR DIRECTION FINDING 8
  2.1 Example of Direction Finding 10
III ANALYTICAL ANALYSIS OF THE AZIMUTH – ELEVATION DIRECTION FINDER 14
  3.1 Data Requirements for the DF data Processing System 14
  3.2 Analytical Results for a Three-Dimensional DF System 19
  3.3 Consideration of Antenna Number and Location 31
IV RIGOROUS METHODS OF DIRECTION FINDING IN THREE DIMENSIONS USING DATA PROCESSING EQUIPMENT 37
  4.1 Three-Dimensional Radiation Pattern of an Individual Antenna 37
  4.2 Vector Addition Method 1 39
  4.3 Vector Addition Method 2 45
  4.4 Determination of the Constants, A, B, C and D for a Specific Distribution 50
  50
V ELEMENT INVESTIGATION 51
  5.1 Spiral Geometry 51
  5.2 Spiral Configuration 52
  5.3 Balun Considerations 53
VI ELECTROMECHANICAL SWITCH 55
VII COMPUTER REQUIREMENTS 56
VIII CONCLUSIONS 59
REFERENCES 61
DD FORM 1473
# List of Illustrations

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Azimuth – Elevation Direction Finder</td>
<td>2</td>
</tr>
<tr>
<td>1-2</td>
<td>Spherical Coordinate System</td>
<td>4</td>
</tr>
<tr>
<td>1-3</td>
<td>Azimuth – Elevation Direction Finder Simplified Computer Flow Design</td>
<td>7</td>
</tr>
<tr>
<td>2-1</td>
<td>Vector Geometry</td>
<td>9</td>
</tr>
<tr>
<td>2-2</td>
<td>Geometry of a Ring of 8 Antennas</td>
<td>10</td>
</tr>
<tr>
<td>2-3</td>
<td>Spiral Antenna Pattern Data for Calculations (Freq. = 1.8 GHz)</td>
<td>13</td>
</tr>
<tr>
<td>3-1</td>
<td>Two-Dimensional Direction Finding with some number &quot;n&quot; of Equally Spaced Antennas</td>
<td>15</td>
</tr>
<tr>
<td>3-2</td>
<td>Co-ordinate System for Azimuth – Elevation Direction Finding Calculations</td>
<td>21</td>
</tr>
<tr>
<td>3-3</td>
<td>Geometry of Azimuth – Elevation Direction Finder with 17 Antennas, Note Antennas Align on Rings in Elevation Plane</td>
<td>22</td>
</tr>
<tr>
<td>3-4</td>
<td>Computed Elevation Angle vs. Actual Elevation Angle for Cosine Antenna Pattern, 17 Antennas</td>
<td>23</td>
</tr>
<tr>
<td>3-5</td>
<td>Graph of Various Types of Far-Field Antenna Patterns Considered in Computer Programs</td>
<td>24</td>
</tr>
<tr>
<td>3-6</td>
<td>Computed Elevation Angle vs. Actual Elevation Angle for Linear 90 Antenna Patterns, 17 Antennas</td>
<td>25</td>
</tr>
<tr>
<td>3-7</td>
<td>Computed Elevation Angle vs. Actual Elevation Angle for Linear 80 Pattern, 17 Antennas</td>
<td>26</td>
</tr>
<tr>
<td>3-8</td>
<td>Computed Elevation Angle vs. Actual Elevation Angle for Linear 70 Antenna Pattern, 17 Antennas</td>
<td>27</td>
</tr>
<tr>
<td>3-9</td>
<td>Computed Elevation Angle vs. Actual Elevation Angle for Cosine Patterns Skewed $10^\circ$ off Normal, 17 Antennas</td>
<td>29</td>
</tr>
<tr>
<td>3-10</td>
<td>Maximum Error in Azimuth vs. the Elevation Angle $\theta$ for Cosine Pattern, 17 Antennas</td>
<td>30</td>
</tr>
<tr>
<td>3-11</td>
<td>Maximum Azimuth Error vs. the Elevation Angle for Cosine Pattern Skeewed $10^\circ$, 17 Antennas</td>
<td>32</td>
</tr>
<tr>
<td>3-12</td>
<td>Spherical Array – X's Mark Elements Utilized for Icosahdron Direction Finding Calculation</td>
<td>33</td>
</tr>
</tbody>
</table>
List of Illustrations, Continued

<table>
<thead>
<tr>
<th>Page</th>
<th>Illustration Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-13</td>
<td>Maximum Azimuth Error vs. Elevation Angle for Icosahedron Geometry with Cosine Patterns</td>
</tr>
<tr>
<td>3-14</td>
<td>Error in Azimuth vs. The Azimuth Angle $\phi$ for $\theta = 80^0$, 17 Elements with Cosine Patterns</td>
</tr>
<tr>
<td>4-1</td>
<td>Local Coordinate System Used</td>
</tr>
<tr>
<td>4-2</td>
<td>Distribution of Antennas on a Hemisphere</td>
</tr>
<tr>
<td>4-3</td>
<td>Illuminated Antennas for all $\theta$ and $\frac{-\beta}{2} \leq \phi \leq \frac{\beta}{2}$</td>
</tr>
</tbody>
</table>
I

INTRODUCTION

Below we present a description of the operating characteristics for the azimuth and elevation direction finder, presently being designed and fabricated by the Radiation Laboratory. A block diagram of the system is shown in Fig. 1-1 consisting of an antenna, electromechanical switch, receiver (GFE), an Automatic Recognition Signal Unit (GFE), an A to D converter, computer, and a visual display.

The antenna is conceived to consist of a hemispherical surface to which will be attached a number of antennas. Each antenna will have associated with it a particular $\theta$ and $\phi$ coordinate of the spherical coordinate system. Direction finding will be accomplished (employing data processing techniques) as a consequence of the relative amplitudes received by each of the antennas. The total number of antennas has not yet been specified. It is probable that the minimum number of antennas required will be 17 and the maximum number 26. The antennas will be interrogated by a rotating electromechanical switch. Tentatively, the switch will have three rotational rates (10, 100 and 1000 rpm). The operator will then be able to select the optimum switching rate which will depend upon the types of signals being interrogated by the azimuth and elevation direction finder. At present the signals which are considered to be of greatest concern are those from search radars.

To optimize the switching rate of the direction finder, consideration is being given to typical search radar pulse widths, rates and antenna scanning rates. An additional consideration is the computer cycling rate, which is to be discussed later. Consideration must also be given to signals that have CW, AM and FM modulation. However, it is generally agreed that these signal types will not present a severe problem to the system.
FIG. 1-1: AZIMUTH-ELEVATION DIRECTION FINDER.
The switch will consist of either 17 or 26 input ports (each to be identified with a particular antenna). In addition to the signal input, an identification signal (Ref. Sig.) must be available from the switch to identify to the computer which signal port is being interrogated. This identification is conceived to be in the form of a light beam to be interrupted as the switch output port slews past each of the input ports. It is anticipated that the switch will interrogate each of the antennas such that the information will be collected and stored in the computer in a sequential format. An example would be for the first antenna to be associated with $\theta = 0^\circ$, $\phi = 0^\circ$ (employing the spherical coordinate system of Fig. 1-2); the second antenna would be at $\theta = 45^\circ$, $\phi = 0^\circ$; the third at $\theta = 45^\circ$, $\phi = 45^\circ$, etc., through the last antenna. For this example, we have assumed a total of 17 antennas, one located at the pole, a ring of 8 antennas at $\theta = 45^\circ$, and the second ring at $\theta = 90^\circ$. Each of the rings would consist of 8 antennas placed at $\phi$ increments of $45^\circ$.

The Ref. Sig. is to be transferred from the switch directly to the computer to identify the signals being inserted in the computer. The signals themselves would first be transferred to the RF receiver, where the signals will be amplified and detected. A video amplifier may be required following the receiver to amplify the detected data to insure proper operation of the computer. In the event more than one frequency is received that is within the bandwidth of the receiver, an Automatic Signal Recognition Unit (ASRU) will be required. It is understood that such units are available and have been employed by the military for this purpose. The data from ASRU is then transferred to the analog-to-digital converter, where it is digitalized in the proper format for the computer.

In the computer, the data is stored sequentially similar to that discussed above. For example, the first storage compartment will consist of data for the pole antenna.
FIG. 1-2: SPHERICAL COORDINATE SYSTEM
The second storage compartment would have data from the antenna located at $\theta = 45^\circ$, $\phi = 0^\circ$, and the third compartment would have data for $\theta = 45^\circ$, $\phi = 45^\circ$, etc. through the 17 antennas (in the event a 17 element antenna is employed).

Within the computer a vector addition of the data is performed from which directional data may be obtained. Figure 1-3 is a simplified flow chart of the computer program to be employed to perform the direction finding analysis. In the first block, data is stored as noted above. Next, the data that has been collected and stored will be scanned to determine the maximum signal ($A_{\text{max}}$) such that all of the remaining data may be normalized with respect to $A_{\text{max}}$ as noted in Fig. 1-2. After normalization the data for each antenna will be separated into three rectangular components (associated with the $\theta$ and $\phi$ coordinates of the particular antenna under consideration) as noted in the third block. The data for the pole antenna would be separated into the $A_{1x}$, $A_{1y}$, and $A_{1z}$ components as follows:

$$A_{1x} = A_1 \sin \theta_1 \cos \phi_1$$  \hspace{1cm} (1.1)

$$A_{1y} = A_1 \sin \theta_1 \sin \phi_1$$  \hspace{1cm} (1.2)

and

$$A_{1z} = A_1 \cos \theta_1.$$  \hspace{1cm} (1.3)

A similar set of data will be obtained for each of the $K$ antennas. From this data a summation of all the $x$, $y$, and $z$ components would be performed as noted in the fourth block of the flow chart. From the summation the predicted angles ($\theta_p$ and $\phi_p$) would be determined employing the expressions.
\[
\phi_p = \tan^{-1} \frac{\sum A_{ky}}{\sum A_{kx}} \quad (1.4)
\]

and

\[
\theta_p = \tan^{-1} \frac{\sum A_{kz}}{\sqrt{\left(\sum A_{kx}\right)^2 + \left(\sum A_{ky}\right)^2}} \quad (1.5)
\]

Above it has been shown how the \(\phi_p\) and \(\theta_p\) would be determined by the computer. Several analytical computer analyses have been made by this laboratory and have shown that the \(\phi_p\) and \(\theta_p\) differ from the actual angles (\(\phi_A\) and \(\theta_A\)). However, it has been found that each of the \(\phi_p\) and \(\theta_p\) angles have a particular error associated with them such that the error can easily be accounted for by the insertion of a correction factor in the computer. Therefore, the final operation within the computer would be to correct the \(\phi_p\) and \(\theta_p\) to obtain the actual (\(\phi_A\) and \(\theta_A\)) angles of the signal under interrogation as shown in the sixth block of the simplified flow chart.

After the actual (\(\phi_A\) and \(\theta_A\)) angles have been determined by the computer, this data will then be read into a digital display system which will consist of nixie tubes to present the angular information in numerical form to the operator. It is felt that a scope display may not be advantageous since any information that would be presented on the scope would come from the digital information that provides the nixie tubes display. For this reason and because of the cost, it is not felt advisable to incorporate a cathode ray tube display in the present system.
FIG. 1-3: Azimuth - Elevation Direction Finder Simplified Computer Flow Design
II
A SIMPLE METHOD FOR DIRECTION FINDING

Direction finding may be catalogued into the following four basic techniques:

1) D. F. by using carefully controlled antenna patterns to produce a null or a pencil beam (Wullenweber type).

2) D. F. by detecting the direction of arrival of the phase front (Adcock type).

3) D. F. by Doppler shift where a rapidly rotating beam produces a Doppler shift of the incoming signal.

4) D. F. by data processing.

The first three methods (which are classical in nature) require antenna patterns that are carefully controlled to achieve the required accuracy.

Contract requirements for the azimuth - elevation direction finder call for detection of broadband (5:1 frequency band) circularly polarized signals. These requirements limit the types of antennas that may be utilized in the antenna system. To satisfy the above requirements consideration was initially given to the use of the cavity backed spirals built for the preceding study [Contract DA 28-043-AMC-01499(E)].

The patterns for these antennas are not felt to be suitable for the classical direction finding techniques, i.e., the first three techniques mentioned above. For example, patterns skewing off axis as a function of frequency and variations in patterns as a function of antenna orientation introduce errors that cannot be readily handled by the classical D. F. methods. Specifications for the subject azimuth - elevation direction finding system along with the required circular polarization tend to negate the classical direction finding techniques.

One of the interesting classical methods is the Watson-Watt D. F. system employing four antennas with four receivers. The outputs from the four antennas are
applied to the four electrostatic deflection plates of a cathode ray oscilloscope (CRO). With the antennas carefully designed (to produce a cosine response) and located on the corners of a square, the system is capable of providing directional information accurate to $\pm 4^\circ$. Employing this geometry, the coordinate systems of adjacent antennas are rotated $90^\circ$ such that one antenna has a $\cos \theta$ pattern and the neighboring antenna has a $\cos (\theta - 90)$ or $\sin \theta$ pattern. Addition of the $\cos \theta$ and $\sin \theta$ information, along with the $90^\circ$ space orientation of the CRO plates, produces a resultant trace on the face of the scope along the direction of the incoming signal as shown in Fig. 2-1 below.

![Triangle Diagram](image)

**FIG. 2-1: VECTOR GEOMETRY**

Following the above technique (employing a hemispherical array of elements) consider only an azimuthal ring of antennas (for this simplified discussion) located at a particular elevation of a spherical surface. These antennas are assumed to be uniformly spaced on the ring, i.e., separated by a constant angle $\phi$. If each antenna points outward (normal to the local tangent of the ring), its coordinate system will be
rotated $\phi^0$ from its neighbor. An incoming signal arriving at each antenna may be represented by a magnitude and an angle corresponding to the pointing direction of the antenna receiving the signal. Vector addition of the signals from all antennas will result in a first approximation of the angle of arrival of the incident energy. To perform the vector addition a computer will be employed. The computer will compare voltages resulting from signals received from all the antennas and normalize the signals to the maximum voltage. For the first approximation, this results in assigning to the antenna with maximum signal, direction information suggesting the signal is arriving normal to its aperture. A correction is introduced by combining with the initial predicted information data that would result from a cosine distribution as demonstrated below.

2.1 Example of Direction Finding

Consider a signal arriving on a ring of 8 antennas arranged as shown in Fig. 2–2.

![Diagram of ring of eight antennas](image)

**FIG. 2–2: GEOMETRY OF RING OF EIGHT ANTENNAS.**

With 8 antennas the angular separation between them becomes $\phi = 45^0$. Let the arrived signal be $22^0$ to the left of antenna No. 1. For this example, the antenna patterns are
considered ideal in that they are all similar but not necessarily a cosine distribution. Further, it is assumed the three dimensional pattern is equivalent to the pattern shown in Fig. 2-3 when rotated about its axis of symmetry. Table II-1 shows the relative signal amplitude at the output of the various antennas for a signal arriving from $\phi = 22^\circ$.

<table>
<thead>
<tr>
<th>Antenna Number</th>
<th>$\theta$ With Respect to Antenna Normal</th>
<th>Signal Level From Fig. 2-3(db)</th>
<th>Amplitude</th>
<th>Normalized Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$22^\circ$</td>
<td>-0.9</td>
<td>0.814</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>$67^\circ$</td>
<td>-8.5</td>
<td>0.141</td>
<td>0.178</td>
</tr>
<tr>
<td>7</td>
<td>$68^\circ$</td>
<td>-8.7</td>
<td>0.135</td>
<td>0.161</td>
</tr>
<tr>
<td>8</td>
<td>$23^\circ$</td>
<td>-1.0</td>
<td>0.795</td>
<td>0.978</td>
</tr>
</tbody>
</table>

In this example all signal levels more than $90^\circ$ from the local normal have been rejected due to their low amplitude level. The signals in Table II-1 are now added vectorially in terms of horizontal and vertical components as shown in Table II-2.

<table>
<thead>
<tr>
<th>Antenna No.</th>
<th>Vertical Component</th>
<th>Horizontal Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.000 \times \cos 0^\circ = 1.000$</td>
<td>$1.000 \times \sin 0^\circ = 0.000$</td>
</tr>
<tr>
<td>2</td>
<td>$0.178 \times \cos 45^\circ = 0.126$</td>
<td>$0.178 \times \sin (45^\circ) + 0.126$</td>
</tr>
<tr>
<td>7</td>
<td>$0.135 \times \cos 90^\circ = 0.000$</td>
<td>$0.135 \times \sin (-90^\circ) = -0.135$</td>
</tr>
<tr>
<td>8</td>
<td>$0.978 \times \cos 45^\circ = 0.691$</td>
<td>$0.978 \times \sin (-45^\circ) = -0.691$</td>
</tr>
</tbody>
</table>

1.817

-0.700
\[ \tan \theta = \frac{1.817}{-0.700} = -2.600, \quad \theta = -69^\circ \quad 90 - 69 = 21^\circ \quad \text{or } 1^\circ \text{ error from true angle.} \]
FIG. 2-3: SPIRAL ANTENNA PATTERN DATA FOR CALCULATIONS
(Frequency = 1.8 GHz).
III

ANALYTICAL ANALYSIS OF THE AZIMUTH – ELEVATION DIRECTION FINDER

It has been decided that vector addition data processing is to be employed to effect azimuth – elevation direction finding. During this interim we have been concerned with several parameters that may affect the system accuracy. Typically the parameters considered were as follows:

1) antenna pattern,
2) number of receiving antennas,
3) antenna location, and
4) the measurement errors particular to the system.

Below we will discuss computer programs that have dealt with the first three variables. More work is necessary on measurement error, this problem will be discussed in a future report.

3.1 Data Requirements for the DF Data Processing System

As noted above, the probability of employing the three-dimensional vector addition for direction finding originated from the standard two-dimensional DF systems (utilizing 4 antennas oriented 90° with respect to each other). Analytical studies that have been conducted for the three-dimensional system have shown that its accuracy is strongly dependent upon the antenna pattern characteristics and is less dependent on the geometrical location of the antennas. Further, there are inherent limitations associated with the three-dimensional system that are not found in the conventional two-dimensional system, (Watson-Watt DF system employing 4 antennas), (Jasik, 1961).

To gain familiarity with the vector addition concept we consider an example which employs a ring of n antennas; separated by equal angles of θ as shown in Fig. 3-1, the antennas are not orthogonal to one another as is the case in the Watson-Watt system.
FIG. 3-1: TWO-DIMENSIONAL DIRECTION FINDING WITH SOME NUMBER "N" OF EQUALLY SPACED ANTENNAS.
To assist in the analytical analysis, Table 3-1 lists the expressions from which the relative amplitude levels (at each antenna) may be calculated (assuming the direction of the unknown signal is arriving from an angle $\theta$).

**TABLE 3-1**

Antenna Responses

1) $\cos (\theta - 2\beta)$
2) $\cos (\theta - \beta)$
3) $\cos \theta$
4) $\cos (\theta + \beta)$
5) $\cos (\theta + 2\beta)$

To illustrate the limitation of this system, we will assume that the unknown signal is arriving at an arbitrary angle $\theta$ and that only three antennas receive data. Further, it is assumed that the three antennas are located at $\beta = 0$ and $\pm 45^\circ$. We have chosen these specific angular locations to simplify the math. However, if one were to employ a more general analysis it would have similar results to those obtained from this simplified analysis. Further, we will show that unless one considers all antennas over a $\beta = 180^\circ$ sector, there will be an inherent discrepancy in the calculation of the predicted angle of arrival of the unknown signal. Table 3-2 is the expression for the vertical and horizontal components of the three antennas noted above. The horizontal and vertical components may then be combined in a ratio to attain $\tan \theta$ as shown in equation (3.1).

$$\tan \theta = \frac{\text{hor. comp.}}{\text{vert. comp.}} = \frac{\cos (\theta - \beta) \sin \beta - \cos (\theta + \beta) \sin \beta}{\cos (\theta - \beta) \cos \beta + \cos \theta + \cos (\theta + \beta) \cos \beta} \quad (3.1)$$
TABLE 3-2

Vertical Component
2) \( \cos (\theta - \beta) \cos \beta \)
3) \( \cos (\theta) \cos 0^o \)
4) \( \cos (\theta + \beta) \cos \beta \)

Horizontal Component
2) \( \cos (\theta - \beta) \sin \beta \)
3) \( \cos \theta \sin 0^o \)
4) \( -\cos (\theta + \beta) \sin \beta \)

Expanding the sum and difference trigonometric expressions, equation (3.2) is obtained, which reduces to a more simple form involving \( \tan \theta \) multiplied by a trigonometric ratio which is dependent on the angular position of the antennas. It is the multiplication factor which causes the discrepancy between the calculated and actual angle of arrival (\( \theta \)).

\[
\frac{(\cos \theta \cos \beta + \sin \theta \sin \beta) \sin \beta - \left[ \cos \theta \cos \beta - \sin \theta \sin \beta \right] \sin \beta}{\left[ \cos \theta \cos \beta + \sin \theta \sin \beta \right] \cos \beta + \cos \theta + \left[ \cos \theta \cos \beta - \sin \theta \sin \beta \right] \cos \beta} = \\
\frac{\sin \theta \cdot 2 \sin^2 \beta}{\cos \theta \cdot 1 + 2 \cos^2 \beta} = \tan \theta \cdot \frac{2 \sin^2 \beta}{1 + 2 \cos^2 \beta} \tag{3.2}
\]

Let us now consider using all of the antennas that are within a \( \beta = 180^o \) sector (\( \pm 90^o \) with respect to the angular position of the unknown signal. For the purposes of this analysis we will assume five antennas (however, one need not be limited to five). Table 3-3 lists the expression for the vertical and horizontal component that would be associated with each of the five antennas.
TABLE 3-3
Vertical Component,

1) \( \cos (\theta - 2\beta) \cos 2\beta \)
2) \( \cos (\theta - \beta) \cos \beta \)
3) \( \cos \theta \cos 0^\circ \)
4) \( \cos (\theta + \beta) \cos \beta \)
5) does not receive signal as \( \theta + 2\beta > 90^\circ \)

Horizontal Component
1) \( \cos (\theta - 2\beta) \sin 2\beta \)
2) \( \cos (\theta - \beta) \sin \beta \)
3) \( \cos \theta \sin 0^\circ \)
4) \( \cos (\theta + \beta) \sin \beta \)
5) does not receive signal as \( \theta + 2\beta > 90^\circ \)

Through the use of equation (3.3), \( \tan \theta \) for the unknown signal is obtained employing all antennas within the \( 180^\circ \) sector.

\[
\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\text{vertical comp}}{\text{horizontal comp}}
\]

\[
= \frac{\cos (\theta - 2\beta) \cos 2\beta + \cos (\theta - \beta) \cos \beta + \cos \theta + \cos (\theta + \beta) \cos \beta}{\cos (\theta - 2\beta) \sin 2\beta + \cos (\theta - \beta) \sin \beta - \cos (\theta + \beta) \sin \beta} \tag{3.3}
\]

Equation (3.4) is the trigometric expansion of this and equation (3.5) has been simplified to a form which gives a trigometric ratio times the tangent of \( \theta \). It may be shown that the trigometric ratio is equivalent to unity for the case of five antennas that are equally spaced (\( \beta = 45^\circ \)). However, had there been more antennas and again equally spaced the trigometric ratio would have been more complex, but it can be shown that it would be equivalent to unity as in the case for five antennas. Therefore, the predicted angle
of arrival is equal to the actual angle of arrival only when all of the sampling antennas are located within a $180^\circ$ sector and these outputs are employed in the data processing. Intuitively, one might reason that this is to be expected since for the purposes of this simplified analysis, we have assumed that the radiation patterns of each of the antennas fits a cosine distribution in the range of $\beta = \pm 90^\circ$.

\[
\frac{(\cos \theta \cos 2\beta + \sin \theta \sin 2\beta) \cos 2\beta + (\cos \theta \cos \beta + \sin \theta \sin \beta) \cos \beta + \cos \theta}{(\cos \theta \cos 2\beta + \sin \theta \sin 2\beta) \sin 2\beta + (\cos \theta \cos \beta + \sin \theta \sin \beta) \sin \beta}
\]

\[
+ \frac{\cos \theta \cos \beta - \sin \theta \sin \beta \cos \beta}{\cos \theta \cos \beta - \sin \theta \sin \beta}
\]

(3.4)

Note $\beta = 45^\circ$ and $\cos 2\beta = 0$, $\sin 2\beta = 1$

\[
\frac{\cos + 2 \cos \theta \cos^2 \beta + \cos \theta}{\sin \theta + 2 \sin \theta \sin^2 \beta} = \frac{\cos \theta}{\sin \theta} \frac{1 + 2 \cos^2 \beta}{1 + 2 \cos \beta}
\]

(3.5)

for $\beta = 45^\circ$, $\cos \beta = \sin \beta$

From the above analysis (for the two-dimensional system) we have attempted to show that if antennas are not located over a $180^\circ$ sector, there will be inherent deviations in the calculation of the angular location of the unknown signals. It must be noted that this deviation is predictable and therefore can be corrected if needed. Below we will discuss several analytical results that have been obtained for the three-dimensional system that will illustrate this point more explicitly.

3.2 Analytical Results for a Three-Dimensional DF System

Several computer programs have been prepared and employed during this interim employing a three-dimensional DF system with antennas located on a hemispherical surface. Data from these programs have shown that there will be a discrepancy in the
predicted angle in both elevation and azimuth. The discrepancy in elevation has been greater because of the non-symmetry of the antennas (antennas are located on a hemisphere of $2\pi$ steradians) which was expected from the above discussion for the simple two-dimensional system. However, the azimuth errors were found to be minimal provided the element patterns were well behaved. We might elaborate a little on the elevation deviations and note again that the discrepancy in elevation results because the spherical area illuminated is less than the desired $2\pi$ steradians. Data from the computer program bear this out; in the case of $\theta = 0^\circ$ only (see Fig. 3.2) will antennas be excited over $2\pi$ steradians.

The first set of data calculated assumed 17 antennas equally spaced over $2\pi$ steradians (at $\theta$ and $\phi$ increments of $45^\circ$, as shown in Fig. 3-3) of the hemisphere and that each antenna had a cosine pattern that was symmetrical about the axis of the antenna. Typical elevation data is shown in Fig. 3-4 which illustrates the deviation of the predicted angle from the actual angle. It will be noted there are two sets of data points which represent the maximum and minimum deviation of the predicted elevation angle as a function of the azimuth angle. The large variations in this predicted angle may be corrected for, within the computer, prior to reading out the information to the display system (see Fig. 1-1).

Because of the discrepancy between the predicted and the actual angular data noted above, we have investigated the effects of element pattern variations and the number of antennas to be employed to effect the necessary direction finding information.

Here we will discuss the variations of cosine versus linear patterns as shown in Fig. 3-5. We will refer to these patterns as the cosine, linear $90^\circ$, $80^\circ$, and $70^\circ$ which are self explanatory from Fig. 3-5. Figures 3-6, 3-7, and 3-8 are respectively the predicted angles for linear $90^\circ$, $80^\circ$ and $70^\circ$ patterns. It is interesting to note there is a very little difference between the data for the cosine pattern (Fig. 3-4) and the linear $90^\circ$ pattern (Fig. 3-6). However, the deviation for the linear $70^\circ$ and
FIG. 3-2: COORDINATE SYSTEM FOR AZIMUTH-ELEVATION DIRECTION FINDING CALCULATIONS.
FIG. 3-3: GEOMETRY OF AZIMUTH-ELEVATION DIRECTION FINDER WITH SEVENTEEN ANTENNAS. NOTE ANTENNAS ALIGN ON RINGS IN ELEVATION PLANE.
FIG. 3-4: COMPUTED ELEVATION ANGLE VERSUS ACTUAL ELEVATION ANGLE FOR COSINE ANTENNA PATTERN; SEVENTEEN ANTENNAS.
FIG. 3-5: VARIOUS TYPES OF FAR FIELD ANTENNA PATTERNS CONSIDERED IN COMPUTER PROGRAMS.
FIG. 3-6: COMPUTED ELEVATION ANGLE VERSUS ACTUAL ELEVATION ANGLE FOR LINEAR 90 ANTENNA PATTERN, SEVENTEEN ANTENNAS.
FIG. 3-7: COMPUTED ELEVATION ANGLE VERSUS ACTUAL ELEVATION ANGLE FOR LINEAR 80 PATTERN, SEVENTEEN ANTENNAS.
FIG. 3-8: COMPUTED ELEVATION ANGLE VERSUS ACTUAL ELEVATION ANGLE FOR LINEAR 70 ANTENNA PATTERN, SEVENTEEN ANTENNAS.
80° (with respect to the cosine data) are appreciably larger. Typical far-field radiation patterns that have been measured for spiral antennas have shown that they exhibit patterns that may be closely approximated by an analytical expression similar to that for the linear 90° distribution.

We have noted many spiral patterns whose maximums are skewed from the axis of the antenna surface. In an attempt to determine the effects of a skewed pattern on the variations that may result between the predicted versus the actual angle of arrival, a computer program was employed assuming the main beam was skewed 10° off axis for a linear 90° pattern. An attempt was made to randomly orient the skewed pattern over the hemispherical surface such that the skewing would not all point in the same direction. Figure 3-9 is a plot of the predicted elevation angles as a function of the actual angle employing the skewed patterns. It is interesting to note that the variation in the predicted angle was a random variation which would be difficult to correct for to obtain the actual angle of arrival. As a consequence, it is felt that the patterns (of the individual element) must be well behaved (have a minimum of skewing \(< 10°\)) both as a function of antenna orientation as well as a function of frequency, i.e., the main lobe must be symmetrical about the axis of the antenna as a function of antenna orientation and frequency.

Above, consideration has been given to discrepancies that are probable in the elevation plane. Here we will discuss azimuthal discrepancies. We will show that because of the lack of antennas below \(\theta = 90°\), there will be discrepancies in the azimuthal angular predictions as a part of the direction finding analysis. Figure 3-10 is a graph of the maximum azimuthal error as a function of the elevation angle (assuming the element patterns satisfy a cosine distribution). It is interesting to note that from \(\theta = 0°\) to 45°, there is no azimuthal discrepancy. However, from \(\theta = 45°\) to 90° there is an azimuthal discrepancy which is a periodic function with the maximum occurring at \(\theta = 80°\). Since the discrepancy is periodic it is amenable to correction within the computer as has been discussed above.
FIG. 3-9: COMPUTED ELEVATION ANGLE VERSUS ACTUAL ELEVATION ANGLE FOR COSINE PATTERNS SKewed 10° OFF NORMAL, SEVENTEEN ANTENNAS.
FIG. 3-10: MAXIMUM ERROR IN AZIMUTH VERSUS ELEVATION ANGLE $\theta$ FOR COSINE PATTERN, SEVENTEEN ANTENNAS.
In the discussion of the elevation angles, consideration was given to the use of antennas whose patterns may be skewed. As a part of the azimuth study, consideration was again given to the use of skewed patterns. An attempt was made to randomly orient the skewed patterns on the hemisphere. Figure 3-11 is a plot of the predicted angle in azimuth as a function of the elevation angle. It is interesting to note that the deviation in the predicted azimuthal angle from the actual angle is greatest near $\theta = 0^\circ$ (top of the hemisphere), where it approaches $10^\circ$ or more. However, when data is collected near the horizon, the discrepancy decreases to approximately $3^\circ$. Although the discrepancy has decreased, it is larger than is acceptable. It may be concluded that the disadvantage of employing elements with skewed patterns for the azimuth – elevation direction finder would be the large discrepancies incurred both in azimuth and elevation and the difficulty in correcting for this discrepancy, since the discrepancy is not analytically expressible.

3.3 Consideration of Antenna Number and Location

Above we have assumed that 17 antennas are employed and are equally distributed over the surface of the hemisphere. A second distribution consisted of antennas located on an icosahedron as shown in Fig. 3-12 and discussed by Sengupta, et al. (1966). The icosahedron distribution has the advantage of achieving a uniform coverage of antennas over the surface of the hemisphere. Early data from a 26 element icosahedron distribution showed there was no advantage to employing it over the 17 element design discussed above. Figure 3-13 is a plot of the maximum azimuth discrepancy as a function of the elevation angle for the icosahedron distribution. The data shown in Fig. 3-13 was calculated in increments of $5^\circ$ for the azimuth angle. A later set of data was collected at $1^\circ$ increments and the data compared with the 17 element system also showed that there was insufficient advantage to employing the icosahedron 26 element distribution. Therefore, we have chosen to employ the 17 element system for the azimuth – elevation direction finder.
FIG. 3-11: MAXIMUM AZIMUTH ERROR VERSUS ELEVATION ANGLE FOR COSINE PATTERN SKewed 10°, SEVENTEEN ANTENNAS.
FIG. 3-12: SPHERICAL ARRAY. X's MARK ELEMENTS UTILIZED FOR ICOSAHEDRON DIRECTION FINDING CALCULATIONS
FIG. 3.13: MAXIMUM AZIMUTH ERROR VERSUS ELEVATION ANGLE FOR ICOSAHEDRON GEOMETRY WITH COSINE PATTERNS.
The maximum discrepancy for the 17 element system with cosine pattern distribution occurs at $\theta=80^\circ$ and midway between elements in the $\phi$-plane. A plot of the discrepancy as a function of $\phi$ for $\theta=80^\circ$ is shown in Fig. 3-14. The maximum discrepancy here is $0.86^\circ$. It may be shown that the discrepancy can either be increased or decreased by respectively decreasing or increasing the number of elements. However, in view of the contractual requirements, it is felt that using a smaller number of elements will decrease the system accuracy and sensitivity. Increasing the number of elements would improve both the accuracy and sensitivity but would further complicate the data processing requirement (computer and associated hardware) thus increasing the costs. In view of these and many other factors involved, it is recommended that the 17 element system be employed for the present feasibility model.
FIG. 3-14: ERROR IN AZIMUTH VERSUS AZIMUTH ANGLE $\phi$ FOR $\theta=80^\circ$ SEVENTEEN ELEMENTS WITH COSINE PATTERNS.
IV

RIGOROUS METHODS OF DIRECTION FINDING IN THREE DIMENSIONS USING DATA PROCESSING EQUIPMENT

Two methods of direction finding in three dimensions are discussed using data processing equipment, vector addition, and assuming cosine radiation patterns for the elements. In the first method, the signals induced in all the antennas, which are distributed on a hemisphere, are added vectorially to determine the direction of arrival of the signal. In the second method, the signals induced in a selected number of antennas are added vectorially. Then techniques to find the direction of arrival by proper data processing is discussed. The errors involved in the first method and the difficulties encountered in correcting them are also discussed. Theoretically, the second method gives correct direction of arrival of the signal.

4.1 Three Dimensional Radiation Pattern of an Individual Antenna

Before we discuss the methods used for direction finding, it is necessary to express the radiation patterns of the individual antennas, which are disposed on a hemisphere, in terms of spherical coordinates with the center of the hemisphere as the origin.

The individual antennas are assumed to be distributed on a hemisphere in such a way that the maximum radiation is directed radially outward and have a cosine radiation pattern which is rotationally symmetric about the radial line passing through the antenna and the center of the hemisphere as shown in Fig. 4-1.

In Fig. 4-1, $x$, $y$, and $z$ represent the rectangular coordinate system referred to the center of the hemisphere. $x'$, $y'$, and $z'$ are the rectangular coordinates referred to the center of the Antenna A, which is located on the hemisphere. $\alpha_o$ and $\beta_o$ are the polar coordinates of the antenna A referred to the unprimed coordinate system. In the primed coordinate system, the antenna pattern is assumed to be $\cos \theta'$.

Using the appropriate coordinate transformation, it can be shown (Sengupta, et al, 1965), that
FIG. 4-1: LOCAL COORDINATE SYSTEM USED.
\[
\cos \theta' = \sin \alpha_0 \sin \theta \cos (\phi - \beta_0) + \cos \alpha_0 \cos \theta \quad (4.1)
\]

The right hand side of the equation (4.1) represents the radiation pattern referred to the unprimed coordinate system. In general, if \( \alpha_n \) and \( \beta_n \) represents the position coordinates of any antenna on the hemisphere, the antenna pattern referred to unprimed coordinates is given by

\[
F_n (\theta, \phi) = \sin \alpha_n \sin \theta \cos (\phi - \beta_n) + \cos \alpha_n \cos \theta \quad (4.2)
\]

Except for a proportionality constant, the signal induced \( E_n \) due to a plane wave arriving from the direction given by \( (\theta, \phi) \) is equal to \( F_n (\theta, \phi) \). Hence,

\[
E_n = \sin \alpha_n \sin \theta \cos (\phi - \beta_n) + \cos \alpha_n \cos \theta \quad (4.3)
\]

Given the position of the antenna \( (\alpha_n, \beta_n) \) and the direction of the incident signal \( (\theta, \phi) \), it is straightforward to find the signal induced in that antenna using equation (4.3).

4.2 Vector Addition Method 1

In this method and the second method to follow, the individual antennas are assumed to be distributed uniformly around the circles of constant elevation angles as shown in Fig. 4-2.

Even though the following discussion applies to any number of elements distributed uniformly on circles of constant elevation angles, the final expressions are specialized for 17 antenna elements distributed as shown in Fig. 4-2.

In this method, the signals induced in different antennas will be added vectorially in three dimensions. To do this each signal will be resolved into the components along
FIG. 4-2: DISTRIBUTION OF ANTENNAS ON A HEMISPHERE.
the cartesian coordinates and then the components are added vectorially. If \( E_n \) is resolved into \( x, y, \) and \( z \) components, we obtain

\[
E_{nx} = E_n \sin \alpha_n \cos \beta_n \quad (4.4)
\]
\[
E_{ny} = E_n \sin \alpha_n \sin \beta_n \quad (4.5)
\]
\[
E_{nz} = E_n \cos \alpha_n \quad (4.6)
\]

If there are \( N \) antennas, the total \( x, y, \) and \( z \) components are given by

\[
E_x = \sum_{n=1}^{N} E_n \sin \alpha_n \cos \beta_n \quad (4.7)
\]
\[
E_y = \sum_{n=1}^{N} E_n \sin \alpha_n \sin \beta_n \quad (4.8)
\]
\[
E_z = \sum_{n=1}^{N} E_n \cos \alpha_n \quad (4.9)
\]

where \( E_n \) is given by equation (4.3).

The direction of the resultant vector is given by

\[
\phi_a = \tan^{-1} \left( \frac{E_y}{E_x} \right) \quad (4.10)
\]

and

\[
\theta_a = \cos^{-1} \left[ \frac{E_z}{\sqrt{E_x^2 + E_y^2 + E_z^2}} \right] \quad (4.11)
\]
The vector addition method assumes that \( \theta_a \) and \( \theta_a \) will represent the direction of the incident signal which is assumed to be \( \theta \) and \( \theta_a \). Later on we will consider a specific example to study the error involved in this method and how to correct it (if possible).

Equations (4.7) through (4.11) can be used for any type of distribution of antennas on the hemisphere. If the antennas are uniformly distributed on circles of constant elevation angles \( \alpha_p \) and if \( \beta \) is the angle between any two antennas on a circle, then

\[
E_x = \sum_{p=1}^{P} \sum_{m=1}^{M/2+1} \left\{ \sin \alpha_p \sin \theta \cos (\beta - m\beta) + \cos \alpha_p \cos \theta \right\} \sin \alpha_p \cos m\beta \quad (4.12)
\]

\[
E_y = \sum_{p=1}^{P} \sum_{m=-M/2}^{M/2+1} \left\{ \sin \alpha_p \sin \theta \cos (\beta - m\beta) + \cos \alpha_p \cos \theta \right\} \sin \alpha_p \sin m\beta \quad (4.13)
\]

\[
E_z = \sum_{p=1}^{P} \sum_{m=-M/2}^{M/2+1} \left\{ \sin \alpha_p \sin \theta \cos (\beta - m\beta) + \cos \alpha_p \cos \theta \right\} \cos \alpha_p + \cos \theta \quad (4.14)
\]

where \( P \) is the number of circles of constant elevation, and \( M \) is the number of elements on each circle minus two.

In using equations (4.7), (4.8), (4.9), or (4.12), (4.13) and (4.14), one should note that only the contributions of the antennas which are illuminated by the incident signal should be taken into consideration. In general, the number of antennas which are illuminated depends both on \( \theta \) and \( \beta \). Hence, it is not possible to simplify the above
equations for all \( \theta \) and \( \phi \). At this point we consider a specific distribution of elements with \( P = 2, \ M = 6, \ \alpha_1 = \frac{\pi}{4}, \) and \( \alpha_2 = \frac{\pi}{2}. \) With this distribution \( \beta = \frac{\pi}{4}. \) From Fig. 2, it is clear that if \( \theta \leq \frac{\pi}{4} \) all the antennas on the circle with \( \alpha_1 = \frac{\pi}{4} \) are illuminated along with the antenna on the horizon and 4 antennas on the circle with \( \alpha_2 = \frac{\pi}{2}. \) For this case the equations (4.12), (4.13), and (4.14) could be simplified and are given below.

\[
E_x = \sum_{m=-3}^{4} \left[ \sin \frac{\pi}{4} \sin \theta \cos \left( \phi - m \frac{\pi}{4} \right) + \cos \frac{\pi}{4} \cos \theta \right] \sin \frac{\pi}{4} \cos m \frac{\pi}{4} \\
+ \sum_{m=-1}^{2} \left[ \sin \frac{\pi}{2} \sin \theta \cos \left( \phi - m \frac{\pi}{4} \right) + \cos \frac{\pi}{2} \cos \theta \right] \sin \frac{\pi}{2} \cos m \frac{\pi}{4} \tag{4.15}
\]

This will be simplified to

\[
E_x = 4 \sin \theta \cos \phi \tag{4.16}
\]

Similarly one can easily show that

\[
E_y = 4 \sin \theta \sin \phi \tag{4.17}
\]

and

\[
E_z = 5 \cos \theta \tag{4.18}
\]

From equations (4.16), (4.17) and (4.18), the apparent azimuth angle and elevation angle are given by

\[
\phi_a = \tan^{-1} \left( \frac{E_y}{E_x} \right) = \tan^{-1} \tan (\phi) = \phi \tag{4.19}
\]

\[
\theta_a = \cos^{-1} \left( \frac{E_z}{\sqrt{E_x^2 + E_y^2 + E_z^2}} \right) = \cos^{-1} \left[ \frac{5 \cos \theta}{\sqrt{16 \sin^2 \theta + 25 \cos^2 \theta}} \right] \tag{4.20}
\]
From (4.19) it is evident that the correct azimuth angle is obtained by the method used here. However, from equation (4.20) it is clear that the elevation angle, even though independent of $\phi$, is in error. Fortunately, it is not difficult to obtain correct elevation angles by a simple modification of vector addition as outlined below.

Let

$$E'_x = \frac{5}{4} E_x = 5 \sin \theta \cos \phi$$

$$E'_y = \frac{5}{4} E_y = 5 \sin \theta \sin \phi$$

(4.21)

Then define

$$\theta_a = \cos^{-1} \frac{E_z}{\sqrt{E'_x^2 + E'_y^2 + E_z^2}}$$

(4.23)

Substituting $E'_x$, $E'_y$, and $E_z$ in equation (4.23), it can easily be verified that

$$\theta_a = \theta$$

(4.24)

From the above discussion it is clear that by simple modification of vector addition, one could predict the correct elevation and azimuth angles, provided $\theta \leq \pi/4$. Even though the specific example given above assumes 8 antennas on each circle of constant elevation, the same procedure could be used, provided the number of antennas on each circle are $4J$ ($J$ an integer) and the
correct elevation and azimuth angles could be predicted (for $\theta = \leq \pi/4$).

The above type of simplification is not possible if $5 > \pi/4$. The reason for this is that the number of antennas illuminated by the incident signal vary with $\theta$ and $\phi$. Consequently no single analytical expression could represent the resultant vector.
Even though the vector addition method 1 is straightforward using a computer, the results will in general be in error and there is no simple modification which can be used for $\theta > \pi/4$ to predict the correct elevation and azimuth angles as was demonstrated for $\theta \leq \pi/4$.

We have computed $\theta_a$ and $\phi_a$ using a computer for the distribution given in Fig. 4-2 for different values of $\theta$ and $\phi$. Our results agree with the results obtained here analytically for $\theta < \pi/4$. For $\theta > \pi/4$ our results show that there is only a weak dependence of $\theta_a$ on $\phi$ and $\phi_a$ on $\theta$. So there is a possibility to use some correcting factors to predict the correct values of $\theta$ and $\phi$. Even after using the correcting factors there will be some errors in the final values. If there is no alternate method of predicting the correct elevation and azimuth, vector addition method 1 may be satisfactory. An alternate method will be given below which will predict the correct elevation and azimuth in the whole range of interest.

4.3 Vector Addition Method 2

In this method, instead of adding vectorially the contributions of all the antennas which are illuminated, as was done in method 1, only the contributions of the maximum number of antennas which are illuminated for all $\theta$ and $\phi$ will be added vectorially.
The advantage of this approach is that the number of antennas contributing to vector addition remains unchanged and results in a single expression for $E_x$, $E_y$, and $E_z$ as a function of $\theta$ and $\phi$. By proper data processing, one could predict the correct $\theta$ and $\phi$. 

45
For this method it is assumed that the number of elements on each circle of
contant elevation are in multiples of 4. In the beginning our discussion applies to
any number of circles of constant elevation angle and any number of elements on
each circle with the previously mentioned assumption.

Let the antennas at $\hat{\theta} = 0$ have the maximum signal compared to other antennas
on each circle. Then choose $M_1 + 1$ antennas on each circle which are illuminated
for all $\theta$, and $-\frac{\beta}{2} \leq \hat{\theta} \leq \frac{\beta}{2}$, where $\beta$ is the angle between any two antennas on a circle.
Let $P$ be the number of circles. If $M_o$ is the total number of elements in any circle,
it is not difficult to show that $M_1 = (M_o - 4)/2$. The sector in which the antennas are
illuminated are shown in Fig. 4–3. We now consider the contributions of antennas
within the sector shown in Fig. 4–3. From equation (4.12), we can write that

$$E_x = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \left[ \sin \alpha_p \sin \theta \cos (\hat{\theta} - m\beta) + \cos \alpha_p \cos \theta \right] \sin \alpha_p \cos m\beta$$

$$= \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \left[ \sin^2 \alpha_p \cos^2 m\beta \sin \theta \cos \hat{\theta} + \cos \alpha_p \cos m\beta \cos \theta \right]$$

(4.25)

On rewriting, we have

$$E_x = A \sin \theta \cos \hat{\theta} + B \cos \theta$$

(4.26)

where

$$A = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \sin^2 \alpha_p \cos^2 m\beta$$
and

\[ B = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \cos \alpha_p \sin \alpha_p \cos m_\beta \]

From equation (4.13) we can write that

\[ E_y = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \left[ \sin \alpha_p \sin \theta \cos (\phi - m_\beta) + \cos \alpha_p \cos \theta \right] \sin \alpha_p \sin m_\beta \]

\[ = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \sin^2 \alpha_p \sin^2 m_\beta \sin \theta \sin \phi \]

(4.27)

on rewriting we have

\[ E_y = C \sin \theta \sin \phi \]

(4.28)

where

\[ C = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \sin^2 \alpha_p \sin^2 m_\beta \]

From equation (4.14), we can write

\[ E_z = \cos \theta + \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \left[ \sin \alpha_p \sin \theta \cos (\phi - m_\beta) + \cos \alpha_p \cos \theta \right] \cos \alpha_p \]
\[
= \cos \theta + \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \sin \alpha_p \cos \alpha_p \cos m\beta \sin \theta \cos \phi + \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \cos \theta \cos^2 \alpha_p
\]

(4.29)

on rewriting we have

\[
E_z = D \cos \theta + G \sin \theta \cos \phi
\]

(4.30)

where

\[
D = 1 + \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \cos^2 \alpha_p
\]

and

\[
G = \sum_{p=1}^{P} \sum_{m=-M_1/2}^{M_1/2} \sin \alpha_p \cos \alpha_p \cos m\beta
\]

Note that \( G \) is also equal to \( B \).

Now we plan to find \( \theta \) and \( \phi \) from equations (4.26), (4.28), and (4.30) using the following data processing. From equations (4.26), and (4.30), we note that

\[
DE_x - BE_z = (DA - BG) \sin \theta \cos \phi
\]

(4.31)

and

\[
(AE_z - GE_x) = (DA - BG) \cos \theta
\]

(4.32)

Now we define

\[
E'_x = \frac{DE_x - BE_z}{DA - BG} \cdot C
\]

(4.33)

and

\[
E'_z = \frac{AE_z - GE_x}{DA - BG} \cdot C
\]

(4.34)
FIG. 4-3: ILLUMINATED ANTENNAS FOR ALL $\theta$ AND $\frac{-\beta}{2} \leq \phi \leq \frac{\beta}{2}$.
It is easy to show that

\[ E'_x = C \sin \theta \cos \phi \]  \hspace{1cm} (4.35)

\[ E'_z = C \cos \theta \]  

We already have

\[ E'_y = C \sin \theta \sin \phi \]  \hspace{1cm} (4.37)

From equations (4.35), (4.36), and (4.37), it is straightforward to find the correct \( \theta \) and \( \phi \) and are given by

\[ \phi_a = \tan^{-1} \left( \frac{E'_y}{E'_x} \right) = \phi \]  \hspace{1cm} (4.38)

and

\[ \theta_a = \cos^{-1} \left( \frac{E'_z}{\sqrt{E'_x^2 + E'_y^2 + E'_z^2}} \right) = \theta \]  \hspace{1cm} (4.39)

In summary, the induced signals in individual antennas in a sector are used to find \( E_x, E_y, \) and \( E_z \). Then knowing the constants \( A, B, C, \) and \( D \) (which depends only on the distribution of antennas on the hemisphere); \( E'_x \) and \( E'_z \) are determined using equations (4.33) and (4.34). Finally \( \theta \) and \( \phi \) are determined from (4.38) and (4.39).

4.4 Determination of the Constants A, B, C and D for a Specific Distribution

The specific distribution is shown in Fig. 4-2 for which \( M_o = 8, \ P = 2 \) with \( \alpha_1 = \frac{\pi}{4} \) and \( \alpha_2 = \frac{\pi}{2} \) and \( \beta = \frac{\pi}{4} \). Substituting these values in the expressions for \( A, B, C \) and \( D \), we obtain

\[ M_1 = 2 \]

\[ A = 3, \quad B = 0.5 + \frac{1}{\sqrt{2}}, \quad C = 1.5, \quad D = 2.5, \quad \text{and} \quad G = B. \]
V

ELEMENT INVESTIGATION

As a result of recent analytical studies with the direction finder, we have found that it is extremely important that the electrical characteristics of the antenna elements be well behaved such that the radiation characteristics do not vary both as a function of angular orientation of a particular antennas as well as the frequency of operation. To ensure that the antennas will have the required consistency in radiation characteristics, both as a function of angular orientation and the frequency of operation, consideration is now being given to the employment of conical spirals having a quadrafiilar element being fed by a stripline balun configuration.

The Naval Ordnance Test Station (NOTS), China Lake, California was contacted and the feasibility of employing spiral antennas as the basic elements for the DF antenna system was discussed. During our discussions three areas were covered that were felt to be of interest to this contract: 1) spiral geometry, 2) spiral configuration, and 3) balun consideration.

5.1 Spiral Geometry

During our discussions of spiral geometry it was noted that several unsuccessful attempts have been made to employ cavity backed spirals for broadband applications. Broadband widths are generally considered to be much greater than the 2:1, e.g., 10:1 are typical. A typical requirement is for the pattern characteristics to remain relatively constant over both the frequency range of interest, and as a function of spiral orientation. It should be noted that NOTS is interested in employing one spiral to operate in both the sum and difference modes which is considered to be a very severe restriction. The sum mode is defined as having a main lobe on axis, while the difference mode has a null on axis (a split beam). An additional requirement is for the gain to remain relatively constant in the frequency range of interest.
The Navy has found for broadband operation (greater than 2:1), one should consider the use of the conical spiral in lieu of a cavity backed spiral. When employing a cavity backed spiral, the cavity is designed to be $\lambda/4$ deep at the center frequency of the frequency band, and at frequencies other than this the cavity deviates from $\lambda/4$. Because of this variation in electrical cavity depth the performance of the antenna deteriorates when operated over bandwidths greater than 2:1. This has been observed by this laboratory in work with spiral antennas both of our own design and other organizations' designs.

5.2 **Spiral Configuration**

The spiral configuration that would best suit the requirements of the DF system, i.e., to achieve broadband pattern characteristics and the required gain, was also discussed. First, the disadvantages of the bifilar spiral configuration were noted. From a simple analysis of the bifilar spiral and the current distribution along the elements of the spiral, we find that higher order current bands are formed when the spiral electrical circumference is equal to $(m + n)\lambda$ (where $m$ is equivalent to 1 for the $\sum$ mode and 2 for the $\Delta$ mode and $n$ is equivalent to the number of filaments) e.g. 1, 3, 5, ..., . Further, it should be noted that the above exists for the $\sum$ mode when the two elements are to be fed $180^\circ$ out of phase. For a multifilar element, it can be seen from the above expression, that the radiating current bands become more randomly distributed as the number of filaments increases. A quadrafilar spiral whose four terminals are fed, 0, 90, 180 and 270$^\circ$ will have a radiating band when the electrical circumference is one wavelength in diameter and the next higher order radiating band occurs when the spiral is five wavelengths in diameter. Therefore, it would appear desirable to employ a multifilar spiral to minimize the number of radiating bands.
It is recommended that we consider as a minimum, a quadraflexar spiral for the DF antenna to ensure good pattern characteristics for the 5:1 frequency band.

5.3 Balun Considerations

We also discussed the problems associated with the construction of a broadband balun. One of the difficulties associated with the present bifilar spiral antenna is the construction of a balun that has good phase and amplitude characteristics over the frequency range of interest (in our case a 5:1 frequency band). Previous work with broadband antenna configurations has employed the balun configuration described by Duncan and Minerva (1960). This balun, although simple to construct, has been found to have unsatisfactory phase and amplitude characteristics over broadbandwidths.

NOTS has given consideration to other techniques by which an unbalanced line may be converted to a balanced line (the construction of a better broadband balun). From efforts they have funded at Radiation Systems, Inc., they found that a stripline 3db coupler and a stripline 90° phase shifter could be constructed to operate over large frequency bands ( > 10:1) to effect a broadband balun. This work has been documented by Shelton and Mosko, (1966). From this investigation, they found that it was possible to broadband such elements. However, one cannot expect to have a constant 3db power split nor a uniform 90° phase shift over the frequency range of interest. However, one may place a ripple factor on both the amplitude and the phase characteristics and build a broadband (greater than 2:1) 3db directional coupler that has equal power split and 90° phase shift within the ripple factor limits. Mosko showed data for a typical 3db coupler that has uniform power split with a very small ripple and a constant phase shift of 90° again with small ripple for a 14:1 frequency band. Similarly, an investigation was conducted to determine how one might broadband a phase shifter. They found (using a technique originally conceived by Shiffman, 1958) that a broadband phase shifter
could be built having a small ripple in the phase as a function of frequency. Both of these components have been developed and a number of them have been fabricated and tested, and appear to be within the state-of-the-art at this time.

By properly arranging power splitters and phase shifters, one can design the required phasing to build a balun for either a conventional bifilar or multifilar spiral. For example, to feed a quadrafilar spiral, one must use three 3db directional couplers and one 90° phase shifter, all of which must be broadband over the frequency range of interest. Each of these can be evaluated and analyzed prior to their construction and after the units have been constructed they may be evaluated such that the experimental data may be compared with the analytical data.
VI
ELECTROMECHANICAL SWITCH

Several laboratory models of the electromechanical switch required for the azimuth - elevation direction finder have been assembled and tested. Preliminary test results have shown that a single pole (n) throw switch can be built to have a VSWR of less than 1.5:1 with respect to 50 $\Omega$. In addition, the isolation between adjacent ports has been found to be greater than 20 db. The above data has been collected at several frequencies in the 5:1 frequency band (600 – 3000 MHz).

During the preliminary study of the switch, consideration has been given to the design of both the switching ports and the rotary joint to be required for the switch. The switching ports will be of a non-contacting capacitively coupled configuration. Likewise the rotary joint will be non-contacting and capacitively coupled. We have discussed with shop personnel the feasibility of fabricating a switch that can rotate at rates up to 1000 rpm. From these discussions it is felt to be within the present state-of-the-art to fabricate such a device.

The results of this study have indicated that a switch can be fabricated that will fit within a cylindrical volume 6" in diameter and 5 or 6 inches high.
We have discussed the requirements of the computer for the azimuth - elevation direction finder with several computer representatives. During these discussions we have learned the most difficult problem is that of sampling the antenna data to be read into the analog-to-digital (A to D) converter. It is important to the proper operation of the azimuth - elevation direction finder, to ensure that all data received during the on mode of each antenna (this is referred to the switch aperture) is read into the A to D converter. Many commercially available computers (suitable to the azimuth - elevation DF system) sample incoming data at a 60kc rate. Presently it is assumed that the sampling switch (of the DF system) will consist of 17 inputs each having on time (switch aperture) of 1.17 milliseconds for the 1000 rpm switching rate. A conventional 60kc sampler would then sample each switch aperture three times during a particular DF scan cycle. In the event the DF data in the switch aperture are from a radar having a high prf rate, there is a possibility that the sampling would occur during a no-signal interval. Therefore, it has been concluded that we must either increase the sampling rate or employ an alternate technique.

Before discussing the above further, we should note another complication (which has been discussed previously) which requires we have some knowledge of the rise time of the receiver so that the pulses will be interrogated properly. In the event the receiver has a poor rise time there will be a leading and trailing edge associated with each pulse and sampling during the rise time would lead to erroneous data. A third problem that must be considered is the possibility of very low prf radars such that only a small portion of a pulse may be intercepted by the antenna during the switch aperture interval. This would give a very narrow pulse for the analog-to-digital converter to sample during the total aperture interval.
To overcome these problems noted above, it is possible to increase the probability of detection of the maximum signal for both low and high prf’s by considering much faster sampling rates in the analog-to-digital converter (e.g., 500 kc). However, at a minimum, there are two undesirable features associated with this choice, i.e., the cost of the unit increases along with a reduction in accuracy of the overall DF system. As for the cost, a 60 kc sampler costs $4500.00 while the cost of a 500 kc sampler would range between $8000.00 and $9000.00. Inaccuracies may occur since data would be sampled at a very high rate and it is conceivable that portions of the data would be sampled during the rise and trailing time of pulse signals thus providing erroneous data for the computer. This would then require an additional operation to be performed by the computer to average the data which in turn increases the complexity of the computer program. Further, we would be requiring the computer to store a considerable amount of non-functional data employing the 500 kc sampling rate.

During discussions with personnel of the University of Michigan Computer Center, they confirmed that the most difficult problem for the computer in so far as the DF system is concerned was sampling the signal during the time interval of the switch aperture as noted above. They expressed concern with regards to employing a 500 kc sampling rate since they felt it was close to the limiting response time of the computer and that it may not be a state-of-the-art item at this time. They suggested an alternate method which would perform the sampling by integrating the voltage within the switch aperture. However, this was discarded because of the erroneous signal which would result particularly for radars having very low prf's. As was noted earlier, there is a high probability that such signals may be present in the switch aperture for a small part of the pulse. The average of this data, when integrated over the entire switch aperture period, would be very low providing erroneous data to the computer. As a consequence, it was recommended that we consider a more complex interface (that would proceed the computer) to determine the maximum signal voltage.
Let us note that the technique employed for reading data into the computer will have an influence on the amount of time allocated for the computer to determine the desired $\theta$ and $\phi$ coordinates of the unknown signal. A second complication would occur if one employs a high sampling rate to read data directly into the computer during the period of the switch aperture; the computer memory will be burdened with absorbing a large amount of non-functional information as noted above.

Therefore, to overcome these undesirable features, it was recommended that we employ a direct access technique allowing the computer to determine the signal maximum during the off time of the switch. However, a more complicated interface would be required which would determine the signal maximum during the period of the switch aperture and that at the end of the aperture, read this information into the computer. Simultaneously the computer would reduce the data obtained during the preceding switch aperture into its respective rectangular components and continuously add them as each switch aperture provides its information to the computer. Having the computer collect, reduce and sum the data simultaneously as the individual switch apertures interrogate the antennas it would be necessary for the computer to perform one calculation to obtain the direction of arrival ($\theta$ and $\phi$ coordinates) of the incoming signal. It is probable that the computer would be capable of calculating the new direction of arrival during every rotation of the switch.

From these discussions, consideration is now being given to two computers for use with the azimuth - elevation direction finder. The computers are the Data Machines, Incorporated, 620I and the Hewlett-Packard, 2115A.
VIII
CONCLUSIONS

During this interim considerable effort has been of an analytical nature in addition to a small experimental effort associated with the design of the electromechanical switch. A number of laboratory models for the switch have been built and evaluated, and presently an engineering model is being assembled. The analytical work has shown that it is extremely important for the antenna elements to be electrically well behaved (patterns and impedance) over the range of frequencies of interest (600 MHz to 3 GHz). Discussions with computer representatives have assured us that the computer requirements of the azimuth - elevation direction finder are well within the present state-of-the-art of computer techniques.

At this time it would appear that the most difficult problem is in the design and development of the feasibility model of the azimuth - elevation direction finder is the development of antenna elements. We are presently investigating the use of multifilar conical spirals. Since there is little in the open literature with respect to multifilar spiral elements, we have discussed them with several investigators. It is generally felt that the element pattern requirements for the azimuth - elevation direction finder will be difficult to achieve. However, there is evidence that broadband (5:1) circularly polarized elements can be developed to satisfy the azimuth - elevation direction finder requirements. Therefore, an effort is being expended on the development of elements to be employed with the subject DF system.

From discussions with computer representatives, it is apparent that the cost of the computer (for the azimuth - elevation direction finder) will be in the neighborhood of $30,000. A present consideration is to determine the optimum method of reading antenna information into the computer. The principal type of data that will be difficult to read into the computer is radar pulse information. We are also giving further consideration to this problem before we make arrangements to purchase a computer.
Although the analytical studies have shown that azimuth - elevation direction finding can be accomplished employing techniques that have been described in this report, there are many problems that will be associated with the hardware. Emphasis must be placed on the problems that will be associated with the design and development of the antenna elements. As the program progresses, we are sure there will be additional problems associated with the system and it is suggested that there be a close liaison maintained between the University and Ft. Monmouth as the program progresses.
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AZIMUTH AND ELEVATION DIRECTION FINDER TECHNIQUES

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Joseph E. Ferris, Boppama L. J. Rao, and Wiley E. Zimmerman

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A block diagram of the proposed DF azimuth-elevation direction finder is presented and discussed. A simple analysis is presented for a two-dimensional direction finder and is extended into a three-dimensional system. As for the three-dimensional azimuth-elevation direction finder analysis, a number of computer programs have been utilized to determine the accuracy of the system. In general it has been found that the system accuracy is strongly dependent upon the element pattern characteristics (i.e. it is important that they have broadband radiation pattern characteristics both as a function of element orientation and frequency). Further, it has been found that because of the elements being distributed over 2π steradians that the predicted elevation angles have associated with them a discrepancy has been found to be predictable and can easily be handled by the computer to provide the true angle. Discussions have been held with several computer representatives and it has been determined that it is feasible to employ a small computer to perform the necessary data processing to obtain the required azimuth-elevation data from the hemisphere antenna system.
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