THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Engineering Mechanics
Department of Mechanical Engineering

Tire and Suspension Systems Research Group

Technical Report No. 11

DIGITAL COMPUTATION OF TWO-PLY ELASTIC CHARACTERISTICS

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NOMENCLATURE

English Letters:

a_{ij}	Elastic constants dependent on both cord angle and elastic properties.
E,F,G	Elastic constants of a single sheet.

Greek Letters:

ϵ	Strain.
σ	Stress.

Subscripts:

x,y	Co-ordinate directions parallel and perpendicular to the cords.
η,ξ	Co-ordinate directions along the bisectors of angles between the cords.

Superscripts:

*,**	Indicating the actual stress carried by the first and second plies in a two-ply laminate, respectively.
'	Indicating interply stress.
+,++	Indicating the sum of externally applied stress and interply stress carried by the first and second plies, respectively.

I. FOREWORD

The study of the elastic interaction of two similar plies bonded together can be carried on in a very simple way, as indicated in previous reports. When the two plies which are bonded together become dissimilar, as occurs when the cords in one ply are unstressed or in compression, while the cords in the other ply are in a state of tension, then this study becomes considerably more complicated and is generally not possible to do efficiently by hand calculation. As part of the general research effort of this group, it is necessary to determine the important characteristics in the elastic response of two dissimilar bonded plies. For that reason, it is necessary to consider the solution of the equations governing this structure in some detail.

It was not originally intended to present the methods of arriving at conclusions concerning the actions of such dissimilar plies but rather only to present the conclusions themselves. Some interest has been shown in the details of these solutions and for this reason they are presented here. They are not intended at this time to be anything other than a research tool to aid in the study of the load-carrying characteristics of such a structure.

II. SUMMARY

The elastic action of two dissimilar plies bonded together in a single laminate may be completely described by nine simultaneous linear algebraic equations in nine unknowns. Since solving these equations would not be easy by hand methods, a digital computer program was constructed, permitting us to solve these equations numerically, and inexpensively and quickly. This report presents the details of that program, along with a sample solution illustrating the way the program was actually used.

III. PHYSICAL CONSIDERATIONS

The physical effects which take place when two dissimilar plies are bonded together and loaded are discussed in Ref. 1. That discussion is also pertinent to this report since, from it, one may see at once that the application of a single load will result in the presence of two interply stresses, as well as general deformation. This means, of course, that normal and shearing effects are completely coupled in such a structure.

When the two plies making up a laminate are identical in their properties, shear and normal effects are no longer coupled together along principal, or orthotropic, axes. In these directions, the application of load or stress results in extension without distortion and vice versa. Here, the situation becomes physically quite simple.[3]

IV. EQUATIONS GOVERNING THE ACTION OF A TWO-PLY LAMINATE

The equations governing the action of two bonded dissimilar plies were given as Eqs. (22) of Ref. 2. These will be repeated here with a slight notation change for completeness.

Ply 1

$$\begin{aligned}
\epsilon_\xi &= [a_{11}(+\alpha)]\sigma_\xi^+ + [a_{12}(+\alpha)]\sigma_\eta^+ + [a_{13}(+\alpha)]\sigma_{\xi\eta}^+ \\
\epsilon_\eta &= [a_{21}(+\alpha)]\sigma_\xi^+ + [a_{22}(+\alpha)]\sigma_\eta^+ + [a_{23}(+\alpha)]\sigma_{\xi\eta}^+ \\
\epsilon_{\xi\eta} &= [a_{31}(+\alpha)]\sigma_\xi^+ + [a_{32}(+\alpha)]\sigma_\eta^+ + [a_{33}(+\alpha)]\sigma_{\xi\eta}^+
\end{aligned}$$

Ply 2

$$\begin{aligned}
\epsilon_\xi &= [a'_{11}(-\alpha)]\sigma_\xi^{++} + [a'_{12}(-\alpha)]\sigma_\eta^{++} + [a'_{13}(-\alpha)]\sigma_{\xi\eta}^{++} \\
\epsilon_\eta &= [a'_{21}(-\alpha)]\sigma_\xi^{++} + [a'_{22}(-\alpha)]\sigma_\eta^{++} + [a'_{23}(-\alpha)]\sigma_{\xi\eta}^{++} \qquad (1) \\
\epsilon_{\xi\eta} &= [a'_{31}(-\alpha)]\sigma_\xi^{++} + [a'_{32}(-\alpha)]\sigma_\eta^{++} + [a'_{33}(-\alpha)]\sigma_{\xi\eta}^{++}
\end{aligned}$$

Equations Linking Both Plies

$$\begin{aligned}
2\sigma_\xi &= \sigma_\xi^+ + \sigma_\xi^{++} = \sigma_\xi^* + \sigma_\xi^{**} \\
2\sigma_\eta &= \sigma_\eta^+ + \sigma_\eta^{++} = \sigma_\eta^* + \sigma_\eta^{**} \\
2\sigma_{\xi\eta} &= \sigma_{\xi\eta}^+ + \sigma_{\xi\eta}^{++} = \sigma_{\xi\eta}^* + \sigma_{\xi\eta}^{**}
\end{aligned}$$

In Eqs. (1), the previous notation involving a single and double asterisk has been replaced by a notation using a super-plus and super-plus-plus. These stresses are taken to mean the sum of the external and interply stress on the first ply, and the difference between the external and interply stress on the second ply, respectively. A conversion equation relating these is given as Eqs. (2).

$$
\begin{aligned}
\sigma_\xi^* + \sigma_\xi' &= \sigma_\xi^+ & \qquad \sigma_\xi^{**} - \sigma_\xi' &= \sigma_\xi^{++} \\
\sigma_\eta^* + \sigma_\eta' &= \sigma_\eta^+ & \qquad \sigma_\eta^{**} - \sigma_\eta' &= \sigma_\eta^{++} \\
\sigma_{\xi\eta}^* + \sigma_{\xi\eta}' &= \sigma_{\xi\eta}^+ & \qquad \sigma_{\xi\eta}^{**} - \sigma_{\xi\eta}' &= \sigma_{\xi\eta}^{++}
\end{aligned}
\tag{2}
$$

In Eqs. (1), the use of the notation given in Eqs. (2) results in lumping together the interply stress and the stress applied by some external agency. Some care must be taken in interpreting this quantity for various loading cases. For example, when one wishes to apply simultaneously three external stresses σ_ξ, σ_η, and $\sigma_{\xi\eta}$, the resulting solutions in terms of the σ^+ and σ^{++} components of each of these stresses do not given any information about the origin of the various interply stresses. To be specific, the number obtained from the solution Eqs. (1) for σ_ξ^+ will contain interply stress components arising from the application of both σ_η and $\sigma_{\xi\eta}$. The individual contributions of these two will not be separable.

Since these equations are linear, superposition always holds. It has been found much more convenient to use these equations by applying one external stress at a time and observing the resulting interply reactions. For example, if one applies only a σ_ξ, the resulting solutions or numerical values for σ_η^+ and σ_η^{++} will represent the η direction normal components of interply stresses generated on each of the two plies due to the application of the σ_ξ alone. Similarly, the numerical values for $\sigma_{\xi\eta}^+$ and $\sigma_{\xi\eta}^{++}$ will represent the shearing components of interply stress due to the application of the σ_ξ alone.

If one wishes to determine the total stress state due to the application of several stresses, effects may be merely added together.

The known quantities in Eqs. (1) are usually taken to be σ_ξ, σ_η and $\sigma_{\xi\eta}$ along with all the a_{ij} and a'_{ij} quantities. This means that one must know the elastic characteristics of each of the plies involved in the laminate as well as the stress state applied to the structure. The resulting unknowns in these nine equations are the three strains ϵ_ξ, ϵ_η, and $\epsilon_{\xi\eta}$, the three stresses on the first ply σ^+_ξ, σ^+_η, and $\sigma^+_{\xi\eta}$, and their counterparts on the second ply σ^{++}_ξ, σ^{++}_η, and $\sigma^{++}_{\xi\eta}$. If some of these quantities go to zero, a corresponding reduction in the number of equations ensues. The three strains are presumed to be the same since the two plies are bonded tightly together.

Using Saint Venant's principle, it is possible to clarify the role of interply stresses in Eqs. (1). First, suppose that certain external stresses σ^+_ξ and σ^{++}_ξ are applied arbitrarily to the edge of a two-ply laminate. A certain portion of this stress σ^*_ξ will then be carried directly by the ply involved while another portion σ'_ξ will be generated due to an interply stress. For the second ply, the part actually transmitted directly through the ply is given by σ^{**}_ξ while the contribution from the interply bond is now $-\sigma'_\xi$. This is concluded from Eqs. (2). Similar conclusions could be reached from the same line of reasoning using either the stresses in the η-direction or the shearing stresses. Thus, in the presence of arbitrary edge loads, interply stresses may be generated in the direction in which the loads are applied. Some distance away from the points of application of these loads, the stresses carried by the two plies adjust themselves so that one ply carries σ^*_ξ while the other carries σ_ξ^{**}; so if one postulates that the edge of the structure is loaded with some average stress σ_ξ and solves for the stresses in each of the two plies, the solution is that given by Eqs. (3).

$$\sigma_\xi^{+} = \sigma_\xi^{*} \quad \sigma_\xi^{++} = \sigma_\xi^{**} \tag{3}$$

Similar equations could be written concerning σ_η and $\sigma_{\xi\eta}$. Thus, some distance away from points of concentrated load application or from free edges, external stresses in the ξ-direction do not generate interply stresses in that direction. Similarly, external stresses in the η-direction do not generate interply stresses in that direction and external stresses $\sigma_{\xi\eta}$ do not generate shear components of the interply stress $\sigma'_{\xi\eta}$. Thus one may always visualize that interply stresses are generated by external loads in directions different from that of the interply stress.

With this concept in mind, one may then use Eqs. (1) to study the various elastic characteristics which result from the bonding together of two plies of cord imbedded in rubber. When the elastic characteristics of both plies are the same, and their thicknesses are the same, Eqs. (1) reduce to a set of two equations in two unknowns since the loads and stresses are distributed equally between the two plies. When the materials or thicknesses of the two plies are different, or when the cords of one ply are in a state of tension while the cords of another ply are in a state of compression, then all nine of Eqs. (1) must be considered. Generally, for the application of only one external stress at a time, this reduces to seven equations in seven unknowns.

The computer program developed is applicable only to the proper two-ply laminate. For instance, the two-ply structure shown in Fig. 1 cannot be analyzed by using this program because the cord half angle of Ply 1 is not equal to the negative cord half angle of Ply 2.

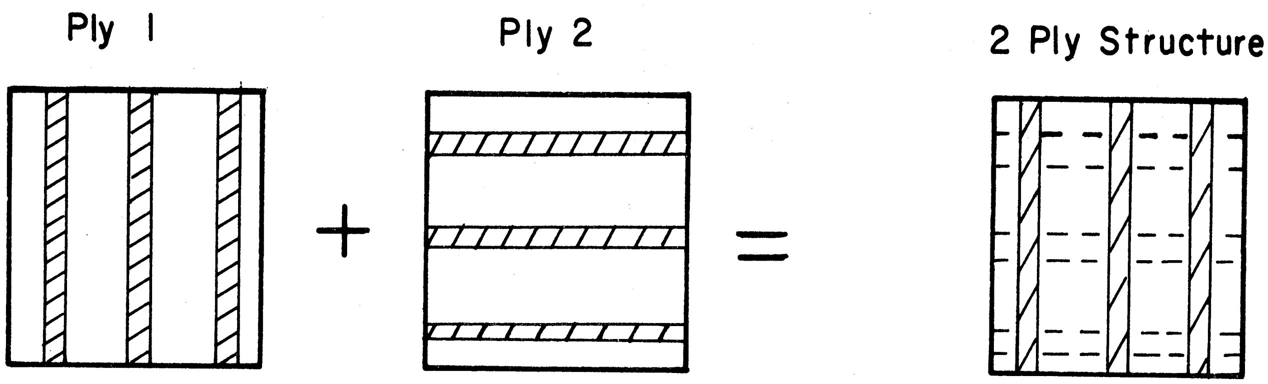


Fig. 1. Two plies at right angles.

E_x	$= 10^4$	E_x	$= 10^4$
E_y	$= 10^2$	E_y	$= 10^2$
F_{xy}	$= 2\times10^4$	F_{xy}	$= 2\times10^4$
G_{xy}	$= .5\times10^2$	G_{xy}	$= .5\times10^2$

where the x-direction is parallel to the cords, y-direction perpendicular to cords.

However, the two-ply laminate in Fig. 1 can be analyzed if it is noted that the two-ply laminate of Fig. 2 is equivalent to that of Fig. 1.

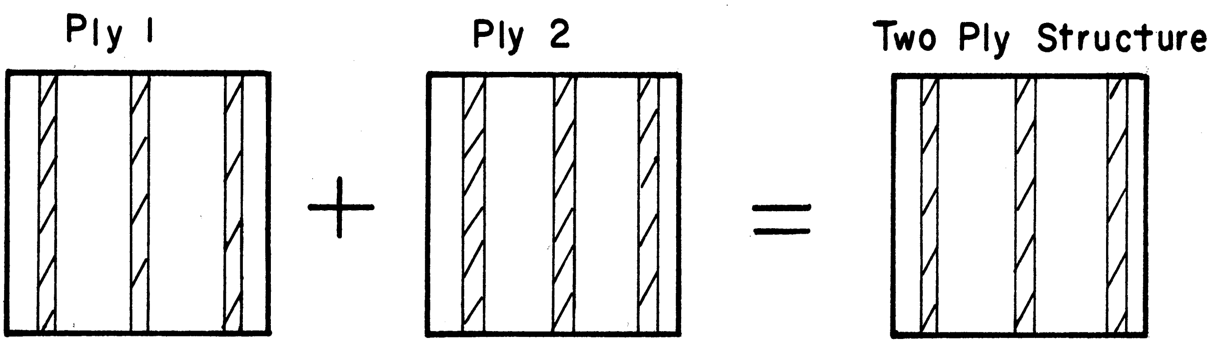


Fig. 2. Two plies at right angles.

$E_x = 10^4$	$E_x = 10^2$
$E_y = 10^2$	$E_y = 10^4$
$F_{xy} = 2x10^4$	$F_{xy} = 2x10^4$
$G_{xy} = .5x10^2$	$G_{xy} = .5x10^2$

where the x-direction is parallel to the cords, y-direction perpendicular to cords.

V. DIGITAL COMPUTER PROGRAM FOR THE SOLUTION OF THE ELASTIC EQUATIONS

The development of the digital computer program for solving Eqs. (1) will become more apparent if the equations are rewritten as shown in Eqs. (1a).

$$
\begin{array}{ccccccccccccccccccc}
a_{11}\sigma_\xi^+ & + & a_{12}\sigma_\eta^+ & + & a_{13}\sigma_{\xi\eta}^+ & + & 0 & + & 0 & + & 0 & - & \epsilon_\xi & + & 0 & + & 0 & = & 0 \\
a_{21}\sigma_\xi^+ & + & a_{22}\sigma_\eta^+ & + & a_{23}\sigma_{\xi\eta}^+ & + & 0 & + & 0 & + & 0 & + & 0 & - & \epsilon_\eta & + & 0 & = & 0 \\
a_{31}\sigma_\xi^+ & + & a_{32}\sigma_\eta^+ & + & a_{33}\sigma_{\xi\eta}^+ & + & 0 & + & 0 & + & 0 & + & 0 & + & 0 & - & \epsilon_{\xi\eta} & = & 0 \\
0 & + & 0 & + & 0 & + & a'_{11}\sigma_\xi^{++} & + & a'_{12}\sigma_\eta^{++} & + & a'_{13}\sigma_{\xi\eta}^{++} & - & \epsilon_\xi & + & 0 & + & 0 & = & 0 \\
0 & + & 0 & + & 0 & + & a'_{21}\sigma_\xi^{++} & + & a'_{22}\sigma_\eta^{++} & + & a'_{23}\sigma_{\xi\eta}^{++} & + & 0 & - & \epsilon_\eta & + & 0 & = & 0 \\
0 & + & 0 & + & 0 & + & a'_{31}\sigma_\xi^{++} & + & a'_{32}\sigma_\eta^{++} & + & a'_{33}\sigma_{\xi\eta}^{++} & + & 0 & + & 0 & - & \epsilon_{\xi\eta} & = & 0 \\
.5\sigma_\xi^+ & + & 0 & + & 0 & + & .5\sigma_\xi^{++} & + & 0 & + & 0 & + & 0 & + & 0 & + & 0 & = & \sigma_\xi \\
0 & + & .5\sigma_\eta^+ & + & 0 & + & 0 & + & .5\sigma_\eta^{++} & + & 0 & + & 0 & + & 0 & + & 0 & = & \sigma_\eta \\
0 & + & 0 & + & .5\sigma_{\xi\eta}^+ & + & 0 & + & 0 & + & .5\sigma_{\xi\eta}^{++} & + & 0 & + & 0 & + & 0 & = & \sigma_{\xi\eta}
\end{array}
\qquad (1a)
$$

The results to be obtained from the computer program are defined below. Whenever the denominator of the following ratios approaches zero, the computation recognize this fact and a comment "NO SIGNIFICANCE" is printed.

The Moduli

XI	Extension Modulus	$= \sigma_\xi / \epsilon_\xi$
ETA	Extension Modulus	$= \sigma_\eta / \epsilon_\eta$
XI-ETA	Shear Modulus	$= \sigma_{\xi\eta} / \epsilon_{\xi\eta}$
XI	Extension Cross Modulus	$= \sigma_\xi / \epsilon_\eta$
ETA	Extension Cross Modulus	$= \sigma_\eta / \epsilon_\xi$
XI	Deformation Modulus	$= \sigma_\xi / \epsilon_{\xi\eta}$
ETA	Deformation Modulus	$= \sigma_\eta / \epsilon_{\xi\eta}$
XI	Shear Cross Modulus	$= \sigma_{\xi\eta} / \epsilon_\xi$
ETA	Shear Cross Modulus	$= \sigma_{\xi\eta} / \epsilon_\eta$

(2a)

The Stress Ratios

For Sigma XI$^+$

Sigma XI	$= \sigma_\xi^+ / \sigma_\xi$
Sigma ETA	$= \sigma_\xi^+ / \sigma_\eta$
Sigma XI-ETA	$= \sigma_\xi^+ / \sigma_{\xi\eta}$

For Sigma ETA$^+$

Sigma XI	$= \sigma_\eta^+ / \sigma_\xi$
Sigma ETA	$= \sigma_\eta^+ / \sigma_\eta$
Sigma XI-ETA	$= \sigma_\eta^+ / \sigma_{\xi\eta}$

For Sigma XI-ETA$^+$

Sigma XI	$= \sigma_{\xi\eta}^+ / \sigma_\xi$
Sigma ETA	$= \sigma_{\xi\eta}^+ / \sigma_\eta$
Sigma XI-ETA	$= \sigma_{\xi\eta}^+ / \sigma_{\xi\eta}$

For Sigma XI^{++}

Sigma XI	$= \sigma_\xi^{++} / \sigma_\xi$
Sigma ETA	$= \sigma_\xi^{++} / \sigma_\eta$
Sigma XI-ETA	$= \sigma_\xi^{++} / \sigma_{\xi\eta}$

For Sigma ETA^{++}

Sigma XI	$= \sigma_\eta^{++} / \sigma_\xi$
Sigma ETA	$= \sigma_\eta^{++} / \sigma_\eta$
Sigma XI-ETA	$= \sigma_\eta^{++} / \sigma_{\xi\eta}$

For Sigma XI-ETA^{++}

Sigma XI	$= \sigma_{\xi\eta}^{++} / \sigma_\xi$
Sigma ETA	$= \sigma_{\xi\eta}^{++} / \sigma_\eta$
Sigma XI-ETA	$= \sigma_{\xi\eta}^{++} / \sigma_{\xi\eta}$

(2b)

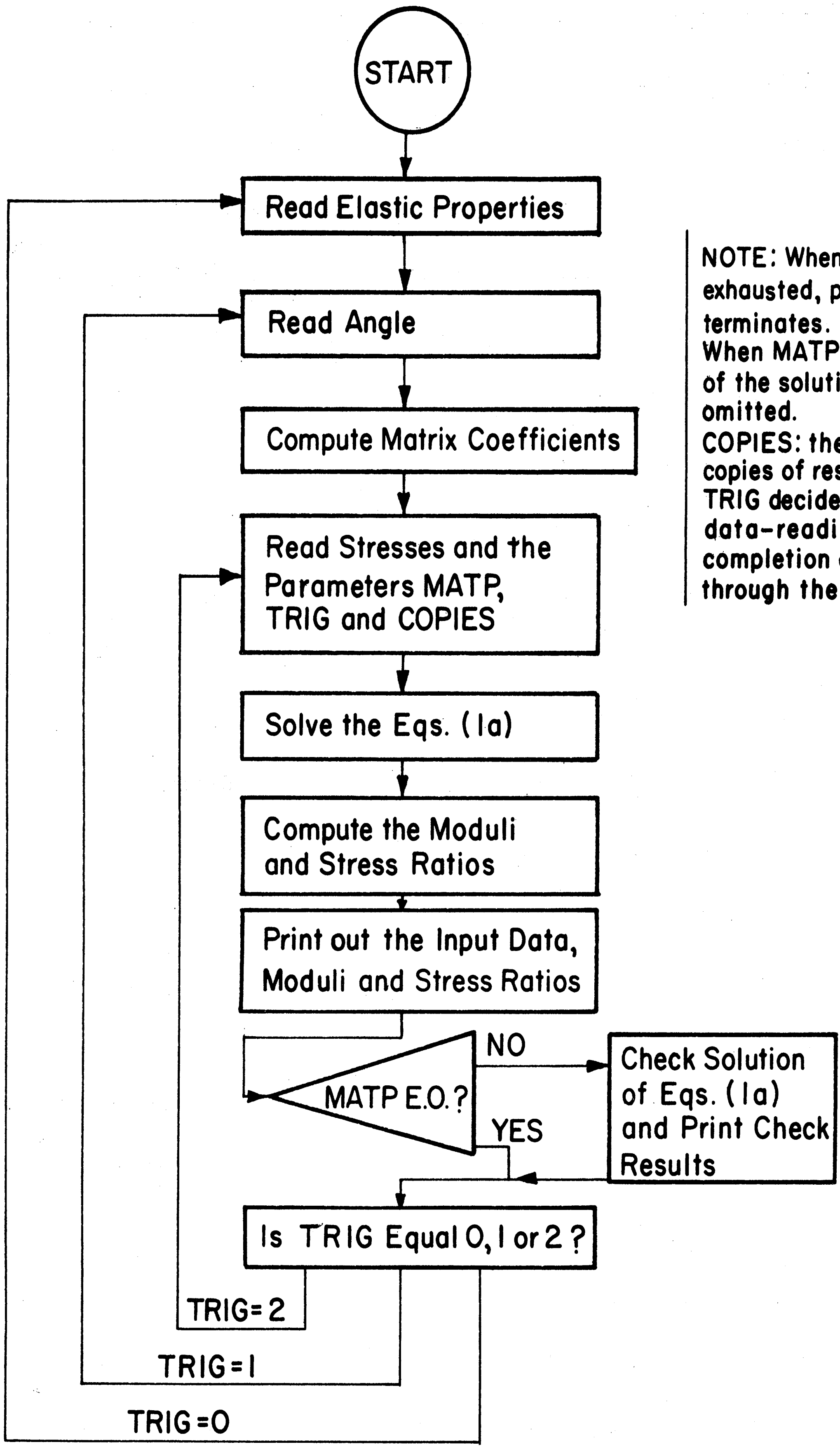
The computer program computes the a-coefficients of the Eqs. (1a), solves the linear simultaneous equations, and computes the results given by Eqs. (2a) and (2b). A flow diagram of the basic parts of the procedure is given in Fig. 3. As input to the program, data must be furnished for the elastic constants, the ply angle, and the stresses which form the right-hand sides of the last three of Eqs. (1a), as well as three parameters used in control of the program. The final output of the program is an image of the input data and the results as given by Eqs. (2a) and (2b). At the option of the user, it is possible to view a check of the coefficient matrix of Eqs. (1a), as well as an accuracy check obtained by back substitution of the solutions into the Eqs. (1a) and noting the difference between the computed right-hand sides and those specified.

A further option available is to print extra copies of the results.

The basic steps of the program are outlined in the flow diagram of Fig. 3. Data are read into the program in three blocks, i.e.,

1. Elastic properties;
2. Ply angle; and
3. Stresses and parameters.

The first (MATP) of the three parameters read in the third block controls the option to check the solution of the equations, while the second parameter (COPIES) will be equal to the number of extra copies of the results to be printed. The last parameter TRIG is used at the end of any complete computation to decide upon the next type of computation. Thus it is possible to leave the elastic properties and cord angle unchanged while making a change in the stresses (and control parameters). Alternately, it is possible



NOTE: Whenever data are exhausted, program automatically terminates.
When MATP is zero, checking of the solution of Eq. (1a) is omitted.
COPIES: the number of extra copies of results to be printed
TRIG decides upon the next data-reading sequences upon completion of the present pass through the program.

Fig. 3. Flow diagram of computer program.

to leave the elastic constants unchanged and read in a new cord angle; in this case new stress data must be read in. Lastly, the whole sequence may be repeated by reading into the program new elastic constants which must be followed by new angle data which, in turn, must be followed with stress data.

A copy of the MAD* program and a typical set of results are included in this report.

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*MAD (Michigan Algorithm Decoder) is an algebraic statement language designed by members of The University of Michigan Computing Center originally for the IBM 704 computer and now available for IBM 709 and 7090. The main features of MAD are very-high-speed compilation and a very general language. Programs produced by MAD are not as fast in execution as those produced by some other compilers; however, this disadvantage can be partially overcome by appropriate programming.

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                 FOR TECHNICAL DISCUSSION OF THIS PROGRAM SEE

                 TECHNICAL REPORT NO. 8

                 DIGITAL COMPUTATION OF

                 TWO-PLY ELASTIC CHARACTERISTICS

                 JULY 1961



                 PROGRAM BY B.HERZOG

                 ORIGINAL**** APRIL 1961

                 REVISED **** AUGUST 1961


                 INFORMATION FOR DATA DECK MAKE UP



                 THE VARIABLE TRIG DECIDES THE CONSEQUENT DATA
                 READING SEQUENCE

                 *************************************************************
                 NORMAL INITIATION OF THE  PROGRAM REQUIRES A COMPLETE
                 SEQUENCE OF DATA AND HENCE NO SELECTION IS REQUIRED

                 AFTER ANY PASS OF THE PROGRAM IT EXPECTS TO READ SOME
                 DATA. IF NO DATA OF ANY KIND IS PRESENT EXECUTION IS
                 AUTOMATICALLY TERMINATED

                 HOWEVER , ANY CONSEQUENT EXECUTION REQUIRES INFORMATION
                 FROM THE PREVIOUS RUN AND THIS INFORMATION IS OBTAINED FORM
                 THE STATEMENT BEGIN(2)

                 THEREFORE THERE MUST APPEAR A 0 (ZERO), 1, OR  2 FOR TRIG.

                 *************************************************************
                         IF ON THE LAST SWEEP OF THE PROGRAM

                             TRIG WAS 2   THEN ONLY SIGMAS ARE READ
                             TRIG WAS 1   THEN NEW ALPHA AND  SIGMAS ARE READ
                             TRIG WAS 0    THEN NEW ELASTIC PROPERTIES,
                                          NEW ALPHA AND NEW SIGMAS ARE READ
```

```
        ****************************************************************
        **  IF AT ANY TIME THE CARDS ARE NOT ARRANGED  IN ACCORDANCE   **
        **  WITH THE EXPECTATIONS THEN THE PROGRAM THROWS ITSELF       **
        **  OFF THE MACHINE                                            **
        ****************************************************************


          EQUIVALENCE (SIGMA,SIG)                                              *001
          DIMENSION R(27),RESULT(9)                                            *002
          DIMENSION MAT(89,DIM),T1(10),T2(10),T3(10),T4(10),                   *003
         1A(89,DIM),RESID(8),RESULT(8),TRIG(30),MD(100),KMNT(10),              *003
         2NORM(10),SRS(35),SIG(3),SIGMA(3),IR(9)                               *003
          VECTOR VALUES DIM=2,0,10                                             *004
          INTEGER TRIG,I,J,K,Z,MD,KMNT,NORM,SRS,NOD,ZZ                         *005
          INTEGER J1,J2,J3,COPIES,TEST,NOR,KKK,KMN,IR,KR                       *006
          BOOLEAN MATP                                                         *007
                 READ ELASTIC PROPERTIES

                FOR PLY(1)
BEGIN     READ FORMAT PROP,TEST,EX,EY,FXY,GXY                                  *008
          WHENEVER TEST.NE.$E$,EXECUTE SYSTEM.                                 *009

                FOR PLY2
          READ FORMAT PROP,TEST,EXP,EYP,FXYP,GXYP                              *010
          WHENEVER TEST.NE.$E$,EXECUTE SYSTEM.                                 *011

                READ AN ANGLE ALPHA WHICH IS THE CHORD HALF ANGLE
BEGIN(1)  READ FORMAT PROP,TEST,ALPHA                                          *012
          WHENEVER TEST.NE.$A$,EXECUTE SYSTEM.                                 *013
                COMPUTE THE TRIGONOMETRIC FUNCTIONS WHICH ARE USED IN
                COMPUTING THE COEFFICIENTS OF MAT
          ALP=ALPHA*PI/180.                                                    *014
          S=SIN.(ALP)                                                          *015
          C=COS.(ALP)                                                          *016
          S2=S*S                                                               *017
          S3=S2*S                                                              *018
          S4=S2*S2                                                             *019
          C2=C*C                                                               *020
          C3=C*C2                                                              *021
          C4=C2*C2                                                             *022
          C2S2=C2*S2                                                           *023
          CS3=C*S3                                                             *024
          C3S=C3*S                                                             *025
          C4PS4=C4+S4                                                          *026
          VECTOR VALUES PI=3.1415927                                           *027
                COMPUTE THE COEFFICIENTS OF SIGMA+ (PLY(1)),
                SEE EQUATION ( 1 ) IN SECTION (1V) OF REPORT
          E1=EX                                                                *028
          E2=EY                                                                *029
          G=GXY                                                                *030
          F=FXY                                                                *031
                MAT(1,1)=F1.(E1,E2)
          MAT=F1.(E1,E2)                                                       *032
                MAT(2,2)=F1.(E2,E1)
          MAT(11)=F1.(E2,E1)                                                   *033
                MAT(3,3)=F3. (0)
          MAT(22)=F3.(0)                                                       *034
          T=F12.(0)                                                            *035
                MAT(1,2)=T
```

```
           MAT(1)=T                                                            *036
                 MAT(2,1)=T
           MAT(10)=T                                                           *037
           K1=1./G-2./F                                                        *038
           K2=2./E1-K1                                                         *039
           K3=K1-2./E2                                                         *040
           T=C3S*K2+CS3*K3                                                     *041
                 MAT(1,3)=T
           MAT(2)=T                                                            *042
                 MAT(3,1)=T
           MAT(20)=T                                                           *043
           T=CS3*K2+C3S*K3                                                     *044
                 MAT(2,3)=T
           MAT(12)=T                                                           *045
                 MAT(3,2)=T
           MAT(21)=T                                                           *046
                 COMPUTE THE COEFFICIENTS OF SIGMA++ (PLY(2)),
                 SEE EQUATION ( 1) IN SECTION  (IV) OF REPORT
           E1=EXP                                                              *047
           E2=EYP                                                              *048
           G=GXYP                                                              *049
           F=FXYP                                                              *050
                 MAT(4,4)=F1.(E1,E2)
           MAT(33)=F1.(E1,E2)                                                  *051
                 MAT(5,5)=F1.(E2,E1)
           MAT(44)=F1.(E2,E1)                                                  *052
                 MAT(6,6)=F3.  (0)
           MAT(55)=F3.(0)                                                      *053
           T=F12.    (0)                                                       *054
                 MAT(4,5)=T
           MAT(34)=T                                                           *055
                 MAT(5,4)=T
           MAT(43)=T                                                           *056
           K1=1./G-2./F                                                        *057
           K2=-2./E1+K1                                                        *058
           K3=K1-2./E2                                                         *059
           T=C3S*K2-CS3*K3                                                     *060
                 MAT(4,6)=T
           MAT(35)=T                                                           *061
                 MAT(6,4)=T
           MAT(53)=T                                                           *062
           T=CS3*K2-C3S*K3                                                     *063
                 MAT(5,6)=T
           MAT(45)=T                                                           *064
                 MAT(6,5)=T
           MAT(54)=T                                                           *065
                  READ THE GIVEN SIGMAS,TRIG AND NUMBER OF EXTRA PRINT COPIES
                 SIG(1),SIG(2) AND SIG(3) ARE THE APPLIED SIGMA EXI, SIGMA ETA
                 AND SIGMA EXI-ETA RESPECTIVELY
BEGIN(2)   READ FORMAT PROP,TEST,SIG(1)...SIG(3),MATP,TRIG,COPIES              *066
           WHENEVER TEST.NE.$S$,EXECUTE SYSTEM.                                *067
           MAT(69)=SIG(1)                                                      *068
           MAT(79)=SIG(2)                                                      *069
           MAT(89)=SIG(3)                                                      *070
                 LOAD INTO A
           THROUGH SUBST,FOR I=0,1,I.G.89                                      *071
SUBST      A(I)=MAT(I)                                                         *072
                  SOLVE THE EQUATIONS
           TRANSFER TO NEXT(SLINEQ.(A,9.,1.,0.,T1,T2,T3,T4))                   *073
NEXT(1)    THROUGH RES,FOR I=1,1,I.G.9                                         *074
```

```
RES      RESULT(I)=A(10*I-1)                                 *075
               BEGIN CALCULATIONS OF MODULI AND RATIOS
         EXECUTE ZERO.(TRIG(1)...TRIG(27))                   *076
         EXECUTE ZERO.(R...R(27))                            *077
               COMPUTE THE MODULII
         WHENEVER.ABS.RESULT(7).LE.1.E-20                    *078
         TRIG(1)=10                                          *079
         TRIG(5)=10                                          *080
         TRIG(8)=10                                          *081
         OTHERWISE                                           *082
               R(1) IS SIGMA EXI/EXI STRAIN
         R(1)=SIGMA(1)/RESULT(7)                             *083
               R(5) IS SIGMA ETA/EXI STRAIN
         R(5)=SIGMA(2)/RESULT(7)                             *084
               R(8) IS SIGMA EXI-ETA/EXI STRAIN
         R(8)=SIGMA(3)/RESULT(7)                             *085
         END OF CONDITIONAL                                  *086

         WHENEVER.ABS.RESULT(8).LE.1.E-20                    *087
         TRIG(2)=10                                          *088
         TRIG(4)=10                                          *089
         TRIG(9)=10                                          *090
         OTHERWISE                                           *091
               R(2) IS SIGMA ETA/ETA STRAIN
         R(2)=SIGMA(2)/RESULT(8)                             *092
               R(4) IS SIGMA EXI/ETA STRAIN
         R(4)=SIGMA(1)/RESULT(8)                             *093
               R(9) IS SIGMA EXI-ETA/ETA STRAIN
         R(9)=SIGMA(3)/RESULT(8)                             *094
         END OF CONDITIONAL                                  *095

         WHENEVER.ABS.RESULT(9).LE.1.E-20                    *096
         TRIG(3)=10                                          *097
         TRIG(6)=10                                          *098
         TRIG(7)=10                                          *099
         OTHERWISE                                           *100
               R(3) IS SIGMA EXI-ETA/EXI-ETA STRAIN
         R(3)=SIGMA(3)/RESULT(9)                             *101
               R(6) IS SIGMA EXI/EXI-ETA STRAIN
         R(6)=SIGMA(1)/RESULT(9)                             *102
               R(7) IS SIGMA ETA/EXI-ETA STRAIN
         R(7)=SIGMA(2)/RESULT(9)                             *103
         END OF CONDITIONAL                                  *104

         THROUGH INTRIZ,FOR I=1,1,I.G.9                      *105
         RI=R(I)                                             *106
         KR=1                                                *107
SMALL    WHENEVER RI.G.1.E+7.AND.TRIG.E.0                    *108
         RI=RI/10                                            *109
         KR=KR*10                                            *110
         TRANSFER TO SMALL                                   *111
         END OF CONDITIONAL                                  *112
         IR=RI                                               *113
         IR(I)=IR*KR                                         *114
INTRIZ   CONTINUE                                            *115
               COMPUTE STRESS RATIOS
         K=9                                                 *116
         THROUGH JOE,FOR J=1,1,J.G.6                         *117
         THROUGH JOE,FOR VALUES OF I=1,2,3                   *118
         K=K+1                                               *119
```

```
              WHENEVER.ABS.SIGMA(I).LE.1.E-20
              TRIG(K)=10
              TRANSFER TO JOE
              END OF CONDITIONAL
              R(K)=RESULT(J)/SIGMA(I)
JOE           CONTINUE
                    PRINT RESULTS
              THROUGH SMELL ,FOR ZZ=0,1,ZZ.G.COPIES
              PRINT FORMAT TITLE
              PRINT FORMAT TITLE1,EX,EXP,EY,EYP,FXY,FXYP,GXY,GXYP
              PRINT FORMAT DATA,ALPHA,SIG(1),SIG(2),SIG(3),TRIG,COPIES
              PRINT FORMAT TITLE 5
              THROUGH MOD,FOR I=1,1,I.G.9
              J=10*I
              WHENEVER TRIG(I).E.10
              K=J+6
              J1=K+1
              J2=K+2
              J3=K+3
              MD(K)=KMNT
              MD(J1)=KMNT(1)
              MD(J2)=KMNT(2)
              MD(J3)=KMNT(3)
              PRINT FORMAT MD(10*I)
              MD(K)=NORM
              MD(J1)=NORM(1)
              MD(J2)=NORM(2)
              MD(J3)=NORM(3)
              OTHERWISE
              PRINT FORMAT MD(J),IR(I),R(I)
MOD           END OF CONDIT IONAL
              PRINT FORMAT TITLE6
              THROUGH STR,FOR I=10,3,I.G.27
              SRS(3)=NOD(I)
              SRS(4)=NOD(I+1)
              J=I+1
              K=I+2
              WHENEVER TRIG(I).E.10
              SRS(12)=KMN
              SRS(13)=KMN(1)
              SRS(14)=KMN(2)
              SRS(15)=KMN(3)
                   WHENEVER TRIG(J).E.10
                   SRS(23)=KMN
                   SRS(24)=KMN(1)
                   SRS(25)=KMN(2)
                   SRS(26)=KMN(3)
                        WHENEVER TRIG(K).E.10
                        SRS(34)=KMN
                        SRS(35)=KMN(1)
                        SRS(36)=KMN(2)
                        SRS(37)=KMN(3)
                        PRINT FORMAT SRS
                        THROUGH FIX,FOR VALUES OF Z=12,23,34
                        SRS(Z)=NOR
                        SRS(Z+1)=NOR(1)
                        SRS(Z+2)=NOR(2)
FIX                     SRS(Z+3)=NOR(3)
                        TRANSFER TO STR
                        END OF C ONDITIONAL
```

```
                PRINT FORMAT SRS,R(K),R(K)
                THROUGH FIX(3),FOR VALUES OF Z=12,23
                SRS(Z)=NOR
                SRS(Z+1)=NOR(1)
                SRS(Z+2)=NOR(2)
FIX(3)          SRS(Z+3)=NOR(3)
                TRANSFER TO STR
                END OF CONDITIONAL
                WHENEVER TRIG(K).E.10
                SRS(34)=KMN
                SRS(35)=KMN(1)
                SRS(36)=KMN(2)
                SRS(37)=KMN(3)
                PRINT FORMAT SRS,R(J),R(J)
                THROUGH FIX(1),FOR VALUES OF Z=12,34
                SRS(Z)=NOR
                SRS(Z+1)=NOR(1)
                SRS(Z+2)=NOR(2)
FIX(1)          SRS(Z+3)=NOR(3)
                     TRANSFER TO STR
                     END OF CONDITIONAL
           PRINT FORMAT SRS,R(J),R(J),R(K),R(K)
           SRS(12)=NOR
           SRS(13)=NOR(1)
           SRS(14)=NOR(2)
           SRS(15)=NOR(3)
           TRANSFER TO STR
           OR WHENEVER TRIG(J).E.10
           SRS(23)=KMN
           SRS(24)=KMN(1)
           SRS(25)=KMN(2)
           SRS(26)=KMN(3)
                WHENEVER TRIG(K).E.10
                SRS(34)=KMN
                SRS(35)=KMN(1)
                SRS(36)=KMN(2)
                SRS(37)=KMN(3)
                PRINT FORMAT SRS,R(I),R(I)
                THROUGH FIX(2),FOR VALUES OF Z=23,34
                SRS(Z)=NOR
                SRS(Z+1)=NOR(1)
                SRS(Z+2)=NOR(2)
FIX(2)          SRS(Z+3)=NOR(3)
                TRANSFER TO STR
                END OF CONDITIONAL
           PRINT FORMAT SRS,R(I),R(I),R(K),R(K)
                SRS(23)=NOR
                SRS(24)=NOR(1)
                SRS(25)=NOR(2)
                SRS(26)=NOR(3)
                TRANSFER TO STR
           OR WHENEVER TRIG(K).E.10
           SRS(34)=KMN
           SRS(35)=KMN(1)
           SRS(36)=KMN(2)
           SRS(37)=KMN(3)
           PRINT FORMAT SRS,R(I),R(I),R(J),R(J)
           SRS(34)=NOR
           SRS(35)=NOR(1)
           SRS(36)=NOR(2)
```

```
         SRS(37)=NOR(3)                                                        *239
         TRANSFER TO STR                                                       *240
         END OF CONDITIONAL                                                    *241
         PRINT FORMAT SRS,R(I),R(I),R(J),R(J),R(K),R(K)                        *242
STR      CONTINUE                                                              *243

               IF MATP IS DIFFERENT FROM ZERO THE COEFFICIENT MATRIX ETC.
              WILL BE PRINTED

         WHEN EVER MATP                                                        *244
              COMPUTE RESIDUES TO CHECK SOLUTION
         THROUGH CHECK,FOR I=1,1,I.G.9                                         *245
         T=-MAT(I,10)                                                          *246
         THROUGH LINE,FOR J=1,1,J.G.9                                          *247
LINE     T=RESULT(J)*MAT(I,J)+T                                                *248
CHECK    RESID(I)=T                                                            *249
              THE COEFFICIENT MATRIX PRINT OUT
         PRINT FORMAT TITLE 3                                                  *250
         THROUGH DOG,FOR I=0,10,I.G.80                                         *251
DOG      PRINT FORMAT CAT,MAT(I)...MAT(I+5)                                    *252
         PRINT FORMAT TITLE 4                                                  *253
         THROUGH BIRD,FOR I=1,1,I.G.9                                          *254
BIRD     PRINT FORMAT CAT,MAT(10*I-4)...MAT(10*I-1),RESULT(I),RESID(I)         *255
         END OF C ONDITIONAL                                                   *256


SMELL    CONTINUE                                                              *257
         TRANSFER TO BEGIN(TRIG)                                               *258


NEXT(2)  PRINT FORMAT SING                                                     *259
              THE SIMOULTANEOUS EQUATIONS APPEAR  *SINGULAR* TO SLINEQ.
         EXECUTE ERROR.                                                        *260


              INTERNAL FUNCTIONS

         INTERNAL FUNCTION F1.(EA,EB)=C4/EA+S4/EB                              *261
        1+C2S2*(1./G-2./F)                                                     *261
         INTERNAL FUNCTION F3.(COWS)=C2S2*(4./E1+4./E2+8./F-2./G)              *262
        1+C4PS4/G                                                              *262
         INTERNAL FUNCTION F12.(BULLS)=C2S2*(1./E1+1./E2-1./G)-C4PS4/F         *263


              VECTOR VALUE STATEMENTS FOR FORMATS ETC.

         VECTOR VALUES CAT=$1H0,6(E19.8)          *$                           *264
         VECTOR VALUES PROP=$C1,F19.5,3F15.5,I5,I2*$                           *265
         VECTOR VALUES TITLE=$1H1,S53,14HTHE INPUT DATA/1H0,118(1H*)//         *266
        1*$                                                                    *266
         VECTOR VALUES TITLE1=                                                 *267
        1$1H0,S10,6HEX    =F13.2,S2,4HPSI.,S35,6HEXP   =F13.2,S2,4HPSI./       *267
        2$,                                                                    *267
        3$1H0,S10,6HEY    =F13.2,S2,4HPSI.,S35,6HEYP   =F13.2,S2,4HPSI./       *267
        4$,                                                                    *267
        5$1H0,S10,6HFXY   =F13.2,S2,4HPSI.,S35,6HFXYP  =F13.2,S2,4HPSI./       *267
        6$,                                                                    *267
        7$1H0,S10,6HGXY   =F13.2,S2,4HPSI.,S35,6HGXYP  =F13.2,S2,4HPSI./       *267
        8*$                                                                    *267
```

```
 VECTOR VALUES DATA=                                                   *268
1$1H0,S46,7HALPHA =F6.1,S2,7HDEGREES/1H0,$,                            *268
2$S10,8HSIGMA1 =F10.2,$,                                               *268
3$S10,8HSIGMA2 =F10.2,$ ,                                              *268
4$S10,8HSIGMA3 =F10.2/I65,I5//*$                                       *268
 VECTOR VALUES TITLE3=$1H1,S42,35H THE COEFFICIENT MATRIX AND          *269
1RESULTS                                                               *269
21H0,118(1H*)///S6,10HFOR SIGMA+.S47,11HFOR SIGMA++//*$                *269
 VECTOR VALUES TITLE4=$1H0///S6,11HFOR EPSILON,S46,                    *270
112HGIVEN SIGMAS,S 7,13HTHE SOLUTIONS,S6,12HTHE RESIDUES//*$           *270
 VECTOR VALUES TITLE5=$1H4,S50,18HTHE ELASTIC MODULI                   *271
1/1H0,118(1H*)//*$                                                     *271
 VECTOR VALUES TITLE6=$1H1,S50,17HTHE STRESS RATIOS                    *272
1/1H ,118(1H*)//*$                                                     *272
 VECTOR VALUES WELL=$1H0,E30.8,E56.8,E20.8*$                           *273
 VECTOR VALUES SING=$1H4,15HSINGULAR RETURN*$                          *274
 VECTOR VALUES SRS=                                                    *275
1$27H0      FOR SIGMA            $,                                    *275
2$/1H ,S5,21(1H*)//1H ,S5,14HSIGMA XI      = $,                        *275
3$F26.4,            E60.8   $,                                         *275
4$/1H ,                      S5,14HSIGMA ETA     = $,                  *275
5$F26.4,            E60.8   $,                                         *275
6$/1H ,                      S5,14HSIGMA XI-ETA = $,                   *275
7$F26.4,            E60.8       ///*$                                  *275
 VECTOR VALUES KMN=$S11,15HNO SIGNIFICANCE  $                          *276
 VECTOR VALUES     NOR=$F26.4,            E60.8   $                    *277
 VECTOR VALUES NOD(10)=$ XI+          $                                *278
 VECTOR VALUES NOD(13)=$ ETA+         $                                *279
 VECTOR VALUES NOD(16)=$ XI-ETA+      $                                *280
 VECTOR VALUES NOD(19)=$ XI++         $                                *281
 VECTOR VALUES NOD(22)=$ ETA++        $                                *282
 VECTOR VALUES NOD(25)=$ XI-ETA++     $                                *283
 VECTOR VALUES KMNT=$S11,15HNO SIGNIFICANCE *$                         *284
 VECTOR VALUES NORM=$F20.2,S2,4HPSI.,E60.8  *$                         *285
 VECTOR VALUES MD(10)=$1H0,29HXI EXTENSION MODULUS          =I20       *286
1,  S2,4HPSI.,E60.8  *$                                                *286
 VECTOR VALUES MD(20)=$1H0,29HETA EXTENSION MODULUS         =I20       *287
1,  S2,4HPSI.,E60.8  *$                                                *287
 VECTOR VALUES MD(30)=$1H0,29HXI-ETA SHEAR MODULUS          =I20       *288
1,  S2,4HPSI.,E60.8  *$                                                *288
 VECTOR VALUES MD(40)=$1H0,29HXI EXTENSION CROSS MODULUS    =I20       *289
1,  S2,4HPSI.,E60.8  *$                                                *289
 VECTOR VALUES MD(50)=$1H0,29HETA EXTENSION CROSS MODULUS =I20         *290
1,  S2,4HPSI.,E60.8  *$                                                *290
 VECTOR VALUES MD(60)=$1H0,29HXI DEFORMATION MODULUS        =I20       *291
1,  S2,4HPSI.,E60.8  *$                                                *291
 VECTOR VALUES MD(70)=$1H0,29HETA DEFORMATION MODULUS       =I20       *292
1,  S2,4HPSI.,E60.8  *$                                                *292
 VECTOR VALUES MD(80)=$1H0,29HXI SHEAR CROSS MODULUS        =I20       *293
1,  S2,4HPSI.,E60.8  *$                                                *293
 VECTOR VALUES MD(90)=$1H0,29HETA SHEAR CROSS MODULUS       =I20       *294
1,  S2,4HPSI.,E60.8  *$                                                *294
 VECTOR VALUES MAT=10.,10.,10.,0.,0.,0.,-1.,0.,0.,0.,10.,10.,1         *295
10.,0.,0.,0.,0.,-1.,0.,0.,10.,10.,10.,0.,0.,0.,0.,0.,-1.,0.,0.         *295
2,0.,0.,11.,11.,11.,-1.,0.,0.,0.,0.,0.,0.,11.,11.,11.,0.,-1.,          *295
30.,0.,0.,0.,0.,11.,11.,11.,0.,0.,-1.,0.,.5,0.,0.,.5,0.,0.,0.,         *295
40.,0.,12.,0.,.5,0.,0.,.5,0.,0.,0.,0.,12.,0.,0.,.5,0.,0.,.5,           *295
50.,0.,0.,12.                                                          *295
 END OF PROGRAM                                                        *296
```

THE INPUT DATA

****

EX =	223500.00 PSI.	EXP =	12400.00 PSI.
EY =	970.00 PSI.	EYP =	970.00 PSI.
FXY =	405000.00 PSI.	FXYP =	14000.00 PSI.
GXY =	308.00 PSI.	GXYP =	308.00 PSI.

ALPHA = 30.0 DEGREES

SIGMA1 = 100.00   SIGMA2 = 0.00   SIGMA3 = 0.00
2 0

THE ELASTIC MODULI

****

XI EXTENSION MODULUS =	4623	PSI.	0.46236108E 04
ETA EXTENSION MODULUS =	-0	PSI.	-0.000000000000
XI-ETA SHEAR MODULUS =	-0	PSI.	-0.000000000000
XI EXTENSION CROSS MODULUS =	-2404	PSI.	-0.24043885E 04
ETA EXTENSION CROSS MODULUS =	0	PSI.	0.000000000000
XI DEFORMATION MODULUS =	-8295	PSI.	-0.82956500E 04
ETA DEFORMATION MODULUS =	-0	PSI.	-0.000000000000
XI SHEAR CROSS MODULUS =	0	PSI.	0.000000000000
ETA SHEAR CROSS MODULUS =	-0	PSI.	-0.000000000000

```
                                             THE STRESS RATIOS
*********************************************************************************************************************

    FOR SIGMA XI+
    ********************

    SIGMA XI     =                 1.0436                                       0.10435629E 01
    SIGMA ETA    =           NO SIGNIFICANCE
    SIGMA XI-ETA =           NO SIGNIFICANCE


    FOR SIGMA ETA+
    ********************

    SIGMA XI     =                 0.0007                                       0.69132327E-03
    SIGMA ETA    =           NO SIGNIFICANCE
    SIGMA XI-ETA =           NO SIGNIFICANCE


    FOR SIGMA XI-ETA+
    ********************

    SIGMA XI     =                 0.5288                                       0.52877135E 00
    SIGMA ETA    =           NO SIGNIFICANCE
    SIGMA XI-ETA =           NO SIGNIFICANCE


    FOR SIGMA XI++
    ********************

    SIGMA XI     =                 0.9564                                       0.95643708E 00
    SIGMA ETA    =           NO SIGNIFICANCE
    SIGMA XI-ETA =           NO SIGNIFICANCE


    FOR SIGMA ETA++
    ********************

    SIGMA XI     =                -0.0007                                      -0.69132327E-03
    SIGMA ETA    =           NO SIGNIFICANCE
    SIGMA XI-ETA =           NO SIGNIFICANCE


    FOR SIGMA XI-ETA++
    ********************

    SIGMA XI     =                -0.5288                                      -0.52877135E 00
    SIGMA ETA    =           NO SIGNIFICANCE
    SIGMA XI-ETA =           NO SIGNIFICANCE
```

THE COEFFICIENT MATRIX AND RESULTS

****

FOR SIGMA+			FOR SIGMA++		
0.67479004E-03	-0.41617152E-03	-0.92216979E-03	0.000000000000	0.000000000000	0.000000000000
-0.41617152E-03	0.11880168E-02	0.33234973E-04	0.000000000000	0.000000000000	0.000000000000
-0.92216979E-03	0.33234973E-04	0.15919434E-02	0.000000000000	0.000000000000	0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.69177637E-03	-0.44498913E-03	0.84283503E-03
0.000000000000	0.000000000000	0.000000000000	-0.44498913E-03	0.11669176E-02	-0.19866139E-04
0.000000000000	0.000000000000	0.000000000000	0.84283503E-03	-0.19866139E-04	0.17525107E-02
0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00	0.000000000000	0.000000000000
0.000000000000	0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00	0.000000000000
0.000000000000	0.000000000000	0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00

FOR EPSILON			GIVEN SIGMAS	THE SOLUTIONS	THE RESIDUES
-0.09999999E 01	0.000000000000	0.000000000000	0.000000000000	0.10435629E 03	0.13969839E-08
0.000000000000	-0.09999999E 01	0.000000000000	0.000000000000	0.69132328E-01	-0.93132257E-09
0.000000000000	0.000000000000	-0.09999999E 01	0.000000000000	0.52877135E 02	0.000000000000
-0.09999999E 01	0.000000000000	0.000000000000	0.000000000000	0.95643708E 02	-0.46566129E-09
0.000000000000	-0.09999999E 01	0.000000000000	0.000000000000	-0.69132328E-01	-0.13969839E-08
0.000000000000	0.000000000000	-0.09999999E 01	0.000000000000	-0.52877135E 02	0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.09999999E 03	0.21628118E-01	-0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.000000000000	-0.41590616E-01	-0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.000000000000	-0.12054510E-01	-0.000000000000

```
                                   THE INPUT DATA
*******************************************************************************************

     EX   =    223500.00  PSI.                        EXP  =     12400.00  PSI.
     EY   =       970.00  PSI.                        EYP  =       970.00  PSI.
     FXY  =    405000.00  PSI.                        FXYP =     14000.00  PSI.
     GXY  =       308.00  PSI.                        GXYP =       308.00  PSI.
                                   ALPHA =  30.0  DEGREES
     SIGMA1 =      0.00        SIGMA2 =      0.00        SIGMA3 =    100.00
                                                    1   0


                                 THE ELASTIC MODULI
*******************************************************************************************

XI EXTENSION MODULUS          =           -0  PSI.                  -0.000000000000
ETA EXTENSION MODULUS         =            0  PSI.                   0.000000000000
XI-ETA SHEAR MODULUS          =         4743  PSI.                   0.47436296E 04
XI EXTENSION CROSS MODULUS    =            0  PSI.                   0.000000000000
ETA EXTENSION CROSS MODULUS   =           -0  PSI.                  -0.000000000000
XI DEFORMATION MODULUS        =            0  PSI.                   0.000000000000
ETA DEFORMATION MODULUS       =            0  PSI.                   0.000000000000
XI SHEAR CROSS MODULUS        =        -8295  PSI.                  -0.82956494E 04
ETA SHEAR CROSS MODULUS       =        25553  PSI.                   0.25553850E 05
```

```
                                        THE STRESS RATIOS
*****************************************************************************************************

     FOR SIGMA XI+
     *********************

     SIGMA XI     =          NO SIGNIFICANCE
     SIGMA ETA    =          NO SIGNIFICANCE
     SIGMA XI-ETA =                   1.6659                                        0.16658786E 01


     FOR SIGMA ETA+
     *********************

     SIGMA XI     =          NO SIGNIFICANCE
     SIGMA ETA    =          NO SIGNIFICANCE
     SIGMA XI-ETA =                   0.5862                                        0.58615185E 00


     FOR SIGMA XI-ETA+
     *********************

     SIGMA XI     =          NO SIGNIFICANCE
     SIGMA ETA    =          NO SIGNIFICANCE
     SIGMA XI-ETA -                   1.0852                                        0.10851838E 01


     FOR SIGMA XI++
     *********************

     SIGMA XI     =          NO SIGNIFICANCE
     SIGMA ETA    =          NO SIGNIFICANCE
     SIGMA XI-ETA =                  -1.6659                                       -0.16658786E 01


     FOR SIGMA ETA++
     *********************

     SIGMA XI     =          NO SIGNIFICANCE
     SIGMA ETA    =          NO SIGNIFICANCE
     SIGMA XI-ETA =                  -0.5862                                       -0.58615185E 00


     FOR SIGMA XI-ETA++
     *********************

     SIGMA XI     =          NO SIGNIFICANCE
     SIGMA ETA    =          NO SIGNIFICANCE
     SIGMA XI-ETA =                   0.9148                                        0.91481622E 00
```

THE INPUT DATA

```
**************************************************************************************************

EX    =   223500.00  PSI.                              EXP  =    12400.00  PSI.
EY    =      970.00  PSI.                              EYP  =      970.00  PSI.
FXY   =   405000.00  PSI.                              FXYP =    14000.00  PSI.
GXY   =      308.00  PSI.                              GXYP =      308.00  PSI.
                              ALPHA =  45.0  DEGREES
SIGMA1 =      0.00        SIGMA2 =      0.00        SIGMA3 =    100.00
                                                 0    0
```

THE ELASTIC MODULI

```
**************************************************************************************************

XI EXTENSION MODULUS         =        -0  PSI.        -0.000000000000
ETA EXTENSION MODULUS        =        -0  PSI.        -0.000000000000
XI-ETA SHEAR MODULUS         =      6585  PSI.         0.65858535E 04
XI EXTENSION CROSS MODULUS   =        -0  PSI.        -0.000000000000
ETA EXTENSION CROSS MODULUS  =        -0  PSI.        -0.000000000000
XI DEFORMATION MODULUS       =         0  PSI.         0.000000000000
ETA DEFORMATION MODULUS      =         0  PSI.         0.000000000000
XI SHEAR CROSS MODULUS       =    -15126  PSI.        -0.15126802E 05
ETA SHEAR CROSS MODULUS      =    -15126  PSI.        -0.15126801E 05
```

```
                                        THE STRESS RATIOS
**************************************************************************************************

FOR SIGMA XI+
*********************

SIGMA XI     =        NO SIGNIFICANCE
SIGMA ETA    =        NO SIGNIFICANCE
SIGMA XI-ETA =                 0.9936                              0.99358482E 00


FOR SIGMA ETA+
*********************

SIGMA XI     =        NO SIGNIFICANCE
SIGMA ETA    =        NO SIGNIFICANCE
SIGMA XI-ETA =                 0.9936                              0.99358482E 00


FOR SIGMA XI-ETA+
*********************

SIGMA XI     =        NO SIGNIFICANCE
SIGMA ETA    =        NO SIGNIFICANCE
SIGMA XI-ETA =                 1.1263                              0.11262749E 01


FOR SIGMA XI++
*********************

SIGMA XI     =        NO SIGNIFICANCE
SIGMA ETA    =        NO SIGNIFICANCE
SIGMA XI-ETA =                -0.9936                             -0.99358482E 00


FOR SIGMA ETA++
*********************

SIGMA XI     =        NO SIGNIFICANCE
SIGMA ETA    =        NO SIGNIFICANCE
SIGMA XI-ETA =                -0.9936                             -0.99358482E 00


FOR SIGMA XI-ETA++
*********************

SIGMA XI     =        NO SIGNIFICANCE
SIGMA ETA    =        NO SIGNIFICANCE
SIGMA XI-ETA =                 0.8737                              0.87372506E 00
```

THE COEFFICIENT MATRIX AND RESULTS

****

FOR SIGMA+			FOR SIGMA++		
0.10693042E-02	-0.55407233E-03	-0.51322675E-03	0.000000000000	0.000000000000	0.000000000000
-0.55407233E-03	0.10693042E-02	-0.51322675E-03	0.000000000000	0.000000000000	0.000000000000
-0.51322675E-03	-0.51322675E-03	0.10403403E-02	0.000000000000	0.000000000000	0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.10538672E-02	-0.56950932E-03	0.47514132E-03
0.000000000000	0.000000000000	0.000000000000	-0.56950932E-03	0.10538672E-02	0.47514132E-03
0.000000000000	0.000000000000	0.000000000000	0.47514132E-03	0.47514132E-03	0.12544300E-02
0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00	0.000000000000	0.000000000000
0.000000000000	0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00	0.000000000000
0.000000000000	0.000000000000	0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00

FOR EPSILON			GIVEN SIGMAS	THE SOLUTIONS	THE RESIDUES
-0.09999999E 01	0.000000000000	0.000000000000	0.000000000000	0.99358482E 02	-0.17462298E-08
0.000000000000	-0.09999999E 01	0.000000000000	0.000000000000	0.99358482E 02	-0.13969839E-08
0.000000000000	0.000000000000	-0.09999999E 01	0.000000000000	0.11262749E 03	-0.93132257E-09
-0.09999999E 01	0.000000000000	0.000000000000	0.000000000000	-0.99358482E 02	0.29103830E-08
0.000000000000	-0.09999999E 01	0.000000000000	0.000000000000	-0.99358482E 02	0.32596290E-08
0.000000000000	0.000000000000	-0.09999999E 01	0.000000000000	0.87372506E 02	-0.93132257E-09
0.000000000000	0.000000000000	0.000000000000	0.000000000000	-0.66107824E-02	0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.000000000000	-0.66107828E-02	0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.09999999E 03	0.15184060E-01	0.000000000000

THE INPUT DATA

```
****************************************************************************************************

          EX    =    200000.00  PSI.                                    EXP   =    200000.00  PSI.

          EY    =      1125.00  PSI.                                    EYP   =      1125.00  PSI.

          FXY   =    421000.00  PSI.                                    FXYP  =    421000.00  PSI.

          GXY   =       775.00  PSI.                                    GXYP  =       775.00  PSI.

                                            ALPHA =   30.0  DEGREES

          SIGMA1 =    100.00         SIGMA2 =       0.00          SIGMA3 =       0.00
                                                               2    0
```

THE ELASTIC MODULI

```
****************************************************************************************************

XI EXTENSION MODULUS         =              12489  PSI.                          0.12489467E 05

ETA EXTENSION MODULUS        =                 -0  PSI.                         -0.000000000000

XI-ETA SHEAR MODULUS         =                 -0  PSI.                         -0.000000000000

XI EXTENSION CROSS MODULUS   =              -4638  PSI.                         -0.46384025E 04

ETA EXTENSION CROSS MODULUS  =                  0  PSI.                          0.000000000000

XI DEFORMATION MODULUS       =          -69905066  PSI.                         -0.71582787E 11

ETA DEFORMATION MODULUS      =                 -0  PSI.                         -0.000000000000

XI SHEAR CROSS MODULUS       =                  0  PSI.                          0.000000000000

ETA SHEAR CROSS MODULUS      =                 -0  PSI.                         -0.000000000000
```

```
                                        THE STRESS RATIOS
*****************************************************************************************************

    FOR SIGMA XI+
    *********************

    SIGMA XI     =              1.0000                                0.09999999E 01
    SIGMA ETA    =         NO SIGNIFICANCE
    SIGMA XI-ETA =         NO SIGNIFICANCE


    FOR SIGMA ETA+
    *********************

    SIGMA XI     =              0.0000                                0.11920928E-08
    SIGMA ETA    =         NO SIGNIFICANCE
    SIGMA XI-ETA =         NO SIGNIFICANCE


    FOR SIGMA XI-ETA+
    *********************

    SIGMA XI     =              0.4692                                0.46915075E 00
    SIGMA ETA    =         NO SIGNIFICANCE
    SIGMA XI-ETA =         NO SIGNIFICANCE


    FOR SIGMA XI++
    *********************

    SIGMA XI     =              1.0000                                0.99999999E 00
    SIGMA ETA    =         NO SIGNIFICANCE
    SIGMA XI-ETA =         NO SIGNIFICANCE


    FOR SIGMA ETA++
    *********************

    SIGMA XI     =             -0.0000                               -0.11920928E-08
    SIGMA ETA    =         NO SIGNIFICANCE
    SIGMA XI-ETA =         NO SIGNIFICANCE


    FOR SIGMA XI-ETA++
    *********************

    SIGMA XI     =             -0.4692                               -0.46915075E 00
    SIGMA ETA    =         NO SIGNIFICANCE
    SIGMA XI-ETA =         NO SIGNIFICANCE
```

THE COEFFICIENT MATRIX AND RESULTS

****

FOR SIGMA+			FOR SIGMA++		
0.29941278E-03	-0.75815873E-04	-0.46753696E-03	0.000000000000	0.000000000000	0.000000000000
-0.75815873E-04	0.74135721E-03	-0.29793321E-03	0.000000000000	0.000000000000	0.000000000000
-0.46753696E-03	-0.29793321E-03	0.99656019E-03	0.000000000000	0.000000000000	0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.29941278E-03	-0.75815873E-04	0.46753696E-03
0.000000000000	0.000000000000	0.000000000000	-0.75815873E-04	0.74135721E-03	0.29793321E-03
0.000000000000	0.000000000000	0.000000000000	0.46753696E-03	0.29793321E-03	0.99656019E-03
0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00	0.000000000000	0.000000000000
0.000000000000	0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00	0.000000000000
0.000000000000	0.000000000000	0.50000000E 00	0.000000000000	0.000000000000	0.50000000E 00

FOR EPSILON			GIVEN SIGMAS	THE SOLUTIONS	THE RESIDUES
-0.09999999E 01	0.000000000000	0.000000000000	0.000000000000	0.09999999E 03	-0.000000000000
0.000000000000	-0.09999999E 01	0.000000000000	0.000000000000	0.11920929E-06	0.23283064E-09
0.000000000000	0.000000000000	-0.09999999E 01	0.000000000000	0.46915075E 02	0.46566129E-09
-0.09999999E 01	0.000000000000	0.000000000000	0.000000000000	0.99999999E 02	-0.69849193E-09
0.000000000000	-0.09999999E 01	0.000000000000	0.000000000000	-0.11920929E-06	0.23283064E-09
0.000000000000	0.000000000000	-0.09999999E 01	0.000000000000	-0.46915075E 02	0.13969839E-08
0.000000000000	0.000000000000	0.000000000000	0.09999999E 03	0.80067466E-02	-0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.000000000000	-0.21559146E-01	-0.000000000000
0.000000000000	0.000000000000	0.000000000000	0.000000000000	-0.13969839E-08	-0.000000000000

```
6 SEPT 1961 090661 733                    WEDNESDAY  5 57 PM     6 01 PM
   HERZOG CLARK             S063F



                    ELAPSED TIMES IN MINUTES
   $  17. TOTAL        .7 EXECUTE       2.8 COMPILE



ALL USERS NOTE - THE 704 WILL BE REPLACED BY A 709 COMPUTER AS OF SEPT. 18.  SEE NOTICE ON THE BULLETIN BOARD.
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VI. EXTENSION TO MULTI-PLY LAMINATES

The equations writen here, and the resulting computer program, may be used for a study of laminates involving even numbers of plies such as four, six, or eight, provided that each alternate pair takes on the same distribution of elastic properties as the first. This would quite generally be the case in a structure where the application of torsion or twist caused the cords in one direction to go into one stress state while those in the other direction went into a stress state of opposite sign. The reason that this same set of equations may be used is simply that here each pair of plies acts as a unit in exactly the same way as the first pair of plies. Thus, the solution for each pair is exactly the same as the solution for the first pair.

When a number of plies are bonded together and each of the plies has a different set of elastic characteristics, it is necessary to write equations similar to Eqs. (1) but now embracing the entire structure. For example, in a four-ply structure made up so that each ply is different from the others, either in angle or in elastic characteristics, three equations would have to be written for each of the four plies, similar to the first three of Eqs. (1). In addition, a set of three equations would be necessary linking together the over-all average stresses and the stresses carried by each separate ply such as the last three of Eqs. (1). One would then be faced with a set of fifteen equations in the fifteen unknowns, such as given by Eqs. (4).

Ply 1

$$\epsilon_\xi = [a_{11}(\alpha_1)]\sigma_{\xi 1} + [a_{12}(\alpha_1)]\sigma_{\eta 1} + [a_{13}(\alpha_1)]\sigma_{\xi\eta 1}$$
$$\epsilon_\eta = [a_{21}(\alpha_1)]\sigma_{\xi 1} + [a_{22}(\alpha_1)]\sigma_{\eta 1} + [a_{23}(\alpha_1)]\sigma_{\xi\eta 1}$$
$$\epsilon_{\xi\eta} = [a_{31}(\alpha_1)]\sigma_{\xi 1} + [a_{32}(\alpha_1)]\sigma_{\eta 1} + [a_{33}(\alpha_1)]\sigma_{\xi\eta 1}$$

Ply 2

$$\epsilon_\xi = [a_{11}(\alpha_2)]\sigma_{\xi 2} + [a_{12}(\alpha_2)]\sigma_{\eta 2} + [a_{13}(\alpha_2)]\sigma_{\xi\eta 2}$$
$$\epsilon_\eta = [a_{21}(\alpha_2)]\sigma_{\xi 2} + [a_{22}(\alpha_2)]\sigma_{\eta 2} + [a_{23}(\alpha_2)]\sigma_{\xi\eta 2}$$
$$\epsilon_{\xi\eta} = [a_{31}(\alpha_2)]\sigma_{\xi 2} + [a_{32}(\alpha_2)]\sigma_{\eta 2} + [a_{33}(\alpha_2)]\sigma_{\xi\eta 2}$$

Ply 3

$$\epsilon_\xi = [a_{11}(\alpha_3)]\sigma_{\xi 3} + [a_{12}(\alpha_3)]\sigma_{\eta 3} + [a_{13}(\alpha_3)]\sigma_{\xi\eta 3}$$
$$\epsilon_\eta = [a_{21}(\alpha_3)]\sigma_{\xi 3} + [a_{22}(\alpha_3)]\sigma_{\eta 3} + [a_{23}(\alpha_3)]\sigma_{\xi\eta 3} \qquad (4)$$
$$\epsilon_{\xi\eta} = [a_{31}(\alpha_3)]\sigma_{\xi 3} + [a_{32}(\alpha_3)]\sigma_{\eta 3} + [a_{33}(\alpha_3)]\sigma_{\xi\eta 3}$$

Ply 4

$$\epsilon_\xi = [a_{11}(\alpha_4)]\sigma_{\xi 4} + [a_{12}(\alpha_4)]\sigma_{\eta 4} + [a_{13}(\alpha_4)]\sigma_{\xi\eta 4}$$
$$\epsilon_\eta = [a_{21}(\alpha_4)]\sigma_{\xi 4} + [a_{22}(\alpha_4)]\sigma_{\eta 4} + [a_{23}(\alpha_4)]\sigma_{\xi\eta 4}$$
$$\epsilon_{\xi\eta} = [a_{31}(\alpha_4)]\sigma_{\xi 4} + [a_{32}(\alpha_4)]\sigma_{\eta 4} + [a_{33}(\alpha_4)]\sigma_{\xi\eta 4}$$

$$\sigma_\xi(h_1 + h_2 + h_3 + h_4) = h_1\sigma_{\xi 1} + h_2\sigma_{\xi 2} + h_3\sigma_{\xi 3} + h_4\sigma_{\xi 4}$$
$$\sigma_\eta(h_1 + h_2 + h_3 + h_4) = h_1\sigma_{\eta 1} + h_2\sigma_{\eta 2} + h_3\sigma_{\eta 3} + h_4\sigma_{\eta 4}$$
$$\sigma_{\xi\eta}(h_1 + h_2 + h_3 + h_4) = h_1\sigma_{\xi\eta 1} + h_2\sigma_{\xi\eta 2} + h_3\sigma_{\xi\eta 3} + h_4\sigma_{\xi\eta 4}$$

In Eqs. (4), it has been assumed that each ply lies at some arbitrary angle given by the symbol α with a subscript used to denote the particular ply in question. Similarly, it is most convenient to denote the stresses in the various plies by means of a subscript showing direction, such as ξ and η, followed by a number indicating the number of the ply involved.

These fifteen equations in fifteen unknowns could be solved in just exactly the same manner used here to solve nine equations in nine unknowns. This could undoubtedly be done most conveniently by means of some standard program for the solution of a large number of simultaneous algebraic equations on a digital computer. In any event, the equations are determinate, could be solved, and are linear. Thus, answers from them could be obtained for quite general four-ply structure, and for a structure made up of any number of plies such as six, eight, or any larger number.

In dealing with laminates involving odd numbers of plies, exactly the same line of reasoning may be used as just given for even numbers of plies. Here, however, it will be necessary simply to write the appropriate equations similar to the first three of Eqs. (1) for each of the plies involved in the structure. Finally, it will be necessary to construct a set of equations similar to the last three of Eqs. (1) involving a relation between the average stress on the structure and the stresses in the individual plies. Within these rules, a set of equations can now be constructed which will allow determination of the interply stresses as well as the loads carried by each of the individual plies in the structure.

From the nature of these equations, it may be seen that the number of them necessary to describe a general, anisotropic, structure having n plies is $3(1+n)$. Since the number of equations only increases linearly with the number of plies, it should be relatively easy to study multi-ply structures involving combinations of wire and textile materials in a very complete fashion. In addition, the effect of using varying cord angles in multi-ply laminates can be easily seen.

VIII. REFERENCES

1. S. K. Clark, Interply Stresses and Load Distribution in Cord-Rubber Laminates, The Univ. of Michigan, Office of Research Administration, Technical Report 02957-8-T, Ann Arbor, Michigan.

2. S. K. Clark, The Plane Elastic Characteristics of Cord-Rubber Laminates, The Univ. of Michigan, Office of Research Administration, Technical Report 02957-3-T, Ann Arbor, Michigan.

IX. DISTRIBUTION LIST

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