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STATISTICAL MEASUREMENTS OF DETECTION OF M ORTHOGONAL SIGNALS KNOWN EXCEPT FOR PHASE

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1. The study reported was not exhaustive.
2. The results presented concern one phase of a continuing study.
3. The study reported was judged to have insufficient scope.

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ABSTRACT

The SIMulated Receiver And Recorder (SIMRAR) equipment of this laboratory has been used to make statistical measurements of detection of M orthogonal signals known except for phase and starting time. The possible starting times of the signals are overlapping. The results are compared to the theoretical detection performance for nonoverlapping signals.

STATISTICAL MEASUREMENTS OF DETECTION
OF M ORTHOGONAL SIGNALS KNOWN EXCEPT FOR PHASE

1. INTRODUCTION

Technical Report No. 13* discusses the optimum receiver for detecting a signal which is one of M orthogonal signals, known except for carrier phase. If the signals are pulses with the possible starting times restricted to being one pulse width apart, this development applies. Here

$$M = \frac{\text{observation time}}{\text{pulse time duration}} . \tag{1}$$

Figure 1 depicts the elements of the optimum receiver.

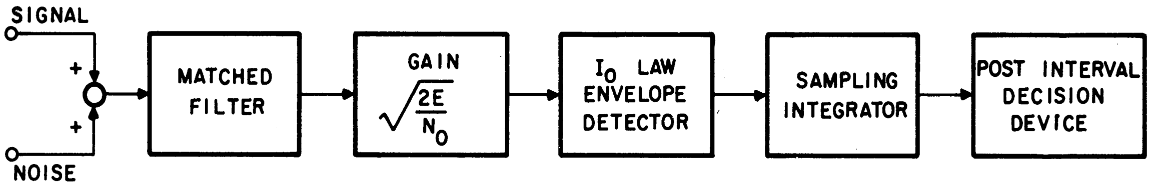

Fig. 1. Optimum receiver for detection of M orthogonal signals known except for phase.

Under the approximation that the distribution of the logarithm of the likelihood ratio is normal, the derivation of TR-13 leads to a detection index,

$$d = \ln \left[1 - \frac{1}{M} + \frac{1}{M} I_0\left(\frac{2E}{N_0}\right)\right] \tag{2}$$

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*W. W. Peterson and T. G. Birdsall, "The Theory of Signal Detectability," Cooley Electronics Laboratory Technical Report No. 13, The University of Michigan Research Institute, June 1950, Part II, pp. 49-53. Also W. W. Peterson, T. G. Birdsall, W. C. Fox, "The Theory of Signal Detectability," Trans. IRE, PGIT, Sept. 1954, pp. 207-209.

The objective of this study was to use the SIMRAR equipment to determine what corrections must be applied to Eq. 2 when the possible signals are overlapping, and hence the integration is continuous and not discrete.[1]

2. SIMULATION PROCEDURE

The pulse signal was simulated by electronically gating an audio oscillator. The ideal matched filter would hence require a square envelope impulse response. The SIMRAR IF filter has an impulse response with an exponentially decaying envelope shape. One problem in the simulation was to choose the signal so that the SIMRAR IF filter would approximate the performance of an ideal IF filter.

The ideal filter is the one which maximizes the peak output signal-to-noise power ratio for a fixed signal energy. The duration of the pulse signal was therefore chosen so that the SIMRAR IF filter peak output power was maximized for a fixed pulse energy. Figure 2 is an experimentally measured curve. Since the input signal amplitude was held constant the signal energy is proportional to its duration. Dividing the output power by the signal duration gives the power for a fixed input energy. The curve shows that the pulse duration should be 14 milliseconds. This is very close to the impulse response time constant of a simple tuned filter having the nominal bandwidth of the SIMRAR filter.

Figure 3 is a block diagram illustrating a technique for simulation of this problem using SIMRAR. The post-detection gain, ρ, is the ratio of peak signal voltage to rms noise at the filter output. For the ideal filter this is $\sqrt{2E/N_o}$.

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[1] D. W. Fife, "SIMRAR: Simulated Receiver and Recorder," Cooley Electronics Laboratory Technical Report No. 118, The University of Michigan Research Institute, January 1961.

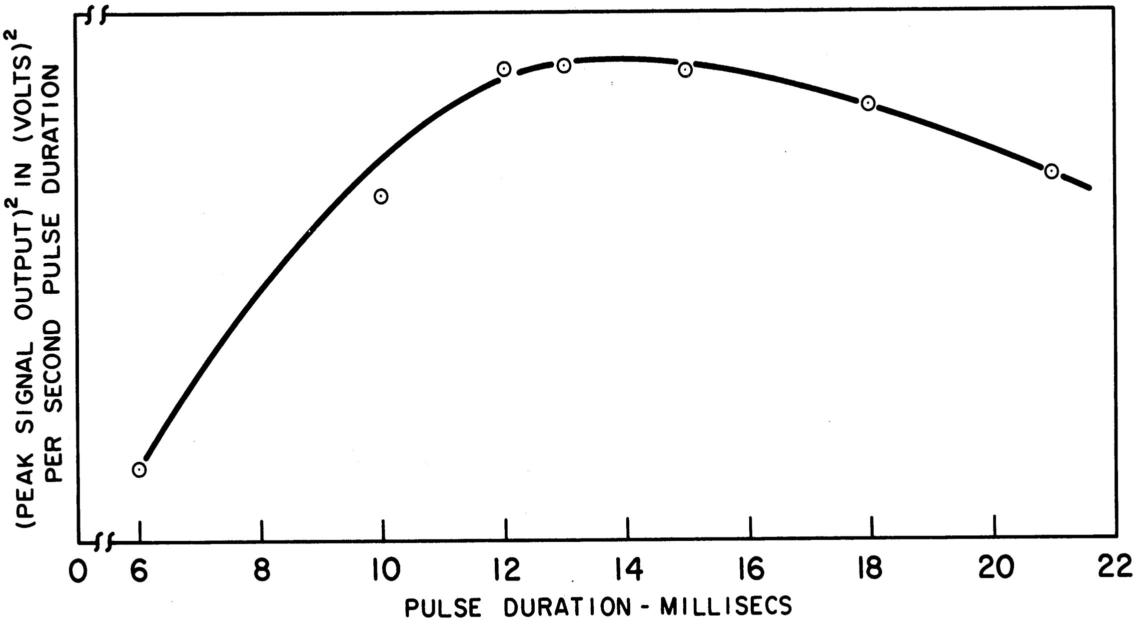

Fig. 2. Pulse matching of SIMRAR 1 kc IF filter.

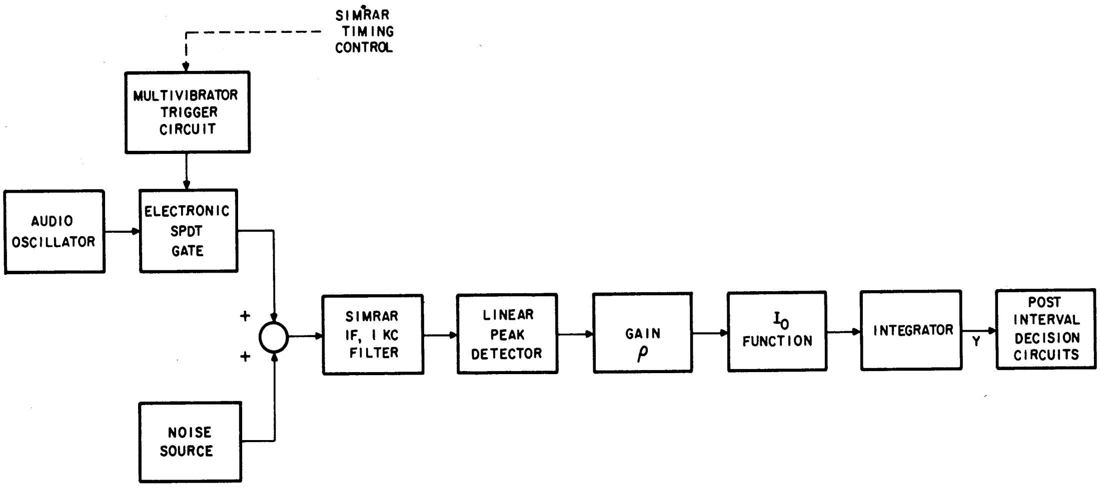

Fig. 3. Simulation of optimum receiver using SIMRAR.

The nature of the I_o function allowed a simplification in the simulation. $I_o(x)$ rises sharply as x becomes fairly large, to such an extent that $I_o(x)$ for x only slightly smaller than the extreme value of the range is insignificant in comparison. Figure 4 illustrates this be-

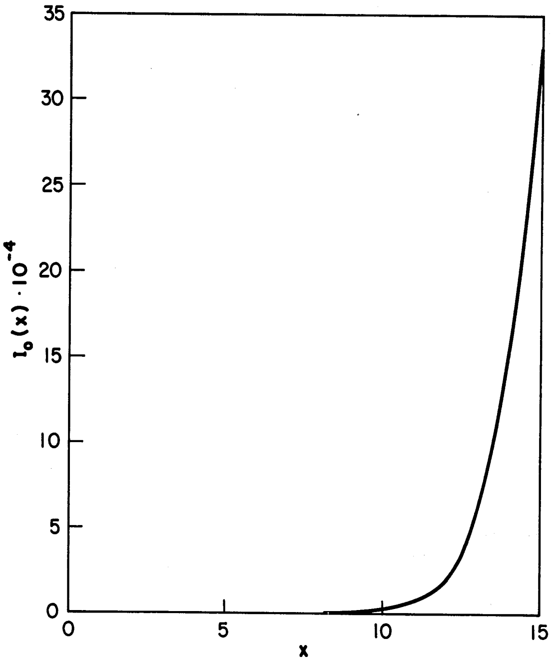

Fig. 4. $I_0(x)$ function.

havior. The $2E/N_0$ values and noise power used were sufficiently large that the instrumentation of $I_0(x)$ for the range of x shown in Fig. 4 was required. But, because of the large slope required at the extreme of the range, this was very difficult to do accurately. However, because the I_0 function strongly emphasizes the peak value of x, the approximation

$$y = \int_0^T I_0(x)\,dt \approx \max I_0(x) \cdot \Delta t = I_0(\max[x]) \cdot \Delta t \qquad (3)$$

appeared reasonable.

Now, since $I_0(x)$ is a monotonic function of x, and Δt can be looked upon as simply a gain, the decision may just as well be based upon max [x] with this approximation. This corresponds to making the detection decision continuously during the observation interval. Figure 5 is the block diagram of the simulation which was used. The 40 cps bandwidth video filter was included primarily because it was conveniently scaled for the proper amplitude of the 2 kc perturbation signal required for recording in SIMRAR. The time constant of this filter is less than the duration of the pulse signal, and therefore inclusion of this filter does not yield a significant integration effect.

It should be emphasized that the approximation which allows removal of the I_0 function and integrator will result in poorer detection performance. The results of this study will indicate the extent of the degradation. The approximation should be better for large $2E/N_o$, since the effect of neglecting integration of secondary peaks of $I_o(x)$ becomes negligible.

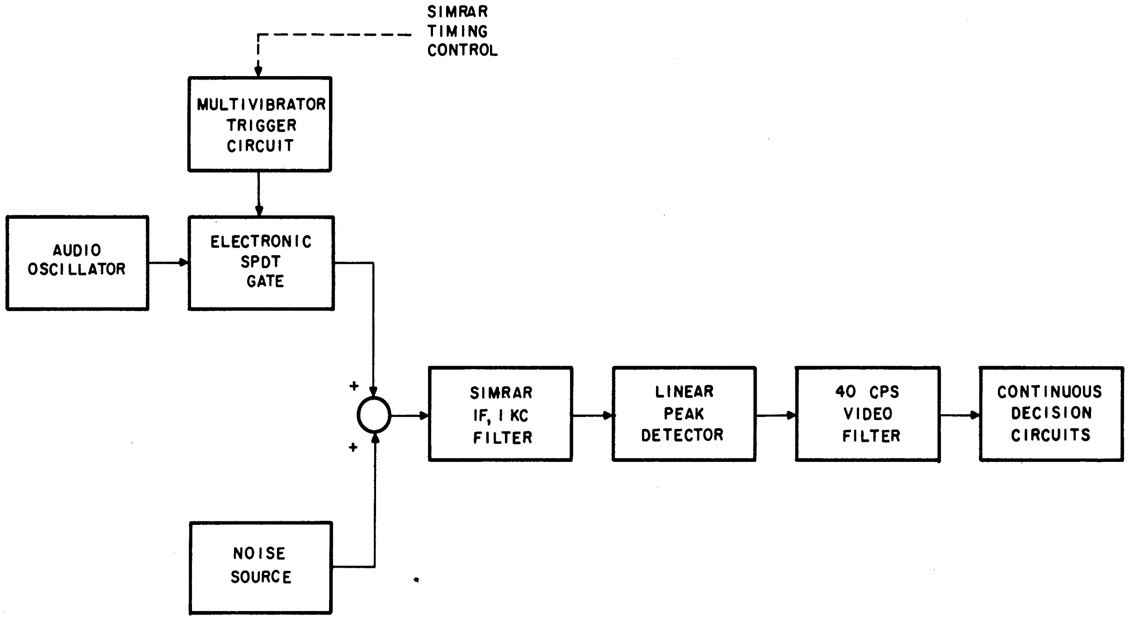

Fig. 5. Actual simulation of receiver.

3. INTERPRETATION OF RESULTS

The receiver operating curves were found for various signal-to-noise power ratios, $(S/N)_{IF}$, at the filter output (before the detector). The d' value for equal conditional errors was taken from the ROC, and squared to give d. For comparison with the results of TR-13 it was necessary to translate the $(S/N)_{IF}$ power ratio into an equivalent $2E/N_o$. The noise bandwidth of the SIMRAR IF filter was required. Experimental determination of the frequency response and graphical integration gave a value of 36.6 cps. (The total 3 db bandwidth of the filter is 22 cps.)

Then

$$\frac{2E}{N_o} = 2\left(\frac{S}{N}\right)_{IF} \tau_D W_N = 1.024 \left(\frac{S}{N}\right)_{IF} , \qquad (4)$$

where: τ_D = pulse duration = .014 sec, and

W_N = IF filter noise bandwidth (cps) .

Figures 6 and 7 give the experimental results for three values of $2E/N_o$. The theoretical curve of d from Eq. 2 is also shown for the same values of $2E/N_o$. The detection performance of the receiver is not degraded primarily by the mismatch of the IF filter. The difference in performance shown by Figs. 6 and 7 arises, for the most part, from the additional uncertainty in the signal position and the approximation to the optimum receiver which was made. Figure 8 compares the experimental results for overlapping signals with half as much signal energy. These curves compare reasonably well. Hence, the conclusion may be drawn that the additional uncertainty introduced by overlapping signals and the degradation due to the approximation, results in roughly the same performance as achieved with nonoverlapping signals 3 db lower in energy.

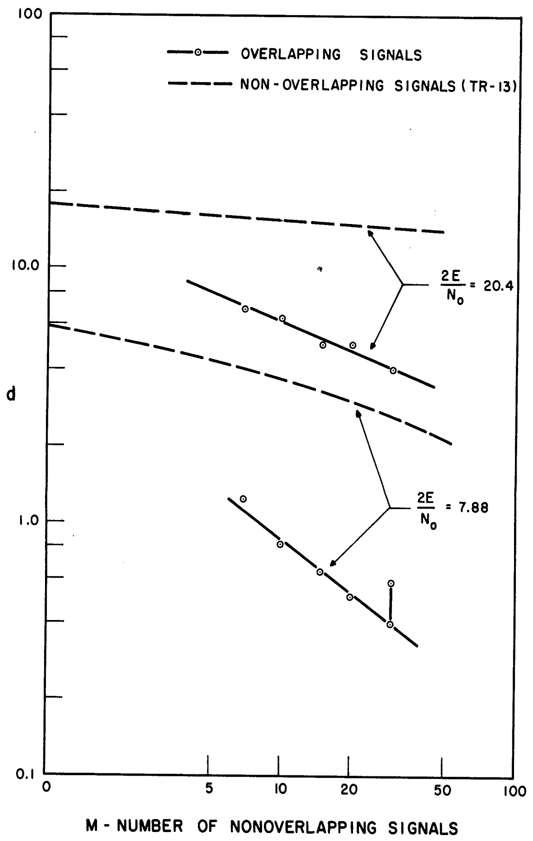

Fig. 6. Measured detection for overlapping signals and theoretical performance for nonoverlapping signals.

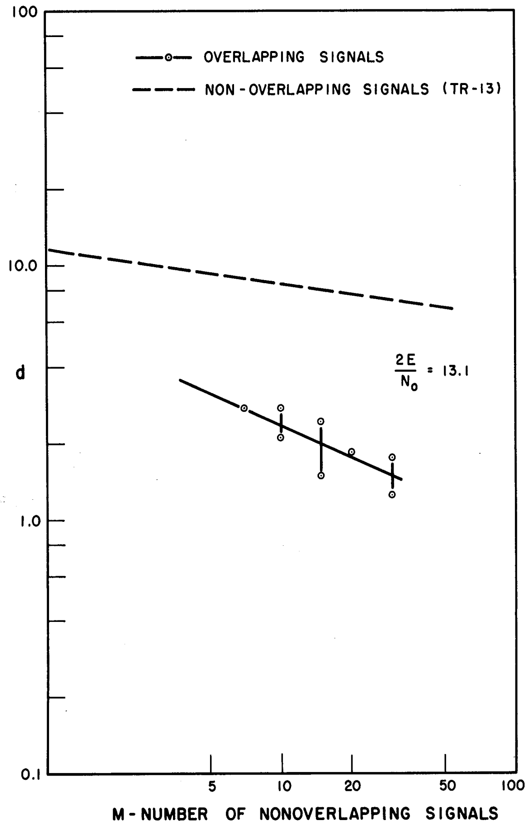

Fig. 7. Measured detection performance for overlapping signals and theoretical performance for nonoverlapping signals.

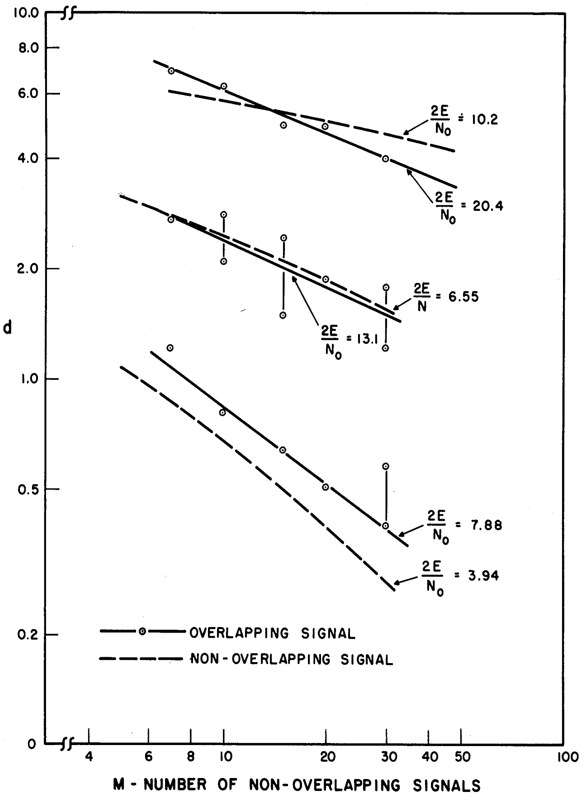

Fig. 8. Measured detection performance for overlapping signals and theoretical performance for nonoverlapping signals 3 db lower in energy.

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