

# Chain inflation in the landscape: 'bubble bubble toil and trouble'

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**Abstract.** In the model of chain inflation, a sequential chain of coupled scalar fields drives inflation. We consider a multidimensional potential with a large number of bowls, or local minima, separated by energy barriers: inflation takes place as the system tunnels from the highest energy bowl to another bowl of lower energy, and so on until it reaches the zero-energy ground state. Such a scenario can be motivated by the many vacua in the stringy landscape, and our model can apply to other multidimensional potentials. The 'graceful exit' problem of old inflation is resolved since reheating is easily achieved at each stage. Coupling between the fields is crucial to the scenario. The model is quite generic and succeeds for natural couplings and parameters. Chain inflation succeeds for a wide variety of energy scales—for potentials ranging from 10 MeV scale inflation to  $10^{16}$  GeV scale inflation.

**Keywords:** cosmological phase transitions, string theory and cosmology, inflation, physics of the early universe

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**Contents**

<b>1. Introduction</b>	<b>2</b>
<b>2. Review of tunnelling in double-well potential and old inflation</b>	<b>6</b>
<b>3. Double-field inflation: time dependent nucleation rate</b>	<b>9</b>
<b>4. Chain inflation</b>	<b>10</b>
4.1. Simplest model: single chain in which all fields have the same parameters .	15
4.1.1. The first field. . . . .	18
4.1.2. Example. . . . .	19
4.1.3. Caveat. . . . .	20
4.1.4. Issues: would the path skip a link in the chain? . . . . .	20
4.2. Generalizing the model . . . . .	21
4.2.1. Arbitrary parameters, a number of chains. . . . .	21
4.2.2. More general interactions. . . . .	24
4.2.3. Thermal activation. . . . .	24
4.2.4. Mixing rollers and tunnellers. . . . .	24
<b>5. Variants</b>	<b>24</b>
5.1. Multiple uncoupled tunnelling fields . . . . .	25
5.2. Two tunnellers . . . . .	25
5.3. Single trigger field catalysing multiple secondary fields to simultaneously tunnel . . . . .	26
<b>6. Issues</b>	<b>27</b>
6.1. Can de Sitter violations of energy conservation send a tunnelling field over the top of the barrier? . . . . .	27
6.2. Fine-tuning . . . . .	27
6.3. Density fluctuations . . . . .	28
6.4. Beyond the thin wall . . . . .	29
<b>7. Conclusion</b>	<b>29</b>
<b>Acknowledgments</b>	<b>30</b>
<b>References</b>	<b>30</b>

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**1. Introduction**

In 1981, Guth [1] proposed an inflationary phase of the early universe to solve several glaring paradoxes of the standard cosmology: the horizon, flatness, and monopole problems. During the inflationary epoch, the Friedmann equation

$$H^2 = 8\pi G\rho/3 + k/a^2 \tag{1}$$

is dominated on the right-hand side by a (nearly constant) false vacuum energy term  $\rho \simeq \rho_{\text{vac}} \sim \text{constant}$ . Here  $H = \dot{a}/a$  is the Hubble parameter and  $a$ , the scale factor of the

universe, expands superluminally,  $a \sim t^p$  with  $p > 1$ . During this period, a small causally connected region of the universe inflates to a sufficiently large scale (the scale factor must increase by about  $10^{27}$  or roughly 60 e-folds of inflation) to successfully resolve these cosmological shortcomings. Since the period of superluminal expansion is an adiabatic process, the temperature of the universe drops precipitously during this phase. Hence, it must be followed by a period of thermalization, in which the vacuum energy density is converted to radiation leading to the standard cosmology.

Inflationary models can be divided into two categories: old inflation-type models, in which a scalar field tunnels from false to true vacuum during a first-order phase transition, and slowly rolling inflation models, in which a scalar field rolls down a flat potential to its minimum. In either case, the vacuum energy density of the field before it reaches its minimum drives the inflationary expansion.

In chain inflation, we generalize the variety of possible models by considering a sequential chain of a large number of tunnelling and/or rolling fields, in which reheating relies on the coupling between the fields. This chain of multiple tunnellers can be interpreted as a path in a multidimensional potential landscape,  $V(\phi_1, \phi_2, \dots, \phi_q)$ , such as may exist in the stringy landscape, where  $q$  is the number of fields. One can think of this path as starting at a bowl (a local minimum) in this multidimensional parameter space, then moving down to a sequence of bowls of ever lower energy, until one reaches the ground state with zero potential. We can model this such that each drop to a lower bowl is equivalent to one of the fields tunnelling.

### Shortcomings of previous inflationary models

Any successful inflationary model must meet two key requirements: (1) there must be sufficient inflation; and (2) the universe must thermalize and reheat. The original old inflation model, in which bubbles of true vacuum nucleate in a false vacuum background, failed [6] because the interiors of expanding spherical bubbles of true vacuum cannot thermalize: the 'graceful exit' problem. Hence this model does not produce a universe such as our own. Shortly after the demise of old inflation, models with slowly rolling fields were proposed [7, 8]. As a scalar field slowly rolls down its potential, superluminal expansion is achieved. Reheating then takes place successfully as the field oscillates around its minimum and decays to radiation. However, slowly rolling models typically suffer from fine-tuning of their potentials in order that they be flat enough to provide sufficient inflation and yet not overproduce density fluctuations; we note that natural inflation [11] is a model in which the required small parameters arise naturally.

### Overcoming the shortcomings with chain inflation

The new framework of chain inflation has several attractive features. First, it can resurrect the basic idea of tunnelling inflation (as in old inflation) in that multiple coupled tunnelling fields can achieve graceful exit. Second, no fine-tuned parameters are required for the potentials (unlike the case for most slowly rolling models). Third, the model is quite generic; it succeeds for a wide variety of parameters and couplings. In particular, chain inflation succeeds for a wide variety of energy scales, ranging from 10 MeV to grand unified scales ( $10^{16}$  GeV). Fourth, it relies upon the fact that the fields are coupled to one another, which, in general, they probably are. We will illustrate these features in the

paper. The idea of taking seriously a model of inflation relying upon hundreds (or more) of scalar fields was motivated by the large number of vacua in the string theory landscape.

Unlike old inflation, by having a chain of multiple fields tunnelling sequentially, we are able to fulfil both requirements for inflation: sixty e-folds of inflation as well as reheating. The key element is that no single stage of inflation is responsible for much inflation. Each stage of inflation gives rise to only a fraction of an e-fold, and it is only due to the large number of stages of tunnelling that the universe inflates sufficiently. Graceful exit from inflation is obtained by coupling the fields together. Once a field has tunneled to its true vacuum, its coupling to secondary field(s) causes a change in the nucleation rate in the secondary field(s); the rapid tunnelling of the secondary field(s) to the true vacuum allows bubble percolation and reheating. A chain of such tunnelling events, each catalysed by a previous tunneller, leads to a homogeneous hot universe.

In chain inflation, the possible range of the energy scales of the potentials can vary from  $10^{16}$  GeV down to any energy scale that allows sufficient reheating and baryogenesis. Thus the potential can have an energy scale possibly as low as 10 MeV, so that the universe can reheat to that energy scale and still experience ordinary Big Bang nucleosynthesis<sup>3</sup>. In short, whereas most traditional rolling models of inflation require large energy scales, often above a Planck scale, in chain inflation it is possible for all the potentials to have much lower energies, e.g., they can all have TeV scales.

In order to illustrate the basic scenario, we first consider a chain of  $q$  tunnelling fields, all with the same parameters. In this simple picture, once the first field tunnels, it catalyses the second field to tunnel (where the second field without the coupling to the first would remain in the metastable vacuum), the second field catalyses the third field to tunnel, etc. The key point is that the universe inflates a fraction of an e-fold at each stage, and the coupling ensures that the sequence of tunnelling events takes place. After discussing the simplest variant of the chain in which each field is coupled only to two others (the previous and subsequent ones in the chain), we turn to a more generic situation in which the potentials are allowed to have a variety of parameters and couplings exist between multiple fields. The same basic behaviour—a sequence of tunnelling events—ensues, as the universe chooses a path to the ground state. We also will assume that the ground state is Minkowski space ( $V = 0$ ).

Whether or not reheating is successful for a tunnelling field depends crucially on the ratio between the expansion rate  $H$  and the bubble-nucleation rate  $\Gamma$  per unit volume. We can define the ratio

$$\beta = \Gamma/H^4. \tag{2}$$

Roughly speaking, if  $\beta \ll 1$ , bubble nucleation is rare, and the universe stays trapped in the false vacuum and inflates. If  $\beta \geq O(1)$ , on the other hand, bubble nucleation is rapid, there is no further inflation, and the phase transition successfully completes via bubble percolation and thermalization. Old inflation, which had a single tunnelling scalar field, required  $\beta \ll 1$ , so that 60 e-folds could be achieved; then for a  $\beta$  that is constant in time, it became impossible for nucleation ever to successfully complete. Double-field inflation,

<sup>3</sup> The lower bound on the energy scale of inflation would be set by baryogenesis, which is currently not understood. For example, if baryogenesis is found to take place at the electroweak scale, then the lower bound on the chain inflation potential would be roughly TeV scale. However, as we do not currently know how baryogenesis works, we can contemplate even lower energy scales.

proposed in 1990 by Adams and Freese [9], solved this problem with a time dependent nucleation rate for a single tunnelling field, so that  $\beta$  started out small and the universe inflated; later due to coupling to a rolling field,  $\beta$  for the tunnelling field became large so that the phase transition suddenly took place and completed throughout the universe. In this paper we use some of the features of this double-field model. We couple a series of tunnelling fields together in such a way that each member of the chain catalyses the next to rapidly nucleate and thermalize.

As a toy model, we will consider many scalar fields, each of which has an asymmetric double-well potential. We do not mean to imply that this is a perfect model for the real potentials of these fields; it is merely an illustrative example. Of course the shape of the multi-dimensional potential is far more complicated than our simple picture. In addition, we do not deal with anti-de Sitter vacua. Despite these reservations, our model has the advantage that it is quite generic. The basic features of the model should survive if one considers more complicated potentials. The model succeeds for a wide variety of parameters and couplings. We allow arbitrary couplings between the fields and find that their potentials do not need to be fine-tuned. Hence the model has the positive feature that it succeeds for potentials that are natural in the context of field theory. To be succinct, if we take a large number of scalar fields which are coupled together and have natural potentials (two criteria which are reasonable in the context of field theory and possibly also in the context of a landscape), we have the necessary conditions for inflation.

The generic feature of our model is this idea that, in a complicated landscape, the system drops through a series of bowls by tunnelling from one to another, until it hits the ground state. We will consider a specific form of couplings between fields in order to allow us to perform calculations and get reasonable estimates. However, we emphasize that the general idea of tunnelling from bowl to bowl does *not* rely on the choices of potential and couplings that we use in order to present a concrete toy model. Our model relies only on the one general feature of tunnelling from bowl to bowl, regardless of the detailed form of the potential.

Since our treatment is entirely field theoretical, this work does not rely on the existence of any stringy landscape. This model may be relevant to many systems with multidimensional potentials, such as condensed matter systems or in particle physics. Previous authors [2,3] and [4] have considered multiple rolling fields, without any tunnelling fields present; our work differs in that we generically have tunnelling fields in the model and in that the couplings between the fields are key to the success of the model. We note that in the future it would also be interesting to combine a number of rolling as well as tunnelling fields. We also plan to shortly publish a paper considering inflation from a chain of vacua in a single field: one might imagine a tilted cosine, and in fact we find that chain inflation can succeed with the QCD axion [5]. The case of a large number of uncoupled scalar fields was previously considered by [2,3]. A chain of rolling fields as the source of inflation was considered by Easther [4] in a model he dubbed 'folded inflation'. Our work, which focuses on tunnelling fields, is hence complementary to his paper.

One important issue we have not yet dealt with in this paper is the question of density fluctuations. One of the outstanding successes of rolling models in inflation is that they are able to produce density fluctuations with a scale invariant spectrum. However, unless their potentials are fine-tuned, they tend to overproduce the amplitude of the

density fluctuations (except in a few models such as natural inflation [11]). In order for chain inflation to be a viable competitor to rolling models, it too should produce density fluctuations of the right spectrum and amplitude. Towards the end of the paper (in the discussion section) we present a partial discussion of the expected results, but far more work must be done on this important issue<sup>4</sup>.

We begin with a review of tunnelling in double-well potentials and the failures of old inflation in section 2. In section 3, we review the double-field model, which revives the idea of old inflation. In section 4, we present the idea of chain inflation: we start with the toy model of identical tunnellers in which all the fields have potentials with identical parameters, and then generalize to coupled tunnellers with different parameters. In section 5, we present several variants on the idea of a chain of tunnellers. In section 6 there is a discussion (including the issue of density perturbations) and we conclude in section 7.

## 2. Review of tunnelling in double-well potential and old inflation

We consider a quantum field theory of a scalar field with a Lagrangian of the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi), \quad (3)$$

where  $V(\phi)$  is an asymmetric potential with metastable minimum  $\phi_-$  and absolute minimum  $\phi_+$  (see figure 1). The energy difference between the vacua is  $\epsilon$ . Bubbles of true vacuum ( $\phi_+$ ) expand into a false vacuum ( $\phi_-$ ) background.

In the zero-temperature limit, the nucleation rate  $\Gamma$  per unit volume for producing bubbles of true vacuum in the sea of false vacuum through quantum tunnelling has the form [13, 14]

$$\Gamma(t) = Ae^{-S_E} \quad (4)$$

where  $S_E$  is the Euclidean action and where  $A$  is a determinantal factor which is generally the energy scale  $\epsilon$  of the phase transition<sup>5</sup>. Guth and Weinberg have shown that the probability of a point remaining in a false de Sitter vacuum is approximately

$$p(t) \sim \exp\left(-\frac{4\pi}{3}\beta Ht\right) \quad (5)$$

where the dimensionless quantity  $\beta$  is defined by

$$\beta \equiv \frac{\Gamma}{H^4}. \quad (6)$$

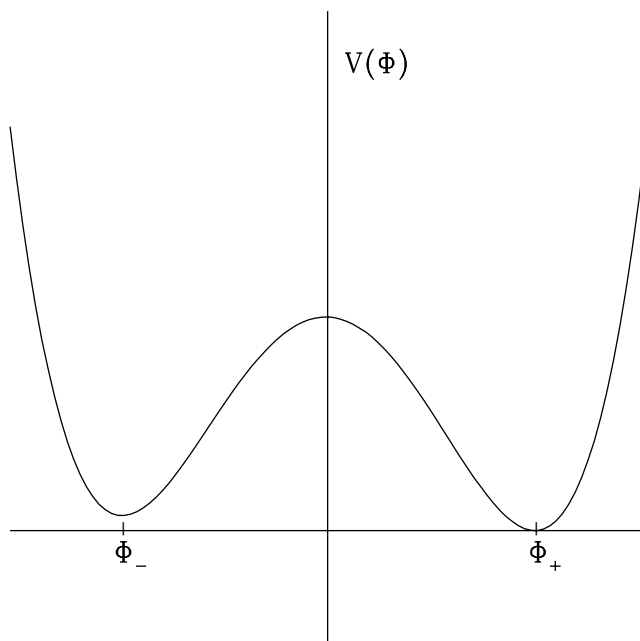
Writing equation (5) as  $p(t) \sim \exp(-t/\tau)$ , we can estimate the lifetime of the field in the metastable vacuum as roughly<sup>6</sup>

$$\tau = \frac{3}{4\pi H\beta} = \frac{3}{4\pi} \frac{H^3}{\Gamma}. \quad (7)$$

<sup>4</sup> In the worst case, one might argue that, as long as chain inflation does not overproduce the perturbations, they might be produced elsewhere in the universe, but it would be preferable to generate them during inflation.

<sup>5</sup> We note that we do not need to include gravitational effects [15] as they would only be relevant for bubbles comparable to the horizon size, whereas the bubbles produced here are much smaller.

<sup>6</sup> There will be a distribution around this typical value, as we will discuss further below.



**Figure 1.** Potential energy density of tunnelling field  $\phi$  as a function of field strength. The energy difference  $\epsilon$  between the false vacuum (at  $\phi_1 = -a$ ) and the true vacuum (at  $\phi_+ = +a$ ) provides the vacuum energy density for inflation.

For definiteness, we consider asymmetric double-well potentials as in figure 1,

$$V(\phi) = \frac{1}{4}\lambda(\phi^2 - a^2)^2 - \frac{\epsilon}{2a}(\phi - a). \quad (8)$$

To leading order, the metastable minimum is at  $\phi = -a$  and the absolute minimum at  $\phi = +a$ . The energy difference between minima is  $\epsilon$ . Throughout the paper we will assume that  $\lambda$  is not too different from 1, as it is the most natural value of this parameter.

In the thin wall limit<sup>7</sup>, the Euclidean action is

$$S_E = 64 \frac{\pi^2 \lambda^2 a^{12}}{6 \epsilon^3}. \quad (9)$$

When  $\beta \ll 1$  (low nucleation efficiency), the phase transition proceeds slowly. The field remains in its metastable minimum and the universe inflates. However, as long as  $\beta$  is small, the phase transition cannot complete. Bubbles of true vacuum do periodically nucleate in various places in the universe, but their production is rare and sporadic. The rate of filling the universe with true vacuum cannot keep up with the exponential expansion of the false vacuum. Percolation of true vacuum bubbles does not take place. The interiors of individual bubbles or the small groups of bubbles that are able to form are unable to thermalize. The latent heat of the phase transition resides entirely in the kinetic energy of the bubble walls and cannot thermalize the interior. The universe ends

<sup>7</sup> The thin wall approximation breaks down for much of the parameter space we are considering. However, as illustrated in the discussion section below, the results of the paper would be relatively unchanged if we were to perform a more careful study in which this approximation is not made. Hence we proceed cautiously using the thin wall limit.

up looking like ‘Swiss cheese’, with isolated empty bubbles of true vacuum of various sizes unable to find one another in the background of false vacuum.

Old inflation, which has constant (time independent) small  $\beta$ , fails. In old inflation, the universe can easily grow the requisite 60 e-foldings, but reheating never takes place.

In the opposite limit of  $\beta > O(1)$  (high nucleation efficiency), the phase transition proceeds very rapidly. If there were only one scalar field, it would not remain in the false vacuum long enough to give rise to sufficient inflation, though the phase transition would complete, leading to percolation and thermalization. Guth and Weinberg [6] as well as Turner *et al* [16] calculated that a critical value of

$$\beta \geq \beta_{\text{crit}} = 9/4\pi \quad (10)$$

is required in order for percolation and thermalization to be achieved. In this paper, we consider a chain of multiple tunnelling fields rather than a single field. Each of these fields will develop a value of  $\beta$  in excess of the critical value required for percolation.

The amount that the universe inflates before the phase transition completes varies inversely to the parameter  $\beta$ . The number of e-foldings due to tunnelling of a single scalar field is

$$N = \int H dt \sim H\tau. \quad (11)$$

Using the first equality in equation (7) for the lifetime  $\tau$ , we then have

$$N = \frac{3}{4\pi\beta}. \quad (12)$$

Sufficient inflation requires the total number of e-foldings to satisfy

$$N_{\text{tot}} > 60. \quad (13)$$

Old inflation can easily achieve this requirement with a single scalar field for a wide variety of parameters. For a double-well potential in the thin wall limit, the only relevant quantity in the potential for determining the total amount of inflation is the ratio  $\epsilon/a^4$ . Hence old inflation can take place at a variety of energy scales, and can be as successful at a TeV as at  $10^{16}$  GeV.

However, using equation (10), we see that percolation and thermalization of any single field require

$$N \leq N_{\text{crit}} = 1/3. \quad (14)$$

For the case of inflation with a single scalar field,  $N_{\text{tot}} = N$  and obviously one cannot simultaneously satisfy both criteria in equations (13) and (14). Hence with time independent  $N$  and  $\beta$ , old inflation fails. However, as we will illustrate in the next section, both of these criteria can be satisfied with a time dependent nucleation rate, in which case  $\beta$  and  $N$  change with time.

This transition from small  $\beta$  (large  $N$ ) to large  $\beta$  (small  $N$ ) can be achieved via a small change in the parameters in the potential. Because of the exponential dependence in equation (9) on the Euclidean action, the values of  $\beta$  and  $N$  are extremely sensitive to the parameters in the potential, in this case to the ratio  $a/\epsilon^{1/4}$ . Because the tunnelling rate is so sensitive to this ratio, the transition from a field that would remain for an inordinate amount of time in the false vacuum to a field that tunnels rapidly requires only a small



change in the ratio  $a/\epsilon^{1/4}$ . As discussed later, this ratio only needs to change by a few per cent to cause a transition from slow to rapid tunnelling for a wide range of energy scales from GUT to TeV.

It is interesting that potentials which have comparable values of  $\epsilon$  and  $a^4$  are near the borderline between rapid transitions and no transition. We will take advantage of this fact in this paper. Such potentials with parameter  $\epsilon \sim O(a^4)$  are quite natural.

### 3. Double-field inflation: time dependent nucleation rate

Adams and Freese [9] solved the problem of old inflation for a single tunnelling field by using a time dependent nucleation rate and hence a time dependent  $\beta$  (see also [10]). The ideal time dependence of  $\beta$  would be a step function. In that case,  $\beta$  is initially small, so one obtains the required 60 e-foldings of inflation, followed by a sudden transition to a large value of  $\beta$  so that all of the universe goes from false to true vacuum at once. Then all the bubbles of true vacuum are roughly of the same size, and they are able to percolate and thermalize (leaving no Swiss cheese). Adams and Freese obtained the appropriate time dependence of  $\beta$  by coupling a tunnelling inflaton field  $\phi$  to a rolling field  $\psi$ .

They took the total potential for the two fields to be

$$V_{\text{tot}} = V_1(\phi) + V_2(\psi) + V_{\text{int}}(\phi, \psi), \quad (15)$$

where the inflaton field  $\phi$  is a tunnelling field with potential  $V_1(\phi)$  given by the form of equation (8), and  $\psi$  is a rolling field with potential  $V_2(\psi)$ . The purpose of the rolling field is to catalyse a change in the tunnelling rate of the tunnelling inflaton. As the interaction, they considered

$$V_{\text{int}}(\phi, \psi) = -\gamma(\phi - a)\psi^3. \quad (16)$$

Clearly many other forms of the potential are possible (e.g.,  $V_{\text{int}} \sim \phi^2\psi^2$ ), but the resulting behaviour should be qualitatively the same. Due to this interaction (in the thin wall limit), bubbles will nucleate at a rate given by equation (4), where the effective energy difference between the vacua is given by

$$\epsilon_{\text{eff}} = \epsilon + 2a\gamma\psi^3. \quad (17)$$

The tunnelling rate is practically zero when the rolling field  $\psi$  is at the top of its potential and abruptly becomes very large as  $\psi$  approaches the minimum of its potential.

Initially, when the value of the rolling field  $\psi \sim 0$  is small, the interaction term  $V_{\text{int}}(\phi, 0) \sim 0$  is negligible, and the energy difference  $\epsilon_{\text{eff}} \sim \epsilon$  between the two minima is small enough that the field is stable in the false vacuum. The parameter  $\beta$  is small and the universe inflates. Then, as  $\psi$  rolls to the bottom of its potential, the energy difference  $\epsilon_{\text{eff}}$  between the minima of the tunnelling field grows and the tunnelling rate given by equations (4) and (9) climbs sharply due to the exponential dependence on the value of  $\epsilon_{\text{eff}}^3$ . Suddenly  $\beta$  becomes larger than the critical value for nucleation and true vacuum bubbles of the tunnelling field appear everywhere at once. The universe successfully completes the phase transition.

The strength of this double-field model is that it saves old inflation. The bubble nucleation is sudden and occurs throughout the universe, so that it can percolate and thermalize. In the double-field model, the rolling field must be flat (just as in new inflation)

in order for the tunnelling field to remain in its false vacuum for a sufficiently long time and in order not to overproduce density fluctuations. To explain this flatness, Freese [17] used a pseudo-Nambu Goldstone boson (shift symmetry), similar to the model of natural inflation [11, 12], but then found that the parameters of the tunnelling inflaton potential needed to be fine-tuned as well.

An alternate approach to obtaining a time dependent  $\beta$  was pursued in the models of extended [18] and hyperextended inflation [19, 20], in which the Hubble constant becomes time dependent due to Brans–Dicke gravity [21]. However, it has been shown [22, 16] that most versions of these models are ruled out due to overproduction of big bubbles in conflict with microwave background [23] and other data.

In this paper we preserve some features of the original double-field inflation model. We couple fields together so as to modify the value of  $\epsilon_{\text{eff}}$  in the double-well potential and thereby modify the nucleation rate. Here, however, no individual tunnelling stage is responsible for more than a fraction of an e-folding, and no fine-tuning of potentials is required. In the simplest variant of our model, only tunnelling fields are implemented, though some of these may be replaced by rolling fields instead.

#### 4. Chain inflation

The model of chain inflation relies on a chain of tunnelling fields. All the fields start out in their false vacua. The chain is set off by a single field tunnelling to its true vacuum; this tunnelling event then catalyses a chain of tunnelling events of the other fields. In the context of a landscape, where there is a multidimensional potential for a large number of scalar fields, one can think of chain inflation in the following way. At some place in the universe, the fields start off in a bowl (metastable minimum) of some energy. Then, the tunnelling of one field in our chain model is equivalent to moving to a bowl of slightly lower energy. In the chain picture, each tunnelling event provides a small amount of inflation (less than one e-fold) but, in the end, by the time the fields reach their collective ground state (which we take to be  $V = 0$ ), more than 60 e-folds have taken place.

We will begin by discussing an extremely simplified version of the chain. We will have the fields tunnel to their true vacua one at a time: first one field tunnels (i.e. bubbles of its true vacuum nucleate), then it catalyses the second field to nucleate, which catalyses the third field to nucleate, etc. Each field couples only to two others: the prior and subsequent ones in the chain. This oversimplified model is like a set of dominoes: once the first domino falls, all the rest follow. After we discuss this extremely simplified model, we will generalize to additional couplings between the fields and a variety of parameters for the potentials and couplings.

Our model relies only on one general feature, of tunnelling from bowl to bowl, regardless of the detailed form of the potential. Chain inflation will happen for a system of scalar fields in a multidimensional potential, which tunnel from bowl to bowl. Our particular choice of couplings allows simple calculations so as to obtain reasonable estimates. However, the coupling certainly need not be of the form presented here. Indeed, all the fields could couple to each in other in a complicated fashion as long as the couplings preserve the bowl type of structure in a multidimensional landscape.

As our toy model, we take all the fields to have double-well potentials. The basic features of the model generalize to other choices of potential. As a concrete example, we will start with a particular form of the interaction which is linear in one of the fields and

cubic in the other (simply because it makes the algebra easy). Qualitatively, the particular choice of interaction term is not important, and other choices such as interactions which are quadratic in both fields, would produce the same behaviour of the chain model. Other choices of potential or changes in the potential could equally well drive a time dependent nucleation rate that allows inflation as well as percolation. In this paper, we use a time dependence of the (effective) energy difference between minima to drive a time dependent nucleation rate. Alternatively, one could use a time-changing barrier height or time dependent potential width to achieve the same time dependence of the nucleation rate and hence of  $\beta$ . There are many ways to achieve the same effect, and we have chosen a cubic coupling influencing the value of the energy difference as a concrete example.

The total potential for the system is

$$V_{\text{tot}}(\phi_1, \phi_2, \dots, \phi_q) = \sum_i V_{\text{tot},i} = \sum_i [V_i(\phi_i) + V_{i,i-1}] \quad (18)$$

where  $0 < i \leq q$ . We take asymmetric double-well potentials

$$V_i(\phi_i) = \frac{1}{4}\lambda_i(\phi_i^2 - a_i^2)^2 - \frac{\epsilon_i}{2a_i}(\phi_i - a_i) \quad (19)$$

and, as a simple example, for interactions between the fields we take

$$V_{i,i-1} = -\frac{\gamma_{i,i-1}}{16}(\phi_i - a_i)(\phi_{i-1} + a_{i-1})^3. \quad (20)$$

The first field in the chain must be treated individually and is discussed in the next section below. All the fields start out in their false vacua,  $\phi_{i,\text{initial}} = -a_i$ . One after the next, in a chain, they tunnel to their true vacua at  $\phi_{i,\text{final}} = +a_i$ . After  $i - 2$  of them have tunnelled to their true vacua, the effective energy difference between minima for field  $i$  is given by

$$\begin{aligned} \epsilon_{\text{eff}} &= V_{\text{tot}}[\phi_0 = a_0, \dots, \phi_{i-2} = a_{i-2}, \phi_{i-1} = \mp a_{i-1}, \phi_i = -a_i, \phi_{i+1} = -a_{i+1}, \dots, \phi_q = -a_q] \\ &\quad - V_{\text{tot}}[\phi_0 = a_0, \dots, \phi_{i-2} = a_{i-2}, \phi_{i-1} = \mp a_{i-1}, \phi_i = +a_i, \phi_{i+1} = -a_{i+1}, \dots, \phi_q = -a_q] \\ &= \epsilon_i + \frac{1}{8}a_i\gamma_{i,i-1}(\phi_{i-1} + a_{i-1})^3 - \gamma_{i,i+1}a_i^3a_{i+1}. \end{aligned} \quad (21)$$

The potential in equation (21) is evaluated at  $\phi_{i-1} = -a_{i-1}$  when the  $(i - 1)$ th field is in its false vacuum, and at  $\phi_{i-1} = +a_{i-1}$  when the  $(i - 1)$ th field is in its true vacuum.

The last term in equation (21) arises due to the fact that, once the field  $i$  tunnels to its true minimum, its interaction with field  $i + 1$  is nonzero, i.e., the field  $i$  tunnels to the location in the landscape where its interaction with the next field in the chain is active. Some of the energy  $\epsilon_i$  must go into this interaction energy rather than into the energy of the true vacuum bubbles. Hence, the interaction with the next field in the chain enters with a minus sign in equation (21). This *decreases* the energy difference between the vacua, and makes tunnelling of field  $i$  to its true vacuum more difficult (lowers the tunnelling rate).

However, it is still true that each field in the chain successfully catalyses the next one to nucleate. When field  $i - 1$  tunnels to its true minimum, it increases the tunnelling rate of the  $i$ th field to the point where it too tunnels to its minimum. At first, when  $\phi_{i-1,\text{initial}} = -a_{i-1}$  is in its false vacuum, there is no interaction with  $\phi_i$  and

$$\epsilon_{\text{eff},i,\text{initial}} = \epsilon_i - \gamma_{i,i+1}a_i^3a_{i+1}. \quad (22)$$

Then, once the  $(i-1)$ th field tunnels to its true vacuum at  $\phi_{i-1,\text{final}} = +a_{i-1}$ , the interaction with field  $\phi_i$  turns on and

$$\epsilon_{\text{eff},i,\text{final}} = \epsilon_i + \gamma_{i,i-1}a_{i-1}^3a_i - \gamma_{i,i+1}a_i^3a_{i+1}. \quad (23)$$

This positive change in the energy difference by an amount  $\gamma_{i,i-1}a_{i-1}^3a_i$  (the energy of interaction between fields  $i-1$  and  $i$ ) is enough to increase the tunnelling rate for field  $i$  to the point where bubbles of its true vacuum nucleate throughout, allowing percolation and thermalization of these bubbles of field  $i$ .

In the language of a landscape, due to the tunnelling of field  $i-1$ , the system moves to a different location in the multi-dimensional potential. It starts in a bowl with negligible tunnelling rate for field  $i$  from false to true vacuum, and moves to another bowl from which the field  $i$  can easily tunnel to its minimum.

Let us consider two of the fields in the chain to illustrate how this works. We will assume that the first field in the chain (subscript 0) has already tunnelled to its minimum. Then the potential terms felt by fields 1 and 2 are as follows:

$$V_{0,1} = -\frac{1}{2}\gamma_{0,1}a_0^3(\phi_1 - a_1), \quad (24)$$

$$V_1 = \frac{\lambda_1}{4}(\phi_1^2 - a_1^2)^2 - \frac{\epsilon_1}{2a_1}(\phi_1 - a_1), \quad (25)$$

$$V_{1,2} = -\frac{1}{16}\gamma_{1,2}(\phi_2 - a_2)(\phi_1 + a_1)^3, \quad (26)$$

$$V_2 = \frac{\lambda_2}{4}(\phi_2^2 - a_2^2)^2 - \frac{\epsilon_2}{2a_2}(\phi_2 - a_2), \quad (27)$$

$$V_{2,3} = -\frac{1}{16}\gamma_{2,3}(\phi_3 - a_3)(\phi_2 + a_2)^3. \quad (28)$$

These are all the terms involving fields 1 and 2. In the last term, we will set  $\phi_3 = -a_3$ , as we are interested in the case (the location in the landscape) where the third field is still in its false minimum. In addition, we can ignore all terms proportional to  $\lambda_i$ , since these terms are zero when evaluated at  $\phi_i = \pm a_i$ . Hence, if we include all the terms involving fields 1 and 2 and ignoring terms with  $\lambda_i$ , we have

$$V_{\text{eff}}(\phi_1, \phi_2) = \frac{-1}{2} \left( \frac{\epsilon_1}{a_1} + \gamma_{0,1}a_0^3 \right) (\phi_1 - a_1) - \frac{1}{2} \frac{\epsilon_2}{a_2} (\phi_2 - a_2) - \frac{\gamma_{1,2}}{16} (\phi_2 - a_2) (\phi_1 + a_1)^3 + \frac{\gamma_{2,3}}{8} a_3 (\phi_2 + a_2)^3. \quad (29)$$

Initially, we start out with  $\phi_{1,\text{initial}} = -a_1$  and  $\phi_{2,\text{initial}} = -a_2$ . At these initial values, there is no interaction between fields 1 and 2, i.e.,  $V_{1,2}(-a_1, -a_2) = 0$ . Then the first field tunnels, so that  $\phi_1 \rightarrow a_1$ . Now the term of interaction between fields 1 and 2 becomes nonzero so that  $\epsilon_{2,\text{eff}}$  increases by an amount  $\gamma_{2,1}a_2a_1^3$ , following equation (23). Now the tunnelling rate in equation (4) is increased to the point where the second field tunnels and reheats. We require that  $\beta_{i,\text{final}} > \beta_{\text{crit}} = 9/4\pi$ .

One can describe this behaviour in the language of a multidimensional landscape as follows. There are four bowls, or local minima of the multidimensional potential: (1) at point A,  $\phi_1 = -a_1, \phi_2 = -a_2$ , (2) at point B,  $\phi_1 = +a_1$  and  $\phi_2 = -a_2$ , (3) at point C,  $\phi_1 = -a_1$  and  $\phi_2 = +a_2$ , and (4) at point D,  $\phi_1 = +a_1, \phi_2 = +a_2$ . Table 1 illustrates the

**Table 1.** All potential terms for fields 1 and 2 in the chain, assuming that field 0 is in its true vacuum and field 3 is still in its metastable false vacuum.

Bowl	$\phi_1$	$\phi_2$	$V_{\text{eff}}$
A	$-a_1$	$-a_2$	$\epsilon_1 + \gamma_{0,1}a_0^3a_1 + \epsilon_2$
B	$a_1$	$-a_2$	$\epsilon_2 + \gamma_{1,2}a_1^3a_2$
C	$-a_1$	$a_2$	$\epsilon_1 + \gamma_{0,1}a_0^3a_1 + \gamma_{2,3}a_2^3a_3$
D	$a_1$	$a_2$	$\gamma_{2,3}a_2^3a_3$

values for  $V_{\text{eff}}(\phi_1 = \pm a, \phi_2 = \pm a)$  in these four bowls. We will show that the path taken in this multidimensional parameter space is  $A \rightarrow B \rightarrow D$ .

The highest energy bowl is at point A, where both fields are in their false vacua; this is the starting point of the system. First, field  $\phi_1$  tunnels from  $-a_1$  to  $+a_1$  and the system moves from bowl A to bowl B. The energy difference between bowls A and B is typically higher than the energy difference between bowls A and C. Thus,  $A \rightarrow B$  has a higher tunnelling rate and provides the preferred path<sup>8</sup>. After the system has taken the path from bowl A to bowl B in the landscape, the second field  $\phi_2$  can now easily tunnel from  $-a_2$  to  $+a_2$ . The system reaches bowl D, the bowl with the lowest energy. (As we do not want to end up with a cosmological constant, we will in fact subtract off the energy of the lowest bowl everywhere.)

In this language of the bowls, the statement that the tunnelling of the first field catalyses the tunnelling of the second field can be given the following interpretation. We can see that the first field catalyses the second one to tunnel. The rate of tunnelling of the second field via  $A \rightarrow C$  (in the location of the landscape where the first field has not yet tunnelled to its true vacuum) is slow and the tunnelling typically does not take place; with

$$\epsilon_{2,\text{eff,initial}} = E_A - E_C = \epsilon_2 - \gamma_{2,3}a_2^3a_3; \quad (30)$$

the system does not choose this path. However, due to the interaction between fields 1 and 2, the (B, D) energy difference is higher than the (A, C) energy difference, so

$$\epsilon_{2,\text{eff,final}} = E_B - E_D = \epsilon_2 - \gamma_{2,3}a_2^3a_3 + \gamma_{1,2}a_1^3a_2 \quad (31)$$

and the rate of tunnelling of the second field via  $B \rightarrow D$  is fast. Hence the path taken by the fields is  $A \rightarrow B \rightarrow D$ . Here we have illustrated the mechanism by which the tunnelling of one field catalyses the tunnelling of the next in the chain via the coupling between them. We reiterate that the exact form of the coupling is irrelevant as long as the general picture of tunnelling from bowl to bowl is successful.

*Constraint.* In order for the interaction term in equation (23) to play a role in changing the energy difference and hence the tunnelling rate, its value must be large enough relative to the original value of the energy difference. The tunnelling rate must be slow without the interaction term, and large when it is important. For the specific form of the interaction

<sup>8</sup> We will presently comment on parameter choices for which this is not the preferred path.

term studied, we need

$$\frac{\gamma_{i,i-1} a_{i-1}^3}{\epsilon_i} \geq \eta, \quad (32)$$

where  $\eta$  is a number whose exact value depends on the parameters of the potential. Earlier we showed that, because of the exponential dependence of the tunnelling rate on these parameters, in particular on the ratio  $(\epsilon_i/a_i^4)^3$ , for reasonable potentials the required change in the value of  $\epsilon$  is extremely small. Hence, we can take  $\eta \sim 1/10$  as an estimate.

Throughout the paper, we will assume that the interaction couplings are not fine-tuned, so that all  $\gamma_{i,i+1} \sim \mathcal{O}(1)$ .

*Reheating.* In figure 2, we have plotted the probability  $p(t) \propto \exp(-(4\pi/3)\beta Ht)$  of any point remaining in the false vacuum for any of the tunnelling scalar fields (other than the first one) involved in chain inflation (see equation (5)). Successful inflation requires small  $\beta$  initially followed by large  $\beta$  at a later time, or, equivalently, large  $p(t)$  initially followed by small  $p(t)$  at a later time. In the ideal case, the transition from large to small  $p(t)$  (small to large  $\beta$ ) would be a step function, so percolation and reheating can easily take place. We have plotted the function  $p(t)$  for both old inflation and chain inflation. While the transition from slow to rapid nucleation (large to small  $p(t)$ ) is too gradual for old inflation, it is virtually a step function for chain inflation, which can thus easily percolate.

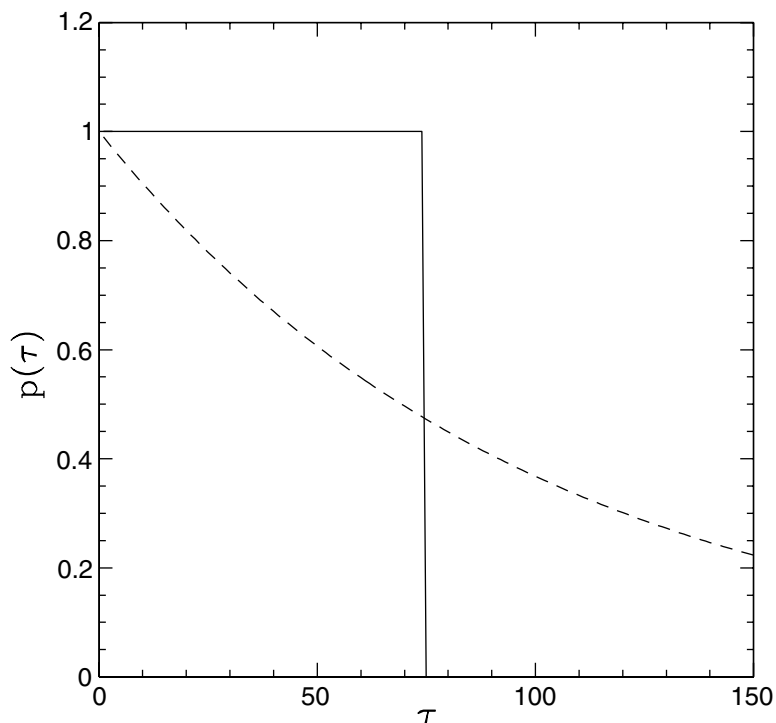
We note that, in the case of tunnelling fields in chain inflation, this time dependence can be closer to the ideal of a step function even than in the case of double-field inflation. Here, with a tunneller as catalyst, there is a two-state system. When the catalyst is in its false vacuum, there is no interaction term whatsoever and no tunnelling; with the catalyst in its true vacuum, the interaction term suddenly turns on and tunnelling is immediate. When a rolling field is the catalyst, on the other hand, the interaction increases more gradually with time so that the time dependence of  $\beta$  is more gradual. Hence with a tunnelling field as catalyst, reheating is easy to achieve.

During each step of the chain, the tunnelling rate is rapid enough that  $\beta > \beta_{\text{crit}}$  and percolation and thermalization take place. However, the particles that are produced by the fields that tunnel during the early stages of inflation are inflated away by the subsequent e-folds of inflation. Only the last field (or two) that tunnel are relevant for reheating. In order to reheat to a high enough temperature to allow ordinary nucleosynthesis to take place, we require a reheating temperature of at least 10 MeV (this is the absolute minimum temperature one could possibly imagine). Another lower bound on the reheating temperature arises from baryogenesis; as the mechanism for baryogenesis is currently not yet understood, we allow flexibility on this value. Hence the energy difference between vacua of the last tunnelling field  $\epsilon^{1/4} \geq 10$  MeV. On the other hand, we also require

$$V_{\text{tot}} < m_{\text{pl}}^4 \quad (33)$$

so as to stay within the bounds of applicability of ordinary effective quantum field theory. Thus, e.g., for 1000 fields, we must take  $\epsilon^{1/4} < m_{\text{GUT}} \sim 10^{16}$  GeV. Hence, for all the fields in the chain, we take

$$10 \text{ MeV} \leq \epsilon^{1/4} \leq 10^{16} \text{ GeV}. \quad (34)$$



**Figure 2.** The probability of any point remaining in the false vacuum for old inflation (dotted line) and chain inflation (solid line). While the transition from slow to rapid nucleation (large to small  $p(t)$ ) is too gradual for old inflation, it is virtually a step function for chain inflation, which can thus easily percolate and reheat.

#### 4.1. Simplest model: single chain in which all fields have the same parameters

In the simplest version of the chain, we take the parameters of the potential to be the same for every field other than the first one in the chain, so that  $\epsilon_i \equiv \epsilon$  and  $a_i \equiv a$  for all  $i > 1$ . The energy of interaction between any two of the fields is  $V_{\text{int}}$ . We also take  $\epsilon \sim a^4$ , the most natural choice for these parameters.

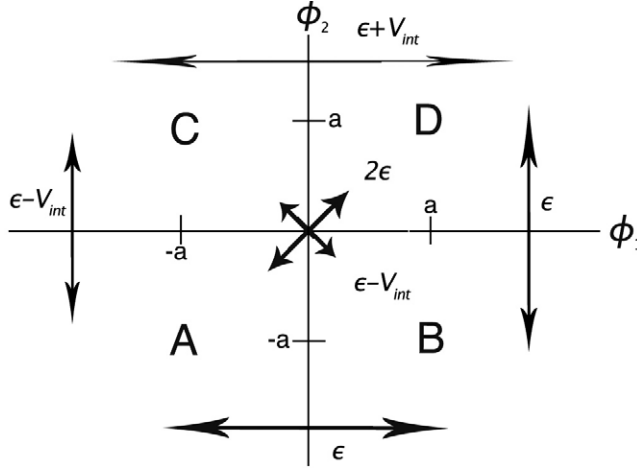
Figure 3 illustrates the four bowls (minima) and energy differences between them, for two fields  $\phi_1$  and  $\phi_2$  for this simple case. Using table 1 and simplifying to the case of equal parameters for all fields, we can evaluate the energies of the four bowls and their energy differences. The energies of the bowls are

$$E_A = 2\epsilon + V_{\text{int}} \quad (35)$$

$$E_B = \epsilon + V_{\text{int}} \quad (36)$$

$$E_C = \epsilon + 2V_{\text{int}} \quad (37)$$

$$E_D = V_{\text{int}} \quad (38)$$



**Figure 3.** The four bowls (minima) in the multidimensional potential for two fields  $\phi_1$  and  $\phi_2$  for the case of identical parameters for the fields. Energy differences between bowls are labelled in the figure. The length of the arrows gives a rough indication of the amplitude of the ratio of (the energy difference to distance<sup>4</sup>) between any two bowls. This quantity determines the tunnelling rate. The system chooses the path with the longest arrow, since its tunnelling rate is the highest.

and the energy differences between bowls are

$$\Delta E_{AB} = \epsilon \quad (39)$$

$$\Delta E_{BC} = \epsilon - V_{\text{int}} \quad (40)$$

$$\Delta E_{CD} = \epsilon + V_{\text{int}} \quad (41)$$

$$\Delta E_{BD} = \epsilon \quad (42)$$

$$\Delta E_{AD} = 2\epsilon \quad (43)$$

$$\Delta E_{AC} = \epsilon - V_{\text{int}}. \quad (44)$$

The highest energy bowl is A, where both fields are in their false vacua. The next highest energy bowl is C, where the second field is at its true vacuum but the first field is still in its false vacuum. The third highest energy bowl is B, where the first field has tunneled to its true vacuum but the second field is still stuck in its false vacuum. The lowest energy bowl is D, with both fields in their true minima. No matter what path the system chooses, the total energy difference between A and D is the same.

The system chooses the path

$$A \rightarrow B \rightarrow D, \quad (45)$$

so that field 1 tunnels first and in so doing catalyses field 2 to tunnel. Here the path  $A \rightarrow C$  is suppressed while the path  $A \rightarrow B$  is fast. This is achieved because the tunnelling rate



is slow with energy barrier  $\epsilon - V_{\text{int}}$  (appropriate to  $A \rightarrow C$ ) but fast with energy barrier  $\epsilon$  (appropriate to  $A \rightarrow B$ ). In this simple case of equal parameters for all potentials, we have

$$\epsilon_{\text{eff,initial}} = \epsilon - V_{\text{int}} \text{ (slow tunnelling)} \quad (46)$$

and

$$\epsilon_{\text{eff,final}} = \epsilon \text{ (fast tunnelling)}. \quad (47)$$

Since  $\epsilon \sim a^4$  is near the border from slow to rapid tunnelling, the model works successfully if we choose parameters in this range.

During each stage of inflation, one of the fields tunnels to its true vacuum, so that the total energy of the system drops by an amount  $\epsilon$ . For successful reheating, the number of e-folds of inflation  $N_n$  at each stage cannot exceed (see equation (14))

$$N_n < N_{\text{crit}} = 1/3, \quad (48)$$

or, equivalently,  $\beta_n > \beta_{\text{crit}}$ , where subscript  $n$  refers to the  $n$ th stage and  $n \geq 0$ . The universe expands by a fraction of an e-folding during each stage. If we add up all the e-folds from all the stages, the total amount of inflation must satisfy

$$N_{\text{tot}} = \sum_n N_n \geq 60. \quad (49)$$

Hence, there must be at least 180 stages of inflation; i.e., the number of fields  $q$  must be at least 180.

During any one of the stages, the number of e-folds of inflation is given following equation (11) as

$$N_n = \int H_n dt \sim H_n \tau_n = \frac{3}{4\pi} \frac{H_n^4}{\Gamma_n}, \quad (50)$$

where

$$H_n^2 = \frac{8\pi}{3m_{\text{pl}}^2} q_n \epsilon \quad (51)$$

and following equation (7), the lifetime  $\tau = (3/4\pi)(H_n^3/\Gamma_n)$ .

Here,  $q_n$  is the number of fields still in their false vacua at stage  $n$ . Since some tunnel at every stage,  $q_n$  drops as time goes on, as does the number of e-folds per stage  $N_n$ . At first all  $q$  fields are in their false vacua, so that  $q_n = q$  and the total energy is  $q\epsilon$ . During each stage, the total energy drops by an amount  $\epsilon$ , and  $q_n \rightarrow q_n - 1$ . Using equations (4), (9), and (51) in equation (50), in the thin wall limit, the number of e-folds at a given stage is then

$$N_n = \frac{3}{4\pi} \left( \frac{8\pi}{3} \right)^2 q_n^2 \frac{\epsilon}{m_{\text{pl}}^4} \exp\left( +64 \frac{\pi^2}{6} \lambda^2 \frac{a^{12}}{\epsilon^3} \right). \quad (52)$$

Thus we find that the amount of inflation  $N_n$  with  $q_n$  multiple fields scales as  $q_n^2$  times the amount of inflation  $N_*$  with a single field in the absence of any other fields,

$$N_n = q_n^2 N_*. \quad (53)$$

Other than the first field to tunnel, the fields remain in their false vacua during several stages. Although the length of each stage is a fraction of an e-folding, the field that tunnels has already been in its false vacuum for a much longer time. For example, the third field to tunnel has remained in its false vacuum during the first three stages. In fact some of the later fields remain in their metastable minima for 60 e-folds. Thus we choose parameters for the potentials that would allow the metastable minima to be stable for a very long time, more than 60 e-folds. In each stage, the potential of one of the fields  $\phi_n$  is modified so as to cause it to rapidly tunnel; as we have seen,  $\epsilon_n$  only needs to change by a very small amount in order for tunnelling to happen. The length  $N_n$  of that stage is set by the modified height and width of the field  $\phi_n$  whose values are changed to their new values by the previous element in the chain.

*4.1.1. The first field.* The first member of the chain must make the transition from false to true vacuum spontaneously, as it is not catalysed to tunnel by any other fields. There are three possibilities for this first field.

(1) One possibility is that the first field to tunnel has (slightly) different parameters to the rest, so that it tunnels rapidly on its own, without assistance from coupling to the other fields. All the subsequent fields, on the other hand, can have exactly the same parameters, and we choose them to have long lived metastable minima until the interactions change the value of  $\epsilon_{\text{eff}}$ . The parameters of the first field do not need to be very different from the rest, due to the extreme (exponential) sensitivity of the tunnelling rate to the parameter values.

We note that this 'first' rapid tunneller does not have to be the first member in the chain. If there are 1000 links in the chain, this rapid tunneller can be number 500 in the chain. Once this field spontaneously tunnels, there is still sufficient inflation (60 e-folds) from fields 501 to 1000 in the chain.

(2) The first field in the chain can be a rolling field which produces but a fraction of an e-fold of inflation. Since it need not be responsible for much inflation, or for density fluctuations, its potential can be arbitrarily steep. Its parameters need not be fine-tuned, unlike the case for slowly rolling fields in most rolling models of inflation. In fact, this first roller can be followed by many more than 60 e-folds of inflation so that any information about its properties is inflated far outside our horizon.

(3) We can imagine that the first field is a tunnelling field with a very slow nucleation rate. In fact, it can have exactly the same parameters as all the fields. Then the first field sits in its metastable vacuum for a very long time before tunnelling. In other words, it behaves much like the single scalar field in the original old inflation model. Hence this first field has  $\beta \ll 1$  at all times, and does *not* percolate and thermalize. Instead there are isolated Swiss cheese bubbles of this first field, i.e., empty bubbles of true vacuum. It is only inside these empty bubble interiors (not in the predominant sea of false vacuum) that the chain of phase transitions continues. The interactions between the first field and the subsequent fields only become nonzero inside the bubble interiors of this first field, so that catalysis of further phase transitions in the chain only takes place inside the bubbles of true vacuum. Hence, our observable universe must live inside a large bubble produced by this first trigger field. Reheating of this large bubble is not a problem, as it can easily happen due to the tunnelling and percolation of subsequent fields in the chain.

There is another issue one must consider if the universe lives inside a single true vacuum bubble of the first tunnelling field. The interior of the first bubble is an infinite

open universe. The appropriate time slices are hyperbolic, determined by slices of constant value of the field. The interior has negative curvature, so that the Friedmann equation becomes

$$H^2 - \frac{1}{a^2} = \frac{8\pi}{3m_{\text{pl}}^2} V_{\text{tot}}. \quad (54)$$

Another sixty e-folds of inflation, subsequent to the production of this trigger field bubble, are required in order to inflate away this negative curvature. This requirement can easily be satisfied by the many subsequent tunnelling fields in the chain, each of which contributes to the total amount of inflation.

In sum, the first field in the chain can either be a spontaneous tunneller, a rolling field, or a slow tunneller. The phase transition of the first field is followed by many e-folds of inflation which erase any unwanted signatures from the first field.

*4.1.2. Example.* As an example, let us consider  $q = 10^4$  fields. We will allow the universe to inflate by 0.1 e-folds per stage (for the first thousand or so stages). Thus the universe inflates 0.1 e-folds before the first field tunnels, then another 0.1 e-folds from that time until the second field reaches its true minimum, then another 0.1 e-folds until the third field reaches its true minimum, etc, with enough stages to obtain a total of at least 60 e-folds.

In order to obtain 0.1 e-folds in the first stage, the number of e-folds that would be obtained by the first tunnelling field (if the other fields were not present) is  $N_* \sim 0.1/q^2 \sim 10^{-5}$ . After each stage, the number of fields participating in the inflation decreases by 1. In the  $n$ th stage, the number of e-foldings is thus

$$N_n = (q - (n - 1))^2 \times 10^{-5}. \quad (55)$$

Thus, for the first 1000 stages,  $N_n \sim 0.1$ . By this time there are already a total of 100 e-foldings (adding up over the first 1000 stages). Eventually enough fields have tunnelled to their minima that the number of e-folds per stage become very small. At the 10000 stage, only  $10^{-5}$  e-folds result. However, the total number of e-folds, summed over all the stages, easily exceeds 60.

We can now ask how reasonable the parameters would be to obtain the above scenario. To honestly do so would require solving for tunnelling in the thick wall limit, as will be considered in a later paper. Typically tunnelling is suppressed in the thin wall limit, but we will discuss here a case where it is not, for illustrative purposes. To tunnel in a thin wall limit requires two things. First, the field should tunnel rather than slow roll; i.e. the energy difference between vacua should be less than the barrier height  $\epsilon < V_0$ . For the asymmetric double well this requirement becomes

$$\frac{\lambda}{2} a^4 > \epsilon. \quad (56)$$

Second, the thin wall condition must be satisfied. Following Coleman [24], the thin wall condition for an asymmetric double well is

$$2\lambda a^4 \gg \epsilon. \quad (57)$$

As an example, to satisfy the above conditions, we will take  $\lambda a^4/\epsilon = 5$ .

The number of e-folds due to a single tunnelling event in the thin wall limit is determined by equation (52). It is indeed possible to find values for which one can both tunnel quickly and satisfy the thin wall conditions. We note that a larger value of  $\lambda$  allows for a quicker transition and also fixes the minimal value of  $a/\epsilon^{1/4}$  to satisfy the above conditions.

We will now consider an example to illustrate that even in this unrealistic case, fairly reasonable parameters can be used. For  $N_n = 0.1$  with  $q = 1000$  and for inflation at the GUT scale  $\epsilon = 10^{16}$  GeV, we find  $a/\epsilon^{1/4} = 0.26$  is required (for the parameters including the effects of coupling to a prior field). Similarly, for inflation at the TeV scale  $\epsilon = 1$  TeV, we find  $a/\epsilon^{1/4} = 0.47$  is required. In addition, to go from a slow tunnelling regime (1000 e-foldings) to a fast regime (0.1 e-foldings) requires a 2% or a 3% change of  $a/\epsilon^{1/4}$  for respectively TeV and GUT scale fields. Despite the artificial nature of the thin wall limit, these values are quite sensible.

*4.1.3. Caveat.* Typically, tunnelling is suppressed in the thin wall limit for any theory (we thank Erick Weinberg for pointing this out to us), though we have discussed above some parameter ranges in which thin wall tunnelling does take place. More generally, a more accurate calculation of tunnelling rates must be performed numerically, in the thick wall limit, where the energy difference  $\epsilon$  is no longer much smaller than the barrier height, following the work of Adams [25]. Of course, in the case where  $\epsilon$  is greater than the barrier height, there is no tunnelling at all and the field simply rolls down the potential. Then too little inflation would ensue. If one were to allow a distribution of parameters, even one peaked about  $\epsilon = a^4$ , then it is plausible that a significant fraction of the fields would remain in their false vacua long enough for the model to work. In particular, as we will see in section 4.2, if there are different paths taken by neighbouring patches of the universe, those regions that inflate more end up much larger than those that inflate little, and our observable universe is more likely to end up within the larger patch. It is only the extreme case where the rapidly tunnelling fields couple to all the intrinsically slowly tunnelling fields and cause them to tunnel rapidly as well (or roll) that is dangerous. Below we will also consider interactions that act in the direction of slowing down the tunnelling, which would assist in this case. (These interactions could exist initially and disappear in time.) However, in general, we do want to warn that there is a range of parameter space for which the potential is not fine-tuned, and yet chain inflation may not work. For example, if the parameter of coupling between the fields  $\gamma_{i,i-1}$  were large, then the resulting tunnelling could become far too rapid (or, depending on the details of the coupling, could become far too slow); note that we have assumed that all couplings are naturally  $O(1)$ . There is one basic requirement for a multidimensional potential in order to obtain successful chain inflation: there must be enough long lived fields, in the sense that they remain in their false vacua for a significant fraction of an e-folding before being catalysed to tunnel to their vacua. Then sufficient inflation will result.

*4.1.4. Issues: would the path skip a link in the chain?* One might ask whether or not the path taken by the system on the way to its ground state would preferentially skip elements in the chain. In a landscape picture, the path could jump directly to a bowl that is lower in energy by several  $\epsilon$ ; this alternative route would be dangerous if it were the quickest path towards the ground state, as fewer e-folds of inflation would result. We can show

that the system does not skip steps for the case where all the potentials have identical parameters; the more general case will be considered in the next section.

Let us consider two fields  $\psi_1$  and  $\psi_2$ , each with a double-well potential of the same parameters (same  $\epsilon$  and  $a$ ). Following the discussion above, in the two-dimensional field space, there are four bowls: (1) at point A,  $\psi_1 = \psi_2 = -a$ , (2) at point B,  $\psi_1 = +a$  and  $\psi_2 = -a$ , (3) at point C,  $\psi_1 = -a$  and  $\psi_2 = +a$ , and (4) at point D,  $\psi_1 = \psi_2 = +a$ . The system starts out at point A, where both fields are in their false vacua. Then, one can ask the question of whether it is faster to proceed from  $A \rightarrow B \rightarrow D$  or directly from  $A \rightarrow D$ . The latter path would be dangerous if it were the fastest, as a path that proceeds quickly to the ground state might not inflate enough. As a reminder, the tunnelling rate scales as  $\Gamma \propto \exp[-F(a^4/\epsilon)^3]$  where  $F = \pi^2\lambda^2/6$ . In comparing the path ( $A \rightarrow B$ ) versus ( $A \rightarrow D$ ), we see that we need to compare  $\epsilon$  versus  $2\epsilon$  and  $a$  versus  $\sqrt{a^2 + a^2} = \sqrt{2}a$ , so that the relevant comparison is between  $a^4/\epsilon$  and  $2a^4/\epsilon$ . We can now compare the times it takes to follow the two paths:

$$\text{Time}(A \rightarrow D) \propto \exp[F8(a^4/\epsilon)^3] \gg 2 \times \exp[F(a^4/\epsilon)^3] \propto \text{Time}(A \rightarrow B) + (B \rightarrow D). \quad (58)$$

Hence the preferred path is to tunnel in the direction in field space in which the energy decreases by a single unit of  $\epsilon$  at a time; i.e., which can be thought of as the tunnelling of a single field.

## 4.2. Generalizing the model

Chain inflation is a general phenomenon in which the system of scalar fields in a multidimensional potential tunnel from bowl to bowl. Our particular choice of couplings allows simple calculations so as to obtain reasonable estimates. However, the coupling certainly need not be of the form presented here. Indeed, all the fields could couple to each in other in a complicated fashion as long as the couplings preserve the bowl type of structure in a multidimensional landscape.

Previously we considered the simplest single-chain model in which all fields have the same parameters. In this section we discuss some simple generalizations of this model which would also work. We restrict our discussion to double-well potentials, but the results should generalize to other choices of potentials.

*4.2.1. Arbitrary parameters, a number of chains.* First, let us consider the effects of keeping the same form of the potential and the interactions, but allowing a variety of parameters  $a_i$  and  $\epsilon_i$  for the different fields. In addition, we will allow for the existence of a number of chains. The first field could couple to a number of secondary fields (next in line in a chain after the first field). Then each of the secondary fields couples to a number of tertiary fields, etc. Some subset of the secondary fields could also couple to each other.

In one patch of real space, the fields follow a single path in (multidimensional) field space; i.e., at each instant, there is a single vacuum expectation value for each of the fields  $\langle\phi_1\rangle, \langle\phi_2\rangle, \dots$ . Of the many choices of direction in field space, e.g. the variety of possible secondary tunnelling fields, the system only chooses one. At each stage, the path *en route* to the ground state that would be chosen would be the fastest one. For example, if the first field couples to ten others, then the system chooses the direction in field space with the fastest rate of tunnelling to a lower energy. The system ‘chooses’ the field that tunnels the most quickly. The other nine fields to which the first field couples become irrelevant.

Here we address three issues that could arise for different choices of parameters in the potentials: (i) fields which tunnel too slowly, (ii) fields which tunnel too rapidly, and (iii) interactions which are so large that the chain sequence is modified.

*Tunnelling too slowly.* Due to the variety of parameters, there might be some fields which, even after their interactions turn on, are still stuck in their false vacua. In the language of the landscape, even at field values in the multidimensional field space where the interaction of the given field with the preceding member of the chain is nonzero, still the tunnelling rate of the field is extremely slow. In that case, the path that the system takes *en route* to the ground state will simply avoid that direction. As long as one of the secondary fields is able to tunnel to a lower minimum, the expectation values of the fields will choose the path that has a series of rapid tunnellers. In a landscape picture, the system will take the path of least resistance to the ground state. In addition, generically there are multiple couplings between fields, and it is likely that one of these couplings will suffice to induce the (otherwise slowly tunnelling) field to quickly tunnel.

Even if the path that is chosen does include a very slow tunneller in the middle of the chain, as long as there are sixty e-folds subsequent to that slow tunneller, any resultant negative curvature will get washed out by the subsequent inflation.

*Tunnelling too rapidly.* A new problem could arise, however. Somewhere down in the chain, e.g. the fiftieth field in the chain, could have parameters (e.g. a large enough  $\epsilon$ ) such that it tunnels spontaneously, without being catalysed by any other field. If we think of the single chain as a series of dominoes, then this case corresponds to the 50th domino falling over on its own. If there are 100 dominoes in total, then the first set of 50 and the second set of 50 will fall over at the same time. It will take half as long for all of them to fall over, i.e., only half as many e-foldings of inflation will result, so the overall vacuum energy will drop down by an amount  $\epsilon$  twice as quickly. However, sufficient inflation should still result as long as (i) there are significantly more than 60 steps in the chains and (ii) relatively natural values of parameters for the potentials are chosen, i.e.,  $\epsilon \leq a^4$  for most of the fields, so that most fields do not tunnel spontaneously (see the discussion in section 4.1.3).

At different spatial points in the universe, it is possible that the system chooses different paths in field space. For example, of the ten possible secondary paths in the above example, it is possible that two of them have the same tunnelling rate and are equally likely to be chosen. Then, in the end, two different patches of the universe that followed slightly different paths could end up at different ground states, both with  $V = 0$  but with different field values. Topological objects such as domain walls could be formed in between these different patches. Of all the possible paths to the ground state, some may be too rapid, in the sense that the total number of e-folds is too small (e.g. because of the effect discussed in the last paragraph). Then clearly we do not live in a patch of the universe that took such a steep, rapid path. Since this underinflated patch of the universe is much smaller than the patches that did inflate, it is quite reasonable that our observable universe lies within the much larger regions that did inflate sufficiently.

*Too large interactions.* Our treatment in equation (18) has assumed that one can break up the potential into a set of asymmetric double-well potentials with interactions. However,

if the interactions are very large,  $V_{\text{int}} > V_i(\phi_i)$  for one of the fields, this basic picture is not quite right. If two subsequent fields in the chain have very different energy scales, then the order in which fields tunnel in the chain may change. For example, if a field with a TeV scale potential is followed in the chain by a field with a GUT scale potential, the GUT scale potential will override the TeV one and will tunnel first.

To illustrate this effect, let us consider two of the fields in the system, 1 and 2, as studied in table 1 and the accompanying discussion. Now let us take the first field to be characterized by a TeV scale,  $\epsilon_1 \sim a_1^4 = \text{TeV}^4$ , while the second field is characterized by a grand unified (GUT) scale  $\epsilon_2 \sim a_2^4 \sim (10^{16} \text{ GeV})^4$ . We will also assume  $a_0, a_3 \ll m_{\text{GUT}}$ . The four bowls (minima) now have energies

$$E_A = m_{\text{GUT}}^4 + \text{TeV}^4 + a_0^3 \text{TeV}, \quad (59)$$

$$E_B = m_{\text{GUT}}^4 + \text{TeV}^3 m_{\text{GUT}}, \quad (60)$$

$$E_C = \text{TeV}^4 + a_0^3 \text{TeV} + m_{\text{GUT}}^3 a_3, \quad (61)$$

$$E_D = m_{\text{GUT}}^3 a_3. \quad (62)$$

The system starts out in bowl A, where both fields are in their false vacua. Ordinarily field  $\phi_1$  would tunnel first and the system would then move to bowl B. Here, however, the interaction is so big that the energy difference between bowls A and B is negative,

$$\Delta E_{AB} = E_A - E_B = \text{TeV}^4 - \text{TeV}^3 m_{\text{GUT}} + a_0^3 \text{TeV} < 0, \quad (63)$$

i.e. bowl B is at a *higher* energy than bowl A. Clearly the fields will not tunnel from A to a state B of higher energy. Instead, the second (larger) field  $\phi_2$  tunnels first. The energy difference between bowls A and C is

$$E_{AC} = m_{\text{GUT}}^4 - m_{\text{GUT}}^3 a_3 \sim m_{\text{GUT}}^4 \quad (\text{for } a_3 \ll m_{\text{GUT}}). \quad (64)$$

Thus the system preferentially takes the path  $A \rightarrow C \rightarrow D$ . The resultant number of e-folds is not significantly different from the number obtained in the normal sequence of the chain as discussed previously. If there is a string of mismatched scales, one may lose some e-foldings but with the large number of fields there should still be no problem getting enough inflation.

*Getting the right amount of inflation.* The chain model works as long as potentials are not fine-tuned. For the double-well example, as long as  $\epsilon \sim a^4$  for a large fraction of the fields, they are on the border between remaining in metastable vacua and rapid tunnelling; in addition, they are able to influence one another to tunnel. If all the fields have  $\epsilon \gg a^4$ , then the system may reach the ground state without inflating enough. If, on the other hand, all the fields have  $\epsilon \ll a^4$ , then one might worry that they all remain in their false vacua too long, although their couplings to other fields in that case will probably induce them to tunnel and percolate. (If there is some discrete symmetry that forces  $\epsilon = 0$ , so that both minima in the double-well potential are degenerate, obviously there is no tunnelling at all.)

4.2.2. *More general interactions.* We have considered a very simple form of the interaction,  $V_{\text{int}} \propto (\phi - a)$ , where  $\pm a$  are the field values at the minima. More generally, the interaction could be of the form

$$V_{\text{int}} \propto (\phi \pm b)^p \quad (65)$$

where  $b$  is an arbitrary field value and  $p$  is an arbitrary power. The fact that the value of  $b$  does not necessarily equal the value of  $a$  does not qualitatively change the model. The amount by which  $\epsilon_{\text{eff}}$  differs from  $\epsilon$  due to the interaction has to be recalculated, but the effect of causing a transition from slow to rapid tunnelling still takes place (one must of course check that the interaction is strong enough to change  $\epsilon$  by 1%).

Due to the possibility of the opposite sign in front of  $b$ , the interaction could have the opposite effect from what we want. It could inhibit the field from tunnelling, rather than catalyse the tunnelling. One could imagine that half of the interactions felt by a given field would serve to increase the tunnelling rate, and half to decrease it. As different trigger fields tunnelled to their minima, they could act in either direction. However, the symmetry is broken by the fact that the field only has to be induced to tunnel once. Once it has tunnelled, the sign of future interactions is irrelevant. Also, the system will take the path of least resistance, i.e., the path along which the interaction serves to speed up rather than slow down the tunnelling.

One could also imagine an interaction which is nonzero only when catalysing field is still in its false vacuum; for example, the catalysing field could prevent tunnelling of another field early on (due to the interaction) and then play no further role once it reaches its minimum.

4.2.3. *Thermal activation.* One could imagine that, depending on the values of the parameters, as one of the fields in a stage of chain inflation successfully reheats, it thermally activates one of the other fields to go over the top of its barrier. Heretofore we have not considered the effects of such thermal activation, which might be interesting to pursue.

4.2.4. *Mixing rollers and tunnellers.* In the paper we have restricted the discussion to a series of tunnelling fields. However, there is no reason to do so. In the landscape, one could imagine a mixed succession of rolling fields and tunnelling fields. The path chosen towards the ground state could involve rolling for a while, followed by a tunnelling event, followed again by a period of rolling down a potential. Such a mixed chain would be interesting to study further.

## 5. Variants

Here we briefly comment on three variants of tunnelling inflation with multiple fields which are alternatives to the chain we have been discussing: (i) a large number of uncoupled tunnelling fields which tunnel simultaneously, (ii) two tunnellers, and (iii) the case where the first tunnelling field in the chain inflates sixty e-folds and then catalyses a large number of fields to simultaneously tunnel and percolate. We will see that all these ideas are fatally flawed if one restricts the discussion to multiple tunnelling fields only.



However, we emphasize that the third idea may succeed if the trigger field is a rolling field (as in the case of the double-field model) which, once it reaches its minimum, catalyses the simultaneous tunnelling of a large number of secondary fields. This idea will be discussed in future work.

### 5.1. Multiple uncoupled tunnelling fields

Here we consider a large number  $q$  of uncoupled tunnelling fields, analogous to the large number of uncoupled rolling fields in assisted inflation [2]. The number of e-folds obtained from any one of the fields is given using equation (11) as

$$N_i \sim \frac{8\pi}{3m_{\text{pl}}^2} \sqrt{\epsilon_i} \tau. \quad (66)$$

If each field produces a small amount of inflation  $N_i \ll 1$ , the total number of e-folds from all the fields together is  $N \sim q^2 N_i$  (see equation (53)). One can easily imagine obtaining sufficient inflation,  $N > 60$ . However, then the parameter  $\beta \sim (1/q^2)\beta_i \ll 1$  is a constant value independent of time, and never reaches  $\beta > \beta_{\text{crit}}$  required for percolation and thermalization. This model is equivalent to inflating with a single field of energy density  $\rho = q\epsilon$  with constant nucleation rate. Hence the required criteria of sufficient inflation (small  $\beta$ ) followed by reheating (large  $\beta$ ) can never be achieved for constant  $\beta$ . In short this model fails for the same reason old inflation does. Thus, although the model of assisted inflation, with multiple uncoupled rolling fields, can succeed, the equivalent model with multiple uncoupled tunnelling fields fails.

### 5.2. Two tunnellers

One might propose an alternative to the double-field model, in which there are two tunnelling fields instead of one roller and one tunneller. Here the first tunnelling field serves as the ‘trigger’ field for catalysing the tunnelling of the second field. At first, both fields are in their false vacua. Then, when the first field tunnels to its true minimum, it catalyses the second field to tunnel as well. In the discussion of the double-field model above, in equation (16) one could simply replace  $\psi^3$  with  $(\phi_1 + a)^3$ , where  $\phi_1$  is the trigger field, whose potential has a false vacuum at  $\phi_1 = -a$  and a true vacuum at  $\phi_1 = +a$ . Hence the interaction term only turns on once the trigger field has tunneled to its true vacuum. At that point, the energy difference between minima in equation (17) of the second field becomes so large that its tunnelling rate becomes very large, allowing bubbles of its true vacuum to nucleate simultaneously throughout.

In this two-tunneller model, the trigger field must remain in its false vacuum for at least sixty e-folds, because once it tunnels to its true vacuum, inflation is over. Hence, this trigger field must have  $\beta \ll 1$ . It does *not* percolate and thermalize. Instead, there are isolated Swiss cheese bubbles of this first field, i.e., empty bubbles of true vacuum of the trigger field. Since the interaction term only becomes nonzero for the bubble interiors from the trigger field, the secondary field only undergoes the phase transition inside these trigger field bubbles. All the bubbles from the secondary field must live inside a large bubble produced by this first trigger field. The idea would be to reheat the interior of one of the bubbles of the trigger fields with the energy density from the bubble collisions of the secondary tunneller. Since the secondary tunneller does have a time dependent nucleation

rate, due to interaction with the trigger field, the bubbles from the second tunneller can easily percolate and thermalize. If  $\epsilon$  is large enough for this second tunneller, there is no problem reheating the inside of the big bubble from the first tunneller.

However, in the case where the trigger field is a tunnelling field, the universe we live in today cannot have originated inside the interior of a true vacuum bubble of the first tunneller. The interior of the bubble is an infinite open universe with negative curvature as in equation (54). Another sixty e-folds of inflation, subsequent to the production of this trigger field bubble, are required in order to inflate away this negative curvature. But in the model where the secondary field tunnels right away once the trigger field bubble comes into existence, these 60 e-folds do not take place. Hence the two-tunneller model fails<sup>9</sup>.

### 5.3. Single trigger field catalysing multiple secondary fields to simultaneously tunnel

Here we consider inflation with multiple fields which are all coupled together in such a way that, once the first one reaches its true minimum, it catalyses all the rest to tunnel immediately. This model is similar to the two-tunneller model discussed above, but with the single secondary field replaced by multiple secondary fields which all tunnel at once. We will see that this model suffers from the same problems as the two-tunneller model above.

In order for this model to work, the first field must remain in its false vacuum for sixty e-folds; once it tunnels, all the other secondary fields simultaneously and very quickly tunnel, thereby ending the inflation. One could imagine, e.g., that the potential for each of the fields is a double well as in equation (8), with interaction terms of the form

$$V_{\text{int},i} = -\frac{1}{16}\gamma_i(\phi_i - a_i)(\phi_1 + a_1)^3. \quad (67)$$

Here  $\phi_1$  is the trigger field. As in the double-field model of Adams and Freese, once the trigger field reaches its minimum,  $\epsilon_i \rightarrow \epsilon_{\text{eff},i}$  and all the fields tunnel at once and successfully reheat. Here,  $\epsilon_{\text{eff},i} = \epsilon_i + \frac{1}{8}a_i\gamma_i(\phi_1 + a_1)^3$ . Once the trigger field reaches its minimum at  $\phi_1 = a_1$ ,  $\epsilon_{\text{eff},i} = \epsilon_i + \gamma_i a_1^3 a_i$ . Due to the abrupt change in tunnelling rate for all the secondary fields from very slow to very fast, these fields can easily nucleate bubbles of true vacuum throughout the universe and hence percolate and thermalize.

However, as in the two-tunneller model, the interiors of bubbles from the initial trigger field are problematic. The trigger field must remain in its false vacuum for at least sixty e-folds, because once it tunnels to its true vacuum all the other fields tunnel as well, and inflation is over. Hence, this trigger field must have  $\beta \ll 1$ , it does not percolate, and empty bubbles result from this first tunnelling. The interaction term in equation (67) is only nonzero inside these empty bubbles, so that the secondary fields only tunnel to their true vacua inside the trigger bubbles. Again, although the interior of the big bubble can successfully reheat due to the percolation and thermalization of the secondary bubbles, the interior does not look like our universe. It is an infinite open universe with negative curvature. Another sixty e-folds of inflation, subsequent to the production of this trigger field bubble, are required in order to inflate away this negative curvature. But in the model where all the secondary fields tunnel right away once the trigger field bubble comes

<sup>9</sup> In the chain picture considered previously, where there is a series of secondary fields, there was sufficient inflation following the trigger tunnelling event.

into existence, these 60 e-folds do not take place. Hence, one must return to the chain picture that we have discussed as the primary model of this paper, in which the first field triggers a chain of tunnellers that approach their minima serially and in the process give rise to another 60 e-folds of inflation.

We note a variant of this idea that avoids the problems discussed above. The trigger field may be a rolling field. The key difference is that the first field is rolling, as in the case of the double-field model. Once this rolling field reaches its minimum, it catalyses the simultaneous tunnelling of a large number of secondary fields. If one uses a rolling field as the initial trigger field, then there is no issue of negative curvature. However, one must address the issue of parameters of the rolling field to demonstrate that one can avoid fine-tuning. This subject is the topic of an upcoming paper.

## 6. Issues

### 6.1. Can de Sitter violations of energy conservation send a tunnelling field over the top of the barrier?

Here we address the question of whether or not de Sitter violations of energy conservation could render the barrier between false and true vacuum in the double wells irrelevant, in that the energy violations are so large that they send the field over the top of the barrier. We show that the effect is not important in our model.

Since the de Sitter metric has no timelike Killing vector globally defined, there are violations of energy conservation of magnitude

$$\Delta E \sim \sqrt{\rho_{\text{vac}}/m_{\text{pl}}^2} \quad (68)$$

where  $\rho_{\text{vac}}$  is the vacuum energy density. For us,  $\rho_{\text{vac}}$  is largest at the beginning of inflation,  $\rho_{\text{vac}} = q\epsilon$  where  $q$  is the number of fields. If  $\Delta E > \text{barrier energy}^{1/4}$ , then the field can hop over the top of the barrier. For this to happen, one would need  $q\epsilon > m_{\text{pl}}^2\epsilon^{1/2}$ . If we take the mass scale of the barrier energy to be the GUT scale, this corresponds to  $\epsilon^{1/2} \sim 10^{-6}m_{\text{pl}}^2$ . Then the condition for the field to be able to hop is never satisfied for  $q < 10^6$ . At potentials with lower energy scales, it is even more difficult for energy violation to cause the field to hop over the barrier, and an even larger number of fields would be required. Thus we conclude that, in the models we have been considering, the field does not go over the top of the barrier due to energy non-conservation.

### 6.2. Fine-tuning

Whereas slowly rolling models of inflation require unnaturally small parameters, the proposed model of chain inflation has the attractive feature that the potential need not be fine-tuned.

In inflationary models with a single scalar field that is slowly rolling, the potential is characterized by its height  $\Delta V$  (the vacuum energy of the inflaton) and width  $\Delta\psi$  (the change in field value during inflation). For inflation to work in this context, it must satisfy certain constraints: there must be sufficient inflation and the amplitude of the density perturbations [26, 27] cannot exceed  $\delta\rho/\rho \leq 10^{-5}$ . A fine-tuning parameter,

$$\chi \equiv \frac{\Delta V}{\Delta\psi^4}, \quad (69)$$

has been introduced to examine these constraints [28]. The most natural value of this ratio in particle physics would be  $\chi \sim \mathcal{O}(1)$ . Instead, in order to satisfy the above two constraints, it has been shown that this parameter must satisfy [28]

$$\chi \leq 10^{-8}, \quad (70)$$

a very unnatural value. This strongly constrains models of slow roll inflation. Very few inflation models (such as natural inflation [11, 12]) can explain this small number.

The potentials in chain inflation do not require fine-tuning. The potentials we have considered have  $\epsilon \sim a^{1/4}$ , i.e., the parameter  $\chi \sim 1$ . This is an advantage of the chain inflation model.

As discussed in section 4.1.3, however, the multidimensional potential in chain inflation must have the right parameters to allow the fields to remain in their false vacua for a long enough time to obtain at least sixty e-folds in all.

### 6.3. Density fluctuations

There are several remaining issues for future work. The most important is the question of generation of density fluctuations. One source of perturbations would be the fact that the lifetime of a field in its false vacuum is given only roughly by equation (7). In fact, there would be a distribution of timescales about this typical value. Some regions of the universe would tunnel to their true minima on a slightly shorter timescale than their neighbours. In different patches of the universe, the phase transition could be a little ahead or a little behind, and this timing difference can lead to density fluctuations. This situation is similar to the density fluctuations in slow roll inflation that are produced as a result of neighbouring regions rolling to their minima at slightly different times (due to quantum fluctuations at the beginning of rolling). As in new inflation, here again the fluctuations that are created sixty to fifty e-folds before the end of inflation are on length scales that today correspond to observable structure. It is not clear what the spectrum of these perturbations would be, or whether they would be adiabatic or isocurvature. This issue must be investigated further.

In those regions which tunnelled the most quickly, the resulting bubbles would have more time to grow than in the neighbouring regions. In [16], the authors worried about the ‘Big Bubble’ problem that resulted in inflation with a single field; the universe could not have tolerated the excessively big bubbles that would arise. Here, however, the size difference between bubbles of any one field is given by  $\exp(N_n)$ , and  $N_n$  is a small number (less than 1/3). Thus the largest bubbles cannot even be twice as large as the typical bubbles, and would not create unduly large inhomogeneities. They would instead potentially create the density fluctuations that we see. It takes an extra amount of time,  $\exp[N_n/2]$ , for energy in the bubble walls to convert to particles. One finds this amount of extra energy near the bubble walls. One can ask whether this extra energy can be smoothed out across the bubbles. The particles (at this point relativistic) move across the bubbles with the speed of light. Hence in a Minkowski background, the particles could easily thermalize across the interior. However, here the background spacetime is still de Sitter after one of the fields has tunnelled to its minimum while some of the other fields have not, and the system is not yet at its ground state. Hence the interior space is expanding superluminally and the particles cannot traverse the interior. One ends up with

spherical shells of particles. Of course, as the universe continues to inflate, the density of these particles is diluted to the point where they are irrelevant. In short, percolating bubbles in the early stages of inflation turn into spherical shells of particles (where the bubble wall was) that cannot thermalize, because the interior is de Sitter and information can only traverse the bubble at the speed of light. However, the situation is different at the very final stage of inflation, when the last field in the chain tunnels down to  $V = 0$ . The bubbles of this last stage of inflation can thermalize so that reheating is successful. As the system has now tunnelled down to Minkowski space, the bubble interiors are not expanding superluminally. The particles produced near the bubble walls can easily traverse the entire bubble in one Hubble time and reheat the interior. In addition, the spherical shells of bubbles produced at earlier inflationary stages can now also spread out throughout the universe, again in a Hubble time.

To reiterate, the key question as regards density fluctuations is the timing of the phase transitions in different patches of the universe, as discussed above and as should be investigated further.

Density perturbations could result from other effects as well. For example, perturbations to the exact  $O(4)$  symmetry of the solution to the Euclidean action might lead to deviations in the density. Other effects leading to perturbations (in rolling models) were discussed in [29] and [30].

#### 6.4. Beyond the thin wall

A more accurate treatment of the chain double-well model discussed in this paper would require going beyond the thin wall limit. We are aware that the thin wall approximation is valid only in the case where  $\epsilon/a^4 \ll 1$ , in which case the bounce action is really small and there is no tunnelling at all. Hence in those cases of interest where tunnelling takes place, one should numerically solve the bounce equation. However, the basic picture presented in the current paper would remain unchanged, though the detailed numerical answers might be different. Adams [25] has previously studied generic quartic potentials in the thick wall limit. The chain model as presented still results (as long as  $\epsilon < a^4$  so that the field is tunnelling rather than rolling).

### 7. Conclusion

In conclusion, we have proposed a model of chain inflation in which a sequential chain of coupled scalar fields can drive inflation. We considered a toy model of a chain of tunnelling fields, each of which catalyses the next to tunnel to its true vacuum. Since each tunnelling stage provides only a fraction of an e-folding, percolation of the true vacuum bubbles and hence reheating is easily achieved. Many fields, at least several hundred, are required in order to achieve enough inflation. Such a large number of fields is motivated by the many vacua in the stringy landscape, but our model can apply to a chain of tunnellers in any multidimensional potential. One can think of each tunnelling event as equivalent to dropping from one bowl in a multidimensional potential to another bowl of lower energy, until the zero-energy ground state is achieved. Chain inflation has the attractive feature that it relies on couplings between the fields, which are likely to exist. We have focused on double-well potentials as a toy model, first with identical parameters and couplings, and then generalized to arbitrary values. However, the idea is quite general, and relies

only on the idea of tunnelling in a multidimensional potential from one minimum to the next, regardless of any details of the potential. Chain inflation works for natural values of parameters and couplings. It can be successful for a wide variety of energy scales for the potential, ranging from values as low as 10 MeV up to a GUT scale at  $10^{16}$  GeV.

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