The age of the universe in Cardassian models

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Abstract. The age of the universe is obtained in a subset of Cardassian models by using WMAP data. Cardassian expansion is a modification to the Friedmann equation that allows the universe to be flat, matter dominated, and accelerating, without a vacuum component. Since this model changes the evolution of the universe, we should not *a priori* expect the Cardassian age to be the same as the WMAP Friedmann derived result of 13.7 ± 0.2 Gyr. However, in the subset of Cardassian models that we consider, we discover that the age of the universe varies from 13.4 to 13.8 Gyr over the range of parameter space that we explore, a result close to that of the standard Λ cold dark matter model. The Hubble constant h, which may also vary in these models, likewise varies little from the Friedmann result.

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Contents

1. Introduction

Over the past five years, cosmic microwave background (CMB) data have shown the universe to be flat [1, 2] and supernova data have indicated that it is accelerating [3, 4]. The matter content of the universe falls well short of the energy density necessary to provide a flat curvature in the standard cosmological model, the Friedmann model. The most popular interpretation of this mismatch is that the 'missing' density is assumed to be present in the form of a vacuum (or dark) energy that provides a pressure leading to the acceleration of the universe.

Alternatively, general relativity may need to be modified. Modification of the standard cosmological model, as in the Cardassian model [5], allows both the flatness and acceleration to be accounted for solely due to the matter content of the universe. This modification changes the evolution of the universe. Hence the CMB anisotropy spectrum will imply a different age of the universe [2].

The Cardassian model [5]–[9] modifies the Friedmann equation by adjusting the right-hand side to a more general function of the energy density. This function returns to the usual Friedmann equation for the early history of the universe, so ordinary nucleosynthesis takes place. However, in the recent past, beginning at a redshift O(1), this function drives the universe to accelerate. Such modifications to the Friedmann equation may arise if our universe exists on a brane embedded in a higher dimensional universe [10]. Other proposed modifications to the Friedmann equations include [11].

In section 2, we will review the Friedmann model and describe how to determine the age of the universe. In section 3, Wilkinson microwave anisotropy probe (WMAP) results are reviewed and the choice of appropriate parameters is discussed. In section 4, we will expand these techniques to the Cardassian model. We note that, since the customary parameters h, Ω_b, and Ω_m are not orthogonal and indeed have values that depend upon the cosmological model itself, more suitable parameters must be extracted from the data in order to obtain the age of the universe in any model. And finally, results for the Cardassian model are discussed in section 5. In this paper, we obtain the age of the universe only for the modified polytropic (MP) Cardassian models defined below.

2. Age in the Friedmann model

In standard cosmology, the evolution of the universe is governed by the Friedmann equation:

$$H^2 = \frac{8\pi G\rho}{3} \tag{1}$$

where H is the Hubble parameter and we have dropped the curvature term as experimental results are consistent with a flat universe [1, 2]. At the current epoch the critical density is

$$\rho_{\mathrm{c}} = \frac{3H_0^2}{8\pi G} = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3} \tag{2}$$

where subscript 0 refers to the present day and $H_0 = 100h$ km s^{-1} Mpc^{-1}. Here, $\Omega \equiv \rho/\rho_{\mathrm{c}}$ is the ratio of the energy density to the critical density, with Ω_i representing the ratio due to the component ρ_i of the density. In the standard picture, an additional component beyond matter and radiation is assumed to reach the critical density. This component is taken to be a vacuum energy: a cosmological constant Λ or a time dependent vacuum energy, or scalar field known as 'quintessence', that evolves dynamically with time [12].

The Hubble parameter can be related to its present day value by

$$H = H_0 E_{\mathrm{F}}(z) \tag{3}$$

$$E_{\mathrm{F}}(z)^2 = \Omega_{0\mathrm{r}}(1 + z)^4 + \Omega_{0\mathrm{m}}(1 + z)^3 + \Omega_{0\mathrm{X}}(1 + z)^{3(1+w_{\mathrm{X}})} \tag{4}$$

where $\Omega_{0\mathrm{r}}$, $\Omega_{0\mathrm{m}}$, and $\Omega_{0\mathrm{X}}$ are the current contributions from radiation, matter, and vacuum respectively, and $w_{\mathrm{X}} = p_{\mathrm{X}}/\rho_{\mathrm{X}}$ is the equation of state.

To determine the age of the universe, we integrate equation (1):

$$t_0 = H_0^{-1} \int_0^\infty \frac{\mathrm{d}z}{(1+z)} \frac{1}{E_{\mathrm{F}}(z)}. \tag{5}$$

3. WMAP parameters

The WMAP Collaboration has determined values for several parameters by mapping the cosmic microwave background [2]; relevant ones are listed in table 1. They have determined the age of the universe in a Λ-CDM (cold dark matter) model which assumes that the dark energy is a cosmological constant,

$$t_0 = 13.7 \pm 0.2 \text{ Gyr} \qquad (\Lambda\text{-CDM model}), \tag{6}$$

where the uncertainty is determined by statistical analysis of their Monte Carlo Markov chain results [13].

Using the WMAP results for h and $\Omega_{0\mathrm{m}}$ in equation (5) yields the correct value, but overestimates the uncertainty. This is due to the fact that the parameters h and $\Omega_{0\mathrm{m}}$ are not orthogonal; that is, their uncertainties are correlated [14, 15]. Instead, the set of (nearly) orthogonal parameters is $\omega_{\mathrm{m}} \equiv \Omega_{0\mathrm{m}}h^2$, $\omega_{\mathrm{b}} \equiv \Omega_{0\mathrm{b}}h^2$ (where b is for baryon), and the acoustic scale ℓ_{A}.

Table 1. Cosmological parameters determined by the WMAP Collaboration from fits to multiple experiments [2].

WMAP parameters	
Hubble constant	$h = 0.71^{+0.04}_{-0.03}$
Baryon density	$\Omega_{0b} = 0.044 \pm 0.004$
	$\omega_b = 0.0224 \pm 0.0009$
Matter density	$\Omega_{0m} = 0.27 \pm 0.04$
	$\omega_m = 0.135^{+0.008}_{-0.009}$
Acoustic scale	$\ell_A = 301 \pm 1$
Redshift of decoupling	$z_{dec} = 1089 \pm 1$

Oscillations in the photon–baryon fluid during the early universe led to peaks in the CMB power spectrum. The first peak in this spectrum is related to the angle θ_A subtended by the conformal distance s a sound wave travelled from the big bang until decoupling at the surface of last scattering, which has a conformal distance denoted by D. In a flat universe,

$$\theta_A = \frac{\pi}{\ell_A} = \frac{s}{D}. \tag{7}$$

The distances s and D are given by

$$s = \frac{1}{H_0} \int_{z_{dec}}^{\infty} dz \, \frac{c_s}{E_F(z)} \tag{8}$$

$$D = \frac{1}{H_0} \int_0^{z_{dec}} dz \, \frac{1}{E_F(z)} \tag{9}$$

where c_s is the speed of sound in the photon–baryon fluid:

$$c_s = \frac{1}{\sqrt{3 \left(1 + \frac{3}{4}(\rho_b/\rho_\gamma)\right)}}. \tag{10}$$

An in-depth discussion of a photon–baryon fluid and the CMB may be found in [16]–[21].

The three parameters ω_m, ω_b, and ℓ_A have nearly orthogonal effects on the CMB power spectrum; ℓ_A is related to the position of the first peak, while the other two are related to the peak heights.

The redshift of the photon–baryon decoupling z_{dec} is determined mainly from the overall temperature of the CMB, with a WMAP value of 1089 ± 1. The redshift of matter–radiation equality z_{eq} is given by

$$1 + z_{eq} = \frac{5464}{1 + \rho_\nu/\rho_\gamma} \frac{\omega_m}{0.135} \left(\frac{T}{2.725 \text{ K}}\right)^4 \tag{11}$$

with a neutrino to photon density ratio of $\rho_\nu/\rho_\gamma = 0.6851$ and an overall CMB temperature of $T = 2.725$ K [15].

While several of the above equations may be greatly simplified in the standard Friedmann model (with closed forms available for some of the integrals) [15], we will

not make these simplifications here. Instead, it will be useful to use numerical techniques that can be applied below to the Cardassian model, where the equivalent integrals do not have closed forms. From equation (7), we can take h to be an implicit function of ω_{m}, ω_{b}, and ℓ_{A}:

$$h \equiv h(\omega_{\mathrm{m}}, \omega_{\mathrm{b}}, \ell_{\mathrm{A}}). \tag{12}$$

That is, given observed values for ω_{m}, ω_{b}, and ℓ_{A} (from WMAP), we can use numerical routines to find the value of h necessary to satisfy $\ell_{\mathrm{A}} = \pi D/s$, where s and D (dependent upon ω_{m}, ω_{b}, and h only) are given by equations (8) and (9), respectively. The value of h thus obtained may then be inserted into the time integral, equation (5) (using $\Omega_{0\mathrm{m}} = \omega_{\mathrm{m}}/h^2$), to obtain an age that is dependent on the three orthogonal parameters.

This procedure correctly reproduces both the Hubble constant h and age of the universe t_0 (including uncertainties) obtained through the WMAP MCMC, listed in table 1. Reproduction of the correct uncertainties affirms that the aforementioned parameters are indeed nearly orthogonal and will be the appropriate parameters to use in more general models.

4. Age in Cardassian models

Cardassian expansion was proposed as a model in which matter alone is sufficient to drive acceleration of the universe; in this model, there is *no* vacuum energy. Instead, the Friedmann equation is modified to give a general function of the energy density on the right-hand side:

$$H^2 = g(\rho). \tag{13}$$

This function returns to the usual Friedmann equation for the early history of the universe, so ordinary nucleosynthesis and evolution results. However, in the recent past, beginning at a redshift O(1), this function drives the universe to accelerate.

We see that the critical density $\tilde{\rho}_{\mathrm{c}}$ is then defined by the relation

$$g(\tilde{\rho}_{\mathrm{c}}) = H_0^2 \tag{14}$$

where we note the critical density is not, in general, the same as in the original Friedmann model (equation (2)). It will be useful to relate the new critical density and other parameters to the old critical density. To prevent confusion, we will use $\tilde{\rho}_{\mathrm{c}}$ to denote the Cardassian critical density, while ρ_{c} will continue to denote the Friedmann critical density. Likewise, $\Omega_i \equiv \rho_i/\rho_{\mathrm{c}}$ will continue to be defined in terms of the Friedmann critical density.

In this section, we will look at a subset of the Cardassian models [5, 6] known as modified polytropic Cardassian (MP Cardassian):

$$H^2 = g(\rho) = \frac{8\pi G \rho}{3} \left[1 + \left(\frac{\rho}{\rho_{\mathrm{card}}} \right)^{q(n-1)} \right]^{1/q} \tag{15}$$

where ρ_{card}, $q \geq 1$, and $n < 2/3$ are free parameters. One of these parameters is fixed by the observed value of Ω_{m0}, and a constraint on the remaining two is obtained by matching supernova data (the redshift at which the second term becomes important). The normalization is fixed to match the Friedmann model at early times.

We will require $n < 2/3$, so that the second term in the brackets of equation (15) increases over time, eventually dominates, and then leads to acceleration of the universal expansion. In this paper we consider only $n \geq 0$; in fact it could be negative as well. The case $q = 1$ is equivalent to the original power law form of the Cardassian model [5], and the case $q = 1$ and $n = 0$ is equivalent to Λ-CDM with the second term representing the vacuum component.

Taking the geometry to be flat, the critical density condition equation (14) gives

$$\tilde{\rho}_{\mathrm{c}} = \rho_{\mathrm{c}} \left[1 + \left(\frac{\rho_0}{\rho_{\mathrm{card}}} \right)^{q(n-1)} \right]^{-1/q} . \tag{16}$$

Here the critical density can be smaller than in the original Friedmann case. We will choose the parameters so that matter alone is sufficient to provide today's critical density, without any vacuum contribution, $\rho_{0\mathrm{m}} \approx \rho_0 = \tilde{\rho}_{\mathrm{c}}$, so that

$$\Omega_{0\mathrm{m}} \equiv \frac{\rho_{0\mathrm{m}}}{\rho_{\mathrm{c}}} = \frac{\tilde{\rho}_{\mathrm{c}}}{\rho_{\mathrm{c}}}. \tag{17}$$

The numerical value of $\Omega_{0\mathrm{m}}$ will be roughly $1/3$ when matching this model to the CMB data (see the discussion in the results section below). The condition in equation (17) is satisfied for parameters satisfying

$$\rho_{\mathrm{card}} = \rho_0 \left[\Omega_{0\mathrm{m}}^{-q} - 1 \right]^{1/q(1-n)} , \tag{18}$$

or, equivalently,

$$1 + z_{\mathrm{card}} = \left[\Omega_{0\mathrm{m}}^{-q} - 1 \right]^{1/3q(1-n)} . \tag{19}$$

After this redshift, the second term on the RHS of equation (15) begins to dominate and (for $n < 2/3$) the universal expansion will accelerate (even without a vacuum component).

With the previous constraints, we may rewrite equation (15) as

$$H = H_0 E_{\mathrm{C}}(z, q, n) \tag{20}$$

$$E_{\mathrm{C}}(z)^2 = \Omega_{0\mathrm{r}}(1 + z)^4 + \Omega_{0\mathrm{m}}(1 + z)^3 + \Omega_{0\mathrm{X}} f_{\mathrm{X}}(z) \tag{21}$$

where (treating $\Omega_{0\mathrm{r}}$ as negligible)

$$\Omega_{0\mathrm{X}} = 1 - \Omega_{0\mathrm{m}} \tag{22}$$

$$f_{\mathrm{X}}(z) \equiv \frac{(1 + z)^3}{\Omega_{0\mathrm{m}}^{-1} - 1} \left\{ \left[1 + (\Omega_{0\mathrm{m}}^{-q} - 1)(1 + z)^{3q(n-1)} \right]^{1/q} - 1 \right\} \tag{23}$$

and we now have only two free parameters, q and n.

Note that at high redshifts z, $E_{\mathrm{C}}(z, q, n) \to E_{\mathrm{F}}(z)$ for all allowable choices of q and n, so at early times, $E_{\mathrm{C}}(z, q, n)$ is equivalent to the Friedmann case (compare equation (3) with equation (20)). Consequently, the evolution of the universe before and around the period of decoupling ($z > 1000$) is the same in the Cardassian model as in the Friedmann model, so ordinary nucleosynthesis and oscillations in the photon–baryon fluid are unaffected.

The age of the universe may be derived in the same manner as in the standard Friedmann model, and we find

$$t_0 = H_0^{-1} \int_0^\infty \frac{\mathrm{d}z}{(1+z)} \frac{1}{E_{\mathrm{C}}(z,q,n)}. \tag{24}$$

This integral does not in general have a closed form and must be evaluated numerically.

Note the CMB peak amplitudes depend upon the parameters ω_{m} and ω_{b}, but are essentially independent of the late time expansion. Thus, the WMAP fits to these two parameters are valid—the Cardassian model does not affect their determination from these peak amplitudes. The values for $\Omega_{0\mathrm{m}}$ and $\Omega_{0\mathrm{b}}$ are not fixed, however, as h is not independently determined from the amplitudes. Instead, h may be determined from the peak positions, expressed by ℓ_{A}, which also depends upon the expansion model. In our analysis, we may fix ω_{m}, ω_{b}, and ℓ_{A} to the WMAP observed values and proceed as before.

Since the sound horizon s in equation (8) is evaluated at $z > 1000$, its value is unaffected by Cardassian modifications which are important only at late times. Hence s is essentially independent of q and n; additionally it has only a minor dependence on h. For a given ω_{b} and ω_{m}, s varies by less than 0.1% over the entire allowed parameter space. However, the distance to the surface of last scattering D is evaluated over a period when the Cardassian modifications are important so that D has a non-trivial dependence on q and n.

Since the CMB peaks are generated prior to and during the time of last scattering, when the usual Friedmann approximation holds, ℓ_{A} remains the same, independently of the Cardassian parameters q and n. Hence ℓ_{A} is fixed by the WMAP data, having the same value in the Cardassian model as in the standard Friedmann case. Due to equation (7), the fact that ℓ_{A} and s are unchanged relative to the standard Λ model constrains the value of D to be unchanged as well. However, since D depends non-trivially on q, n, and h, as described in the previous section, we must allow all three parameters to vary. In particular, the value of h must be allowed to vary in such a way that D remains constant; i.e.,

$$h \equiv h(\omega_{\mathrm{m}}, \omega_{\mathrm{b}}, \ell_{\mathrm{A}}, q, n). \tag{25}$$

That is, for WMAP observed values for ω_{m}, ω_{b}, and ℓ_{A}, and for chosen values of the parameters q and n, numerical routines can be used to find the value of h necessary to satisfy $\ell_{\mathrm{A}} = \pi D/s$. The chosen parameters q and n and the numerical solution for h may then be used to determine the age of the universe using equation (24).

5. Results

We have obtained the age of the universe appropriate to the MP Cardassian modification to the Friedmann equation given in equation (15). Our results are displayed in figures 1 and 2. Since the best fit value of the Hubble constant now depends on the new parameters n and q, we present a contour plot of h over this parameter space in figure 1. A contour plot of the age is shown in figure 2. The contours were generated using the central values for ω_{m}, ω_{b}, and ℓ_{A} observed by WMAP; uncertainties for several choices of the parameters are shown in table 2.

Constraints from SNe Ia data on the free parameters q and n, which were obtained by Wang *et al* [8], are illustrated in both the h and age contour plots. These constraints
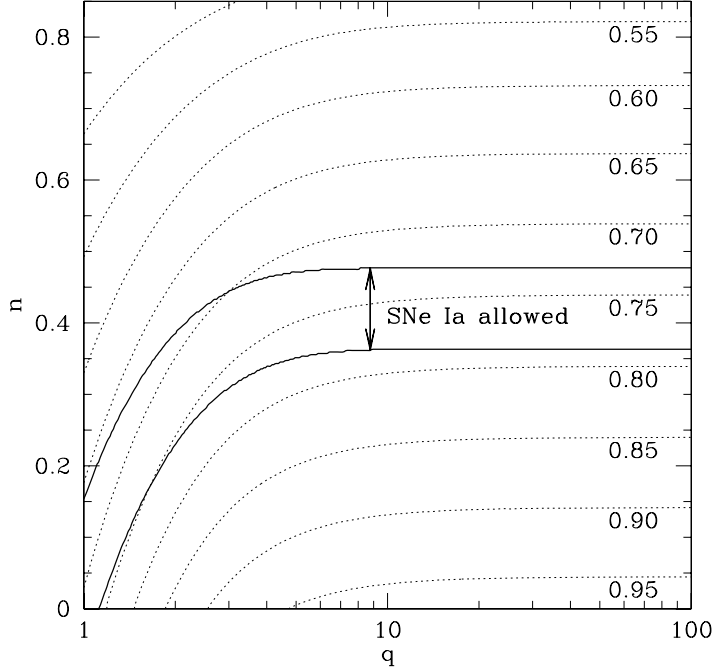
Figure 1. Hubble constant h contours (in units of 100 km s^{-1} Mpc^{-1}) for various values of q and n in the MP Cardassian model (defined in equation (15)), based upon central values of WMAP data. Uncertainties associated with several cases may be found in table 2. The solid lines show the supernovae Ia allowed region of parameter space determined by Wang *et al* [8].

Table 2. Values for the Hubble constant h and age of the universe t_0 for selected SNe Ia allowed parameters in the Cardassian model [8].

q	n	h	t_0 (Gyr)
1	0	0.71 ± 0.04	13.7 ± 0.2
1.5	0.2	0.72 ± 0.04	13.6 ± 0.2
2	0.3	0.72 ± 0.03	13.6 ± 0.2
10	0.4	0.76 ± 0.03	13.4 ± 0.2
100	0.4	0.77 ± 0.03	13.4 ± 0.2

were derived using fixed $\Omega_{0m} = 0.3$; recall, however, that we have fixed ω_m in our analysis, in which case Ω_{0m} varies. It would likewise be more appropriate to fix ω_m for deriving SNe Ia constraints. The constraints in this case, though, would not significantly differ. For $n \approx 0$ and $q \approx 1$, the MP Cardassian model is nearly equivalent to the Λ-CDM model and $\Omega_{0m} = 0.3$ is valid; hence the SNe Ia constraints in this region should remain valid. For $q \gg 1$, the luminosity distances to the supernovae are essentially independent of Ω_{0m} and the constraints shown in this region are also valid[3]. With the fixed Ω_{0m} constraints

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[3] From equations (21)–(23), one can show for $q \gg 1$ that $E_C(z)^2 \to (1+z)^{3n}$, independently of Ω_{0m}, when $\Omega_{0m}(1+z)^{3(1-n)} < 1$. For the $z < 1$ redshifts of the SNe Ia measurements and n and Ω_{0m} values of the SNe Ia constraints, the above constraint is satisfied (note the above limit is *not* valid for much larger z). The luminosity distance to the SNe, $d_L(z) = (1+z) \int_0^z dz' [E(z')]^{-1}$, is therefore independent of Ω_{0m}.
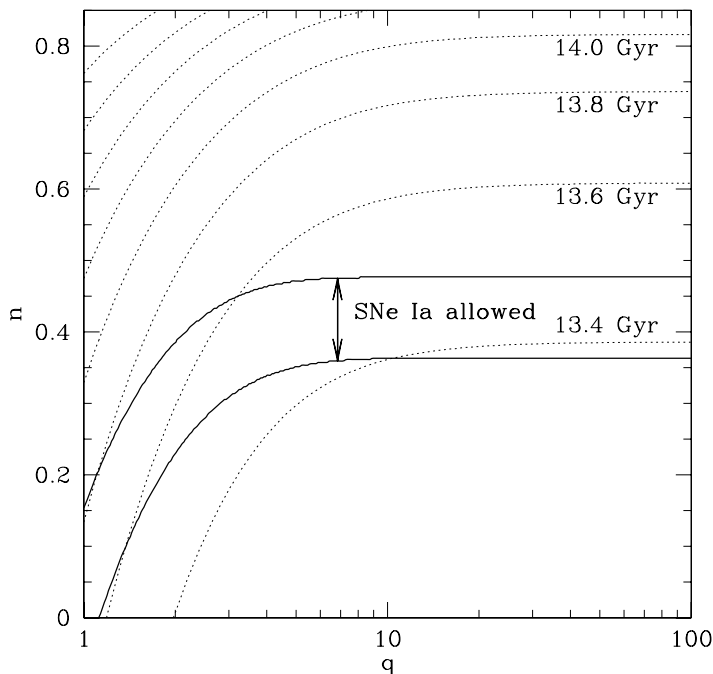
Figure 2. Age of the universe contours (in Gyr) for various values of q and n in the MP Cardassian model (defined in equation (15)), based upon central values of WMAP data. Uncertainties and the SNe Ia allowed region are as described in figure 1.

valid for $q \approx 1$ and $q \gg 1$, the intermediate q constraints are unlikely to be significantly different from the fixed ω_{m} values (the constraints would otherwise have to exhibit fairly pathological behaviour). So while the SNe Ia constraints shown here are not rigorously correct for our analysis, they are fairly close to the 'true' constraints and should still be valid.

Other comparisons of the MP Cardassian model to observational data have also been made [8], [22]–[27]. In addition to the SNe Ia data, Wang *et al* determined constraints on the q–n parameter space from the shift in the CMB angular spectrum [8]; these constraints are weaker than those of the SNe Ia data, except at $q < 2$, where the upper bound on n is somewhat reduced (see figure 2 of their paper). Amarzguioui *et al* have combined CMB and SNe Ia measurements to determine constraints on the parameter space very similar to the Wang *et al* SNe Ia constraints, but with a preference for lower values of q [22] (see figure 3 of their paper; the 1σ contour gives $q \lesssim 4$). Note: because the CMB spectrum was included with the SNe Ia observations, the concerns over the validity of the Wang *et al* limits discussed in the previous paragraph do not apply to the limits given by Amarzguioui *et al*.

Values for h and t_0 are shown in table 2 for several cases of q and n lying within the SNe Ia allowed region. The case $q = 1$ and $n = 0$, which is equivalent to the Λ-CDM model, reproduces the WMAP results with a Hubble parameter h of 0.71 ± 0.04 and an age of 13.7 ± 0.2 Gyr.

Even over the entire parameter space that we considered for the MP Cardassian model (with q ranging from 1 to 100), the age varies very little, ranging from 13.2 to 14.6 Gyr.

The SNe Ia allowed band roughly follows the contours and contains values ranging from 13.4 to 13.8 Gyr. This range is consistent with other constraints on the age [28]–[32]; in particular, it is above the minimum of $t_0 > 12.6^{+3.4}_{-2.4}$ Gyr determined from globular cluster ages by Krauss and Chaboyer [33] as well as a minimum of $t_0 > 12.5 \pm 3$ Gyr determined from radioisotope studies by Cayrel *et al* [32].

While the CMB determination of h depends on the overall model, other techniques measure its value more directly. The Hubble Space Telescope (HST) Key Project analysed Cepheids over distances of 60–400 Mpc to determine a value of 0.72 ± 0.08 [34]. Measurements over such short distances are not significantly affected by the variations in the cosmological model. While the value of h obtained in figure 1 does vary significantly over the entire allowed ($n < 2/3$) parameter space, ranging from 0.50 to 0.97, we can see that the SNe Ia allowed band roughly follows the contours and contains values between 0.66 and 0.78. This allowed band is thus consistent with the HST results. Thus we find the MP Cardassian model to be in general agreement with other independent determinations and limits for the Hubble constant and age of the universe.

We now have estimates of the matter density in these models as well. From the value of $\omega_{\rm m}$ measured by the CMB (given in table 2), together with the estimates of the Hubble constant given in figure 1, we can extract $\Omega_{\rm m}$. For the full $n < 2/3$ parameter space discussed in the last paragraph, we find $0.14 \leq \Omega_{\rm 0m} \leq 0.54$. The range is far more restricted if we consider only those values consistent with the SNe Ia allowed band: $0.22 \leq \Omega_{\rm 0m} \leq 0.31$; this result is in agreement with that of Amarzguioui *et al* [22] ($\Omega_{\rm 0m} \sim 0.3$ in their analysis).

In summary, we have shown how to determine the age of the universe in a MP Cardassian model from CMB data, which requires the appropriate choice of CMB derived parameters. We find that the CMB implied age is entirely consistent with other observational constraints for essentially the entire MP Cardassian parameter space examined here (alternatively, observational constraints on the age do not constrain the Cardassian parameter space). When restricting the parameter space to that consistent with SNe Ia measurements [8], the age of the universe falls somewhere between 13.4 and 13.8 Gyr, a range that includes the age in a Λ-CDM model.

Acknowledgments

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