The meaning of group delay in barrier tunnelling: a re-examination of superluminal group velocities

Herbert G Winful

Department of Electrical Engineering and Computer Science,
University of Michigan, 1301 Beal Avenue,
Ann Arbor, MI 48109-2122, USA
E-mail: winful@eecs.umich.edu

Abstract. We show that the group delay in tunnelling is not a traversal time but a lifetime of stored energy or stored probability escaping through both ends of the barrier. Because it is a lifetime associated with both forward (transmitted) and backward (reflected) fluxes, it cannot be used to define a group velocity for forward transit in cases where a wavepacket is mostly reflected. For photonic tunnelling barriers the group delay is identical to the dwell time which is also a property of an entire wavefunction with reflected and transmitted components. Theoretical predictions and experimental reports of superluminal group velocities in barrier tunnelling are re-interpreted.

Contents

1. Introduction

The question of superluminality in barrier tunnelling dates back to MacColl's 1932 paper which noted, on the basis of an approximate solution to the time-dependent Schrödinger equation, that there is no appreciable delay in the transmission of a wavepacket through a potential barrier [1]. Later work by Bohm [2], Eisenbud [3] and Wigner [4] provided a recipe for calculating the delay time in tunnelling: namely that the energy derivative of the transmission phase shift yields the delay time, known variously as the phase time, Wigner time, or group delay. Hartman used this recipe to calculate the delay in tunnelling through a rectangular barrier and found that the delay time for a tunnelling particle is shorter than the delay time for a particle traversing the same distance in a barrier-free region [5]. Furthermore, in what has become known as the Hartman effect [6], Hartman showed that the tunnelling delay time saturates with increasing barrier length [5]. If one assumes that the group delay in tunnelling is a transit time, then division of the barrier length L by the delay time τ_g yields a tunnelling group velocity $v_g = L/\tau_g$ [6, 7]. The result of this exercise suggests that the group velocity in tunnelling can become arbitrarily large, and indeed superluminal, as the barrier length increases, while the delay stays fixed [6, 7]. Such a result is not an artefact arising from the use of the non-relativistic Schrödinger equation but is also present in the fully relativistic Dirac equation [8]. The presence of these rather large tunnelling velocities occasions some discomfort as relativity forbids massive particles to travel faster than the speed of light in vacuum.

To test these predictions of quantum theory, experiments were proposed in the early 1990s using electromagnetic waves which can tunnel as evanescent waves through forbidden regions in a manner analogous to that of quantum wavepackets [9]–[11]. The philosophy behind these tests is the identity between the time-independent Schrödinger equation for quantum particles and the Helmholtz equation for electromagnetic waves. These experiments, carried out by many [12]–[19], all show the result that the group delay for the tunnelling pulse is shorter than that of a non-tunnelling pulse traversing the same distance. They also confirm that the group delay indeed describes the arrival of the peak of the tunnelled pulse and that it saturates with barrier length. Indeed, even tunnelling sound waves manifest similar anomalous delays [20, 21] (phenomena we might term supersonic), thus showing the generality of these phenomena as a consequence of wave behaviour. These experiments, and their agreement with the predictions of MacColl and Hartman, have led to the conclusion that the process of barrier tunnelling is, in some sense, superluminal [6]–[8], [12]–[14], [16]–[18].

In recent papers, we have suggested that the calculated and measured group delays in tunnelling are not transit times [22]–[27]. This has made it possible to explain the Hartman effect as arising from the saturation of stored energy [22] or the number of particles under the barrier [24]. In this paper, we expand on this suggestion and demonstrate explicitly that the theoretical and measured delays in barrier tunnelling are cavity lifetimes that describe the leakage of energy (or integrated probability density) from both ends of the barrier. In other words, the barrier acts as an evanescent mode cavity with a finite lifetime. We prove the identity between the tunnelling group delay and cavity lifetime and support this proof with numerical simulations of energy decay within the barrier. Because group delay describes a simultaneous escape process through both transmission and reflection channels, it should not be used as a measure of forward traversal time in barrier tunnelling. We show that it is a property of an entire wavefunction with transmitted and reflected components, a status it shares with the dwell time, which likewise cannot be interpreted as a transit time or used to compute a tunnelling velocity [30]. As noted by Büttiker and Landauer

[31] of the dwell time, 'This time is the average dwell time of a particle in the barrier, and is not the traversal time, if most particles are reflected'. For photonic barriers the dwell time and group delay are both the length of time the incident photon flux has to act to build up the accumulated photon density or stored energy within the barrier. Under quasi-static conditions, this time is also the cavity lifetime, the lifetime of stored energy escaping through both ends of the barrier. Our work provides an alternative approach to understanding paradoxically short group delays in tunnelling without appealing to superluminal velocities. Other attempts to explain the anomalously short group delays invoke pulse reshaping [32]–[35] or weak measurements [36, 37]. In our approach, since the group delay is not a transit time from input to output, issues of superluminality, causality, or the speed of information transfer [38] do not even arise.

Our focus is on wavepackets that tunnel without distortion, albeit with significant attenuation as a result of reflection. (We are concerned here only with *tunnelling*, i.e. transport in classically forbidden or evanescent regions and do not consider the negative group velocities seen with allowed propagation in gain media [39, 40].) As we have shown [23, 25], true tunnelling is a quasi-static phenomenon requiring pulses whose spatial extent exceeds the length of the barrier. The quasi-static approximation says that the pulse temporal envelope changes very slowly in the time it takes a light front travelling at c to traverse the length of the barrier. As a result, at any instant in time, near steady-state conditions obtain.

2. Group delay, flux delays and dwell time

A barrier acts as a filter, rejecting some frequency components and transmitting others. It also imparts a phase shift to each frequency component. The barrier is thus characterized by a complex transmission coefficient $|T(\omega)|e^{i\phi_t(\omega)}$ and a complex reflection coefficient $|R(\omega)|e^{i\phi_r(\omega)}$. These are steady-state concepts that assume that the incident wave has existed long enough for the interferences that give rise to selective transmission to be established. Because the subject of tunnelling time is still controversial [27, 35], we will be purposely redundant in reviewing certain known properties of filters and time delay [41] as they pertain to tunnelling. We will then introduce a new relation (equations (13) and (15)) between group delays, dwell time and flux delays that demonstrates the status of group delay in tunnelling as a measure of bidirectional energy transport. While the results apply equally to electromagnetic and quantum tunnelling, we couch the description in terms of electromagnetic waves since the most convincing tunnelling time experiments have been done with those waves.

If the Fourier transform of an incident time-dependent field $E_i(t)$ is $E_i(\omega)$, the transform of the transmitted field is

$$E_t(\omega) = E_i(\omega)|T(\omega)| \exp i\tilde{\phi}_t(\omega), \tag{1}$$

where $\tilde{\phi}_t(\omega) = \phi_t(\omega) + k(\omega)L$ and k is the wavenumber. Similarly the transform of the reflected field is

$$E_r(\omega) = E_i(\omega)|R(\omega)| \exp i\phi_r(\omega). \tag{2}$$

Suppose the incident field is sufficiently narrowband that the magnitudes of the reflection and transmission coefficients are constant over the bandwidth of the pulse. Expanding the phase

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terms to first order about the centre frequency ω_0, we have

$$\phi_r(\omega) \approx \phi_r(\omega_0) + (\omega - \omega_0)\phi_r'(\omega_0) + O(\omega^2), \tag{3a}$$

$$\tilde{\phi}_t(\omega) \approx \tilde{\phi}_t(\omega_0) + (\omega - \omega_0)\tilde{\phi}_t'(\omega_0) + O(\omega^2). \tag{3b}$$

The transmitted field in the time domain is

$$E_t(t) = |T(\omega_0)| \exp i[\tilde{\phi}_t(\omega_0) - \omega_0\tilde{\phi}_t'(\omega_0)] \int_{-\infty}^{\infty} E_i(\omega) \exp\left[-i(\omega[t - \tilde{\phi}_t'(\omega_0))]\right] d\omega$$

$$= |T(\omega_0)| \exp i[\tilde{\phi}_t(\omega_0) - \omega_0\tilde{\phi}_t'(\omega_0)]E_i(t - \tau_{gt}), \tag{4}$$

where $\tau_{gt} = \tilde{\phi}_t'$ is the transmission group delay. Similarly, the reflected field is given by

$$E_r(t) = |R(\omega_0)| \exp i[\phi_r(\omega_0) - \omega_0\phi_r'(\omega_0)]E_i(t - \tau_{gr}), \tag{5}$$

where $\tau_{gr} = \phi_r'$ is the reflection group delay. A filter with a constant amplitude transmission and a linear phase shift over the pulse bandwidth thus gives rise to a pure time delay without distortion. Any distortion or reshaping will have to come from the neglected higher order terms. Thus 'reshaping' cannot be seen as a mechanism for pure time delay. Since the power carried by the wave is $P(t) \propto |E(t)|^2$, for narrowband pulses the reflected and transmitted pulses are simply delayed and attenuated versions of the incident pulse

$$P_t(t) = |T|^2 P_i(t - \tau_{gt}), \tag{6a}$$

$$P_r(t) = |R|^2 P_i(t - \tau_{gr}). \tag{6b}$$

The group delays can be related to power flow and stored energy in the barrier [25, 26]. For the electromagnetic barrier, Poynting's theorem provides a local statement of continuity of energy density and power flow

$$\nabla \cdot \mathbf{S}(\mathbf{r}, t) = -\frac{\partial u(\mathbf{r}, t)}{\partial t}. \tag{7}$$

Here, $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ (W m^{-2}) is the Poynting vector and $u(\mathbf{r}, t) = (\varepsilon\mathbf{E} \cdot \mathbf{E} + \mu\mathbf{H} \cdot \mathbf{H})/2$ is the electromagnetic energy density. For quantum wavepackets, a similar continuity relation holds between the probability current $j(r, t)$ and the probability density $\rho(x, t)$

$$\nabla \cdot \bar{j} = -\frac{\partial \rho}{\partial t}.$$

The integral form of Poynting's theorem then tells us that

$$-\oint_s \mathbf{S} \cdot \hat{\mathbf{n}} \, da = \frac{dU}{dt}, \tag{8}$$

with $U = \int_{vol} u \, dv$. Here, for the one-dimensional system, the surface integral on the left-hand side reduces to an integration over the input and output faces of the barrier. Thus evaluation of

the surface integral in (8) yields

$$P_i(t) - P_r(t) - P_t(t) = \frac{dU}{dt}. \tag{9}$$

This relation simply says that the rate of increase of stored electromagnetic energy in the cavity is given by the incident power (P_i) minus the sum of the transmitted (P_t) and reflected (P_r) power. This equation represents a non-local relationship between incident and reflected fields on the one hand as measured at the input plane $z = 0$ and the transmitted field measured at the exit plane $z = L$, all measurements taken at the same instant t.

The group delay in transmission (τ_{gt}) has the well-defined meaning of the time at which a pulse peak appears at $z = L$, given that the peak of the incident pulse is at $z = 0$ at $t = 0$. Similarly, the group delay in reflection (τ_{gr}) is the time at which a reflected pulse peak appears at $z = 0$, given that the peak of the incident pulse is at $z = 0$ at $t = 0$. Because of the absence of pulse distortion, the reflected and transmitted pulses are simply delayed, attenuated versions of the incident pulse, with possibly different delays. In practice, these pulses are measured some distance away from the barrier, but assuming free propagation outside the barrier region, the peaks can always be extrapolated to the boundaries of the barrier. To first order in the delay, assumed much smaller than the pulse width, we can write

$$P_i(t - \tau) \approx P_i(t) - \tau \, dP_i/dt.$$

Equation (9) then becomes

$$P_i(t) - |R|^2 \left[P_i(t) - \tau_{gr} \frac{dP_i}{dt} \right] - |T|^2 \left[P_i(t) - \tau_{gt} \frac{dP_i}{dt} \right] = \frac{dU}{dt}. \tag{10}$$

Since $|R|^2 + |T|^2 = 1$ for a lossless barrier, equation (10) reduces to

$$[|R|^2 \tau_{gr} + |T|^2 \tau_{gt}] \frac{dP_i}{dt} = \frac{dU}{dt}. \tag{11}$$

Upon integrating both sides of equation (11) and setting the arbitrary constant equal to zero, we obtain the result

$$|R|^2 \tau_{gr} + |T|^2 \tau_{gt} = \tau_d \equiv \frac{U}{P_i}. \tag{12}$$

The last identity above defines the dwell time τ_d. The relation between the dwell time and the weighted sum of certain transmission and reflection delays is one that is often postulated as a criterion for the validity of those delays [30]. It has also been criticized as not being justified for quantum tunnelling since it neglects interference terms which are significant at low particle energies [42]. For the photonic barrier in the slowly varying approximation, those interference terms disappear and thus (12) is an exact relation. For quantum tunnelling the interference terms also disappear if the delays are averaged over the wave number spectrum of the incident wavepacket [43].

The dwell time is a weighted average of reflection and transmission delays and does not distinguish which of the two channels the energy stored from the input flux escapes through [30]. Obviously, since it describes an escape process through *both* channels, it cannot be ascribed to

only the transmitted pulse nor used to assign a forward traversal velocity whenever there is some reflection. Upon dividing equation (12) by τ_d, we obtain the relation

$$1 = \frac{\tau_{gr}}{\tau_r} + \frac{\tau_{gt}}{\tau_t}, \tag{13}$$

where the transmitted flux delay is defined as

$$\tau_t = \frac{\tau_d}{|T|^2} = \frac{U}{P_t}, \tag{14a}$$

and the reflected flux delay is defined as

$$\tau_r = \frac{\tau_d}{|R|^2} = \frac{U}{P_r}. \tag{14b}$$

The transmitted flux delay is the time it takes for all the stored energy in the barrier to leave the barrier in the direction of the transmitted flux. The reflected flux delay is the time it takes for all the stored energy to leave in the direction of the reflected flux.

Equations (14) provide one way to distribute the dwell time between the reflection and transmission channels since it follows from the conservation law $|R|^2 + |T|^2 = 1$ that $1/\tau_r + 1/\tau_t = 1/\tau_d$ [44]. For a symmetric barrier it can be shown that $\tau_{gr} = \tau_{gt} \equiv \tau_g$ [30]. In that case equation (13) reduces to

$$\frac{1}{\tau_g} = \frac{1}{\tau_r} + \frac{1}{\tau_t}, \tag{15}$$

and equation (12) becomes

$$\tau_g = \tau_d. \tag{16}$$

Thus as we have shown previously [22], the group delay and dwell time are identical for a symmetric photonic barrier. Equation (15) says that the sum of the rate of escape through the reflection channel and the transmission channel equals the inverse of the group delay. The group delay therefore represents the time it takes to empty the barrier through *both* channels.

3. Group delay equals cavity lifetime

An important point we wish to make is this: in the presence of reflections the group delay and the dwell time relate to the simultaneous escape of energy through both ends of the barrier. Neither of these times can be assigned to just the transmitted pulse or just the reflected pulse, in the sense of the time it takes a well-defined pulse to travel from A to B. Indeed, the group delay is just the lifetime of stored energy escaping through both ends of the barrier. It is a cavity lifetime.

To see the connection to cavity lifetime, first recall the standard definition of the Q of a cavity [45]

$$Q = \omega \frac{U}{P_d}, \tag{17}$$

which is the time average stored energy divided by the power dissipated per cycle P_d. As a result of dissipation, the cavity mode has a finite lifetime, a $1/e$ lifetime of stored energy which is given by [45]

$$\tau_c \equiv \frac{Q}{\omega} = \frac{U}{P_d}. \tag{18}$$

For a cavity with no internal losses the power dissipated is the power that escapes through the ends of the cavity. At steady state, this power lost equals the incident power, or $P_d = P_i$. Thus the cavity lifetime can be written

$$\tau_c = \frac{U}{P_i} = \tau_d, \tag{19}$$

which shows that the cavity lifetime and the dwell time are identical. Furthermore, for a symmetric barrier all three quantities, the group delay, the dwell time, and the cavity lifetime are one and the same object.

As an important example, we consider a one-dimensional photonic bandgap structure with a refractive index profile $n(z) = n_0 + n_1 \cos(2n_0\omega_B z/c)$, where ω_B is the Bragg frequency and n_1 is the amplitude of the index perturbation. For convenience, we collect some known results [22]. The envelopes of the forward and backward fields propagating inside the structure satisfy the coupled-mode equations [46]

$$\frac{\partial E_F}{\partial z} + \frac{1}{v}\frac{\partial E_F}{\partial t} = i\kappa E_B e^{-i2\Delta\beta z}, \tag{20a}$$

$$\frac{\partial E_B}{\partial z} - \frac{1}{v}\frac{\partial E_B}{\partial t} = -i\kappa E_F e^{i2\Delta\beta z}. \tag{20b}$$

Here, $\kappa = n_1 n_0 \omega_B / 2c$ is a coupling constant related to the strength of the refractive index perturbations, $\Delta\beta = n_0\Omega/c$, $\Omega = \omega - \omega_B$ and $v = c/n_0$. The steady-state solutions are

$$E_F(z) = E_0[\gamma \cosh\gamma(z - L) + i(\Omega/v)\sinh\gamma(z - l)]/g, \tag{21a}$$

$$E_B(z) = -i[E_0\kappa \sinh\gamma(z - L)]/g, \tag{21b}$$

where $\gamma = \sqrt{\kappa^2 - (\Omega/v)^2}$ and $g = \gamma\cosh\gamma L - i(\Omega/v)\sinh\gamma L$. The barrier amplitude transmission coefficient is $T = E_F(L)/E_o = (\gamma/|g|)e^{i\phi}$, the phase of which is given by

$$\phi_t = \tan^{-1}[(\Omega/\gamma v)\tanh\gamma L].$$

For an incident power

$$P_i = (1/2)\varepsilon_0 n_0 c |E_0|^2 A, \tag{22}$$

the stored energy in the barrier is

$$U = U_0\left[\frac{(\kappa^2/\gamma^2)(\tanh\gamma L)/\gamma L - (\Omega/\gamma v)^2 \sec h^2\gamma L}{1 + (\Omega/\gamma v)^2 \tanh^2\gamma L}\right], \tag{23}$$
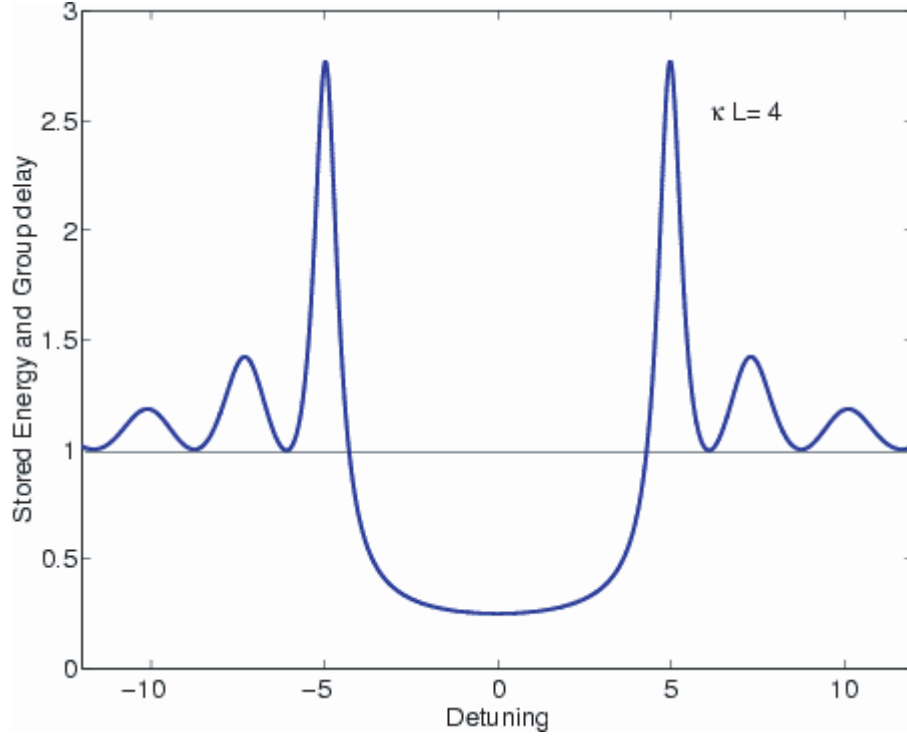
Figure 1. Normalized group delay, dwell time, cavity lifetime, and stored energy versus frequency detuning $\Omega L/v$ for a symmetric photonic bandgap structure.

where

$$U_0 = \tfrac{1}{2}\varepsilon_0 n_0^2 E_0^2 AL \tag{24}$$

is the energy stored in a barrier-free region of the same length. From the expressions for stored energy and input power we calculate the group delay,

$$\tau_g = \frac{U}{P_i} = \tau_0 \left[\frac{(\kappa^2/\gamma^2)(\tanh \gamma L)/\gamma L - (\Omega/\gamma v)^2 \sec \mathrm{h}^2 \gamma L}{1 + (\Omega/\gamma v)^2 \tanh^2 \gamma L} \right] = \frac{\mathrm{d}\tilde{\phi}_t}{\mathrm{d}\Omega}, \tag{25}$$

where $\tau_0 = L/v$. Figure 1 shows the normalized stored energy and group delay for a photonic bandgap structure as a function of detuning. The group delay is also the frequency-dependent cavity lifetime. Indeed at resonances of the barrier, where the transmission is unity, the expression for group delay reduces to

$$\tau_g = \frac{L}{v}\left[1 + \left(\frac{\kappa L}{m\pi}\right)^2\right], \tag{26}$$

which has the L^3 dependence first postulated for the cavity lifetime of distributed feedback lasers by Chinn [47]. Near these transmission resonances, constructive interference of the forward scattered wavelets results in an enhancement of the stored energy over the free-propagation value. Consequently, the group delay is greater than the free-propagation value in the vicinity of those resonances. Conversely, within the stop band, the interferences that give rise to coherent Bragg
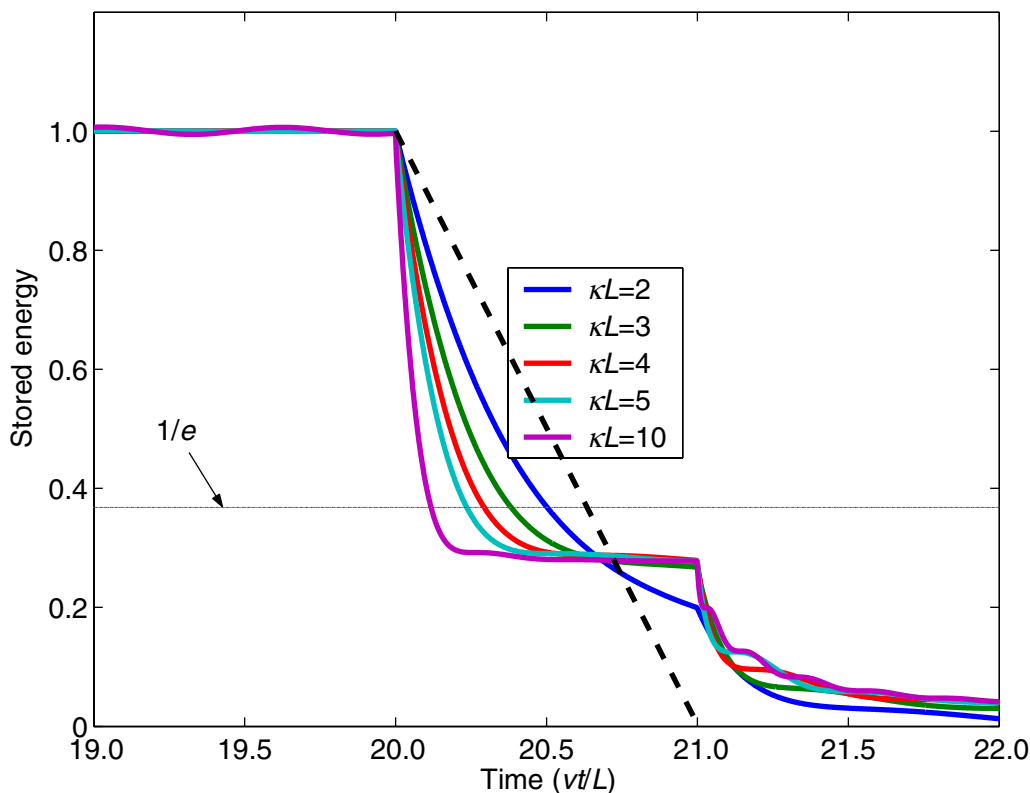
Figure 2. Decay of the normalized stored energy versus time. The steady state incident beam is turned off at $vt/L = 20$. Here $\Omega = 0$. The black dashed line shows the decay of stored energy in the barrier-free case.

reflection result in a suppression of the stored energy below the value it would have had in the absence of the barrier. It is within this band that the group delay is less than the free-propagation value. It reaches a minimum value at midgap of

$$\tau_{\mathrm{g}} = \frac{L}{v} \left[\frac{\tanh \kappa L}{\kappa L} \right]. \tag{27}$$

This delay can be arbitrarily short for strong enough barriers.

We will now show, by means of direct numerical solution of the coupled-mode equations, that the group delay is indeed the $1/e$ lifetime of stored energy escaping through both ends of the barrier. The coupled-mode equations for forward and backward waves are integrated for a unit step function input until steady state is reached. The input is then switched off and the evolution of the integrated stored energy is monitored as a function of time. In figure 2, we show the decay of the stored energy for several values of the coupling strength κL. Since the stored energy is numerically equal to the group delay for unit input power, we have normalized the plot of stored energy by the appropriate group delay. Hence what is plotted is $U(t)/\tau_{\mathrm{g}}$. It is seen that the stored energy drops rapidly after turn off of the input. This rapid drop is largely due to the escape of the backward component through the input face of the barrier. Note that in steady state most of the energy is stored near the input end of the barrier (see figure 3(b)). The larger the coupling strength, the smaller the stored energy ($U \sim 1/\kappa L$) and thus the faster decay. The temporal decay
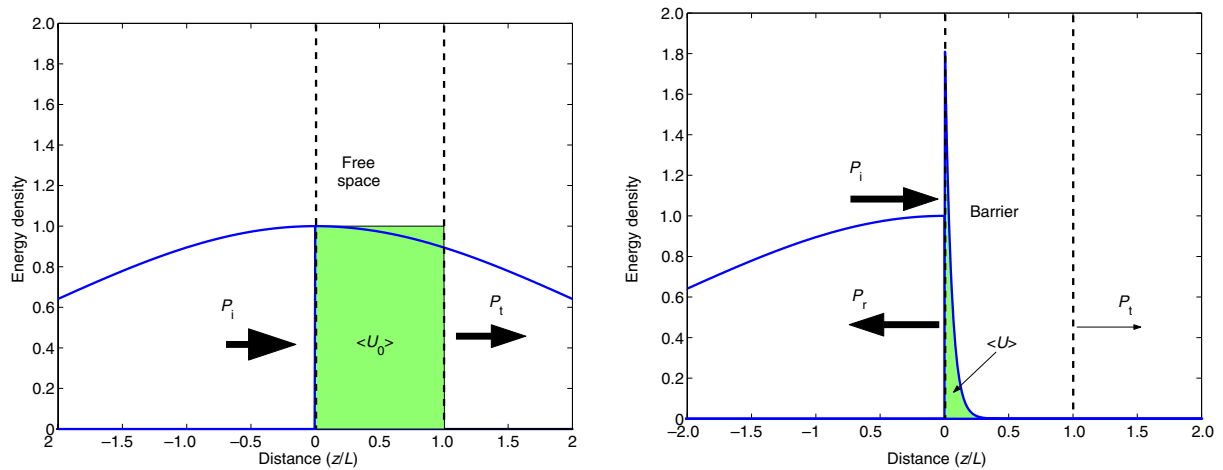
Figure 3. (a) Snapshot of a pulse traversing a region of free space. The shaded area shows the stored energy corresponding to the peak incident power. (b) Snapshot of a pulse interacting with a barrier. The shaded area shows the stored energy corresponding to the peak incident power. In front of the barrier is shown only the incident power but not the reflected power.

in the initial phase is approximately exponential. The thin horizontal line in the figure indicates the $1/e$ level of the stored energy. Upon reading off the values of time at the intersection of this line with the plots of $U(t)$, we find that those times are close to the group delays calculated through equation (27). Thus the group delay is indeed the lifetime of stored energy leaving the cavity through both ends. After the initial rapid drop, there is a plateau which occurs because most of the energy in the backward wave has left, leaving behind the forward going component whose energy is approximately constant as it is no longer coupled to the backward wave. When the front of the forward component leaves the barrier at the exit after one transit time, the stored energy suddenly drops again. We have shown through the definition of Q and through direct numerical simulations that the group delay is the $1/e$ lifetime of stored energy escaping through both ends of the barrier.

4. Reinterpretation of tunnelling experiments

With this interpretation of the group delay as a cavity lifetime, it is now possible to explain every aspect of 'superluminal' tunnelling experiments without ever mentioning the words 'group velocity'. In the typical pulsed tunnelling time experiment, a pulse of electromagnetic energy is sent through a barrier-free region of length L. The spatial extent of the pulse exceeds L. The arrival time of the peak of this pulse at a detector is used as a reference time. A barrier of length L is next inserted in the path of the pulse. The arrival time of the peak of a transmitted pulse is then compared to the reference time. Let the delay in traversing the barrier-free region be τ_0. Let the delay due to the barrier be τ_1. It is important to note that these experiments measure *time delays* (or, in interferometric measurements, mirror *positions* [13]). They do not directly measure velocity. Velocity is an inferred quantity. In order to relate a measured time delay to an inferred group velocity, one must ascertain that the measured delay is a propagation delay that can be assigned to the propagation of a forward pulse through a region.

Consider first the reference pulse. Figure 3(a) shows a snap shot of the energy density in the pulse as a function of position at the instant when the peak of the pulse arrives at the input plane. At that instant the flux of energy crossing that plane is the incident power P_i. Some time later, that flux of energy leaves the region. For the reference pulse, all the entering energy leaves the region at the exit plane $z = L$ since there is no reflection or absorption. The time it takes for all the energy to leave is given by the stored energy divided by the rate at which energy enters: $\tau_0 = U_0/P_i$. The time average stored energy in the transparent region of volume $V = LA$ is just U_0 as given in equation (24). The net energy flux transmitted in the forward direction through this lossless, reflectionless region is just the input power P_i, given in equation (22). Upon dividing U_0 by P_i we get $L/v \equiv \tau_0$, the time it takes for all the energy stored in the region of length L to leave that region in the direction of the net flux and with velocity v. Here, because all the energy that enters leaves later in the forward direction, one can infer a sensible velocity $v = L/\tau_0$.

Now consider a pulse incident on a barrier. It should be noted right away that unlike the case of the reference pulse the incident and transmitted pulses are not the same entity. The incident pulse creates a cavity field which is made up of a sum of forward and backward propagating components that have undergone various amounts of multiple scattering within the barrier. This cavity field then gives rise to a transmitted pulse and a reflected pulse. The transmitted pulse is not the delayed incident pulse. It is the released barrier field. One cannot associate a given temporal point within the transmitted or reflected pulse with a given temporal point in the incident pulse. Again we calculate the stored energy in the barrier at the moment the pulse peak arrives. From figure 3(b), it is clear that this stored energy represented by the shaded area is much smaller than in the case of free space propagation. The energy density decays almost exponentially with distance. It consists of a forward and backward component whose sum, just inside the barrier exceeds the incident power. Most of the stored energy leaves the barrier in the backward direction and a small amount is transmitted in the forward direction. The group delay, the time it takes for this stored energy to escape through both forward and backward channels is $\tau_1 = U/P_i$. Because the stored energy is much smaller than in the free space case, for the same input power the delay time for this stored energy is much less than in the free space case. Note that this delay is not the time it takes for the input peak to propagate to the exit since the pulse does not really propagate through the barrier. What is really measured is the lifetime of stored energy escaping through both ends: the cavity lifetime. This explains why the group delay is shorter for stronger barriers (stronger barriers store less energy), why it saturates with barrier length (stored energy saturates with barrier length), and why it is less than the free space delay (barrier stores less energy than free space for the same input power). Because the group delay is associated with both forward and backward components, it cannot be considered a delay time for forward traverse. Because it is not a delay time for forward transit, it is inappropriate to use this delay to define a group velocity for forward pulse propagation as distance L divided by delay. In short, $v_g \neq L/\tau_g$ when the delay is a delay associated with fluxes propagating in both directions. Even in regions of allowed propagation, when there are reflections and standing waves it becomes problematic to define a velocity of propagation and hence a traversal time. Only when the reflection is zero (as at transmission resonances) can one associate this delay with a forward traversal velocity.

We conclude this section with a look at the experiment of Longhi *et al* [18] which is the cleanest, most quantitative pulsed tunnelling-time measurement on a photonic band gap structure. A 380 ps long pulse is tuned to the stop band of a 2 cm long fibre Bragg grating that transmits 1.5% of the incident power. The pulse is much longer than the 67 ps transit time of a light front traversing that distance at speed c in free space and hence the interaction is quasi-static [23, 25].

By comparing the peak of the weak transmitted pulse with the peak of a reference pulse tuned outside the stop band (barrier-free region), a superluminal group velocity of $1.97c$ was inferred. In our interpretation, we consider the stored energy in the structure for a wave tuned to the centre of the stop band $U = (U_0 \tanh \kappa L)/\kappa L$. The group delay and dwell time for this structure are identical, given by the ratio of stored energy to input power: $\tau_g = \tau_d = U/P_i = \tau_0 (\tanh \kappa L)/\kappa L$. Obviously the ratio of the barrier group delay to barrier-free group delay is just the ratio of the stored energies

$$\frac{\tau_g}{\tau_0} = \frac{U}{U_0} = \frac{\tanh \kappa L}{\kappa L}. \tag{28}$$

where $\tau_0 = L/v$ is the barrier-free delay. The parameters of the experiment [18] yield $v = c/1.452$ and $\kappa L = 2.8$, from which we find that the barrier only stores 35% of the energy that would be stored in an equivalent volume without a barrier (given the same incident power). The calculated group delays are $\tau_0 = 97$ ps and $\tau_g = 34$ ps. It takes 34 ps to empty the energy stored in the barrier through both ends (with most of it leaving in the backward direction) compared to the 97 ps, it takes to empty the energy stored in the equivalent barrier-free region, with all of it exiting in the forward direction. This is all the experiment is saying. It does not imply that anything is being transported through the barrier at faster than the speed of light. This calculation yields a delay time difference of 63 ps which is exactly what is observed in the experiment. The barrier group delay is 35% shorter because it stores 35% less energy than the barrier-free region. This explanation applies to all the reported superluminal tunnelling experiments including the 'single-photon' measurements of Steinberg *et al* [13]. As noted by the authors themselves [7], for purely linear phenomena such as tunnelling, single photons exhibit the same behaviour as classical pulses. 'Propagation effects are then governed by the classical wave equations, and quantization merely affects detection *statistics* and higher order effects' [7].

5. Absence of reshaping

The existence of 'superluminal' group velocities in tunnelling has been attributed to a reshaping phenomenon in which the barrier transmits the early parts of the incident pulse and rejects the later parts, acting in essence as a time-dependent shutter [32]–[35]. This would imply that the transmission of the barrier is a function of time. A barrier whose transmission is time dependent would necessarily distort an incident pulse. However, the exact numerical solutions of the coupled mode equations show that the transmission is the same for all parts of the delayed input pulse, at least over the detectable bulk of the pulse. This constancy of the transmission is also in agreement with experimental observations for pulses whose bandwidth is narrow compared to the stop band of the barrier.

Figure 4(a) shows the incident and transmitted pulses for a long pulse tunnelling through a barrier of strength $\kappa L = 4$. Also shown is a reference pulse traversing a barrier-free region. In figure 4(b), the two normalized pulses are overlaid so that their shapes can be compared. On this scale their shapes are identical. Notwithstanding claims to the contrary, there is no reshaping seen in theory or experiment. For a slowly varying input pulse, every portion of the main part of the transmitted pulse (after an initial transient) is delayed by the same amount from the incident pulse. Any distortion or reshaping is due to the higher order terms in the expansion of the transmission
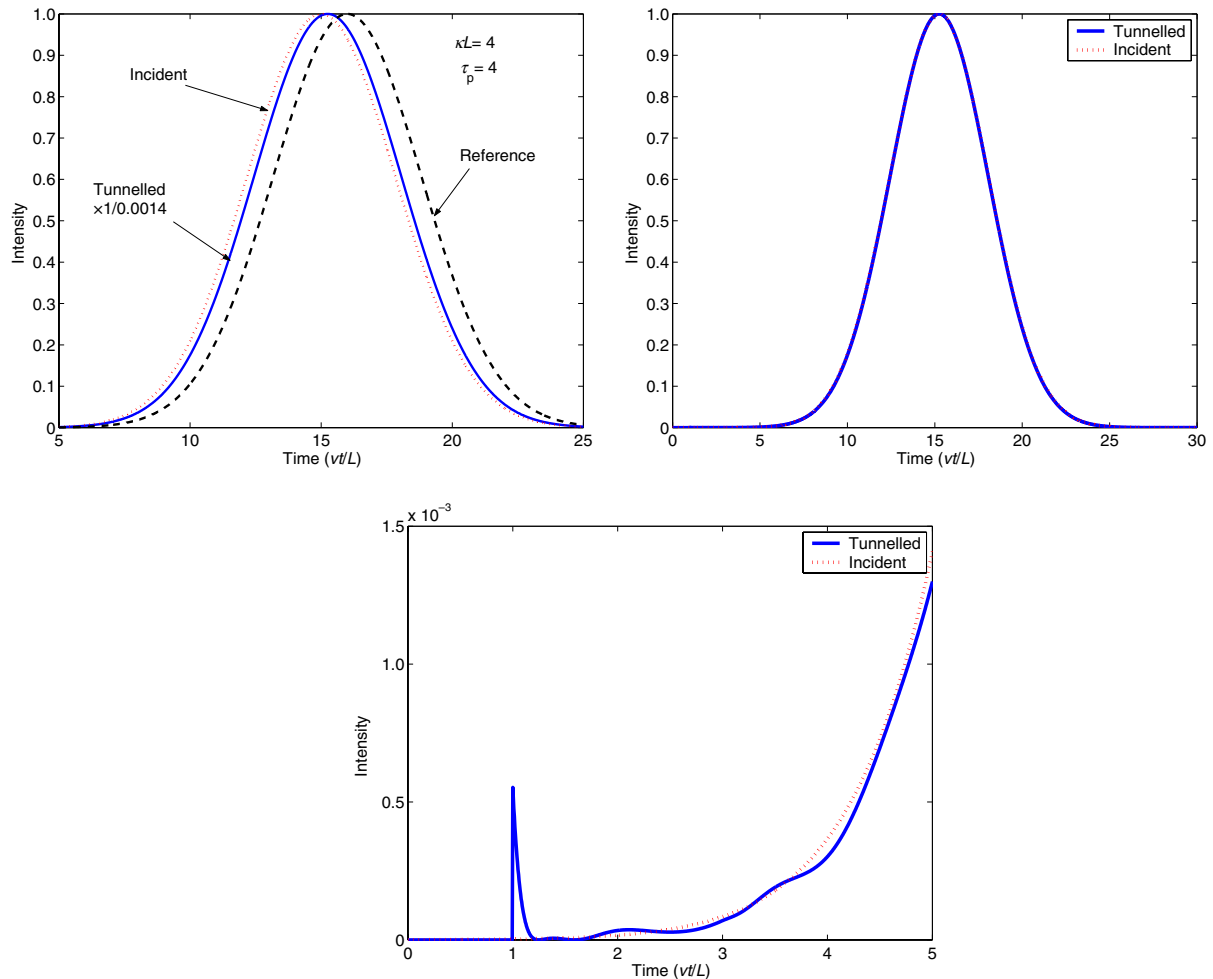
Figure 4. (a) Incident, reference and transmitted pulses for a barrier of strength $\kappa L = 4$. The reference pulse travels the equivalent distance in a barrier-free region. (b) Comparison of incident and transmitted pulse shapes. (c) Enlarged portion of (b) showing the evolution of the pulse front. The tunnelled pulse has been scaled up by a factor of 1/0.0014.

phase. It is not a mechanism for the prompt appearance of the transmitted peak. A pulse that is sufficiently long will not experience any reshaping. Thus 'reshaping' cannot be seen as an essential part of tunnelling dynamics. It is rather a sign of an approximation gone wrong. If the group delay is seen as a lifetime and not a transit time then there is no superluminality to explain away.

It does take time for the barrier reflectivity to build up to its steady-state value. That build up process, however, occurs in the far wings of the input pulse, long before the main part of the pulse arrives. That part of the pulse, the front or 'turn on' part contains the high frequency components which do not tunnel because they lie outside the stop band. That portion is necessarily 'reshaped'. However, that is not what is normally meant by the reshaping of a pulse. Furthermore, that portion has nothing to do with the tunnelling process. Figure 4(c) shows the evolution of the pulse front and the approach to quasi-steady-state transmission. Note that the intensities here are three orders

of magnitude smaller than in figures 4(a) and (b) which show the bulk of the pulse. The front propagates at c and the reflectivity builds up in a couple of transit times.

6. Implications for quantum tunnelling

In the foregoing we have couched the interpretation of tunnelling group delay in terms of electromagnetic wavepackets. Because of the analogy between the Schrödinger and Helmholtz equations, these results also hold for quantum wavepackets. Hence the group delay in quantum tunnelling should also be seen as a lifetime and not a traversal time. The question then is, what is it a lifetime of? For electromagnetic waves for which the slowly varying envelope approximation holds, this lifetime was just the lifetime of stored energy escaping through both ends of the barrier. For quantum wavepackets, the equivalent of stored energy in the barrier is the integral of the probability density in the barrier

$$W = \int_0^L |\psi(x)|^2 \, \mathrm{d}x,$$

where $\psi(x)$ is the wavefunction. For an incident particle flux j_i, the dwell time is given by [28, 29]

$$\tau_\mathrm{d} = W/j_\mathrm{i}.$$

Mathematically, the dwell time is the lifetime of the integrated probability density within the barrier irrespective of the escape channel. Since the integrated probability density is proportional to the number of particles, the dwell time can be also interpreted as the storage time of particles within the barrier, averaged over all incoming particles. If most of the particles are reflected, this lifetime will obviously be very short. For quantum wavepackets, in addition to the dwell time inside the barrier, there is a contribution to the tunnelling delay time that arises from the interference between incident and reflected portions of the wavepacket in front of the barrier. The group delay takes into account this excess dwell time due to self-interference. The general relation between group delays and dwell times is now [24, 43, 48]

$$|R|^2 \tau_\mathrm{gr} + |T|^2 \tau_\mathrm{gt} = \tau_\mathrm{d} + \tau_\mathrm{i},$$

where $\tau_\mathrm{i} = -\mathrm{Im}(R)/kv$ is the self-interference delay. To emphasize its role as an additional dwell time, we include the self-interference delay in an overall dwell time defined as

$$\tilde{\tau}_\mathrm{d} = \tau_\mathrm{d} + \tau_\mathrm{i}.$$

For a symmetric barrier, the group delay is thus seen as the duration of stored probability density (particles) in the barrier plus a contribution from any excess stored density due to interference in front of the barrier. Because the stored probability in front of the barrier arises from the interference between forward and backward waves, its contribution to the delay cannot be associated with either wave separately. The group delay is again a measure of bidirectional fluxes and cannot be associated with either the transmitted or reflected particles separately. At barrier resonances the group delay is just the lifetime of metastable states, whereas for energies below the barrier height one can associate the delay with the lifetime of a highly transient virtual state.

For a wavepacket whose energy spread ΔE is much less than the barrier height V_0, the ratio of the limiting group delay to the wavepacket temporal extent τ_p is given roughly by

$$\tau_\mathrm{g}/\tau_\mathrm{p} = \delta x/\Delta x = \Delta E/V_0 \ll 1.$$

The spatial delay δx caused by the barrier is a very small fraction of the uncertainty in the particle position Δx. Because this shift is such a small fraction of the wavepacket's length, a more meaningful measure of the duration of the tunnelling process is the duration of the wavepacket itself.

7. Conclusions

For many years the anomalously short delays seen with tunnelling quantum wavepackets and electromagnetic pulses have been taken to mean that these entities propagate with superluminal velocity through the barrier. We have now demonstrated that the group delay time in barrier tunnelling is actually the lifetime of stored energy (or stored particles) leaking through both ends of the barrier. The group delay is a property of an entire wavefunction with forward and backward components. Indeed, without the reflected component there would be no net tunnelling flux. Because it represents a bidirectional flow of energy, it cannot be associated with a forward traversal time. We have also shown that 'pulse reshaping' is not the mechanism for the short group delay: every part of the main portion of the pulse suffers the same delay. This interpretation of group delay as a lifetime should help resolve most of the outstanding paradoxes in the physics of tunnelling time [49, 50].

Acknowledgment

References

[1] MacColl L A 1932 *Phys. Rev.* **40** 621
[2] Bohm D 1951 *Quantum Theory* (New York: Prentice-Hall)
[3] Eisenbud L 1948 *Dissertation* Princeton (June)
[4] Wigner E P 1955 *Phys. Rev.* **98** 145
[5] Hartman T E 1962 *J. Appl. Phys.* **33** 3427
[6] Olkhovsky V S and Recami E 1992 *Phys. Rep.* **214** 339
[7] Chiao R Y and Steinberg A M 1997 *Progress in Optics* vol XXXVII, ed E Wolf (Amsterdam: Elsevier) p 345
[8] Krekora P, Su Q and Grobe R 2001 *Phys. Rev.* A **63** 032107
[9] Ranfagni A, Mugnai D, Fabeni P and Pazzi G P 1991 *Appl. Phys. Lett.* **58** 774
[10] Chiao R Y, Kwiat P G and Steinberg A M 1991 *Physica* B **175** 257
[11] Martin Th and Landauer R 1992 *Phys. Rev.* A **45** 2611
[12] Enders A and Nimtz G 1992 *J. Phys. I (France)* **2** 1693
[13] Steinberg A M, Kwiat P G and Chiao R Y 1993 *Phys. Rev. Lett.* **71** 708
[14] Spielmann Ch, Szipöcs R, Stingl A and Krausz F 1994 *Phys. Rev. Lett.* **73** 2308
[15] Balcou P and Dutriaux L 1997 *Phys. Rev. Lett.* **78** 851

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[16] Mojahedi M, Schamiloglu E, Hegeler F and Malloy K J 2000 *Phys. Rev.* E **62** 5758
[17] Carey J J, Zawadzka J, Jaroszynski D A and Wynne K 2000 *Phys. Rev. Lett.* **84** 1431
[18] Longhi S, Marano M, Laporta P and Belmonte M 2001 *Phys. Rev.* E **64** 055602
[19] Hache A and Poirier L 2002 *Appl. Phys. Lett.* **80** 518
[20] Robertson W M, Ash J and McGaugh J M 2002 *Am. J. Phys.* **70** 689
[21] Yang S, Page J H, Liu Z, Cowan M L, Chan C T and Sheng P 2002 *Phys. Rev. Lett.* **88** 104301
[22] Winful H G 2002 *Opt. Express* **10** 1491 (http://www.opticsinfobase.org/abstract.cfm?URI=oe-10-25-1491)
[23] Winful H G 2003 *Phys. Rev. Lett.* **90** 023901
[24] Winful H G 2003 *Phys. Rev. Lett.* **91** 260401
[25] Winful H G 2003 *IEEE J. Sel. Top. Quantum Electron.* **9** 17
[26] Winful H G 2003 *Phys. Rev.* E **68** 016615
[27] Winful H G 2003 *Nature* **424** 638
[28] Smith F T 1960 *Phys. Rev.* **118** 349
[29] Büttiker M 1983 *Phys. Rev.* B **27** 6178
[30] Hauge E H and Støvneng J A 1989 *Rev. Mod. Phys.* **61** 917
[31] Büttiker M and Landauer R 1982 *Phys. Rev. Lett.* **49** 1739
[32] Chiao R Y, Kwiat P G and Steinberg A M 1993 *Sci. Am.* **269** 52
[33] Wang Y P and Wang D L 1995 *Phys. Rev.* A **52** 2597
[34] Japha Y and Kurizki G 1996 *Phys. Rev.* A **53** 586
[35] Büttiker M and Washburn S 2003 *Nature* **422** 271
[36] Steinberg A M 1995 *Phys. Rev. Lett.* **74** 2405
[37] Sokolovski D, Msezane A Z and Shaginyan V R 2005 *Phys. Rev.* A **71** 064103
[38] Stenner M D, Gauthier D J and Neifeld M A 2003 *Nature* **425** 695
[39] Gehring G M, Schweinsberg A, Barsi C, Kostinski N and Boyd R W 2006 *Science* **312** 895
[40] Wang L J, Kuzmich A and Dogariu A 2000 *Nature* **406** 227
[41] Lathi B P 2005 *Linear Systems and Signals* (New York: Oxford University Press)
[42] Landauer R and Martin Th 1994 *Rev. Mod. Phys.* **66** 217
[43] Hauge E H, Falck J P and Fjeldly T A 1987 *Phys. Rev.* B **36** 4203
[44] Goto M, Iwamoto H, de Aquino V M, Aguilera-Navarro V C and Kobe D H 2004 *J. Phys. A: Math. Gen.* **37** 3599
[45] Jackson J D 1975 *Classical Electrodynamics* 2nd edn (New York: Wiley) p 357
[46] Winful H G 1985 *Appl. Phys. Lett.* **46** 527
[47] Chinn S R 1976 *Opt. Commun.* **19** 208
[48] Winful H G, Ngom M and Litchinitser N 2004 *Phys. Rev.* A **70** 052112
[49] Winful H G 2005 *Phys. Rev.* E **72** 0466808
[50] Muga J G, Sala-Mayato R and Egusquiza I L (ed) 2002 *Time in Quantum Mechanics* (New York: Springer)