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$(1+p)$-dimensional open D$(p-2)$-brane theories

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Abstract: The dynamics of a Dp-brane can be described either by an open string ending on this brane or by an open D$(p-2)$-brane ending on the same Dp-brane. The ends of the open string couple to a Dp-brane worldvolume gauge field while the boundary of the open D$(p-2)$-brane couples to a $(p-2)$-form worldvolume potential whose field strength is Poincaré dual to that of the gauge field on the Dp-brane worldvolume. With this in mind, we find that the Poincaré dual of the fixed rank-2 magnetic field used in defining a $(1+p)$-dimensional noncommutative Yang-Mills (NCYM) gives precisely a near-critical electric field for the open D$(p-2)$-brane. We therefore find $(1+p)$-dimensional open D$(p-2)$-brane theories along the same line as for obtaining noncommutative open string theories (NCOS), OM theory and open Dp-brane theories (ODp) from NS5 brane. Similarly, the Poincaré dual of the near-critical electric field used in defining a $(1+p)$-dimensional NCOS gives a fixed magnetic-like field. This field along with the same bulk field scalings defines a $(1+p)$-dimensional noncommutative field theory. In the same spirit, we can have various (1+5)-dimensional noncommutative field theories resulting from the existence of ODp if the description of open D$(4-p)$-brane ending on the NS5 brane is insisted.

Keywords: D-branes, String Duality.

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1. Introduction

By now we know that there exists not only the big M-theory but also a little m-theory. The latter is particularly interesting since it shares many properties of the big M-theory and yet appears as a decoupled theory without gravity. Therefore, we have a better hand on this theory and hopefully we can learn new things and gain insights for the big M-theory in the process of uncovering more on this little m-theory.

The purpose of this paper is to show the existence of new non-gravitational theories which are closely related to the recently discovered decoupled noncommutative Yang-Mills theories (NCYM) [1]–[4], noncommutative open string theories (NCOS) [5, 6, 7], OM theory and open Dp-brane (ODp) theories [8, 9, 10].

In particular, we will show that the Dp-brane worldvolume Poincaré dual of the fixed rank-2 magnetic field used in defining a $(1+p)$-dimensional NCYM gives a critical $(p-1)$-form electric field[1] for an open D$(p-2)$-brane ending on the Dp-brane.

[1]The scalings for the bulk fields such as the metric and the closed string coupling remain the same.

We therefore find $(1+p)$-dimensional open D$(p-2)$-brane theories in the same spirit as for OM theory, NCOS and ODp. In other words, with the same bulk (the metric and the closed string coupling) scaling limit, we can end up with either a $(1+p)$-dimensional open D$(p-2)$-brane theory from the open D$(p-2)$-brane perspective or a $(1+p)$-dimensional NCYM from the open string perspective. Moreover, the open D$(p-2)$ brane theory provides a completion of the NCYM if the latter is nonrenormalizable. In this sense, the former is in general a better description.

By the same token, we find that the existence of a $(1+p)$-dimensional NCOS implies also a $(1+p)$-dimensional "noncommutative" field theory.[2] The corresponding noncommutative geometry is determined through the quantization of the boundary action which is obtained from a topological one for the open D$(p-2)$-brane. For the particular $p = 3$ case, the new noncommutative field theory is also a NCYM resulting from an open D-string ending on a D3 brane and can actually be identified with the usual NCYM resulting from an open F-string ending on the same base D3 brane.

The above results are consistent with the compactification of OM theory on either a magnetic circle or an electric circle. The usual picture is: the compactification of OM theory on a magnetic circle gives the usual $(1+4)$-dimensional NCYM while on an electric circle it gives the $(1+4)$-dimensional NCOS. As we will show in section 5, the actual path is: The magnetic-circle compactification of OM theory gives our $(1+4)$-dimensional open D2 brane theory which provides a completion of the effective $(1+4)$-dimensional NCYM. The electric-circle compactification gives the $(1+4)$-dimensional NCOS which provides a completion of the new effective $(1+4)$-dimensional noncommutative tensor field theory mentioned above. We will elaborate these in section 5.

Along the similar line, we should also have new $(1+5)$-dimensional noncommutative field theories given the existence of the ODp theories from NS5 brane for $p \leq 5$. We will discuss this in section 6. All these new non-gravitational theories are consistent with U-duality, therefore lending support to the notion that U-duality is inherited to the little m-theory without gravity.

This paper is organized as follows: in section 2, we give a rather detailed motivation for the work presented in this paper. In section 3, we show that the fixed rank-2 magnetic field used in defining a usual $(1+p)$-dimensional NCYM from the open string perspective gives precisely a critical $(p-1)$-form electric field for an open D$(p-2)$ brane theory if the dynamics of the base Dp-brane is described in terms of the ending D$(p-2)$-brane. We also discuss the relationship between the open D$(p-2)$-brane theory and the corresponding NCYM. In section 4, we follow the same line as in section 3 but now for a $(1+p)$-dimensional NCOS. We will show that the resulting limit gives a noncommutative field theory with a noncommutative

[2] Here for $p > 3$, the noncommutative geometry is also expected to be nonassociative as well.

geometry determined by the boundary action for the D$(p-2)$-brane. In section 5, we give a detailed picture on the compactification of OM theory on either a magnetic or an electric circle. We will show that the results obtained in the previous sections are consistent with the compactifications of OM theory. In section 6, we first argue the proper limits for ODp theories from NS5 brane. Then we show that the $(1+p)$-dimensional open D$(p-2)$-brane theories discussed in section 3 are U-duality related to the ODp's. We also show that the bulk decoupling limits for ODp from NS5 brane give ones for noncommutative field theories living on NS5 brane in a similar spirit as discussed in section 3 and 4. In section 7, we discuss S-duality between the $(1+3)$-dimensional NCOS and our open D-string theory, and the implication for the existence of a $(1+3)$-dimensional open (p,q)-string theory.

2. Motivation

Strominger some time ago in [11] concluded that a D$(p-2)$-brane can end on a Dp-brane (also M2 brane on M5 brane) without violating charge conservation along the similar line for a fundamental string on a Dp-brane. This same conclusion was also reached by Townsend in [12] from the analysis of Chern-Simons terms in $D = 10$ and $D = 11$ supergravity theories. From the D-brane worldvolume perspective, the end of a fundamental string (or F-string) appears as a point electric charge which couples to the worldvolume U(1) field. The magnetic charge (or monopole) with respect to the U(1) field implies actually a (p - 2)-dimensional extended object carrying an electric-like charge which couples to a worldvolume (p - 1)-form field strength (Poincaré dual to the U(1) gauge field strength) in the Poincaré-dual picture. Therefore, a $(1+p)$-dimensional NCYM as a decoupled theory of Dp-branes with a magnetic field in the F-string picture implies the existence of a different decoupled theory of the Dp brane in an electric-like $(p-1)$-form field strength in the open D$(p-2)$- brane picture. This new theory is just our $(1+p)$-dimensional open D$(p-2)$- brane theory which will be discussed in the following section. Similarly, a $(1+p)$-dimensional NCOS as a decoupled theory of Dp brane with a near-critical electric field in the F-string picture implies also the existence of a different field theory of Dp brane with a magnetic-like $(p-1)$-form field strength in the open D$(p-2)$- brane picture. This new theory is a "noncommutative" field theory defined on a noncommutative geometry.

Let us elaborate the above further. The dynamics of Dp-brane with a constant magnetic flux in it can be described by the open F-string ending on the Dp-brane with its boundary coupled to this background. In the decoupling limit, the kinetic term of the string theory can be ignored and the dynamics is described by a topological term [4]. This topological term can be expressed as a boundary one and the quantization of this boundary action gives rise to spatial noncommutativity along the directions with nonvanishing magnetic field on the Dp-brane worldvolume.

What is the picture if we look from the description in terms of the open D$(p-2)$-brane ending on the Dp-brane with the same scalings for the bulk metric and the closed string coupling as those for NCYM? As is well known that Dp-branes with a constant magnetic flux represent a non-threshold bound state of Dp-branes with smeared D$(p-2)$-branes along the two co-dimensions [13, 14, 15]. The smeared D$(p-2)$-branes are within the Dp-brane worldvolume rather than end on them. As discussed in [16], in the decoupling limit for NCYM, if we view the smeared D$(p-2)$-branes as periodic vortices along the two co-dimensions, each vortex will decouple from the rest. Therefore we need to consider only one vortex, for example, the one in the origin of the coordinate system for the two co-dimensions. In other words, we have localized D$(p-2)$-branes within the Dp-brane worldvolume in the decoupling limit for NCYM. We now know that in terms of the open D$(p-2)$-brane picture, this system should also decouple from the bulk in the decoupling limit and its dynamics is described by the open D$(p-2)$-branes which couple to a Dp-brane worldvolume $(p-1)$-form field strength. The very fact that the D$(p-2)$-branes reside within Dp brane worldvolume must imply that the background $(p-1)$-form electric field reach its critical value.[3] We will show that this is indeed true as expected.

The above picture is along the same line as for the decoupling limits for NCOS, OM theory and those ODp from NS-5-branes. In particular, the gravity systems used for their gravity descriptions [8, 7, 17, 18, 10] in the respective decoupling limits are nothing but the corresponding non-threshold bound states. For example, for OM theory, the gravity system is the (M5, M2) bound state [19]. For NCOS, the gravity systems are the (F, Dp) bound state [20]. The gravity description of the present open D$(p-2)$-brane theory is the same as the corresponding one of the usual $(1+p)$-dimensional NCYM except that we have traded the asymptotic B-field for NCYM with the asymptotic RR $(p-1)$-form potential through the Dp-brane worldvolume Poincaré duality.[4]

We have the following two additional pieces of evidence to support the existence of the open D$(p-2)$-brane theories found in this paper. First, OM theory results from a critical electric 3-form H_{012} field limit. The non-linear self-duality constraint for this 3-form field implies also a non-vanishing H_{345}. As discussed in [8], this theory reduces to a usual $(1 + 4)$-dimensional NCYM upon compactification on a magnetic circle. The H_{345} gives a rank-2 magnetic field which gives rise to the noncommutativity in the NCYM theory.

[3]This conclusion can only be drawn in the decoupling limit. From NCYM side, we know that in the decoupling limit the open string massive modes decouple and the dynamics is described by its massless modes, i.e., the gauge modes, which live on the brane. So we expect that the dynamical degrees of freedom should also remain on the brane if the open D$(p-2)$-brane description is adopted. Here what left in the decoupling limit is the D$(p-2)$-branes and therefore the background field must reach its critical value.

[4]We will use the constant bulk B-field or RR $(p-1)$-potential only when we discuss the gravity dual descriptions. Otherwise, we always use the worldvolume fields to avoid possible confusions.

Upon the reduction on a circle along one of the M5 worldvolume directions, the 3-form field strength on M5-brane will give either a 2-form gauge field strength or a 3-form field strength but not both on the D4-brane worldvolume. Otherwise, we double counting the degrees of freedom for the worldvolume field since the two are not independent but related through a constraint inherited from the self-duality on M5 brane. This is familiar for the self-dual 5-form field strength in the dimensional reduction of type IIB supergravity on a circle to the $N = 2$ nine dimensional supergravity.

The usual (1 + 4)-dimensional NCYM is nonrenormalizable and therefore this description is an effective one which is good for relevant energy much smaller than the inverse of the gauge coupling $g^2_{\rm NCYM}$. If this effective description is valid, we can choose to keep the 2-form gauge field strength rather than 3-form field strength.

Note that the magnetic-circle compactification of OM theory is along a direction transverse to the open membrane which is used to define OM theory. One must be wondering where is the open membrane and naturally expects an open membrane theory in (1 + 4)-dimensions. In other words, we expect OM theory to reduce to an open membrane theory in (1 + 4)-dimensions when the compactification radius is invisible to the OM theory (i.e., the KK modes are too heavy in comparison to the OM theory scale). This theory is also expected to provide a complete description in (1 + 4)-dimension. As we will show in section 5, this is indeed true. This open membrane theory is just our (1 + 4)-dimensional open D2-brane theory which we will discuss in the following section. This theory provides the completion of the usual (1 + 4)-dimensional NCYM. In other words, OM theory implies the existence of the (1 + 4)-dimensional open D2-brane theory. For this theory, we need to keep instead the 3-form H_{012} upon the reduction. Starting with this (1 + 4)-dimensional open D2-brane theory, we can obtain in general $(1 + p)$-dimensional open D$(p - 2)$-brane theories by T-duality along a direction either common or transverse to both of D$(p - 2)$- and Dp-branes. We limit ourselves to $p \leq 5$ in this paper because for $p > 5$, the corresponding $(1 + p)$ NCYM cannot decouple from the bulk [21]. This might imply that we have only decoupled open Dp-brane theories for $p \leq 3$.

By the same token, we may expect a new noncommutative tensor field theory upon the compactification of OM theory on an electric circle when the spatial 3-form H_{345} can be kept instead. We will discuss this possibility in section 4.

The ODp theories from NS-5-brane discovered in [8, 10] also imply the existence of the $(1 + p)$-dimensional open D$(p - 2)$- brane theories found in this paper. As discussed in [8], one direct evidence for ODp theories is from the fact that an open string ending on a D5-brane is S-dual to a D-string ending on a NS-5 brane in type IIB string theory. The former gives the (1 + 5)-dimensional NCOS in the critical electric field limit. The S-dual of this gives OD1 now also in the corresponding critical electric field limit. This can also be understood as the electric force, due to the near-critical electric field, acting at the two ends of the D-string on the NS

-5-brane almost balances the D-string tension. As a result, the D-string decouples from the bulk and is confined on the NS-5-brane worldvolume. T-dualities along NS5-brane directions on this OD1 give in general ODp for $p \leq 5$. In other words, these ODp are just the results of open Dp-branes ending on the base NS5 in the corresponding critical electric field limits.

The direct connection between these ODp and the ones found in this paper occurs for OD3. Since the tension and the near-critical electric field associated with the open D3-brane, and the scalings for the closed string parameters (metric and closed string coupling) remain the same under S-duality, we conclude that the S-dual of OD3 gives another OD3 since the D3-brane itself is intact under S-duality.[5] This new OD3 theory is now from an open D3-brane ending on D5-branes in the critical 4-form electric field limit. Therefore, this OD3 theory is our present (1 + 5)-dimensional open D3-brane theory. T-dualities along the D3-brane directions therefore give also our (1 + p)-dimensional open D$(p-2)$-brane theories.

The field theories resulting from the existence of NCOS or ODp can be discussed in a similar fashion and we will not repeat them here.

3. $(1+p)$-Dimensional open D$(p-2)$-brane theories

In this section, we will show that the decoupling limit for a (1 + p)-dimensional NCYM with rank-2 noncommutative matrix from the open string perspective gives precisely a critical field limit for an open D$(p-2)$-brane theory if this open D$(p-2)$-brane description of Dp-brane is insisted. Let us begin with a summary of the decoupling limit for NCYM [4]:

$$\begin{aligned}
\tilde{\alpha}' &= \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}\,,\\
\tilde{g}_s &= \frac{\tilde{\alpha}'^{\frac{3-p}{2}}_{\rm eff}\,\tilde{g}^2_{\rm NCYM}}{(2\pi)^{p-2}}\epsilon^{\frac{3-(p-2)}{4}}, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1,\cdots(p-2))\,,\\
g_{ij} &= \epsilon\delta_{ij}\,, \quad (i,j = (p-1),p)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\
2\pi\tilde{\alpha}' B_{(p-1)p} &= \epsilon^{1/2}\,,
\end{aligned} \tag{3.1}$$

where $\tilde{g}^2_{\rm NCYM}$ is the fixed noncommutative Yang-Mills coupling. We know that with the presence of Dp-brane, the worldvolume gauge invariant quantity is $\mathcal{F} = 2\pi\tilde{\alpha}'(B+F)$ with F the worldvolume gauge field. For the purpose of performing the worldvolume Poincaré duality in the following, we replace the constant rank-2 B-field in Eq. (3.1) by a constant rank-2 gauge field strength using a gauge choice. As a result, we have now

$$2\pi\tilde{\alpha}' F_{(p-1)p} = \epsilon^{1/2}\,, \qquad B = 0\,. \tag{3.2}$$

[5] Some parameters of the original OD3 theory such as the effective open D3-brane coupling are transformed under S-duality but the theory is not. This conclusion differs from that given in [8] where the S-duality gives (1 + 5)-dimensional NCYM. We will reconcile this difference in section 6.

The Dp-brane worldvolume Poincaré dual of the above magnetic background gives an electric-like worldvolume (p - 1)-form field strength $H_{012\cdots(p-2)}$ which is associated with the D$(p-2)$- brane ending on the Dp-brane. Note that the relevant Dp-brane lagrangian for the purpose of obtaining such an electric-like background field $H_{012\cdots(p-2)}$ is

$$\mathcal{L}_{DBI} = -\frac{1}{(2\pi)^p \tilde{\alpha}'^{(1+p)/2}\tilde{g}_s}\sqrt{-\det(g_{\alpha\beta} + 2\pi\tilde{\alpha}' F_{\alpha\beta})}\,, \tag{3.3}$$

where $\alpha, \beta = 0, 1, \cdots p$. We then have

$$\sqrt{-\det g}\frac{H_{012\cdots(p-2)}}{2\pi} = -\frac{1}{2}\frac{\epsilon_{012\cdots(p-2)ij}}{\sqrt{-\det g}}\frac{\partial\mathcal{L}_{DBI}}{\partial F_{ij}}\,, \tag{3.4}$$

where we define $\epsilon_{\alpha_0\cdots\alpha_p} = g_{\alpha_0\beta_0}\cdots g_{\alpha_p\beta_p}\epsilon^{\beta_0\cdots\beta_p}$ with $\epsilon^{01\cdots p} = 1$.

Using the scalings for $\tilde{g}_s$, the metric in eq. (3.1) and the magnetic background in eq. (3.2), we have from the above

$$H_{012\cdots(p-2)} = \frac{1}{(2\pi)^{p-2}\tilde{\alpha}'^{\frac{(p-2)+1}{2}}_{\text{eff}}\tilde{G}^2_{o(p-2)}}\left(\frac{1}{\epsilon} - \frac{1}{2}\right), \tag{3.5}$$

where we have defined

$$\tilde{G}^2_{o(p-2)} = \frac{\tilde{g}^2_{\text{NCYM}}\tilde{\alpha}'^{(3-p)/2}_{\text{eff}}}{(2\pi)^{p-2}}\,. \tag{3.6}$$

The scalings for the metric and the closed string coupling remain the same as those given in eq. (3.1). The form of the above electric (p - 1)-form field strength indicates that it reaches its critical limit as $\epsilon \to 0$. Let us confirm this. The effective action of an open D$(p-2)$-brane ending on a Dp-brane can be written in its simplest form as

$$S_{(p-2)} = -\frac{1}{(2\pi)^{p-2}\tilde{\alpha}'^{(p-1)/2}\tilde{g}_s}\int_{M^{p-1}} d^{p-1}\sigma\sqrt{-\det(\hat{g}_{\mu\nu} + 2\pi\tilde{\alpha}' F_{\mu\nu})} + \int_{M^{p-1}}\mathcal{H}_{p-1} + \cdots\,, \tag{3.7}$$

where we have

$$\mathcal{H}_{p-1} = C_{p-1} + H_{p-1}\,, \tag{3.8}$$

with C_{p-1} the pull-back of the bulk RR (p - 1)-form potential and H_{p-1} is the aforementioned Dp-brane worldvolume $(p-1)$-form field strength which comes from the conversion of the open D$(p-2)$-brane boundary term to its worldvolume along the Dp-brane directions. The $\cdots$ terms are irrelevant for the discussion of this paper and for this reason we drop them from now on. The D$(p-2)$- brane worldvolume gauge field $F_{\mu\nu}$ is also irrelevant and we drop it for the following discussion. In the above, the gauge invariant quantity is now $\mathcal{H}_{p-1}$. Once again, we see that in the presence of this D$(p-2)$-brane, given $\mathcal{H}_{p-1}$ and H_{p-1}, C_{p-1} cannot be arbitrary but fixed according to the above equation.[6] For the choice of eq. (3.5), we have $C_{01\cdots(p-2)} = 0$.

With the above, let us calculate the effective proper (also coordinate) tension for a D$(p-2)$-brane along $12\cdots(p-2)$ directions with the metric and the closed string coupling given in eq. (3.1) and with the $H_{01\cdots(p-2)}$ given in eq. (3.5), we then have

$$-\frac{1}{(2\pi)^{p-2}\tilde{\alpha}'^{(p-1)/2}\tilde{g}_s} + \epsilon^{01\cdots(p-2)}H_{01\cdots(p-2)} = -\frac{1}{2(2\pi)^{p-2}\tilde{\alpha}'^{(p-1)/2}_{\rm eff}\tilde{G}^2_{o(p-2)}}\,, \tag{3.9}$$

which indicates that our $H_{01\cdots(p-2)}$ is indeed a near-critical electric field. The near-critical electric force stretches the boundary of the D$(p-2)$-brane to balance its original tension such that a finite tension as given above is obtained. As a result, the D$(p-2)$-brane is now confined within the Dp-brane worldvolume. The conventional discussion implies that we end up with an open D$(p-2)$-brane theory for $p \leq 5$. For later use, let us summarize the decoupling limit for a $(1+p)$-dimensional open D$(p-2)$- brane theory:

$$\begin{aligned}
&\tilde{\alpha}' = \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}\,, \quad \tilde{g}_s^{(p-2)} = \epsilon^{\frac{3-(p-2)}{4}}\tilde{G}^2_{o(p-2)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1,\cdots(p-2))\,,\\
&g_{ij} = \epsilon\delta_{ij}\,,\ (i,j = (p-1),p)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\
H_{012\cdots(p-2)} &= \frac{1}{(2\pi)^{p-2}\tilde{\alpha}'^{\frac{(p-2)+1}{2}}_{\rm eff}\tilde{G}^2_{o(p-2)}}\left(\frac{1}{\epsilon} - \frac{1}{2}\right),
\end{aligned} \tag{3.10}$$

where the coupling $\tilde{G}_{o(p-2)}$ for the open D$(p-2)$-brane theory is related to the gauge coupling through (3.6).

Let us briefly discuss each of the open D$(p-2)$-brane theories for $2 \leq p \leq 5$.

Open D0 theory: This case can be discussed similarly following that for the OD0 theory from NS5-brane given in [8]. The present open D0-brane theory results from a D2-brane in the presence of a worldvolume near-critical 1-form field strength $H_0 = \frac{1}{\epsilon\sqrt{\tilde{\alpha}'_{\rm eff}}\tilde{G}^2_{o(0)}}(1 - \frac{\epsilon}{2})$. This field strength can be traded to a 1-form bulk RR potential C_0. The dynamical objects in this theory are the light D0 branes. Again, the light excitations of this open D0-brane theory carry a conserved charge.

If we lift this open D0-brane theory to eleven dimensions on a transverse circle, the D2-brane now becomes an M2-brane. We have the eleven-dimensional Planck mass and the compactified radius as

$$R_{11} = \sqrt{\tilde{\alpha}'}\tilde{g}_s^{(0)} = \epsilon\sqrt{\tilde{\alpha}'_{\rm eff}}\tilde{G}^2_{o(0)} \equiv \epsilon R\,, \quad M_p = \frac{1}{\sqrt{\tilde{\alpha}'}(g_s^{(0)})^{1/3}} = \epsilon^{-1/2}\tilde{M}_{\rm eff}\,, \tag{3.11}$$

[6]This example indicates that we cannot choose the asymptotic values as we wish for bulk potentials whether they are NSNS or RR origins in the presence of various kinds of D branes

where $\tilde{M}_{\rm eff} = \frac{1}{\sqrt{\tilde{\alpha}'_{\rm eff}\tilde{G}^{2/3}_{o(0)}}}$. Choosing the fixed coordinate in the 11-th direction such that $x^{11} \sim x^{11} + 2\pi R$, the bulk 11-dimensional metric is

$$ds^2_M = -(dx^0)^2 + R^2_{11}\left(\frac{dx^{11}}{R} - C_0 dx^0\right)^2 + \epsilon dx^2_\perp = \epsilon\left[-(dx^0)^2 - dx^{11}dx^0 + dx^2_\perp\right], \tag{3.12}$$

where we have dropped a term proportional to ϵ^2. Note that the lifted theory is defined with respect to the metric ds^2_M/ϵ and now the compactified 11-th direction is light-like. We have now the bulk Planck scale $\tilde{M}_{\rm eff}$ which is the same as the proper tension for the open D0-brane theory.

In other words, the open D0-brane theory with N units of D0-brane charge is a DLCQ compactification of M theory with N units of DLCQ momentum in the presence of a transverse M2-brane.

Open D1 theory: The decoupling limit for this theory can be summarized as

$$\begin{aligned}
\tilde{\alpha}' &= \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}\,, \quad \tilde{g}^{(1)}_s = \epsilon^{\frac{1}{2}}\tilde{G}^2_{o(1)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1)\,,\\
g_{ij} &= \epsilon\delta_{ij}\,, \ (i,j = 3,4)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\
H_{01} &= \frac{1}{(2\pi)\tilde{\alpha}'_{\rm eff}\tilde{G}^2_{o(1)}}\left(\frac{1}{\epsilon} - \frac{1}{2}\right).
\end{aligned} \tag{3.13}$$

For this particular case, given the relation between the open D-string and the open F-string, we expect that the open D-string metric and noncommutative parameter can be obtained from the usual Seiberg-Witten relations for open F-string ending on a D-brane through the following replacements:

$$\tilde{\alpha}' \to \tilde{\alpha}'\tilde{g}^{(1)}_s\,, \quad \tilde{g}^{(1)}_s \to \frac{1}{\tilde{g}^{(1)}_s}\,, \quad F_{\alpha\beta} \to H_{\alpha\beta}\,, \tag{3.14}$$

i.e., we have now

$$\begin{aligned}
G_{\alpha\beta} &= g_{\alpha\beta} - (2\pi\tilde{\alpha}'\tilde{g}^{(1)}_s)^2 (Hg^{-1}H)_{\alpha\beta}\,,\\
\Theta^{\alpha\beta} &= 2\pi\tilde{\alpha}'\tilde{g}^{(1)}_s\left(\frac{1}{g + 2\pi\tilde{\alpha}'\tilde{g}^{(1)}_s H}\right)^{\alpha\beta}_A\,,
\end{aligned} \tag{3.15}$$

where A in $()_A$ denotes the anti-symmetric part of the matrix and $\alpha,\beta = 0,1,2,3$. Using the above scalings, we have the open D-string metric and the nonvanishing noncommutative parameter as

$$G_{\alpha\beta} = \epsilon\eta_{\alpha\beta}\,, \quad \Theta^{01} = 2\pi\tilde{\alpha}'_{\rm eff}\tilde{G}^2_{o(1)}\,. \tag{3.16}$$

As expected, we have $\tilde{\alpha}'\tilde{g}^{(1)}_s G^{\alpha\beta} = \tilde{\alpha}'_{\rm eff}\tilde{G}^2_{o(1)}\eta^{\alpha\beta}$. This is a well-defined perturbative theory for small $\tilde{G}^2_{o(1)}$. The usual $(1+3)$-dimensional NCYM is believed to be renormalizable and therefore it is a well-defined perturbative noncommutative field theory for small coupling $\tilde{g}^2_{\rm NCYM}$. Further we have $\tilde{G}^2_{o(1)} = \tilde{g}^2_{\rm NCYM}/(2\pi)$ which implies that the two perturbative theories break down at the same time when either of the couplings is strong. As mentioned earlier, the two have basically the same gravity dual description. Note that the NCYM can have T-duality, and therefore it is not really a field theory since it does not have a well-defined energy-momentum tensor. All these indicate that the usual (1 + 3)-dimensional NCYM and the (1 + 3)-dimensional open D-string theory are just two different descriptions of the same physics.

Open D2 theory: This theory is related to OM theory compactified on a small magnetic circle and provides a completion of the usual (1 + 4)-dimensional NCYM. We will discuss this case in detail in section 5.

Open D3 theory: The decoupling limit for this theory contains D5-branes in the presence of a near-critical 4-form worldvolume field strength $H_{0123} = \frac{1}{(2\pi)^3\epsilon\tilde{\alpha}'^2_{\rm eff}\tilde{G}^2_{o(3)}}(1-\frac{\epsilon}{2})$. The bulk scalings are

$$\begin{aligned} &\tilde{\alpha}' = \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}\,, \quad \tilde{g}_s^{(3)} = \tilde{G}^2_{o(3)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1,2,3)\,, \\ &g_{ij} = \epsilon\delta_{ij}\,,\ (i,j = 4,5)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,. \end{aligned} \tag{3.17}$$

The coupling for this theory is related to the usual (1 + 5)-dimensional NCYM coupling as

$$\tilde{G}^2_{o(3)} = \frac{\tilde{g}^2_{\rm NCYM}\tilde{\alpha}'^{-1}_{\rm eff}}{(2\pi)^3}\,. \tag{3.18}$$

The usual (1 + 5)-dimensional NCYM is nonrenormalizable and as such it is an effective theory. The present open D3-brane theory provides a completion of this NCYM. Therefore this is an example that the open D3 brane description is better than the usual NCYM one (or the F-string description). As we will discuss this case further in section 6, this open D3-brane theory is actually self-dual under S-duality.

In a similar fashion as discussed in [8], different (1 + p)-dimensional open D$(p-2)$-brane theories here can be related to each other either by a T-duality along a direction of the D$(p-2)$- brane or by a T-duality along a direction transverse to both this D$(p-2)$-brane and the parent Dp-brane. However, a T duality along any codimension gives a D$(p-1)$-brane which no longer lives inside the parent D$(p-1)$-brane. This indicates that such a T-duality may render the open D$(p-1)$-brane undecoupled. If we compactify the x^{p-2}-direction with the identification $x^{p-2} \sim x^{p-2} + 2\pi R_{p-2}$, the usual transformations of bulk quantities under a T-duality along this direction give the following

$$H_{01\cdots(p-3)} = 2\pi R_{p-2}H_{01\cdots(p-2)}\,, \quad R'_{p-2} = \frac{\tilde{\alpha}'_{\rm eff}}{R_{p-2}}\,, \quad \tilde{G}^2_{o(p-3)} = \frac{\sqrt{\tilde{\alpha}'_{\rm eff}}}{R_{p-2}}\tilde{G}^2_{o(p-2)}\,, \tag{3.19}$$

where R'_{p-2} is the T-dual coordinate radius. One can check that the resulting decoupling limit is for a $(1+(p-1))$-dimensional open D$(p-3)$-brane theory.

4. $(1+p)$-Dimensional noncommutative field theories

We follow the same steps as what we did in the previous section but now for a $(1+p)$-dimensional NCOS rather than for a (1 + p)-dimensional NCYM. From the open string perspective, the critical electric field limit gives a $(1+p)$-dimensional NCOS. The question is: what is the corresponding decoupled theory with the same bulk scalings but now from the open D$(p-2)$-brane perspective? As we will argue below, the answer seems a decoupled $(1+p)$-dimensional "noncommutative" field theory defined on a noncommutative geometry which is in general different from that for the usual (1 + p)-dimensional NCYM.

The decoupling limit for a $(1+p)$-dimensional NCOS can be given collectively as [8]:

$$\begin{aligned}\alpha' &= \epsilon\alpha'_{\text{eff}}\,, \quad g_s = \frac{G_o^2}{\sqrt{\epsilon}}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1)\,,\\ g_{ij} &= \epsilon\delta_{ij}\ (i,j = 2,\cdots p)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\ 2\pi\alpha'\epsilon^{01}F_{01} &= 1 - \frac{\epsilon}{2}\,,\end{aligned} \tag{4.1}$$

where the scaling parameter $\epsilon \to 0$ and the NCOS parameters α'_{eff} and G_o remain fixed.

The Dp-brane worldvolume Poincaré dual of F_{01}, i.e., $H_{2\cdots p}$, can be obtained, following the same steps as those given in the previous section, as

$$\sqrt{-\det g}\,\frac{H_{2\cdots p}}{2\pi} = -\frac{1}{2}\frac{\epsilon_{2\cdots p\mu\nu}}{\sqrt{-\det g}}\frac{\partial\mathcal{L}_{DBI}}{\partial F_{\mu\nu}}\,. \tag{4.2}$$

Using the scaling limit given in (4.1) for the metric, the closed string coupling and the near-critical electric field, we have

$$H_{2\cdots p} = \frac{1}{(2\pi)^{p-2}\alpha'^{\frac{p-1}{2}}_{\text{eff}}G_o^2}\,, \tag{4.3}$$

which remains fixed.

In summary, from the open D$(p-2)$-brane perspective, we have now the following scaling limits:

$$\begin{aligned}\alpha' &= \epsilon\alpha'_{\text{eff}}\,, \quad g_s = \frac{G_o^2}{\sqrt{\epsilon}}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1)\,,\\ g_{ij} &= \epsilon\delta_{ij}\ (i,j = 2,\cdots p)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\ H_{2\cdots p} &= \frac{1}{(2\pi)^{p-2}\alpha'^{\frac{p-1}{2}}_{\text{eff}}G_o^2}\,.\end{aligned} \tag{4.4}$$

Let us inspect the action (3.7) proposed in the previous section for the open D$(p-2)$-brane ending on the Dp-brane which moves in the background given in (4.4).

For convenience, we write it down here as

$$S_{(p-2)} = -\frac{1}{(2\pi)^{p-2}\alpha'^{(p-1)/2}g_s}\int_{M^{p-1}} d^{p-1}\sigma\sqrt{-\det \hat{g}_{\alpha\beta}} + \int_{M^{p-1}} \mathcal{H}_{p-1}\,, \tag{4.5}$$

where we have dropped the D$(p-2)$-brane worldvolume U(1) field for the reason mentioned in the previous section, the D$(p-2)$-brane worldvolume indices $\alpha, \beta = 0, 1, \cdots (p-2)$ and the induced worldvolume metric

$$\hat{g}_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N g_{MN}\,, \tag{4.6}$$

where the metric g_{MN} is the bulk spacetime one with $M, N = 0, 1, \cdots 9$. The above Nambu-Goto-type action is not convenient for considering the scaling behavior of the action. We here follow the procedure given in [22] to introduce the auxiliary worldvolume metric $\gamma_{\alpha\beta}$ and recast the above action in Polyakov form as

$$S_{(p-2)} = -\frac{1}{2(2\pi)^2\alpha' g_s}\int_{M^{p-1}} d^{p-1}\sigma\sqrt{-\det\gamma}\left(\gamma^{\alpha\beta}\partial_\alpha X^M\partial_\beta X^N g_{MN} - (2\pi)^2(p-3)\alpha'\right) + \\ + \int_{M^{p-1}} H_{p-1}\,, \tag{4.7}$$

where we have again followed [9] by insisting the worldvolume coordinates σ^α as dimensionless. One can check that the equation of motion for $\gamma_{\alpha\beta}$ gives the induced metric and if substituting this back to the above action, we end up with the Nambu-Goto action (4.5). In the following, we consider the scaling behavior of the above action under the scaling limit (4.4). As it is understood that the coordinates X^M are now fixed. The D$(p-2)$-brane coordinates σ^α as well as its intrinsic metric $\gamma_{\alpha\beta}$ are also fixed. With these, we have

$$\begin{aligned} S_{(p-2)} = &-\frac{1}{2(2\pi)^2\alpha'_{\text{eff}}G_o^2}\int_{M^{p-1}} d^{p-1}\sigma\sqrt{-\det\gamma}\,\times \\ &\times\Big[\epsilon^{-1/2}\gamma^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu} + \epsilon^{1/2}\gamma^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j\delta_{ij} + \\ &+ \epsilon^{1/2}\gamma^{\alpha\beta}\partial_\alpha Y^m\partial_\beta Y^n\delta_{mn} - \epsilon^{1/2}(2\pi)^2\alpha'_{\text{eff}}(p-3)\Big] + \\ &+\frac{1}{(p-1)!}\int_{M^{p-1}} d^{p-1}\sigma\epsilon^{\alpha_0\alpha_1\cdots\alpha_{p-2}}\partial_{\alpha_0}X^{i_1}\partial_{\alpha_1}X^{i_2}\cdots\partial_{\alpha_{p-2}}X^{i_{p-1}}\mathcal{H}_{i_1 i_2\cdots i_{p-1}}\,, \end{aligned} \tag{4.8}$$

where Y^m denote the bulk modes in directions transverse to the base Dp-brane. From the above, we have the following:

1. The bulk modes X^μ for $\mu = 0, 1$ are frozen out.

2. The action for the bulk modes X^i and Y^m vanishes.

Since the bulk field $\mathcal{H}_{2\cdots p}$ as given in (4.4) is a fixed constant, the bulk theory is now described by the following topological action

$$S_{(p-2)} = \frac{1}{(p-1)!}\int_{M^{p-1}} d^{p-1}\sigma\epsilon^{\alpha_0\alpha_1\cdots\alpha_{p-2}}\partial_{\alpha_0}X^{i_1}\partial_{\alpha_1}X^{i_2}\cdots\partial_{\alpha_{p-2}}X^{i_{p-1}}\mathcal{H}_{i_1 i_2\cdots i_{p-1}}\,, \tag{4.9}$$

which in turn can be expressed as the following boundary action for $p \geq 3$

$$\frac{1}{(p-1)!}\int_{\partial M^{p-1}} d^{p-2}\xi\epsilon^{\alpha_0\alpha_1\cdots\alpha_{p-3}}\partial_{\alpha_0}X^{i_1}\partial_{\alpha_1}X^{i_2}\cdots\partial_{\alpha_{p-3}}X^{i_{p-2}}X^{i_{p-1}}\mathcal{H}_{i_1 i_2\cdots i_{p-1}}\,, \tag{4.10}$$

where ξ^α with ($\alpha = 0, 1, \ldots (p-3)$) denote now the local coordinates for the boundary $(p-3)$-brane and X^i are the embedding fields of the boundary $(p-3)$-brane.

The boundary degrees of freedom for the D$(p-2)$-brane are governed by the above action. For $p = 2$, we can see that the action (4.9) has no local dynamics for a constant H_i. We therefore don't expect the noncommutative geometry to arise for this case. For $p = 3$, the quantization of the above action gives $[X^i, X^j] \neq 0$, therefore implying the spatial noncommutative geometry of the base D3-brane along the line as for the usual NCYM discussed in [4]. For $p = 4, 5$, we may follow [22] to discuss the corresponding spatial noncommutativity geometries of the base Dp-branes. However, for the $p = 5$ case, the S-dual of the resultant theory does not appear to decouple from the bulk as we will discuss in section 6. This may indicate that the present theory is not well-defined, either. For this reason, we postpone to study this case carefully elsewhere, not pursuing it further in this paper. Therefore, except for the $p = 2$ case, we expect in general that we have a noncommutative geometry for the base Dp-brane upon the quantization of the above action. The remaining question is: what is the decoupled theory at hand with the decoupling limit (4.4)?

Our current knowledge is that a decoupled open brane theory requires usually a near-critical electric-like background field while a decoupled field theory requires a fixed magnetic-like background field (with respect to the fixed coordinates). With this, we might expect that the decoupling limits (4.4) describe decoupled field theories defined on noncommutative geometries determined through the quantization of the action (4.10). Naively, we may take the field theory modes on Dp-branes as super Yang-Mills multiplet. This would imply that the above decoupled field theories are also “noncommutative” Yang-Mills theories but now defined on noncommutative geometries which are in general different from those for the usual NCYM.

Given that the decoupled field theory is obtained from the open D$(p-2)$-brane perspective and the noncommutative geometry is determined through the fixed Dp-brane worldvolume H_{p-1}-form, the resultant decoupled theory is naturally expected to be a tensor field theory since the field theory modes on a single Dp-brane is a tensor multiplet[7] which is Poincaré dual to the U(1) gauge modes on the brane. If such a field theory for $p > 3$ exists indeed, the question is: Can we use the (1 + p)-dimensional Poincaré dual to map this decoupled field theory to a NCYM? To address this, we first need to know if it is consistent to Poincaré dual the dynamical tensor field while leaving the "noncommutative" geometry intact. If this is true, we can end up with a U(1) gauge field defined on a "noncommutative" geometry determined by the boundary action (4.10). If this is not true, we don't expect that we can end up with a field theory since the Poincaré dual of spatial "noncommutative" geometry would imply a time-space one. The expected theory should be the $(1 + p)$-dimensional NCOS but we cannot get it by performing the Poincaré dual on the decoupled tensor field theory since the later is expected to be an incomplete description of the underlying physics while the former is a complete description for $p > 3$. Work on this issue for $p = 4$ case is in progress.

In spite of what has been said above, directly confirming the existence of the $(1 + p)$-dimensional "noncommutative" tensor field theories may not be easy since we need to know the effective open D$(p - 2)$-brane metric which is hardly available for $p > 3$. For $p = 3$, however, we are reasonably sure that we end up with a $(1+3)$-dimensional noncommutative Yang-Mills which is actually identical to the usual (1 + 3)-dimensional NCYM if their parameters are properly identified.

Let us give some detail about this theory. As discussed above, quantization of the boundary action (4.10) for $p = 3$ gives

$$[x^2, x^3] = -i2\pi\alpha'_{\rm eff} G_o^2 \,. \tag{4.11}$$

Therefore, we have the spatial noncommutative parameter $\Theta^{23} = -2\pi\alpha'_{\rm eff} G_o^2$. The present decoupled theory is obtained from the open D-string ending on D3-branes in the decoupling limit (4.4) for $p = 3$. Given the relation between D-string and F-string, we expect that the low energy Born-Infeld action for D3-branes with the open D-string ending on them can be obtained from that for D3-branes with a F-string ending on them through the following replacements

$$g_s \to \frac{1}{g_s}\,, \qquad \alpha' \to \alpha' g_s\,, \qquad F_{\alpha\beta} \to H_{\alpha\beta}\,, \tag{4.12}$$

where $F_{\alpha\beta}$ is the worldvolume gauge field in the F-string picture while $H_{\alpha\beta}$ is the corresponding one in the D-string picture. With the above, the decoupling limit (4.4) is essentially the same as the one for the usual NCYM as given in (3.1) in the previous section. Given the above, let us make a consistent check on the open D-string metric, the noncommutative parameter and the gauge coupling using the corresponding Seiberg-Witten relations for the present noncommutative Yang-Mills

[7] For $p > 3$, we know only how to deal with a single Dp-brane since at present we don't know how to generalize an abelian tensor multiplet to its non-abelian one.

theory. They are now

$$G_{\alpha\beta} = g_{\alpha\beta} - (2\pi\alpha' g_s)^2 (Hg^{-1}H)_{\alpha\beta}\,,$$
$$\Theta^{\alpha\beta} = 2\pi\alpha' g_s \left(\frac{1}{g + 2\pi\alpha' g_s H}\right)_A^{\alpha\beta}\,,$$
$$\frac{1}{g^2_{\rm NCYM}} = \frac{g_s}{2\pi}\left(\frac{\det(g + 2\pi\alpha' g_s H)}{\det G}\right)^{1/2}\,, \tag{4.13}$$

where A in $()_A$ denotes the anti-symmetric part of the matrix. Using the decoupling limit (4.4) for $p = 3$, we have from the above

$$G_{\alpha\beta} = \eta_{\alpha\beta}\,, \quad \Theta^{23} = -2\pi\alpha'_{\rm eff} G_o^2\,, \quad \frac{1}{g^2_{\rm NCYM}} = \frac{G_o^2}{2\pi}\,. \tag{4.14}$$

The noncommutative parameter Θ^{23} is the same as the one obtained above and the open string metric is also expected. The Yang-Mills coupling is inversely related to the open string coupling for NCOS. This is quite different from that between the open D-string coupling and the usual NCYM coupling as given in (3.6) for $p = 3$.

Under S-duality, we expect that our open D-string theory discussed in the previous section is mapped to the present NCOS via

$$\tilde\alpha'_{\rm eff} \to \alpha'_{\rm eff} = \tilde\alpha'_{\rm eff} \tilde G^2_{o(1)}\,, \qquad \tilde G^2_{o(1)} \to G_o^2 = \frac{1}{\tilde G^2_{o(1)}}\,, \tag{4.15}$$

which are obtained from $\tilde\alpha' \to \alpha' = \tilde\alpha' \tilde g_s$, $\tilde g_s \to g_s = 1/\tilde g_s$.

With the above relation, we have the same parameters for the usual NCYM and the above NCYM. Therefore, they are identical theories. In other words, the NCYM keeps intact under S-duality. This is just the consequence of S-duality given the two S-duality related bulk scalings and the relation $F_{23} = H_{23}$. In other words, the low energy dynamics of the open F-string ending on the base D3-branes with background F_{23} is identical to that of the open D-string ending on the same D3-branes with background H_{23}.

Note that the above S-duality for the NCYM is induced from that for the bulk type IIB string theory. This is different from the usual one which requires in addition a worldvolume Poincaré duality for the background field. The usual S-duality maps the usual NCYM directly to the NCOS as discussed in [6]. In terms of our interpretation, the NCYM keeps intact under S-duality.

At low energies, the NCOS, our open D-string theory and the NCYM are all expected to reduce to the corresponding usual Yang-Mills theories. The question is: What are the relations among the three usual Yang-Mills theories. Let us find them out. For the NCOS, we have the gauge coupling from [8] as $g^2_{\rm YM} = 2\pi G_o^2$. For the NCYM, the gauge coupling is just $g^2_{NCYM} = \tilde g^2_{\rm NCYM} = 2\pi G^2_{(1)} = 2\pi/G_o^2$. For our open D-string theory, we can calculate $g^2_{\rm YM} = 2\pi/G^2_{(1)}$. Given $G_o^2 = 1/G^2_{(1)}$, we have the same low energy Yang-Mills theory for the NCOS and our open D-string theory since the gauge coupling is the same. However, we have $g^2_{\rm YM} = (2\pi)^2/g^2_{\rm NCYM}$. In other words, the low energy Yang-Mills theory from either the NCOS or our open D-string theory is strong-weakly related to that from the NCYM. This is the manifestation of the S-duality for the usual (1 + 3)-dimensional YM. This result is consistent with the S-duality relation between the NCOS and the usual NCYM discussed in [6] even though our interpretation here is different as mentioned above.

5. Compactification of OM theory on a circle and (1 + 4)-dimensional theories

In this sub-section, we try to make connections of the (1 + 4)-dimensional open D2-brane theory and the new (1 + 4)-dimensional NCYM discussed in the previous two sections to the compactification of OM theory on a (either magnetic or electric) circle. We will see that the dimensional reduction of OM theory on either a magnetic circle or an electric circle indicates the existence of the open D2-brane theory or the new (1 + 4)-dimensional NCYM.

5.1 OM Theory on a magnetic circle and 5-D open D2-brane theory

In this section, we try to show that OM theory describes the strong coupling of the usual (1 + 4)-dimensional open D2-brane theory discussed in section 2. We also show that this open D2-brane theory provides a UV completion of the (1 + 4)-dimensional NCYM.[8]

As discussed in [8], OM theory on a magnetic circle gives NCYM with rank-2 noncommutative matrix with the following parameters

$$\tilde\alpha' = \frac{1}{L\sqrt{2M^2_{\rm eff}M^3_p}}\,, \quad \tilde g_s = \left(\frac{2L^2M^2_{\rm eff}}{M_p}\right)^{3/4}\,, \quad \tilde g^2_{\rm NCYM} = 4\pi^2 L\,,$$
$$g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1,2)\,, \quad g_{ij} = 2\frac{M^3_{\rm eff}}{M^3_p}\delta_{ij}\,,\ (i,j = 4,5), \quad F_{45} = \frac{LM^3_{\rm eff}}{\pi}\,, \quad (5.1)$$

where L is the coordinate radius of the magnetic circle, $M_{\rm eff}$ is the energy scale for the OM theory and M_p is the eleven-dimensional Planck scale which is sent to infinity in the decoupling limit for OM theory. It was also concluded in that paper that OM theory provides a completion of the (1 + 4)-dimensional NCYM. The detailed path, as we show below, is that the (1 + 4)-dimensional open D2-brane found in this paper provides a completion of the NCYM and OM theory describes the strong coupling of this open D2-brane theory.

[8]That the UV completion of the (1 + 4)-dimensional NCYM is an open D2-brane theory was also briefly mentioned in a recent paper [23]. An open D3-brane theory as the UV completion of the (1 + 5)-dimensional NCYM was also mentioned there. The author would like to thank R.-G. Cai for bringing his attention to this reference.

Comparing the above with the decoupling limit for NCYM with $p = 4$ in eq. (3.1), we have also

$$\epsilon = 2\frac{M_{\rm eff}^3}{M_p^3}\,, \qquad \tilde{\alpha}'_{\rm eff} = \frac{1}{2LM_{\rm eff}^3}\,. \tag{5.2}$$

The (1 + 4)-dimensional NCYM is nonrenormalizable and therefore this theory does not have a complete (1 + 4)-dimensional description. However, when $L \ll 1/M_{\rm eff}$, the magnetic circle is invisible to OM theory. We should end up with a (1 + 4)-dimensional open membrane theory which provides a completion of the NCYM. We will show below that this open membrane theory is our open D2-brane theory.

As discussed in the Introduction, an alternative description of this compactification of OM theory is via the open membrane since the compactification along the magnetic circle is transverse to the open membrane which is used in defining the OM theory. With this in mind, we have from $\tilde{g}_s = \epsilon^{1/4}\tilde{G}^2_{o(2)}$ and the relations given in eqs. (5.1) and (5.2)

$$\tilde{G}^2_{o(2)} = \frac{\tilde{g}^2_{\rm NCYM}\tilde{\alpha}'^{-1/2}_{\rm eff}}{(2\pi)^2} = (2LM_{\rm eff})^{3/2}\,. \tag{5.3}$$

The scalings of other parameters for the OM theory can be read from [8] as[9]

$$\begin{aligned}
\tilde{\alpha}' &= \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}\,, \quad \epsilon = e^{-2\beta} = 2\frac{M_{\rm eff}^3}{M_p^3} \\
g_{\mu\nu} &= \eta_{\mu\nu}\ (\mu,\nu = 0,1,2)\,, \quad g_{ij} = \epsilon\delta_{ij}\ (i,j = 4,5)\,, \\
H_{012} &= \frac{M_p^3\tanh\beta}{(2\pi)^2} = \frac{1}{(2\pi)^2\tilde{\alpha}'^{3/2}_{\rm eff}\tilde{G}^2_{o(2)}}\left(\frac{1}{\epsilon} - \frac{1}{2}\right).
\end{aligned} \tag{5.4}$$

The above parameters and scalings are precisely what we used to define our (1 + 4)-dimensional open D2-brane theory in section 3. If we examine the coupling $\tilde{G}_{o(2)}$ of our open D2-brane theory given (5.3), we have $\tilde{G}_{o(2)} \ll 1$ if $L \ll 1/M_{\rm eff}$ and $\tilde{G}_{o(2)} \gg 1$ if $L \gg 1/M_{\rm eff}$. The former implies that the magnetic circle is invisible to OM theory while the latter says that the circle appears to be uncompactified to OM theory. Therefore, our open D2-brane theory is OM theory on a magnetic circle when $L \ll 1/M_{\rm eff}$ and provides a completion of the usual (1 + 4)-dimensional NCYM. Its strong coupling is OM theory.

In summary, when $L \ll 1/M_{\rm eff}$ and the relevant energy scale $\ll 1/\tilde{g}^2_{\rm NCYM}$, both OM theory and our open D2-brane theory can be effectively described by the usual (1 + 4)-dimensional NCYM. When we have only $L \ll 1/M_{\rm eff}$, OM theory reduces to our open D2-brane theory. In other words, OM theory provides a completion of our open D2-brane in coupling while our open D2-brane provides an completion of the usual (1 + 4)-dimensional NCYM in energy.

[9] Our convention here for H_{012} differs from that used in [8] by a factor of $(2\pi)^2$.

5.2 OM theory on an electric circle and 5-D noncommutative tensor field theory

As discussed in [8], the compactification of OM theory on an electric circle (say in the 2 direction) with proper (also coordinate) radius R gives (1 + 4)-dimensional NCOS with the following parameters:

$$
\begin{aligned}
\alpha' &= \frac{1}{RM_p^3}\,, \quad g_s = (RM_p)^{3/2}\,, \quad 2\pi\alpha' F_{01} = \alpha' R H_{012} = 1 - \frac{M_{\rm eff}^3}{M_p^3}\,, \\
g_{\mu\nu} &= \eta_{\mu\nu}\ (\mu,\nu = 0,1)\,, \quad g_{ij} = \frac{2M_{\rm eff}^3}{M_p^3}\delta_{ij}\ (i,j = 3,4,5)\,, \\
g_{mn} &= \frac{2M_{\rm eff}^3}{M_p^3}\delta_{mn}\,, \ (m,n = \text{transverse})\,,
\end{aligned} \tag{5.5}
$$

where $M_p \to \infty$ is understood. Comparing with the decoupling limit for NCOS given in (4.1) for $p = 4$, we have

$$
\epsilon = \frac{2M_{\rm eff}^3}{M_p^3}\,, \quad \alpha'_{\rm eff} = \frac{1}{2RM_{\rm eff}^3}\,, \quad G_o^2 = \sqrt{2}(RM_{\rm eff})^{3/2}\,. \tag{5.6}
$$

It is not difficult to see that $G_o \gg 1$ implies $R \gg 1/M_{\rm eff}$. In other words, the circle appears uncompactified. Therefore, OM theory provides a completion of the (1 + 4)-dimensional NCOS in coupling. On the other hand, if $G_o \ll 1$, we have $R \ll 1/M_{\rm eff}$. This is to say that the circle is invisible to OM theory. Since one of the dimensions of the open membrane in OM theory is wrapped on this circle, we therefore end up with the above NCOS theory.

Again as discussed in the Introduction, we can instead focus on the magnetic 3-form field H_{345} rather than on the electric one. The question is: what is the decoupled theory in this case? Let us examine the decoupling limit. Since the change here is to replace F_{01} by H_{345}, we therefore have the following:

$$
\begin{aligned}
\alpha' &= \epsilon\alpha'_{\rm eff}\,, \quad g_s = \frac{G_o^2}{\sqrt{\epsilon}}, \quad H_{345} = \left(\frac{2M_{\rm eff}^3}{M_p}\right)^{3/2}\frac{\sinh\beta}{(2\pi)^2} = \frac{2M_{\rm eff}^3}{(2\pi)^2} = \frac{1}{(2\pi)^2\alpha'^{3/2}_{\rm eff}G_o^2}\,, \\
g_{\mu\nu} &= \eta_{\mu\nu}\ (\mu,\nu = 0,1)\,, \quad g_{ij} = \epsilon\delta_{ij}\ (i,j = 3,4,5)\,, \\
g_{mn} &= \epsilon\delta_{mn}\,, \ (m,n = \text{transverse})\,,
\end{aligned} \tag{5.7}
$$

where the parameters $\epsilon, \alpha'_{\rm eff}$ and G_o are given in (5.6). Note that our convention for the above H_{345} differs from that given in [8]: our H_{234} corresponds to $-H_{345}/(2\pi)^2$ used in [8]. With this in mind, the above limit gives precisely the one in (4.4) for $p = 4$. As discussed in the previous section, this limit gives a (1 + 4)-dimensional tensor field theory defined on a noncommutative geometry which is determined upon the quantization of the boundary action (4.10).

This (1 + 4)-dimensional tensor field theory is expected to be an effective theory and its completion is the (1 + 4)-dimensional NCOS.

6. Relation to ODp theories from NS5-branes

As discussed in the Introduction, the existence of ODp theories from NS5-branes for $p \leq 5$, as discovered independently in [8, 10], can be traced back to the fact that an open Dp-brane can end on NS5-branes. These ODp theories are also related to the known NCOS theories (for example, the $(1+5)$ NCOS) and to each other through S- and T-dualities and their other properties have also been discussed in [8].

The scaling limits for these ODp are given in [8] as

$$\begin{aligned}
\bar{\alpha}' &= \epsilon^{1/2}\bar{\alpha}'_{\rm eff}\,, \quad g_s^{(p)} = \epsilon^{(3-p)/4}\bar{G}^2_{o(p)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu},\ (\mu,\nu = 0,1,\cdots,p)\,,\\
g_{ij} &= \epsilon\delta_{ij}\ (i,j = (p+1),\cdots 5)\,, \quad g_{mn} = \epsilon\delta_{mn},\ (m,n = \text{transverse})\,,\\
\epsilon^{01\cdots p}C_{01\cdots p} &= \frac{1}{(2\pi)^p\bar{G}^2_{o(p)}\bar{\alpha}'^{(p+1)/2}_{\rm eff}}\left(\frac{1}{\epsilon} - \frac{1}{2}\right),\\
C_{(p+1)\cdots 5} &= \frac{1}{(2\pi)^{4-p}\bar{G}^2_{o(p)}\bar{\alpha}'^{(5-p)/2}_{\rm eff}}\,. \qquad (6.1)
\end{aligned}$$

In the above, both a RR $(1+p)$-form and a RR (5 - p)-form potentials are included for defining the ODp. These constant RR potentials can be traded to the corresponding NS5-brane worldvolume $(1+p)$-form field strength $H_{01\cdots p}$ and (5 - p)-form field strength $H'_{(5-p)\cdots 5}$. Given the fact that the two are related to each other by the worldvolume Poincaré duality for $p = 2$ case, we expect that the two are related so for a general $p \leq 5$. In other words, the (1 + p)-form field strength H_{1+p} and the (5 - p)-form field strength $H'_{(5-p)}$ are not independent to each other but related by the worldvolume Poincaré duality. This is consistent with the low energy field contents on a NS5-brane in either IIA or IIB string theory for which we don't have such two independent field strengths living on the NS5-brane worldvolume at the same time. To avoid doubly counting degrees of freedom, we allow only one of them present at one time except for the case of $p = 2, 5$. For the $p = 2$ case, we still have only one 3-form field strength but with two nonvanishing components related to each other by the non-linear worldvolume Poincaré duality. For the $p = 5$ case, neither the 6-form field strength nor the the 0-form one carries local dynamics on the NS5 brane. For this reason, they are allowed to present at the same time. We therefore interpret that the decoupling limit for ODp given in [8] should include only the $C_{01\cdots p}$ not the $C_{(p+1)\cdots 5}$ one except for $p = 2, 5$ cases. This will affect the interpretations for some of the ODp theories given in [8].

For different ODp, the origin of the worldvolume background field $H_{01\cdots p}$ is different. Let us explain this briefly. For $p = 0$, the D0-brane used in defining OD0 theory couples to a 1-form field strength. This 1-form must be a derivative of one of the five scalars in the (2, 0) tensor multiplet. Since this scalar interacts with D0 brane charge and therefore must be the zero mode associated with the compactified direction transverse to the original M5-brane which is now the NS5-brane in IIA. The Poincaré dual of this 1-form field strength on the NS5-brane worldvolume gives a 5-form field strength whose potential couples to the boundary of the open D4 brane ending on the NS5-brane. The critical electric field limit of this 5-form field strength, which is actually Poincaré dual to a magnetic-like 1-form H_5, defines the OD4 theory. For even p, only the OD2 theory is defined as the critical field limit of the self-dual field strength H_{012} in the (2, 0) tensor multiplet.

For odd p, the NS5-brane is in type-IIB string theory. The low energy field content on the NS5-brane is the (1, 1) vector multiplet. The OD1 theory results from the critical electric field strength H_{01} whose potential is in the (1, 1) vector multiplet. The OD3 theory results from a near-critical 4-form field strength H_{0123} which is Poincaré dual to the magnetic-like 2-form field strength H_{45}. So the origin of this 4-form field strength is also clear. However, we have neither a 6-form field strength nor a 0-form field strength in the (1, 1) vector multiplet. Actually, a 6-form or a 0-form field strength in (1 + 5)-dimensions carries no local dynamics. For this reason, both of the 6-from and the 0-form can appear at the same time. So for OD5, we can also have both the 6-form H_{012345} and a 0-form H. Because of this, we don't have a well-defined S-dual of OD5 as discussed in [8].

One of purposes in this section is to show that the open Dp-brane and the NCYM theories discussed in sections 3 and 4 are also implied by the ODp theories given our above interpretation for the NS5-brane worldvolume fields. For convenience, we rewrite the scaling limits for ODp except for $p = 2, 5$ case using our interpretation as

$$\begin{aligned}
\bar{\alpha}' &= \epsilon^{1/2}\bar{\alpha}'_{\rm eff}\,, \quad g_s^{(p)} = \epsilon^{(3-p)/4}\bar{G}^2_{o(p)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu},\ (\mu,\nu = 0,1,\cdots,p)\,,\\
g_{ij} &= \epsilon\delta_{ij}\ (i,j = (p+1),\cdots,5)\,, \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,,\\
\epsilon^{01\cdots p}H_{01\cdots p} &= \frac{1}{(2\pi)^p\bar{G}^2_{o(p)}\bar{\alpha}'^{(p+1)/2}_{\rm eff}}\left(\frac{1}{\epsilon}-\frac{1}{2}\right),
\end{aligned}\tag{6.2}$$

Let us point out first that except for the dimensionality (here it is (1 + 5)-dimensions), the scalings for the OD$(p-2)$- theories in eq. (6.2) look exactly the same as those for our $(1+p)$-dimensional open D$(p-2)$-brane theories discussed in section 3 for $p \leq 5$. We now explore the connection between these two.

For this purpose, let us consider $p = 3$ in eq. (6.2). The decoupling limit for this OD3 is

$$\begin{aligned}
\bar{\alpha}' &= \epsilon^{1/2}\bar{\alpha}'_{\rm eff}\,, \quad g_s^{(3)} = \bar{G}^2_{o(3)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu},\ (\mu,\nu = 0,1,\cdots,3)\,,\\
g_{ij} &= \epsilon\delta_{ij}\ (i,j = 4,5)\,, \quad g_{mn} = \epsilon\delta_{mn},\ (m,n = \text{transverse})\,,\\
\epsilon^{0123}H_{0123} &= \frac{1}{(2\pi)^2\bar{G}^2_{o(3)}\bar{\alpha}'^2_{\rm eff}}\left(\frac{1}{\epsilon}-\frac{1}{2}\right).
\end{aligned}\tag{6.3}$$

For a better understanding of what follows, we digress to give a discussion of the little string sector in ODp theory. As discussed in [8], there is a little closed string sector in each of the ODp theories which provides the completion of the low energy field theory for each of the ODp theories. The existence of the little closed strings can be deduced as follows: the low energy theory for both the (1 + 5)-dimensional NCOS and OD1 is the (1 + 5)-dimensional Yang-Mills. The instanton solution of this Yang-Mills is a closed string with its tension inversely proportional to the Yang-Mills coupling, i.e., $1/g_{\rm YM}^2 \sim 1/\bar{\alpha}'_{\rm eff}$. (Note that OD1 has parameters $\bar{\alpha}'_{\rm eff}, \bar{G}_{(1)}$ and the NCOS has parameters $\alpha'_{\rm eff}, G_o$ with $\bar{G}_{(1)} = 1/G_o, \bar{\alpha}'_{\rm eff} = \alpha'_{\rm eff} G_o^2$ due to the S-dual relation between the two. Both the OD1 and NCOS have the same tension $T_{\rm eff} = 1/4\pi\alpha'_{\rm eff} = 1/4\pi\bar{G}_{(1)}^2\bar{\alpha}'_{\rm eff}$.)

For NCOS, in the limit $\alpha'_{\rm eff} \to 0, G_o \to \infty$ with $g_{\rm YM}^2$ held fixed, the noncommutative parameter $\Theta^{01} = 2\pi\alpha'_{\rm eff} \to 0$ and the NCOS tension blows up. We therefore end up with a complete Lorentz invariant theory in this limit. As we know that the (1 + 5)-dimensional Yang-Mills is incomplete and the little string remains light in this limit. For this reason, it was conjectured in [8] that the NCOS in this limit reduces to the little string theory.

In the S-dual theory of the above, i.e., OD1, the above limit says that $\bar{G}_{(1)} \to 0$ with $\bar{\alpha}'_{\rm eff}$ kept fixed. The above conjecture implies that the OD1 in this limit reduces to type IIB little string theory since $\bar{G}_{(1)} = 0$ at fixed $\bar{\alpha}'_{\rm eff}$. Since the tension for little strings in OD1 theories remains unchanged under T-dualities, it was concluded in [8] that there is a closed little string sector in each of the ODp theories.

We now return to discuss the OD3 theory. There is a closed little string sector in this theory with the string tension $T = 1/4\pi\bar{\alpha}'_{\rm eff}$. As we will show later in this section, we can have two identical (1 + 5)-dimensional Noncommutative Yang-Mills theories, one is obtained from open F-string ending on D5-branes while the other from open D-string ending on NS5-branes in their respective decoupling limits. The two theories are once again S-dual invariant and have the same noncommutative parameter $\Theta^{45} = 2\pi\bar{\alpha}'_{\rm eff}\bar{G}_{o(3)}^2 = 2\pi\tilde{\alpha}'_{\rm eff}$ and the same gauge coupling $g_{\rm NCYM}^2 = \tilde{g}_{\rm NCYM}^2 = (2\pi)^3\bar{\alpha}'_{\rm eff}$. In the above, we have $\bar{G}_{o(3)} = 1/\tilde{G}_{o(3)}, \bar{\alpha}'_{\rm eff} = \tilde{\alpha}'_{\rm eff}\tilde{G}_{o(3)}^2$. In the limit $\bar{G}_{o(3)} \to 0$ with $\bar{\alpha}'_{\rm eff}$ held fixed (or $\tilde{G}_{o(3)} \to \infty, \tilde{\alpha}'_{\rm eff} \to 0$ but with $\tilde{\alpha}'_{\rm eff}\tilde{G}_{o(3)}^2$ held fixed), the NCYM reduces to the (1 + 5)-dimensional Yang-Mills which is actually the same as the low energy theory of both (1 + 5)-dimensional NCOS and OD1 which will be discussed later in this section.

The complete description of the (1 + 5)-dimensional ordinary Yang-Mills is given by the type IIB little string theory. In other words, in the above limit $\bar{G}_{o(3)} \to 0$ with $\bar{\alpha}'_{\rm eff}$ kept fixed, the complete description of the NCYM is given by the little string theory. In the same limit, the tension for OD3-brane is $\sim 1/\bar{G}_{o(3)}^2\bar{\alpha}'^2_{\rm eff}$ which blows up while the tension for the closed little strings remains finite. Therefore the little strings are light and we expect that the OD3 reduces to the little string theory. In other words, the complete description of both OD3 and NCYM in this limit is in terms of type IIB little string theory.

When the noncommutative parameter remains nonvanishing, i.e., with both $\bar{\alpha}'_{\rm eff}$ and $\bar{G}_{o(3)}$ finite, what is the complete description of the NCYM? It cannot be the little string theory any more and the natural answer is the OD3 theory.

For this reason, using our decoupling limit eq. (6.3) and the further discussion in what follows, we interpret the OD3 theory to be self-dual rather than to be S-dual to the usual (1 + 5)-dimensional NCYM since a complete theory cannot be mapped to an incomplete one under S-duality. This case is quite different from that in (1 + 3)-dimensions where the NCYM is also a complete theory.

If we S-dual this OD3 theory, we end up with another OD3 theory whose scalings look identical to the original ones except for some changes for the fixed parameters $\bar{\alpha}'_{\rm eff}, \bar{G}^2_{o(3)}$. This is due to the fact that the D3-brane is intact under S-duality.[10] The only possible effects associated with the base NS5-brane in the decoupling limit are on the closed string constant $\bar{\alpha}'$ and the closed string coupling $g_s^{(3)}$. It turns out that their scalings remain the same under S-duality for this case, a welcome and yet expected result. If we denote with $\tilde{A}$ as the S-dual of quantity A which is not invariant under S-duality, we have

$$\bar{\alpha}' \to \tilde{\alpha}' = \bar{\alpha}' g_s^{(3)} = \epsilon^{1/2} \tilde{\alpha}'_{\rm eff}\,, \quad g^{(3)} \to \tilde{g}^{(3)} = \frac{1}{g^{(3)}} = \frac{1}{\bar{G}^2_{o(3)}} = \tilde{G}^2_{o(3)}\,, \tag{6.4}$$

for which we insist that the closed string metric remains the same as before.[11] This also implies that the D3 brane tension $\sim 1/(\bar{\alpha}'^2 g_s^{(3)})$ remains invariant under S-duality, again a welcome and yet expected result. This further implies that the OD3 tension $\sim 1/(\bar{\alpha}'^2_{\rm eff} \bar{G}^2_{o(3)})$ also remains invariant under S-duality which is consistent with the fact that H_{0123} (or C_{0123}) is intact under S-duality. Given that the closed string metric, the proper tension of the D3-brane ending on the NS5-brane and the near-critical electric field C_{0123} all remain unchanged under S-duality, we therefore still

[10]This is manifest by the fact that the near-critical electric field H_{0123} is intact under S-duality. This becomes more clear if we use C_{0123} rather than the worldvolume H_{0123}.

[11]The notion that the string constant α' transforms under S-duality is due to our choice that the asymptotic string-frame metric does not change under S-duality. This is an effective way in implementing S-duality which is also useful. The original S-duality requires the Einstein-frame metric and α' to be invariant under S-duality. Let us demonstrate the above two cases in the following simple examples: a) If we insist that the asymptotic string metric remain the same but the $\alpha' \to \tilde{\alpha}' = \alpha' g_s$, we have $(1/\alpha') \int \partial X^M \partial X^N g_{MN} \to (1/\tilde{\alpha}') \int \partial X^M \partial X^N g_{MN}$ under S-duality. This basically says that a fundamental string with its parameter α' is mapped to another fundamental string with its parameter $\tilde{\alpha}' = \alpha' g_s$. However, if we interpret this new string in its original α', it is a D-string. b) If we insist that only Einstein metric and α' remain invariant under S-duality, we have $(1/\alpha') \int \partial X^M \partial X^N e^{\phi/2} g^E_{MN} \to (1/\alpha') \int \partial X^M \partial X^N e^{-\phi/2} g^E_{MN} = (1/g_s\alpha') \int \partial X^M \partial X^N g_{MN}$ where we have used the relation $g = e^{\phi/2} g^E$ in relating the original string-frame metric g to its Einstein-frame metric g^E in the last step. We have also used $\phi \to -\phi$ under S-duality. If we interpret this string in the original string metric, this S-dual string is a D-string because of the tension is now $\sim 1/(\alpha' g_s)$. However, it is still a fundamental string if we use the S-dual string metric which is now $\tilde{g} = g/g_s$. Therefore the above two pictures don't lead to any inconsistency. It is merely a choice of attributing the change to the metric or to the string constant α'.

have an open D3-brane theory under S-duality as claimed above with the following decoupling limit:

$$\begin{aligned}
\tilde{\alpha}' &= \epsilon^{1/2}\tilde{\alpha}'_{\rm eff}\,, \quad \tilde{g}_s^{(3)} = \tilde{G}^2_{o(3)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}\ (\mu,\nu = 0,1,2,3)\,, \\
g_{ij} &= \epsilon\delta_{ij}\ (i,j = 4,5), \quad g_{mn} = \epsilon\delta_{mn}\ (m,n = \text{transverse})\,, \\
\epsilon^{0123}H_{0123} &= \frac{1}{(2\pi)^2\tilde{G}^2_{o(3)}\tilde{\alpha}'^2_{\rm eff}}\left(\frac{1}{\epsilon} - \frac{1}{2}\right),
\end{aligned} \tag{6.5}$$

where we have

$$\tilde{\alpha}'_{\rm eff} = \bar{\alpha}'_{\rm eff}\bar{G}^2_{o(3)}\,, \quad \tilde{G}^2_{o(3)} = \frac{1}{\bar{G}^2_{o(3)}}\,, \tag{6.6}$$

which implies $\tilde{\alpha}'^2_{\rm eff}\tilde{G}^2_{o(3)} = \bar{\alpha}'^2_{\rm eff}\bar{G}^2_{o(3)}$. This new open D3-brane theory has the same tension as the original one but its coupling $\tilde{G}^2_{o(3)}$ is inversely related to the original one as indicated above. Therefore when one OD3 theory is strongly coupled, the other is weakly coupled and vice-versa. This new OD3 theory is just the open D3 brane theory discussed in section 3. Our above discussion on the relation between the OD3 and the NCYM from the decoupling of the open D-string ending on NS5-branes is also consistent with that between our open D3-brane theory and the NCYM from the decoupling of the F-string ending on D5-branes discussed in section 3. In other words, either OD3 or our open D3-brane theory provides the complete description of the corresponding NCYM. This further implies that there should exist a little string sector in our open D3-brane theory. In other words, our open D3-brane theory reduces to the little string theory in the limit $\tilde{G}_{o(3)} \to \infty$ but with $\tilde{\alpha}'_{\rm eff}\tilde{G}^2_{o(3)}$ held fixed. One can check indeed that the closed little strings remain light while our open D3-brane tension blows up in this limit.

Subsequent applications of T-duality on this new open D3-brane theory along x^3, x^2, x^1 as described in section 3 will give our open Dp brane theories for $p \leq 3$. Therefore, the OD$(p-2)$- theories from NS5-branes also imply the existence of those $(1+p)$-dimensional open D$(p-2)$-brane theories discovered in this paper.

It is clear now that the OD$(p-2)$- theories from NS5-branes and those found in this paper are U-duality related. Let us make some further comparisons between them. First for $p \leq 5$, our open D$(p-2)$-brane theories live in $(1+p)$-dimensions while those from NS5-brane always live in $(1+5)$-dimensions. Assuming the respective compactification radii to be the same, we have the ratio $\tilde{G}^2_{o(p-2)}/\bar{G}^2_{o(p-2)} = 1/\bar{G}^{p-1}_{o(3)}$. If $\bar{G}_{o(3)} > 1$, then $\tilde{G}_{o(p)} < \bar{G}_{o(p)}$ and the other way around if $\bar{G}_{o(3)} < 1$. Further $\tilde{G}^2_{o(p-2)}\tilde{\alpha}'^{(p-1)/2}_{\rm eff} = \bar{G}^2_{o(p-2)}\bar{\alpha}'^{(p-1)/2}_{\rm eff}$. This implies that our open D$(p-2)$-brane theory and that from NS5-brane have the same proper tension and the same near-critical electric field $H_{01\cdots(p-2)}$. The bulk metric in both cases remain the same. Therefore, the reason that our open D$(p-2)$-brane theory can only see $(1+p)$-dimensions while those from NS5-brane always see $(1+5)$-dimensions may be due to the difference in their couplings.

For $p = 5$, as discussed in section 3, the open D3-brane provides a completion of the usual (1 + 5)-dimensional NCYM. Our discussion above says that the S-duality of this open D3-brane theory is the OD3. This indicates that the S-duality of the usual (1 + 5)-dimensional NCYM gives another (1 + 5)-dimensional NCYM. It is for this case that our interpretation differs from that given in [8] where the S-duality of OD3 was interpreted to give the usual (1 + 5)-dimensional NCYM. The question is: what is the new (1 + 5)-dimensional NCYM? This is the topic to which we turn next.

Following the discussion given in section 3 and 4, we expect that we might have noncommutative field theories for $p = 0, 1, 3, 4$ if the open D$(4-p)$-brane description is insisted with the following scaling limits

$$\begin{aligned} \bar{\alpha}' &= \epsilon^{1/2}\bar{\alpha}'_{\rm eff}\,, \quad g_s^{(p)} = \epsilon^{(3-p)/4}\bar{G}^2_{o(p)}\,, \quad g_{\mu\nu} = \eta_{\mu\nu}, \ (\mu,\nu = 0,1,\cdots,p)\,, \\ g_{ij} &= \epsilon\delta_{ij} \ (i,j = (p+1),\cdots 5)\,, \quad g_{mn} = \epsilon\delta_{mn}, \ (m,n = \text{transverse})\,, \\ H_{(p+1)\cdots 5} &= \frac{1}{(2\pi)^{4-p}\bar{G}^2_{o(p)}\bar{\alpha}'^{(5-p)/2}_{\rm eff}}\,. \end{aligned} \tag{6.7}$$

Let us examine the action of open D$(4-p)$-brane ending on NS5-branes:

$$\begin{aligned} S_{(4-p)} &= -\frac{1}{2(2\pi)^2\bar{\alpha}' g_s^{(p)}}\int_{M^{5-p}} d^{5-p}\sigma\sqrt{-\det\gamma}\left(\gamma^{\alpha\beta}\partial_\alpha X^M\partial_\beta X^N g_{MN} - (2\pi)^2(3-p)\alpha'\right) + \\ &\quad + \int_{M^{5-p}} H_{5-p}\,. \end{aligned} \tag{6.8}$$

With the scaling limits (6.7), we have

$$\begin{aligned} S_{(4-p)} = &-\frac{1}{2(2\pi)^2\bar{\alpha}'_{\rm eff}\bar{G}^2_{o(p)}}\int_{M^{5-p}} d^{5-p}\sigma\sqrt{-\det\gamma}\,\times \\ &\times\Big[\epsilon^{-(p-5)/4}\gamma^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}\epsilon^{(p-1)/4}\gamma^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j\delta_{ij} + \\ &\quad + \epsilon^{(p-1)/4}\gamma^{\alpha\beta}\partial_\alpha Y^m\partial_\beta Y^n\delta_{mn} - \epsilon^{(p-3)/4}(2\pi)^2(3-p)\Big] + \\ &\quad + \int_{M^{5-p}} H_{5-p}\,. \end{aligned} \tag{6.9}$$

where we denote Y^m as the bulk modes along the directions transverse to the NS5-brane.

Except for the $p = 1$ case, the only finite part of the above action is the bulk topological term which can be expressed in terms of the following boundary action (except for the $p = 4$ case)

$$\frac{1}{(5-p)!}\int_{\partial M^{5-p}} d^{4-p}\xi\epsilon^{\alpha_0\alpha_1\cdots\alpha_{3-p}}\partial_{\alpha_0}X^{i_1}\partial_{\alpha_1}X^{i_2}\cdots\partial_{\alpha_{3-p}}X^{i_{4-p}}X^{i_{5-p}}\mathcal{H}_{i_1 i_2\cdots i_{5-p}}\,. \tag{6.10}$$

In other words, we can have noncommutative field theories for $p = 0, 3$ upon the quantization of the above action which determines the geometry of the base NS5-brane. For $p = 0$, this appears to be a noncommutative (2, 0) theory. Since the background field used in defining this theory comes from the magnetic dual of the derivative of the scalar in (2, 0) theory, whether we indeed have such a noncommutative field theory needs further investigation. For $p = 3$, we end up with the aforementioned NCYM which can actually be identified with the usual NCYM. We will show this later on.

The $p = 4$ case does not give noncommutativity and therefore we expect that we end up with the usual (2, 0) theory. For $p = 1$, the bulk modes X^i, Y^m remain even with the decoupling limit. This may indicate that we don't have a decoupled noncommutative field theory. This also indicates that the (1 + 5)-dimensional noncommutative tensor field theory discussed in section 4 may not be well-defined either since it is expected to be related to the present one by S-duality.

We now discuss the $p = 3$ case mentioned above. The quantization of the boundary action (6.10) for this case gives

$$[x^4, x^5] = -i2\pi\bar{\alpha}'_{\text{eff}}\bar{G}^2_{(3)} \,, \tag{6.11}$$

which gives the noncommutative $\Theta^{45} = -2\pi\bar{\alpha}'_{\text{eff}}\bar{G}^2_{(3)}$.

Given the S-dual relation between the open F-string ending on D5 branes and open D-string ending on NS5-branes, we expect, as before, that the open D-string metric, the noncommutative parameter and the gauge coupling can be calculated with the scaling limit (6.7) using the following Seiberg-Witten relations:

$$\begin{aligned} G_{\alpha\beta} &= g_{\alpha\beta} - (2\pi\bar{\alpha}' g_s^{(3)})^2 (Hg^{-1}H)_{\alpha\beta} \,, \\ \Theta^{\alpha\beta} &= 2\pi\bar{\alpha}' g_s^{(3)} \left(\frac{1}{g + 2\pi\bar{\alpha}' g_s^{(3)} H} \right)_A^{\alpha\beta} \,, \\ \frac{1}{g^2_{\text{NCYM}}} &= \frac{g_s^{(3)}}{(2\pi)^3 (\bar{\alpha}' g_s^{(3)})} \left(\frac{\det(g + 2\pi\bar{\alpha}' g_s^{(3)} H)}{\det G} \right)^{1/2} \,, \end{aligned} \tag{6.12}$$

where $\alpha, \beta = 0, 1, \ldots, 5$. We find

$$G_{\alpha\beta} = \eta_{\alpha\beta} \,, \quad \Theta^{45} = -2\pi\bar{\alpha}'_{\text{eff}}\bar{G}^2_{(3)} \,, \quad g^2_{\text{NCYM}} = (2\pi)^3\bar{\alpha}'_{\text{eff}} \,. \tag{6.13}$$

The fixed open D-string metric indicates that we indeed end up with a noncommutative field theory. The noncommutative parameter is the same as the one calculated above from the quantization of the boundary action. Let us understand the above Yang-Mills coupling. Since an open D-string ending on NS5-branes are S-dual to an open F-string ending on D5-branes, we expect that the bulk scaling limits for this NCYM are S-dual to those for the usual NCYM. This further implies that the parameters for the two decoupled NCYM are related to each other. Let us find these relations. The scaling limits for the usual (1 + 5)-dimensional NCYM are given in (3.1). Under S-duality, we have

$$\tilde{\alpha}' \to \bar{\alpha}' = \tilde{\alpha}' \tilde{g}_s^{(3)}\,, \quad \tilde{g}_s^{(3)} \to g_s^{(3)} = \frac{1}{\tilde{g}_s^{(3)}}\,. \tag{6.14}$$

From the above, we have

$$\bar{G}_{o(3)}^2 = \frac{(2\pi)^3 \tilde{\alpha}'_{\rm eff}}{\tilde{g}_{\rm NCYM}^2} = \frac{1}{\tilde{G}_{(3)}^2}\,, \quad \bar{\alpha}'_{\rm eff} = \frac{\tilde{g}_{\rm NCYM}^2}{(2\pi)^3} = \tilde{\alpha}'_{\rm eff} \tilde{G}_{(3)}^2\,. \tag{6.15}$$

With this, we have

$$\Theta^{45} = 2\pi \bar{\alpha}'_{\rm eff} \bar{G}_{o(3)}^2 = 2\pi \tilde{\alpha}'_{\rm eff}\,, \quad g_{\rm NCYM}^2 = (2\pi)^3 \bar{\alpha}'_{\rm eff} = \tilde{g}_{\rm NCYM}^2\,. \tag{6.16}$$

In other words, the two NCYM theories have the same parameters and they can actually be identified. Again this is just the consequence of S-duality. We have seen this for the two (1 + 3)-dimensional NCYM theories discussed in section 4. In other words, the NCYM keeps intact under S-duality.

At low energies, all these (1 + 5)-dimensional decoupled theories (i.e., the NCOS, OD1, OD3, our open D3-brane theory and the NCYM) from type IIB string theory are expected to give the usual (1 + 5)-dimensional Yang-Mills. The question is: Can we have a unique usual Yang-Mills? We can check this at least for the NCOS, OD1 and the NCYM. For the NCYM, from the above, we can see that the low energy limit can be achieved by insisting $\tilde{\alpha}'_{\rm eff} \to 0$ while keeping $\bar{\alpha}'_{\rm eff}$ fixed. This in turn implies that we set $\bar{G}_{(3)}^2 \to 0$.

For the NCOS, it reduces to the usual Yang-Mills with gauge coupling $g_{\rm YM}^2 = (2\pi)^3 G_o^2 \alpha'_{\rm eff}$ as given in [8]. For the OD1, it reduces to

$$\begin{aligned} S &= \frac{\bar{G}_{(1)}^2}{4(2\pi)^3 \bar{\alpha}' g_s^{(1)}} \int d^6x \sqrt{-G} G^{AC} G^{BD} \hat{H}_{AB} \hat{H}_{CD}\,, \\ &= \frac{1}{4(2\pi)^3 \bar{\alpha}'_{\rm eff}} \int d^6x \eta^{AC} \eta^{BD} \hat{H}_{AB} \hat{H}_{CD}\,, \end{aligned} \tag{6.17}$$

where the open D-string metric $G_{AB} = \epsilon \eta_{AB}$ has been used. From the above, we have $g_{\rm YM}^2 = (2\pi)^3 \bar{\alpha}'_{\rm eff}$.

Since the NCOS (with parameters $\alpha'_{\rm eff}, G_o$) is S-dual to OD1 (with parameters $\bar{\alpha}'_{\rm eff}, \bar{G}_{(1)}$), we have the following

$$\bar{G}_{(1)}^2 = \frac{1}{G_o^2}\,, \quad \bar{\alpha}'_{\rm eff} = \alpha'_{\rm eff} G_o^2\,. \tag{6.18}$$

This implies that the low energy Yang-Mills theories from the above three different theories are actual the same since the gauge coupling is the same. This is different from the (1 + 3)-dimensional case discussed at the end of section 4.

7. $(1+3)$-dimensional open (p, q)-string theory

The discussion given in the previous sections hints already that we have interesting story in (1 + 3)-dimensions. For example, our (1 + 3)-dimensional open D-string theory discussed in section 3 is equivalent to the usual (1 + 3)-dimensional NCYM. We intend to give explanations for related issues in this section.

In [6], it was shown that the S-duality of (1 + 3)-dimensional NCYM gives (1 + 3)-dimensional NCOS. This conclusion, in spite of its correctness, does raise the following puzzles: a) Why is this true only for the (1 + 3)dimensional NCYM, not for the (1 + 5)-dimensional one, for example? b) How can we reconcile this with the belief that the non-perturbative quantum $\mathrm{SL}(2, \mathbb{Z})$ symmetry of the parent type IIB string theory is actually inherited to its decoupled sub-theory (we call it the little type IIb string theory) without gravity?

As we know that the existence of D-string or in general a (p, q)-string is a consequence of this $\mathrm{SL}(2, \mathbb{Z})$ symmetry in the non-perturbative type IIB string theory. By the same token, if we have $\mathrm{SL}(2, \mathbb{Z})$ symmetry for the little type IIb string theory, the existence of (1 + 3)-dimensional NCOS should imply a (1 + 3)-dimensional open D-string or in general a (1 + 3)-dimensional open (p, q)-string theory. However, the above conclusion given in [6] says that the S-dual of the NCOS is the usual (1 + 3)-dimensional NCYM.

The (1 + 3)-dimensional open D-string found in section 3 resolves this puzzle. First the existence of this theory is consistent with the S-duality. Second that this theory is equivalent to the usual (1 + 3)-dimensional NCYM as discussed in section 3 is also consistent with the S-duality between the (1 + 3)-dimensional NCOS and the usual NCYM. Our interpretation for S-duality is a bit different from that given in [6] where a worldvolume Poincaré duality is also employed as discussed in section 4. In terms of our interpretation, the (1 + 3)-dimensional NCYM is actually S-dual invariant while our open D-string theory is S-dual to the NCOS.

Our picture of S-duality for the decoupled theories from the parent type IIB string theory is as follows: In general, a decoupled open brane theory is S-dual to another decoupled open brane theory while a decoupled field theory is S-dual to another decoupled field theory. The examples are: a) (1 + 3)-dimensional NCOS is S-dual to the (1 + 3)-dimensional open D-string theory in this paper, (1 + 5)-dimensional NCOS is S-dual to the (1 + 5)-dimensional OD1 theory and (1 + 5)-dimensional OD3 is S-dual to the (1 + 5)-dimensional open D3-brane theory in this paper. The usual (1 + 3)-dimensional NCYM is S-dual to the (1 + 3)-dimensional NCYM discussed in section 4 (actually self-dual), the usual (1 + 5)-dimensional NCYM is S-dual to the (1 + 5)-dimensional NCYM discussed in the previous section. As discussed in the previous section, an open brane theory should not be in general S-dual to a field theory since the latter may not be complete (due to nonrenomalizability) while the former is generally complete.

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As mentioned above, the reason that the usual (1 + 3)-dimensional NCYM can be S-dual (using the interpretation of [6]) to the (1 + 3)-dimensional NCOS is due to that this NCYM is a complete theory and is equivalent to the (1 + 3)-dimensional open D-string theory. This is, however, not the case in (1 + 5)-dimensions.

Now the remaining question is: Does a general (1 + 3)-dimensional (p, q) open string theory exist? The answer should be yes if the type IIB $SL(2, \mathbb{Z})$ is inherited to the little type-IIb string theory. The existences of both (1 + 3)-dimensional NCOS and open D-string theories, both (1 + 5)-dimensional NCOS and OD1 and the two versions of open D3-brane theory related by S-duality also strongly support this. Given that an open (p, q)-string can end on D3-branes, one expects that a force due to a proper background field can balance the tension. For examples, in the simplest context, if we apply only a near-critical electric field B_{01},

$$2\pi\alpha'\epsilon^{01}B_{01} = 1 - \frac{\epsilon}{2}, \tag{7.1}$$

with the usual scaling limit for NCOS,

$$\alpha' = \epsilon\alpha'_{\rm eff}, \quad g_s = \frac{G_o^2}{\sqrt{\epsilon}}, \tag{7.2}$$

we have

$$-\frac{1}{2\pi\alpha'}\sqrt{p^2 + q^2/g_s^2} + p\epsilon^{01}B_{01} = -\frac{1}{4\pi p\alpha'_{\rm eff}}\left(p^2 + \frac{q^2}{G_o^4}\right) \tag{7.3}$$

which is finite and is the tension for the decoupled theory which is still a NCOS. Similarly, we can have only a near-critical RR C_{01} and with the decoupling limit for the open D-string theory, we can also end up with a deformed open D-string theory.

Recall that an open (p, q)-string is a non-threshold bound and its ends carry both NSNS and RR charges (or electric and magnetic charges with respect to the D3-brane worldvolume gauge field). So both the background NSNS B_{01} and C_{01} apply forces on this string. A genuine open (p, q)-string theory requires the presence of both the near-critical field B_{01} and C_{01}. Further each of these two fields along with the proper scalings for the closed string coupling and the bulk metric must act in a non-trivial way such that we can end up with a finite tension for the (p, q)-string theory. One can check easily that a naive critical field limit following that for either open D-string theory or NCOS does not work. The investigation on this is in progress and we hope to report this elsewhere. Nevertheless, the finite tension is expected to be

$$T_{(p,q)} = \frac{1}{2\pi\alpha'_{\rm eff}}\sqrt{p^2 + \frac{q^2}{G_o^4}}. \tag{7.4}$$

This should also be true for the (1 + 5)-dimensional open (p, q)-string theory. We expect that the (p, q)-string action proposed in [24, 25] may be useful.

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Note added: during the course of writing up, we become aware that when the spatial directions of the D$(p-2)$-brane are compactified, our $(1+p)$-dimensional open D$(p-2)$-brane theory may be related to the Galilean D$(p-2)$-brane theory discovered in [26] (see also [27, 28]). However, there are differences between these two theories. Let us mention a few: 1)The spatial directions of the brane for our open D$(p-2)$-brane theory can be either non-compact or compact while by definition the spatial directions of the brane for the Galilean D$(p-2)$-brane theory found in [26, 27] must be compact due to the absence of the base D-brane. 2) As a result, our open D$(p-2)$-brane theory lives on $(1+p)$-dimensional Dp-brane worldvolume while the Galilean D$(p-2)$-brane theory lives on the $(1+9)$-dimensional spacetime. 3) The starting points are completely different.

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