Reduction of open membrane moduli

Eric Bergshoeff

Institute for Theoretical Physics, University of Groningen
Nijenborgh 4, 9747 AG Groningen, The Netherlands
E-mail: `bergshoeff@phys.rug.nl`

Jan Pieter van der Schaar

Michigan Center for Theoretical Physics
Randall Laboratory, Department of Physics, University of Michigan
Ann Arbor, MI 48109-1120, USA
E-mail: `jpschaar@umich.edu`

Abstract: We perform a general reduction of the open membrane metric in a worldvolume direction of the M5-brane. Using reduction rules analogous to the bulk, we show that the open membrane metric leads to the standard open string metric and open string coupling constant on the D4-brane only for an "electric" reduction in which case the open membrane metric has no off-diagonal components and the Born-Infeld curvature tensor is a matrix of rank 2. Instead, if we perform a general reduction, with nonzero off-diagonal components of the open membrane metric, we obtain a rank 4 Born-Infeld tensor corresponding to a bound state of an open string with an open D2-brane. Next, we identify and reduce a 3-form open membrane "theta tensor" on the M5-brane which reduces to the open string noncommutativity tensor on the D4-brane provided we constrain ourselves to an "electric" or a "magnetic" reduction.

Contents

1. Introduction

One of the recent intriguing developments in M-theory has been the insight that a certain decoupling limit of the elusive M5-brane leads to an open membrane theory [1, 2]. This open membrane theory, shortly called OM-theory, is a generalization of the non-commutative open strings found on D-branes in string theory [3]. The geometry of OM-theory is rather unclear at the moment. In the same way that the non-commutative open strings give rise to a non-commutative geometry one expects that the open membrane theory gives rise to a non-commutative loop space describing a generalized non-commutative (and perhaps non-associative) geometry [4]. The non-commutative geometry of non-commutative open string theory is characterized by a non-commutativity tensor θ^{ab}. In this paper we will discuss the possibility that one can introduce a 3-form tensor Θ^{abc} for the open membrane that is related to the open string non-commutativity tensor θ^{ab} via dimensional reduction. In the absence of a solid mathematical identification of the non-commutative and perhaps non-associative geometry of open membrane theory we will refrain from calling this tensor a "non-commutativity" tensor but instead refer to it as a (generalized) theta tensor. The possibility of introducing theta parameters for the open membrane has been discussed independently in [5].

The main purpose of this paper is to take the expressions for the open membrane metric and 3-form theta parameter and show under which conditions they reduce to the standard expressions for the open string metric/coupling constant and non-commutativity parameter. This work is a generalization of an earlier work by one of us [6] where, in order to fix the conformal factor of the open membrane metric, a specific reduction of the open membrane metric was considered leading to a rank 2 Born-Infeld matrix on the D4-brane. In this paper we will analyze in which sense the results of [6] can be generalized to the rank 4 case.

Our starting point is the M5-brane. For some basic M5-brane preliminaries we refer to [6]. We consider the following six-dimensional symmetric tensor defined on a single (abelian) M5-brane ($\hat{a}, \hat{b} \in (0, 1, \ldots, 5)$)[1]

$$\hat{C}^{\hat{a}\hat{b}} = \frac{1}{K} \left[\left(1 + \frac{1}{12}\hat{\mathcal{H}}^2 \right) \hat{g}^{\hat{a}\hat{b}} - \frac{1}{4}(\hat{\mathcal{H}}^2)^{\hat{a}\hat{b}} \right] . \tag{1.1}$$

The gauge invariant 3-form field strength

$$\hat{\mathcal{H}}_{\hat{a}\hat{b}\hat{c}} \equiv \partial_{[\hat{a}} \hat{B}_{\hat{b}\hat{c}]} + \hat{A}_{\hat{a}\hat{b}\hat{c}} \tag{1.2}$$

is defined in terms of a 2-form gauge field $\hat{B}$ living on the M5-brane and the 3-form gauge field $\hat{A}$ of $D = 11$ supergravity. The 3-form $\hat{\mathcal{H}}$ satisfies the following nonlinear self-duality equation on the M5-brane [7]:

$$\hat{C}_{\hat{a}}{}^{\hat{d}} \hat{\mathcal{H}}_{\hat{d}\hat{b}\hat{c}} = \frac{\sqrt{-\det\hat{g}}}{3!} \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}\hat{f}} \hat{\mathcal{H}}^{\hat{d}\hat{e}\hat{f}} . \tag{1.3}$$

We have also defined $(\hat{\mathcal{H}}^2)^{\hat{a}\hat{b}} \equiv \hat{\mathcal{H}}^{\hat{a}\hat{c}\hat{d}} \hat{\mathcal{H}}^{\hat{b}}{}_{\hat{c}\hat{d}}$, $\hat{\mathcal{H}}^2 \equiv \hat{g}_{\hat{a}\hat{b}}(\hat{\mathcal{H}}^2)^{\hat{a}\hat{b}}$ and introduced a function K given by

$$K = \sqrt{1 + \frac{1}{24}\hat{\mathcal{H}}^2} . \tag{1.4}$$

In order that the tensor (1.1) has the correct (OM-metric) signature we will only consider the positive branch of the square root. This implies that $K \geq 1$ if we assume, which we will do from now on, that the 6-dimensional M5-brane worldvolume is flat, i.e. $g^{\hat{a}\hat{b}} = \eta^{\hat{a}\hat{b}}$. The symmetric tensor (1.1) reduces to the so-called Boillat metric of nonlinear DBI electrodynamics [8]. We will refer to this tensor as the M5-brane Boillat metric.

The nonlinear self-duality equation (1.3) leads to the following constraint on $(\hat{\mathcal{H}}^4)^{\hat{a}\hat{b}} \equiv (\hat{\mathcal{H}}^2)^{\hat{a}\hat{c}}(\hat{\mathcal{H}}^2)^{\hat{b}}_{\hat{c}}$:

$$(\hat{\mathcal{H}}^4)^{\hat{a}\hat{b}} = \frac{2}{3}\hat{\mathcal{H}}^2 \left[\hat{\eta}^{\hat{a}\hat{b}} + \frac{1}{2}(\hat{\mathcal{H}}^2)^{\hat{a}\hat{b}} \right] . \tag{1.5}$$

Tracing this equation we find

$$\frac{1}{4}\hat{\mathcal{H}}^4 = \hat{\mathcal{H}}^2 \left(1 + \frac{1}{12}\hat{\mathcal{H}}^2 \right) . \tag{1.6}$$

Using (1.5) one can verify that the inverse of $\hat{C}^{\hat{a}\hat{b}}$ is given by[2]

$$(\hat{C}^{-1})_{\hat{a}\hat{b}} = \frac{1}{K} \left[\hat{\eta}_{\hat{a}\hat{b}} + \frac{1}{4}(\hat{\mathcal{H}}^2)_{\hat{a}\hat{b}} \right] . \tag{1.7}$$

We note that the traces of $\hat{C}^{\hat{a}\hat{b}}$ and its inverse are both equal to $6K$ and that we have, according to [8], the remarkable identity $\det (\hat{C}^{-1})_{\hat{a}\hat{b}} = \det \hat{g}_{\hat{a}\hat{b}}$. The open membrane co-metric is conformal to the M5-brane Boillat metric:

$$\hat{G}^{\hat{a}\hat{b}}_{\text{OM}} = z\, \hat{C}^{\hat{a}\hat{b}} . \tag{1.8}$$

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[1]We use a mostly plus signature convention for the metric. The 3-form fields are dimensionless. We use hats to distinguish the $D = 6$ fields and indices from the $D = 5$ fields and indices.

[2]Because indices are raised and lowered with the worldvolume metric $\hat{g}_{\hat{a}\hat{b}}$, we must clearly distinguish between inverse metrics and co-metrics, e.g. $(\hat{C}^{-1})_{\hat{a}\hat{b}} \neq \hat{C}_{\hat{a}\hat{b}} \equiv \hat{g}_{\hat{a}\hat{c}}\hat{g}_{\hat{b}\hat{d}}\hat{C}^{\hat{c}\hat{d}}$.

The conformal factor z was determined to be equal to [6]

$$z = \left(K - \sqrt{K^2 - 1} \right)^{-1/3} . \tag{1.9}$$

Because $K \geq 1$ we find that $0 < z^{-3} \leq 1$. The expression (1.9) for the conformal factor was recently confirmed using the requirement of deformation independence [5].

2. General reduction of the open membrane metric

To reduce the open membrane co-metric we split the $D = 6$ indices into $\hat{a} = (a, y)$ where y corresponds to a compact direction in the worldvolume of the M5-brane. Next, we identify the $D = 5$ (dimensionless) 2-form and 3-form fields as follows

$$\hat{\mathcal{H}}_{aby} \equiv \mathcal{F}_{ab} \, ,$$
$$\hat{\mathcal{H}}_{abc} \equiv \mathcal{H}_{abc} \, . \tag{2.1}$$

As a consequence of the nonlinear self-duality equation in $D = 6$ the 3-form $\mathcal{H}$ and the 2-form $\mathcal{F}$ are related through a set of nonlinear duality equations given by

$$\hat{C}_a{}^d \mathcal{H}_{dbc} + \hat{C}_a{}^y \mathcal{F}_{bc} = \frac{1}{2} \epsilon_{abcde} \mathcal{F}^{de} \, , \tag{2.2}$$

$$\hat{C}_y{}^d \mathcal{H}_{dab} + \hat{C}_y{}^y \mathcal{F}_{ab} = -\frac{1}{3!} \epsilon_{abdef} \mathcal{H}^{def} \, , \tag{2.3}$$

$$\hat{C}_a{}^d \mathcal{F}_{db} = -\frac{1}{3!} \epsilon_{abdef} \mathcal{H}^{def} \, . \tag{2.4}$$

The different components of $\hat{C}^{\hat{a}\hat{b}}$ are given by[3]

$$\hat{C}^{ab} \equiv C^{ab} = \frac{1}{K} \left[\left(1 + \frac{1}{12} \mathcal{H}^2 - \frac{1}{4} \text{tr} \mathcal{F}^2 \right) \eta^{ab} - \frac{1}{4} (\mathcal{H}^2)^{ab} + \frac{1}{2} (\mathcal{F}^2)^{ab} \right] , \tag{2.5}$$

$$\hat{C}^{ay} = \frac{-1}{4K} \mathcal{V}^a \, , \tag{2.6}$$

$$\hat{C}^{yy} = \frac{1}{K} \left(1 + \frac{1}{12} \mathcal{H}^2 \right) \eta^{yy} \, . \tag{2.7}$$

In deriving the expressions (2.5), (2.6) and (2.7) we have used that $\hat{\mathcal{H}}^2 = \mathcal{H}^2 - 3\text{tr}\,\mathcal{F}^2$ and $(\hat{\mathcal{H}}^2)^{ab} = (\mathcal{H}^2)^{ab} - 2(\mathcal{F}^2)^{ab}$. Furthermore, we have defined a 5-dimensional vector $\mathcal{V}^a$ as follows:

$$\mathcal{V}^a \equiv \mathcal{H}^a{}_{cd} \mathcal{F}^{cd} = (\hat{\mathcal{H}}^2)^{ay} \, . \tag{2.8}$$

Because of the duality equations this vector is a null eigenvector of $\mathcal{F}_{ab}$, i.e. $\mathcal{V}^a \mathcal{F}_{ab} = 0$. This implies that in 5 dimensions, for a generic rank 4 background $\mathcal{F}_{ab}$, the vector $\mathcal{V}^a$ has only one nonzero component. At first sight it seems that for a rank 2 background $\mathcal{F}_{ab}$ the fact

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[3]Our conventions slightly differ from those of [6] in the sense that we use a matrix multiplication convention for the two-form $\mathcal{F}_{ab}$. This means that $(\mathcal{F}^2)_{ab}$ in these notes is $-(\mathcal{F}^2)_{ab}$ in [6] and we will use $\text{tr}\,\mathcal{F}^2$ to denote $-\mathcal{F}^2$ in [6] and also to distinguish it from ordinary Lorentz scalar contractions $\mathcal{F}^2 \equiv \mathcal{F}_{ab}\mathcal{F}^{ab}$, that differ from $\text{tr}\,\mathcal{F}^2$ by a sign.

that $\mathcal{V}^a$ is a null eigenvector of $\mathcal{F}_{ab}$ is less restrictive. However, using the definition (2.8) and expressing $\mathcal{V}^a$ in terms of $\mathcal{F}_{ab}$ only, it is not hard to see that $\mathcal{V}^a$ vanishes in that case. Therefore, a rank 2 reduction implies that the off-diagonal vector component of the open membrane metric vanishes. As pointed out in [6], and which will become clear as we go along, this is also true vice versa.

One can obtain another set of relations, consistent with the 5-dimensional duality equations, by reducing the constraint (1.5) on $(\hat{\mathcal{H}}^4)^{\hat{a}\hat{b}}$:

$$\left((\mathcal{H}^2)^a_{\ c} - 2\,(\mathcal{F}^2)^a_{\ c}\right)\left((\mathcal{H}^2)^{cb} - 2(\mathcal{F}^2)^{cb}\right) + \mathcal{V}^a\mathcal{V}^b = \frac{2}{3}(\mathcal{H}^2 - 3\mathrm{tr}\mathcal{F}^2)\times$$
$$\times \left(\eta^{ab} - (\mathcal{F}^2)^{ab} + \frac{1}{2}(\mathcal{H}^2)^{ab}\right)\,, \quad (2.9)$$

$$\left[\frac{1}{3}\mathcal{H}^2\eta^{ac} - \left((\mathcal{H}^2)^{ac} - 2(\mathcal{F}^2)^{ac}\right)\right]\mathcal{V}_c = 0\,, \quad (2.10)$$

$$\mathcal{V}^2 - \frac{2}{3}(\mathcal{H}^2 - 3\mathrm{tr}\mathcal{F}^2) + \frac{1}{3}\mathcal{H}^2\mathrm{tr}\mathcal{F}^2 = 0\,. \quad (2.11)$$

According to [7] the most general solution to the 5-dimensional duality equations is given by[4]

$$\frac{1}{3!}\epsilon_{abcde}\mathcal{H}^{cde} = \frac{\mathcal{D}_0\mathcal{F}_{ab} + (\mathcal{F}^3)_{ab}}{\sqrt{\mathcal{D}}}\,. \quad (2.12)$$

We have introduced two 5-dimensional scalars, $\mathcal{D}$ and $\mathcal{D}_0$, which are defined by

$$\mathcal{D} \equiv -\det(\eta_{ab} + \mathcal{F}_{ab}) = \mathcal{D}_0 + \frac{1}{8}(\mathrm{tr}\,\mathcal{F}^2)^2 - \frac{1}{4}\mathrm{tr}\,\mathcal{F}^4\,, \quad (2.13)$$

$$\mathcal{D}_0 \equiv 1 - \frac{1}{2}\mathrm{tr}\,\mathcal{F}^2\,. \quad (2.14)$$

To avoid an imaginary DBI action we need $\mathcal{D} > 0$ and $\mathcal{D}_0 > 0$.[5] Furthermore, for electric fields we have $\mathcal{D}_0 < 1$ whereas magnetic fields correspond to $\mathcal{D}_0 > 1$. In the following we will make use of the following identity for antisymmetric matrices in 5 dimensions:

$$(\mathcal{F}^5)_{ab} = (\mathcal{D}_0 - \mathcal{D})\,\mathcal{F}_{ab} + (1 - \mathcal{D}_0)\,(\mathcal{F}^3)_{ab}\,. \quad (2.15)$$

Using all the previous formulae one can calculate the following useful tensors, enabling us to eliminate $\mathcal{H}_{abc}$ in terms of $\mathcal{F}_{ab}$:

$$\frac{1}{6}\mathcal{H}^2\,\eta_{ab} - \frac{1}{2}(\mathcal{H}^2)_{ab} = \frac{\left[(\mathcal{D}_0)^2 + (\mathcal{D}_0 - \mathcal{D})\right](\mathcal{F}^2)_{ab} + (\mathcal{D}_0 + 1)(\mathcal{F}^4)_{ab}}{\mathcal{D}}\,, \quad (2.16)$$

$$\frac{1}{6}\mathcal{H}^2 = (1 - \mathcal{D}_0) + \frac{4(\mathcal{D}_0 - \mathcal{D}) + (1 - \mathcal{D}_0)^2}{\mathcal{D}}\,, \quad (2.17)$$

$$\mathcal{V}^2 = \frac{(\mathcal{D}_0 + 1)^2}{\mathcal{D}}4(\mathcal{D}_0 - \mathcal{D})\,. \quad (2.18)$$

We note that $\mathcal{V}^2$ is proportional to $\mathcal{D}_0 - \mathcal{D} = \frac{1}{4}\mathrm{tr}\,\mathcal{F}^4 - \frac{1}{8}(\mathrm{tr}\,\mathcal{F}^2)^2$. Upon imposing the

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[4]Since our main purpose in this paper is to investigate under which conditions the reduction of the open membrane leads to open strings, we have solved $\mathcal{H}_{abc}$ in terms of $\mathcal{F}_{ab}$. To analyze when the open membrane reduces to open D2-branes it would be convenient at this point to solve $\mathcal{F}_{ab}$ in terms of $\mathcal{H}_{abc}$.

[5]The critical electric field limit corresponds to $\mathcal{D}_0 \downarrow 0$.

constraint $\mathcal{D}_0 - \mathcal{D} = 0$, or equivalently $\mathcal{V}^2 = 0$, we find that $\mathrm{tr}\,\mathcal{F}^4 = \frac{1}{2}(\mathrm{tr}\,\mathcal{F}^2)^2$. By going to a skew-symmetric basis one can show that this restricts us to a matrix $\mathcal{F}_{ab}$ of rank 2. For this special case we recover the results of [6], in particular $\mathcal{V}^a = 0$. For later reference, we emphasize that the 5-dimensional scalar $\mathcal{V}^2$ transforms under 6-dimensional Lorentz rotations and therefore the constraint $\mathcal{D}_0 - \mathcal{D} = 0$ can be removed by performing a 6-dimensional Lorentz transformation on the M5-brane worldvolume.

The above results can be used to relate the 6-dimensional Lorentz scalar K, see (1.4), to the 5-dimensional Lorentz scalars $\mathcal{D}$ (2.13) and $\mathcal{D}_0$ (2.14) as follows:

$$K = \frac{\frac{1}{2}(\mathcal{D}_0 + 1)}{\sqrt{\mathcal{D}}} . \tag{2.19}$$

Although the right hand side of this equation is only manifestly invariant under 5-dimensional Lorentz transformations, this equation tells us that it is actually invariant under 6-dimensional Lorentz transformations. The conformal factor $z(K)$, see (1.9), which is obviously a 6-dimensional Lorentz scalar, can be written in terms of the 5-dimensional scalars $\mathcal{D}$ and $\mathcal{D}_0$ as follows

$$z^{-3} = \frac{\frac{1}{2}(\mathcal{D}_0 + 1) - \sqrt{\frac{1}{4}(\mathcal{D}_0 - 1)^2 + (\mathcal{D}_0 - \mathcal{D})}}{\sqrt{\mathcal{D}}} . \tag{2.20}$$

Upon a rank 2 truncation ($\mathcal{D}_0 = \mathcal{D}$) we find

$$z^{-3} = \frac{\frac{1}{2}(\mathcal{D}_0 + 1) - \frac{1}{2}|\mathcal{D}_0 - 1|}{\sqrt{\mathcal{D}_0}} . \tag{2.21}$$

We see that an electric reduction leads to $z^{-3} = \sqrt{\mathcal{D}_0}$ whereas a magnetic reduction gives $z^{-3} = 1/\sqrt{\mathcal{D}_0}$.

For our later purposes we give the expression for the 5-dimensional components of the Boillat co-metric (1.1) in terms of the 2-form $\mathcal{F}_{ab}$ only:

$$C^{ab} = \frac{1}{\sqrt{\mathcal{D}}}\left[\mathcal{D}\,\eta^{ab} + \mathcal{D}_0\,(\mathcal{F}^2)^{ab} + (\mathcal{F}^4)^{ab} \right]. \tag{2.22}$$

One can check that the (5-dimensional) inverse of this matrix is given by

$$(C^{-1})_{ab} = \frac{1}{\sqrt{\mathcal{D}}}\left[\eta_{ab} - (\mathcal{F}^2)_{ab} \right], \tag{2.23}$$

which is indeed the D4-brane Boillat metric [9], confirming the result first reported in [7] and more recently in [8].

We are now ready to reduce the open membrane metric (1.8), using an appropriate ansatz, and then write the reduced open membrane metric in terms of $\mathcal{F}_{ab}$ only. Before actually doing this it is instructive to make a few observations. The basic result of [6] was to show that the conformal factor of the open membrane metric could be determined using an uplifting procedure. The analysis of [6] was based upon the assumption that the off-diagonal components of the open membrane metric vanished (i.e. $\mathcal{V}^2 = 0$), restricting the procedure of [6] to a rank 2 reduction.

We have noted that the difference between the rank 2 and rank 4 cases is nothing but a 6-dimensional Lorentz rotation. The conformal factor (1.9) is a six-dimensional Lorentz scalar and hence is invariant under such a Lorentz rotation and therefore the same for rank 2 and rank 4 reductions. However, this does not imply we should get the same results in a general rank 4 reduction as we did in a rank 2 reduction. One should keep in mind that the open membranes ending on the M5-brane align themselves along the electric directions of the 3-form background, e.g. along the 1 and 2 directions, with $\mathcal{H}_{012} \neq 0$. This implies that only an "electric" rank 2 reduction, i.e. a reduction along the 1 or 2 direction, will lead to open strings ending on the D4-brane. On the other hand, a "magnetic" rank 2 reduction, i.e. a reduction along the 3,4 or 5 direction, will give open D2-branes ending on the D4-brane [5]. From this perspective it seems rather unlikely that we will find the open string metric and coupling constant in a magnetic rank 2 or a general rank 4 reduction. Instead, in a magnetic rank 2 reduction it will presumably be more appropriate to rewrite everything in terms of the 3-form $\mathcal{H}_{abc}$ instead of the 2-form $\mathcal{F}_{ab}$ and define a reduction ansatz involving an open D2-brane metric and an open D2-brane coupling constant (which would naturally be associated with the open membrane metric Kaluza-Klein scalar, see also [5]). In a general rank 4 reduction, i.e. an oblique reduction along a linear combination of the (1,2) and (3,4,5) directions, we expect to find a bound state of an open string with an open D2-brane. We do not know much about the description of this system but a priori there is no reason to expect that it can be described by open string parameters only. The purpose of the analysis below is to check the above sketched scenario by performing a general rank 4 reduction of the open membrane parameters and to investigate under which conditions we can identify, using a reduction ansatz analogous to a bulk M-IIA reduction, the Seiberg-Witten open string metric and coupling constant. In section 3 we will apply a similar analysis to the open string non-commutativity tensor.

We now proceed and put forward a general reduction ansatz for the open membrane metric (or co-metric), in analogy with the bulk M-IIA ansatz like in [6], using the conformal factor (1.9). More specific, we propose the following reduction ansatz for the open membrane co-metric $\hat{G}_{\text{OM}}^{\hat{a}\hat{b}} \equiv z\,\hat{C}^{\hat{a}\hat{b}}$:

$$\lambda_{\text{os}}^{2/3} (G_{\text{os}})^{ab} = \hat{G}_{\text{OM}}^{ab}\,, \tag{2.24}$$

$$\lambda_{\text{os}}^{-4/3} = \frac{\hat{G}_{\text{OM}}^{yy}}{\left(1 - \hat{G}_{\text{OM}}^{ay}(\hat{G}_{\text{OM}}^{-1})_{ay}\right)}\,. \tag{2.25}$$

Note that $\hat{G}_{\text{OM}}^{ay}(\hat{G}_{\text{OM}}^{-1})_{ay} = \hat{C}^{ay}(\hat{C}^{-1})_{ay} = -\frac{1}{16K^2}\mathcal{V}^2 = \mathcal{D}_0 - \mathcal{D}$. To ensure that reducing the open membrane metric (instead of the inverse open membrane co-metric) gives the inverse result we require the following ansatz for the open membrane metric $(\hat{G}_{\text{OM}}^{-1})_{\hat{a}\hat{b}} \equiv z^{-1}\,(\hat{C}^{-1})_{\hat{a}\hat{b}}$:

$$\lambda_{\text{os}}^{-2/3}(G_{\text{os}}^{-1})_{ab} = (\hat{G}_{\text{OM}}^{-1})_{ab} - \frac{(\hat{G}_{\text{OM}}^{-1})_{ay}(\hat{G}_{\text{OM}}^{-1})_{by}}{(\hat{G}_{\text{OM}}^{-1})_{yy}}\,, \tag{2.26}$$

$$\lambda_{\text{os}}^{4/3} = (\hat{G}_{\text{OM}}^{-1})_{yy}\,. \tag{2.27}$$

The open string metric and coupling we would like to obtain after reducing the open membrane metric are equal to[6] [10]:

$$(G_{\text{os}}^{-1})_{ab} = \eta_{ab} - (\mathcal{F}^2)_{ab}, \tag{2.28}$$

$$\lambda_{\text{os}} = \sqrt{\mathcal{D}}. \tag{2.29}$$

We observe that the open string coupling constant is not a 6-dimensional Lorentz invariant object, i.e. its value changes when transforming from a rank 2 to a rank 4 background using a 6-dimensional Lorentz rotation.

Let us focus our attention on the open string coupling λ_{os}, as defined by the open membrane metric (2.27). We find

$$\lambda_{\text{os}}^{4/3} = (\hat{G}_{\text{OM}}^{-1})_{yy} = z^{-1}(\hat{C}^{-1})_{yy} = \left(\frac{\frac{1}{2}(\mathcal{D}_0 + 1) - \sqrt{\frac{1}{4}(\mathcal{D}_0 - 1)^2 + (\mathcal{D}_0 - \mathcal{D})}}{\sqrt{\mathcal{D}}} \right)^{1/3} \sqrt{\mathcal{D}}, \tag{2.30}$$

which is obviously not equal to the expression (2.29). In fact, any attempt to modify the ansatz (2.27) (to include for example arbitrary functions of λ_{os}), or modify the conformal factor $z(K) \neq 1$, see eq. (1.9), will fail to reproduce (2.29) for the simple reason that we need to obtain a function of $\mathcal{D}$ only. As we already discussed, the conformal factor z has to be a 6-dimensional Lorentz scalar and therefore it can never be a function of $\mathcal{D}$ only, as is confirmed by eq. (2.19). The only exception is the case $z(K) = 1$, when we find $(\hat{G}_{OM}^{-1})_{yy} = (\hat{C}^{-1})_{yy} = \sqrt{\mathcal{D}}$, which is unsatisfactory because it does not reproduce the required scaling of the open membrane metric in the OM-theory decoupling limit [1, 2]. Furthermore, the reduction ansatz that would be required does not follow the bulk ansatz. Any failure to reproduce the open string coupling also means that the open string metric (2.28) is not reproduced using the proposed ansatz (2.26).[7]

To remind the reader, when imposing the constraint $\mathcal{D}_0 - \mathcal{D} = 0$ in (2.30), we reproduce the open string coupling $\lambda_{\text{os}} = \sqrt{\mathcal{D}_0}$, as well as the open string metric, defined through (2.26) [6], but only when we consider an electric reduction.[8] What happens when we perform a 6-dimensional Lorentz rotation is that the open string coupling as defined by (2.27) will transform into (2.30), whereas the rank 4 Seiberg-Witten open string coupling equals (2.29). Only when we truncate to rank 2 do the two expressions coincide. Another way to put this is that when $z(K) \neq 1$ the Seiberg-Witten open string coupling (2.29) can not be related to the Kaluza-Klein scalar of the open membrane metric because it does not appropriately transform under 6-dimensional Lorentz transformations.

We conclude that only an electric rank 2 reduction leads to open strings. Instead, a magnetic rank 2 reduction or a general rank 4 reduction naturally involves open D2-branes. One cannot expect that these reductions lead to open string moduli only. The analysis of

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[6]Throughout this paper we will suppress any closed string coupling dependence, i.e. $g_s = 1$.

[7]One does of course always find something conformal to the open string metric.

[8]Taking the square root in (2.30) we obtain the *absolute* value of $\mathcal{D}_0 - 1$, which is equal to $1 - \mathcal{D}_0$ in an electric reduction. In that case we exactly find $\lambda_{os} = \sqrt{\mathcal{D}_0}$. On the other hand, for a rank 2 magnetic reduction, with $\mathcal{D}_0 > 1$, we find $\lambda_{os} = (\mathcal{D}_0)^{1/4}$.

this section shows that, when open D2-branes are involved, it is not possible to view the open string coupling constant as the Kaluza-Klein scalar of an open membrane metric.

3. General reduction of the open membrane theta parameter

Besides the open string metric and open string coupling one needs the open string non-commutativity parameter to define the non-commutative open string theory (NCOS). The open string non-commutativity parameter θ^{ab} has an analogue in the D3-brane DBI theory as the dual Maxwell field P^{ab}. According to [9] these objects are related in the following way

$$\theta^{ab} = \frac{-1}{\sqrt{\mathcal{D}}} P^{ab} \,, \tag{3.1}$$

where the dual Maxwell field P^{ab} equals

$$P^{ab} = \frac{1}{\sqrt{\mathcal{D}}} \left(\mathcal{D}_0 \mathcal{F}^{ab} + (\mathcal{F}^3)^{ab} \right) \,. \tag{3.2}$$

This expression for the dual Maxwell field P^{ab} can actually be written as (using (2.22) and (2.15))

$$P^{ab} = \mathcal{F}^{ad} C_d^b \,, \tag{3.3}$$

which means that $\theta^{ab} = -\mathcal{F}^{ad} (G_{\text{os}})_d^b$.

The question arises whether one can similarly find a 3-form tensor on the M5-brane that corresponds to an appropriately generalized, so far unknown, notion of non-commutativity. Theta parameters of rank 3 from a different point of view have also been discussed in e.g. [11]. One property of such a 3-form tensor should be that it reduces to $\theta^{ab} \propto P^{ab}$ on the D4-brane. Motivated by the expression (3.3) we first introduce the following 3-form tensor on the M5-brane which is made out of the 3-form $\hat{\mathcal{H}}_{abc}$ and *one* Boillat co-metric:

$$\hat{P}^{\hat{a}\hat{b}\hat{c}} \equiv \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{d}} \hat{C}_{\hat{d}}^{\hat{c}} = \frac{1}{K} \left[\left(1 + \frac{1}{12} \hat{\mathcal{H}}^2 \right) \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}} - \frac{1}{4} (\hat{\mathcal{H}}^3)^{\hat{a}\hat{b}\hat{c}} \right] \,, \tag{3.4}$$

where $(\hat{\mathcal{H}}^3)^{\hat{a}\hat{b}\hat{c}} \equiv \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{d}} (\hat{\mathcal{H}}^2)_{\hat{d}}^{\hat{c}}$. Because of the self-duality condition (1.3) we have $\hat{P}_{\hat{a}\hat{b}\hat{c}} = \frac{1}{3!} \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}\hat{f}} \hat{\mathcal{H}}^{\hat{d}\hat{e}\hat{f}}$. Upon reduction, using results of the previous section (most importantly (2.22) and (2.15)), we find the following 2-form

$$\hat{P}^{yab} \equiv P^{ab} = \frac{1}{\sqrt{\mathcal{D}}} \left[\mathcal{D}_0 \, \mathcal{F}^{ab} + (\mathcal{F}^3)^{ab} \right] \,, \tag{3.5}$$

which is indeed equal to (3.2). In order that this tensor exactly reduces to the non-commutativity parameter θ^{ab} we need to fix its conformal factor. Assuming a rank 2 electric reduction, i.e. $\mathcal{D} = \mathcal{D}_0$ and $z^{-3} = \sqrt{\mathcal{D}_0}$, it is easy to see that this conformal factor has to be equal to z^3 (see (1.9)):

$$\theta^{ab} = -z^3 P^{ab} \,. \tag{3.6}$$

Because we had to introduce the conformal factor z^3, see (2.20), we conclude that a magnetic rank 2 reduction or a general rank 4 reduction of the three-form $z^3 \, \hat{P}^{\hat{a}\hat{b}\hat{c}}$ yields the

open string non-commutativity tensor (3.1) only up to a conformal factor. The explanation for this obstruction is the same as in our discussion of the reduction of the open membrane metric in the previous section. These reductions also lead to open D2-branes and therefore one should presumably not expect to obtain the theta parameter of open strings only. Note that the 3-form (3.6) does not even reproduce the open string non-commutativity parameter in a magnetic rank 2 reduction. In any reduction of an M5-brane 3-form we will get a 2-form and a 3-form on the D4-brane, which should be dual to each other. If the M5-brane 3-form is really the generalized open membrane theta parameter we expect that the 2-form that arises after a rank 2 electric or magnetic reduction gives the open string non-commutativity tensor.

There is another argument to discard the 3-form $-z^3 \hat{P}^{\hat{a}\hat{b}\hat{c}}$ (3.6) as the open membrane theta parameter. By definition, the 3-form $\hat{P}^{\hat{a}\hat{b}\hat{c}}$ satisfies a *nonlinear* self-duality condition. This implies that it does not scale homogeneously in the OM-theory limit $\ell_p \to 0$ (which is directly related to $\hat{\mathcal{H}}$ not scaling homogeneously, see [2]). This does not conform with our expectation for a generalized M5-brane theta tensor; in the OM-theory decoupling limit we expect to find a fixed theta parameter scale in all directions on the M5-brane. We therefore conclude that the 3-form (3.6) cannot be the generalized open membrane theta parameter we are looking for, even though it does give the open string non-commutativity tensor in an electric rank 2 reduction.

Fortunately, there exists another 3-form tensor $\hat{W}^{\hat{a}\hat{b}\hat{c}}$, first put forward in [5], that also reduces to the open string non-commutativity tensor θ^{ab} under an electric rank 2 reduction. The tensor $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ has the special feature that it reduces to θ^{ab} (in a rank 2 reduction) *without* the need to introduce an extra conformal factor $f(z)$. The absence of such a conformal factor implies that both the electric and the magnetic rank 2 reduction lead to the open string non-commutativity tensor on the D4-brane.

The 3-form tensor $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ is made out of the 3-form $\hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}}$ and *two* Boillat co-metrics [5]:

$$\hat{W}^{\hat{a}\hat{b}\hat{c}} \equiv \hat{\mathcal{H}}^{\hat{a}\hat{d}\hat{e}} \hat{C}^{\hat{b}}_{\hat{d}} \hat{C}^{\hat{c}}_{\hat{e}} = \hat{P}^{\hat{a}\hat{b}\hat{d}} \hat{C}^{\hat{c}}_{\hat{d}} . \tag{3.7}$$

Expressed in terms of the 3-form $\hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}}$ only we find the expression (using a constraint on $(\hat{\mathcal{H}}^5)^{\hat{a}\hat{b}\hat{c}}$ that can be deduced from (1.5))

$$\hat{W}^{\hat{a}\hat{b}\hat{c}} = \left(1 + \frac{1}{6}\hat{\mathcal{H}}^2 \right) \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}} - \frac{1}{2}(\hat{\mathcal{H}}^3)^{\hat{a}\hat{b}\hat{c}} . \tag{3.8}$$

Note that the expressions (3.4) and (3.8) for the tensors $\hat{P}^{\hat{a}\hat{b}\hat{c}}$ and $\hat{W}^{\hat{a}\hat{b}\hat{c}}$, respectively, in terms of $\hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}}$, only differ in the numerical factors. Actually, the three 3-forms $\hat{P}^{\hat{a}\hat{b}\hat{c}}$, $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ and $\hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}}$ are related in the following way:

$$\hat{W}^{\hat{a}\hat{b}\hat{c}} = 2K \, \hat{P}^{\hat{a}\hat{b}\hat{c}} - \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}} = 2K \, {}^{\star}\hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}} - \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}} . \tag{3.9}$$

Under the assumption that the off-diagonal terms in the Boillat co-metric vanish, i.e. $\mathcal{D} = \mathcal{D}_0$, we find that the 3-form $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ reduces to

$$\hat{W}^{aby} = \hat{\mathcal{H}}^{ady} \hat{C}^b_d \hat{C}^y_y = \hat{P}^{ab} \hat{C}^y_y . \tag{3.10}$$

This exactly reproduces the open string non-commutativity tensor θ^{ab} for the rank 2 case (electric or magnetic),[9] without the need to fix the conformal factor (except for a minus sign). We next wish to check what happens to this result beyond the rank 2 case. Using the antisymmetry of $\hat{W}^{\hat{a}\hat{b}\hat{c}}$, we find that

$$\hat{W}^{aby} = \hat{W}^{yab} \equiv \hat{\mathcal{P}}^{yad}\,\hat{C}^b_d. \tag{3.11}$$

We already know the expressions (3.5) for $\hat{\mathcal{P}}^{yad}$ and (2.22) for $\hat{C}^b_d$. By repeatedly using the identity (2.15) for antisymmetric matrices in 5 dimensions we find

$$\hat{W}^{yab} = \frac{1}{\mathcal{D}}\left[(\mathcal{D}_0^2 + (\mathcal{D}_0 - \mathcal{D}))\mathcal{F}^{ab} + (\mathcal{D}_0 + 1)(\mathcal{F}^3)^{ab}\right], \tag{3.12}$$

which does not reproduce the open string non-commutativity tensor (3.1). One can check that when imposing $\mathcal{D} = \mathcal{D}_0$ the expression reproduces (3.1),[10] as we already concluded using eq. (3.11). Like in the previous section the explanation for this result is that a general rank 4 reduction will give a bound state of an open D2-brane and an open string ending on the D4-brane. The 3-form obtained after a magnetic rank 2 reduction should naturally be related to a 3-form open D2-brane theta tensor on the D4-brane [5]. We conclude that the tensor $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ only reduces to the open string non-commutativity tensor under a rank 2 electric or magnetic reduction.

Although the property of $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ to reduce to the open string theta parameter for both electric and magnetic rank 2 is perhaps preferable, it is not enough to conclude unambiguously that $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ is the open membrane theta parameter. In order to establish this more convincingly we should verify whether the tensor $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ has the expected behavior in the OM-theory decoupling limit. In [5] it was shown that the 3-form $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ on the M5-brane is unique in the sense that it is the only 3-form that satisfies a *linear* self-duality condition with respect to the open membrane metric. The discussion below will only focus on the behavior of the 3-form $\hat{W}^{\hat{a}\hat{b}\hat{c}}$ in the OM-theory limit (see also [5]). Introducing the correct dimensions $[length]^3$ and rewriting in terms of the open membrane co-metric (1.8) we (re-)define

$$\hat{\Theta}^{\hat{a}\hat{b}\hat{c}} \equiv -\ell_p^3 \hat{W}^{\hat{a}\hat{b}\hat{c}} = -\ell_p^3\, z^{-2}\, \hat{g}^{\hat{a}\hat{k}} \hat{\mathcal{H}}_{\hat{k}\hat{l}\hat{m}}\,(\hat{G}_{OM})^{\hat{b}\hat{l}}\,(\hat{G}_{OM})^{\hat{c}\hat{m}}. \tag{3.13}$$

We want to check that this tensor gives a fixed homogeneous theta parameter, relative to the Planck length ℓ_p, in the OM-theory limit $\ell_p \to 0$. In the OM-theory limit the conformal factor z^{-2} and the open membrane co-metric scale homogeneously as follows

$$z^{-2} \sim (\ell_p/\ell_g)\,, \qquad (\hat{G}_{OM})^{\hat{a}\hat{b}} \sim (\ell_g/\ell_p)^2\,, \tag{3.14}$$

where we introduced ℓ_g as the fundamental length-scale of OM-theory [1, 2]. Turning on a nontrivial constant 3-form background naturally induces a $3+3$ split and in the OM-theory

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[9]Remember that $\hat{C}^y_y = \frac{1}{\sqrt{\mathcal{D}_0}}$.

[10]One has to use the fact that the identity (2.15) for antisymmetric matrices in 5 dimensions reduces to a constraint on $(\mathcal{F}^3)_{ab}$ (relating it to $\mathcal{F}_{ab}$). This works for both the electric and magnetic rank 2 reduction. Unlike in the reduction of the open membrane metric there is no square root involved which could lead to a different result for the electric and magnetic cases.

decoupling limit the (bulk) metric and the 3-form $\hat{\mathcal{H}}$ scale (inhomogeneously) as follows ($\alpha, \beta \in (0, 1, 2)$ and $i, j \in (3, 4, 5)$)

$$
\begin{aligned}
\hat{g}^{\alpha\beta} &\sim 1, & \hat{g}^{ij} &\sim (\ell_g/\ell_p)^3, \\
\hat{\mathcal{H}}_{012} &\sim 1, & \hat{\mathcal{H}}_{345} &\sim (\ell_p/\ell_g)^3.
\end{aligned}
\tag{3.15}
$$

Using these scalings one can check that $\hat{g}^{ak}\hat{\mathcal{H}}_{\hat{k}\hat{l}\hat{m}}$ is actually fixed (~ 1) in all directions (homogeneous) in the OM-theory limit. This means we end up with the following homogeneously fixed expressions of $\hat{\Theta}$, see (3.13), in the OM-theory limit $\ell_p \rightarrow 0$

$$
\hat{\Theta}^{012} \sim \ell_g^3,
\tag{3.16}
$$

$$
\hat{\Theta}^{345} \sim \ell_g^3.
\tag{3.17}
$$

This is exactly what one would expect of a generalized M5-brane theta tensor in the OM-theory limit [5].

We conclude that there exist two 3-form tensors on the M5-brane, $-z^3 \hat{P}^{\hat{a}\hat{b}\hat{c}}$ and $\hat{\Theta} = -\ell_p^3 \hat{W}^{\hat{a}\hat{b}\hat{c}}$, that both reduce to the open string non-commutativity tensor under an electric rank 2 reduction. One of them, $\hat{\Theta}^{\hat{a}\hat{b}\hat{c}}$, also reduces to the open string non-commutativity tensor using a magnetic rank 2 reduction. Both tensors do not reduce to the open string non-commutativity tensor in a general rank 4 reduction. An OM-theory scaling argument [5] (and possibly the fact that only $\hat{\Theta}^{\hat{a}\hat{b}\hat{c}}$ gives the open string non-commutativity tensor in an electric *and* magnetic reduction) shows that only the 3-form tensor $\hat{\Theta}^{\hat{a}\hat{b}\hat{c}}$ can be identified with the open membrane generalized theta parameter. The precise expression for this theta parameter is given by

$$
\hat{\Theta}^{\hat{a}\hat{b}\hat{c}} = -\ell_p^3 z^{-2} \hat{g}^{\hat{a}\hat{k}} \hat{\mathcal{H}}_{\hat{k}\hat{l}\hat{m}} (\hat{G}_{OM})^{\hat{b}\hat{l}} (\hat{G}_{OM})^{\hat{c}\hat{m}} = -\ell_p^3 \left(\left(1 + \frac{1}{6}\hat{\mathcal{H}}^2\right) \hat{\mathcal{H}}^{\hat{a}\hat{b}\hat{c}} - \frac{1}{2}(\hat{\mathcal{H}}^3)^{\hat{a}\hat{b}\hat{c}} \right).
\tag{3.18}
$$

It is important to realize that we can now study this theta parameter without considering a particular limit (although generically the M5-brane theory will not be decoupled from the bulk). For example, in [4] a large h (which is the variable parameterizing nonlinearly self-dual constant solutions, see [4]) limit was studied in which a commutator (in the magnetic directions) describing a non-commutative loop space proportional to h^{-1} was found. Going outside the limit one expects to find corrections to the h^{-1} behavior that make it nonsingular for $h \rightarrow 0$. We expect to find something like $\frac{h}{1+h^2\ell_p^6}$ such that $\hat{\Theta} \rightarrow 0$ for $h \rightarrow 0$ and $\hat{\Theta} \sim h^{-1}$ for $h \rightarrow \infty$. This is what happens in the non-commutative D-brane cases. These and possibly other interesting considerations related to (3.18) will be left for a future investigation.

4. Conclusions

We have performed a general rank 4 reduction of the open membrane metric. The difference with the rank 2 reduction, discussed by one of us [6], is that the rank 4 reduction contains an extra parameter which can be associated with an angle that describes the reduction

of the open membrane with respect to a fixed background. The electric rank 2 reduction of [6] corresponds to a zero angle and a reduction of the open membrane to an open string. On the other hand, the generic rank 4 reduction corresponds to a nonzero angle and leads to open D2-branes as well. We have shown in this work that for those reductions the open string coupling constant can *not* be interpreted as the Kaluza-Klein scalar of an open membrane metric.

We also discussed the introduction of a generalized theta tensor on the M5-brane, see also [5] for an independent discussion. We showed that there are two 3-form tensors that reduce to the open string non-commutativity tensor under an electric rank 2 reduction. Only one of them, i.e. the one of [5], has the correct behavior in the OM-theory decoupling limit. This tensor also reduces to the open string non-commutativity tensor under a magnetic rank 2 reduction. It would be interesting to work out this magnetic reduction in more detail and analyze how the appropriate open D2-brane parameters are related to the different components of the open membrane metric.

In analogy with open strings, it is natural to conjecture that the 3-form open membrane theta tensor is somehow related to 3-point functions of the open membrane. It would be interesting to see what the geometric properties are of a space deformed by such a 3-form tensor and whether it has something to do with a non-associativity of the M5-brane worldvolume. So far, not much progress has been made in describing such a non-associative geometry although we expect a relation with non-commutative loop space [4].

Although all open membrane or OM-theory parameters have now been identified, i.e. the OM-metric [2, 5, 6] and more recently the OM-theta parameter [5], it is unknown what the microscopic (open membrane) origin of these objects is. This in contrast with D-branes where the open string 2-point function gives us both the open string metric and the theta parameter [10]. We hope that the future will see some progress in our understanding of the microscopic degrees of freedom of OM-theory. Without doubt this will lead to a better understanding of M-theory.

Acknowledgments

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