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Consistent reductions of IIB*/M* theory and de Sitter supergravity

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Abstract: We construct consistent non-linear Kaluza Klein reduction ansatze for a subset of fields arising from the reduction of IIB* and M* theory on $dS_5 \times H^5$ and $dS_4 \times AdS_7$, respectively. These reductions yield four and five-dimensional de Sitter supergravities, albeit with wrong sign kinetic terms. We also demonstrate that the ansatze may be used to lift multi-centered de Sitter black hole solutions to ten and eleven dimensions. The lifted dS_5 black holes correspond to rotating E4-branes of IIB* theory.

Keywords: M-Theory, Supergravity Models.

Contents

1. Introduction

The recent observations on the acceleration of the universe has led to renewed interest in de Sitter backgrounds in cosmology. At the same time, this has led to much debate on whether de Sitter space itself is in fact compatible with M-theory. Although several arguments may be made against this possibility, it should be noted that de Sitter spaces naturally arise as backgrounds for the * theories of [1]–[5]. These * theories are obtained through a timelike T-duality of the ordinary string theories, and admit an unconventional effective field theory description involving wrong sign kinetic energy terms for the RR fields. Because of this and other problems, the field theory limits of the * theories appear ill-defined. Nevertheless, so long as one allows T-duality along a timelike circle, one must allow for the existence of such * theories as a component of the full M-theory. It is in this spirit that we choose to investigate the de Sitter supergravities which arise as consistent dimensional reductions of IIB* and M* theory [5].

Conventional wisdom indicates that de Sitter space is incompatible with supersymmetry. This may be seen, for example, in the standard classification of possible supersymmetry algebras, which allows for both Poincaré and anti-de Sitter superalgebras, but not in

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general for the possibility of a de Sitter superalgebra. Thus in order to obtain de Sitter supersymmetry, one must relax one or more of the usual assumptions. Such possibilities were first generally studied in [6, 7]. Subsequently, de Sitter and variant supergravities have been considered in refs. [8, 9]; the resulting models may be obtained by analytic continuation of anti-de Sitter supergravities, and have wrong sign kinetic energy terms for some of the fields. In the present case, the de Sitter supergravities which we investigate are similarly unconventional in that they also have wrong sign kinetic energy terms as well as non-compact gaugings. Of course, unitary theories do exist with non-compact gaugings. However, as will be explained below, the non-compact symmetries which we consider here are of a different nature, and yield mixed sign kinetic terms for the gauge fields. While these properties of the de Sitter supergravities are clearly undesirable, they follow as a natural consequence of the underlying * theory. As a result, we leave open the possibility that such de Sitter supergravities would be of relevance for the * theory beyond the field theory limit. It is, however, possible that the effective field theories which we explore, being unstable, do not provide an adequate description of the full * theory. Nevertheless, one may hope to gain additional insight into de Sitter space and supersymmetry regardless of the ultimate fate of the * theories.

For the case of anti-de Sitter supersymmetry, it has long been known that a generalized Freund-Rubin compactification yields backgrounds of the form $AdS \times$ Sphere. Furthermore, a linearized Kaluza-Klein analysis indicates that the zero mode fluctuations about the $AdS \times$ Sphere background gives rise to a maximal gauged supergravity in the lower dimensional AdS space. More recently, various full and truncated non-linear Kaluza-Klein reductions have been constructed, demonstrating the consistent embedding of the corresponding gauged supergravities in the higher dimensional theory [10, 11]–[20]. Based on the observation that such gauged supergravities yield a negative cosmological constant of the form $\Lambda \sim -g^2$, one may at least formally obtain a de Sitter supergravity through the analytic continuation to imaginary coupling constant, namely $g \to i\hat{g}$. While we follow this approach in spirit, it is important to note that the de Sitter supergravities discussed here (being descended from * theories) have only real bosonic fields and real gauge couplings (although Majorana conditions on the fermions may have to be relaxed). In particular, it was noted in [5] that reductions of IIB* on $\mathrm{dS}_5 \times H^5$ and M* on $dS_4 \times AdS_7$ yield $D = 5$, $\mathrm{SO}(5,1)$ and $D = 4$, $\mathrm{SO}(6,2)$ de Sitter supergravities, respectively. Furthermore, these reductions are consistent, as they may be obtained by analytic continuation of the corresponding sphere reductions of IIB and M theory [5].

The non-linear Kaluza-Klein ansatze for the conventional sphere reductions allow the embedding of various lower-dimensional solutions into the underlying higher-dimensional theories. For example, R-charged AdS black holes may be lifted to ten and eleven dimensions, where they take on the nature of rotating branes [11]. Similarly, the consistent reductions constructed below allow us to lift dS black holes into the original * theory. In particular, multi-centered dS black holes have been constructed in [21, 22]; these were furthermore shown to satisfy an unconventional supersymmetry involving the imaginary coupling constant mentioned above [22, 23]. Here we demonstrate that such dS black holes provide an interesting cosmological background for * theory and furthermore investigate

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their lifting to ten or eleven dimensions. Such multi-centered black holes have also been considered recently by Behrndt and Cvetič as examples of time-dependent backgrounds in the analytically continued de Sitter supergravity [24].

In the following section 2 we describe the general procedure of obtaining consistent reduction ansatze for * theories. Then in sections 3 and 4 we turn to the specific cases of IIB* and M* reductions, respectively. The latter model is particularly interesting, as it admits a $dS_4 \times AdS_7$ background, whereupon either dS_4 or AdS_7 may be viewed as the lower-dimensional spacetime. Since both cases arise from the same eleven-dimensional background, this hints at some form of a dS_4/AdS_7 duality [4, 25]. Finally, we consider the lifting of dS black holes in section 5, and conclude in section 6.

2. Generalized sphere reductions of IIB* and M* theory

It has been an important observation that non-dilatonic branes, in particular the D3, M2 and M5 branes, serve as interpolating solutions between asymptotic Minkowski and near horizon $AdS \times$ Sphere geometries ($AdS_5 \times S^5$, $AdS_4 \times S^7$ and $AdS_7 \times S^4$, respectively). Alternatively, the $AdS \times$ Sphere geometry directly arises from a generalized Freund-Rubin compactification. In several cases, the complete non-linear Kaluza-Klein reduction corresponding to such geometries is known. However, it is generally more common that only a truncated ansatz (often to a maximal abelian subgroup of the full gauge group, or with a subset of scalars and higher-rank potentials) has been constructed. Such truncated ansatze are often sufficient for the lifting of solutions such as AdS black holes to the higher dimensional theory.

While it would be desirable to obtain a complete reduction, in this paper we exclusively focus on the truncation to the sector arising from the higher dimensional metric and p-form fields. To motivate our approach, consider the Kaluza-Klein sphere reduction ansatz to $AdS_d \times S^n$. In this case, the metric has the general form [17]–[19]

$$ds_D^2 = \Delta^{2/(d-1)} ds_d^2 + g^{-2} \Delta^{-(d-3)/(d-1)} (T^{-1})^{ij} D\mu^i D\mu^j \,. \tag{2.1}$$

Here, i and j run from 1 to $n+1$, and the μ^i's satisfy the constraint $\sum_i \mu^{i\,2} = 1$, corresponding to a parametrization of S^n. The n-sphere itself is given by the coset space $\mathrm{SO}(n+1)/\,\mathrm{SO}(n)$, while T_{ij} is a symmetric unimodular matrix consisting of $\frac{1}{2}n(n+3)$ scalar degrees of freedom parameterizing the coset $\mathrm{SL}(n+1,R)/\,\mathrm{SO}(n+1)$. The isometry of S^n gives rise to a $\mathrm{SO}(n+1)$ gauge symmetry, with gauge potentials $A_{(1)}^{[ij]}$. The gauge covariant derivative is given by, e.g., $D\mu^i = d\mu^i + gA_{(1)}^{ij}\mu^j$. Associated with the metric ansatz (2.1) is a corresponding one on the p-form potential (F_4 for $D = 11$, or F_5 for IIB). This will be considered in more detail below, when we specialize to the various cases at hand.

The above analysis of the near-horizon dynamics of non-dilatonic branes may be generalized to encompass the * theories of [1, 2]. For example, the IIA* and IIB* theories admit branes which are the timelike T-duals of ordinary D branes, while M* theory admits generalized M2 and M5 branes, all of which may have unusual signatures on their

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world sheets [3, 4]. In themselves, these branes are all legitimate solutions of the * theories. However, it has been noted that they may be obtained from the ordinary brane solutions via appropriate analytic continuations.[1] This fact will be important to us below in constructing the generalized sphere reductions.

Just as the ordinary non-dilatonic branes serve as interpolating solutions between maximally symmetric spaces, the branes of * theory serve a similar role. However, in this case, the near horizon limits are generalizations of the $AdS \times$ Sphere geometries to different signatures and different signs of the spacetime and internal space curvatures. In all such cases, the resulting geometries for either spacetime or the internal manifold have the maximally symmetric coset form $\mathrm{SO}(s+1,t)/\mathrm{SO}(s,t)$ or $\mathrm{SO}(s,t+1)/\mathrm{SO}(s,t)$ where s and t denote space and time dimensions, respectively [4]. It should be noted that the internal spaces are often non-compact, and may include time-like coordinates. In particular, M* theory admits an interesting $dS_4 \times AdS_7$ vacuum, which may be obtained as the near horizon of either a M2 $(3,0,-)$ or a M5 $(5,1,+)$ brane. This has led to speculation on a possible AdS/CFT duality between the worldvolume theories of M2 and M5, with the roles of spacetime and internal space interchanged [4, 25].

In order to obtain the non-linear Kaluza-Klein ansatz for reduction on either $\mathrm{SO}(s+1,t)/\mathrm{SO}(s,t)$ or $\mathrm{SO}(s,t+1)/\mathrm{SO}(s,t)$, we may analytically continue away from the sphere, S^n, corresponding to $\mathrm{SO}(n+1)/\mathrm{SO}(n)$. More specifically, starting with the homogeneous embedding of S^n in R^{n+1}, given by $(\mu^1)^2 + (\mu^2)^2 + \cdots + (\mu^{n+1})^2 = 1$, we analytically continue an appropriate subset of μ^i's, namely[2] $\mu^i \to i\hat\mu^i$, while at the same time changing the signature of the embedding space in the natural manner. This generalization to a non-compact internal space is conveniently encoded in terms of the lorentzian metric, η_{ij}, on the embedding space, with the hyperbolic embedding specified by the constraint $\eta_{ij}\hat\mu^i\hat\mu^j = -1$. In this case, the generalization of the metric reduction ansatz, (2.1), takes the essentially identical form

$$ds_D^2 = \Delta^{2/(d-1)} ds_d^2 + \hat g^{-2}\Delta^{-(d-3)/(d-1)}\eta_{ij}(\hat T^{-1})^{jk}\eta_{kl} D\hat\mu^i D\hat\mu^l \,. \tag{2.2}$$

Note, however, that while the scalars are still represented by a symmetric matrix $\hat T_{ij}$, they now parametrize the coset $\mathrm{SL}(s+1+t,R)/\mathrm{SO}(s+1,t)$ or $\mathrm{SL}(s+t+1,R)/\mathrm{SO}(s,t+1)$ as appropriate to a generalized signature internal space. As a result, $\hat T_{ij}$ may have negative eigenvalues, and $\hat T_{ij} = \eta_{ij}$ corresponds to the vacuum with no scalar excitations.

At this point, it is important to realize that our claim of negative eigenvalues for $\hat T_{ij}$ yields an unconventional version of non-compact gauging. In an ordinary gauged supergravity theory, one may choose to gauge a non-compact group, say, $\mathrm{SO}(p,q)$. Nevertheless, all fields have conventional kinetic energies, and the theory remains unitary. In particular, the equivalent of $\hat T_{ij}$ for the conventional supergravity never has negative eigenvalues, regardless of the compact versus non-compact nature of the gauging. Furthermore, when the gauging is removed by taking $g \to 0$, one recovers the standard ungauged supergravity.

[1] That this is the case simply follows from the fact that the supergravity description of the * theories may be obtained by suitable Wick rotations and analytic continuations of the usual supergravities.

[2] Throughout this paper, we use a caret to distinguish quantities involved in the generalized reduction from those of the usual sphere reduction.

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For a conventional non-compact gauging, the gauge group is spontaneously broken to its maximal compact subgroup. In this case, it has been shown in [26, 27] that such theories arise from dimensional-reductions on non-compact spaces, and that they may be obtained via analytical continuation in the same spirit that we are advocating here. The difference with the present case, however, is that here we choose to reduce the * theory on the maximally symmetric vacuum. In particular, this means that, even in the ungauged limit, we retain both compact and non-compact gauge fields (with both positive and negative kinetic terms, related to the eigenvalues of η_{ij}). In the framework of [26], we would be expanding about an unstable vacuum. However we leave this issue as one that must be resolved by the underlying * theory.

While (2.2) is the universal form of the metric ansatz for a generalized internal space, we have at the moment left the nature of the p-form field unspecified. To proceed, we must specialize to a particular theory. This is what we carry out in the next section (for IIB* theory) and the subsequent one (for M* theory).

3. The H^5 reduction of IIB* supergravity

The bosonic sector of IIB supergravity consists of the metric, dilaton and 3-form field strength $H_{(3)}$ in the NSNS sector, as well as $F_{(1)}$, $F_{(3)}$ and $F_{(5)}$ in the RR sector. The complete S^5 reduction of type IIB supergravity gives rise to five-dimensional SO(6) gauged $N = 8$ supergravity. However, by considering the truncation of IIB supergravity to only the metric and self-dual 5-form, one ends up in five dimensions with a truncation of $N = 8$ supergravity to a bosonic system with SO(6) gauge fields and 20 scalars. The consistent S^5 reduction in this subsector (which is the one most directly relevant to the D3-brane) was given in [19], and has the form

$$\begin{aligned} ds_{10}^2 &= \Delta^{1/2} ds_5^2 + g^{-2}\Delta^{-1/2}(T^{-1})^{ij} D\mu^i D\mu^j \,, \\ F_{(5)} &= G_{(5)} + {*}G_{(5)} \,, \\ G_{(5)} &= -gU\epsilon_{(5)} + g^{-1}((T^{-1})^{ij} \bar{*} DT_{jk}) \wedge (\mu^k D\mu^i) - \frac{1}{2} g^{-2}(T^{-1})^{ik}(T^{-1})^{jl} \bar{*} F_{(2)}^{ij} \wedge D\mu^k \wedge D\mu^l \,, \end{aligned} \tag{3.1}$$

where

$$U = 2T_{ij}T_{jk}\mu^i\mu^k - \Delta T_{ii} \,, \qquad \Delta = T_{ij}\mu^i\mu^j \,. \tag{3.2}$$

Here, $\epsilon_{(5)}$ is the volume 5-form in the spacetime. As usual, the field strength $F_{(2)}^{[ij]}$ and gauge covariant derivatives are given in terms of the SO(6) gauge fields $A_{(1)}^{[ij]}$ by

$$\begin{aligned} F_{(2)}^{ij} &= dA_{(1)}^{ij} + gA_{(1)}^{ik} \wedge A_{(1)}^{kj} \,, \\ DT_{ij} &= dT_{ij} + gA_{(1)}^{ik} T_{kj} + gA_{(1)}^{jk} T_{ik} \,, \\ D\mu^i &= d\mu^i + gA_{(1)}^{ij} \mu^j \,. \end{aligned} \tag{3.3}$$

We mostly follow the notation of [19], except that we reserve the caret to denote quantities relevant to * theory. The coordinates, μ^i, are subject to the constraint $\delta_{ij}\mu^i\mu^j = 1$, and parametrize the internal S^5.

We now recall that IIB* supergravity with signature $(9,1)$ may be obtained from IIB theory by changing the signs of the RR kinetic terms. In particular, the wrong-sign kinetic term for the self-dual 5-form gives rise to the ten-dimensional Einstein equation

$$R_{MN} = -\frac{1}{2\cdot 2\cdot 4!}\hat{F}_{MPQRS}\hat{F}_N{}^{PQRS}\,, \tag{3.4}$$

which in turn leads to a $\mathrm{dS}_5 \times H^5$ Freund-Rubin vacuum. This simply corresponds to an interchange of positive and negative curvatures between spacetime and internal space (as is evident from the extra sign in the above Einstein equation). It should now be evident how the appropriate IIB* reduction ansatz may be obtained by the analytic continuation of (3.1). To generate the wrong-sign kinetic term, we take $F_{(5)} \to i\hat{F}_{(5)}$. However, since $G_{(5)} = -gU\epsilon_{(5)} + \cdots$, we avoid a resulting imaginary $\hat{G}_{(5)}$ by simultaneously taking $g \to i\hat{g}$. At this stage, we find

$$\begin{aligned} ds_{10}^2 &= \Delta^{1/2} ds_5^2 - \hat{g}^{-2}\Delta^{-1/2}(T^{-1})^{ij} D\mu^i D\mu^j\,,\\ \hat{G}_{(5)} &= -\hat{g}U\epsilon_{(5)} - \hat{g}^{-1}((T^{-1})^{ij} {*}DT_{jk})\wedge(\mu^k D\mu^i) - \frac{i}{2}\hat{g}^{-2}(T^{-1})^{ik}(T^{-1})^{jl} {*}F_{(2)}^{ij}\wedge D\mu^k\wedge D\mu^l\,. \end{aligned}$$

Here, we implicitly assume that both U and Δ remain real quantities throughout the analytic continuation. This intermediate result for the metric and 5-form is unsatisfactory in that the internal five-dimensional space has the wrong signature and that $\hat{G}_{(5)}$ is still complex because of the factor of i in the last term. Both issues may be solved by the analytic continuation

$$\begin{aligned} &\mu^i \to i\hat{\mu}^i\,, && \mu^6 \to \hat{\mu}^6\,,\\ &A_{(1)}^{ij} \to -i\hat{A}_{(1)}^{ij}\,, && A_{(1)}^{[i6]} \to -\hat{A}_{(1)}^{[i6]} \qquad (i,j = 1,\ldots,5)\,. \end{aligned} \tag{3.5}$$

It is this step that changes the μ^i's from parameterizing S^5 as $\mathrm{SO}(6)/\,\mathrm{SO}(5)$ to parameterizing H^5 as $\mathrm{SO}(5,1)/\,\mathrm{SO}(5)$. At the same time, the analytic continuation of the gauge fields leads to the non-compact gauge group $\mathrm{SO}(5,1)$ [5].

In order for U and Δ to be real, we finally continue appropriate entries in the T_{ij} matrix, namely

$$T_{ij} \to \hat{T}_{ij}\,, \qquad T_{(i6)} \to i\hat{T}_{(i6)}\,, \qquad T_{66} \to -\hat{T}_{66} \qquad (i,j = 1,\ldots,5)\,, \tag{3.6}$$

with corresponding inverse

$$\begin{aligned} &(T^{-1})^{ij} \to (\hat{T}^{-1})^{ij}\,, \qquad (T^{-1})^{(i6)} \to -i(\hat{T}^{-1})^{(i6)}\,, \qquad (T^{-1})^{66} \to -(\hat{T}^{-1})^{66}\\ &(i,j = 1,\ldots,5)\,. \end{aligned} \tag{3.7}$$

Note that $\hat{T}$ has a single negative eigenvalue, and $\det\hat{T} = -1$. We reiterate that this negative eigenvalue results in both correct and wrong-sign gauge kinetic energy terms showing up, depending on the non-compact versus compact nature of the corresponding generator. The resulting reduction ansatz for IIB* theory on $dS_5 \times H^5$ is most conveniently given in terms of the SO(5,1) metric $\eta_{ij} = \mathrm{diag}(+,+,+,+,+,-)$. We find

$$
\begin{aligned}
ds_{10}^2 &= \Delta^{1/2} ds_5^2 + \hat{g}^{-2}\Delta^{-1/2}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl} D\hat{\mu}^i D\hat{\mu}^l \,, \\
\hat{F}_{(5)} &= \hat{G}_{(5)} + *\hat{G}_{(5)} \,, \\
\hat{G}_{(5)} &= -\hat{g} U \epsilon_{(5)} + \hat{g}^{-1}\left(\eta_{ij}\left(\hat{T}^{-1}\right)^{jk} \bar{*} D\hat{T}_{kl}\right) \wedge (\hat{\mu}^l D\hat{\mu}^i) - \\
&\quad - \frac{1}{2}\hat{g}^{-2}\eta_{ij}(\hat{T}^{-1})^{jk}\eta_{kl}\eta_{mn}\left(\hat{T}^{-1}\right)^{np}\eta_{pq}\bar{*}\hat{F}_{(2)}^{im} \wedge \left(D\hat{\mu}^l \wedge D\hat{\mu}^q\right) ,
\end{aligned} \tag{3.8}
$$

where

$$
U = -2\hat{T}_{ij}\eta^{jk}\hat{T}_{kl}\hat{\mu}^i\hat{\mu}^l - \Delta\eta^{ij}\hat{T}_{ij} \,, \qquad \Delta = -\hat{T}_{ij}\hat{\mu}^i\hat{\mu}^j \,, \qquad \eta_{ij}\hat{\mu}^i\hat{\mu}^j = -1 \,, \tag{3.9}
$$

and the gauge covariant derivatives are given by

$$
\begin{aligned}
\hat{F}_{(2)}^{ij} &= d\hat{A}_{(1)}^{ij} + \hat{g}\eta_{kl}\hat{A}_{(1)}^{ik} \wedge \hat{A}_{(1)}^{lj} \,, \\
D\hat{T}_{ij} &= d\hat{T}_{ij} + \hat{g}\eta_{ik}\hat{A}_{(1)}^{kl}\hat{T}_{lj} + \hat{g}\eta_{jk}\hat{A}_{(1)}^{kl}\hat{T}_{il} \,, \\
D\hat{\mu}^i &= d\hat{\mu}^i + \hat{g}\eta_{jk}\hat{A}_{(1)}^{ij}\hat{\mu}^k \,.
\end{aligned} \tag{3.10}
$$

Although the structure of the non-compact gauging is perhaps obvious in this reduction, to avoid confusion we will always retain explicit factors of the SO(5,1) metric η_{ij}.

Since this reduction of IIB* supergravity on $\mathrm{dS}_5 \times H^5$ was obtained by appropriate continuation of the ansatz given in [19], it follows that the resulting five-dimensional lagrangian may similarly be obtained through analytic continuation. The resulting lagrangian has the form [19]

$$
\begin{aligned}
\mathcal{L}_5 &= R\,\bar{*}1 - \frac{1}{4}(\hat{T}^{-1})^{ij}\bar{*}D\hat{T}_{jk} \wedge (\hat{T}^{-1})^{kl} D\hat{T}_{li} + \frac{1}{4}\eta_{ij}(\hat{T}^{-1})^{jk}\eta_{kl}\eta_{mn}(\hat{T}^{-1})^{np}\eta_{pq}\bar{*}\hat{F}_{(2)}^{im} \wedge \hat{F}_{(2)}^{lq} - \\
&\quad - V\bar{*}1 - \frac{1}{48}\epsilon_{i_1 i_2 i_3 i_4 i_5 i_6}\Big(\hat{F}_{(2)}^{i_1 i_2}\hat{F}_{(2)}^{i_3 i_4}\hat{A}_{(1)}^{i_5 i_6} - \hat{g}\hat{F}_{(2)}^{i_1 i_2}\hat{A}_{(1)}^{i_3 i_4}\hat{A}_{(1)}^{i_5 j}\eta_{jk}\hat{A}_{(1)}^{k i_6} + \\
&\qquad + \frac{2}{5}\hat{g}^2\hat{A}^{i_1 i_2}\hat{A}^{i_3 j}\eta_{jk}\hat{A}^{k i_4}\hat{A}^{i_5 l}\eta_{lm}\hat{A}^{m i_6}\Big) \,.
\end{aligned} \tag{3.11}
$$

This corresponds to a truncation of maximally symmetric SO(5,1) gauged de Sitter supergravity. Note that the kinetic terms for the gauge fields in the compact subgroup $\mathrm{SO}(5) \subset \mathrm{SO}(5,1)$ have the wrong sign, as expected in the $*$ theory. In addition, the potential

$$
V = -\frac{1}{2}\hat{g}^2\left(2\hat{T}_{ij}\eta^{jk}\hat{T}_{kl}\eta^{li} - (\eta^{ij}\hat{T}_{ij})^2\right) , \tag{3.12}
$$

has opposite sign from the sphere case, and yields a maximally symmetric dS_5 vacuum, invariant under the de Sitter supergroup SU*(4|4). Note that this is the opposite sign of the SO(6) but *not* the SO(5,1) potential of [28, 29, 30] since in the present case $T_{ij}\eta^{jk}$ has all positive eigenvalues.

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3.1 Truncation to $D = 5$, $N = 2$ de Sitter supergravity

The maximal $N = 8$ SO(6) gauged anti-de Sitter supergravity has a natural $\mathrm{U}(1)^3$ truncation to $N = 2$ gauged supergravity coupled to two vector multiplets. The bosonic fields of this truncation comprise the metric, two scalars and three gauge fields. The three gauge fields are naturally taken to be the mutually commuting subset A^{12}, A^{34} and A^{56} of the full SO(6) gauge fields. At the same time, the two scalars originate from the parametrization of T_{ij} as $T = \mathrm{diag}(X_1, X_1, X_2, X_2, X_3, X_3)$, with the constraint $X_1X_2X_3 = 1$.

Turning to the de Sitter supergravity, on the other hand, the maximal compact subgroup $\mathrm{SO}(5) \subset \mathrm{SO}(5, 1)$ does not admit a $\mathrm{U}(1)^3$ truncation. Nevertheless, we may perform an analogous truncation to two compact and one non-compact U(1) gauge fields. To do so, we let

$$A^1 = \hat{A}^{12}\,, \qquad A^2 = \hat{A}^{34}\,, \qquad A^3 = \hat{A}^{56}\,, \tag{3.13}$$

where A^3 is the non-compact gauge field. In addition, the scalars may be given by $\hat{T} = \mathrm{diag}(X_1, X_1, X_2, X_2, X_3, -X_3)$, with $X_1X_2X_3 = 1$.

Since the choice of $\mathrm{U}(1)^3$ truncation corresponds to taking mutually commuting rotation (boost) planes along the 1-2, 3-4 and 5-6 directions, it is natural to parametrize the hyperboloid H^5 by taking

$$\hat{\mu} = \{\mu_1 \sin\phi_1, \mu_1 \cos\phi_1, \mu_2 \sin\phi_2, \mu_2 \cos\phi_2, \mu_3 \sinh\phi_3, \mu_3 \cosh\phi_3\}\,. \tag{3.14}$$

In this case, the constraint $\eta_{ij}\hat{\mu}^i\hat{\mu}^j = -1$ turns into $\mu_1^2 + \mu_2^2 - \mu_3^2 = -1$, and the reduction ansatz (3.8) becomes

$$\begin{aligned} ds_{10}^2 &= \Delta^{1/2} ds_5^2 + \hat{g}^{-2}\Delta^{-1/2} \sum_{i=1}^{3} X_i^{-1}(\eta_i d\mu_i^2 + \mu_i^2(d\phi_i + \hat{g}A^i)^2)\,, \\ \hat{F}_{(5)} &= \hat{G}_{(5)} + {*\hat{G}_{(5)}}\,, \\ \hat{G}_{(5)} &= 2\hat{g}\sum_{i=1}^{3}(\eta_i X_i^2\mu_i^2 + \Delta X_i)\epsilon_{(5)} + \frac{1}{2}\hat{g}^{-1}\sum_{i=1}^{3}\eta_i X_i^{-1} \bar{*} dX_i \wedge d(\mu_i^2) + \\ &\quad + \frac{1}{2}\hat{g}^{-2}\sum_{i=1}^{3}\eta_i X_i^{-2} d(\mu_i^2) \wedge (d\phi_i - \hat{g}A^i) \wedge \bar{*}F^i\,, \end{aligned} \tag{3.15}$$

where $\Delta = -\sum_{i=1}^{3}\eta_i X_i \mu_i^2$ and $\eta_i = (+1, +1, -1)$, signifying the non-compact nature of H^5. Inserting the above $\mathrm{U}(1)^3$ truncation into (3.11), we obtain the lagrangian describing the bosonic sector of the $N = 2$ theory:

$$e^{-1}\mathcal{L}_5 = R\bar{*}1 - \frac{1}{2}\bar{*}d\varphi_1\wedge d\varphi_1 - \frac{1}{2}\bar{*}d\varphi_2\wedge d\varphi_2 - V\bar{*}1 + \frac{1}{2}\sum_{i=1}^{3}\eta_i X_i^{-2}\bar{*}F^i\wedge F^i - F^1\wedge F^2\wedge A^3\,. \tag{3.16}$$

Here we have chosen to parametrize the scalars in terms of two dilatons $\vec{\varphi} = \{\varphi_1, \varphi_2\}$ according to

$$X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\varphi}}\,, \qquad \vec{a}_i\cdot\vec{a}_j = 4\delta_{ij} - \frac{4}{3}\,. \tag{3.17}$$

Note that the first two compact gauge fields in (3.16) have wrong sign kinetic terms, while the non-compact gauge field, which would ordinarily have come in with the wrong sign, now enters with the proper one. The potential is positive definite, and has the form $V = 4\hat{g}^2 \sum_i X_i^{-1}$. While we have started with a truncation of the $D = 5$, $N = 8$ theory, the $N = 2$ content is complete. Thus from an $N = 2$ perspective, we have obtained a consistent reduction of IIB* theory (followed by truncation) to $N = 2$ de Sitter supergravity in five dimensions coupled to two vector multiplets, at least in the bosonic sector.

We may further truncate the bosonic lagrangian (3.16) by setting $F^1 = F^2 = F/\sqrt{2}$ along with $X_1 = X_2 = X_3^{-1/2} = e^{-\frac{1}{\sqrt{6}}\varphi}$. The resulting lagrangian is that of $N = 2$ de Sitter supergravity coupled to a single vector multiplet

$$\begin{aligned} e^{-1}\mathcal{L}_5 = {}& R{*}1 - \frac{1}{2}{*}d\varphi \wedge d\varphi - 4\hat{g}^2 \left(2e^{\frac{1}{\sqrt{6}}\varphi} + e^{-\frac{2}{\sqrt{6}}\varphi}\right) {*}1 + \frac{1}{2}e^{\frac{2}{\sqrt{6}}\varphi}{*}F \wedge F - \\ & - \frac{1}{2}e^{-\frac{4}{\sqrt{6}}\varphi}{*}\mathcal{F} \wedge \mathcal{F} - \frac{1}{2}F \wedge F \wedge \mathcal{A}. \end{aligned} \tag{3.18}$$

Here $\mathcal{A} = A^3$ denotes the non-compact gauge field.

The anti-de Sitter supergravity, where both F and $\mathcal{F}$ have proper kinetic terms, admits one further truncation to eliminate the remaining vector multiplet by setting $\varphi = 0$ and $A = \sqrt{2}\mathcal{A}$. However, in this case, a consistent truncation to $\varphi = 0$ involves satisfying the condition ${*}F \wedge F + {*}\mathcal{F} \wedge \mathcal{F} = 0$, which arises from the φ equation of motion. Since this condition cannot be met for real gauge fields, we are unable to reduce the de Sitter theory of (3.18) any further.

It should be noted, of course, that were one to simply analytically continue from the truncated $N = 2$ anti-de Sitter supergravity lagrangian to obtain the pure supergravity truncation of (3.16), one would have to set $iF^1 = iF^2 = F^3 = F/\sqrt{3}$ in order to obtain

$$e^{-1}\mathcal{L}_5 = R{*}1 - 12\hat{g}^2{*}1 - \frac{1}{2}{*}F \wedge F + \frac{1}{3\sqrt{3}}F \wedge F \wedge A\,. \tag{3.19}$$

While this appears to yield a desirable theory with proper sign kinetic term and positive cosmological constant, we emphasize that this cannot be viewed as a reduction of IIB nor of IIB* theory, since this would necessarily involve imaginary (or in general complex) bosonic fields.

4. Reductions of M^*-theory

In the previous section we have examined the H^5 reduction of IIB* supergravity, which gives rise to a de Sitter supergravity in five dimensions. Of course, similar techniques may be applied for the reduction of M^* theory. In particular, we now turn to the $dS_4 \times AdS_7$ reduction of $M^*_{(9,2)}$ theory. Since this theory has two time directions, an interesting feature arises in that we have the freedom of regarding either dS_4 or AdS_7 as the spacetime, with the other factor considered as an internal space. We now examine both possibilities in detail.

4.1 The AdS$_7$ reduction of M* theory

By taking AdS$_7$ as the 'compactification space', the resulting reduction will yield a four-dimensional de Sitter supergravity theory with non-compact gauge symmetry SO$(6,2)$ [5]. As before, we begin with the truncated S^7 reduction ansatz of M-theory, given by [17]

$$
\begin{aligned}
ds_{11}^2 &= \Delta^{2/3} ds_4^2 + g^{-2}\Delta^{-1/3} T_{ij}^{-1} D\mu^i D\mu^j\,, && (4.1)\\
F_{(4)} &= -gU\epsilon_{(4)} + g^{-1}((T^{-1})^{ij} {\bar{*}} DT_{jk}) \wedge (\mu^k D\mu^i) - \\
&\quad -\frac{1}{2} g^{-2}(T^{-1})^{ik}(T^{-1})^{jl} {\bar{*}} F_{(2)}^{ij} \wedge D\mu^k \wedge D\mu^l\,, && (4.2)
\end{aligned}
$$

where

$$
U = 2T_{ij}T_{jk}\mu^i\mu^k - \Delta T_{ii}, \qquad \Delta = T_{ij}\mu^i\mu^j\,, \tag{4.3}
$$

and $\delta_{ij}\mu^i\mu^j = 1$ (with $i,j = 1,2,\ldots,8$) so that the μ^i coordinates parametrize a seven-sphere. This ansatz retains the full set of SO(8) Yang-Mills fields, $A_{(1)}^{[ij]}$, which satisfy the standard relations given by (3.3). In addition, there are 35 scalars represented by the symmetric unimodular matrix T_{ij}, and which are described by the coset SL$(8,R)/$SO(8). Note that the 35 pseudo-scalars of $D=4$, $N=8$ have been truncated away, and as a result this is technically not a consistent truncation. Nevertheless, this ansatz may be used to lift a large class of four-dimensional solutions without axions.

The M$^*_{(9,2)}$ supergravity may be formally obtained from M-theory by analytically continuing the four-form, $F_{(4)} \to i\hat{F}_{(4)}$, while simultaneously performing a Wick rotation on one of the spatial coordinates (so as to yield a theory with two times). For the Freund-Rubin ansatz leading to $AdS_4 \times S^7$, it is clear that the Wick rotation should be performed on one of the seven sphere coordinates, thus yielding $AdS_4 \times dS_7$. The analytic continuation on $F_{(4)}$ then finally gives the $dS_4 \times AdS_7$ solution of M$^*_{(9,2)}$. As a result, this leads us to make the continuation $g \to i\hat{g}$ along with a reparametrization of the sphere coordinates

$$
\mu^i \to i\hat{\mu}^i, \qquad \mu^m \to \hat{\mu}^m \qquad (i = 1,\ldots,6, \quad m = 7,8)\,. \tag{4.4}
$$

The resulting $\hat{\mu}$'s now parametrize AdS$_7$ in terms of an SO$(6,2)/$SO$(6,1)$ coset.

In addition, we must analytically continue the gauge fields

$$
A_{(1)}^{ij} \to -i\hat{A}_{(1)}^{ij}\,, \quad A_{(1)}^{[im]} \to -\hat{A}_{(1)}^{[im]}\,, \quad A_{(1)}^{[mn]} \to i\hat{A}_{(1)}^{[mn]} \quad (i,j = 1,\ldots,6, \quad m,n = 7,8)\,, \tag{4.5}
$$

as well as the scalar matrix

$$
T_{ij} \to \hat{T}_{ij}\,, \quad T_{(im)} \to i\hat{T}_{(im)}\,, \quad T_{mn} \to -\hat{T}_{mn} \quad (i,j = 1,\ldots,6, \quad m,n = 7,8)\,. \tag{4.6}
$$

The analytic continuation of the gauge fields leads to a non-compact gauge group SO$(6,2)$ [5]. As a result, the reduction ansatz for M$^*_{(9,2)}$ theory on dS$_4 \times AdS_7$ is given by

$$
\begin{aligned}
ds_{11}^2 &= \Delta^{2/3} ds_4^2 + \hat{g}^{-2}\Delta^{-1/3}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl} D\hat{\mu}^i D\hat{\mu}^l\,,\\
\hat{F}_{(4)} &= -\hat{g}U\epsilon_{(4)} + \hat{g}^{-1}\left(\eta_{ij}\left(\hat{T}^{-1}\right)^{jk} {\bar{*}} D\hat{T}_{kl}\right) \wedge \left(\hat{\mu}^l D\hat{\mu}^i\right) - \\
&\quad -\frac{1}{2}\hat{g}^{-2}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl}\eta_{mn}\left(\hat{T}^{-1}\right)^{np}\eta_{pq} {\bar{*}} \hat{F}_{(2)}^{im} \wedge \left(D\hat{\mu}^l \wedge D\hat{\mu}^q\right)\,, \qquad (4.7)
\end{aligned}
$$

where

$$U = -2\hat{T}_{ij}\eta^{jk}\hat{T}_{kl}\hat{\mu}^i\hat{\mu}^l - \Delta\eta^{ij}\hat{T}_{ij}\,, \qquad \Delta = -\hat{T}_{ij}\hat{\mu}^i\hat{\mu}^j, \qquad \eta_{ij}\hat{\mu}^i\hat{\mu}^j = -1\,. \tag{4.8}$$

Here, $\eta_{ij} = \mathrm{diag}(+,+,+,+,+,+,-,-)$ is the SO(6, 2) invariant metric. The SO(6, 2) covariant derivatives may be written in a straightforward manner using η_{ij} when appropriate, and have the same structure as those of (3.10).

The resulting four-dimensional lagrangian has the form

$$\begin{aligned}\mathcal{L}_4 = {} & R\,\bar{*}1 - \frac{1}{4}\left(\hat{T}^{-1}\right)^{ij}\bar{*}D\hat{T}_{jk}\wedge\left(\hat{T}^{-1}\right)^{kl}D\hat{T}_{li} + \\ & + \frac{1}{4}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl}\eta_{mn}\left(\hat{T}^{-1}\right)^{np}\eta_{pq}\bar{*}\hat{F}^{im}_{(2)}\wedge\hat{F}^{lq}_{(2)} - V\,\bar{*}1\,,\end{aligned} \tag{4.9}$$

and corresponds to a truncation (without pseudoscalars) of the bosonic sector of $N = 8$, SO(6, 2) gauged de Sitter supergravity. The potential is given by

$$V = -\frac{1}{2}\hat{g}^2\left(2\hat{T}_{ij}\eta^{jk}\hat{T}_{kl}\eta^{li} - (\eta^{ij}\hat{T}_{ij})^2\right), \tag{4.10}$$

and yields a maximally symmetric dS_4 vacuum. In addition, the gauge fields in the compact subgroup $\mathrm{SO}(6)\times\mathrm{SO}(2)\subset\mathrm{SO}(6,2)$ have wrong sign kinetic terms.

4.2 Truncation to $D = 4$, $N = 2$ de Sitter supergravity

Although the $N = 8$ de Sitter supergravity has a non-compact SO(6, 2) gauge group, it nevertheless admits a natural $\mathrm{U}(1)^4\subset\mathrm{SO}(6)\times\mathrm{SO}(2)\subset\mathrm{SO}(6,2)$ truncation to $N = 2$ de Sitter supergravity coupled to three vector multiplets. The four gauge fields are naturally taken as

$$A^1 = \hat{A}^{12}\,, \qquad A^2 = \hat{A}^{34}\,, \qquad A^3 = \hat{A}^{56}\,, \qquad A^4 = \hat{A}^{78}\,, \tag{4.11}$$

where we follow the notation of the previous section that $i, j = 1, \ldots, 6$ are SO(6) indices, while $m, n = 7, 8$ are SO(2) indices. This choice of parametrization also suggests that we take

$$\hat{\mu} = \{\mu_a\sin\phi_a, \mu_a\cos\phi_a\}\,, \qquad a = 1, \ldots, 4\,. \tag{4.12}$$

Note that ϕ_4 has the conventional interpretation as a periodic AdS_7 time coordinate. Thus A^4, the SO(2) gauge field, is necessarily connected to gauging the isometry related to the second time direction of the $M^*_{(9,2)}$ theory.

Corresponding to this choice of truncation, we take the surviving scalars to be given by $\hat{T} = \mathrm{diag}(X_1, X_1, X_2, X_2, X_3, X_3, -X_4, -X_4)$, where $X_1X_2X_3X_4 = 1$. The background AdS_7 geometry is determined by the constraint $\eta_{ij}\hat{\mu}^i\hat{\mu}^j = -1$, or equivalently $\mu_1^2 + \mu_2^2 + \mu_3^2 - \mu_4^2 = -1$. The truncation of (4.7) is then

$$\begin{aligned} ds_{11}^2 = {} & \Delta^{2/3}ds_4^2 + \hat{g}^{-2}\Delta^{-1/3}\sum_{i=1}^{4}X_i^{-1}\eta_i\left(d\mu_i^2 + \mu_i^2\left(d\phi_i + \hat{g}A^i\right)^2\right), \\ \hat{F}_{(4)} = {} & 2\hat{g}\sum_{i=1}^{4}\left(\eta_iX_i^2\mu_i^2 + \Delta X_i\right)\epsilon_{(4)} + \frac{1}{2}\hat{g}^{-1}\sum_{i=1}^{4}\eta_iX_i^{-1}\bar{*}dX_i\wedge d\left(\mu_i^2\right) + \\ & + \frac{1}{2}\hat{g}^{-2}\sum_{i=1}^{4}\eta_iX_i^{-2}d\left(\mu_i^2\right)\wedge\left(d\phi_i - \hat{g}A^i\right)\wedge\bar{*}F^i\,, \end{aligned} \tag{4.13}$$

where $\Delta = -\sum_{i=1}^4 \eta_i X_i \mu_i^2$ and $\eta_i = (+1,+1,+1,-1)$. The resulting theory may be described by the lagrangian

$$e^{-1}\mathcal{L}_4 = R\bar{*}1 - \frac{1}{2}\sum_{\alpha=1}^{3} \bar{*}d\varphi_\alpha \wedge d\varphi_\alpha + \frac{1}{2}\sum_{i=1}^{4} X_i^{-2}\bar{*}F^i \wedge F^i - V\bar{*}1\,, \tag{4.14}$$

where

$$V = 4\hat{g}^2 \sum_{i<j} X_i X_j = 8\hat{g}^2 \sum_{\alpha=1}^{3} \cosh\varphi_\alpha \tag{4.15}$$

is positive definite. Here the X_i scalars are parametrized in terms of three dilatons $\vec{\varphi} = \{\varphi_1, \varphi_2, \varphi_3\}$ according to

$$X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\varphi}}\,, \qquad \vec{a}_i \cdot \vec{a}_j = 4\delta_{ij} - 1\,. \tag{4.16}$$

This lagrangian is simply the $\mathrm{U}(1)^4$ truncation of (4.9). Unlike the $D = 5$, $N = 2$ de Sitter theory of (3.18), here all four U(1) fields are compact (and all have wrong sign kinetic terms). Note furthermore that, as already mentioned previously, (4.14) cannot be viewed as a consistent $N = 2$ reduction of $M^*_{(9,2)}$ theory, as it is missing three axionic scalars. Hence the vector multiples are incomplete. This truncation with only dilatonic scalars is similar to that considered in [31] for the truncated $N = 8$ anti-de Sitter supergravity.

We may further truncate the bosonic lagrangian (4.14) by setting $F^1 = F^2 = F/\sqrt{2}$ and $F^3 = F^4 = \mathcal{F}/\sqrt{2}$ along with $X_1 = X_2 = X_3^{-1} = X_4^{-1} = e^{\frac{1}{2}\varphi}$. The resulting lagrangian is that of $N = 2$ de Sitter supergravity coupled to a single vector multiplet

$$e^{-1}\mathcal{L}_4 = R\bar{*}1 - \frac{1}{2}\bar{*}d\varphi \wedge d\varphi - 8\hat{g}^2(2+\cosh\varphi)\bar{*}1 + \frac{1}{2}e^{-\varphi}\bar{*}F\wedge F + \frac{1}{2}e^{\varphi}\bar{*}\mathcal{F}\wedge\mathcal{F}\,. \tag{4.17}$$

While this theory is still incomplete in that it lacks an axion, one further truncation to pure $N = 2$ de Sitter supergravity is possible by equating the U(1) fields and setting $\varphi = 0$. The pure supergravity lagrangian is

$$e^{-1}\mathcal{L}_4 = R\bar{*}1 - 24\hat{g}^2\bar{*}1 + \frac{1}{2}\bar{*}F\wedge F\,, \tag{4.18}$$

and concisely captures both features of * theory, namely a positive cosmological constant and wrong-sign kinetic term for the graviphoton.

4.3 The dS$_4$ reduction of M* theory

We now consider the second possibility for interpreting the $dS_4 \times AdS_7$ compactification of $M^*_{(9,2)}$ theory, namely where we view the resulting theory as a seven dimensional anti-de Sitter supergravity with SO(4,1) gauging. This case is especially interesting in that the *complete* consistent Kaluza-Klein reduction of ordinary eleven-dimensional supergravity to seven dimensions is known from the work of [12, 13]. Thus, unlike in the cases considered above, there is no need to truncate.

To obtain the dS_4 reduction of $M^*_{(9,2)}$ theory, we analytically continue and Wick rotate the ansatz obtained in [13, 32], which is given by

$$\begin{aligned} ds_{11} &= \Delta^{1/3} ds_7^2 + g^{-2}\Delta^{-2/3} T^{-1}_{ij} D\mu^i D\mu^j \,, \\ *_{11}F_{(4)} &= -gU\epsilon_{(7)} - g^{-1}\left(T^{-1}_{ij}\bar{*}DT_{jk}\right)\wedge(\mu^k D\mu^i) + \frac{1}{2}g^{-2}T^{-1}_{ik}T^{-1}_{jl}\bar{*}F^{ij}_{(2)}\wedge D\mu^k\wedge D\mu^l + \\ &\quad + g^{-4}\Delta^{-1}T_{ij}S^i_{(3)}\mu^j\wedge W - \frac{1}{6}g^{-3}\Delta^{-1}\epsilon_{ijl_1l_2l_3}\bar{*}S^m_{(3)}T_{im}T_{jk}\mu^k\wedge D\mu^{l_1}\wedge D\mu^{l_2}\wedge D\mu^{l_3}\,, \end{aligned}$$

where

$$\begin{aligned} U &= 2T_{ij}T_{jk}\mu^i\mu^k - \Delta T_{ii}\,, \qquad \Delta = T_{ij}\mu^i\mu^j\,, \\ W &= \frac{1}{4!}\epsilon_{i_1i_2i_3i_4i_5}\mu^{i_1}D\mu^{i_2}\wedge D\mu^{i_3}\wedge D\mu^{i_4}\wedge D\mu^{i_5}\,. \end{aligned} \tag{4.19}$$

The full reduction gives rise to $N=4$ gauged SO(5) supergravity in seven dimensions. The bosonic fields consist of 14 scalars given by the symmetric unimodular T_{ij} which describe the coset $\mathrm{SL}(5,R)/\mathrm{SO}(5)$, the SO(5) gauge fields $A^{[ij]}_{(1)}$, and 5 three-form potentials $S^i_{(3)}$ transforming in the fundamental of SO(5). Note that the reduction is given on the dual of the eleven-dimensional four-form, $*_{11}F_{(4)}$, with $\epsilon_{(7)}$ corresponding to the volume 7-form in spacetime. In addition, the coordinates μ^i, subject to the constraint $\delta_{ij}\mu^i\mu^j = 1$, parameterize the unit four-sphere.

The analytic continuation to * theory is different from the previous cases, since here this is no need to take $g \to i\hat{g}$. This is because the simultaneous continuation $F_{(4)} \to i\hat{F}_{(4)}$ along with Wick rotation of one of the S^4 coordinates leaves the present four-form ansatz unchanged. To see this, consider the pure $AdS_7 \times S^4$ solution, given essentially by $F_{(4)} = g\,dx^7\wedge dx^8\wedge dx^9\wedge dx^{10}$. While $F_{(4)}$ picks up an i, so does, say, dx^{10}. The only important modification is then to replace the S^4 coset structure $\mathrm{SO}(5)/\mathrm{SO}(4)$ by the dS_4 coset $\mathrm{SO}(4,1)/\mathrm{SO}(3,1)$ by analytic continuation on the μ_i's. The combined transformation, including that of 1-form and 3-form potentials is given by

$$\begin{aligned} &\mu^i \to \hat{\mu}^i, \qquad \mu^5 \to i\hat{\mu}^5\,, \\ &S^i_{(3)} \to \hat{S}^i_{(3)}, \qquad S^5_{(3)} \to i\hat{S}^5_{(3)}\,, \qquad A^{[ij]}_{(1)} \to \hat{A}^{[ij]}_{(1)}\,, \qquad A^{[i5]}_{(1)} \to i\hat{A}^{[i5]}_{(1)} \qquad (i,j=1,\ldots,4)\,. \end{aligned}$$

In order for U and Δ to be real, we also continue appropriate entries in the T_{ij} matrix, namely

$$T_{ij} \to \hat{T}_{ij}\,, \qquad T_{(i5)} \to -i\hat{T}_{(i5)}\,, \qquad T_{55} \to -\hat{T}_{55} \qquad (i,j=1,\ldots,4)\,. \tag{4.20}$$

The resulting reduction ansatz for $M^*_{(9,2)}$ theory on $dS_4 \times AdS_7$ may be given in terms of the SO(4,1) metric $\eta_{ij} = \mathrm{diag}(+,+,+,+,-)$. We find

$$\begin{aligned} ds^2_{11} &= \Delta^{1/3}ds_7^2 + g^{-2}\Delta^{-2/3}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl}D\hat{\mu}^i D\hat{\mu}^l\,, \\ *_{11}\hat{F}_{(4)} &= -gU\epsilon_{(7)} - g^{-1}\left(\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\bar{*}D\hat{T}_{kl}\right)\wedge\left(\hat{\mu}^l D\hat{\mu}^i\right) + \\ &\quad + \frac{1}{2}g^{-2}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl}\eta_{mn}\left(\hat{T}^{-1}\right)^{np}\eta_{pq}\bar{*}\hat{F}^{im}_{(2)}\wedge D\hat{\mu}^l\wedge D\hat{\mu}^q + \\ &\quad + g^{-4}\Delta^{-1}\hat{T}_{ij}\hat{S}^i_{(3)}\hat{\mu}^j\wedge\hat{W} - \\ &\quad - \frac{1}{6}g^{-3}\Delta^{-1}\hat{\epsilon}_{ijl_1l_2l_3}\eta^{in}\eta^{jp}\bar{*}\hat{S}^m_{(3)}\hat{T}_{nm}\hat{T}_{pk}\mu^k\wedge\hat{D}\mu^{l_1}\wedge\hat{D}\mu^{l_2}\wedge\hat{D}\mu^{l_3}\,, \end{aligned} \tag{4.21}$$

where

$$U = 2\hat{T}_{ij}\eta^{jk}\hat{T}_{kl}\hat{\mu}^i\hat{\mu}^l - \Delta\eta^{ij}\hat{T}_{ij}\,, \qquad \Delta = \hat{T}_{ij}\hat{\mu}^i\hat{\mu}^j\,, \qquad \eta_{ij}\hat{\mu}^i\hat{\mu}^j = 1\,,$$
$$\hat{W} = \frac{1}{4!}\hat{\epsilon}_{i_1 i_2 i_3 i_4 i_5}\hat{\mu}^{i_1}\hat{D}\mu^{i_2}\wedge\hat{D}\mu^{i_3}\wedge\hat{D}\mu^{i_4}\wedge\hat{D}\mu^{i_5}\,. \tag{4.22}$$

Note that $\hat{\epsilon}_{ijklm}$ is the SO(4,1) invariant antisymmetric tensor.

The resulting lagrangian has the form

$$\begin{aligned}\mathcal{L}_7 = {} & R\,\bar{*}1 - \frac{1}{4}\left(\hat{T}^{-1}\right)^{ij}\bar{*}D\hat{T}_{jk}\wedge\left(\hat{T}^{-1}\right)^{kl}D\hat{T}_{li} - \\ & -\frac{1}{4}\eta_{ij}\left(\hat{T}^{-1}\right)^{jk}\eta_{kl}\eta_{mn}\left(\hat{T}^{-1}\right)^{np}\eta_{pq}\bar{*}\hat{F}^{im}_{(2)}\wedge\hat{F}^{lq}_{(2)} - \frac{1}{2}\hat{T}_{ij}\bar{*}\hat{S}^i_{(3)}\wedge\hat{S}^j_{(3)} + \\ & +\frac{1}{2g}\eta_{ij}\hat{S}^i_{(3)}\wedge\hat{H}^j_{(4)} - \frac{1}{8}\hat{\epsilon}_{ij_1j_2j_3j_4}\hat{S}^i_{(3)}\wedge\hat{F}^{j_1j_2}_{(2)}\wedge\hat{F}^{j_3j_4}_{(2)} - V\,\bar{*}1 + (\text{Chern-Simons})\,,\end{aligned} \tag{4.23}$$

where

$$\hat{H}^i_{(4)} = d\hat{S}^i_{(3)} + g\hat{A}^{ik}\eta_{kj}\hat{S}^j_{(3)}\,. \tag{4.24}$$

This resulting theory is simply an unusual non-compact gauging of maximal $D = 7$ supergravity. In particular, both the SO(4,1) gauge fields and the potential

$$V = \frac{1}{2}g^2\left(2\hat{T}_{ij}\eta^{jk}\hat{T}_{kl}\eta^{li} - (\eta^{ij}\hat{T}_{ij})^2\right), \tag{4.25}$$

have their usual signs, despite the $M^*_{(9,2)}$ origin of this theory.

4.4 Truncation of the $D = 7$ theory

The $D = 7$, $N = 4$ theory constructed above admits a truncation to $N = 2$ supergravity coupled to an $N = 2$ vector. The fields in the supergravity multiplet consist of the metric $g_{\mu\nu}$, three gauge bosons in the adjoint of $SU(2)_+ \subset SO(4) \subset SO(4,1)$, namely $\hat{A}^{\overline{ij}}_{(1)} + \frac{1}{2}\epsilon^{\overline{ijkl}}\hat{A}^{\overline{kl}}_{(1)}$, a single three-form potential $\hat{S}^5_{(3)}$, and a dilaton scalar φ. In addition, the vector multiplet consists of a single U(1) gauge potential contained in $SO(2)_-$ and three axionic scalars. In principle, it is straightforward to consistently truncate the above reduction to yield the $N = 2$ theory. However, the presence of the non-abelian graviphotons and the axionic scalars results in some complication. We choose here not to pursue this truncation, but instead focus on a two U(1) truncation with $U(1)^2 \subset SO(4,1)$, while simultaneously retaining two dilatonic scalars [11]. While this is not a consistent truncation, it may still be used to lift solutions to eleven dimensions.

To obtain this $U(1)^2$ truncation of the full theory, we parametrize the internal dS_4 by $(\mu_1\sin\phi_1, \mu_1\cos\phi_1, \mu_2\sin\phi_2, \mu_2\cos\phi_2, \mu_0)$. Only two gauge fields, $A^1 = \hat{A}^{12}_{(1)}$ and $A^2 = \hat{A}^{34}_{(1)}$, are kept as well as $\hat{T} = \mathrm{diag}(X^1, X^1, X^2, X^2, -X^0)$. The rest of the fields are truncated. The reduction ansatz, (4.21), then becomes

$$ds^2_{11} = \Delta^{1/3}ds^2_7 + g^{-2}\Delta^{-2/3}\left(-X_0^{-1}d\mu_0^2 + \sum_{i=1}^{2}X_i^{-1}\left(d\mu_i^2 + \mu_i^2\left(d\phi_i + gA^i\right)^2\right)\right),$$

$$
\begin{aligned}
*_{11}\hat{F}_{(4)} = & \; 2g\sum_{i=1}^{2}\left(X_i^2\mu_i^2-\Delta X_i\right)\epsilon_{(7)} - g\left(2X_0^2\mu_0^2-\Delta X_0\right)\epsilon_{(7)} + \\
& + \frac{1}{2}g^{-1}\sum_{i=1}^{2}X_i^{-1}\bar{*}dX_i\wedge d(\mu_i^2) - \frac{1}{2}g^{-1}X_0^{-1}\bar{*}dX_0\wedge d\left(\mu_0^2\right) + \\
& + \frac{1}{2}\hat{g}^{-2}\sum_{i=1}^{2}\eta_i X_i^{-2} d\left(\mu_i^2\right)\wedge\left(d\phi_i-\hat{g}A^i\right)\wedge\bar{*}F^i\,, \qquad (4.26)
\end{aligned}
$$

where $\Delta = -X_0\mu_0^2+\sum_{i=1}^{2}X_i\mu_i^2$. Inserting the above $\mathrm{U}(1)^2$ truncation into (4.23), we obtain the bosonic lagrangian

$$
e^{-1}\mathcal{L}_7 = R\bar{*}1 - \frac{1}{2}\bar{*}d\varphi_1\wedge d\varphi_1 - \frac{1}{2}\bar{*}d\varphi_2\wedge d\varphi_2 - \frac{1}{2}\sum_{i=1}^{2}X_i^{-2}\bar{*}F^i\wedge F^i - V\bar{*}1\,. \qquad (4.27)
$$

Here we have chosen to parametrize the scalars in terms of two dilatons $\vec{\varphi} = \{\varphi_1,\varphi_2\}$ according to

$$
X_i = e^{-\frac{1}{2}\vec{a}_i\cdot\vec{\varphi}}\,, \qquad \vec{a}_i\cdot\vec{a}_j = 4\delta_{ij}-\frac{8}{5}\,, \qquad X_0 = (X_1X_2)^{-2} \qquad (i,j=1,2)\,. \qquad (4.28)
$$

The potential is given by

$$
V = -g^2\left(4X_1X_2+2X_0X_1+2X_0X_2-\frac{1}{2}X_0^2\right), \qquad (4.29)
$$

corresponding to the AdS_7 vacuum. Further truncation is possible by setting $\varphi_1 = 0$ and $F^1 = F^2 = F/\sqrt{2}$. In this case, the resulting lagrangian becomes

$$
e^{-1}\mathcal{L}_7 = R\bar{*}1 - \frac{1}{2}\bar{*}\partial\varphi\wedge\partial\varphi - \frac{1}{2}X^{-2}\bar{*}F\wedge F + g^2\left(4X^2+4X^{-3}-\frac{1}{2}X^{-8}\right)\bar{*}1\,. \qquad (4.30)
$$

Note that both truncations (4.27) and (4.30) have gauge fields contained in the maximally compact subgroup $\mathrm{SO}(4)\subset\mathrm{SO}(4,1)$. As a result, they both have proper kinetic energy terms and a standard potential admitting the AdS vacuum. So, after appropriate truncation, we have in fact obtained a unitary theory (indistinguishable from the corresponding truncation of ordinary $N=4$ gauged supergravity in seven dimensions) from a non-unitary one. Nevertheless, this fact is perhaps of limited usefulness, as the model before truncation inherits the usual drawbacks of the underlying field theory description of * theory.

5. Lifting de Sitter black hole solutions

It is an interesting observation that, even in the absence of obvious supersymmetry, multi-centered de Sitter black hole solutions are known to exist, and have been constructed in [21, 22] in the context of Einstein-Maxwell theory with a positive cosmological constant. These solutions resemble traditional BPS objects in that they satisfy a set of first order equations [22]–[24] related to the analytic continuation of the Killing spinor equation in

AdS gauged supergravity. In the d-dimensional Reissner-Nordstrom case (for $d \geq 4$), the solution may be written in terms of a cosmological metric

$$ds^2 = -H^{-2}(t,\vec{x}\,)dt^2 + H^{2/(d-3)}(t,\vec{x}\,)\,a^2(t)d\vec{x}^{\,2}\,, \tag{5.1}$$

where the scale factor is given by $a(t) = e^{\hat{g}t}$ and the harmonic function has the multi-center form

$$H(t,\vec{x}\,) = 1 + \sum_i \frac{q_i}{(a(t)|\vec{x}-\vec{x}_i|)^{d-3}}\,. \tag{5.2}$$

It is apparent that while the background is time-dependent, this time dependence is rather trivial and is simply a reflection of the cosmological expansion in de Sitter space. For this reason, these black holes are natural candidates to lift to the higher dimensional * theory, where they may be interpreted as possible fundamental objects of the * theory.

It was demonstrated in [23] that this Reissner-Nordstrom black hole, (5.1), may be generalized to the analytically continued $D=5$, $N=2$ supergravity, and in particular the STU model corresponding to the anti-de Sitter version of (3.16). Further generalizations to Einstein-Maxwell-dilaton theories in arbitrary dimensions are also possible.

5.1 $D=5$ dS black holes

We first examine the five-dimensional case obtained by the hyperbolic reduction of IIB*. The U$(1)^3$ truncated theory of (3.16) admits a three-charge black hole solution, given by

$$\begin{aligned} ds_5^2 &= -(H_1H_2H_3)^{-2/3}dt^2 + (H_1H_2H_3)^{1/3}e^{2\hat{g}t}d\vec{x}^{\,2}\,, \\ X_i &= H_i^{-1}(H_1H_2H_3)^{1/3}\,, \\ A_{(1)}^{1,2} &= i\left(1-\frac{1}{H_{1,2}}\right)dt\,, \qquad A_{(1)}^3 = \left(1-\frac{1}{H_3}\right)dt\,, \end{aligned} \tag{5.3}$$

where

$$H_i(t,\vec{x}\,) = 1 + e^{-2\hat{g}t}\sum_j \frac{q_i^j}{|\vec{x}-\vec{x}_j|^2}\,. \tag{5.4}$$

This solution was given implicitly in [23], at least up to analytic continuation to the present * theory vacuum. The continuation is chosen here so that the metric remains real at the expense of introducing imaginary background gauge fields.[3]. In fact, the complexification of the solution compensates for the wrong sign kinetic terms in (3.16) in just such a way that the black hole solution has positive energy compared with the vacuum.

There is a potential difficulty associated with lifting the solution (5.3) to ten dimensions, in that the imaginary gauge fields $A_{(1)}^1$ and $A_{(1)}^2$ would lead to a complex ten dimensional metric (as well as F_4). To avoid this, we turn off the first two gauge fields, and lift the single charge ($A_{(1)}^3$) black hole to IIB* theory. The resulting metric is

$$ds_{10}^2 = \widetilde{\Delta}^{1/2}\left(-H^{-1}dt^2 + e^{2\hat{g}t}d\vec{x}^{\,2}\right) + \hat{g}^{-2}\widetilde{\Delta}^{-1/2}ds^2(\widetilde{H}^5)\,, \tag{5.5}$$

[3] The de Sitter black hole solution is a real solution for positive cosmological constant and correct sign kinetic terms for the fields. Since the first two U(1)'s of (3.16) have the wrong sign, the corresponding gauge potentials become imaginary in (5.3).

where $\widetilde{\Delta} \equiv H^{2/3}\Delta = 1 + (1-H)\sinh^2\alpha$ and

$$ds^2\left(\widetilde{H}^5\right) = d\alpha^2 + \sinh^2\alpha\, d\Omega_3^2 + (1-H)\sinh^2\alpha\, d\alpha^2 + H\cosh^2\alpha(d\psi - \hat{g}H^{-1}(1-H)dt)^2 \quad (5.6)$$

is the metric on the distorted internal hyperbolic space. Note that we have chosen an explicit parametrization of the maximally symmetric H^5 as

$$ds^2(H^5) = d\alpha^2 + \sinh^2\alpha\, d\Omega_3^2 + \cosh^2\alpha\, d\psi^2\,. \quad (5.7)$$

After applying a change of coordinates, $\hat{g}\tau = e^{\hat{g}t}$, and a rearrangement of terms, the lifted solution (5.5) may be written in the form

$$ds_{10}^2 = \widetilde{\mathcal{H}}_0^{-1/2} d\vec{x}^{\,2} + \widetilde{\mathcal{H}}_0^{1/2}\left[-\widetilde{\Delta}H^{-1}d\tau^2 + \tau^2 ds^2(\widetilde{H}^5)\right], \quad (5.8)$$

where

$$\widetilde{\mathcal{H}}_0 = \lim_{\tau\to 0}\widetilde{\mathcal{H}}\,, \qquad \widetilde{\mathcal{H}} = 1 + \frac{1}{\hat{g}^4\tau^4\widetilde{\Delta}}\,. \quad (5.9)$$

We have written $\widetilde{\mathcal{H}}$ in this form in anticipation of the brane interpretation of this lifted solution.

In fact, we recall that the IIB* theory admits an E4-brane solution of the form [1]

$$ds_{10}^2 = \mathcal{H}^{-1/2} d\vec{x}^{\,2} + \mathcal{H}^{1/2}(-dt^2 + dr^2 + r^2 d\Omega_4^2)\,, \quad (5.10)$$

where

$$\mathcal{H} = 1 + \frac{1}{\hat{g}^4|t^2 - r^2|^2}\,. \quad (5.11)$$

This may be most readily compared to the lifted single charge black hole solution of (5.8) by taking the near brane limit $t \to r$ with $t^2 > r^2$. In this case, we may make a change of variables, $t = \tau\cosh\beta$, $r = \tau\sinh\beta$, and drop the one in the harmonic function. The resulting $\mathrm{dS}_5 \times H^5$ metric has the form

$$ds_{10}^2 = \mathcal{H}_0^{-1/2} d\vec{x}^{\,2} + \mathcal{H}_0^{1/2}[-d\tau^2 + \tau^2(d\beta^2 + \sinh^2\beta\, d\Omega_4^2)]\,, \quad (5.12)$$

where $\mathcal{H}_0 = 1/\hat{g}^4\tau^4$. It should now be apparent that (5.8) is a generalization of this solution to the case of non-zero R-charge in five dimensions. Recalling that the R-charged AdS black holes may be interpreted as rotating branes [11], we see here that a similar picture holds, albeit with time as a transverse as opposed to a longitudinal coordinate.

5.2 $D = 4$ dS black holes

Turning now to four dimensions, we note that the $\mathrm{U}(1)^4$ truncation, (4.14), admits a four charge dS black hole solution, given by

$$\begin{aligned} ds_4^2 &= -(H_1H_2H_3H_4)^{-1/2}dt^2 + (H_1H_2H_3H_4)^{1/2}e^{2\hat{g}t}d\vec{x}^{\,2}\,, \\ X_i &= H_i^{-1}(H_1H_2H_3H_4)^{1/4}\,, \\ A_{(1)}^i &= i\left(1 - \frac{1}{H_i}\right)dt\,, \end{aligned} \quad (5.13)$$

where

$$H_i(t,\vec{x}\,) = 1 + e^{-\hat{g}t}\sum_j \frac{q_i^j}{|\vec{x}-\vec{x}_j|}\,. \qquad (5.14)$$

Again we are faced with the difficulty of imaginary gauge potentials. However, unlike the five-dimensional case, all fields in the $\mathrm{U}(1)^4$ truncation have the wrong sign. Hence there is no truncation (short of setting all the charges to zero) that makes the solution real. For this reason, any lifting of these black holes to the M* theory would result in a complex metric, and hence it is unclear what the physical significance of these solutions is.

Nevertheless, we note that the analytic continuation of the Reissner-Nordstrom-de Sitter solution is obtained by setting all four charges equal in (5.13):

$$\begin{aligned} ds_4^2 &= -H^{-2}dt^2 + H^2 e^{2\hat{g}t}d\vec{x}^{\,2}\,,\\ A_{(1)} &= i\left(1-\frac{1}{H}\right)dt\,. \end{aligned} \qquad (5.15)$$

This may be viewed as a solution to the pure $N=2$ de Sitter supergravity of (4.18) where $A_{(1)}$ is taken as the graviphoton.

5.3 $D=7$ AdS black holes

Finally, we note that the $\mathrm{U}(1)^2$ truncation of the seven-dimensional theory, (4.27), admits a two charge AdS black hole solution given by

$$\begin{aligned} ds_7^2 &= -(H_1H_2)^{-4/5}dt^2 + (H_1H_2)^{1/5}e^{-2igt}d\vec{x}^{\,2}\,,\\ X_i &= H_i^{-1}(H_1H_2)^{2/5}\,,\\ A_{(1)}^{1,2} &= \left(1-\frac{1}{H_{1,2}}\right)dt\,, \end{aligned} \qquad (5.16)$$

where

$$H_i(t,\vec{x}\,) = 1 + e^{4igt}\sum_j \frac{q_i^j}{|\vec{x}-\vec{x}_j|^4}\,. \qquad (5.17)$$

Although the gauge fields have correct sign kinetic terms, the negative cosmological constant formally results a complex metric, as noted in [23]. The further truncated theory of (4.30) also admits a solution by setting all $H_i = H$ in (5.16):

$$\begin{aligned} ds_7^2 &= -H^{-8/5}dt^2 + H^{2/5}e^{-2igt}d\vec{x}^{\,2}\,,\\ X &= H^{-1/5}\,,\\ A_{(1)} &= \left(1-\frac{1}{H}\right)dt\,. \end{aligned} \qquad (5.18)$$

6. Discussion

So far we have focused only on the reduction of the bosonic sector of * theories. Nevertheless, it should be possible to handle the fermions in a similar manner through analytic continuation. Unlike the conventional case, where analytic continuation from an anti-de

Sitter to a de Sitter theory would complexify the fermions and destroy the matching between bosonic and fermionic degrees of freedom [7], here the * theories have a twisted supersymmetry built in (at the expense of wrong sign kinetic terms). Thus the resulting fermion sectors should not have any doubling problem.

Although we do not examine the fermions in detail, the structure of the de Sitter supersymmetry transformations may be derived by an appropriate continuation of the anti-de Sitter supergravities. For example, it is well known that the S^5 compactification of IIB supergravity will lead to maximal SO(6) gauged supergravity in five dimensions [28]–[30]. In addition, the non-compact $\mathrm{SO}(p, 6-p)$ gauged case has also been investigated in [30]. Turning to the H^5 compactification of IIB* supergravity, we have seen in section 3 that the appropriate analytic continuation requires sending g to $i\hat{g}$. This results in an unconventional de Sitter supersymmetry with noncompact $\mathrm{SO}(5,1)$ gauge group.

Recall that the $D=5$ ungauged maximal supergravity fields consists of one graviton, eight gravitini ψ_μ^a, 27 vector fields $A_\mu^{[ab]}$, 48 spin-1/2 fields λ^{abc} and 42 scalars φ^{abcd}, where $a, b, \ldots$ are USp(8) indices. The complete scalars parametrize the noncompact coset space $E_{6(6)}/\,\mathrm{USp}(8)$. However, for the truncation considered in section 3, we specialize to the subgroup $\mathrm{SO}(5,1) \subset \mathrm{SL}(6,R) \times \mathrm{SL}(2,R) \subset E_{6(6)}$, with scalars parametrized by a 15-bein ${V_{AB}}^{cd}$. This 15-bein may be constructed by starting with a $\mathrm{SL}(6,R)$ matrix S and then taking

$$\begin{aligned} {U^{IJ}}_{KL} &= 2{S_{[K}}^{[I}{S_{L]}}^{J]}\,, & V^{IJab} &= \frac{1}{8}\left(\Gamma^{KL}\right)^{ab}{U^{IJ}}_{KL}\,, \\ {U_{I\alpha}}^{J\beta} &= {S_I}^J{\delta_\alpha}^\beta\,, & {V_{I\alpha}}^{ab} &= \frac{1}{2\sqrt{2}}\left(\Gamma_{K\beta}\right)^{ab}{U_{I\alpha}}^{K\beta}\,, \end{aligned} \tag{6.1}$$

where the SO(5,1) Dirac matrices satisfy $\{\Gamma_I, \Gamma_J\} = 2\eta_{IJ}$ with $\eta_{IJ} = \mathrm{diag}(+,+,+,+,+,$ $-)$. In addition, $\Gamma_{I\alpha} \equiv \Gamma_I$ $(\alpha = 1)$; $\Gamma_{I\alpha} \equiv i\Gamma_I\Gamma_0$ $(\alpha = 2)$ where Γ_0 anticommutes with the first six Dirac matrices. Here we are following the notations and conventions of [30].

The scalar kinetic terms ${P_\mu}^{abcd}$ and the composite connection ${Q_{\mu a}}^b$ are defined through

$${\tilde{V}_{cd}}{}^{AB} D_\mu {V_{AB}}^{ab} = 2{Q_{\mu[c}}^{[a}\delta^{b]}_{d]} + {P_\mu}^{ab}{}_{cd}\,, \tag{6.2}$$

where ${\tilde{V}_{cd}}{}^{AB}$ is the inverse of ${V_{AB}}^{cd}$. In addition, the T-tensor is defined as

$$\begin{aligned} T^a{}_{bcd} &= \left(2V^{IKae}\tilde{V}_{beJK} - {V_{J\alpha}}^{ae}{\tilde{V}_{be}}{}^{I\alpha}\right)\eta^{JL}\tilde{V}_{cdIL}\,, \\ T_{ab} &= T^c{}_{abc}\,. \end{aligned} \tag{6.3}$$

Note that, the matrix S is related to $\hat{T}_{ij}$ introduced in section 3 by

$$\left(\hat{T}^{-1}\right)^{ij} = {S_I}^i{S_J}^j\eta^{IJ}\,. \tag{6.4}$$

This simply corresponds to the unconventional * supersymmetry with scalars parameterizing $\mathrm{SL}(6,R)/\,\mathrm{SO}(5,1)$.

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Besides introducing this noncompact gauging, we also analytically continue $g \to i\hat{g}$ and $F_{(2)} \to -i\hat{F}_{(2)}$ in order to obtain the lagrangian (3.11). Therefore the unconventional supersymmetry transformation rules for the gravitini and spin-1/2 fermions are

$$
\begin{aligned}
\delta\psi_{\mu a} &= D_\mu \epsilon_a - \frac{2i}{45}\hat{g} T_{ab}\gamma_\mu \epsilon^b + \frac{i}{6}\left(\gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu \gamma^\rho\right)\hat{F}_{\nu\rho ab}\epsilon^b\,,\\
\delta\lambda_{abc} &= \sqrt{2}\gamma^\mu P_{\mu\,abcd}\epsilon^d - \frac{i}{\sqrt{2}}\hat{g} T_{d[abc]}\epsilon^d + \frac{3i}{2\sqrt{2}}\gamma^{\mu\nu}\hat{F}_{\mu\nu[ab}\epsilon_{c]}\,,
\end{aligned}
\tag{6.5}
$$

where

$$
\begin{aligned}
\hat{F}_{\mu\nu ab} &= \hat{F}_{\mu\nu IJ} V^{IJ}{}_{ab} = \frac{1}{4}\hat{F}_{\mu\nu}^{ij}\eta_{im}\eta_{jn} S_K{}^m S_L{}^n (\Gamma_{KL})^{ab}\,,\\
D_\mu \epsilon_a &= \partial_\mu \epsilon_a + Q_{\mu a}{}^b \epsilon_b\,.
\end{aligned}
\tag{6.6}
$$

One should be able to obtain these transformations by direct reduction of the IIB* transformations, although we have not directly verified this.

The multi-centered dS black holes of the previous section are half-BPS solutions of this twisted supersymmetry. However it is not clear if this is sufficient to demonstrate their stability. Unlike for BPS objects in an ordinary supergravity theory, here the wrong-sign kinetic terms allows the possibility of excitations above the supersymmetric background that nevertheless have negative energy. On the other hand, the existence of multi-centered solutions is at least suggestive that there may be a hidden symmetry ensuring their stability.

Although the de Sitter supergravities which we have investigated involve wrong sign kinetic terms and are hence ill-behaved as field theories, such problems were already present in the underlying * theory. Thus we may expect that whatever stringy phenomenon cures the behavior of * theory would also stabilize the ensuing de Sitter supergravities. On the other hand, it is possible that the reduction on non-compact internal manifolds may yield inherently unstable lower dimensional theories. Of course, consistent truncations are possible in a standard $AdS \times$ Sphere reduction, even when states are not well separated by e.g. charge or mass. Hence even if the full lower dimensional de Sitter theory would be unstable, it is possible that a stable truncation would exist. An example of this is given by the truncated seven-dimensional lagrangians (4.27) and (4.30).

While presently we have only examined multi-centered de Sitter black holes, it would be of interest to investigate and lift other de Sitter backgrounds to the underlying IIB* or M* theory. In fact, such lifted solutions may additionally be T-dualized along the time direction so that they become conventional solutions in ten or eleven dimensions. For the case of the lifted $D = 5$ de Sitter black holes, it may be seen that the T-dual of the rotating E4-brane solution, (5.8), would involve D4-branes as well as NSNS flux.

Finally, it remains an open issue whether multi-centered anti-de Sitter black holes may be constructed in ordinary gauged supergravities without resorting to complexification or analytic continuation. To do so, one would have to overcome the fact that in an ordinary supergravity, a background preserving some fraction of the supersymmetries necessarily admits a timelike or null Killing vector. This is in direct contradiction to the expectation that a multi-centered anti-de Sitter black hole configuration would be time dependent, due to the focusing effect of geodesics in anti-de Sitter space. On the other hand, there is no obstruction to the multi-centered de Sitter black holes, as the unconventional signs in the superalgebra relax the condition of having a timelike Killing vector. This hints, at least, that the unconventional de Sitter supergravities investigated here may play a crucial role in the better understanding of de Sitter cosmologies and time dependent backgrounds for string theory.

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