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Instanton effects and linear-chiral duality

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Abstract: We discuss duality between the linear and chiral dilaton formulations, in the presence of super-Yang-Mills instanton corrections to the effective action. In contrast to previous work on the subject, our approach appeals directly to explicit instanton calculations and does not rely on the introduction of an auxiliary Veneziano-Yankielowicz superfield. We discuss duality in the case of an axion that has a periodic scalar potential, and find that the bosonic fields of the dual linear multiplet have a modified interpretation. We note that symmetries of the axion potential manifest themselves as symmetries of the equations of motion for the linear multiplet. We also make some brief remarks regarding dilaton stabilization. We point out that corrections recently studied by Dijkgraaf and Vafa can be used to stabilize the axion in the case of a single super-Yang-Mills condensate.

Keywords: Solitons Monopoles and Instantons, Supersymmetric Effective Theories.

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1. Introduction

Quite impressive and reliable results have been obtained for the instanton generated non-perturbative superpotential in super-Yang-Mills (SYM) and super-QCD [1]. These results have been further refined by computations of corrections due to decoupled matter, sparked by recent work [2, 3] that is currently the subject of intense interest and activity.

Here we discuss the duality between the dilaton described by a linear multiplet [4] L and the dilaton described by a chiral multiplet S, in rigid $N = 1$ 4d supersymmetry. While this has been discussed at length with regard to anomaly cancellation [5], in the present article we aim to describe instanton effects in the dual formalism. Such effects play a crucial role in string-inspired models of moduli stabilization; for example, [6]. In contrast to previous work, such as in [5, 6], we discuss this duality without introducing a Veneziano-Yankielowicz (VY) superfield [7]. This is an auxiliary superfield that produces the known instanton superpotential when it is integrated out.[1] We avoid the VY superfield because we would prefer to understand the duality without ever "integrating in" this superfluous field in the first place.[2]

[1]The VY superfield can also be regarded as a background field of the 2PI effective action [8].

[2]In section 7, we will make some remarks in regard to the superpotential corrections computed by Dijkgraaf and Vafa. At that juncture it will be convenient to make use of the VY superfield in order to make contact with their notation.

We now summarize the content of our paper and our key results:

- In section 2 we briefly describe the motivations for studying nonperturbative corrections to the dilaton potential: first because they are certainly present, as a consequence of instanton configurations, and second because these corrections can play an important role in stabilizing the vacuum.
- Using straightforward manipulations in the superfield formalism, we find in section 3 that when the linearity of the multiplet L is sufficiently modified, this formalism is exactly equivalent to the one involving the chiral multiplet S.
- We have verified our results by also performing the duality transformation at the component field level. In section 4 we discuss the translation between the two formalisms in terms of component fields.
- In section 5, we address an apparent inconsistency between the two formalisms which has appeared in the literature [6]. We point out a simple calculus error that was made, which when corrected, resolves the apparent difficulty for exact duality.
- In section 6 we study the equations of motion for the bosonic fields in the modified linear multiplet. We find that the traditional interpretation of the 1-form that is contained in the linear multiplet — as the Hodge dual of a field strength for a 2-form — is modified if the axion has a potential.
- We find that symmetries of the axion potential are re-expressed in the dual formalism as symmetries of the equations of motion for the 1-form. This is also discussed in section 6.
- We point out that the corrections computed by Dijkgraaf and Vafa [2] may be used to stabilize the axion using a single SYM condensate. Details on this matter may be found in section 7.
- In section 8 we give our concluding remarks and comment on issues for further research.

2. Effective theory and motivations

In this section we review certain well-known facts regarding instanton corrections in pure SYM. Here, the pure SYM theory must be understood as resulting from a more fundamental theory with $N_f = N_c - 1$ flavors of fundamental matter, where N_c is the number of colors; i.e., SU(N_c) super-QCD. As is well-known, instantons generate a nonperturbative superpotential in the $N_f = N_c - 1$ theory [1]. One can then obtain an effective pure SYM, together with a nonperturbative superpotential, valid below a scale μ, by studying the theory along a flat direction where all of the flavors obtain masses of order μ. For a review and citations to the original literature, we refer the reader to [9, 10].

We will work in the chiral dilaton formulation, where these results are most familiar. The scalar component s of the chiral dilaton field S determines the effective gauge coupling and the effective theta angle at a scale μ through its vacuum expectation value (*vev*):

$$\langle s \rangle = \frac{1}{g^2} - i\frac{\theta}{8\pi^2}\,. \tag{2.1}$$

In the case of pure SYM the effective superpotential for S is expressed in terms of a superfield extension of the ordinary dynamical YM scale:

$$\Lambda = \mu \exp\left(-\frac{8\pi^2 S}{b}\right), \qquad W(S) = \tilde{c}\Lambda^3\,, \tag{2.2}$$

where for example $b = 3N_c$ for pure $\mathrm{SU}(N_c)$. Generally we write the Kähler potential as

$$K(S+\overline{S}) = \mu^2 k(S+\overline{S})\,. \tag{2.3}$$

In examples we will examine specific forms for $k(S+\overline{S})$. We always assume that K is a function of $S+\overline{S}$, but not of $S-\overline{S}$. Consequently, $\partial K/\partial S = \partial K/\partial \overline{S} = \partial K/\partial(S+\overline{S})$, or in a more abbreviated notation $K_S = K_{\overline{S}} = K'$.

Quite often in the literature on supergravity and string-inspired effective theories, only the leading order Kähler potential, $-\mu^2 \ln(S+\overline{S})$, is used; however, just as the superpotential receives instanton corrections, the Kähler potential will likewise be modified by nonperturbative effects. Due to a lack of holomorphy, it is difficult to obtain any reliable information on the form of K in the nonperturbative regime. However, instanton corrections are certainly present. For example, if we start from the leading order $k = -\ln(S+\overline{S})$, and the nonperturbative superpotential (2.2), the 1-loop corrections to the Kähler potential take the form

$$\delta K \propto (k'')^{-2}|W''|^2 \propto (S+\overline{S})^4(\Lambda\overline{\Lambda})^3\,. \tag{2.4}$$

Higher orders in perturbation theory and nonperturbative effects will generalize this result in a way that we cannot presently determine. For example, nonperturbative corrections might allow for integral powers of $\Lambda\overline{\Lambda}$ that are not multiples of 3, since they would not necessarily derive from the superpotential, as was the case with (2.4). We can, however, be confident that the leading order Kähler potential is just an approximation that in some regimes may prove to be inaccurate.

Rather than ambling along with a form of K that ignores this reality, we find it more logical to explore various "reasonable" forms for the nonperturbative Kähler potential; and, to classify the qualitative results that follow. Indeed, intuition leads us to believe that some *ad hoc* assumptions are better motivated than others.

As an example, we find it entirely sensible to include instanton effects in the Kähler potential in the most naive way, based on dimensional analysis:

$$K(S+\overline{S}) = \mu^2 k(S+\overline{S}) = -\mu^2 \ln(S+\overline{S}) + c(S+\overline{S})\Lambda\overline{\Lambda} + \mathcal{O}\left(\frac{\Lambda^2\overline{\Lambda}^2}{\mu^2}\right), \tag{2.5}$$

with $c(S+\overline{S})$ a slowly varying function of $S+\overline{S}$ and the $\mathcal{O}(\Lambda^2\overline{\Lambda}^2/\mu^2)$ terms presumably negligible. Of course more general assumptions exist; say, fractional powers of $\Lambda\overline{\Lambda}$ appearing in (2.5), or functions of $\Lambda+\overline{\Lambda}$. Such generalizations are captured by allowing $k(S+\overline{S})$ to be arbitrary.

Our interest in nonperturbative corrections to K is not just academic. In rigid supersymmetry, the scalar potential that determines the vacuum is given by

$$\begin{aligned} V_{\rm rigid} &= \mu^4 \frac{|24\pi^2\tilde{c}|^2}{b^2 k''(s+\overline{s})} \exp\left[-\frac{24\pi^2}{b}(s+\overline{s})\right] \\ &= \mu^4 \frac{|\tilde{c}|^2}{\partial_x^2 k(x)} \exp(-x)\,, \qquad x=\frac{24\pi^2}{b}(s+\overline{s})\,. \end{aligned} \tag{2.6}$$

Generally, the effect of corrections such as assumed in (2.5) is merely[3] to slightly shift the location and height of the maximum of $V_{\rm rigid}$. As an example, we consider the case where the function $c(s+\overline{s})$ amounts to what is essentially a polynomial in g^2:

$$k(x) = \ln(24\pi^2/b) - \ln(x) + (c_1 + c_2 x^{-1}) \exp\left(-\frac{x}{3}\right). \tag{2.7}$$

(The constant term is due to the replacement of $s+\overline{s}$ with x; it is irrelevant here but must be kept track of for supergravity considerations below.) Eq. (2.7) is just the first two terms in (2.5) with $c(s+\overline{s}) \propto c_1 + c_2 b/24\pi^2(s+\overline{s})$, with constants c_1 and c_2. Thus, the correction behaves like

$$\delta K \sim \left(c_1 + c_2\frac{bg^2}{48\pi^2}\right) \exp\left(-\frac{8\pi^2}{bg^2}\right). \tag{2.8}$$

We believe this to be a reasonable assumption, though we have no inkling about the magnitude of c_1 or c_2 (except that the naive loop factor has been scaled out of c_2, so that this sort of suppression does not seem implied for c_2). Higher orders in g^2 certainly will appear in $c(s+\overline{s})$, but for our illustration, which is only meant to be qualitative, we neglect them as small. For convenience, we define $\hat{V}_{\rm rigid}(x) = V_{\rm rigid}(x)/\mu^4|\tilde{c}|^2$. In figure 1 the dashed line shows this quantity as a function of x for the case of $c_1=0$ and $c_2=1$. It can be seen that the well-known runaway behavior to vanishing and infinite couplings is retained.

On the other hand if we generalize to supergravity, the scalar potential reads instead

$$V_{\rm sugra} = \mu^4|\tilde{c}|^2 e^{-x+k(x)} \left[(\partial_x^2 k(x))^{-1}(1-\partial_x k(x))^2 - 3\right]. \tag{2.9}$$

The supergravity corrections have important effects given the assumed instanton-induced corrections (2.7). For convenience, we define $\hat{V}_{\rm sugra}(x) = 24\pi^2 V_{\rm sugra}(x)/b\mu^4|\tilde{c}|^2$. In figure 1 the dotted line shows this quantity as a function of x, again for the case of $c_1=0$ and $c_2=1$. In the regime where $x^{-1}\exp(-x/3)$ competes with the "leading order," i.e. where $x \leq \mathcal{O}(1)$, the modification is so great as to create a deep minimum — and, to completely lift the infinite coupling runaway.

[3] We do not consider corrections that are radically different from (2.5); for example, in [11] it was found that a form of $K(S+\overline{S})$ can be engineered to yield a nontrivial minimum in the rigid susy case. Our results for chiral-linear duality, however, may also be applied to this $K(S+\overline{S})$.

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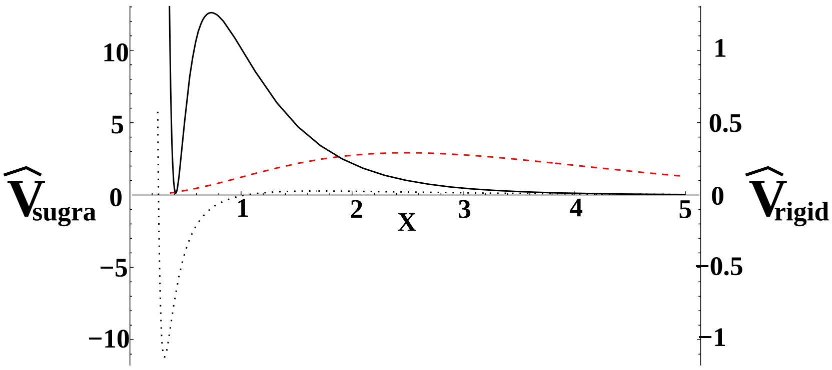

Figure 1: A comparison of the effects of nonperturbative corrections to K for the rigid supersymmetry and supergravity scalar potentials, as a function of $x \propto s+\overline{s}$. The dashed line represents the rigid supersymmetry potential for $c_1 = 0, c_2 = 1$, while the dotted line is the supergravity potential for the same choice of parameters. On the other hand, the solid line represents the supergravity potential with $c_1 = 3.27$ and $c_2 = 1$; this yields a vanishing cosmological constant.

Clearly, the $g^4 \sim x^{-2}$ corrections to $c(s+\overline{s})$ that have been neglected in (2.7) will modify the details of the strong coupling behavior. It is also true that in this same regime $\Lambda\overline{\Lambda}/\mu^2 = \exp(-x/3) = \mathcal{O}(1)$, so the truncation of (2.5) is not justified. Nevertheless, as higher powers of $\Lambda\overline{\Lambda}/\mu^2$ are added in, it is a generic feature that the strong coupling runaway behavior tends to be removed and nontrivial (typically local) minima are created.

Note that for $c_1 = 0$ and $c_2 = 1$ the vacuum energy of is order $-\mu^4$; it is well-known that further refinements and fine-tuning can be used to manipulate this so-called *Kähler stabilization* of the dilaton [12, 6]. Indeed, it is possible to obtain an approximately vanishing cosmological constant and to stabilize at a "weak coupling" minimum. As an example, we have fixed $c_2 = 1$ and then tuned c_1 to a value such that the minimum occurs at vanishing cosmological constant. We find that $c_1 = 3.27$ works well; the corresponding potential is indicated in figure 1 by the solid line. Naturally the nonperturbative corrections to the Kähler potential do not alter the asymptotic behavior at weak coupling, where they are totally negligible; the weak coupling runaway persists. However, with the cosmological constant tuned to zero the barrier height is considerable: $E_{\text{barrier}} \approx 10\mu(b|\tilde{c}|^2/24\pi^2)^{1/4}$. If μ is of order the 4d Planck scale, 2.4×10^{18} GeV, and if $|\tilde{c}|$ is not too small, the effects of the approximately degenerate vacuum at $x \to \infty$ would presumably be neglible, for suitable cosmological initial conditions.

While the supergravity potential (2.9), together with assumptions for K such as (2.7), may be used to stabilize $s+\overline{s}$, it can be seen that the axion $(s-\overline{s})/i$ is absent and thus remains a flat direction. To stabilize the axion requires slightly more complicated assumptions about the form of the effective theory. As a very well-known example, suppose the relevant gauge group has 2 simple factors with $b_1 \neq b_2$. Eq. (2.2) is generalized to

$$\Lambda_1 = \mu \exp\left(-\frac{8\pi^2 S}{b_1}\right), \qquad \Lambda_2 = \mu \exp\left(-\frac{8\pi^2 S}{b_2}\right), \qquad W(S) = \tilde{c}_1\Lambda_1^3 + \tilde{c}_2\Lambda_2^3\,. \tag{2.10}$$

V is no longer just a function of $s+\overline{s}$. In this case the axion $(s-\overline{s})/i$ is also stabilized. Another possibility is to add heavy matter that has a nontrivial vacuum. When this matter is integrated out, as will be seen in section 7, it is possible to obtain corrections to (2.2) that stabilize the axion.

In summary, instanton effects — however they might appear in the Kähler potential — play an important role in the stabilization of the dilaton and axion. Thus it may be of interest to translate between the chiral dilaton formulation and the linear dilaton formulation when these effects are present. Furthermore, we would like to be able to do so without introducing additional machinery, such as the Veneziano-Yankielowicz auxiliary superfield. In the next section we elucidate how this is to be done in the case of rigid supersymmetry.

3. Linear-chiral duality

We now want to address these instanton corrections in the the dual linear dilaton formulation. To do this we want to begin with the chiral dilaton formulation and translate to an equivalent system; this is the so-called *duality transformation.* It is essential that field redefinitions are made that respect both: (i) the equations of motion, and (ii) any constraint equations. To this end we write a first-order lagrangian whose equations of motion contain both (i) and (ii): the constraints are imposed dynamically.

We begin with the dilaton effective lagrangian in the chiral formulation, written in superspace notation:[4]

$$\mathcal{L} = \int d^4\theta\, K(S+\overline{S}) + \left[\int d^2\theta\, W(S) + \text{h.c.}\right]. \tag{3.1}$$

We replace this with a first-order lagrangian that, as will be shown below, imposes the chirality constraints, using Lagrange multiplier superfields $U, \overline{U}$:

$$\begin{aligned}\mathcal{L}_{FO} = &\int d^4\theta\, K(S+\overline{S}) + \int d^4\theta \left[U\left(S - \frac{1}{4}\overline{D}^2\Sigma\right) + \overline{U}\left(\overline{S} - \frac{1}{4}D^2\overline{\Sigma}\right)\right] + \\ &+ \int d^2\theta\, W\left(\frac{1}{4}\overline{D}^2\Sigma\right) + \int d^2\overline{\theta}\, \overline{W}\left(\frac{1}{4}D^2\overline{\Sigma}\right).\end{aligned} \tag{3.2}$$

Note that in (3.2) all superfields $S, \overline{S}, U, \overline{U}, \Sigma, \overline{\Sigma}$ are unconstrained.

The simplest superfield equations of motion occur for fields that appear only in D-density terms; i.e., only under $\int d^4\theta$. The superfield equations of motion obtained from varying U and $\overline{U}$ yield the chirality and antichirality constraints:

$$S = \frac{1}{4}\overline{D}^2\Sigma\,, \qquad \overline{S} = \frac{1}{4}D^2\overline{\Sigma}\,. \tag{3.3}$$

Varying with respect to S and $\overline{S}$ yields simply

$$0 = U + K'(S+\overline{S}) = \overline{U} + K'(S+\overline{S})\,. \tag{3.4}$$

Thus when we impose the equations of motion that follow from (3.2), we obtain the on-shell projection to a real multiplet L:

$$L = U = \overline{U} = -K'(S+\overline{S})\,. \tag{3.5}$$

[4]Our conventions are those of [13].

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Note that if we had just used L in place of U and $\overline{U}$ from the start, we would not have the two equations in (3.3) independently. Instead, the equations of motion would only require

$$S + \overline{S} = \frac{1}{4}\left(\overline{D}^2\Sigma + D^2\overline{\Sigma}\right). \tag{3.6}$$

Since S and $\overline{S}$ are unconstrained superfields in the first order lagrangian (3.2), the constraint (3.6) does not enforce (anti-)chirality by its equations of motion. Of course, the identification (3.3) is *a particular* solution to (3.6). But for the duality to be faithful, the equations of motion must have (3.3) as a *unique* solution. In this respect our duality transformation is more restrictive than the one that has previously been imposed in the literature [5, 6].

While (3.5) implicitly tells us how to replace $S + \overline{S}$ with L in the lagrangian, the superpotential terms will involve S and $\overline{S}$ separately. This is particularly important where the axion has a potential. In the following manipulations we will see that the necessary data is obtained from variation of (3.2) with respect to Σ and $\overline{\Sigma}$, which leads to modified linearity conditions for L. Varying with respect to Σ and $\overline{\Sigma}$ requires that we handle a mixture of D-density and F-density terms. To perform the analysis we rewrite the D-density terms that contain these fields using

$$\int d^4\theta U\overline{D}^2\Sigma = -\frac{1}{4}\int d^2\theta\overline{D}^2\left(U\overline{D}^2\Sigma\right) \tag{3.7}$$

and similarly for the $D^2\overline{\Sigma}$ term.[5]

We vary Σ to obtain

$$\begin{aligned}
\delta\mathcal{L}_{FO} &= \frac{1}{16}\int d^2\theta\left[\overline{D}^2\left(U\overline{D}^2\delta\Sigma\right) + 4W'\left(\frac{1}{4}\overline{D}^2\Sigma\right)\overline{D}^2\delta\Sigma\right] \\
&= \frac{1}{16}\int d^2\theta\left[\overline{D}^2\left(\overline{D}^2U\delta\Sigma\right) + 4W'(S)\overline{D}^2\delta\Sigma\right] \\
&= -\frac{1}{4}\int d^4\theta\left[\delta\Sigma\left(\overline{D}^2U + 4W'(S)\right)\right].
\end{aligned} \tag{3.8}$$

In the second line we use the equations of motion (3.3) and the identity

$$0 = \overline{D}^2\left[\overline{D}^2U\delta\Sigma - U\overline{D}^2\delta\Sigma\right]. \tag{3.9}$$

In the 3rd line we use $\overline{D}^2S \propto \overline{D}^2\overline{D}^2\Sigma = 0$ and reverse the type of manipulation that led to (3.7). A similar analysis leads us to write the variation with respect to $\overline{\Sigma}$ as:

$$\delta\mathcal{L}_{FO} = -\frac{1}{4}\int d^4\theta\left[\delta\overline{\Sigma}\left(D^2\overline{U} + 4\overline{W}'(\overline{S})\right)\right]. \tag{3.10}$$

Vanishing of these two variations leads to the constraints:

$$\overline{D}^2U + 4W'(S) = 0\,, \qquad D^2\overline{U} + 4\overline{W}'(\overline{S}) = 0\,. \tag{3.11}$$

[5] See for example p. 67 of [13].

Taking into account (3.5) we arrive at the modified linearity conditions

$$\overline{D}^2 L = -4W'(S)\,, \qquad D^2 L = -4\overline{W}'(\overline{S})\,. \tag{3.12}$$

These, together with (3.5), are sufficient to (implicitly) redefine the components of $S, \overline{S}$ in terms of the components of L.

4. Component fields

We have verified all of the above superfield relations at the level of component fields. This straightforward exercise begins by writing out the unconstrained superfields appearing in (3.2) in terms of $\theta, \overline{\theta}$ expansions. For example, keeping only bosons, the real part of U is given by

$$L = \ell + \theta\sigma^m\overline{\theta}h_m + \theta^2 Z + \overline{\theta}^2\overline{Z} + \theta^2\overline{\theta}^2\left(D + \frac{1}{4}\Box\ell\right). \tag{4.1}$$

Recall that at this point we want L to be unconstrained, except for the fact that it is defined as $L = (U + \overline{U})/2$. Thus L contains independent fields $Z, \overline{Z}$ and the 1-form h^m is completely general. These degrees of freedom will be subject to constraints — i.e., the modified linearity conditions — when we go on shell. It is straightforward to substitute analogous expressions for the unconstrained superfields into (3.2) and to work out the component expansion. From there, one may work out the equations of motion. There are many, and we will spare the details. Here we will just state key results.

The lagrangian dual to (3.1), neglecting fermions, is merely:

$$\begin{aligned}\mathcal{L}(\ell, h^m) &= -\frac{1}{4}\ell\Box\left[s(\ell,\partial_m h^m) + \overline{s}(\ell,\partial_m h^m)\right] - \frac{1}{K''(s(\ell,\partial_m h^m) + \overline{s}(\ell,\partial_m h^m))}\times \\ &\quad\times\left(\frac{1}{4}h^m h_m + |W'(s(\ell,\partial_m h^m))|^2\right).\end{aligned} \tag{4.2}$$

Here we have shown that $s = s(\ell, \partial_m h^m)$ wherever it appears. It remains to specify how this is obtained.

Implicitly, we can replace $s + \overline{s}$ everywhere using

$$\ell = -K'(s + \overline{s})\,. \tag{4.3}$$

Implicitly, we can replace $s - \overline{s}$ using the additional constraint

$$\partial^m h_m = \frac{i}{K''(s+\overline{s})}\left(W'(s)\overline{W}''(\overline{s}) - \overline{W}'(\overline{s})W''(s)\right). \tag{4.4}$$

Up to some factors that involve $s + \overline{s}$, one sees that $\partial^m h_m$ is identified with $V'(a)$, the axion force term obtained from the potential $V(a)$ for the axion $a = (s - \overline{s})/i$. This is not surprising, because we have the $\theta\overline{\theta}$ part of the constraint (3.5):

$$h^m = \frac{1}{i}K''(s+\overline{s})\partial^m(s - \overline{s}) \tag{4.5}$$

Thus $\partial^m h_m$ should involve $\Box a$. But the equations of motion for the axion relate $\Box a$ to $V'(a)$.

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For the θ^2 and $\overline{\theta}^2$ components of L we have

$$Z = L|_{\theta^2} = \overline{W}'(\overline{s}) \,, \qquad \overline{Z} = L|_{\overline{\theta}^2} = W'(s) \,. \tag{4.6}$$

This is the $\theta = \overline{\theta} = 0$ part of the modified linearity conditions (3.12). In fact it is straightforward to check that (3.12) is consistent with the component field equations of motion to all orders in $\theta, \overline{\theta}$. We remark that in the presence of the instanton effects, the modified linear multiplet contains auxiliary fields. In particular, (4.6) indicates that L contains the F-term component of the chiral dilaton and its conjugate, F_S and $F_{\overline{S}}$. This, of course, has been noted before in formalisms that relied on a Veneziano-Yankielowicz superfield [5, 6, 14].

5. A brief remark

We note in passing that an apparent inequivalence between the two formalisms was noted in the appendix of [6]. There it was found in the linear dilaton formalism that the *Kähler moduli* t^I of a string-inspired effective supergravity were stabilized at the *self-dual* values (with respect to an $\mathrm{SL}(2, \mathbb{Z})$ isometry of the scalar manifold) of 1 or $e^{i\pi/6}$. A duality transformation was made in the appendix of [6], and it was found that "the minimum is shifted slightly away" [from the self-dual value]. However, we find that the apparent conflict with linear-chiral duality is resolved by noting a simple error that was made by the authors of [6] in obtaining their eq. (A.16) from their eq. (A.14). They have kept too many quantities constant in performing the differentiation — simultaneously both $s + \overline{s}$ and ℓ. But since $t^I + \overline{t}^I$ mixes with $s + \overline{s}$ to give ℓ, in their eq. (A.1), this is not right. Once the chain rule is properly applied, it is not hard to show that their dual chiral formulation also predicts stabilization of t^I at the self-dual values. (We do not provide further details because they just involve elementary manipulations. However, we thought it important to resolve this apparent problem for lineal-chiral duality.)

6. Dual description for the axion

If we have only one condensate and a superpotential of the form (2.2) then $|W'(s)|^2$ is independent of $s - \overline{s}$; the axion has no potential and is massless. Equivalently, the right-hand side of (4.4) vanishes identically. This constraint equation has the general solution

$$h^m = \epsilon^{mnpq} \partial_n b_{pq} \,. \tag{6.1}$$

Thus, as has been known for a very long time, the 1-form in the linear multiplet is Hodge dual to a 2-form field strength [4].

In more general situations the axion gets a potential from the instanton physics. In this case the right-hand side of (4.4) does not vanish. eq. (6.1) is inconsistent with the constraints. The 1-form in the linear multiplet must be reinterpreted. It is no longer just the Hodge dual of a 2-form field strength. Instead, it is the Hodge dual of a massive 3-form [15].

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6.1 Massive axion dual

Temporarily we oversimplify and consider just an "axion" with a constant, field independent mass. Thus we assume

$$\mathcal{L}(a) = -\frac{1}{2}\partial_m a \partial^m a - \frac{1}{2}m^2 a^2 \,. \tag{6.2}$$

The duality is obtained with the identification

$$h^m \equiv \partial^m a \quad \Rightarrow \quad \partial^m h_m = \Box a = m^2 a \,, \tag{6.3}$$

where in the second step we use the equation of motion for a. This equation of motion and the constraint (relating the 1-forms h_m and $\partial_m a$) are obtained from the first order lagrangian

$$\mathcal{L}_{FO} = \frac{1}{2}\partial_m a \partial^m a - \frac{1}{2}m^2 a^2 + h^m \left(h_m - 2\partial_m a\right). \tag{6.4}$$

It is easy to check that the dual theory obtained by eliminating a from $\mathcal{L}_{FO}$ through *its* equations of motion is given by

$$\mathcal{L}(h_m) = -\frac{1}{2m^2}\left(m^2 h^m h_m + \partial^m h_m \partial^n h_n\right). \tag{6.5}$$

The equations of motion that follow from $\mathcal{L}(h_m)$ are

$$\partial^m \partial^n h_n - m^2 h^m = 0 \,. \tag{6.6}$$

The general solution is nothing but (6.3). Thus, *the axion just parameterizes the general solution to the 1-form equations of motion.* We will see that this is likewise true in the more interesting circumstance of the 1-form dual to an *interacting* axion.

Note that (6.6) is not the usual equation of motion for a massive vector boson. In Fourier space the mode expansion coefficients $a^m_{\mathbf{p}}$ of h^m are not independent. Instead they satisfy

$$a^m_{\mathbf{p}} = \frac{p^m}{\sqrt{\mathbf{p}^2 + m^2}} a^0_{\mathbf{p}} \,. \tag{6.7}$$

In the rest frame, the spatial components of h^m vanish;[6] there is only 1 on shell degree of freedom. From this we understand how the 1-form can be equivalent to a 0-form. In fact, this is precisely the behavior of a massive 3-form [15].

6.2 Axion with periodic potential

Here we suppose

$$\mathcal{L}(a) = -\frac{1}{2}\partial_m a\, \partial^m a + m^4 \cos\left(\frac{a}{m}\right). \tag{6.8}$$

The degenerate vacua are labeled by an integer n, indicating $\langle n|a|n\rangle = 2\pi n$. The mass of a fluctuation about any of these vacua is m. It is interesting to see how this circumstance is reflected in the dual 1-form theory. To this end, we write down an equivalent, first order lagrangian:

$$\mathcal{L}_{FO} = \frac{1}{2}\partial_m a \partial^m a + m^4 \cos\left(\frac{a}{m}\right) + h^m \left(h_m - 2\partial_m a\right). \tag{6.9}$$

[6] This can also be seen directly from (6.6).

The equations of motion obtained from $\mathcal{L}_{FO}$ are

$$h^m = \partial^m a\,, \qquad 0 = \Box a + m^3 \sin\left(\frac{a}{m}\right) - 2\partial_m h^m\,, \tag{6.10}$$

clearly equivalent to the equation of motion that follows from (6.8). Next we eliminate a to obtain the equivalent 1-form lagrangian. Differentiating the second equation in (6.10) and contracting with h^n it is easy to show

$$m^4 \cos\left(\frac{a}{m}\right) = m^2 \frac{h^n \partial_n \partial_m h^m}{h_p h^p}\,. \tag{6.11}$$

With this result, one finds

$$\mathcal{L}(h) = -\frac{1}{2} h^m h_m + m^2 \frac{h^n \partial_n \partial_m h^m}{h_p h^p}\,. \tag{6.12}$$

The equations of motion that follow from (6.12) are not illuminating, and we need not write them here. The sole thing worth noting about them is that because of the duality transformation that has been made, we are guaranteed that they have a general solution $h^m = \partial^m \psi$, where ψ is an integral of the differential equation

$$\Box\psi = m^3 \sin\left(\frac{\psi}{m}\right). \tag{6.13}$$

But this is nothing other than the axion equation of motion. Thus, *the periodicity of the axion potential is reflected in a degeneracy of solutions to the equivalent 1-form equations of motion.* This degeneracy is not immediately apparent (to us) upon inspection of (6.12). For this reason, the chiral formulation seems advantageous for understanding the pseudoscalar vacuum of the theory.

7. Single condensate stabilization

We now discuss stabilization of the dilaton and axion using only a single SYM condensate. To achieve this, we appeal to the corrections — to the Veneziano-Yankielowicz (VY) superpotential in the case where a very heavy adjoint chiral superfield is present — worked out recently by Dijkgraaf and Vafa (DV) [2]. We use the DV result that the VY superpotential, which contains the VY auxiliary superfield U, can be written

$$W(S,U) = NU \left[\ln\frac{U}{\mu^3} + \frac{8\pi^2}{N} S + f(U)\right]. \tag{7.1}$$

Here f is a calculable power series in the VY superfield:

$$f(U) = \sum_{n>0} c_n \left(\frac{U}{\mu^3}\right)^n. \tag{7.2}$$

The equations of motion for the VY superfield are:

$$\frac{\partial W(S,U)}{\partial U} = 0 = N\left[\ln\frac{U}{\mu^3} + \frac{8\pi^2}{N} S + f(U) + U\frac{\partial f(U)}{\partial U} + 1\right]. \tag{7.3}$$

The solution to (7.3) can be written implicitly as

$$U = \Lambda^3 \exp\left(-f - U\frac{\partial f}{\partial U}\right), \qquad \Lambda^3 = \mu^3 e^{-1} \exp\left(-\frac{8\pi^2}{N}S\right). \tag{7.4}$$

Note that (up to an unimportant factor e^{-1}) we have retained the leading order definition (2.2) of Λ, the uncorrected dynamical ("QCD") scale. The VY superfield is auxiliary, in that it has no kinetic term. It thus has a singular metric, and consequently an effectively infinite mass. We will therefore eliminate U with its equations of motion.

As an aside, we note that, upon imposing the equation of motion (7.3),

$$\frac{\partial^2 W(S,U)}{\partial U^2} = \frac{N}{\Lambda^3}\left(1 + \mathcal{O}\left(\frac{\Lambda^3}{\mu^3}\right)\right). \tag{7.5}$$

Thus even if U *had* a (canonical) kinetic term, it would have a very large mass m_U:

$$m_U = \mathcal{O}\left(\frac{N\mu^3}{\Lambda^3}\right)\cdot \mu \gg \mu\,. \tag{7.6}$$

At energies of order Λ^3 it is essentially static, and certainly should be integrated out in discussing the low energy effective theory.[7] The case where the gaugino bilinear has been treated as a dynamical field has been studied in much greater detail in [16], where a similar conclusion was reached about the effective mass scale of this composite degree of freedom.

Eliminating U with its equations of motion, we obtain:

$$W(S) \equiv W(S,U(S)) = -NU\left(1 + U\frac{\partial f(U)}{\partial U}\right)\bigg|_{U(S)}. \tag{7.7}$$

Provided $\Lambda \ll \mu$ and the coefficients c_n are not unreasonably large (a more precise statement will be given shortly), we need keep only the leading term in (7.2):

$$f \approx c_1 \frac{U}{\mu^3} \approx c_1 \frac{\Lambda^3}{\mu^3}\,. \tag{7.8}$$

The coefficient c_1 depends on the massive matter representations that have implicitly be integrated out to obtain (7.2). As an example we use the DV *perturbative* superpotential with a single adjoint chiral superfield:

$$W = \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3 \quad \Rightarrow \quad c_1 = 2\lambda^2\left(\frac{\mu}{m}\right)^3. \tag{7.9}$$

We implement the approximation (7.8) to obtain:

$$\begin{aligned} U\frac{\partial f}{\partial U} &\approx f\,, \qquad U \approx \Lambda^3\exp(-2f) \approx \Lambda^3\left(1 - 2c_1\frac{\Lambda^3}{\mu^3}\right), \\ W(S) &\approx -N\Lambda^3\left(1 - c_1\frac{\Lambda^3}{\mu^3}\right) = -N\Lambda^3\left(1 - 2\lambda^2\frac{\Lambda^3}{m^3}\right). \end{aligned} \tag{7.10}$$

[7] We thank Erich Poppitz for emphasizing this to us.

Now we can be more specific about the smallness of corrections: we require $m^3 \gg \lambda^2\Lambda^3$ in order that the approximations be valid. *It is worth emphasizing that it is not enough to have $\Lambda \ll \mu$.*

To understand the vaccum in the presence of the DV corrections, note that

$$\frac{\partial \Lambda^3}{\partial S} = -\frac{8\pi^2}{N}\Lambda^3 \,. \tag{7.11}$$

Thus

$$W'(S) \approx 8\pi^2\Lambda^3\left(1 - 2c_1\frac{\Lambda^3}{\mu^3}\right) = 8\pi^2\Lambda^3\left(1 - 4\lambda^2\frac{\Lambda^3}{m^3}\right). \tag{7.12}$$

This theory has a supersymmetric vacuum, and therefore a global minimum, at

$$2c_1\Lambda^3 \approx \mu^3 \quad \Leftrightarrow \quad m^3 \approx 4\lambda^2\Lambda^3 \,. \tag{7.13}$$

Unless $\lambda \neq \mathcal{O}(1)$, *we find stabilization at $\Lambda \approx m$ using only a single condensate and without reference to nonperturbative corrections to the Kähler potential.* Regardless of the value of λ, the "minimum" is outside of the regime of validity of the approximation (7.8) made above. This is clear from the far right-hand side of (7.12): the "correction" must cancel against the leading order to have $W'(S) \approx 0$. Thus it is not possible to draw any firm conclusions without pursuing the higher order corrections in (7.2). Nevertheless it is interesting that a nontrivial minimum does exist for the truncated correction (7.8). In particular, we find it significant that $|W'(S)|^2$ depends on *both* $S + \overline{S}$ and $S - \overline{S}$. Thus the axion can be stabilized with just a single condensate. *This* qualitative result should continue to hold even when the additional corrections in (7.2) are taken into account.[8]

8. Outlook

In rigid supersymmetry, we have arranged for the equations of motion of the first-order system to enforce *all* constraint equations of the chiral and linear systems. Thus we are assured that the two formulations are in every way equivalent: they have equivalent equations of motion *and* constraint equations. This same approach may be applied to locally supersymmetric extensions, which have phenomenological applications to string-inspired effective supergravity theories. The sort of approach taken here will connect the two formulations in a way that is a faithful translation.

We have elucidated how the component fields of the modified linear multiplet can accomodate the features of a chiral multiplet with an axion potential. In essence, the linear multiplet becomes a more general sort of real multiplet when it couples to the SYM instantons. Its component field content has a more general structure and modified interpretation.

[8] At the final stages of preparing this manuscript, we became aware of [17], where nontrivial vacua due to the DV corrections are also studied.

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We have suggested how the DV corrections can lead to stabilization with a single condensate. This should come as no surprise. We are not *forced* to integrate out the adjoint chiral superfield Φ that appears in (7.9). If we leave the field in it has nontrivial vacua and this additional condensate plays a role in the stabilization of S.

It is interesting to consider the DV corrections in the context of string-inspired effective supergravity. Here it may be possible to achieve a stable minimum where the DV corrections are small ($\Lambda \ll m$), unlike the case above. (For this to be true, however, it would seem that the supergravity effects would have to dominate over those that arise from the exchange of the adjoint chiral superfield Φ.) In this case the truncations made above become reasonable approximations and the corrections may have some modest effects on, for example, soft supersymmetry breaking operators in the low energy effective theory.

We are currently investigating these and related issues.

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