

# Penrose limits of non-standard brane intersections

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## Abstract

The non-standard intersection of two 5-branes and a string can give rise to  $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ . We consider the Penrose limit of this geometry and study the supersymmetry of the resulting pp-wave solution. There is a one-parameter family of Penrose limits associated with the orthogonal rotation of the two foliating circles within the two 3-spheres. Supernumerary Killing spinors arise only when the rotation angle is  $45^\circ$ , for which case we obtain the corresponding light-cone string action that has linearly-realized supersymmetry. We also obtain Penrose limits of other non-standard intersections that give rise to the product of  $\text{AdS}_3$  or  $\text{AdS}_2$  and 2-spheres. The resulting pp-waves are supported by multiple constant field strengths.

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## 1. Introduction

The pp-waves arising from the Penrose limits of  $\text{AdS}_p \times S^q$  spacetime provide simple backgrounds on which to study string and M-theory. The maximally supersymmetric pp-wave with a constant 4-form field strength was first constructed in [1] for eleven-dimensional supergravity. Recently, the maximally supersymmetric pp-wave in type IIB supergravity with a constant self-dual 5-form field strength was obtained in [2]. These solutions are actually the Penrose limits of  $\text{AdS}_p \times S^q$  spacetimes with  $(p, q) = (4, 7), (7, 4)$  and  $(5, 5)$ , respectively [3]. In the type IIB case, the light-cone string action for the pp-wave [4, 5], is exactly solvable. This makes it possible to study the AdS/CFT correspondence on the level of full string theory in the Penrose limit [5], which has generated considerable interest recently [6–26].

$\text{AdS}_p \times S^q$  arises as the near-horizon geometry of non-dilatonic branes or intersecting branes, for  $(p, q) = (4, 7), (7, 4), (5, 5), (3, 3), (3, 2), (2, 3)$  and  $(2, 2)$ . AdS structure also arises from ‘non-standard’ intersections, for which the harmonic function  $H$  for each brane component depends on the coordinates of the relative transverse space rather than those of the overall transverse space [27–29]. The simplest example is the non-standard intersection of two 5-branes plus a string in the common worldvolume, whose near-horizon geometry is  $\text{AdS}_3 \times S^3 \times S^3 \times S^1$  [30].

In section 2, we perform the Penrose limit on the above solution. Since the product space involves two 3-spheres, there exists a one-parameter family of Penrose limits of this solution, parametrized by the rotation of the two foliating  $S^1$  within the two 3-spheres. In section 3, we study the supersymmetry of the resulting pp-wave solution. We find that there are 16 standard Killing spinors associated with any pp-wave solution, but supernumerary Killing spinors arise only for a specific choice of the parameter, associated with a  $45^\circ$  rotation of the two  $S^1$ . For this case, there are four supernumerary Killing spinors, which are all independent of the  $x^+$  coordinate. Furthermore, 4 out of the 16 standard Killing spinors are also independent of  $x^+$ . We then obtain the supersymmetric light-cone string action in this pp-wave background. In section 4, we  $T$ -dualize the pp-wave solution and show that it is related to the D3-brane with one of the worldvolume coordinates fibred over the six-dimensional transverse space. In section 5, we obtain the Penrose limits of more examples of non-standard intersections which give rise to  $AdS_2$  or  $AdS_3$ . Again, all of these examples have a one-parameter family of Penrose limits. We conclude our paper in section 6.

**2. 5-5-1 system and Penrose limit**

The 5-5-1 system is supported by a 3-form field strength in  $D = 10$ . The relevant Lagrangian is given by

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{-\phi}F_{(3)}^2. \tag{1}$$

Since our system involves only the metric, the dilaton and the 3-form field strength, the results apply to types IIA, IIB and heterotic string theories. The solution is given by [30]

$$\begin{aligned} ds_{10}^2 &= K^{-3/4}(H\tilde{H})^{-1/4}(-dt^2 + dx^2 + KH dy_i^2 + K\tilde{H} d\tilde{y}_i^2), \\ F_{(3)} &= e^\phi *(\tilde{H} dt \wedge dx \wedge d^4\tilde{y} \wedge dH^{-1}) + e^\phi *(H dt \wedge dx \wedge d^4y \wedge d\tilde{H}^{-1}) \\ &\quad + dt \wedge dx \wedge dK^{-1}, \quad \phi = -\frac{1}{2}\log[K/(H\tilde{H})], \quad K = H\tilde{H}, \end{aligned} \tag{2}$$

where  $H$  and  $\tilde{H}$  are harmonic functions in the relative transverse  $y_i$  and  $\tilde{y}_i$  spaces, respectively. The metric can be represented by the following diagram:

	$t$	$x$	$y_1$	$y_2$	$y_3$	$y_4$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	
5-brane	×	×	×	×	×	×	—	—	—	—	$\tilde{H}$
5-brane	×	×	—	—	—	—	×	×	×	×	$H$
string	×	×	—	—	—	—	—	—	—	—	$K$

5-5-1 system

We parametrize the coordinates as  $dy_i^2 = dy^2 + y^2 d\Omega_3^2$  and  $d\tilde{y}_i^2 = dy^2 + y^2 d\tilde{\Omega}_3^2$ . The isotropic solution is then given by

$$H = 1 + \frac{Q}{y^2}, \quad \tilde{H} = 1 + \frac{\tilde{Q}}{\tilde{y}^2}. \tag{3}$$

For simplicity, we take  $Q = \tilde{Q} = \lambda^2$ . The near-horizon geometry of the solution is  $AdS_3 \times S^3 \times S^3 \times S^1$  [30]. Expressing the  $AdS_3$  in global coordinates, and each  $S^3$  as a foliation of two circles, we have

$$\begin{aligned} ds_{10}^2 &= d\varphi^2 + \lambda^2(\frac{1}{2}(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\gamma^2) + (\cos^2 \theta d\psi^2 + d\theta^2 + \sin^2 \theta d\phi^2) \\ &\quad + (\cos^2 \tilde{\theta} d\tilde{\psi}^2 + d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2)), \end{aligned} \tag{4}$$

$$F_{(3)} = \lambda^2(\epsilon_{(3)} + 2\Omega_{(3)} + 2\tilde{\Omega}_{(3)}), \tag{5}$$

where  $\epsilon_{(3)}$ ,  $\Omega_{(3)}$  and  $\tilde{\Omega}_{(3)}$  are the volume-forms of the unit  $\text{AdS}_{(3)}$  and 3-spheres:

$$\begin{aligned} \epsilon_{(3)} &= \cosh \rho \sinh \rho \, dt \wedge d\rho \wedge d\gamma, & \Omega_{(3)} &= \cos \theta \sin \theta \, d\psi \wedge d\theta \wedge d\phi, \\ \tilde{\Omega}_{(3)} &= \cos \tilde{\theta} \sin \tilde{\theta} \, d\tilde{\psi} \wedge d\tilde{\theta} \wedge d\tilde{\phi}. \end{aligned} \tag{6}$$

After rotating the coordinates as follows,

$$\psi = \cos \alpha \psi_1 + \sin \alpha \psi_2, \quad \tilde{\psi} = -\sin \alpha \psi_1 + \cos \alpha \psi_2, \tag{7}$$

the Penrose limit can be taken to be

$$\begin{aligned} \rho &\longrightarrow \frac{\rho}{\lambda}, & \theta &\longrightarrow \frac{\theta}{\lambda}, & \tilde{\theta} &\longrightarrow \frac{\tilde{\theta}}{\lambda}, \\ t = x^+ + \frac{x^-}{\lambda^2}, & \psi_1 = \frac{1}{\sqrt{2}} \left( x^+ - \frac{x^-}{\lambda^2} \right), & \psi_2 &\longrightarrow \frac{\psi_2}{\lambda}, \end{aligned} \tag{8}$$

with the constant  $\lambda$  set to infinity. By this means, the solution becomes the pp-wave

$$\begin{aligned} ds_{10}^2 &= -2 \, dx^+ dx^- + H \, dx^{+2} + dz_i^2, \\ F_{(3)} &= dx^+ \wedge \Phi_{(2)}, \end{aligned} \tag{9}$$

where  $\Phi_{(2)}$  is a constant 2-form, given by

$$\Phi_{(2)} = (2 \, dz_1 \wedge dz_2 + \sqrt{2} \cos \alpha \, dz_3 \wedge dz_4 - \sqrt{2} \sin \alpha \, dz_5 \wedge dz_6), \tag{10}$$

and

$$H = -\sum_{i=1}^8 \mu_i^2 z_i^2 \equiv -(z_1^2 + z_2^2) - \frac{1}{2} \cos^2 \alpha (z_3^2 + z_4^2) - \frac{1}{2} \sin^2 \alpha (z_5^2 + z_6^2). \tag{11}$$

The coordinates  $z_i$  are defined to be

$$\begin{aligned} z_1 &= \frac{1}{\sqrt{2}} \rho \cos \gamma, & z_2 &= \frac{1}{\sqrt{2}} \rho \sin \gamma, & z_3 &= \theta \cos \phi, & z_4 &= \theta \sin \phi, \\ z_5 &= \tilde{\theta} \cos \tilde{\phi}, & z_6 &= \tilde{\theta} \sin \tilde{\phi}, & z_7 &= \psi_2, & z_8 &= \varphi. \end{aligned} \tag{12}$$

Note that it is straightforward to introduce a parameter  $\mu$  by scaling  $x^+ \rightarrow \mu x^+$  and  $x^- \rightarrow x^-/\mu$ . In this paper we shall set  $\mu = 1$ .

The above 5-5-1 system involves only the metric, the dilaton and the 3-form field strength, and is thus valid for the type IIA, type IIB and heterotic theories. It can be further lifted to  $D = 11$  as the 5-5-2 system. The near-horizon structure  $\text{AdS}_3 \times S^3 \times S^3 \times S^1$  given in (4) can also be supported by a 4-form field strength of the type IIA theory; it is given by

$$F_{(4)} = \lambda^2 (\epsilon_{(3)} + \Omega_{(3)} + \tilde{\Omega}_{(3)}) \wedge d\varphi. \tag{13}$$

However, the brane interpretation becomes obscured in this case. The Penrose limit is now supported by the 4-form  $F_{(4)} = dx^+ \wedge \Phi_{(2)} \wedge dz_8$ . Performing  $T$ -duality on the coordinate  $\varphi$  leads to a solution supported by the R-R 3-form in type IIB theory.

### 3. Supersymmetry and string actions

#### 3.1. Supersymmetry

*3.1.1. M-theory and type II viewpoints.* A simple way of studying the supersymmetry of the pp-wave solution obtained in the previous section is to lift the solution to  $D = 11$ . The resulting pp-wave is supported by the 4-form in  $D = 11$ , given by

$$F_{(4)} = dx^+ \wedge \Phi_{(2)} \wedge dz_9, \tag{14}$$

where  $z_9$  is the 11th direction<sup>3</sup>. This 4-form configuration is a special case of a general class of M-theory pp-waves considered in [23] where the supersymmetry of these solutions was discussed. It is straightforward to apply the general formalism of [23] to our solution. To do so, we introduce

$$W = \frac{1}{2}\Phi_{ij}\Gamma^{ij9} = 2\Gamma^{12} + \sqrt{2}\cos\alpha\Gamma^{34} - \sqrt{2}\sin\alpha\Gamma^{56}. \quad (15)$$

As discussed in [23], there are 16 standard Killing spinors associated with a generic pp-wave solution, which satisfy the projection

$$\Gamma_- \epsilon = 0. \quad (16)$$

In addition, since  $\mu_{8,9} = 0$ , supernumerary Killing spinors arise if  $W$  has zero eigenvalues, which occurs only for

$$\cos^2\alpha = \sin^2\alpha = \frac{1}{2}. \quad (17)$$

Using the formalism of [23] we find that, for the above  $\alpha$ , there are four supernumerary Killing spinors, all of which are independent of  $x^+$  and  $z^i$  for  $i = 7, 8, 9$ . The 16 standard Killing spinors are also independent of  $z^i$  for  $i = 7, 8, 9$ , but only 4 out of the 16 are independent of  $x^+$ . For other values of  $\alpha$ , there are no supernumerary Killing spinors, and all the standard Killing spinors depend on  $x^+$ . For this reason, we mainly focus on the case with  $\alpha$  given by (17).

Since the Killing spinors are independent of  $z_{7,8,9}$ , we can perform dimensional reduction and  $T$ -duality on these directions without breaking any supersymmetry, even at the level of field theory. Thus, the above analysis of supersymmetry carries over to the type IIA and IIB theories.

*3.1.2. Heterotic viewpoint.* To study the number of Killing spinors of the solution in the heterotic theory, we can use the same Killing-spinor calculations as above. However, we must impose an additional ten-dimensional chirality condition on the Killing spinor:

$$\epsilon = \Gamma_9 \epsilon, \quad (18)$$

where  $z_9$  is our 11th coordinate. This has the effect of reducing the number of standard Killing spinors down to eight, two of which are independent of  $x^+$ . The number of supernumerary Killing spinors is now two. If we had chosen a different convention for the chirality projection, then there would not be any supernumerary Killing spinors at all. This implies that the sign choice of  $F_{(3)}$  is important for the supernumerary Killing spinor in the heterotic theory.

### 3.2. String action

The pp-waves for which the function  $H$  is quadratic in the transverse space coordinates are of particular interest because they provide backgrounds for string theory that are exactly solvable. The light-cone string action of the pp-wave arising from the Penrose limit of  $\text{AdS}_5 \times S^5$  was studied in [4, 5]. The action associated with the Penrose limit of the D1/D5 system can be found in [5, 6]. In [31], the Green–Schwarz action for type IIA and IIB strings in an arbitrary bosonic background was derived in component form up to second order in the fermionic coordinates. From these, type IIA and IIB string actions were obtained in the light-cone gauge on the background of a large class of pp-wave solutions [15, 23]. Applying the formalism

<sup>3</sup> If we lift the pp-wave supported by the 4-form in  $D = 10$ ,  $z_9$  is replaced by  $z_8$ , and hence these two solutions are totally equivalent in  $D = 11$ .

in [15, 23] to our example, one finds that the solution supported purely by R–R fields is particularly simple. The bosonic string action is given by

$$\mathcal{L}_B = -\frac{1}{2}\dot{z}^2 - \frac{1}{2}z'^2 - \frac{1}{2}\mu_i^2 z_i^2, \tag{19}$$

where the masses of the bosonic fields  $\mu_i$  are defined in (11), given by

$$\mu_i^2 = \left\{1, 1, \frac{1}{4}\frac{1}{4}\frac{1}{4}, 0, 0\right\}. \tag{20}$$

For the solution supported by the type IIA 4-form field strength, the fermionic Lagrangian is given by

$$\mathcal{L}_F = \bar{\Psi}\Gamma_+(\not{\partial} + \mu\varrho_0\Gamma_8 W)\Psi, \tag{21}$$

where  $\varrho_i$ , with  $i = 0, 1, 2$ , are the worldsheet Dirac matrices, acting on the upper and lower 16 components of the column vector  $\Psi$ , and  $\varrho_2$  is the chirality operator. The fermionic Lagrangian associated with the type IIB solution supported by the R–R 3-form has a similar structure, given by

$$\mathcal{L}_F = \bar{\Psi}\Gamma_+(\not{\partial} + \mu\varrho_2 W)\Psi. \tag{22}$$

As discussed in [5, 15, 23], the existence of  $x^+$ -independent supernumerary Killing spinors ensures that the corresponding string action has linearly-realized supersymmetry. In our case, the masses of the bosonic fields  $z_i$  are given by (20). This precisely matches the masses of the fermionic fields, given by the eigenvalues of  $W$ . This is consistent with the supersymmetry. If we choose a different value of the rotation angle  $\alpha$ , two bosonic scalars remain massless, whilst all the fermions become massive, thereby breaking the supersymmetry. Indeed, for generic values of  $\alpha$ , there are no supernumerary Killing spinors.

For solutions supported by the NS–NS 3-form, the string action is slightly more complicated, since the NS–NS 3-form couples to the worldsheet as well. It is given by

$$\mathcal{L} = \sum_{i=1}^8 \left(\frac{1}{2}\dot{z}^2 - \frac{1}{2}z'^2 + \frac{1}{2}\mu\Phi_{ji}z^j z'_i - \frac{1}{2}\mu_i^2 z_i^2\right) + \bar{\Psi}(\not{\partial} + \frac{1}{4}\varrho W)\Psi. \tag{23}$$

#### 4. T-duality and $S^1$ -wrapped D3-brane

The pp-wave solution in  $D = 10$  has an eight-dimensional transverse space. In our case, the function  $H$  is independent of the coordinates  $z_{7,8}$ . Thus, the ‘natural’ transverse space of our solution is six dimensional. We can perform  $T$ -duality and  $S$ -duality to relate our solution to a D3-brane whose transverse space has six dimensions.

First, we perform  $T$ -duality along the  $\partial/\partial x^+$  Killing direction and obtain an NS–NS string solution given by

$$\begin{aligned} ds_{10}^2 &= H^{-3/4}(-dt^2 + (dx + \mathcal{A}_{(1)})^2) + H^{1/4} dz_i^2, \\ F_{(3)}^{\text{NS}} &= d(H^{-1} dt \wedge (dx + \mathcal{A}_{(1)})) + \mu(dx + \mathcal{A}_{(1)}) \wedge \Phi_{(2)}, \\ d\mathcal{A}_{(1)} &= \Phi_{(2)}, \quad \phi = \frac{1}{2} \log H. \end{aligned} \tag{24}$$

Note that, in this solution, the worldsheet coordinate  $x$  is fibred over the transverse space. This type of  $S^1$ -fibred string solution was first obtained in [32]. Next, we perform  $S$ -duality such that the string is supported by the R–R 3-form of type IIB theory.  $T$ -dualizing the  $z_{7,8}$  directions yields an  $S^1$ -wrapped D3-brane given by

$$\begin{aligned} ds_{10}^2 &= H^{-1/2}(-dt^2 + (dx_1 + \mathcal{A}_{(1)})^2 + dx_2^2 + dx_3^2) + H^{1/2} dz_i^2, \\ F_{(5)} &= d(H^{-1} dt \wedge (dx_1 + \mathcal{A}_{(1)}) \wedge dx_2 \wedge dx_3) + \mu(dx_1 + \mathcal{A}_{(1)}) \wedge dx_2 \wedge dx_3 \wedge \Phi_{(2)} + \text{dual}, \\ d\mathcal{A}_{(1)} &= \Phi_{(2)}. \end{aligned} \tag{25}$$

An  $S^1$ -wrapped D3-brane was first constructed in [33] in order to resolve the singularity while maintaining the supersymmetry of the associated D3-brane on the resolved conifold. Here, when the D3-brane charge is turned on, the more general solution for  $H$  is given by

$$H = 1 + \frac{Q}{r^4} - \sum_{i=1}^6 \mu_i^2 z_i^2, \tag{26}$$

where  $r^2 = z^i z^i$ . The terms associated with  $\mu_i$  break the conformal symmetry. They introduce a naked singularity at some finite  $r$ , which can be argued [15] to be associated with a phase transition from type IIB theory to type IIB\* theory, introduced in [34]. The solution has four Killing spinors for non-vanishing  $Q$ . For  $Q = 0$ , or in the limit  $r \rightarrow \infty$ , four additional Killing spinors emerge.

**5. Further examples**

In this section, we consider further examples of non-standard intersections that give rise to AdS structure and study their Penrose limits. These examples can be generated from the above  $\text{AdS}_3 \times S^3 \times S^3$  by noting that  $\text{AdS}_3$  and  $S^3$  can be (locally) expressed as a  $U(1)$  bundle over  $\text{AdS}_2$  and  $S^2$  respectively, i.e.,

$$\begin{aligned} ds_{\text{AdS}_3}^2 &= \frac{1}{4}(dz + \mathcal{A}_{(1)})^2 + \frac{1}{4} ds_{\text{AdS}_2}^2, & d\mathcal{A}_{(1)} &= \epsilon_{(2)}, \\ d\Omega_3^2 &= \frac{1}{4}(dz + \mathcal{B}_{(1)})^2 + \frac{1}{4} d\Omega_2^2, & d\mathcal{B}_{(1)} &= \Omega_{(2)}, \end{aligned} \tag{27}$$

where  $\epsilon_{(2)}$  and  $\Omega_{(2)}$  are the volume-forms of the unit  $\text{AdS}_2$  and  $S^2$ , respectively. The bundle can be naturally generated by considering a wave in  $\text{AdS}_3$  or a Taub–NUT in  $R^4$ . From the  $D = 10$  perspective, the wave and NUT in eleven dimensions become a D0-brane and D6-brane, respectively. The intersecting systems of this section can be found in [35].

*5.1. Type IIA examples*

*D0/D4/D4/NS1 system.* The solution for the D0/D4/D4/NS1 system is given by [35]

$$\begin{aligned} ds_{10}^2 &= H_0^{-7/8} H_4^{-3/8} \tilde{H}_4^{-3/8} H_1^{-3/4} (-dt^2 + H_0 H_4 \tilde{H}_4 dx^2 + H_0 H_4 H_1 dy_i^2 + H_0 \tilde{H}_4 H_1 d\tilde{y}_i^2), \\ F_{(4)} &= e^{-\frac{1}{2}\phi} * (\tilde{H}_4 dt \wedge d^4 \tilde{y} \wedge dH_4^{-1}) + e^{-\frac{1}{2}\phi} *(H_4 dt \wedge d^4 y \wedge d\tilde{H}_4^{-1}), \\ F_{(3)} &= dt \wedge dx \wedge dH_1^{-1}, & F_{(2)} &= dt \wedge dH_0^{-1}, \\ \phi &= -\frac{1}{4} \log [H_4 \tilde{H}_4 H_1^2 / H_0^3], & H_0 &= H_1 = H_4 \tilde{H}_4, \end{aligned} \tag{28}$$

where  $H_4 = H_4(y_i)$  and  $\tilde{H}_4 = \tilde{H}_4(\tilde{y}_i)$ . The solution can be represented diagrammatically as follows:

	$t$	$x$	$y_1$	$y_2$	$y_3$	$y_4$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	
D0	×	—	—	—	—	—	—	—	—	—	$H_0$
D4	×	—	×	×	×	×	—	—	—	—	$\tilde{H}_4$
D4	×	—	—	—	—	—	×	×	×	×	$H_4$
NS1	×	×	—	—	—	—	—	—	—	—	$H_1$

D0/D4/D4/NS1 system

We parametrize the coordinates as  $dy_i^2 = dy^2 + y^2 d\Omega_3^2$  and  $d\tilde{y}_i^2 = d\tilde{y}^2 + \tilde{y}^2 d\tilde{\Omega}_3^2$ , so that  $H_4$  and  $\tilde{H}_4$  are given by (3). For simplicity, we have equal charges. The near-horizon geometry is  $\text{AdS}_2 \times S^3 \times S^3 \times T^2$  [35], namely

$$ds_{10}^2 = \lambda^2 \left( \frac{1}{8} ds_{\text{AdS}_2}^2 + d\Omega_3^2 + d\tilde{\Omega}_3^2 \right) + d\varphi_1^2 + d\varphi_2^2, \tag{29}$$

$$F_{(4)} = 2\lambda^2 (\Omega_{(3)} + \tilde{\Omega}_{(3)}) \wedge d\varphi_1, \quad F_{(3)} = \frac{\lambda}{\sqrt{8}} \epsilon_{(2)} \wedge d\varphi_1, \quad F_{(2)} = \frac{\lambda}{\sqrt{8}} \epsilon_{(2)}.$$

Performing the Penrose limit, we have

$$ds_{10}^2 = -2 dx^+ dx^- + H dx^{+2} + dz_i^2, \tag{30}$$

$$F_{(4)} = dx^+ \wedge \Phi_{(3)}, \quad F_{(3)} = dx^+ \wedge \Phi_{(2)}, \quad F_{(2)} = dx^+ \wedge \Phi_{(1)},$$

where the constant form fields are given by

$$\Phi_{(3)} = \frac{1}{\sqrt{2}} (\cos \alpha dz_2 \wedge dz_3 - \sin \alpha dz_4 \wedge dz_5) \wedge dz_7, \tag{31}$$

$$\Phi_{(2)} = dz_7 \wedge dz_1, \quad \Phi_{(1)} = dz_1,$$

and

$$H = -z_1^2 - \frac{1}{8} \cos^2 \alpha (z_2^2 + z_3^2) - \frac{1}{8} \sin^2 \alpha (z_4^2 + z_5^2). \tag{32}$$

It is worth observing that in this pp-wave solution, all the form fields of the type IIA theory are turned on.

Note that we can  $T$ -dualize along the  $\partial/\partial x^+$  Killing direction and obtain a deformed  $S^1$ -wrapped type IIB NS string solution. Then, by  $T$ -dualizing along  $z_8$  and lifting to  $D = 11$ , one obtains a deformed  $T^2$ -wrapped M2-brane solution of M-theory given by

$$ds_{11}^2 = H^{-2/3} (-dt^2 + (dx_1 + \mathcal{A}_{(1)})^2 + (dx_2 + \mathcal{B}_{(1)})^2) + H^{1/3} dz_i^2, \tag{33}$$

$$F_{(4)} = d(H^{-1} dt \wedge (dx_1 + \mathcal{A}_{(1)}) \wedge (dx_2 + \mathcal{B}_{(1)})) + [\Phi_{(3)} \wedge dz_8 + 8\text{-dim dual}],$$

$$d\mathcal{A}_{(1)} = \Phi_{(2)}, \quad d\mathcal{B}_{(1)} = \Phi_{(1)} \wedge dz_8,$$

*D2/D4/D6/NS5 system.* The D2/D4/D6/NS5 solution is given by [35]

$$ds_{10}^2 = H_2^{-5/8} H_4^{-3/8} H_6^{-1/8} H_5^{-1/4} (-dt^2 + dx_1^2 + H_4 H_5 dx_2^2 + H_2 H_6 H_5 dy_i^2 + H_2 H_4 dz_j^2),$$

$$F_{(4)} = dt \wedge dx_1 \wedge dx_2 \wedge dH_2^{-1} + e^{-\frac{1}{2}\phi} * (dt \wedge dx_1 \wedge d^3 y \wedge dH_4^{-1}), \tag{34}$$

$$F_{(3)} = e^\phi * (dt \wedge dx_1 \wedge d^4 z \wedge dH_5^{-1}), \quad F_{(2)} = e^{-\frac{3}{2}\phi} * (dt \wedge dx_1 \wedge dx_2 \wedge d^4 z \wedge dH_6^{-1}),$$

$$\phi = \frac{1}{4} \log [H_2 H_5^2 / (H_4 H_6^3)], \quad H_2 = H_4(z_j) H(y_i),$$

where  $H_4 = H_4(z_j)$ ,  $H_6 = H_5 = H(y_i)$ . The solution can be represented diagrammatically as follows:

	$t$	$x_1$	$x_2$	$y_1$	$y_2$	$y_3$	$z_1$	$z_2$	$z_3$	$z_4$	
D2	×	×	×	—	—	—	—	—	—	—	$H_2$
D4	×	×	—	×	×	×	—	—	—	—	$H_4$
D6	×	×	×	—	—	—	×	×	×	×	$H_6$
NS5	×	×	—	—	—	—	×	×	×	×	$H_5$

The D2/D4/D6/NS5 system

The near-horizon geometry is  $\text{AdS}_3 \times S^3 \times S^2 \times T^2$  [35], given by

$$ds_{10}^2 = \lambda^2 \left( \frac{1}{2} dS_{\text{AdS}_3}^2 + d\Omega_3^2 + \frac{1}{4} d\tilde{\Omega}_2^2 \right) + d\varphi_1^2 + d\varphi_2^2, \tag{35}$$

$$F_{(4)} = \lambda^2 (\epsilon_{(3)} + 2\Omega_{(3)}) \wedge d\varphi_1, \quad F_{(3)} = \frac{1}{2} \lambda \tilde{\Omega}_{(2)} \wedge d\varphi_1, \quad F_{(2)} = \frac{1}{2} \lambda \tilde{\Omega}_{(2)}.$$

The Penrose limit is given by

$$ds_{10}^2 = -2 dx^+ dx^- + H dx^{+2} + dz_i^2, \tag{36}$$

$$F_{(4)} = dx^+ \wedge \Phi_{(3)}, \quad F_{(3)} = dx^+ \wedge \Phi_{(2)}, \quad F_{(2)} = dx^+ \wedge \Phi_{(1)},$$

where the constant form fields are given by

$$\begin{aligned} \Phi_{(3)} &= (2 dz_1 \wedge dz_2 + \sqrt{2} \cos \alpha dz_3 \wedge dz_4) \wedge dz_6, \\ \Phi_{(2)} &= \sqrt{2} \sin \alpha dz_5 \wedge dz_6, \quad \Phi_{(1)} = \sqrt{2} \sin \alpha dz_5, \end{aligned} \tag{37}$$

and

$$H = -(z_1^2 + z_2^2) - \frac{1}{2} \cos^2 \alpha (z_3^2 + z_4^2) - 2 \sin^2 \alpha z_5^2. \tag{38}$$

As in the previous example, by  $T$ -duality this solution is related to a deformed  $T^2$ -wrapped M2-brane solution of M-theory.

*D2/D4/D4/NS1/NS5/NS5 system.* The maximal number of intersecting components that give rise to an AdS structure is six, namely the D2/D4/D4/NS1/NS5/NS5 solution, which is given by [35]

$$\begin{aligned} ds_{10}^2 &= H_2^{-5/8} H_4^{-3/8} \tilde{H}_4^{-3/8} H_1^{-3/4} H_5^{-1/4} \tilde{H}_5^{-1/4} (-dt^2 + H_2 H_4 \tilde{H}_4 dx_1^2 + H_4 H_1 \tilde{H}_5 dx_2^2 \\ &\quad + \tilde{H}_4 H_1 H_5 dx_3^2 + H_2 H_4 H_1 H_5 dy_i^2 + H_2 \tilde{H}_4 H_1 \tilde{H}_5 d\tilde{y}_i^2), \\ F_{(4)} &= dt \wedge dx_2 \wedge dx_3 \wedge dH_2^{-1} + e^{-\frac{1}{2}\phi} * (\tilde{H}_4 dt \wedge dx_3 \wedge d^3 \tilde{y} \wedge dH_4^{-1}) \\ &\quad + e^{-\frac{1}{2}\phi} * (H_4 dt \wedge dx_2 \wedge d^3 y \wedge d\tilde{H}_4^{-1}), \\ F_{(3)} &= dt \wedge dx_1 \wedge dH_1^{-1} + e^\phi * (\tilde{H}_5 dt \wedge dx_1 \wedge dx_2 \wedge d^3 \tilde{y} \wedge dH_5^{-1}) \\ &\quad + e^\phi * (H_5 dt \wedge dx_1 \wedge dx_3 \wedge d^3 y \wedge d\tilde{H}_5^{-1}), \end{aligned} \tag{39}$$

$$\phi = \frac{1}{4} \log [H_2 (H_5 \tilde{H}_5)^2 / (H_1^2 H_4 \tilde{H}_4)], \quad H_2 = H_1 = H(y_i) \tilde{H}(\tilde{y}_i),$$

where  $H_4 = H_5 = H(y_i)$  and  $\tilde{H}_4 = \tilde{H}_5 = \tilde{H}(\tilde{y}_i)$ . The solution can be represented diagrammatically as follows:

	$t$	$x_1$	$x_2$	$x_3$	$y_1$	$y_2$	$y_3$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	
D2	×	—	×	×	—	—	—	—	—	—	$H_2$
D4	×	—	—	×	—	—	—	×	×	×	$H_4$
D4	×	—	×	—	×	×	×	—	—	—	$\tilde{H}_4$
NS1	×	×	—	—	—	—	—	—	—	—	$H_1$
NS5	×	×	×	—	—	—	—	×	×	×	$H_5$
NS5	×	×	—	×	×	×	×	—	—	—	$\tilde{H}_5$

D2/D4/D4/NS1/NS5/NS5 system

We parametrize the coordinates as  $dy_i^2 = dy^2 + y^2 d\Omega_2^2$  and  $d\tilde{y}_i^2 = d\tilde{y}^2 + \tilde{y}^2 d\tilde{\Omega}_2^2$ , so that

$$H_4 = H_5 = 1 + \frac{\lambda}{y}, \tag{40}$$

and likewise for  $\tilde{H}_4$  and  $\tilde{H}_5$ . The near-horizon geometry is  $AdS_2 \times S^2 \times S^2 \times T^4$  [35]. Explicitly, we have

$$\begin{aligned} ds_{10}^2 &= \frac{1}{4} \lambda^2 \left( \frac{1}{2} ds_{AdS_2}^2 + d\Omega_{(2)}^2 + d\tilde{\Omega}_{(2)}^2 \right) + d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2 + d\varphi_4^2, \\ F_{(4)} &= \frac{1}{2} \lambda \left( \Omega_{(2)} \wedge d\varphi_1 \wedge d\varphi_3 + \tilde{\Omega}_{(2)} \wedge d\varphi_1 \wedge d\varphi_4 + \frac{1}{\sqrt{2}} \epsilon_{(2)} \wedge d\varphi_3 \wedge d\varphi_4 \right), \\ F_{(3)} &= \frac{1}{2} \lambda \left( \Omega_{(2)} \wedge d\varphi_4 + \tilde{\Omega}_{(2)} \wedge d\varphi_3 + \frac{1}{\sqrt{2}} \epsilon_{(2)} \wedge d\varphi_4 \right). \end{aligned} \tag{41}$$



We find that the Penrose limit of this system is given by

$$\begin{aligned} ds_{10}^2 &= -2 dx^+ dx^- + H dx^{+2} + dz_i^2, \\ F_{(4)} &= dx^+ \wedge \Phi_{(3)}, \quad F_{(3)} = dx^+ \wedge \Phi_{(2)}, \end{aligned} \tag{42}$$

where the constant form fields are given by

$$\begin{aligned} \Phi_{(3)} &= \sqrt{2} (dz_6 \wedge dz_7 \wedge dz_1 + \frac{1}{2} \cos \alpha dz_2 \wedge dz_5 \wedge dz_6 - \frac{1}{2} \sin \alpha dz_3 \wedge dz_5 \wedge dz_7), \\ \Phi_{(2)} &= \sqrt{2} (dz_5 \wedge dz_1 + \frac{1}{2} \cos \alpha dz_2 \wedge dz_7 - \frac{1}{2} \sin \alpha dz_3 \wedge dz_6), \end{aligned} \tag{43}$$

and

$$H = -z_1^2 - \frac{1}{2} \cos^2 \alpha z_2^2 - \frac{1}{2} \sin^2 \alpha z_3^2. \tag{44}$$

Note that this solution can be easily lifted to  $D = 11$  as the Penrose limit of the M2/M2/M5/M5/M5/M5 system. There are six terms in the 4-form field strength in the eleven-dimensional pp-wave solution. This is a special case of the general class of M-theory pp-wave solutions obtained in [23].

### 5.2. Type IIB examples

*D3/D1/D5/NS1/NS5 system.* The near-horizon geometry of this system is  $AdS_2 \times S^3 \times S^2 \times T^3$  [35]. Explicitly, we find that

$$\begin{aligned} ds_{10}^2 &= \lambda^2 \left( \frac{1}{8} ds_{AdS_2}^2 + d\Omega_3^2 + \frac{1}{4} d\tilde{\Omega}_2^2 \right) + d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2, \\ F_{(5)} &= 2\lambda^2 \Omega_{(3)} \wedge d\varphi_1 \wedge d\varphi_3 + \text{dual}, \\ F_{(3)}^{RR} &= \frac{1}{2} \lambda \left( \tilde{\Omega}_{(2)} \wedge d\varphi_1 + \frac{1}{\sqrt{2}} \epsilon_{(2)} \wedge d\varphi_3 \right), \\ F_{(3)}^{NS} &= \frac{1}{2} \lambda \left( \tilde{\Omega}_{(2)} \wedge d\varphi_3 + \frac{1}{\sqrt{2}} \epsilon_{(2)} \wedge d\varphi_1 \right). \end{aligned} \tag{45}$$

The Penrose limit is given by

$$\begin{aligned} ds_{10}^2 &= -2 dx^+ dx^- + H dx^{+2} + dz_i^2, \\ F_{(5)} &= dx^+ \wedge \Phi_{(4)}, \quad F_{(3)}^{RR} = dx^+ \wedge \Phi_{(2)}^{RR}, \quad F_{(3)}^{NS} = dx^+ \wedge \Phi_{(2)}^{NS}, \end{aligned} \tag{46}$$

where the constant form fields are given by

$$\begin{aligned} \Phi_{(4)} &= \frac{1}{\sqrt{2}} \cos \alpha dz_2 \wedge dz_3 \wedge dz_6 \wedge dz_8, \\ \Phi_{(2)}^{RR} &= dz_1 \wedge dz_8 - \frac{1}{\sqrt{2}} \sin \alpha dz_4 \wedge dz_6, \\ \Phi_{(2)}^{NS} &= dz_1 \wedge dz_6 - \frac{1}{\sqrt{2}} \sin \alpha dz_4 \wedge dz_8, \end{aligned} \tag{47}$$

and

$$H = -z_1^2 - \frac{1}{8} \cos^2 \alpha (z_2^2 + z_3^2) - \frac{1}{2} \sin^2 \alpha z_4^2. \tag{48}$$

*D3/D5/D5/NS5/NS5 system.* The near-horizon geometry of this system is  $AdS_3 \times S^2 \times S^2 \times T^3$  [35]. Explicitly, we have

$$\begin{aligned} ds_{10}^2 &= \lambda^2 \left( \frac{1}{2} ds_{AdS_3}^2 + \frac{1}{4} d\Omega_2^2 + \frac{1}{4} d\tilde{\Omega}_2^2 \right) + d\varphi_1^2 + d\varphi_2^2 + d\varphi_3^2, \\ F_{(5)} &= \lambda^2 \epsilon_{(3)} \wedge d\varphi_1 \wedge \varphi_3 + \text{dual}, \\ F_{(3)}^{RR} &= \frac{1}{2} \lambda \left( \Omega_{(2)} \wedge d\varphi_1 + \tilde{\Omega}_{(2)} \wedge d\varphi_3 \right), \\ F_{(3)}^{NS} &= \frac{1}{2} \lambda \left( \Omega_{(2)} \wedge d\varphi_3 + \tilde{\Omega}_{(2)} \wedge d\varphi_1 \right), \end{aligned} \tag{49}$$

The Penrose limit is given by

$$ds_{10}^2 = -2 dx^+ dx^- + H dx^{+2} + dz_i^2, \\ F_{(5)} = dx^+ \wedge \Phi_{(4)}, \quad F_{(3)}^{\text{RR}} = dx^+ \wedge \Phi_{(2)}^{\text{RR}}, \quad F_{(3)}^{\text{NS}} = dx^+ \wedge \Phi_{(2)}^{\text{NS}}, \quad (50)$$

where the constant form fields are given by

$$\Phi_{(4)} = 2 dz_1 \wedge dz_2 \wedge dz_6 \wedge dz_8, \\ \Phi_{(2)}^{\text{RR}} = \sqrt{2}(\cos \alpha dz_3 \wedge dz_6 - \sin \alpha dz_4 \wedge dz_8), \\ \Phi_{(2)}^{\text{NS}} = \sqrt{2}(\cos \alpha dz_3 \wedge dz_8 - \sin \alpha dz_4 \wedge dz_6), \quad (51)$$

and

$$H = -(z_1^2 + z_2^2) - 2 \cos^2 \alpha z_3^2 - 2 \sin^2 \alpha z_4^2. \quad (52)$$

## 6. Conclusions

We have considered the Penrose limits of  $\text{AdS}_3$  and  $\text{AdS}_2$  spacetimes that arise from various non-standard brane intersections in type IIA, IIB and heterotic theories. We have focused on the non-standard intersection of two 5-branes and one string, whose near-horizon geometry is  $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ . This provides a new exact solvable string background for studying  $\text{AdS}_3/\text{CFT}_2$  correspondence. A new feature is that the product spacetime involves two 3-spheres, implying the isometry group is  $SO(4) \times SO(4)$ . After making an orthogonal rotation of the two foliating circles of the two 3-spheres, we obtain a one-parameter family of Penrose limits. Through the study of the supersymmetry of the resulting pp-wave, we find that supernumerary Killing spinors arise only for a  $45^\circ$  rotation. Thus, there are four supernumerary Killing spinors in addition to the 16 standard Killing spinors. The existence of the supernumerary Killing spinors ensures that the corresponding light-cone string action has linearly-realized supersymmetry. It is of interest to further investigate the role of the variable rotation parameter for supersymmetry breaking.

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