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ESTABLISHMENT OF STABILITY BY COLLECTIVE INTERACTIONS IN A PLASMA WITH COLLISIONS

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The purpose of this report is to show that a double humped initially unstable plasma will spontaneously approach stability if the high energy hump is held fixed. The frequently used Vlasov-Poisson equations represent a self-consistent system of nonlinear time-reversible equations. Because of the latter property, however, it is obviously unrealistic to apply those equations to problems involving the approach to equilibrium. An irreversible kinetic equation for plasmas has recently been independently derived by Balescu[1] and Guernsey[2,3]. Guernsey carried out a double expansion of the distribution function in terms of the strength of the electromagnetic interaction $4\pi e^2$ and the parameter $(4\pi e^2/V)$, where $(1/V) = n$ is the number density of particles. The latter parameter is a measure of the interparticle correlations. By equating terms of the same order in $4\pi e^2$ while summing over all powers of $(4\pi e^2/V)$ he obtained a hierarchy of equations. In this way correlations of the particles to all orders are taken into account at each step in the approximation with respect to $4\pi e^2$. Guernsey treated extensively the first order equation in $4\pi e^2$ and proved that it describes an irreversible approach of the distribution function to the Maxwell distribution. In addition he showed that the Vlasov equation is the zeroth order equation in this hierarchy. Thus, the Vlasov equation correctly takes into account interparticle correlations to all orders. However, neither the Guernsey equation nor therefore the Vlasov equation include the effects of close collisions. In fact the interaction term in the Guernsey equation diverges as the momentum transfer becomes large. The reason for this omission is twofold. First, the Coulomb force does not hold down to arbitrarily small distances and secondly electron-neutral collisions are not considered since Guernsey treats a fully ionized gas. For the purpose of investigating the effects of close collisions a phenomenological term is frequently introduced into the kinetic equation. Such an equation has recently been applied to plasma oscillations by Platzman and Buchsbaum[4].

These authors use a velocity independent relaxation model which is a good approximation, for example, in describing effects due to momentum transfer collisions between electrons and neutral particles. In our preliminary calculations the double humped distribution has been applied to the theory developed by Platzman and Buchsbaum[4]. Following these authors we consider a system described by the following equations:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{eE}{m} \frac{\partial f}{\partial v} = - \nu_c f', \tag{1}$$

$$\frac{\partial E}{\partial x} = - \frac{e}{\varepsilon_o} \int f' dv, \tag{2}$$

where

$$f = f_o + f', \tag{3}$$

$$f_o = f_{o1} + f_{o2}, \tag{4}$$

$$f_{o1}(v) = \frac{n_o}{w_o (2\pi)^{1/2}} \exp \left(- \frac{v^2}{2w_o^2} \right), \tag{5}$$

$$f_{o2}(v) = \frac{n_1}{w_1 (2\pi)^{1/2}} \exp \left[- \frac{(v - v_d)^2}{2w_1^2} \right], \tag{6}$$

$$w_o^2 = \kappa T/m,$$

ν_c = constant collision frequency of electrons of energy $w_o^2/2m$,

e = electronic charge,

m = electronic mass,

ε_o = dielectric constant in vacuo,

v_d = drift velocity of high energy electrons,

w_1 = velocity spread of high energy electrons,

κ = Boltzmann's constant,

T = electron temperature

with $n_1 \ll n_o$ and $w_o \ll v_d$. The state of the system is described by the double-humped distribution (4), (5), (6) plus a superposed disturbance f' which is considered to be a longitudinal plasma wave. It has been shown[5] that the kinetic equation (1) satisfies an H-theorem.

Each time an electron collides, it is thrown out of phase with the plasma wave. This means that a certain amount of ordered energy of the wave has been converted into random kinetic energy. The rate of energy transfer is given by the equation

$$\int \nu_c \, v^2 \, f' dv = \frac{d}{dt} \int v^2 \, f_{o1} \, dv$$

$$= 2 \, n_o w_o \, \frac{dw_o}{dt}. \tag{7}$$

Eq. (7) is not an additional assumption but merely a consequence of the collisional term in Eq. (1).

It is well known that a double-humped distribution of the form (4), (5), (6) may be unstable. In that case any small perturbation will excite growing waves in a certain frequency interval. The larger f' becomes, the faster the temperature rises. It is for this reason that the system has been assumed to be describable at all times by a distribution function of the form (3), (4), (5), (6) . The energy needed for this heating is supplied by maintaining f_{o2} constant. Without an external source of energy the hump f_{o2} would diffuse[6]. We express this explicitly by requiring that

$$\frac{\partial f_{o2}}{\partial t} = 0. \tag{8}$$

Using Eq's. (4), (5), (7), and (8) one obtains

$$\frac{\partial f_o}{\partial t} = \frac{\partial f_{o1}}{\partial t} = \frac{v^2 - w_o^2}{w_o^3} f_{o1} \frac{dw_o}{dt}$$

$$(9) \qquad = \frac{\nu_c}{2} \frac{v^2 - w_o^2}{w_o^4} \frac{f_{o1}}{n_o} \int v^2 f' dv.$$

If Eq's. (3) and (9) are substituted into Eq. (1) there results

$$(10) \qquad \frac{\partial f'}{\partial t} + v \frac{\partial f'}{\partial x} - \frac{eE}{m} \frac{\partial f}{\partial v} = - \nu_c \left[f' + \frac{v^2 - w_o^2}{2w_o^4} \frac{f_{o1}}{n_o} \int v^2 f' dv \right].$$

As the temperature rises, the secondary hump f_{o2} becomes gradually absorbed by the main distribution f_{o1}, and the plasma becomes stable. In order to calculate the temperature values for which the system makes the transition from the unstable to the stable regime we observe that the boundary can be approached from the stable region. Hence it suffices to use the linearized form of Eq. (10) as long as one is careful not to draw any conclusions applicable to the unstable regime.

The second term on the right hand side of Eq. (10) is nonlinear.*

Hence, the linearized kinetic equation is

$$(11) \qquad \frac{\partial f'}{\partial t} + v \frac{\partial f'}{\partial x} - \frac{eE}{m} \frac{\partial f_o}{\partial v} = - \nu_c f'.$$

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*The second term on the right hand side of Eq. (10) is of the order of $\int v^2 f' dv$, which is proportional to the average kinetic energy of the particles in the disturbance. However, the average kinetic energy associated with the wave must be of the same order as the average potential energy which is of the order E^2, i.e. of the second order in the perturbation.

For the simultaneous solution of Equations (2) and (11) the customary Ansatz

$$f', E \propto \exp[i(kx - \omega t)]$$

is being used.

Then one obtains the well known dispersion relations[4]

$$(12) \qquad \omega_r^2 = \omega_p^2 + k^2 \langle v^2 \rangle .$$

$$(13) \qquad \gamma = \frac{\pi}{2} \left[1 - \frac{k}{\omega_r} \frac{d\omega_r}{dk} \right] \frac{\omega_r \omega_p^2}{k^2} \frac{1}{n_o} \left. \frac{\partial f_o}{\partial v} \right|_{v=u} - \frac{\nu_c}{2} ,$$

where

$$\omega = \omega_r + i\gamma,$$

$$\omega_p^2 = n_o e^2/\varepsilon_o m,$$

$$u = \omega_r/k.$$

The boundary separating the stable and unstable regions is defined by the equation

$$(14) \qquad \gamma = 0.$$

Substituting Eq's. (4), (5), (6), (12), and (13) into Eq. (14) and simplifying one obtains

$$\frac{1}{w_o^3} \exp\left(-\frac{u^2}{2w_o^2}\right) - \frac{n_1}{n_o}\frac{v_d - u}{u\, w_1^3} \exp\left[-\frac{(v_d - u)^2}{2w_1^2}\right] + \left(\frac{2}{\pi}\right)^{1/2} \frac{\nu_c}{\omega_p}\frac{1}{u^3}\left[1 + \frac{3}{4}\frac{w_o^2}{u^2}\right] = 0, \tag{15}$$

where it has been assumed that $w_o^2/u^2 \ll 1$. This means that the thermal velocity of the electrons is small compared with the phase velocity of the plasma wave.

For fixed f_{o2} the quantities n_1, v_d, and w_1 are constant. In addition ν_c and n_o (and therefore ω_p) are considered constants. Eq. (15) can, therefore, be considered as the relation between the phase velocity and the thermal velocity for which the imaginary component of the frequency vanishes. Fig. 1 shows a typical graph of Eq. (15). The area under the curve is the unstable region, and the area above the curve is the stable region. $T_s = m\, w_{os}^2/\kappa$ is the temperature above which the plasma is stable. In the case of an initially unstable plasma, f' will grow but so will the temperature according to Eq. (7). As soon as the temperature T_s is reached, the system relaxes toward the distribution f_o given by Eq's. (4), (5), and (6) with $w_o^2 = \kappa T_s/m$. Since in the neighborhood of the boundary between the stable and unstable regions the system is adequately described by the linearized Eq. (11), the value of T_s is independent of the term $\partial f_{o1}/\partial t$ and therefore of the dissipation mechanism of Eq. (7). However, the physical significance of this mechanism does not lie in its influence on the stability temperature but rather in the fact that it is responsible for the irreversible rise in temperature of the system to T_s if its initial temperature lies below T_s.

As an example, Eq. (15) has been applied to the results of a plasma beam experiment recently performed by Singh and Rowe[7]. Fig. 1 shows the curve of w_o vs. u obtained from Eq. (15). The experimentally measured temperature[7] was 28500°K which corresponds to a value of w_o of about 6.6×10^5 m/sec, while the electron temperature corresponding to the calculated thermal velocity $w_{os} = 7.9 \times 10^5$ m/sec is about 40000°K. The main reason for this discrepancy is that in Eq. (7) it has been assumed that all of the plasma wave energy dissipated is converted into internal energy, while in the experiment of Singh and Rowe heat is being lost to the outside.

In summary the essential conclusions are as follows: Our equations describe a system in which part of the kinetic energy of the high energy particles is converted into the electric energy of the plasma waves of the main plasma by means of collective interactions. The electric energy in turn is dissipated into random internal energy through close collisions. As the plasma temperature increases, the growth parameter γ decreases until it approaches zero and stability is achieved. Hence, both the collective interactions due to the long range Coulomb force and also close collisions play a significant role in the behavior of the plasma.

In cases where a velocity dependent collision frequency is appropriate, the correct expression has to be substituted into Eq. (7). The temperature will increase again. In such a case the dispersion relation would, of course, be different from Eq's. (12), (13). Hence, the stability temperature depends on the functional form of the collision frequency. But the essential conclusions are not affected.

Finally, it is of interest to point out that the relaxation time for the heating of the main plasma up to the temperature T_s is determined by the collision frequency of the thermal electrons rather than that of the high energy electrons.

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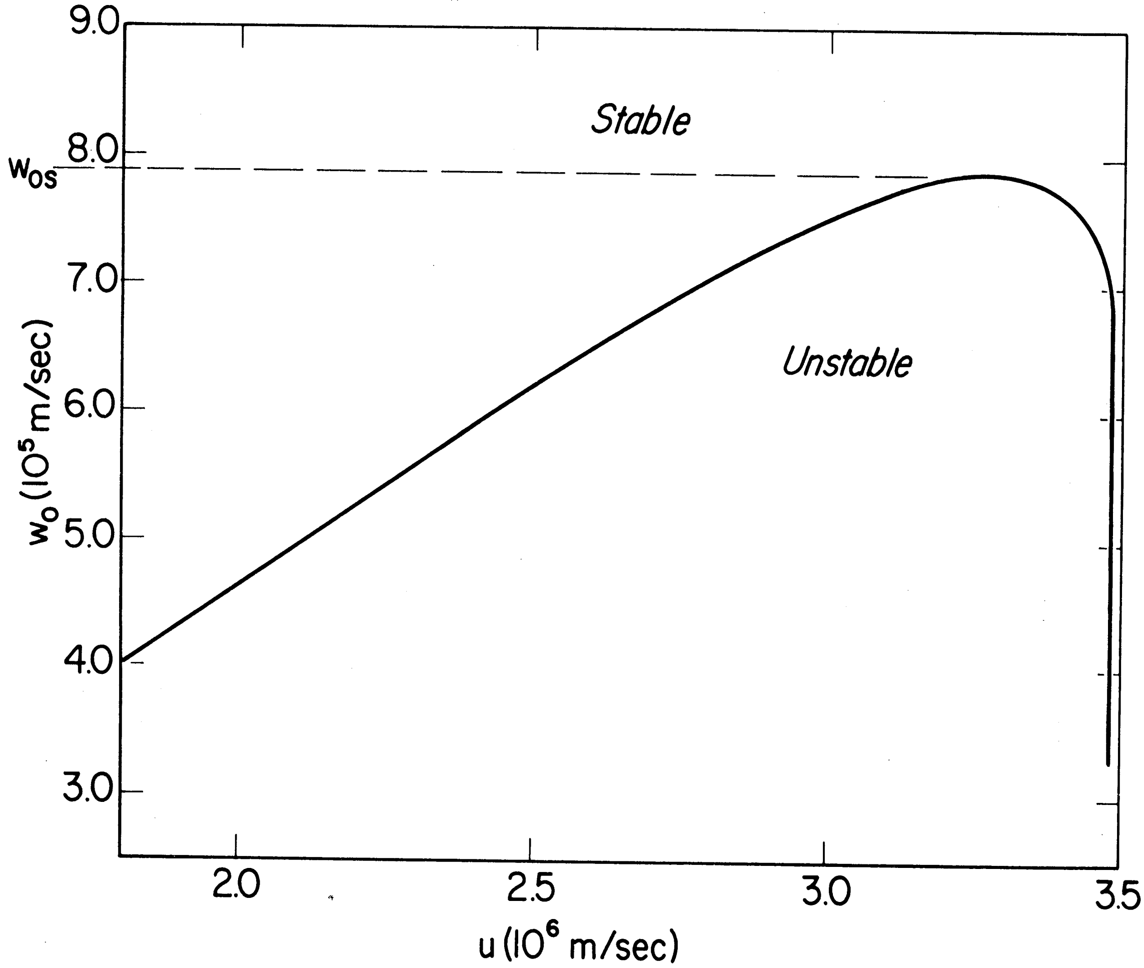

Fig. 1 Plasma stability as a function of the thermal velocity for a double humped distribution with the following parameters[8]: $n_o = 5.0 \times 10^{18}$ m^{-3}, $n_1 = 2.36 \times 10^{16}$ m^{-3}, $v_d = 3.5 \times 10^6$ m/sec, $w_1 = 9 \times 10^5$ m/sec, $\nu_c = 2.08 \times 10^8$ sec^{-1}.