THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Electrical Engineering
Space Physics Research Laboratory

Semi-Annual Report

THEORETICAL AND EXPERIMENTAL INVESTIGATION OF PLASMA WAVES, SPACE VEHICLE PLASMA SHEATHS, AND IONOSPHERIC ELECTRON TEMPERATURES

Prepared by:

Ernest G. Fontheim
Andrew F. Nagy

ORA Project 06106

supported by:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GRANT NO. NsG-525
WASHINGTON, D.C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ABOR

November 1966

Engu
uHR
1.25

RESEARCH ACTIVITIES DURING THE REPORTING PERIOD

This is the semi-annual status report under Grant No. NsG-525 covering the period from 1 April 1966 to 30 September 1966.

The work on the theory of the Langmuir probe in the absence of a magnetic field has been completed. The basic assumptions underlying Langmuir's classical theory have been critically reviewed. It has been shown that Langmuir's volt-ampere relations can be obtained only for a certain class of potentials. A volt-ampere relation for a more general class of potentials has been derived. It has also been shown that the volt-ampere relations for an arbitrary potential can be expressed in terms of a power law potential of the form $\varphi = \varphi_c(r_c/r)^\alpha$ for suitably chosen α, where r_c is the probe radius and φ_c the probe potential. The volt-ampere relations for the latter class of potentials have been computed numerically for a series of values of α, and these results have been compared with the numerical results of Laframboise* who used a self-consistent method to solve the Boltzmann and Poisson equations simultaneously. An extensive report based on this work is being drafted at the present time.

Work on the theory of the cylindrical Langmuir probe in the presence of a constant coaxial magnetic field is nearing completion. The effect of collisions has been included in the theory through a mathematical model which assures continuous replenishment of electron orbits beyond a given radius. Volt-ampere curves for different values of the magnetic field have been calculated.

The research on the problems of the heating and cooling mechanisms in the ionosphere has been continued. A preliminary calculation has been carried out to obtain an estimate of the expected temperature relaxation time. In this calculation the temperature relaxation after sunset has been obtained for a highly simplified mathematical model of the topside ionosphere. Heat loss has been assumed to take place only through downward conduction along magnetic field tubes, which were represented by vertical columns. The electron density has been taken to be constant in time. The problem was then to calculate the temperature profile as a function of time of a vertical plasma column which at time $t = 0$ is suddenly connected to a heat sink at its lower boundary, and into which no heat is being conducted at the upper boundary. The lower and upper boundaries have been taken at 200 km and 16000 km, respectively. The temperature is then given by the equation

---

*J. G. Laframboise, "Theory of Spherical and Cylindrical Langmuir Probes in a Collisionless Maxwellian Plasma at Rest," University of Toronto, Institute for Aerospace Studies, UTIAS Report No. 100 (1966).

$$\frac{\partial}{\partial t}(cnT) = \frac{\partial}{\partial x}\left(KT^{5/2}\frac{\partial T}{\partial x}\right) ,$$

where

$c = 3k$

k = Boltzmann's constant

$K = 1.23 \times 10^{-11}$ MKS units

n = constant electron number density.

After a change of variable $\theta = \frac{2}{7} KT^{7/2}$ the above equation becomes

$$\frac{\partial \theta}{\partial t} = r \frac{\partial^2 \theta}{\partial x^2} ,$$

where

$$r = \frac{K}{cn} \theta^{5/7} .$$

This equation has been solved numerically for the density and initial temperature profiles given in Table I and for the boundary conditions T(200 km) = 800°K = constant, representing the heat sink, and $\partial T/\partial x$ (16000 km) = 0 to satisfy the condition of no heat conduction at the upper boundary. The result of this calculation is presented in Figure 1 which shows the change of the temperature profile as a function of time.

TABLE I

Altitude, km	Density, cm^{-3}	Temperature, °K	Altitude, km	Density, cm^{-3}	Temperature, °K
200	2.0×10^5	800	2800	4.3×10^3	2070
400	5.0×10^4	1400	3400	3.5×10^3	2120
600	1.0×10^4	1600	4000	3.0×10^3	2150
800	8.7×10^3	1720	8000	1.0×10^3	2270
1000	8.0×10^3	1780	12000	3.3×10^2	2330
1600	6.2×10^3	1930	16000	1.1×10^2	2360
2200	5.0×10^3	2020			

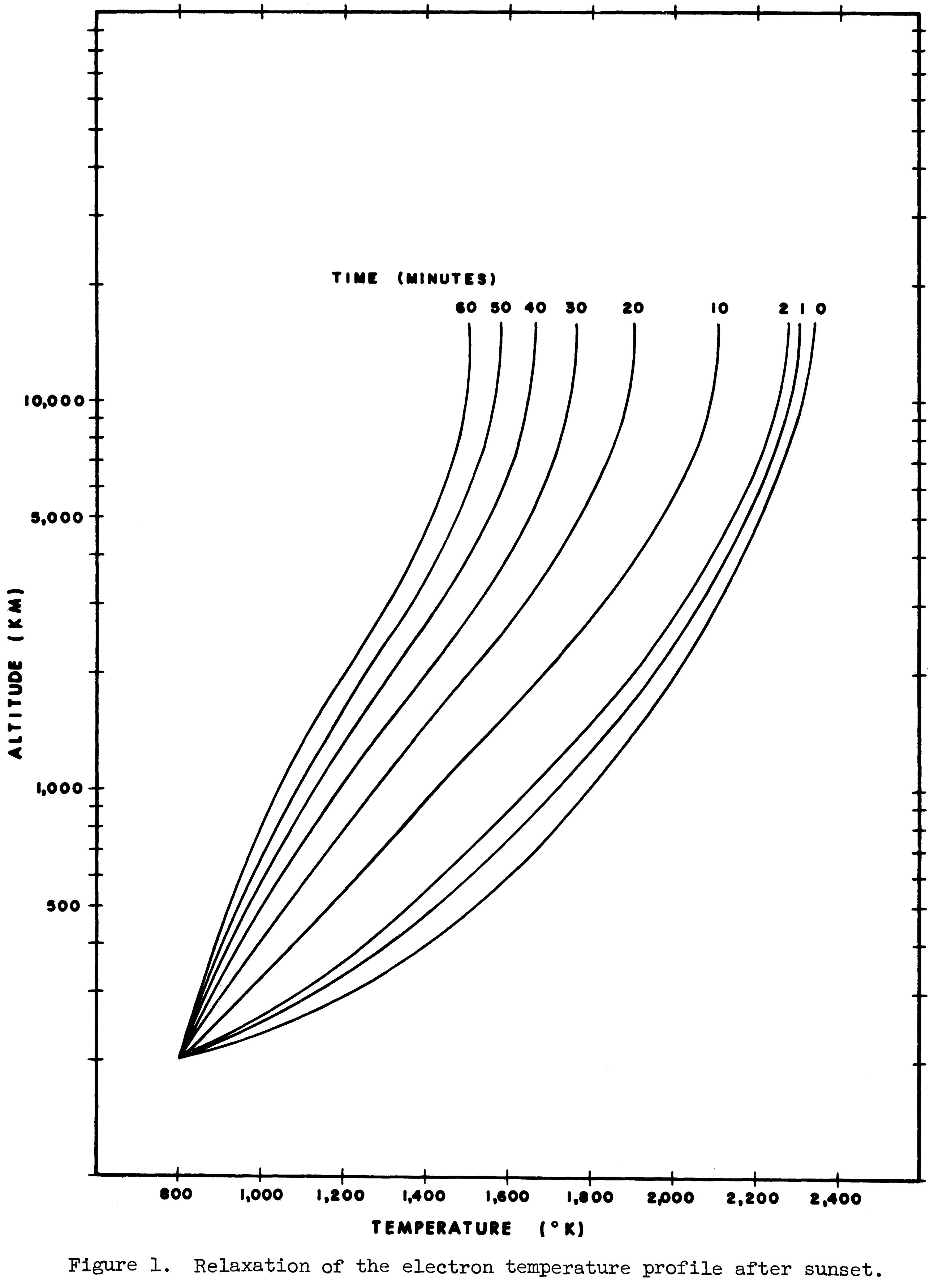

Figure 1. Relaxation of the electron temperature profile after sunset.

At the same time work on the complete problem, including both variable density and temperature, has been continued. A computer program has been written for the simultaneous solution of the continuity and nonlinear heat equations to obtain the electron density and temperature profiles along the magnetic field lines in the protonosphere as a function of time.

The spherical ion-trap was launched from Wallops Island, Virginia, on August 24, 1966, as part of the Nike Apache 14-272 payload, which was integrated by GCA Corporation. The transmitter power output dropped by about 40 db at T + 3.5 seconds. The cause of this transmitter failure has now been traced to faulty construction techniques of the manufacturer. Initial runback of the telemetry tape, using conventional phase lock discriminators, provided data up to about T + 40 seconds (up to which time the experiment was working perfectly) and it is hoped that using a more sophisticated approach useful data can be retrieved up to about T + 100 seconds. This will be useful for engineering evaluation of the system but no geophysical information of significance is expected to be obtained.

Studies on the bandwidth requirements for transmission of Langmuir probe characteristics is progressing. The theoretical results obtained earlier are now being compared with experimental results which are obtained by analog computer simulation of the volt-ampere characteristics. It is expected that this work will be completed by the end of this year, and a technical report will be forthcoming early next year.

The grant also contributes to the professional training of students in the area of ionospheric physics. The research reported above includes work on a doctoral dissertation and on a thesis for a Professional Degree in Electrical Engineering. In addition three graduate students are contributing to various phases of the geophysical aspects of the work and to the theory of the Langmuir probe.

PERSONNEL PARTICIPATING DURING THE REPORTING PERIOD

Pierre Bauer
Duane Beechler
Jose Busnardo-Neto
William G. Dow
Ahmad Z. Faruqi
Ernest G. Fontheim
Madhoo Kanal
James G. Laframboise
Andrew F. Nagy

MONTHLY COST BREAKDOWN

Month	Wages		Overhead	Materials & Supplies	Travel
	Student	Non-Student			
April	$1,378.60	$ 3,905.37	$1,232.82	$ 575.93	$ 304.20
May	942.25	3,924.84	1,162.81	166.74	780.23
June	1,273.85	2,006.48	667.58	36.99	20.60
July	723.75	2,333.81	702.92	158.08	298.95
August	847.62	4,832.98	1,248.99	348.40	215.95
September	721.75	4,867.01	1,130.92	65.74	.08
Totals	$5,887.82	$21,870.49	$6,146.04	$1,351.88	$1,620.01

Remaining Balance as of March 31, 1966: $37,041.00

Expenses Reported in this Semi-Annual Report: $36,876.24

Balance as of September 30 1966: $164.76