THE UNIVERSITY OF MICHIGAN
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A THEORETICAL STUDY OF THE EFFECT OF COLLECTIVE INTERACTIONS ON THE ELECTRON TEMPERATURE IN THE IONOSPHERE AND OF THE LANGMUIR PROBE CHARACTERISTICS IN THE PRESENCE OF A MAGNETIC FIELD

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I. INTRODUCTION

This is the first semi-annual report under grant No. NsG-525 covering the period from 1 October 1963 to 31 March 1964. This grant has been awarded for the purpose of a theoretical investigation of space charge waves in the ionosphere and of space vehicle plasma sheaths.

II. RESEARCH EFFORT DURING THE PERIOD

Research has been carried out both on the subject of instability against growing plasma waves in the ionosphere and on the effect of the plasma sheath on the current collection characteristics of a moving probe in a magnetic field.

In the course of the investigation of the effect of plasma instabilities in the ionosphere, it has been shown that the temperature of certain plasmas which are initially unstable automatically increases toward the value for which the system becomes stable.

In this treatment a one-dimensional Maxwellian plasma with a superposed high energy hump in the distribution function has been considered. High momentum transfer electron-neutral collisions have been included by means of a relaxation term in the kinetic equation. The treatment has appeared as a University of Michigan Scientific Report[1] and has been included as Appendix A of this semi-annual report. This scientific report is a somewhat modified version of a paper[2] which has been presented to the Fifth Annual Meeting of the Division of Plasma Physics of the American Physical Society in San Diego, California, 6-9 November 1963.

In order to apply the above mentioned theory to the ionosphere it became necessary to investigate the high energy tail of the electron distribution. Because of the presence of photoelectrons and of other high energy electrons which penetrate into the ionosphere from above (possibly from the Van Allen belt) the high energy tail is not Maxwellian. Since data on the penetration of corpuscular streams into the ionosphere are still very incomplete,[3] the investigation has been restricted to the effect of the photoelectrons. The energy which a particular photoelectron carries away depends on the final state of the ion. Maximum photoelectron energy occurs if the ion is left in its ground state. The photoelectrons lose their energy through collisions with the ambient gas particles. This results in a spread of the photoelectron energy distribution. Appendix B contains the calculation of the partial distribution due to those photoelectrons created with a definite initial energy corresponding to the ionization process of the oxygen atom $O\ ^3P \rightarrow O^+\ ^4S$. The total energy distribution of the photoelectrons is obtained by summing over all the partial distributions due to all the possible initial photoelectron energies. The energy loss calculations are based solely on those processes caused by binary collisions and neglect collective electron-electron interactions. Under those assumptions, the calculations show that the partial distribution under consideration gives rise to a hump on the high energy tail of the electron energy distribution. Effects which cause an anisotropic distribution have not yet been included in the calculations. Recently, Drummond[4] showed that an instability can arise even in the case of an isotropic distribution, when the distribution has the shape of an energy shell in phase space.

The stability criterion used in Appendix A is based on the conventional plane wave expansion of the perturbation. However, the assumption of a plane wave restricts the validity of the criterion. For example, the case of spherical waves is not included in the treatment. The excitation of spherical plasma waves in the ionosphere by point disturbances is a distinct possibility. Therefore, an investigation has been started of the stability condition for the case of spherical waves.

The Langmuir probe characteristics in the presence of a magnetic field have been investigated. This work is a continuation of the study begun under Contract No. NASr-15. This study was motivated by the need to obtain the correct relation between the current collected by a probe and the ionospheric parameters to be measured. The results for the case of a cylindrical probe in a magnetic field parallel to its axis are contained in Appendix C of this report. An attempt has been made to calculate the current characteristics for the case of arbitrary angles between magnetic field and collector axis. However, the mathematics turned out to be prohibitively complicated so that the attempt had to be abandoned. At the present time work is in progress on the relation between sheath thickness and probe potential. This problem is not only of great general interest but is an important part of the investigation of current characteristics of a probe as is pointed out in Appendix C.

III. FINANCIAL REPORT

GRANT AMOUNT:		$30,125.00
EXPENDITURES:		
A. Salaries and Wages, Students	$2,412.02	
Academic	4,000.19	
		6,412.21
Vacation Accural		336.09
Recharge Units		53.87

B.	University Contribution to OASI and Annuity	$ 282.64	
C.	Indirect Costs 25% of total costs	1,946.63	
D.	Supplies	132.09	
E.	Equipment	---	
F.	Travel	569.57	
Total Expenditures 10/1/63 - 3/31/64			$ 9,733.10
Total Expenditures Prior Periods			---
Total Expenditures through 3/31/64			9,733.10
Balance 3/31/64			$20,391.90

IV. FUTURE WORK

The work reported in Appendix A will be extended to include the magnetic field and a more realistic collision term in the kinetic equation. This will result in a modification of the equation determining the stability temperature T_s. Furthermore, a non-linear (or at least quasi-linear) method of solving Eq. (A-10) will be investigated in order to obtain the time-dependent behavior of the system.

The calculation of the high energy tail of the electron distribution in the ionosphere will be continued and particularly the anisotropy of the velocity distribution of the high energy electrons will be taken into account.[5,6] After that the theory extended from Appendix A will be applied to such a distribution and the computed temperature T_s compared with the measured electron temperature.

The investigation of stability against the excitation of spherical waves will be continued and the effect of this stability condition on electron temperature will be studied.

Work on the derivation of the potential distribution inside the plasma sheath has been started and will be continued.

APPENDIX A

ESTABLISHMENT OF STABILITY BY COLLECTIVE INTERACTIONS IN A PLASMA WITH COLLISIONS

Prepared by Ernest G. Fontheim

The purpose of this report is to show that a double humped initially unstable plasma will spontaneously approach stability if the high energy hump is held fixed. The frequently used Vlasov-Poisson equations represent a self-consistent system of nonlinear time-reversible equations. Because of the latter property, however, it is obviously unrealistic to apply those equations to problems involving the approach to equilibrium. An irreversible kinetic equation for plasmas has recently been independently derived by Balescu[7] and Guernsey.[8,9] Guernsey carried out a double expansion of the distribution function in terms of the strength of the electromagnetic interaction $4\pi e^2$ and the parameter $(4\pi e^2/V)$, where $(1/V) = n$ is the number density of particles. The latter parameter is a measure of the interparticle correlations. By equating terms of the same order in $4\pi e^2$ while summing over all powers of $(4\pi e^2/V)$ he obtained a hierarchy of equations. In this way correlations of the particles to all orders are taken into account at each step in the approximation with respect to $4\pi e^2$. Guernsey treated extensively the first order equation in $4\pi e^2$ and proved that it describes an irreversible approach of the distribution function to the Maxwell distribution. In addition he showed that the Vlasov equation is the zeroth order equation in this hierarchy. Thus, the Vlasov equation correctly takes into account interparticle correlations to all orders. However, neither the Guernsey equation nor therefore the Vlasov equation include the effects of close collisions. In fact the interaction term in the Guernsey equation diverges as the momentum transfer becomes large. The reason for this omission is two-fold. First, the Coulomb force does not hold down to arbitrarily small distances and secondly electron-neutral collisions are not considered since Guernsey treats a fully ionized gas. For the purpose of investigating the effects of close collisions a phenomenological term is frequently introduced into the kinetic equation. Such an equation has recently been applied to plasma oscillations by Platzman and Buchsbaum.[10] These authors use a velocity independent relaxation model which is a good approximation, for example, in describing effects due to momentum transfer collisions between electrons and neutral particles. In our preliminary calculations the double humped distribution has been applied to the theory developed by Platzman and Buchsbaum.[10] Following these authors we consider a system described by the following equations:

$$\frac{\partial f}{\partial t} + v\,\frac{\partial f}{\partial x} - \frac{eE}{m}\,\frac{\partial f}{\partial v} = -\,\nu_c f', \qquad (A-1)$$

$$\frac{\partial E}{\partial x} = -\,\frac{e}{\epsilon_o}\int f' dv, \qquad (A-2)$$

where

$$f = f_o + f', \qquad (A-3)$$

$$f_o = f_{o1} + f_{o2}, \qquad (A-4)$$

$$f_{o1}(v) = \frac{n_o}{w_o(2\pi)^{1/2}} \exp\left(-\,\frac{v^2}{2w_o^{\,2}}\right), \qquad (A-5)$$

$$f_{o2}(v) = \frac{n_1}{w_1(2\pi)^{1/2}} \exp\left[-\,\frac{(v-v_d)^2}{2w_1^{\,2}}\right], \qquad (A-6)$$

$$w_o^{\,2} = \kappa\, T/m,$$

ν_c = constant collision frequency of electrons of energy $w_o^2/2m$,

e = electronic change,

m = electronic mass,

ϵ_o = dielectric constant in vacuo,

v_d = drift velocity of high energy electrons,

w_1 = velocity spread of high energy electrons,

κ = Boltzmann's constant,

T = electron temperature

with $n_1 \ll n_o$ and $w_o \ll v_d$. The state of the system is described by the double-humped distribution (A-4), (A-5), (A-6) plus a superposed disturbance f' which is considered to be a longitudinal plasma wave. It has been shown[11] that the kinetic Eq. (A-1) satisfies an H-theorem.

Each time an electron collides, it is thrown out of phase with the plasma wave. This means that a certain amount of ordered energy of the wave has been converted into random kinetic energy. The rate of energy transfer is given by the equation

$$\int \nu_c \, v^2 f' dv = \frac{d}{dt} \int v^2 f_{o1} \, dv = 2 \, n_o w_o \frac{dw_o}{dt}. \qquad \text{(A-7)}$$

Equation (A-7) is not an additional assumption but merely a consequence of the collisional term in Eq. (A-1).

It is well known that a double-humped distribution of the form (A-4), (A-5), (A-6) may be unstable. In that case any small perturbation will excite growing waves in a certain frequency interval. The larger f' becomes, the faster the temperature rises. It is for this reason that the system has been assumed to be describable at all times by a distribution function of the form (A-3), (A-4), (A-5), (A-6). The energy needed for this heating is supplied by maintaining f_{o2} constant. Without an external source of energy the hump f_{o2} would diffuse.[12] We express this explicitly by requiring that

$$\frac{\partial f_{o2}}{\partial t} = 0. \qquad \text{(A-8)}$$

Using Eqs. (A-4), (A-5), (A-7), and (A-8) one obtains

$$\frac{\partial f_o}{\partial t} = \frac{\partial f_{o1}}{\partial t} = \frac{v^2 - w_o^2}{w_o^3} f_{o1} \frac{dw_o}{dt} = \frac{\nu_c}{2} \frac{v^2 - w_o^2}{w_o^4} \frac{f_{o1}}{n_o} \int v^2 f' dv. \qquad \text{(A-9)}$$

If Eqs. (A-3) and (A-9) are substituted into Eq. (A-1) there results

$$\frac{\partial f'}{\partial t} + v \frac{\partial f'}{\partial x} - \frac{eE}{m} \frac{\partial f}{\partial v} = - \nu_c \left[f' + \frac{v^2 - w_o^2}{2 w_o^4} \frac{f_{o1}}{n_o} \int v^2 f' dv \right]. \qquad \text{(A-10)}$$

As the temperature rises, the secondary hump f_{o2} becomes gradually absorbed by the main distribution f_{o1}, and the plasma becomes stable. In order to calculate the temperature values for which the system makes the transition from the unstable to the stable regime we observe that the boundary can be approached from the stable region. Hence it suffices to use the linearized form of Eq. (A-10) as long as one is careful not to draw any conclusions applicable to the unstable regime. The second term on the right hand side of Eq. (A-10) is nonlinear.* Hence, the linearized kinetic equation is

$$\frac{\partial f'}{\partial t} + v\,\frac{\partial f'}{\partial x} - \frac{eE}{m}\frac{\partial f_o}{\partial v} = -\nu_c f'. \tag{A-11}$$

For the simultaneous solution of Eqs. (A-2) and (A-11) the customary Ansatz

$$f',\ E \propto \exp\,[i(kx-\omega t)]$$

is being used. Then one obtains the well known dispersion relations[10]

$$\omega_r^{\,2} = \omega_p^{\,2} + k^2 \langle v^2\rangle \tag{A-12}$$

$$\gamma = \frac{\pi}{2}\left[1 - \frac{k}{\omega_r}\frac{d\omega_r}{dk}\right]\frac{\omega_r\omega_p^{\,2}}{k^2}\,\frac{1}{n_o}\,\frac{\partial f_o}{\partial v}\bigg|_{v=u} - \frac{\nu_c}{2}, \tag{A-13}$$

where

$$\omega = \omega_r + i\gamma,$$

$$\omega_p^{\,2} = n_o e^2/\epsilon_o m,$$

$$u = \omega_r/k.$$

The boundary separating the stable and unstable regions is defined by the equation

$$\gamma = 0. \tag{A-14}$$

Substituting Eqs. (A-4), (A-5), (A-6), (A-12) and (A-13) into Eq. (A-14) and simplifying one obtains

*The second term on the right hand side of Eq. (A-10) is of the order of $\int v^2 f'dv$, which is proportional to the average kinetic energy of the particles in the disturbance. However, the average kinetic energy associated with the wave must be of the same order as the average potential energy which is of the order E^2, i.e. of the second order in the perturbation.

$$\frac{1}{w_o^3} \exp\left(-\frac{u^2}{2w_o^2}\right) - \frac{n_1}{n_o}\frac{v_d-u}{uw_1^3} \exp\left[-\frac{(v_d-u)^2}{2w_1^2}\right] + \left(\frac{2}{\pi}\right)^{1/2} \frac{\nu_c}{\omega_p}\frac{1}{u^3}\left[1 + \frac{3}{4}\frac{w_o^2}{u^2}\right] = 0, \tag{A-15}$$

where it has been assumed that $w_o^2/u^2 \ll 1$. This means that the thermal velocity of the electrons is small compared with the phase velocity of the plasma wave.

For fixed f_{o2} the quantities n_1, v_d, and w_1 are constant. In addition ν_c and n_o (and therefore ω_p) are considered constants. Equation (A-15) can, therefore, be considered as the relation between the phase velocity and the thermal velocity for which the imaginary component of the frequency vanishes. Figure 1 shows a typical graph of Eq. (A-15). The area under the

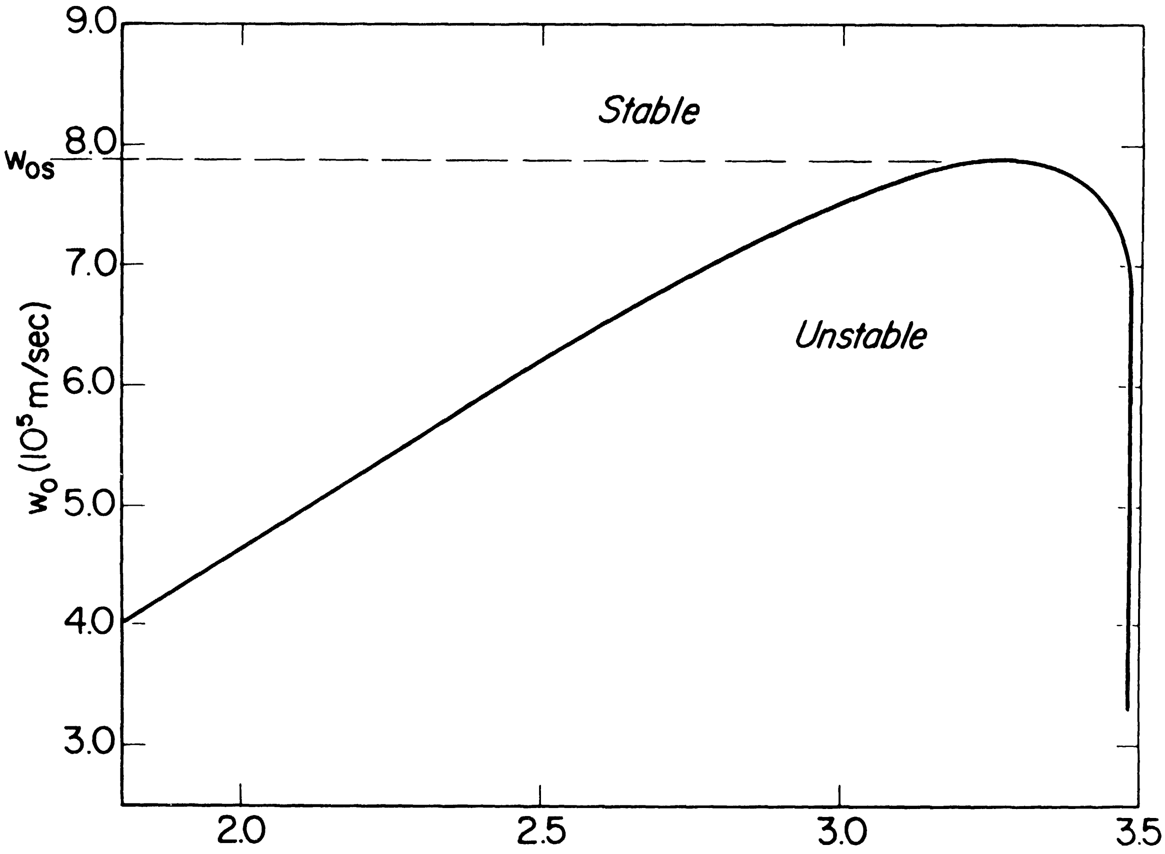

Fig. 1. Plasma stability as a function of the thermal velocity for a double humped distribution with the following parameters:[13] $n_o = 5.0 \times 10^{18}\ m^{-3}$, $n_1 = 2.36 \times 10^{16}\ m^{-3}$, $v_d = 3.5 \times 10^6$ m/sec, $w_1 = 9 \times 10^5$ m/sec, $\nu_c = 2.08 \times 10^8\ sec^{-1}$.

curve is the unstable region, and the area above the curve is the stable region. $T_s = mw_{os}^2/\kappa$ is the temperature above which the plasma is stable. In the case of an initially unstable plasma, f' will grow but so will the temperature according to Eq. (A-7). As soon as the temperature T_s is reached, the system relaxes toward the distribution f_o given by Eqs. (A-4), (A-5), and (A-6) with $w_o^2 = \kappa T_s/m$. Since in the neighborhood of the boundary between the stable and unstable regions the system is adequately described by the linearized Eq. (A-11), the value of T_s is independent of the term $\partial f_{o1}/\partial t$ and therefore of the dissipation mechanism of Eq. (A-7). However, the physical significance of this mechanism does not lie in its influence on the stability temperature but rather in the fact that it is responsible for the irreversible rise in temperature of the system to T_s if its initial temperature lies below T_s.

As an example, Eq. (A-15) has been applied to the results of a plasma beam experiment recently performed by Singh and Rowe.[13] Figure 1 shows the curve of w_o vs. u obtained from Eq. (A-15). The experimentally measured temperature[13] was 28500°K which corresponds to a value of w_o of about 6.6×10^5 m/sec, while the electron temperature corresponding to the calculated thermal velocity $w_{os} = 7.9 \times 10^5$ m/sec is about 40000°K. The main reason for this discrepancy is that in Eq. (A-7) it has been assumed that all of the plasma wave energy dissipated is converted into internal energy, while in the experiment of Singh and Rowe heat is being lost to the outside.

In summary the essential conclusions are as follows: Our equations describe a system in which part of the kinetic energy of the high energy particles is converted into the electric energy of the plasma waves of the main plasma by means of collective interactions. The electric energy in turn is dissipated into random internal energy through close collisions. As the plasma temperature increases, the growth parameter γ decreases until it approaches zero and stability is achieved. Hence, both the collective interactions due to the long range Coulomb force and also close collisions play a significant role in the behavior of the plasma.

In cases where a velocity dependent collision frequency is appropriate, the correct expression has to be substituted into Eq. (A-7). The temperature will increase again. In such a case the dispersion relation would, of course, be different from Eqs. (A-12), (A-13). Hence, the stability temperature depends on the functional form of the collision frequency. But the essential conclusions are not affected.

Finally, it is of interest to point out that the relaxation time for the heating of the main plasma up to the temperature T_s is determined by the collision frequency of the thermal electrons rather than that of the high energy electrons.

APPENDIX B

PHOTOELECTRON ENERGY DISTRIBUTION

Prepared by Walter R. Hoegy

The electrons in the ionosphere are not in an equilibrium state because of the various processes which are continually injecting particles and energy. However, they are evidently in a quasi-steady-state configuration. One of the ionosphere input processes affecting the steady-state condition is the flux of solar radiation which has the effect of injecting high-energy photoelectrons into the ionosphere plasma. It is this process and the consequent energy loss of the photoelectrons in collisions which is treated in this report—the aim being to compute the resultant steady-state photoelectron energy distribution.

The method of computing this distribution function is to treat the photoelectrons as high-energy particles injected into a Maxwellian plasma at fixed energy, which are consequently slowed down by collisions with the ambient particles—electrons, ions, and neutrals. The rate of energy loss of the electrons dE/dt, determines the number residing at each energy.

Essentially two types of data are needed in this calculation—the photoelectron production rates, and the energy loss rates per electron. Once these data have been obtained, the distribution can be computed from the formula derived below in a straight-forward manner. However, the difficulty of the problem lies in obtaining this information from the basic data. For this one must use the best available basic data on cross sections, photon-flux, and chemical composition in the ionosphere. The data on photoionization cross sections and solar flux are taken from Watanabe and Hinteregger;[14] the chemical composition and reaction data are taken from Dalgarno.[15]

The formula for the photoelectron energy distribution in terms of the production rate q and energy loss rate r is as follows. Let $\tau(E_i \to E_f)$ be the time it takes an electron to suffer energy loss from E_i to E_f, and $q(E_i)$ be the production rate at energy E_i. It follows that the product $q(E_i)\tau(E_i \to E_f)$ represents the number of electrons in the energy range from E_f to E_i which were produced at energy E_i. Let $f_i(E)dE$ be the energy distribution function in the range E to E+dE due to electrons produced at E_i, then

$$\int_{E_f}^{E_i} f_i(E)dE = q(E_i)\tau(E_i \to E_f). \qquad \text{(B-1)}$$

Since,

$$\tau(E_i \to E_f) = \int_{E_i}^{E_f} \frac{dE}{r} \,, \qquad \text{(B-2)}$$

where

$$r = \frac{dE}{dt} \,, \qquad \text{(B-3)}$$

it follows that,

$$f_i(E) = -\frac{q(E_i)}{r(E)}.$$

Since the electrons are produced at several energies, the net distribution function, giving the number of photoelectrons in the infinitesimal range E to E + dE, is the sum over production rates for energies greater than E,

$$f(E) = -\frac{\sum\limits_{E<E_i} q(E_i)}{r(E)} . \qquad \text{(B-4)}$$

This is the formula used in the calculation of the photoelectron energy distribution.

At present, the calculations of production rates are not complete; however they will be completed shortly. The energy loss rates have all been computed. Since expression (B-4) for the energy distribution is linear in the production rates, it is possible to compute a partial of the net distribution which is representative of the final result. This has been done for the single ionization process

$$O(1s^2 2s^2 2p^4)\ {}^3P + h\nu \rightarrow O^+(1s^2 2s^2 2p^3)\ {}^4S + e,$$

and the result represents the energy distribution for this process alone. The net distribution is the sum of this partial distribution and the partial distributions computed for the other pertinent ionization processes.

The complete loss rates include energy loss due to collisions with neutrals, vibrational and rotational excitation of N_2, and electron-electron scattering. Only the latter process can be represented by a simple analytical expression. The other collision processes are too difficult to be described in simple mathematical terms and, one must rely heavily on observational data for the energy dependence of these processes.

The results of the calculations are presented in Fig. 2. The details of

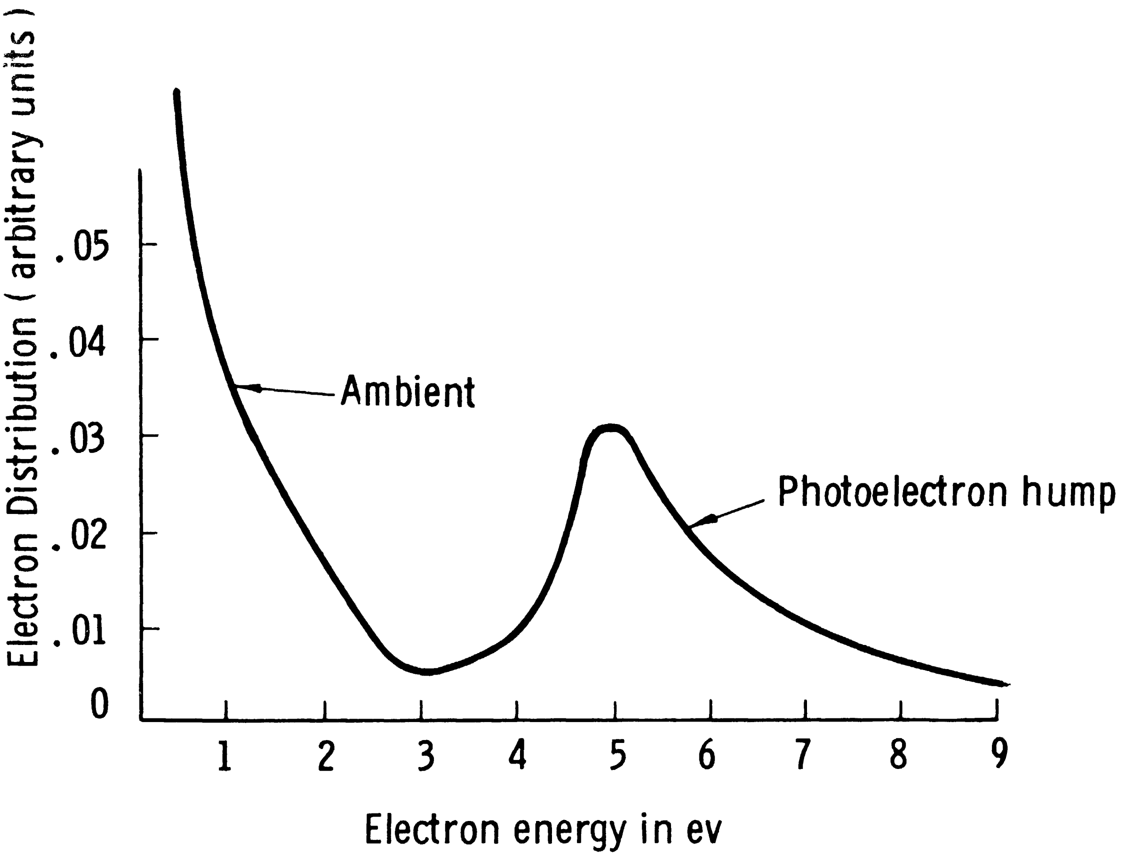

Fig. 2. Photoelectron energy distribution for process O $^3P \rightarrow O^+ \ ^4S$.

computation will be included in the final report along with the complete production rate data. The photoelectron distribution computed here for the ionization process O $^3P \rightarrow O^+ \ ^4S$ gives a hump or maximum at an energy of 5 ev with a height of about 1.3% of the Maxwellian maximum and a halfwidth of about 1.5 ev. The existence of this hump depends of course on the data used in the calculations and on the assumption that all energy loss processes are due to binary collisions.

Since the above treatment deals with the energy distribution, it does not contain any information concerning a possible anisotropy in the electron velocity distribution. The original photoelectrons have of course the well known $\sin^2\theta$ distribution of velocities, where θ is the angle between the direction of incidence of the photons and the direction of the velocity of the emitted electrons. It is reasonable to assume that the collisions completely randomize the distribution so that the electrons in the hump can be considered to be isotropic. In addition, there is corpuscular streaming into the ionosphere which adds to the higher energy electron distribution in an anisotropic manner.

APPENDIX C

THE VOLT-AMPERE CHARACTERISTICS FOR A PROBE IN A MAGNETIC FIELD

Prepared by Madhoo Kanal

The influence of the geomagnetic field on the current characteristics of a Langmuir probe in the ionosphere is generally assumed to be a second order effect and is therefore ignored. As a preliminary step toward testing the validity of this assumption, a theory has been developed for the case of a cylindrical probe in the presence of a magnetic field parallel to its axis.

It is evident that the current characteristics of a cylindrical probe depend strongly on the character of the sheath surrounding the probe. Since a consistent theoretical solution of the sheath problem is not yet available it is necessary to postulate a model for the sheath which approximates the real situation and lends itself to an analytical solution of the problem of current collection. It has been assumed that the sheath is a cylindrical shell concentric with the cylindrical conductor and with a definite boundary between sheath and ambient plasma called the sheath edge. A schematic diagram showing the collector-sheath configuration is shown in Fig. 3. The electric

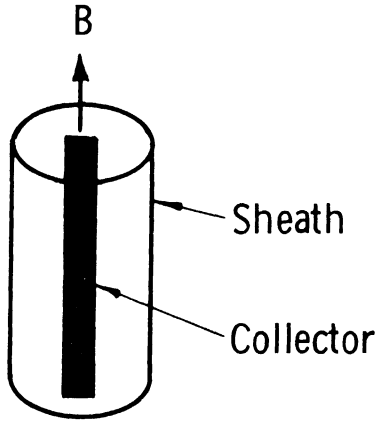

Fig. 3. Diagram showing sheath-collector configuration.

potential is assumed to be a general central field inside the sheath approaching zero at the sheath edge. The charged particles are assumed to suffer no collisions inside the sheath.

Since the ion Larmor radius is much larger than the electron Larmor radius, the effect of the magnetic field on the ion current collection is very small compared to the effect on the electron current collection and will therefore be ignored in this treatment. Hence, from now on only the electron current will be considered. The two cases of a probe at positive and at negative potential with respect to the undisturbed plasma have to be treated separately. They give rise to the so-called accelerated and retarded electron current characteristics respectively. Using the laws of conservation of energy and angular momentum, and assuming a Maxwellian distribution for the electrons at the sheath edge the following expressions have been obtained:

(1) Accelerated Electron Current (probe at positive potential with respect to ambient plasma)

$$I_a = \frac{1}{2} I_o \left[\frac{a}{r} \left\{ \mathrm{erf}\left(\sqrt{\tau^2 V_o + \frac{\omega^2 r^2}{4c^2}} - \frac{\omega a}{2c} \right) + \mathrm{erf}\left(\sqrt{\tau^2 V_o + \frac{\omega^2 r^2}{4c^2}} + \frac{\omega a}{2c} \right) \right\} \right.$$

$$\left. + \exp(V_o) \left\{ \mathrm{erfc}\left(\sqrt{(1+\tau^2) V_o + \frac{\omega^2 a^2}{4c^2}} - \frac{\omega r}{2c} \right) + \mathrm{erfc}\left(\sqrt{(1+\tau^2) V_o + \frac{\omega^2 a^2}{4c^2}} + \frac{\omega r}{2c} \right) \right\} \right] \quad \text{(C-1)}$$

where,

a = radius of the sheath

r = radius of the collector

τ^2 = $r^2/(a^2-r^2)$

V_o = qV/KT

q = electron charge

K = Boltzmann's constant

T = electron temperature

V = voltage across the sheath

m = mass of the electron

c = $\sqrt{2KT/m}$ (most probable velcoity)

$$\omega = qB/m \text{ (cyclotron frequency)}$$

$$B = \text{magnetic field intensity}$$

$$I_o = \sqrt{KT/2m\pi}\; Nq\; 2\pi\; rL$$

$$N = \text{electron number density}$$

$$L = \text{length of the collector}$$

$$\text{erf}(x) = 2/\sqrt{\pi} \int_o^x e^{-t^2}\, dt$$

$$\text{erfc}(x) = 1 - \text{erf}(x).$$

(2) Retarded Electron Current (probe at negative potential with respect to ambient plasma)

(a) $|V| < \dfrac{qB^2(a^2-r^2)}{8m}$

$$I_{r1} = \frac{1}{2} I_o \left[\frac{a}{r} \left\{ \text{erf}\left(\sqrt{\frac{\omega^2 r^2}{4c^2} - \tau^2 V_o} - \frac{\omega a}{2c}\right) + \text{erf}\left(\sqrt{\frac{\omega^2 r^2}{4c^2} - \tau^2 V_o} + \frac{\omega a}{2c}\right)\right\}\right.$$

$$\left. + \exp(-V_o) \left\{ \text{erfc}\left(\sqrt{\frac{\omega^2 a^2}{4c^2} - (1+\tau^2)V_o} - \frac{\omega r}{2c}\right) + \text{erfc}\left(\sqrt{\frac{\omega^2 a^2}{4c^2} - (1+\tau^2)V_o} + \frac{\omega r}{2c}\right)\right\}\right] \tag{C-2}$$

(b) $|V| > \dfrac{qB^2(a^2-r^2)}{8m}$

$$I_{r2} = I_o\, e^{-V_o} \tag{C-3}$$

A detailed derivation of the above expressions is contained in a forthcoming University of Michigan Scientific Report.

According to Eqs. (C-2) and (C-3) the retarded electron current depends on the sheath radius a only in the voltage range $|V| < qB^2(a^2-r^2)/8m$. For larger values of $|V|$ the retarded current is independent of a. In the absence of a magnetic field,[16,17] the retarded electron current is independent of a for all values of V.

In order to obtain a numerical value for the voltage $qB^2(a^2-r^2)/8m$, which separates the regions of validity of Eqs. (C-2) and (C-3), it must be remembered that the ratio of sheath radius to collector radius a/r is not an independent variable but is a function of V. This function involves obviously the solution of the sheath problem. In the absence of such a solution for the cylindrical case the relation derived for a planar probe has been used.* This calculation has been included only with great reservations. However, it is believed that it will yield at least an approximate criterion for the range of V for which Eq. (C-2) is applicable. If d is the sheath thickness, then the expression for the planar case is[18]

$$d = a - r = \sqrt{\frac{\epsilon_0 |V|}{qN}} \tag{C-4}$$

where ϵ_0 is the permittivity of free space and the other symbols have been defined earlier. Using typical ionospheric parameters and a probe radius of the order of 0.01 cm, the voltage range in which Eq. (C-2) is applicable turns out to be $|V| < 10^{-6}$ volt, while the usual range of operation of such a probe is between 0 and -3 volt. The error in using Eq. (C-3) for the entire voltage range is therefore negligible.

Equation (C-4) has also been applied to the expression for the accelerating potential, Eq. (C-1). Figure 4 shows the volt-ampere characteristics for both accelerating ($V>0$) and retarding potential ($V<0$). For the latter case Eq. (C-3) has been used as explained above. It is immediately evident that the magnetic field exerts a strong influence on the collector current in the positive voltage region only. In particular strong magnetic fields cause a rapid decay of the accelerated current. These conclusions depend of course critically on the assumption of the validity of Eq. (C-4). It is possible that the values of a obtained from that relation are too high for the cylindrical case. In that case the probe current would not show the rapid decrease for large values of B presented in Fig. 4. We believe that a more realistic expression for the sheath thickness would result in considerably different current characteristic curves from those presented in Fig. 4. An investigation of the relation between probe potential and sheath thickness for the cylindrical case has been started recently. It is hoped that the results of this investigation can be presented in the annual report.

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*After completion of the calculations reported here a paper by H. A. Whale (J. Geophys. Res. 69, 447, 1964) has come to my attention which contains a derivation of the relation between sheath radius and collector potential for a cylindrical geometry using similar assumptions as have been used in this report.

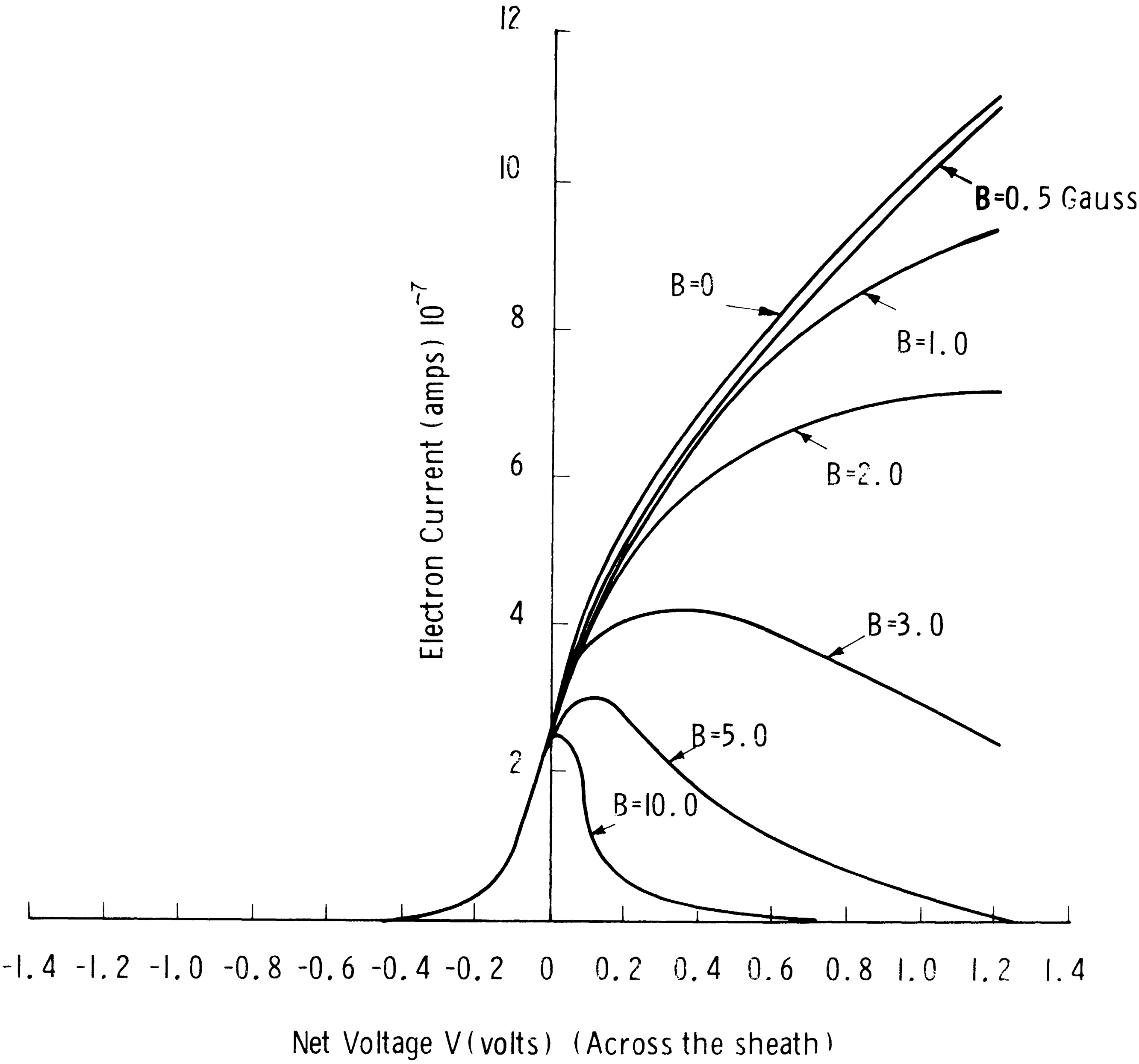

Fig. 4. Predicted electron current to a cylindrical probe vs. net voltage across the sheath. T = 1000°K, N = 10^{11} particles/m^3. B = 0, 0.5, 1.0, 2.0, 3.0, 5.0, and 10.0 (Gauss).

REFERENCES

1. E. G. Fontheim, "Establishment of Stability by Collective Interactions in a Plasma with Collisions," Office of Research Administration Scientific Report TS-1, University of Michigan (1964).

2. E. G. Fontheim, Bull. Am. Phys. Soc. 9 329 (1964).

3. F. Mariani, J. Atm. Sci. 20, 479 (1963). This paper presents a very thorough investigation of the correlation between maximum electron density and solar activity over a 20-year period and on that basis formulates a hypothesis on the penetration of corpuscular radiation into the ionosphere. Some experimental evidence of such corpuscular radiation seems to be implied from preliminary data obtained by the IMP-I satellite.

4. J. E. Drummond, et al., "Collision-Induced Instability in a Plasma with an Isotropic Velocity-Space Distribution," Boeing Scientific Laboratories, Report D1-82-0273.

5. F. Mariani, J. Geophys. Res. 69, 556 (1964).

6. W. Bernstein, J. Geophys. Res. 69, 1201 (1964).

7. R. Balescu, Phys. Fluids 3, 52 (1960).

8. R. L. Guernsey, Thesis, University of Michigan, 1960.

9. T. Y. Wu and R. L. Rosenberg, Can. J. Phys. 40, 463 (1962).

10. P. M. Platzman and S. J. Buchsbaum, Phys. Fluids 4, 1288 (1961). [There are two typographical errors in Eq. (6b) of Ref. 10. The factor $1/n_0$ is missing in the first term and the factor 1/2 in the second term. Both have been corrected in Eq. (A-13) of the present paper.]

11. H. Grad, Handbuch der Physik (S. Flugge, ed.), Vol. 12, 205, Springer 1958, Berlin.

12. W. E. Drummond and D. Pines, Nuclear Fusion - 1962 Supplement, (Conference Proceedings Salzburg, 4-9 September 1961), p. 1049.

13. A. Singh and J. E. Rowe (to be published).

14. H. Watanabe and H. E. Hinteregger, J. Geophys. Res. 67, 999 (1962).

15. A. Dalgarno et al., Planetary Space Sci. 11, 463 (1963).

REFERENCES (Concluded)

16. M. Kanal, "Theory of Current Collection of Moving Cylindrical Probes," Office of Research Administration Scientific Report GS-3, University of Michigan (1962).

17. M. Kanal, J. Appl. Phys. (to be published in May 1964 issue).

18. [illegible] B. Hanson and D. D. McKibbin, J. Geophys. Res. 66, 1667 (1961).