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FIRST PROGRESS REPORT

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MATERIALS LABORATORY

WRIGHT AIR DEVELOPMENT CENTER

ON

NOTCH SENSITIVITY OF HIGH-TEMPERATURE ALLOYS

by

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SUMMARY

The history has been reviewed of the research leading up to the current study. A first aim of the present contract is to evaluate a calculation method developed in earlier work for predicting notched-bar rupture life from test data on conventional unnotched specimens. A program is presented for adapting the method to a digital computer, using mathematical expressions derived from extensive experimental data on one lot of A-286 alloy tested at 1200°F.

Results of the calculations for this material are to be presented in a forthcoming report, along with a discussion of other phases of the current research. These latter include: (1) an experimental study of creep-rupture of A-286 alloy under variable tension-tension and/or tension-compression, and (2) microscopic study of fractured notched bars for possible new clues into the mechanism of fracture at elevated temperatures.

NOTCH SENSITIVITY OF HIGH-TEMPERATURE ALLOYS

The present studies under Contract AF 33(616)-5775 are an outgrowth of research under two past contracts with the Materials Laboratory, WADC. The first investigation (Contract AF 18(600)-62, started in 1952) sought to establish and analyze data on comparative rupture strengths in tension for notched and unnotched specimens of three representative heat-resistant alloys. S-816, Waspaloy and Inconel X-550 were all studied at 1350°F; the Waspaloy material was also evaluated at 1500°F. The general objective was to determine the reason for notch strengthening and notch weakening in metals under creep conditions. Special attention was directed toward ascertaining the role of creep-relaxation to notch-rupture behavior. Additional factors considered included elongation and reduction of area for the material in normal rupture tests on unnotched specimens, the notch geometry, and the influence of metallurgical variables.

Experimental results, supported by a review of the then-available technical literature, suggested that no one such factor characterizes the notch response of an alloy. Rather, notched-bar rupture behavior should be the composite reflection of the initial stress concentration, the redistribution of this stress by creep relaxation at the test temperature and the cumulative "consumption" of the available rupture life as the creep progresses. Presence of a notch was reasoned to add nothing inherently new into alloy properties, but merely to change the stress-strain history of metal fibers in the notched specimen. Under such a premise, notched-bar rupture life should be amenable to analysis and correlation in terms of normal smooth bar properties. Development of such an analysis was treated in the three-part report of Reference 1.

Part 1 demonstrated qualitative agreement between notch strengthening and rates of stress leveling in relaxation tests on unnotched specimens. Rudiments of an engineering statement of creep-rupture behavior under variable stress were also established through a series of experiments in which a conventional creep specimen was run under one constant stress for a portion of the rupture life and then changed to other constant stress levels for additional portions of the test. Rupture in these tests occurred approximately when the sum of the fractions $\left(\frac{\text{Actual time at a given stress}}{\text{Rupture life for this stress}}\right)$ reached unity. Furthermore, the general shape of the creep curve during any portion of these multiple-stress tests seemed to depend only on the momentary stress and stage of the test, independent of the stress levels of prior portions of the test.

Part 2 of Reference 1 proposed a tentative stepwise mathematical procedure to analyze notched-bar rupture behavior in terms of experimental properties of smooth specimens. This procedure assumed quantitative addibility of rupture-time fractions and considered the effective stress of the shear-stress invariant theory to be a suitable measure of the stress controlling creep rupture behavior under multiaxial stresses. Calculations completed for one condition of notch strengthening and one of notch weakening indicated reasonable agreement with experimental findings. In these calculations, momentary creep properties were read off plots of stress versus creep rate with the cumulative percent of expired life shown as a parameter.

The final report under the initial contract covered extensions of the investigation to new materials (a Cr-Si-Mo-V steel and an age-hardening aluminum alloy) and to flat specimens with notches at the edges. The flat notched specimens, with biaxial stressing compared with triaxial stressing of round notched bars, consistently exhibited rupture times slightly below those for round specimens with the same theoretical stress concentration factors. However, no radical new behavior was found. The elevated-temperature rupture characteristics of all notched specimens under steady tensile load were concluded to depend on three major factors:

1. The distribution and level of the initial stress pattern, determined by the notch configuration and tensile characteristics of the alloy.

2. The rate at which the variable creep rates at different locations in the cross section can relax the peak stress originally concentrated near the notch. (Under multiaxial stressing, the effective stress can easily become less than the nominal value for alloys with low creep resistance).

3. Rupture characteristics of the material for the stress-strain-time histories experienced by different fibers in the notched bar. (If too large a portion of the total life is used up at the initial high stresses, the remaining service should be short even for a low final stress level.)

Calculated versus experimental notch-bar rupture times were compared for thirteen conditions chosen at random from among those tested in the program. Agreement was satisfactory for materials tested under conditions where they are metallurgically stable. In some cases where agreement was less favorable, observed deviations could be explained with the aid of fragmentary data on changes in creep-rupture behavior of smooth specimens subjected to a history of variable stress designed to approximate conditions at different positions of a notched specimen. However, for none of the original alloys studied was sufficient stock available to permit complete evaluation of the material's smooth-bar behavior under all types of stress history to be expected in the notched rupture specimens studied.

Research under the second contract (AF 33(616)-3380) was designed primarily to overcome this deficiency by permitting extensive work on a single alloy. For this particular study, A-286 alloy was chosen in the belief that both notch strengthening and notch weakening should be obtainable with the same lot of material by merely altering the solution temperature used in the heat treatment. Stock produced by the vacuum consumable electrode process was selected since it represented the most advanced form of the alloy available and should typify materials which would be produced for future applications.

Survey tests at 1200°F included specimens solution treated at temperatures covering the range from 1650° to 2300°F. For stresses of 60,000-70,000 psi, smooth and notched specimens alike exhibited a steady increase in rupture times for increasing solution temperatures up to about 2000-2200°F and then the strength fell for both types of specimens. Quite unexpected was the finding that for a moderate notch acuity (K_t=1.9) the same order of notch strengthening was obtained for all solution temperatures, despite a drop in smooth-bar rupture ductility from 8-10% for the 1650°F treatment to 1-2% for solution at 2150-2200°F. (See part 1 of Ref. 2).

Major effort was directed toward more intensive investigation for solution temperatures of 1800° and 2200°F, which give roughly comparable smooth-specimens rupture lives but widely-different rupture ductilities. The higher solution temperature resulted in a coarse grain size but was selected to study notch behavior for conditions where elongation at fracture would be 1% or less in long-time tests on unnotched specimens.

Original plans called for extensive tests with a single lot (Heat 21,030) of A-286 to define the basic properties and conditions for the different degrees of notch sensitivity expected to be introduced by altering the heat treatment. When initial tests all showed notch strengthening for typical stress levels and specimen geometries, the study was broadened to include material from other lots of A-286 alloy for which limited tests by others suggested these lots might be more prone to notch sensitivity than was Heat 21,030. The final report on this work (Part 2 of Ref. 2) has still not been reproduced for distribution at the time of the present progress report.

A prime result in the study of Reference 2 is that A-286 alloy from a single lot and with a single heat treatment (Heat 21,030 material with 1800°F solution plus aging at 1325°F) was found to exhibit transition from pronounced notch strengthening to notch weakening by lowering the nominal stress. Moreover, this transition varied with notch geometry.

The number of tests on unnotched specimens in the same condition was sufficient and the scatter of results small enough to establish a good set of experimental creep properties under constant and variable stress. Questions remain concerning rupture behavior under a general state of stress or even for variable simple tension, but normal creep and rupture characteristics could be stated in mathematical forms which lend themselves to machine computation.

Such calculations along the lines suggested in earlier reports are the first objective of the current research (Contract AF 33(616)-5775) and constitute the principle subject of the present report. Additional phases of study called for under the present contract include:

1. Experimental investigation of the pertinent factors which control creep rupture under variable biaxial tension-tension and/or tension-compression.

2. Metallographic examination of fractured notched specimens in search of possible new clues into the mechanisms of fracture at creep temperatures. A recent Russian publication (Ref. 3) and other literature are to be reviewed and re-evaluated in terms of findings in the present research and its predecessors.

Discussion of these latter subjects is being deferred until the second part of reference 2 and an expected English translation of reference 3 become available for study. A forthcoming progress report will attempt to correlate these phases with completed results of the computer calculations, which are not still too incomplete for proper evaluation.

A PROGRAM FOR MACHINE COMPUTATION OF NOTCHED-BAR RUPTURE TIMES FROM SMOOTH-BAR DATA

Gradual leveling of initial stress gradients by creep relaxation is to be expected when a notched specimen is held under constant load at elevated temperature. In the actual specimen, the stress levels will vary smoothly from point to point without discontinuities, but to facilitate calculations the cross section at the notch will be imagined divided into a sufficient number of concentric rings such that conditions at the centroid of any given ring are quite representative of that entire ring. Further, the actual continuous change in stress pattern will be replaced by an equivalent series of time intervals over each of which the creep rate and stress in a given ring may be considered to be nearly constant.

The fraction of rupture life expended during each interval is to be calculated for each ring, assuming the life fraction rule to apply to the effective stress acting. When the cumulative fraction of life expired for any ring reaches unity, rupture at that location should occur and failure on the entire specimen cross section is imminent.

Immediately on loading, any fiber in the specimen has a unique effective creep rate determined by the initial effective stress. The component of the plastic creep strain in any direction may result in elongation (creep) of the body, but it could also replace initial elastic strain, with resultant drop in the stress level of the fiber (relaxation). How the total plastic deformation splits between creep and relaxation depends on the extent of stress gradients in the structure.

In a conventional creep test, where all fibers are subjected to the same simple-tension stress until necking occurs, the body can creep as a unit with no reaction of one fiber on another. Such is not the case, however, where the stresses vary continuously from one fiber to the next.

Consider a flat bar with three parallel bands having axial stresses $S_3 > S_2 > S_1$ at their respective center lines. Corresponding axial creep rates, if each band were separate from its neighbors, would be $C_3 > C_2 > C_1$. For continuity to be maintained between filaments, the same total deformation must exist on the two sides of a common interface. This does not say that the deformations at the two opposite edges of a particular band will be the same. The creep rate at different points across any such band will deviate slightly from the rate at its center, but this latter value should be quite representative if the band chosen is not too wide.

When band 2 has a total creep in excess of band 1, the difference in plastic strain must be made up by elastic strains in the two bands, so long as fibers of 1 do not become stressed above their proportional limit. This elastic interaction gives a stress reduction (or relaxation) in band 2 and a stress rise in 1. The absolute values of these two elastic changes will be distributed inversely as the areas of the two bands concerned.

In relaxation, plastic strains replace equal but opposite elastic strains initially present, so that relaxing fibers exhibit behavior characteristic of both plastic and elastic deformation. In the elastic state the relationship between a principal stress (S_i) and the corresponding principal strain (e_i) may be expressed

$$S_i = \lambda e_{ii} + 2G\, e_i, \qquad (1)$$

where λ is a proportionality constant, G is the shear modulus, and e_{ii} is the volume expansion, equal to the sum of the three principal strains.

By the assumption of plastic incompressibility inherent to the shear-stress invarient criterion of yielding and of creep adopted in the analysis, e_{ii} can be set equal to zero. Therefore, changes in principal stresses are equal to 2G times the corresponding principal strains. This same relationship holds for elastic changes involving effective stresses and effective strain. The factor 2G is related to the elastic modulus(E) and Poisson's ratio (ν) by:

$$2G = E / (1 + \nu) \qquad (2)$$

Available data for A-286 alloy at 1200°F indicate a value of about 0.32 for ν and 21.1×10^6 psi/in/in. for E.

Simultaneously with the interaction between bands 1 and 2, the elastic stress in band 3 is relaxing and that in 2 increasing due to a like interaction at the 2-3 interface. The net stress change for band 2 is the difference between the gain from 3 and the loss to 1.

Such a procedure can be applied in turn to strain rates and stresses for each pair of concentric rings in the plane of a notch. Stress exchanges at each interface are first found for the same short-time interval. When all changes are known, the new stress level and cumulative life expenditure in each ring are calculated and the process repeated.

Scheme Used to Divide the Cross Section for Calculation Purposes

The initial stress distribution in a notched tension specimen can normally be approximated by applying Neuber's equations (Ref. 4) to get a ficticious elastic stress pattern and then correcting for the yielding that must accompany any stresses indicated to be above the proportional limit for the alloy and test temperature. For most common notches, initial stress gradients are steep near the notch root and quite flat near the specimen axis, especially when little or no yielding occurs during load application. Under such a pattern, any imaginary ring near the notch root must be extremely narrow if the ring is to have an essentially-uniform stress level. At smaller radii a much wider ring could be considered without undue variation of conditions over its width. If the cross section is imagined to consist of concentric rings each with half the circular cross section of the adjacent ring toward the axis, a minimum of six such rings appears to give a satisfactory coverage of the total range of stresses across the section. In a specimen with 0.3-0.5 inch diameter at the notch, the smallest-area ring would have about 0.003-inch width. Increasing to nine rings total, the ring at the notch root would be only about 0.0005 inch in width.

The gradation in stress from ring to ring with the selected distribution of area is quite uniform during the critical initial period when creep is rapid and creep rates are very unequal at different radii. Moreover, the outermost ring approaches the surface in level of effective stress. Under the scheme employed, elastic stress changes at any interface are always distributed in the ratio 2/1 for the two rings involved, the larger change occurring in the smaller ring. For flat notched specimens, bands with the same 2/1 area ratios were employed. However, each "band" except the one at the center now consisted of two halves, one toward each notch root.

Proposed Computer Program

Figure 1 shows a block diagram for the proposed computer program. Indexing operations are shown in diamond shapes, branch selections in triangles, and other computation steps in rectangular blocks. Both read (input) and type (output) statements are indicated by a block shaped like a data card; i.e., a rectangle with one corner cut off. When repetitive or cyclic operations are called for, return to an earlier location in the program is directed by the use of Greek letters shown in a circle. The same convention is used to show where the program precedes from one line of the diagram to another.

The actual program used at the University of Michigan was designed for the IBM Type 650 computer, but the block diagram applies to any digital computer. By convention, integers are designated by the capital letter I plus an identifying number. Basic variable terms are indicated by Y plus an identification while other numbers (either input constants or computed values) are represented by C plus an identifying number.

Use of Indices: In the computation method developed, the same set of mathematical operation must be performed successively for each of the rings into which the notch cross section is imagined to be divided. Considerable simplification and shortening of the flow diagram was achieved by the use of a simple system of index groups:

I0 = 1, 2, ...9
I1 = I0 + 10 = 11, 12,...19
I2 = I0 + 20 = 21, 22,...29
I3 = I0 + 30 = etc.

These index groups identify the particular value of a Y variable under consideration. If the current value of I0 happens to be 3, then I2 = 23; the value of YI0 for the present value of I0 is listed under Y3, and the value of YI2 under Y23.

Index groups assigned to seven major variables are presented in Table 1. Input mathematical constants, material constants, limits used to control the course of the program, and computed constants (both program outputs and quantities computed during intermediate steps) have all been assigned storage space as C's according to the listing included in Table 1.

Details of Program Flow Sheet

At the start of the computation cycle, the number of rings (I8) and the initial stress pattern (YI 0) are entered into the computer, along with values of necessary constants (C1 through C37). In order to allow for situations and materials where plastic prestrain on load application reduces subsequent rupture life at lower stress levels, provision has been made to introduce a correction factor (YI2) for each ring. The use of such a single factor for each degree of plastic prestrain is in accordance with experimental findings established by tests at 1350°F on several heats of Waspaloy material (See Part 2 of Ref. 2). Convenient identification of the alloy and its heat treatment, as well as the notch configuration and test conditions, is achieved by including this information with each set of input data and then reading it right out again before the mathematical operations commence.

Initial computations center around selection of the time increments (C38) to be employed in the successive calculation cycles. The smaller the size of the individual steps, the more nearly the calculations approach the actual stepless relaxation process in the specimen. But the total number of computation cycles cannot be allowed to increase without limit or else the operating time per set of calculations becomes excessive for a computer with magnetic-drum storage.

The allowable size of any time increment is limited first by the need for the computed stress correction between a pair of adjacent rings to be small relative to both the existing stress gradient between the rings and the nominal stress level. Otherwise excessive overcorrections could give false indication of negative stresses, and computed conditions for any given ring would vacillate severely instead of following the continuous gradual changes in the actual test piece.

A second limitation to the time increment employed rests on the requirement that the fraction of rupture life consumed in a single step must not be so large as to negate the premise that property changes during each interval are small, so that initial properties may be assumed to hold approximately for the entire interval.

On the flow sheet the initial overall stress gradient (YI 8-Y1) between extreme rings is first compared with the nominal stress (C26) and the smaller of these stored as C28. Starting with the lowest-stress rings (I0=1), the rupture life (YI1) and creep rate (YI 3) are found in turn for the existing conditions in each of the rings. As set up, these calculations presume that normal creep and rupture properties can be represented respectively, by two intersecting straight line segments on a plot of log stress versus log rupture life and a plot of log stress versus log minimum creep rate. Moreover, the ratio of actual creep rate to minimum creep rate has been expressed as the sum of five terms of the form Ae^{BF} where A and B are constants and F is the cumulative fraction of rupture life expired to date. This type of relationship was found to be satisfactory for the entire stress range needed for A-286 Heat 21030 at 1200°F test temperature. If data for other materials so indicate, two or more such equations could be used for different stress ranges.

Closer study of the steps involved in finding YI1 and YI3 should clarify the cyclic iteration process employed repeatedly in this program. Entering the first line (at γ), the indices I1, I2, I3 and I5 are first set to their corresponding values, after which the logarithm of the stress is computed. Proceeding to the second line, a determination is made as to which branch of the rupture time curves is to be used; i. e., whether the existing stress level (YI 0) is greater or less than the stress at the intersection of the two segments on the stress-rupture plot. The normal rupture life (C43) corresponding to the present stress is then corrected for the prestrain factor, YI2.

Two separate operations have been used in the determination C45 (the ratio of actual to minimum creep rate) only because the length of a single statement that will be accepted by the computer is limited. The test to select the proper branch of the stress-creep rate curve corresponds to that just discussed for stress-rupture life. Once the actual creep rate (YI3) has been found as the product of the minimum creep rate (C47) and the factor C45, the index I0 is increased by one. The new value of I0 is then compared with the number of rings, I8. If I0 is still less than or equal to I8, the above calculation steps are repeated until values of YI1 and YI3 have been obtained for all rings. The next indexing operation raises the value of I0 above I8, and the program then continues along the alternate path to the end of the third line of the diagram.

Starting anew by setting indices I3 and I4 to correspond with ring number 1, the differential in creep rate between rings 1 and 2 is computed and the absolute magnitude of this difference is compared against the number stored in location C49. Since no entry for C49 was included with the input data, the value of C48 = /Y14/ - C49 is positive, wherefore C49 is set equal to /Y14/ as the next operation. Indexing I3 to its next value, the question asked at the end of the third line is answered "no" and the program doubles back to (δ) to find the next creep-rate difference; i.e., between rings 2 and 3.

Both the new /YI4/ and C49 now have finite values, so that their difference (C48) can be either positive or negative. If positive, the new (larger) creep-rate difference replaces the old value of C49, but if the C48 is negative, the original value of C49 is retained. Finally, after all of the (I8-1) creep-rate differences between adjacent ring pairs have been compared, and location C49 contains the largest value, I3 reaches a value of (I8 + 30) and the program moves on to selection of tentative time increments for the successive computation cycles.

Since the procedure for picking the first time increment differs from that in later cycles, a simple switching operation has been introduced immediately after (ε). Location C31 was set to zero by the input data, wherefore the first time the fourth line is entered, a trial value of C38 is computed giving the time which would be needed at a creep rate of Y14 to permit relaxation equal to the fraction C29 of stress C28. But before this time interval is accepted, a check is made as to what fraction of rupture life this time represents at the existing stress near the notch root; i.e., at the stress of ring I8. If the computed life fraction (C50) is too large, it is replaced by the allowable limit(C30)

At this point, the life-fraction expenditure for ring I8 during the first cycle has been fixed. If the cumulative fraction of rupture life consumption for this ring is made to equal unity in C36 steps, the required number of computation cycles will be either C36 or slightly fewer, depending upon whether fracture is approached first in ring I8 or one of its neighbors. One simple way to achieve a desired sum in a stated number of terms with the first one fixed is to use an arithmetic progression. If a is the first term, d the common difference, n the number of terms, f the final term and s the sum of n terms, then

$$s = \frac{n}{2} \ (a + f), \text{ and } f = a + (n - 1)d.$$

Rearranging, $f = \left(\frac{2s}{n}\right) - a$ and $d = \left(\frac{f-a}{n-1}\right)$, which forms have been used on the flow sheet.

This particular portion of the flow diagram has served its purpose once a and d (C50 and C52) are known, wherefore "switch" C31 is set equal to unity and the alternate route from (E) will be followed in later cycles until a new set of data is introduced, resetting C31 to zero. In each successive cycle the rupture life fraction to be expended in ring I8 is to be made equal to the fraction used in the preceding cycle, plus the common difference C52. Under an unfortunate combination of input values this procedure might produce unduly large time periods in later cycles.

In order to prevent C50 from exceeding a reasonable maximum value, a control limit (C32) has been introduced. The life fraction (C50) is converted back to a time interval (C38), by multiplying it by the rupture life corresponding to the existing stress in ring I8. In turn, this time interval multiplied by 2G and by the largest creep-rate difference (C49) defines the largest stress correction that would occur between any pair of adjacent rings. A final control limit requires that this stress correction be no more than the fraction C33 of stress C28 found earlier. Once the time interval is finally accepted, its value is entered in C38 and also in storage space C60 for later reference.

Cumulative rupture-life consumption and prevailing stress levels at the end of the current time increment are computed during the remainder of the program cycle. Indices are set for ring number 1 and the cumulative rupture life fraction (YI 5) evaluated as the sum of the previous value and the fraction:

$$\frac{\text{duration of current time interval}}{\text{rupture life at current stress}}$$

During early cycles YI5 is much smaller than unity, so that the calculations immediately index up one number and repeatedly return to (β) until all I8 values of YI5 are known. At this stage, switch C37 is still zero, so that the computer determines the cumulative time (C39) and types it out along with current values of C38, YI0 and YI5 after each cycle. The product C55 = (2G) (Δ time) is now found, and the indices are set for ring I8. The total stress correction, Y(I6-1), between rings I8 and I8-1 is computed as C55 times the corresponding differential creep rate Y(I4-1). Under successive reductions of the index in unit steps, the same calculations are performed for all (I8-1) pairs of adjacent rings. Each time, the starting stress at the center of a ring is increased by 1/3 of the stress leveling between it and the next higher-numbered ring and reduced by 2/3 of the stress leveling between it and the next lower-numbered ring. When all stress levels (YI0) have been corrected, the index is set back to ring 1 and the program returned to (γ) for the next computation cycle.

Eventually in some computation cycle a value of cumulative rupture-life expenditure (YI5) will be found to be greater than unity. This event means that the last computation cycle is in progress and that the full value of the tentative time increment C38 would probably be too large. All of the remaining operations have been designed to choose the critical ring where the cumulative rupture life fraction will first reach unity and to correct C38 to make this fraction reach exactly 1.0 in the final calculation cycle.

The possibility exists that more than one ring might have reached a value of YI5>1 in the same calculation cycle. When the first such ring is encountered, both it and all subsequent ones are examined to find the largest degree of "overshoot". The term (YI5-1) represents the amount of overshoot (or else the amount by which the ring still fails to reach the end of its life), expressed as a fraction of life. Multiplying by the rupture life corresponding to the current stress, the extent of overshoot (or undershoot) is expressed as a time. Subtracting this amount of excess (or deficiency) of time from C38 gives the proper length of time interval (C61) to bring the particular ring's YI5 to 1.0. The value of C61 is compared with C60, which started equal to C38 when this portion of the flow diagram was entered. The value of C60 is changed each time a lower value of C61 is found until, by the time all rings have been examined, it represents the minimum allowable time interval for the final computation cycle.

"Switch" C37 was set to unity when the first YI5>1 was found, so that the lower branch of the last line of the flow diagram is now followed. The total cumulative time (C39) is augmented by C60 to get the final answer and each of the rupture life fractions (YI5) just found on a previous line are reduced to eliminate the portion of life represented by the excess time (C38-C60) included in their calculation. After corrections have been applied for all 18 rings, the corrected total time duration is typed as an output, along with the values of C60 and of all YI5's. The computer then calls for a new set of input data and the start of calculations for the new problem.

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TABLE 1 - STORAGE ASSIGNMENTS FOR PROGRAM VARIABLES AND CONSTANTS

Storage Location	Corresponding Variable or Constant
I0 - I7	Numerical indices used to identify Y's with a particular ring.
I8	The number of concentric rings into which the notched cross section is imagined to be divided.
YI0 = Y1, Y2,... Y(I8+0)	Existing stress in each ring.
YI1 = Y11, Y12,... Y(I8 + 10)	Rupture life at the existing stress.
YI2 = Y21, Y22,... Y(I8 + 20)	Factor to correct rupture life for the initial plastic loading strain in each ring.
YI3 = Y31, Y32,... Y(I8 + 30)	Existing creep rate in each ring.
YI4 = Y41, Y42,... Y(I8 + 39)	Difference in creep rate between adjacent ring pairs; i.e., between rings (I0 + 1) and I0.
YI5 = Y51, Y52,... Y(I8 + 50)	Cumulative fraction of rupture life expired for each ring.
YI6 = Y61, Y62,... Y(I8 + 59)	Total stress correction to be made between rings (I0 + 1) and I0 in a given computation step.
YI7 = Y71, Y72,... Y(I8 + 70)	(Spare index).
	Input Mathematical Constants
C1	2.7182818 (base of natural logarithms).
C2	(Spare).
	Input Material Constants
C3	2G = Twice the shear modulus.
C4 - C8	Empirical constants and range boundary in equation for normal rupture life versus stress: $\text{log rupture life} = \frac{\text{C4-log stress}}{\text{C6}}$ for stress $\leq$ C8; $\text{log rupture life} = \frac{\text{C5-log stress}}{\text{C7}}$ for stress > C8.
C9	(Spare).
C10 - C19	Empirical constants in equation to convert minimum creep rate to actual rate in terms of cumulative fraction (YI5) of rupture life expired: $\frac{\text{Actual Creep Rate}}{\text{Minimum Creep Rate}} = (C10)\ e^{(C11 \times YI5)} + (C12)\ e^{(C13 \times YI5)} + (C14)\ e^{(C15 \times YI5)} + (C16)\ e^{(C17 \times YI5)} + (C18)\ e^{(C19 \times YI5)}$
C20 - C24	Empirical constants and range in equation for minimum creep rate versus stress: $\text{log min. creep rate} = \frac{\text{(stress) - C23}}{\text{C22}}$ for stress $\leq$ C24; $\text{log min. creep rate} = \frac{\text{(stress) - C21}}{\text{C20}}$ for stress > C24.
C25	(Spare).
	Input Limits Used for Program Control
C26	Nominal stress at notch cross section.
C27	Initial stress gradient between outermost and innermost rings.
C28	Smaller of C26 and C27.
C29	Fraction of C28 used in computing time interval for initial calculation cycle: (Initial time interval) = (C29) (C28)/(2G) (Max. differential creep rate between adjacent rings).
C30	Limit placed on the maximum life-fraction expenditure to be permitted in first calculation cycle.
C31	A "switch" used to bypass an initial portion of the program after the first calculation cycle.
C32	Limit placed on the maximum rupture life fraction to be expended in ring number I8 during any cycle after the first.
C33	Limit placed on the maximum fraction of stress C28 which may be leveled between adjacent rings in a single step after the first cycle.
C34 - C35	(Spare).
C36	Approximate number of computation cycles to be used in completing a set of calculations.
C37	A "switch" to set into action an interpolation process used in the final cycle of the calculations.
	Computed Program Outputs
C38	The time increment used for any computation cycle.
C39	Cumulative time for completed computation cycles.
	Quantities Computed for Intermediate Steps
C40	Difference between the initial overall stress gradient and the nominal stress.
C41	Common logarithm of stress level.
C42	Common logarithm of normal rupture life.
C43	Normal rupture life at existing stress (before correction for effects of any plastic prestrain).
C44	Intermediate term in C45.
C45	Ratio: $\frac{\text{Actual Creep Rate}}{\text{Minimum Creep Rate}}$ at existing stress level and cumulative fraction of rupture-life expenditure.
C46	logarithm of C47.
C47	Minimum creep rate at existing stress level.
C48	Intermediate difference used in finding C49.
C49	Largest absolute value of creep-rate differences between pairs of adjacent rings.
C50	Fraction of rupture life of ring I8 expended during the time interval of the first calculation cycle.
C51	Rupture-life fraction to be expended in ring I8 during the final cycle for an arithmetic progression of rupture time fractions in ring I8 to reach unity in C36 steps.
C52	The common difference between terms in the arithmetic progression of C51.
C53	Magnitude of stress that could be leveled between the pair of rings with largest creep-rate difference during a single calculation step.
C54	Difference between C53 and an allowable limit placed on the amount of stress leveling per step.
C55	Product: (Time increment) (2G)
C56 - C59	(Spare).
C60 - C61	Intermediate values used in determining time interval for final step.

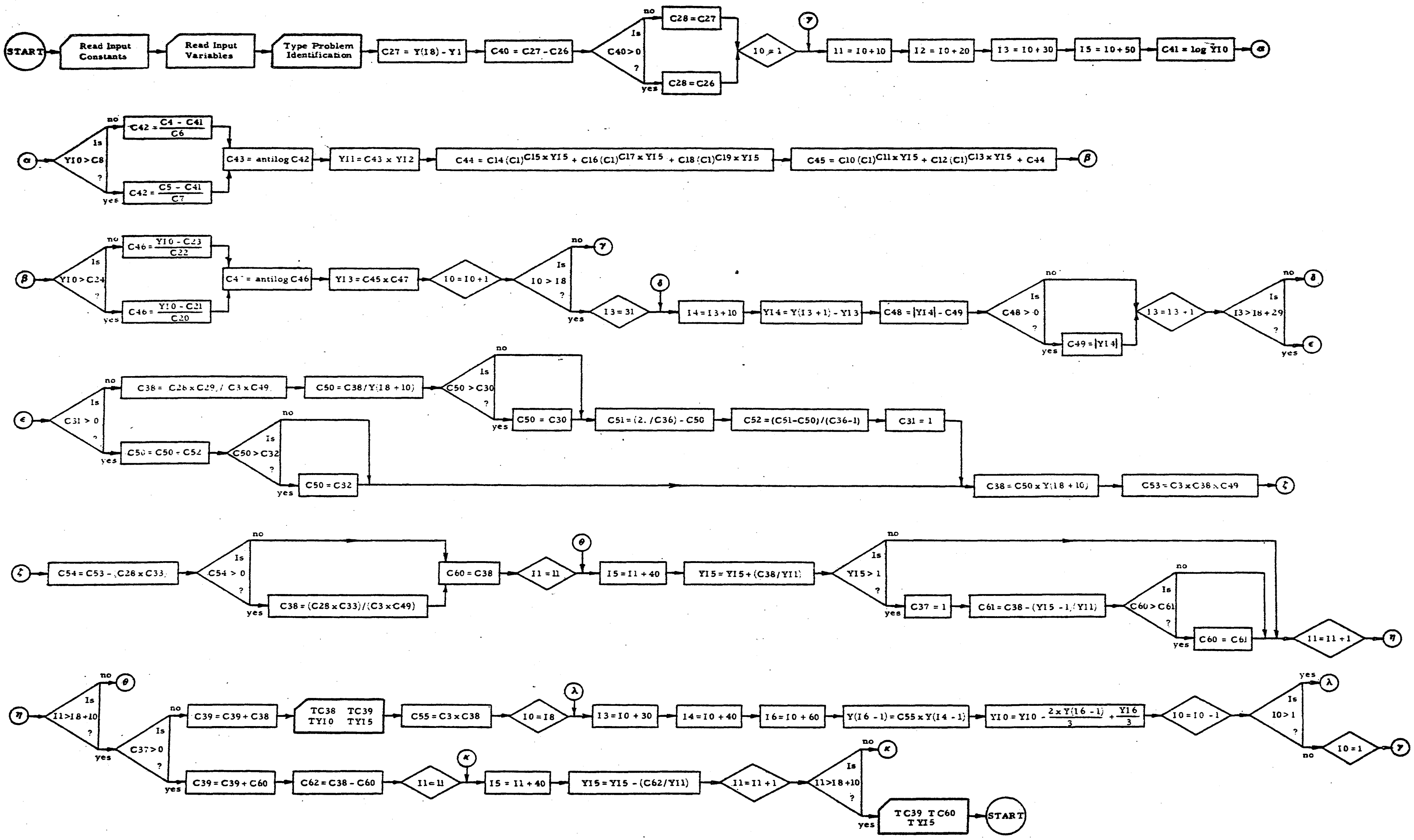

FIG. 1 - FLOW DIAGRAM FOR COMPUTER PROGRAM