THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

THE DESIGN OF POSITION AND VELOCITY SERVOS FOR MULTIPLYING AND FUNCTION GENERATION

Edward O. Gilbert

March, 1959

IP-362

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I. INTRODUCTION

The use of servo positioned potentiometers for multiplication and function generation in analog computation and simulation is wide-spread. Typically, the potentiometers are driven by a two-phase servo-motor through a gear train. The motor in turn is driven by an ac amplifier that receives a compensated error signal in modulated form from a synchronous chopper or other modulated waveform source. The performance of such a system is specified by such factors as bandwidth, static resolution, and velocity and acceleration limits.

The design problem is to select suitable components, such as motor, gear train, and potentiometers, so that performance specifications may be realized or optimized. The particular difficulty of design is that these performance requirements are interrelated, and it is not generally possible to specify arbitrarily all the performance characteristics. It is the purpose of this paper to express the interrelationships so that compatible requirements may be understood and a successful design accomplished.

II. COMPONENTS

There are a number of component characteristics that are significant in design. Many of these characteristics are primarily determined by factors such as accuracy, reliability, availability, economy, and the state of the art. For example, a high resolution, high linearity requirement would specify multiple turn, wire-wound potentiometers. This in turn would set quite definite values for the potentiometer inertia and frictional torque. In other cases, less stringent requirements would allow a more flexible selection of components, and hence, a wider range of component characteristics. Individual components will now be discussed.

Potentiometers

Important characteristics for servo design are:

T_p = total potentiometer frictional torque.

I_p = total potentiometer inertia including potentiometer coupling.

η = resolution of the reference potentiometer.

= number of potentiometer wires in full scale.

n_p = number of shaft revolutions in full scale.

For film potentiometers, η is essentially infinite. Desirable characteristics are low T_p and high η , which have rather definite limits set by potentiometer size, resistance, accuracy and n_p. As will be seen, I_p is not usually significant in design.

Gear Train

Important gear train characteristics are:

n_g = gear reduction ratio between motor shaft and reference potentiometer.

I_g = gear inertia referred to the motor shaft.

T_g = maximum gear train frictional torque referred to the motor shaft.

ζ = gear backlash at the poteniometer shaft expressed as a fraction of full scale potentiometer travel.

The gear reduction ratio is an important parameter fixed by design. The inertia I_g is a function of n_g but may be considered independent of n_g over a limited range which is sufficient for design considerations. The gear train precision is specified by T_g and ζ. Gear friction is a rather erratic function of gear position, and T_g should be small relative to potentiometer and tachometer friction referred to the motor shaft in order to insure jerk-free operation. The backlash ζ should be considerably less than δ, the static resolution error expressed as a fraction of full scale. δ is discussed more fully at a later point. When some of the potentiometers are driven through gearing from other potentiometers, the reference potentiometer should be the one most directly geared to the motor, minimizing backlash in the closed-loop. The gear backlash between all potentiometers should be considerably less than $1/\eta$ so that wire-to-wire oscillation of the reference potentiometer is not possible without the frictional torque load of the other potentiometers.

Tachometer

A tachometer may be used for damping or as the velocity reference in a velocity servo. The tachometer is coupled directly to the motor shaft to eliminate gear backlash in the velocity feedback loop. Important characteristics are:

T_t = tachometer frictional torque.

I_t = tachometer inertia including coupling.

K_t = voltage constant in v./rpm.

Amplifier

Important characteristics are:

K = linear region static gain in v./full scale error.

e_s = saturation voltage of amplifier.

$Y_a(s)$ = equivalent transfer function of amplifier in linear range, where s is the Laplace transform variable.

The required amplifier gain K will be determined by design. The amplifier should have sufficient power capability so that the saturation voltage e_s approximately equals the maximum motor voltage e_{max}.

The existence and determination of the transfer function $Y_a(s)$ is open to question. This is apparent from the ac amplifier gain $|\overline{Y}_a(j\omega)|$ shown in Fig. 1. Here ω_o is the carrier frequency and $\omega_o + \omega_m$ and $\omega_o - \omega_m$ are the side band frequencies corresponding to a sinusoidal modulation frequency of ω_m. For Y_a to exist, $\overline{Y}_a(\pm j\omega_o \pm j\omega) = Y(\pm j\omega)$. This is certainly not true for $\omega_m \geq 1/\tau_a$ due to the nonsymmetrical gain characteristic about the frequency ω_o. The exact effect of the nonsymmetric gain, including the demodulation characteristics of the motor, would be a worthy subject of analysis.

For the design problem considered here, the equivalent transfer function

$$Y_a(s) = \frac{K}{\tau_a s+1} \qquad (1)$$

has proven adequate. $1/\tau_a$ is taken as the lower break frequency of the ac amplifier. Obviously, $1/\tau_a < \omega_o$, and as $1/\tau_a$ approaches ω_o, the transfer function given by (1) becomes less valid, and additional time lags should be considered.

In some servos, a magnetic amplifier provides modulation and amplification. Typically, additional gain is provided by a dc amplifier. The above transfer function is still valid where τ_a is now the time

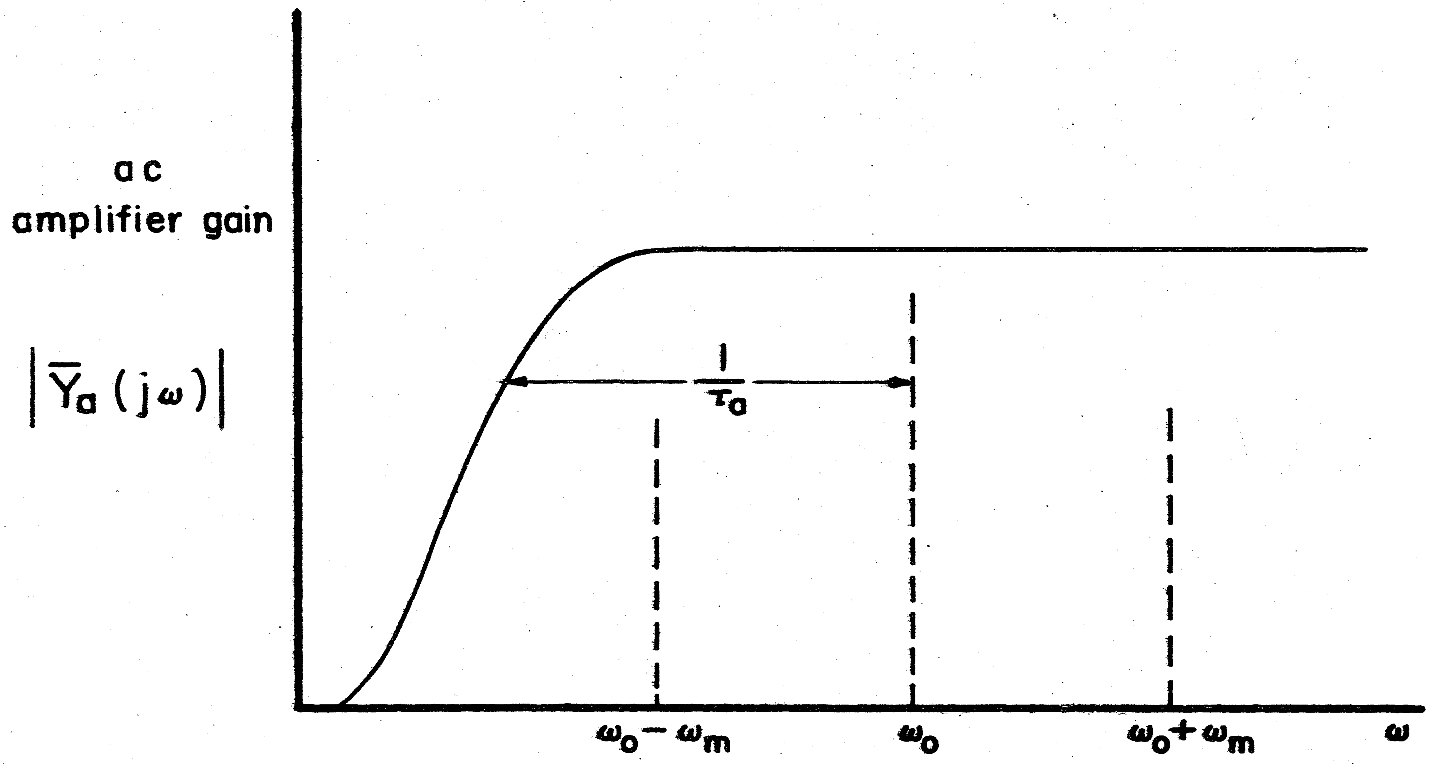

Figure 1. Typical ac Amplifier Gain Characteristic

constant of the control winding. Magnetic amplifiers commonly suffer two disadvantages: 1. the bandwidth $1/\tau_a$ is considerably less than ω_o, and 2. the dynamic characteristics are poor in the saturated state. The bandwidth may frequently be improved using current feedback in the control winding. The poor saturated amplifier dynamics are characterized by long recovery times from the saturated state, which means that the full linear bandwidth is not usable. Such poor dynamics are minimized by good amplifier design or by introducing saturation at a point before the magnetic amplifier.

Two-Phase Servomotor

The important servomotor characteristics are obtained from the speed-torque curves for the motor. A typical family of such curves is shown in Fig. 2 in dimensionless form, where T is the motor torque, $\dot{\theta}_m$ is the motor angular velocity in rad./sec., and e is the rms voltage applied to the variable phase. $\dot{\theta}_{m\ max}$ and T_{max} are the maximum velocity and torque when $e = e_{max}$, the maximum motor voltage.

The family defines a torque function $T = f(\dot{\theta}_m, e)$. Neglecting frictional torques

$$I \ddot{\theta}_m = f(\dot{\theta}_m, e) , \tag{2}$$

where $I = I_m + I_g + I_t + I_p/n_g^2$. I_m is the motor inertia. This nonlinear differential equation may be linearized when $|\dot{\theta}_m| \ll \dot{\theta}_{m\ max}$, which is frequently the case when the servo amplifier operates in the linear range, i.e., when $|e| < e_s \approx e_{max}$. Then (2) becomes

$$I \ddot{\theta}_m = - \gamma \frac{T_{max}}{\dot{\theta}_{m\ max}} \dot{\theta}_m + \frac{T_{max}}{e_{max}} e , \tag{3}$$

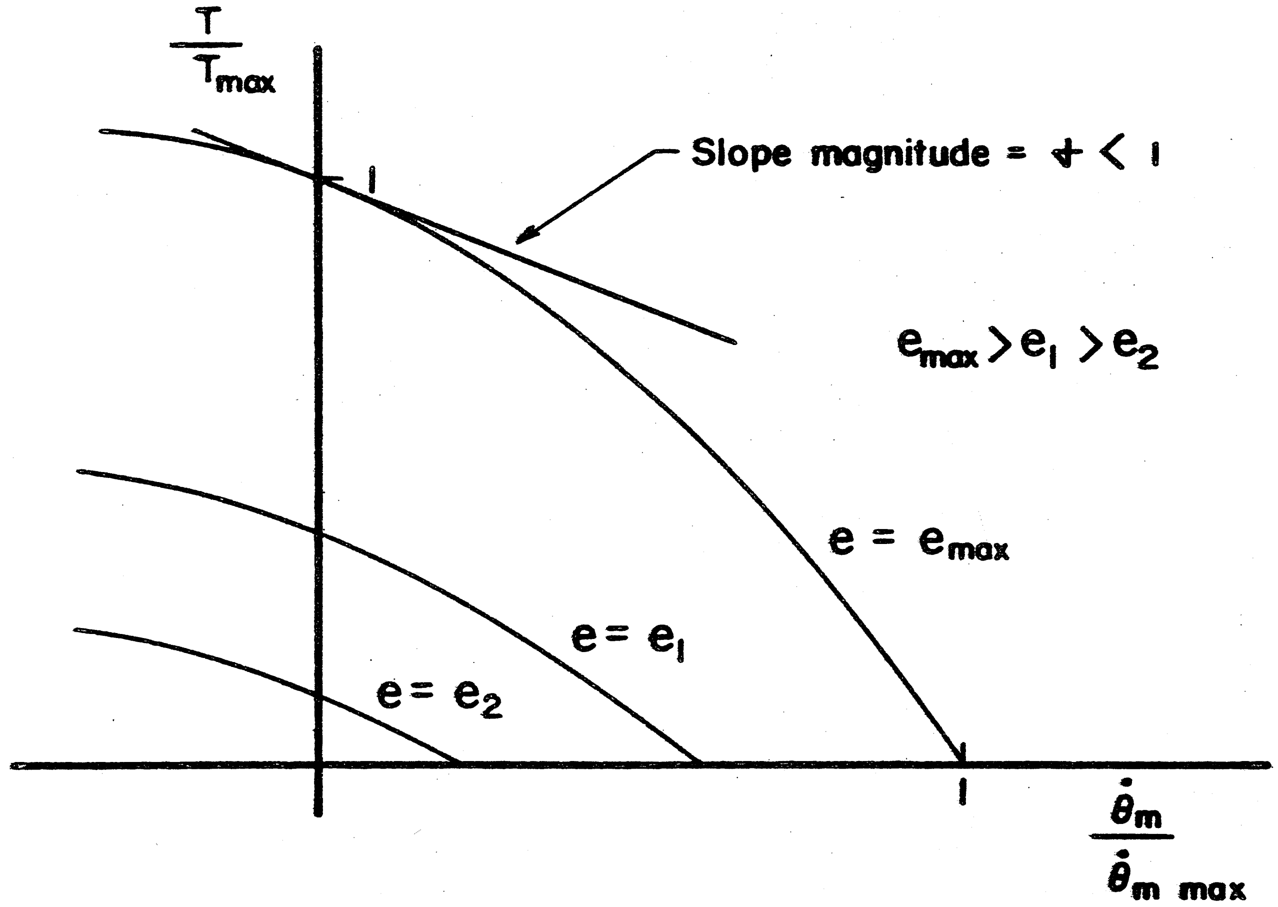

Figure 2. Two-Phase Servomotor Speed-Torque Curves

where γ , generally less than one in magnitude, is the slope parameter of the speed-torque curves defined in Fig. 2. Equation (3) defines the motor transfer function

$$Y_m(s) = \frac{K_m}{s(\tau_m s+1)} , \qquad (4)$$

where the constants K_m and τ_m are expressed in terms of the parameters of the speed-torque curve for $e = e_{max}$ by

$$K_m = \frac{1}{\gamma} \frac{\dot{\theta}_{m\ max}}{e_{max}} = \frac{2\pi N_{max}}{60\gamma\ e_{max}} \frac{rad.}{v.sec.} ,$$

$$\tau_m = \frac{1}{\gamma} \frac{\dot{\theta}_{m\ max}\ I}{T_{max}} = \frac{2\pi N_{max}\ I}{60\ \gamma\ T_{max}} \ sec. , \qquad (5)$$

$$\frac{K_m}{\tau_m} = K_\infty = \frac{T_{max}}{I\ e_{max}} \frac{rad.}{v.sec.^2} .$$

N_{max} is the maximum motor velocity in rpm, and K_∞ is the motor gain constant for frequencies above the break frequency $1/\tau_m$. Expression of the transfer function parameters in the above form is important since it allows design equations to be expressed in terms of readily available speed-torque curve parameters.

III. THE POSITION SERVO

Block Diagram

Fig. 3 shows the block diagram for the position servo. The dimensionless input x and output y are selected so that the full scale range on x and y is unity. The input compensation $Y_i(s)$ and the feedback compensation $Y_f(s)$ are defined to have unit magnitude at zero frequency, i.e., $Y_i(o) = Y_f(o) = 1$. For series compensation, $Y_i(s) = Y_f(s)$.

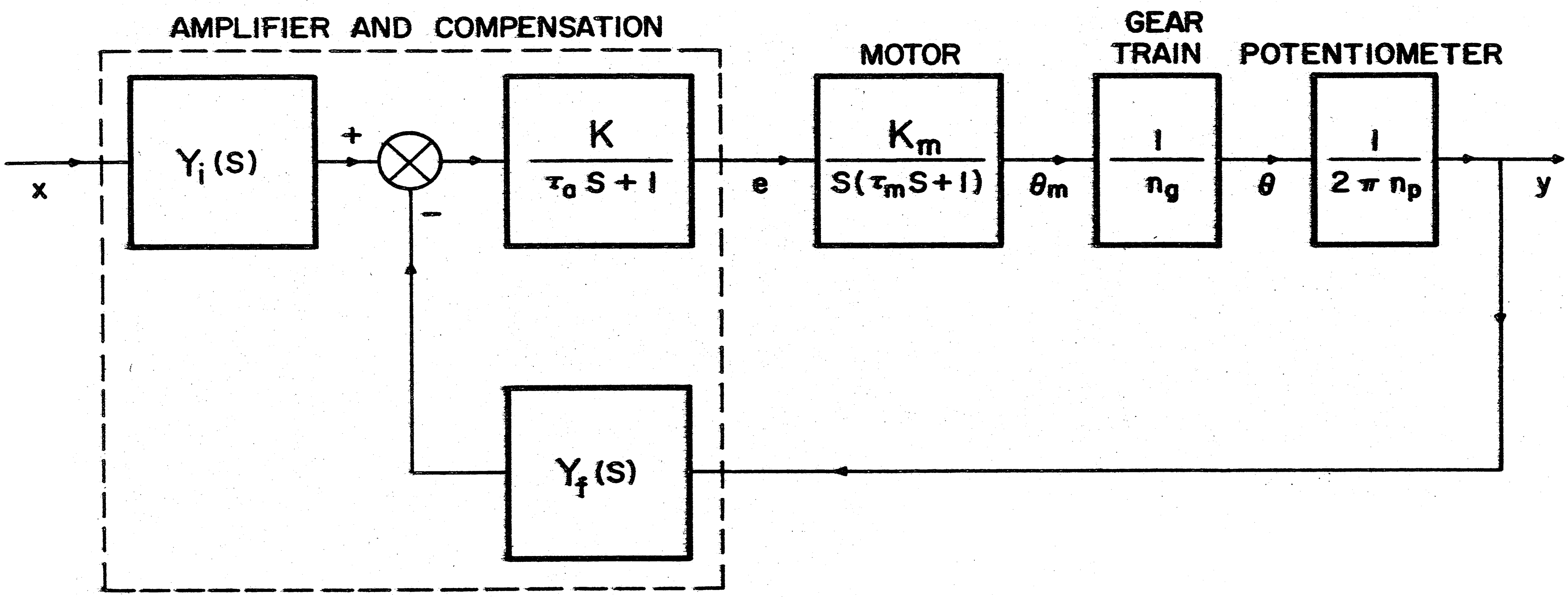

Figure 3. Position Servo Block Diagram

The nonlinear characteristics of the amplifier, motor, gear train, and potentiometer are not shown.

Nonlinear Performance Requirements

A number of the performance requirements are nonlinear in nature and are particularly important in the selection of a motor and gear train. In this section, these requirements will be defined in terms of the component parameters. Later, their full significance in design will be considered.

Two requirements are the velocity and acceleration limits $\dot{y}_{max}$ and $\ddot{y}_{max}$. From the block diagram and definition of motor parameters

$$\dot{y}_{max} = \frac{N_{max}}{60\ n_g\ n_p} \qquad (6)$$

and

$$\ddot{y}_{max} = \frac{T_{max}}{2\pi\ n_g\ n_p\ I} \qquad (7)$$

In (6) and (7), the frictional torque is assumed small compared with T_{max}.

Another parameter that may be important is y_o, the first overshoot magnitude that results from a large step input. Exact determination of y_o is difficult, but y_o may be estimated as follows. Assume that servo amplifier inputs not causing saturation are small compared to y_o, that the torque reverses to full negative value when $\epsilon = x - y = 0$, that the deceleration to zero velocity is constant at $\ddot{y}_{max}$, and that velocity when $\epsilon = 0$ is $\dot{y}_{max}$. Then $2\ \ddot{y}_{max}\ y_o = \dot{y}^2_{max}$. For an actual system, deceleration (proportional to torque in the negative velocity range in Fig. 2) may be somewhat greater than $\ddot{y}_{max}$, which occurs for $T = T_{max}$; also, for typical damping characteristics and linear range, torque reversal is earlier. Thus a somewhat smaller y_o than predicted results and

$$y_o < \frac{\dot{y}_{max}^2}{2\,\ddot{y}_{max}} = \pi \frac{(\frac{N}{60})^2\ I}{n_g\ n_p\ T_{max}} \quad . \tag{8}$$

Determination of the magnitude and number of successive overshoots for a large step input is difficult unless extreme simplifications are made. However, if y_o is less than several times the amplifier input causing saturation, few excursions result with reasonable system damping. For a given motor, y_o is decreased by increasing n_g or n_p . Considering I to be independent of n_g (a reasonable approximation, typically), this means lower $\dot{y}_{max}$ and $\ddot{y}_{max}$.

Since frictional torque exists, there is a static error corresponding to motor torques less than the frictional torque referred to the motor shaft. To reduce this error to allowable values, a minimum gain is required. Thus, if δ is the allowed static resolution error,

$$\frac{T_{max}}{e_{max}} K\,\delta \geq T_T + \frac{T_p}{n_g} \quad , \tag{9}$$

where $T_T = T_g + T_t$, the frictional torques acting at the motor shaft. Solving as an equality yields the minimum gain for a specified static resolution

$$K_{min} = \frac{1}{\delta}\ \frac{T_T + \frac{T_p}{n_g}}{T_{max}}\ e_{max} \quad . \tag{10}$$

When tachometer damping is not used, T_T is small and the approximation

$$K_{min} = \frac{1}{\delta} \frac{1}{n_g} \frac{T_p}{T_{max}} e_{max} , \quad T_T \ll \frac{T_p}{n_g} \tag{11}$$

is valid. It will be seen that K is limited by amplifier bandwidth, and therefore increasing n_g is the only way of meeting a high resolution requirement for given potentiometers and motor.

Selection of the Gear Reduction Ratio

Evaluation of the above performance figures requires the gear reduction ratio. This ratio should be selected to optimize the performance in some sense.[1] Here, some of the different factors will be considered and a minimum gear reduction ratio will be determined.

Frequently, the gear ratio is chosen to maximize the acceleration $\ddot{y}_{max}$ subject to the torque limitation T_{max}. If the frictional torques are neglected, this leads to the familiar result

$$n_g = \sqrt{\frac{I_p}{I_m + I_t + I_g}} \quad . \tag{12}$$

Often, this yields a small n_g. In fact, n_g may be such that the motor torque T_{max} is less than $T_T + T_p/n_g^2$, the frictional torque, so that the assumption of negligible frictional torques is certainly not valid.

Neglecting T_T but including T_p gives maximum $\ddot{y}_{max}$ when

$$n_g = \frac{T_p}{T_{max}} + \sqrt{\left(\frac{T_p}{T_{max}}\right)^2 + \frac{I_p}{I_m + I_t + I_g}} \quad . \tag{13}$$

In many cases the inertia ratio in (13) is small compared with the squared torque ratio and $n_g \approx 2 \; T_p/T_{max}$. Thus the maximum motor torque is approximately twice the frictional torque at the motor shaft.

For smooth tracking of a slowly varying input, the frictional torque must be small compared to the available motor torque. Since smooth tracking is important in accurate analog computation, the gear ratio should be selected accordingly. Experience has shown that maximum motor torque should exceed the frictional torque by a factor of at least five. Thus

$$T_{max} > 5 \left(T_T + \frac{T_p}{n_g}\right) \tag{14}$$

or

$$n_g > \frac{T_p}{\dfrac{T_{max}}{5} - T_T} \quad . \tag{15}$$

When tachometer damping is used, T_T is small and (15) becomes

$$n_g > 5 \frac{T_p}{T_{max}} \quad , \; T_T \ll \frac{T_p}{n_g} \quad . \tag{16}$$

Inequality (15) or (16) frequently sets a minimum gear ratio that is considerably greater than the gear ratio for maximum acceleration, in which case the potentiometer inertia I_p is not significant in design.

If $\dot{y}_{max}$, $\ddot{y}_{max}$, and good tracking characteristics were the only performance figures, (12), (13), (15), or (16) would set the minimum gear ratio for the constraint of a given motor and potentiometer. However, requirements on y_o or static resolution may require an increased minimum gear ratio. Thus (8) bounding y_o may require higher n_g to obtain an acceptable value of y_o. Similarly, specified K_{min} and δ may require higher n_g to satisfy (11). Linear analysis will show that K_{min} is indeed limited by the servo amplifier bandwidth.

With the above limitations in mind, a minimum gear ratio may be determined for specified motor and potentiometers. If the requirements for velocity and acceleration limits are not met with this n_g , a motor with greater maximum torque and/or velocity must be selected, and a new set of performance figures must be computed.

Compensation of the Linear System

To assure smooth tracking, the gear ratio has been selected so that motor torque exceeds frictional torque by a factor of at least five. This means that there is at least a range of five-to-one where motor torque can exceed frictional torque and be proportional to amplifier input. In this range, system design on a linear basis will yield reasonable results. In this section different compensation techniques to achieve specified linear characteristics such as bandwidth, damping, and steady-state errors will be considered. In certain instances, fundamental limitations on the characteristics exist and may influence the selection of components and the gear reduction ratio. These limitations will be apparent from expressions relating the component parameters and linear and nonlinear performance figures.

First, consider series compensation where $Y_i = Y_f$ is the series compensating function. The closed-loop transfer function $Y(s)$ relating the Laplace transform of y to that of x is then

$$Y(s) = \frac{Y_i}{Y_f} \frac{Y_o}{1 + Y_o} = \frac{Y_o}{1 + Y_o} \quad , \tag{17}$$

where

$$Y_o(s) = Y_f \frac{K\ K_m}{2\pi\ n_g n_p \quad s(\tau_m s+1)(\tau_a s+1)} \quad . \tag{18}$$

For simplicity, perfect error rate or lead compensation,

$$Y_f = Y_i = 1 + C_e s \ , \tag{19}$$

will be used. Imperfect and physically realizable compensation yields similar results.

Three cases will be analyzed. The first and most common occurs when $25/\tau_m < 1/\tau_a$. The second and third cases are defined by $5/\tau_m < 1/\tau_a < 25/\tau_m$ and $1/\tau_m < 1/\tau_a < 5/\tau_m$. Analysis will be based on plots of db. magnitude of Y_o/K vs. log ω with compensation selected so that Y_o has a slope of approximately -9 db./octave when $|Y_o| = 1$. This will give a phase margin of about 45° and hence a reasonable resonant peak in the closed-loop transfer function magnitude. The peaking frequency ω_p will be estimated as the frequency where $|Y_o| = 1$. The accuracy of this procedure will be sufficient to determine the required interrelationships between system parameters.

Fig. 4 shows the first case where C_e has been selected to maximize K. Here

$$\frac{1}{C_e} = \frac{1}{5} \frac{1}{\tau_a} \ ,$$

$$\omega_p = \frac{3}{5} \frac{1}{\tau_a} \ , \tag{20}$$

$$\frac{1}{C_e} > 5 \frac{1}{\tau_m} \quad \text{since} \quad \frac{1}{\tau_a} > 25 \frac{1}{\tau_m} \quad .$$

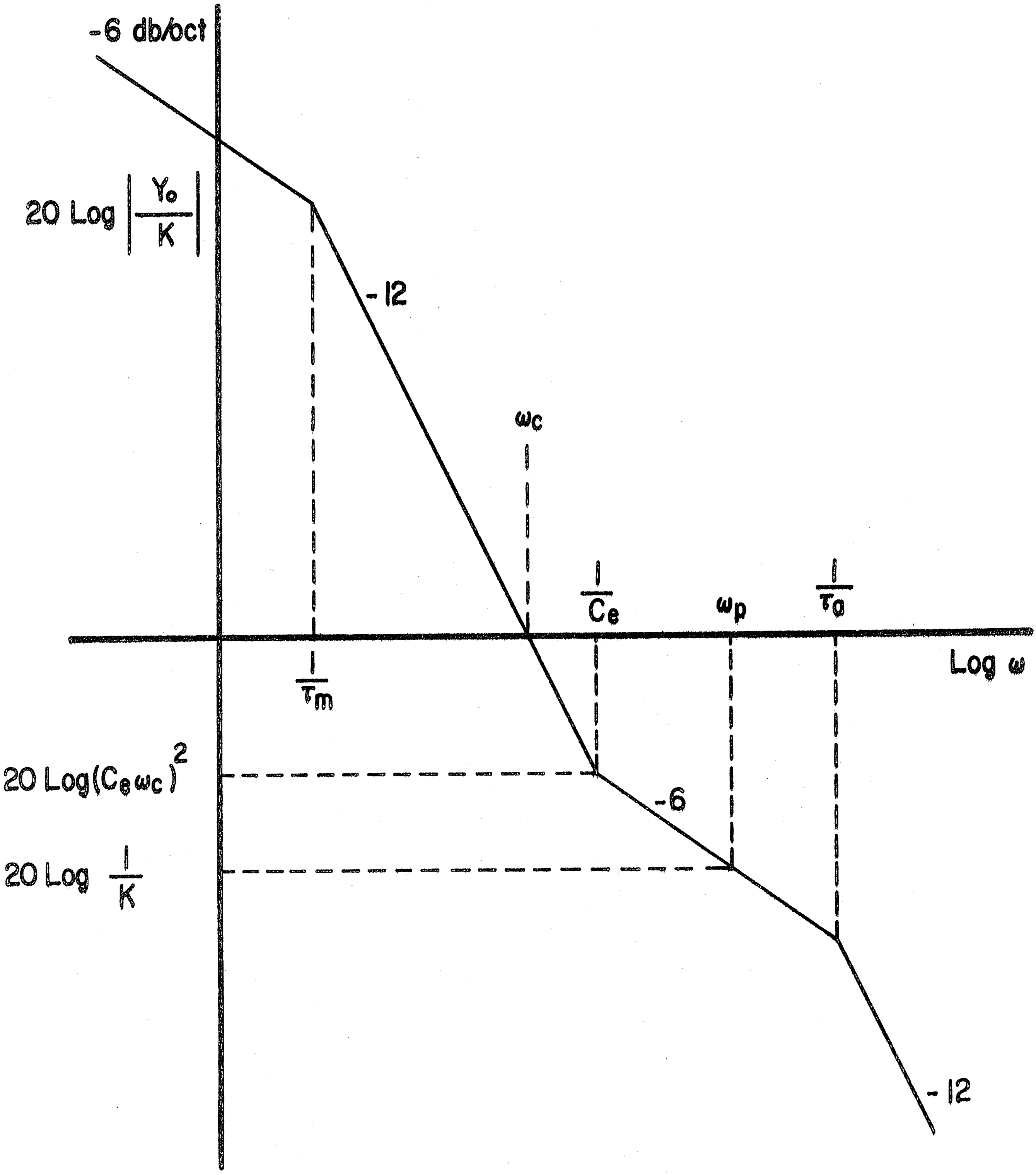

Figure 4. Open-Loop Gain for Position System with Error Rate Compensation When $25/\tau_m < 1/\tau_a$

Since $\omega_p = 3/C_e$, it is apparent from Fig. 4 that

$$\frac{1}{K} = \frac{1}{3}(C_e\omega_c)^2 , \tag{21}$$

where

$$\omega_c = \sqrt{\frac{K_m}{2\pi\, n_g\, n_p\, \tau_m}} = \sqrt{\frac{K_\infty}{2\pi\, n_g\, n_p}} . \tag{22}$$

Substituting (22) and (20) into (21) gives

$$K = \frac{3}{25}\;\frac{2\pi\, n_g\, n_p\, I\, e_{max}}{\tau_a^2\, \tau_{max}} . \tag{23}$$

For given motor, potentiometer, amplifier, and gear train, this is the maximum amplifier gain.

If the above K value equals or exceeds K_{min} determined by (10) or (11), then the specified resolution δ is obtained. If K does not exceed K_{min}, the resolution requirement cannot be met unless certain system parameters are changed. The interrelationship of these parameters is most easily seen by substituting in $K > K_{min}$ from (23) and (10) leading to the inequality

$$\frac{1}{\tau_a} > \sqrt{\frac{25}{6\pi}\;\frac{T_T + \frac{T_P}{n_g^2}}{n_g\, n_p\, I\, \delta}} \tag{24}$$

or

$$\frac{1}{\tau_a} > \frac{1}{n_g}\sqrt{\frac{25}{6\pi}\;\frac{T_P}{n_p\, I\, \delta}} \quad , \; T_T \ll \frac{T_P}{n_g} \tag{25}$$

These inequalities are useful in selecting components that are compatible with specified static resolution. For example, if potentiometers, motor, and amplifier are given, (24) determines a minimum n_g for specified δ. This n_g may very well exceed that given in (13) or (15). The advantage of high amplifier bandwidth and low-frictional torques is clear. It is also clear that nonlinear characteristics cannot be divorced from the linear system design.

The above discussion concerns the limitations placed on gain and static resolution by the compensation restrictions imposed by amplifier bandwidth; the converse is sometimes true. For example, suppose 2δ equals the resolution range $1/\eta$ of the reference potentiometer; certainly, 2δ cannot be less than the wire-to-wire jump $1/\eta$. In this case, K_{min} also becomes a maximum $K = K_{max}$ since larger K could result in wire-to-wire hunting, causing excessive potentiometer wear. That is to say, an error corresponding to one-half of a potentiometer wire can produce torque exceeding frictional torque when K is greater than $K_{min} = K_{max}$. When K_{max} is less than that given by (23), the compensation of Fig. 4 must be changed. By increasing C_e over the value of (20), K can be decreased while ω_p is maintained near $1/\tau_a$.

In the above work, it was assumed that $25/\tau_m < 1/\tau_a$. For relatively low speed, high torque to inertia motors with $\gamma \approx 1$, τ_m given by (5) is small and the inequality may not be valid requiring a modification of the results. For $5/\tau_m < 1/\tau_a < 25/\tau_m$, Fig. 4 is still correct in form, but ω_p and K may be increased somewhat because of the additional phase lead contributed by the motor transfer function. Suppose, for example, that $5/\tau_m \approx 1/\tau_a$; then K may be increased so that $\omega_p \approx 1/\tau_a$ and the three is replaced by five in (21) and (23). In (24) and (25), the factor 25/6 is replaced by 5/2.

When $1/\tau_m < 1/\tau_a < 5/\tau_m$, a different compensation must be used to maximize K and ω_p. This is shown in Fig. 5 where

$$\frac{1}{C_e} = \frac{1}{\tau_m} ,$$
$$\omega_p = \frac{1}{\tau_a} , \qquad (26)$$

and

$$K = \frac{\frac{1}{\tau_a}}{\frac{K_m}{2\pi\ n_g\ n_p}} = n_g\ n_p \frac{1}{\tau_a}\ \frac{60}{N_{max}}\ e_{max} \qquad (27)$$

Again using $K > K_{min}$, it is found that

$$\frac{1}{\tau_a} > \frac{1}{n_g\ n_p}\ \frac{N_{max}}{60\gamma}\ \frac{T_T + \frac{T_p}{n_g}}{T_{max}}\ \frac{1}{\delta} , \qquad (28)$$

or

$$\frac{1}{\tau_a} > \frac{1}{n_g^2\ n_p}\ \frac{N_{max}}{60\ \gamma}\ \frac{T_p}{T_{max}}\ \frac{1}{\delta} ,\ T_T \ll \frac{T_p}{n_g} . \qquad (29)$$

An important figure of merit in linear system design is the steady state error for ramp inputs. In determining this error, it is useful to take the following more general, physically realizable forms for Y_i and Y_f:

$$Y_i = 1 + \frac{C_e s}{\alpha_1 C_e s+1} ,$$
$$Y_f = 1 + \frac{C_d s}{\alpha_2 C_d s+1} . \qquad (30)$$

For series compensation, $C_d = C_e$ and $\alpha_2 = \alpha_1$. Using the final value theorem gives the linear-system steady-state error

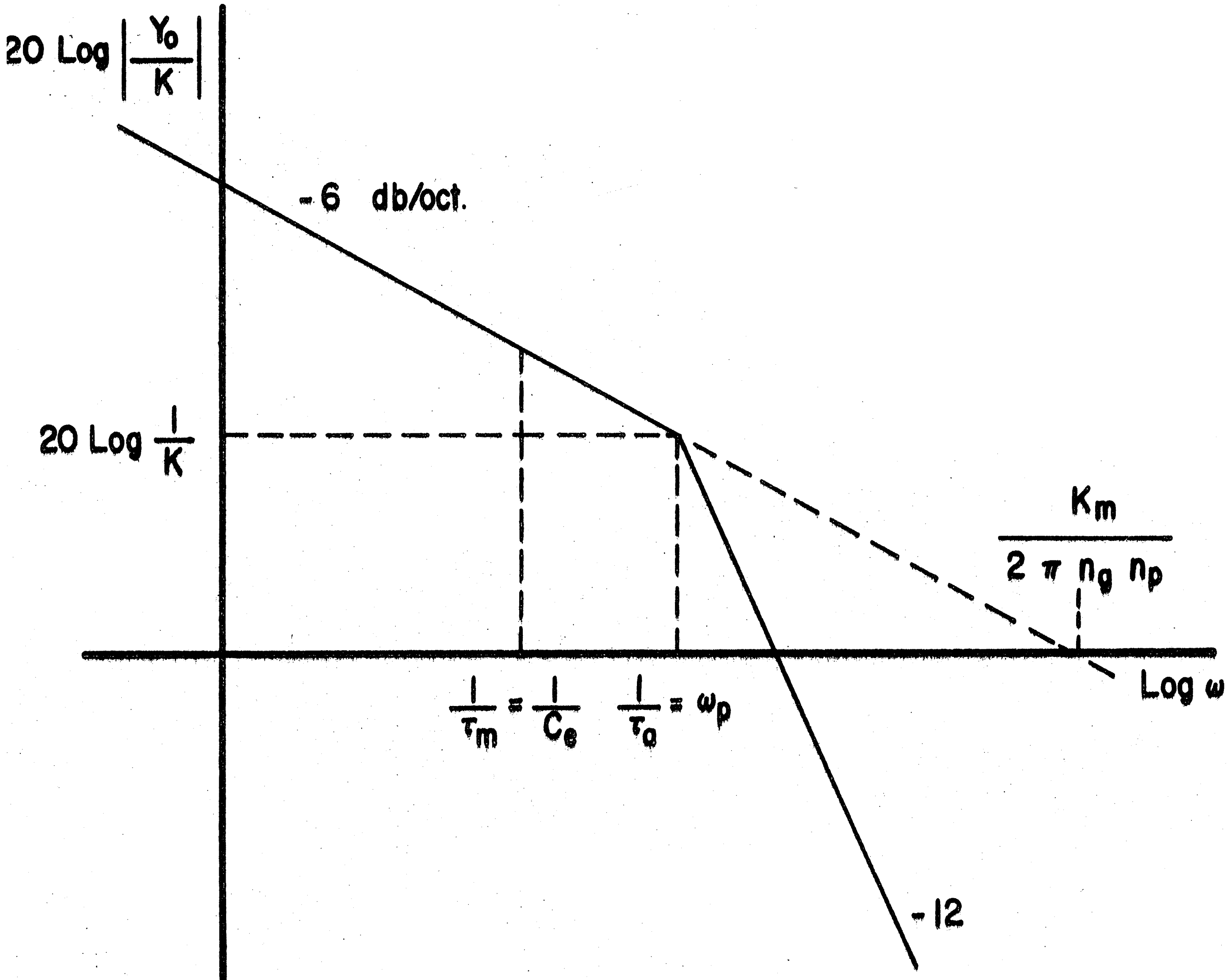

Figure 5. Open-Loop Gain for Position System with Error Rate Compensation when $1/\tau_m < 1/\tau_a < 5/\tau_m$

$$\epsilon_{\ell} = \left[\frac{2\pi \, n_g \, n_p}{K \, K_m} + C_d(1 - \alpha_2) - C_e(1 - \alpha_1) \right] V \tag{31}$$

for the input $x = Vt$. For series compensation the last two terms cancel. In certain input circuity, Y_i and Y_f can be adjusted separately and C_e and α_1 may be selected to make $\epsilon_{\ell} = 0$. This does not change the previous results on series compensation since Y_i and Y_f will differ little for $\epsilon_{\ell} = 0$.

Zero steady-state error for the ramp input is equivalent to zero slope of phase vs. frequency at zero frequency in the closed-loop system. This characteristic is particularly desirable in servos used for analog computation. However, in many cases the first term of (31) is small and cancellation is not warranted.

In addition to the linear-system error, there is the nonlinear steady-state error,

$$\epsilon_n = \frac{V}{|V|} \delta , \tag{32}$$

for the input $x = Vt$. Thus an error equal to δ must exist before motor torque is available to exceed frictional torque. Since this error is independent of $|V|$, it cannot be cancelled out for all V.

Derivative control through tachometer feedback is another commonly used form of compensation. In this case

$$Y_i = 1 \; ,$$
$$Y_f = 1 + \frac{\tau s}{\tau s + 1} C_d \, s \; , \tag{33}$$

where the high pass filter $\tau s/(\tau s + 1)$ in the tachometer path is used to eliminate the steady-state error for ramp inputs [the second term in (31)]. For $\tau > C_d$ ($\tau \approx C_d$ usually gives acceptable damping) the approximation

$$Y\,(j\omega) \approx 1 + C_d j\omega \quad , \ \omega > \frac{1}{C_d} \tag{34}$$

is useful since $Y(j\omega)$ then becomes

$$Y(j\omega) = \frac{Y_i}{Y_f}\ \frac{Y_o}{1 + Y_o} \approx \frac{1}{1 + C_d j\omega}\ \frac{Y_o}{1 + Y_o} , \ \omega > \frac{1}{C_d} \ . \tag{35}$$

The factor $1/(1 + C_d j\omega)$ gives increasing attenuation for $\omega > 1/C_d$ so that the $|Y(j\omega)|$ may not exhibit great peaking even though $|Y_o|/|1 + Y_o|$ does become large. This is apparent from Fig. 6 where the plots of db. gain vs. log ω for Y_o/K and Y are shown for a value of K much greater than that given by (23). The phase margin for Y_o is now very small resulting in a large peaking in $|Y_o|/|1 + Y_o|$ at $\omega = \omega_p$. However, $|Y|$ does not greatly exceed one at $\omega = \omega_p$, because of the factor $1/(1 + C_d j\omega)$. Evidently, the derivative control allows greater gain than series compensation. The useful bandwidth is $1/C_d$.

The gain advantage of derivative control would indicate that static resolution was not limited by amplifier bandwidth for this form of compensation. Unfortunately, large increase of gain is not usually possible because of additional time lags neglected in the analysis. Such lags could reduce the phase margin to a negative value resulting in an unstable system. The maximum increase in K through derivative control can be obtained by a more complete analysis considering the additional time lags and using an inverse Nyquist plot. It is difficult to generalize on the possible gain increase due to the wide variation of

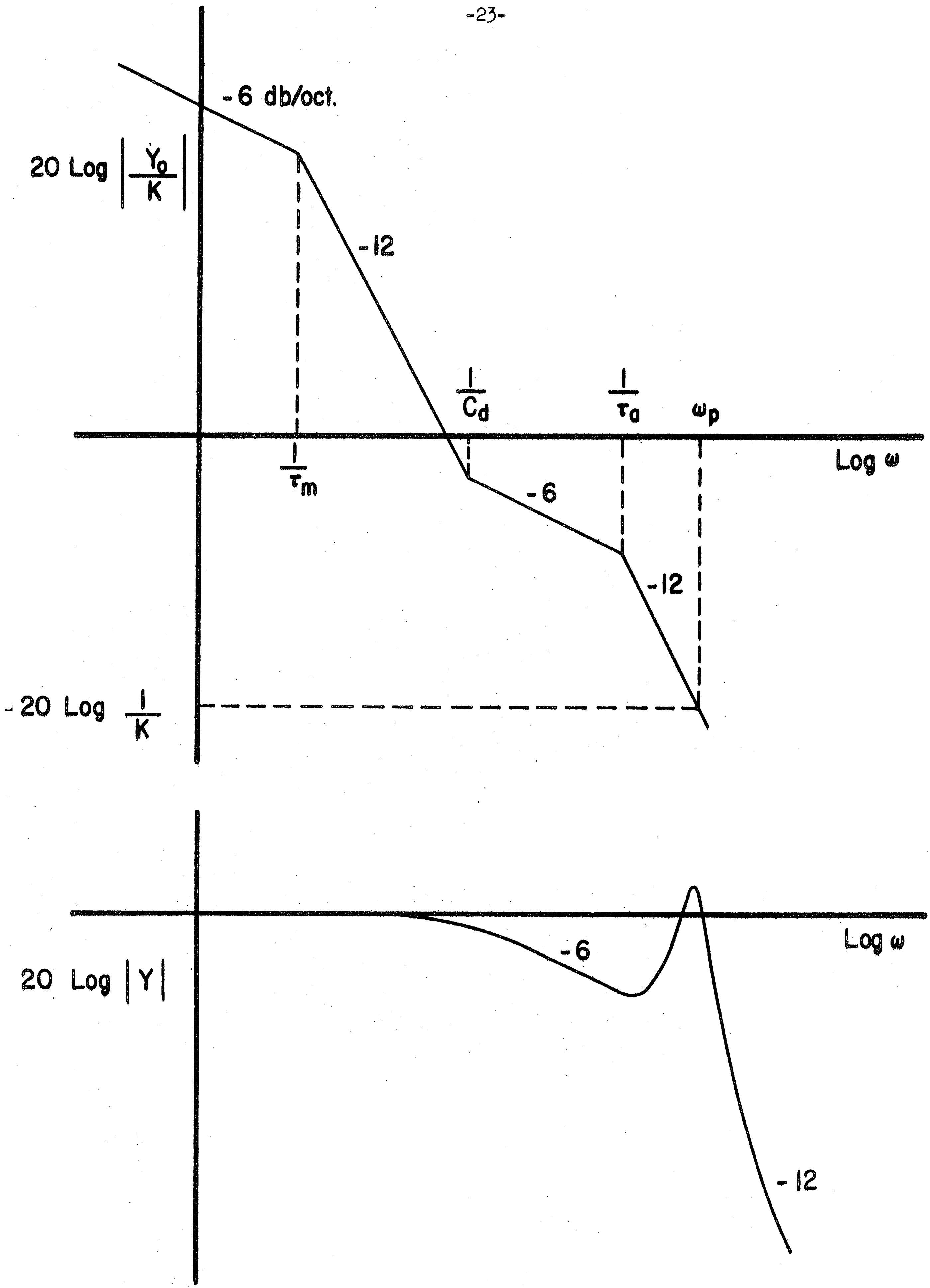

Figure 6. Open and Closed-Loop Gain for Position System with Tachometer Compensation

neglected time lags, but a factor of two to ten above the gain for series compensation is not uncommon.

The increase in K does not necessarily mean a corresponding decrease in the static resolution error. This is apparent from (10) since the added frictional torque from the tachometer requires increased K to maintain a given δ. In addition, the added tachometer inertia reduces the acceleration limit. The lower cost and great reliability of series RC compensation may also outweigh any advantage obtained with tachometer damping.

Another means of increasing K above the value given by (23) is to use a series lead-lag compensating function

$$Y_i = Y_f = \left(\alpha_1 \frac{1 + \frac{C_i}{s}}{1 + \frac{\alpha_1 C_i}{s}}\right)\left(\frac{1 + C_d s}{1 + \alpha_2 C_d s}\right) . \qquad (36)$$

The second factor approximates the error rate control; the first factor approximates integral control and permits an increase in K by a factor of about $1/\alpha_1$ for correctly chosen C_i. Such compensation has two main disadvantages. First, the compensating network is more complicated and may require excessively large capacitor values for realization. Second, lag action introduces additional open-loop negative phase shift that may cause a nonlinear instability or long recovery time for inputs causing amplifier saturation. For these reasons, such compensation has limited use.

IV. THE VELOCITY SERVO

Block Diagram

Fig. 7 shows the block diagram for the velocity servo. A tachometer voltage proportional to the motor shaft velocity is the feedback so that shaft velocity is ideally proportional to input voltage. The tachometer is mounted on the motor shaft to eliminate gear backlash in the feedback. The dimensionless output y is again selected so that the full scale range on y is unity. The input and feedback compensation are defined to have unit gain at zero frequency, i.e., $Y_i(o) = Y_f(o) = 1$. K_i is chosen so that the output velocity $\dot{y}$ is proportional to the input voltage e_i by the desired factor.

For design purposes, it will be useful to consider the equivalent block diagram of Fig. 8 where μ is a dimensionless input voltage and

$$K_v = \frac{K_i}{60\ n_g\ n_p\ K_t} \tag{37}$$

System Design

Much of the previous work applies for the velocity system. Thus (6) and (7) still give correct values for $\dot{y}_{max}$ and $\ddot{y}_{max}$. y_o no longer has significance. The same approach for gear reduction ratio determination is valid and (12), (13), or (15) gives the minimum value for n_g.

The velocity system has a velocity resolution error instead of a position error. That is, the velocity error $\epsilon = u - \dot{y}$ must exceed a

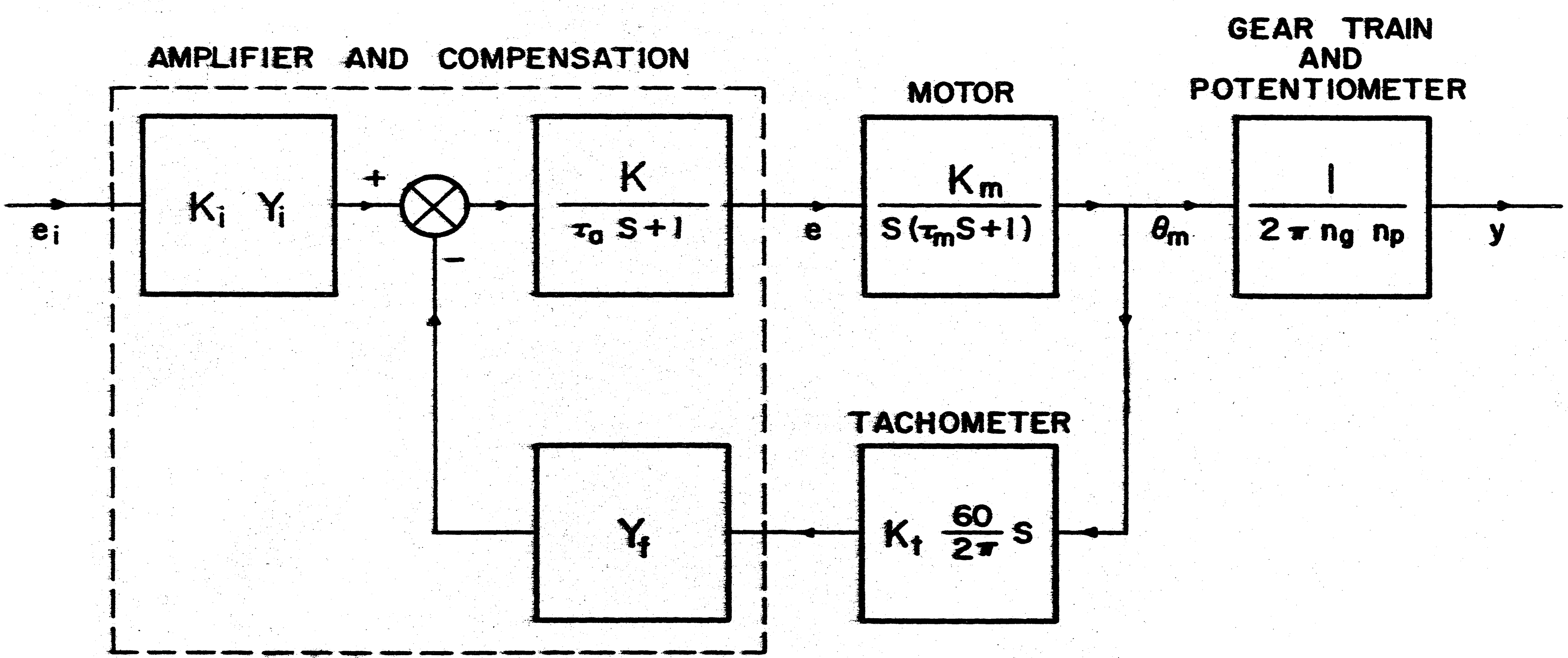

Figure 7. Velocity Servo Block Diagram

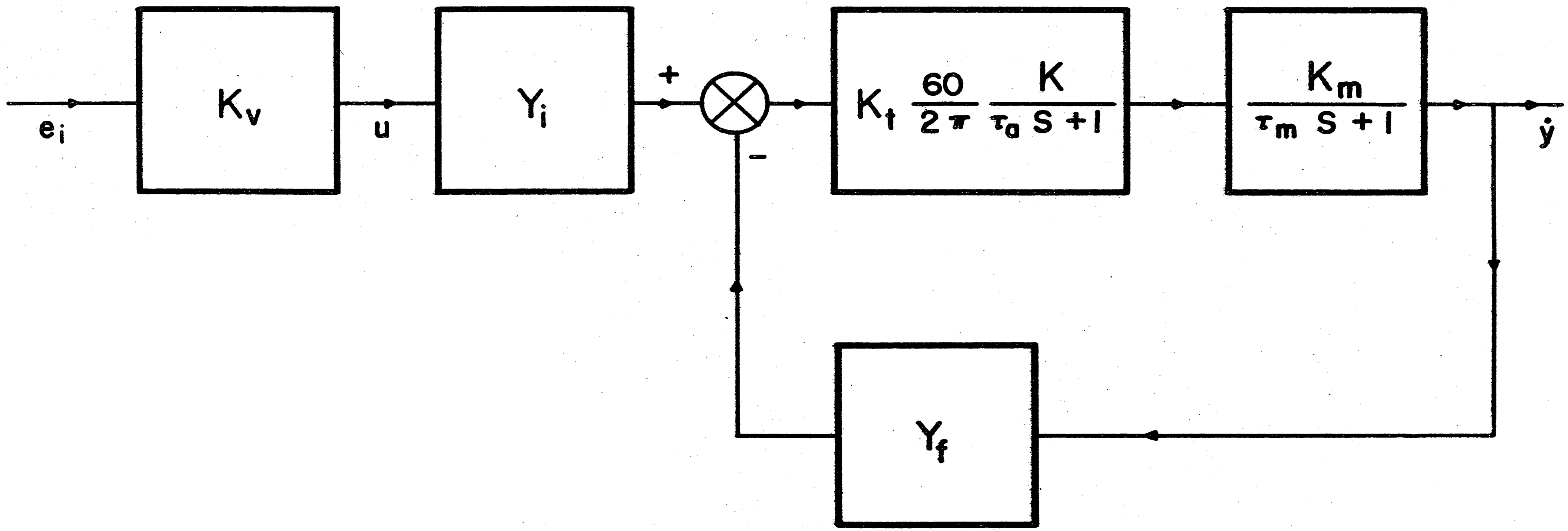

Figure 8. Equivalent Block Diagram for Velocity Servo

certain value before the motor torque exceeds frictional torque. This velocity resolution error δ_v is given by

$$\delta_v = \frac{T_T + \frac{T_p}{n_g}}{\frac{T_{max}}{e_{max}} K K_t \frac{60}{2\pi}} \tag{38}$$

K must be large enough so that δ_v is sufficiently small. It will be seen that there is no particular limit on K as there was previously.

Since the velocity servo is a type zero system, it has a steady-state linear-system error for $u = U$, a constant. The error is given by

$$\epsilon_\ell = \frac{U}{1 + \frac{60}{2\pi} K_t K K_m}$$

$$= \frac{U}{\frac{60}{2\pi} K_t K K_m} = \frac{e_{max}}{K_t K N_{max}} U, \quad 1 \ll \frac{60}{2\pi} K_t K K_m \tag{39}$$

In addition there is a nonlinear resolution error

$$\epsilon_n = \frac{U}{|U|} \delta_v \tag{40}$$

Typically, $|\epsilon_n| \gg |\epsilon_\ell|$. In fact when $U = \dot{y}_{max}$, ϵ_n is usually larger than ϵ_ℓ. Thus design for specified δ_v usually assures steady-state errors not greatly exceeding δ_v .

The compensation of the velocity servo is easier than the position system since the open-loop system no longer has a $1/s$ term present. The main problem is to achieve sufficient gain to meet a requirement on δ_v. Frequently this can be done with a simple series lead compensation network. The more complicated compensation given by (36) may be necessary. In this case there is no stability problem for saturating inputs because of the simpler open-loop characteristics.

V. AN EXAMPLE

To illustrate the above principles, a particular design will now be considered. The problem is to design a position servo with five multiplying potentiometers. Line frequency and the choice of a ten watt magnetic amplifier designate a 60 cps., two pole, 4 oz. in. servomotor. The potentiometer linearity requirement is 0.1% and the static resolution error is 0.05% of full scale. For simplicity and reliability, a simple series RC lead compensating network is to be used. Under these restrictions, the servo is to be designed for the maximum possible velocity and acceleration limits and bandwidth.

Examination of (6) and (7) shows that for a given motor, highest $\dot{y}_{max}$ and $\ddot{y}_{max}$ are obtained for smallest n_p. This dictates single revolution potentiometers and an $n_p = 1$. Consistent with reliability, cost, and similar considerations, a six gang potentiometer is selected for minimum T_p and $\eta > 1000$. Ganged 20,000 ohm, wirewound potentiometers with the following characteristics meet the specifications.

$$T_p = 4 \text{ oz. in.} = 2.08 \cdot 10^{-2} \text{ lb. ft.},$$

$$I_p = 120 \text{ gm. cm.}^2 = 8.8 \cdot 10^{-4} \text{ slug ft.}^2,$$

$$\eta = 1800,$$

I_p includes the inertia of the potentiometer coupling.

The high torque-to-inertia-ratio servomotor has

$$T_{max} = 4 \text{ oz. in.} = 2.08 \cdot 10^{-2} \text{ lb. ft.},$$

$$N_{max} = 3400 \text{ rpm.},$$

$$e_{max} = 115 \text{ v.},$$

$$\gamma = .8,$$

$$I_m = 5 \text{ gm. cm.}^2 = 3.66\cdot10^{-7} \text{ slug ft.}^2 .$$

The maximum power input per phase is 9 watts so the magnetic amplifier has sufficient power capability.

The gear train has $T_g \ll 4$ oz. in. and $\zeta \ll 1/\eta$. For a fairly large range in n_g, the gear train inertia referred to the motor shaft is

$$I_g = 2 \text{ gm.cm.}^2 = 1.47\cdot10^{-7} \text{ slug ft.}^2 .$$

Design for maximum acceleration, neglecting frictional torque, is given by (12) yielding

$$n_g = 4.1 .$$

Consideration of frictional torque using (13) gives

$$n_g = 5.3 .$$

The minimum T_{max} to T_p/n_g ratio of five requires

$$n_g > 5 .$$

Thus design for maximum acceleration considering frictional torque is possible, neglecting resolution limitations.

To determine the effect of the resolution requirement, it is first necessary to compare $1/\tau_a$ and $1/\tau_m$. With current feedback, the magnetic amplifier has $1/\tau_a = 125$ rad./sec. By (5)

$$\frac{1}{\tau_m} = 91 \frac{1}{1 + \frac{17.1}{n_g^2}}$$

and for $n_g \geq 5.3$, $1/\tau_m < 1/\tau_a < 5/\tau_m$. Thus (29) is appropriate; substitution of the specified $\delta = 5\cdot10^{-4}$ and other parameters gives the inequality

$$n_g > 33.6$$

Hence the linear system design and resolution specification require a much larger n_g than that for maximum acceleration. For $n_g < 33.6$, it is not possible to achieve design resolution and acceptable stability. Setting $n_g = 35$ gives

$$C_e = \tau_m = \frac{1}{89.5} \approx .01$$

and by (27)

$$K = 7100$$

Computation of performance figures yields

$$\dot{y}_{max} = 1.62$$

$$\ddot{y}_{max} = 182$$

$$y_o < .0072$$

For a ramp input $x = Vt$ the steady state linear-system and nonlinear errors are given by (31) and (32).

$$\epsilon_\ell = .008\ V$$

$$\epsilon_n = .0005 \frac{V}{|V|}$$

The linear-system error may be cancelled by designing the input circuity so Y_i and Y_f differ by an appropriate amount.

Footnote

(1) The following authors have determined motor suitability and the gear reduction ratio considering $\dot{y}_{max}$ and $\ddot{y}_{max}$ requirements and frictional torque limitations but neglecting the other performance figures and their relation to linear system design.

Harris, H., "A Comparison of Two Basic Servomechanism Types," A.I.E.E., Vol. 66, Pt. II, pp. 83-93, 1947.

Newton, G. C., Jr., "What Size Motor?," Machine Design, Vol. 22, N. 11, pp. 125-130, November, 1950.