THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Aeronautical and Astronautical Engineering

# QUARTERLY PROGRESS REPORT
for October, November, December 1961

by
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ORA Project 04487

Prepared for George C. Marshall Space Flight Center
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## Introduction

This report describes work accomplished in the period of October through December 1961 in the contractor's research program on various aspects of space vehicle booster control systems.

The study of recovery regions of linear systems with limited control effort described herein is by doctoral student Joseph L. LeMay under the supervision of Professor Elmer G. Gilbert.

## 2. Regions of Recoverability of Linear Systems with Input Constraints

In a linear system, feedback can be used to form a closed-loop stable system from an open-loop unstable plant. However, if the control effort is limited (saturating controller), the closed-loop stability may be regional rather than global in system state space. For example, for a space vehicle booster; the pitch (and the yaw) equations, linearized about a point of a reference trajectory, contain roots with positive real parts. The input thrust vector in pitch (and in yaw) is limited in magnitude because of limited thrust and limited gimballing angle. The closed-loop stability region consists of off-reference initial conditions that are capable of being controlled in some defined sense. Determination of this region sets upper bounds on disturbances and permits evaluation of sub-optimal controllers.

A constant linear system that is nth order with m eigenvalues that have real part positive is called an n - m system. The maximum region of recoverability, RRM, is the set of states that can be driven to zero via an input that is amplitude limited. It turns out that knowledge of RRM for an m - m system implies knowledge of RRM for an n - m system. In canonical coordinates the latter is a hypercylinder with m-dimensional cross section equal to the former.

For the real root case, RRM for a general m - m system has been found for scalar forcing. For the complex root case, RRM for a 2 - 2 system and scalar input has been delineated. The determination of

RRM for general vector inputs is computationally complex, but conceptually straightforward. An example of a second-order two-input system has been completely worked out.

For the second-order real-root case, some sub-optimal schemes have been investigated. The regions of recoverability associated with a bang-bang controller with a linear phase plane switching line have been found for 2 - 1 and 2 - 2 systems for all switching line slopes. Also, for the 2 - 1 and 2 - 2 systems, a class of control laws has been described which will realize RRM.

Further work will include the determination of RRM for the yaw (or pitch) control of a space vehicle booster.

## 3. Multivariable Control Systems

Professor Elmer G. Gilbert completed research on multivariable control systems which was well along before the start of the contract. The work has been written up as a technical paper and has been submitted for publication. Copies of the paper are included with this report.

Statement of Man-Hours Expended
and
Summary of Expenditures
for October, November, December 1961

| | October 1961 | November 1961 | December 1961 | Three Month Period |
|---|---|---|---|---|
| Man-Hours Expended | | | | |
| Faculty Participants | 72 | 76 | 100 | 248 |
| Graduate Students | 5 | 4 | 55 | 64 |
| Total | 77 | 80 | 155 | 312 |
| Salaries and Wages | | | | |
| Faculty Participants | $566.00 | $ 628.00 | $ 818.76 | $2,012.76 |
| Graduate Students | 16.25 | 13.00 | 178.75 | 208.00 |
| Total | $582.25 | $ 641.00 | $ 997.51 | $2,220.76 |
| Overhead | $291.13 | $ 343.56 | $ 508.01 | $1,142.70 |
| Materials and Supplies | 0.00 | 28.23 | 12.11 | 40.34 |
| Reports | 0.00 | 46.12 | 18.50 | 64.62 |
| Travel | 0.00 | 276.73 | 156.11 | 432.84 |
| Totals | $873.38 | $1,335.64 | $1,692.24 | $3,901.26 |