ENGINEERING RESEARCH INSTITUTE
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DC ANALOGUE COMPUTER SOLUTION OF B-47 AND F-86D LONGITUDINAL FLIGHT EQUATIONS

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DC ANALOGUE COMPUTER SOLUTION OF B-47 AND F-86D LONGITUDINAL FLIGHT EQUATIONS

1. Introduction

The equations for longitudinal motion for the F-86D and B-47 aircraft were set up and solved on the dc electronic differential analyzer of the Department of Aeronautical Engineering, University of Michigan. Results and conclusions of this investigation are presented.

The simulation of the longitudinal motion was complete. All nonlinearities in the equations were included with the exception of the variations produced in aerodynamic force coefficients by air density and Mach number. The assumption of constant density and Mach number is valid for the changes in altitude and velocity encountered during a single computer run. Scaling was adequate to allow maximum aircraft motion. Thus, results obtained for small motions are realistic from the standpoint of complete simulation.

Computer solutions indicate that duplication of small motion dynamic characteristics by the general purpose dc differential analyzer is good. Since accurate simulation of the longitudinal characteristics is most difficult, it would appear that similar accuracy would be obtained in the complete simulation of all motions.

2. Computing Equipment Used

The computer used was of the REAC* dc analogue type. Resolvers and multipliers were standard Reeves servo units, S-101 Mod 4, capable of good resolution and fair dynamic response (natural frequency of 3 cps). The maximum velocity and acceleration ratings of 50 volts/sec and 500 volts/sec^2 respectively were somewhat inadequate for the largest scale motions. All runs were selected to avoid these limitations. The summing and integrating amplifiers were chopper stabilized and of good bandwidth with computing components of .1% accuracy. Brush recorders were used to chart important variables.

*Reeves Electronic Analogue Computer, Reeves Instrument Corp., N.Y. 28, N.Y.

3. F-86D Equations and Computer Realization

The longitudinal equations of motion are obtained from those of page I-11 of the previous final report for this project* by setting V, P, and R and all their time derivatives equal to zero. Bank (ϕ) and heading (ψ) angles are arbitrarily assumed zero. The equations of equilibrium then become

$$\dot{U} = \frac{X + P_x + X_c}{m} - g \sin \theta - WQ \tag{1}$$

$$\dot{W} = \frac{Z + P_z + Z_c}{m} + g \cos \theta + UQ \tag{2}$$

$$\dot{Q} = \frac{M + T_y + M_c}{I_{yy}} \tag{3}$$

The angle of attitude is given by

$$\dot{\theta} = Q \tag{4}$$

and the rate of climb by

$$- \dot{s}_z = U \sin \theta - W \cos \theta \ . \tag{5}$$

The angle of attack is given within a good approximation by

$$\alpha \cong \frac{W}{U} \tag{6}$$

The force and moment equations were obtained from a report by the Engineering and Research Corporation (ERCO Report No. 6208-20) and are those used in the F-86D flight simulator. In the notation of the ERCO report

$$Z_t = - 288 \ q \ C_L \tag{7}$$

$$X_t = - 288 \ q \ (a + b\alpha + c\alpha^2) \tag{8}$$

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* Reference Project 2164, Final Report

$$M_t = 2329\, q\, [\,.030 - f_8\,(M_a)\, C_L{}' - f_{13}\,(M_a)\, \delta_{SE} - f_{11}\,(q)\, T \cdot 10^{-3} - f_6\,(M_a)\, \frac{Q}{U} - \frac{0.162\, \dot{\alpha}}{U}\,] \quad (9)$$

$$X + P_x + X_c = X_t \cos \alpha - Z_t \sin \alpha + T \cong X_t - Z_t\, \alpha + T \quad (10)$$

$$Z + P_z + Z_c = Z_t \cos \alpha + X_t \sin \alpha \cong Z_t + X_t\, \alpha \quad (11)$$

$$M + T_y' + M_c = M_t - Zd \quad (12)$$

where

$q = 1/2\, \rho\, V_p^2 \cong 1/2\, \rho\, U^2$	dynamic pressure
M_a	Mach number
δ_{SE}	elevator, stick deflection in degrees
T	thrust in pounds
C_L, $C_L{}'$	lift coefficient (function of α and M_a)
$a, b, c, f_6, f_8, f_{11}, f_{13}$	functions of M_a
d	distance from C.G. to reference point

The computer set up diagram is shown in Figure 1. All variables have been scaled to allow the maximum range of variation as given in the ERCO report. Computer symbols are conventional and indicated variables are in volts. Angles are in radians except at inputs of resolvers. Coefficient potentiometer settings are given in Table 1. The airplane is in clean configuration with a gross weight of 15,500 pounds.

4. Results of F-86D Simulation

Computer runs were made at the following combinations of altitude and Mach number: 10,000 ft., Mach .5; 10,000 ft., Mach .7; 30,000 ft.,

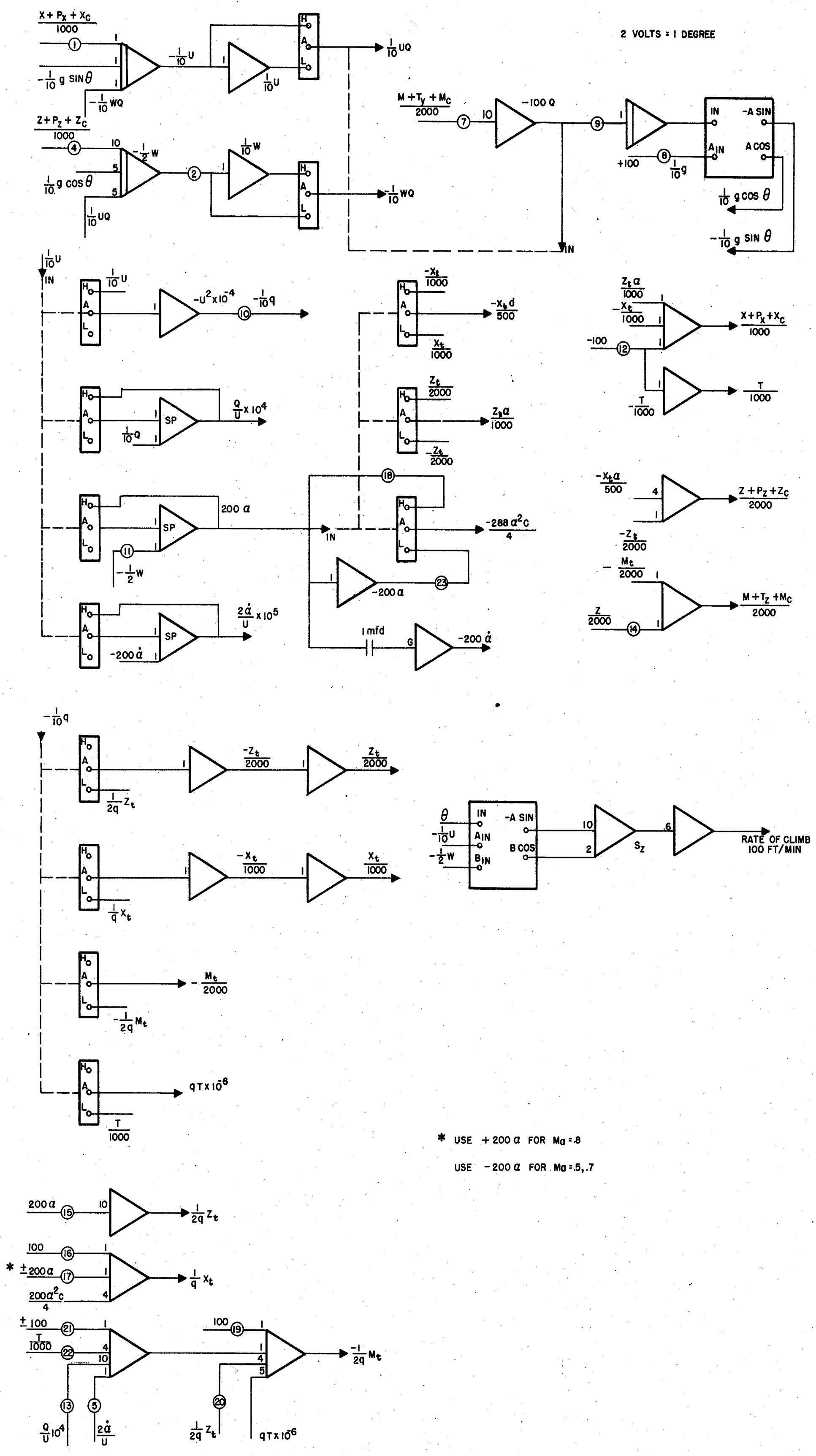

2 VOLTS = 1 DEGREE
IN
-A SIN
A COS
B COS
SP
1 mfd
RATE OF CLIMB
100 FT/MIN
* USE +200 α FOR Ma =.8
USE -200 α FOR Ma =.5,.7

Figure 1

Table 1

Pot. No.	Pot. Setting
1	$100/m = .208$
2	.200
4	$100/m = .208$
5	.002
7	$20,000/I_{yy} = .680$
8	$g/1000 = .032$
9	.268
10	$1/2 \times 10^3 \rho = .876$ (10,000 ft), .444 (30,000 ft) .230 (45,000 ft)
11	.400
12	$T \times 10^{-5}$
13	$.0239\ f_6 = .675\ (M_a = .5)\ .735\ (M_a = .7)\ .756\ (M_a = .8)$
14	$d = .513$
15	$.286\ (M_a = .5),\ .310\ (M_a = .7),\ .351\ (M_a = .8)$
16	$2.88\ a = .043\ (M_a = .5),\ .043\ (M_a = .7),\ .045\ (M_a = .8)$
17	$82.5\ b = .007\ (M_a = .5)\ .010\ (M_a = .7)\ .005\ (M_a = .8)$
18	$592\ c = .243\ (M_a = .5)\ .272\ (M_a = .7)\ .367\ (M_a = .8)$
19	.700
20	$4.02\ f_8 = .830\ (M_a = .5, .7, .8)$
21	$23.29\ f_{13}\ \delta_{SE}$
22	.96

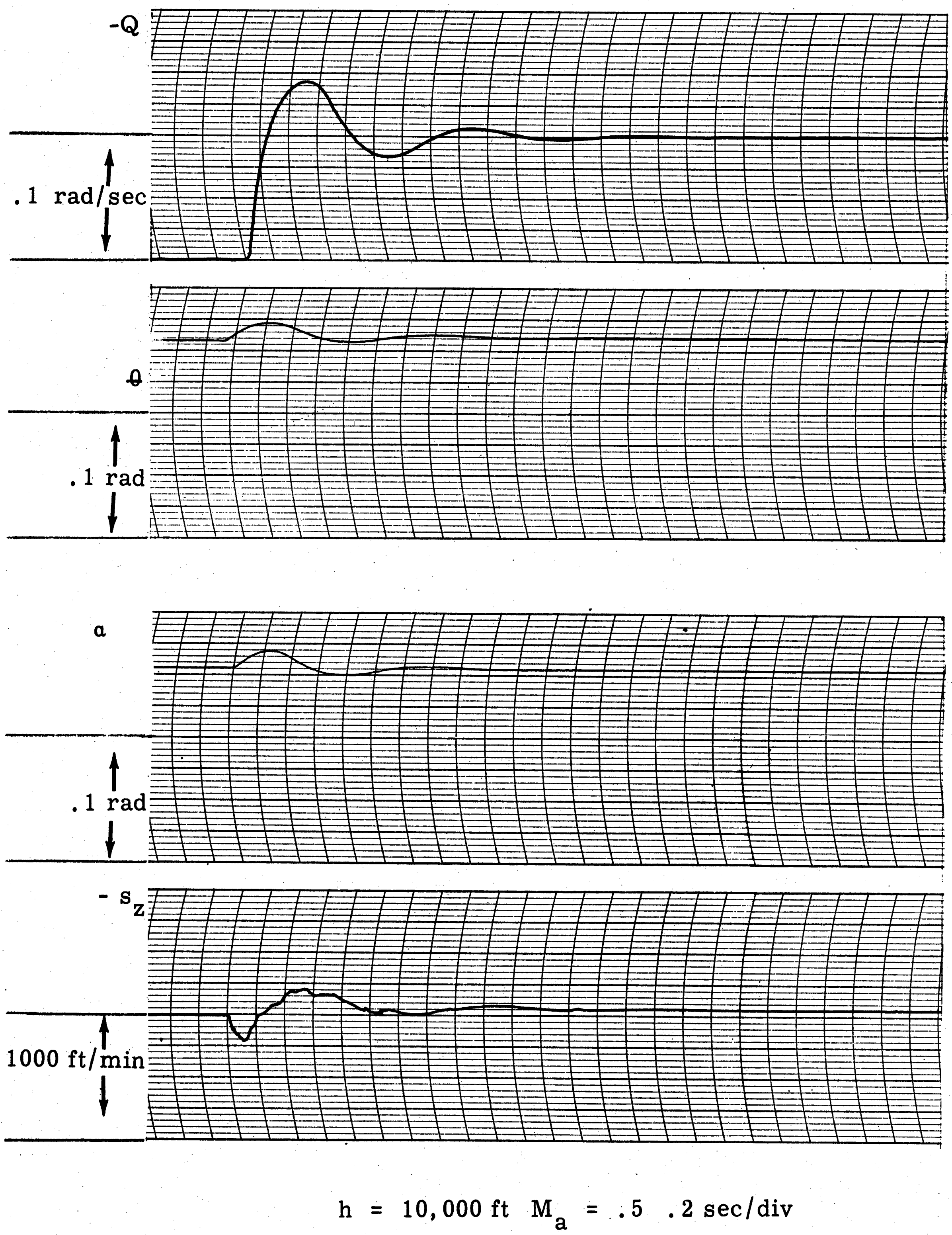
Mach .8; 45,000 ft., Mach .8. Altitude and speed variations were limited for each combination so that aerodynamic force coefficients, as functions of Mach number and air density, could be assumed constant. For each combination Q, θ, α, and $\dot{s}_z$ were recorded, first, for an initial pitch rate, and second, for a step in elevator deflection. Sample recordings are shown in Figure 2 and Figure 3.

In all cases results were excellent. The computer was easily adjusted to maintain level flight or a given rate of climb. Solution stability was good (e.g., a 1000 ft/min rate of climb would hold within $\pm$ 50 ft/min over a three minute period with no computer adjustment). Figure 2 and Figure 3 indicate accurate dynamic simulation and fine resolution. Transient response also checks favorably with previous solutions of the linearized equations. Figure 4 shows that the long period longitudinal oscillation was faithfully reproduced. This is a direct result of excellent resolution, especially in integration. None of the present ac simulators tested by this group have approached such performance in the duplication of small motion, long period oscillations.

The only trace in Figure 2 which shows the effect of finite resolution is the rate of climb. This is due to granularity and sticky friction in the resolving servo which implements Equation (5). Other multiplications and resolutions are less critical as they are all followed by integrations which smooth fine irregularities. This advantage is only present when the integrations themselves are linear for small motions. It is believed that the superior simulation achieved in this study is due in large part to the excellence of dc electronic integration.

Figure 5a illustrates more fully the errors due to resolution in rate of climb. Though jumps appear quite large none of them are much greater than 100 ft/min. It should be noted that the rate of climb does not appear directly in any simulator presentation. Figure 5b shows how rate of climb would be indicated on a panel instrument which has a linear system first order time delay of three seconds. The resulting smoothing makes the response completely acceptable. Similar action would take place upon integration of rate of climb to obtain altitude. It was proved that the irregularities of the trace in Figure 5a were due to poor trigonometric resolution of Equation (5) by using the approximation (for small θ)

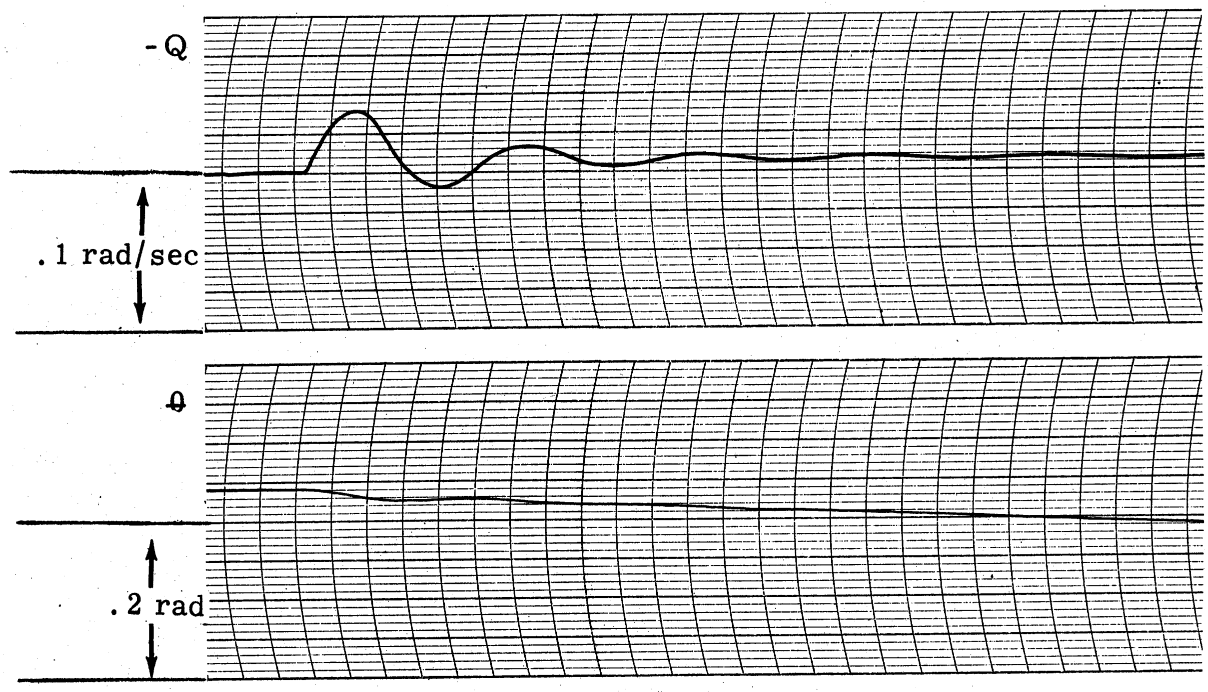
$$-\dot{s}_z \cong U\theta - W \,. \qquad (13)$$



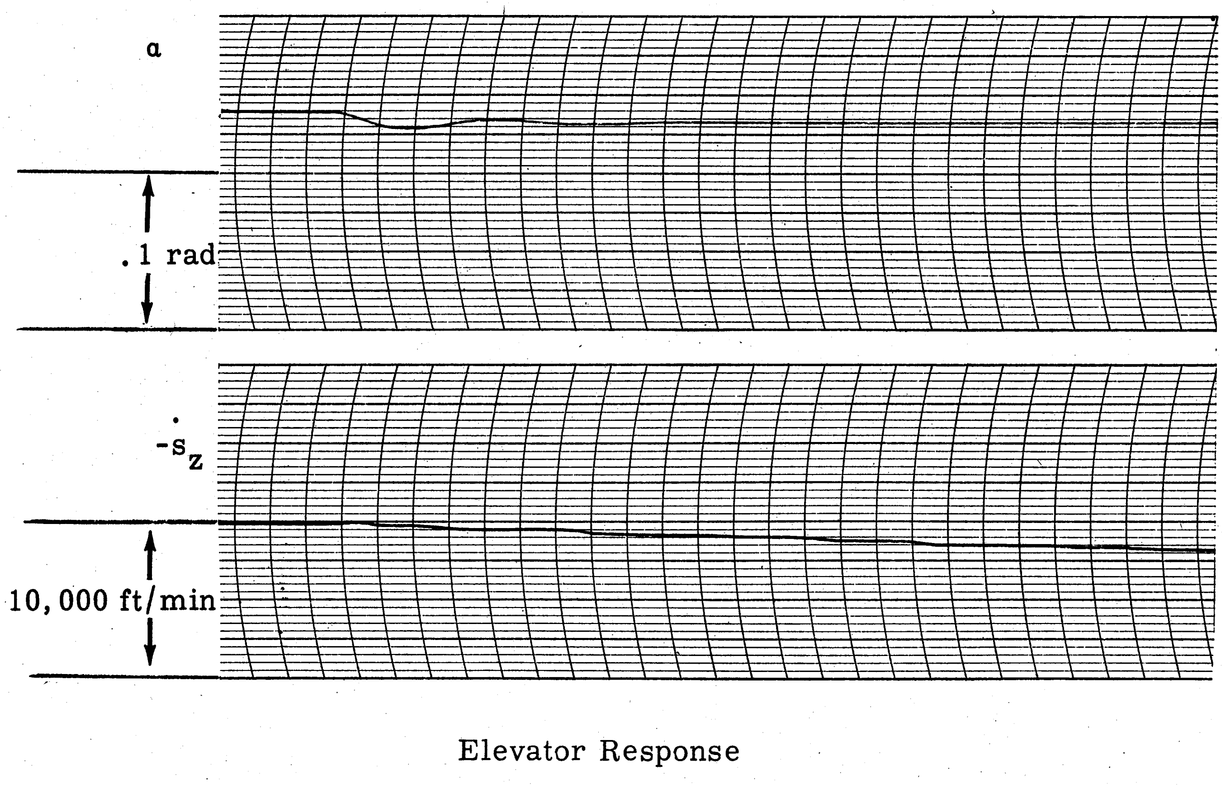
h = 10,000 ft M_a = .5 .2 sec/div

Initial Pitch Rate Response

Figure 2

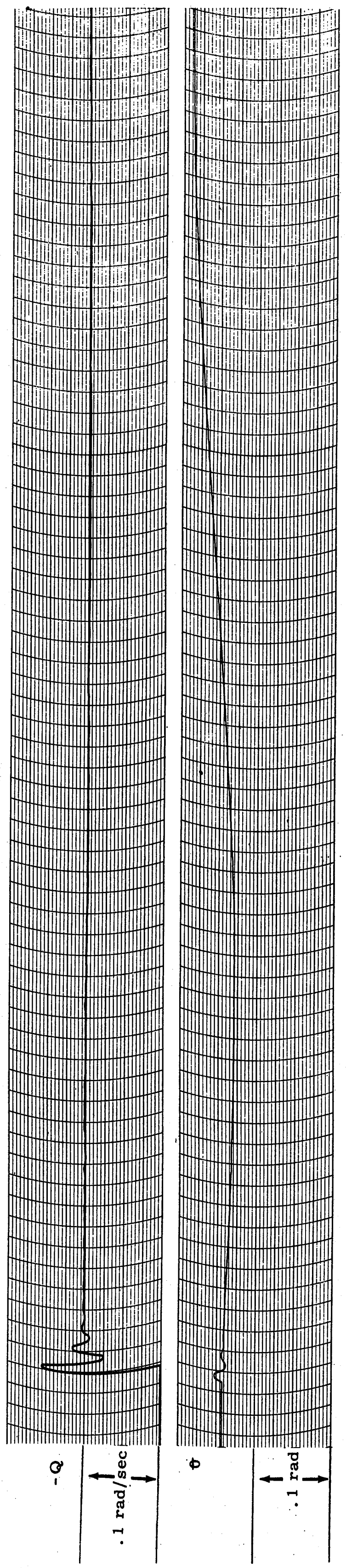


h = 30,000 ft M_a = .8 δ_{SE} from .40 to .09 .2 sec/div



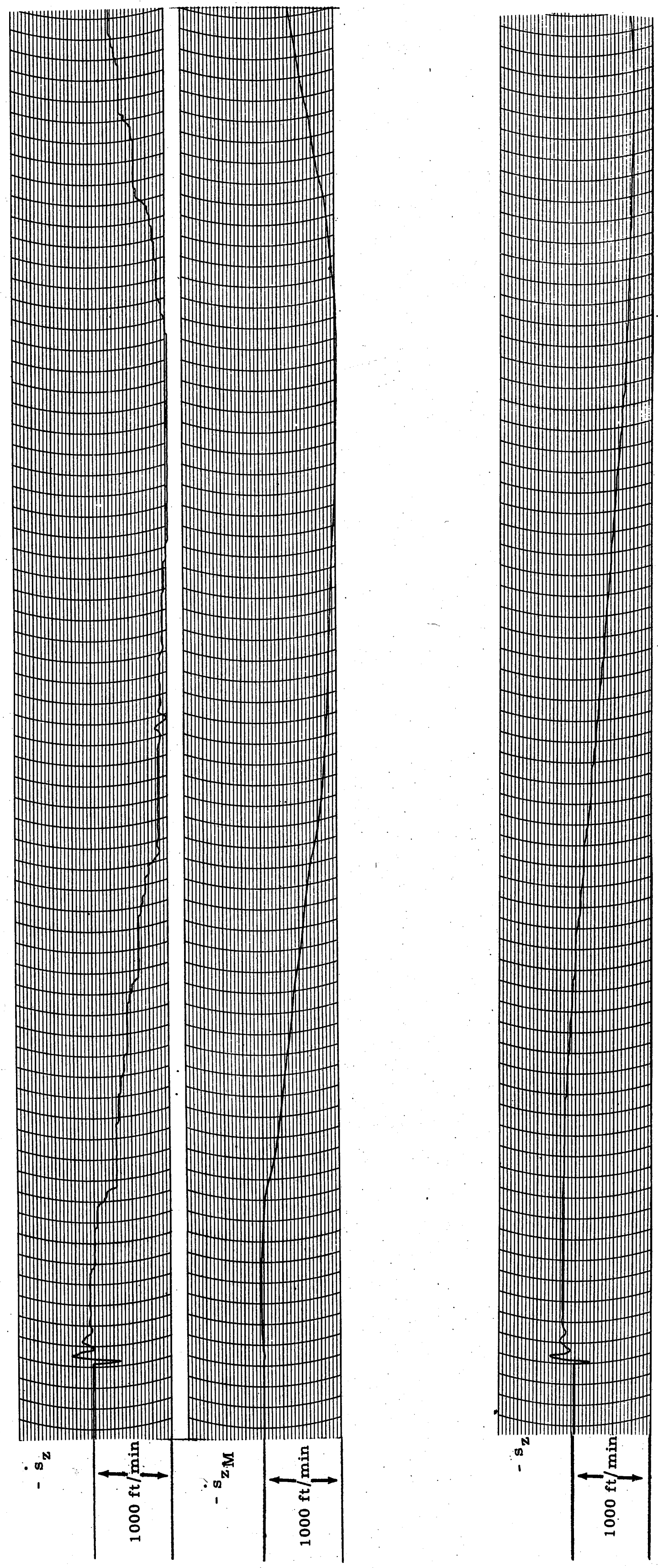
Elevator Response

Figure 3



h = 30,000 ft M_a = .8 1 div/sec

Figure 4. Initial Pitch Rate Response



$$3\,\dot{s}_{z_M} + s_{z_M} = s_z$$

h = 10,000 M_a = .7 1 div/sec

Initial pitch rate .1 rad/sec

Figure 5. Initial Pitch Rate Response

Scaling was such that the product $U\theta$ was performed with high accuracy. Figure 5b shows the improvement. In practice small motion resolution could be improved by letting

$$\sin\theta = \theta + (\sin\theta - \theta) .$$

The term $(\sin\theta - \theta)$ would be generated by a function potentiometer and would be effective for large angle motions only. The linear term in θ would provide high resolution for small angles.

5. B-47 Equations and Computer Realization

To simulate B-47 longitudinal motion it was decided to use Link Aviations' simplified equations. In this system an additional attitude angle is defined. The angle, β, is measured in a vertical plane between the horizontal plane and flight path velocity $\overline{V}_p$. Approximate equations are used to determine accelerations

$$\dot{U}_G \cong \frac{X + P_x + X_c}{m} \qquad (14)$$

$$\dot{W}_G \cong \frac{Z + P_z + Z_c}{m} \qquad (15)$$

$$\dot{Q} = 57.3 \frac{M + T_y + M_c}{I_{yy}} \; * \qquad (16)$$

Note that $\dot{U}_G$ and $\dot{W}_G$ do not include gravity terms. The angle of attitude is given as before

$$\dot{\theta} = Q . \qquad (17)$$

The angle of attack for longitudinal motion is given simply by

$$\alpha = \theta - \beta . \qquad (18)$$

Approximate equations yield β and the flight path velocity, V_p,

$$\dot{\beta} \cong -57.3 \frac{\dot{W}_G + g\cos\beta}{V_p} \qquad (19)$$

$$\dot{V}_p = \dot{U}_G - g\sin\beta \qquad (20)$$

* In this section all angles and their rates are measured in degrees

Rate of climb is simply

$$-s_z = V_p \sin \beta \tag{21}$$

The force and moment equations are taken directly from Link Aviation sources.

$$X + P_x + X_c = q\,[-.376\,(\alpha + 2.7)^2 - 26.85 - 1428\, f_1\,(M_a)] + T \tag{22}$$

$$Y + P_y + Y_c = -109\, q\,(\alpha + 2.7) \tag{23}$$

$$M + T_y + M_c = q\,[-643\,\alpha - 466\,\delta_{SE} - 61000\,(\frac{Q}{V_p}) + 109\,(\alpha + 2.7)\,d + 10{,}000\,(M_a - .6)^2 + 210] + 1.77\,T \tag{24}$$

The computer diagram is shown in Figure 6. The statements of Section 3 regarding symbols, scaling, etc. apply here. Coefficient potentiometer settings are given in Table 2.

6. Results of B-47 Simulation

Computer runs were made under two sets of conditions: altitude - 20,000 ft., Ma = .7, gross weight - 192,000 pounds; altitude - 40,000 ft., Ma = .8, gross weight - 112,000 pounds. The tests and conditions of Section 4 were duplicated. The pitch rate Q and rate of climb were recorded. No attempt was made to determine the errors introduced by the approximate Link equations. It seems likely that the simplification is justifiable when aircraft motion is restricted to near level flight.

Figures 7 and 8 indicate that excellent simulation was again achieved. Characteristics were similar to those of the F-86D simulation and need no further elaboration. Figure 9 compares a response made during this study with one obtained from the Link training simulator. Considering the somewhat different flight conditions agreement is good. There is a noteworthy (about one second) lag in Link response. This might result from the mechanical devices used in multiplication and integration.

7. Conclusions

The longitudinal equations of motion, the most difficult to simulate accurately, were solved with all important non-linearities on the dc electronic analogue computer. Results obtained were far superior to those of any present ac trainer type simulation. Outstanding characteristics of both the F-86D and

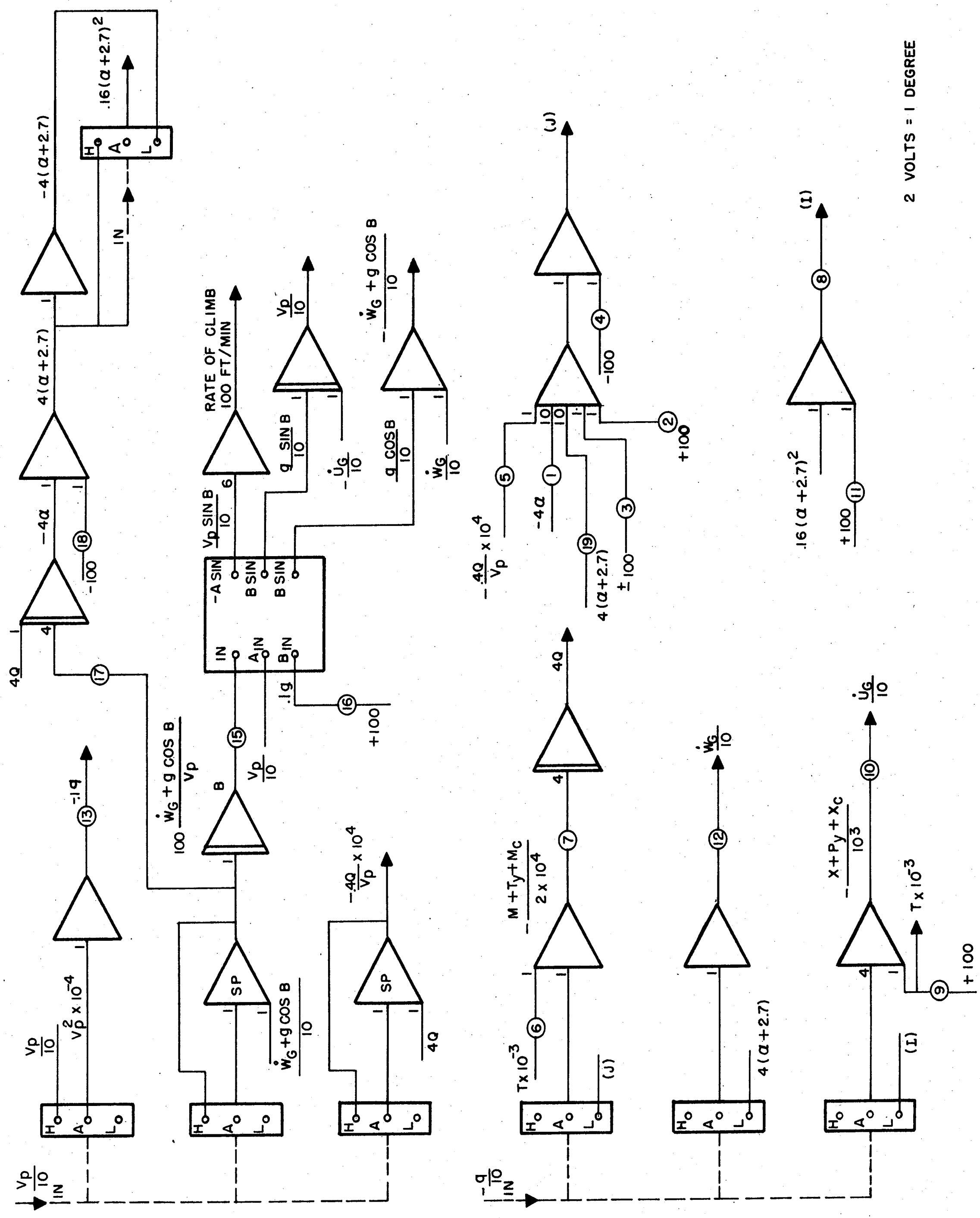


Figure 6

TABLE 2

Pot. No.	Pot. Setting
1	.802
2	$5 (M_a .6)^2$
3	$.233\ \delta_{SE}$
4	.105
5	.760
6	.088
7	.730
8	.586
9	$T \times 10^{-5}$
10	100/m = .017 (192,000 lbs), .029 (112,000 lbs)
11	.119 (M_a = .7), .162 (M_a = .8)
12	$1.07 \times 10^3/4m$ = .445 (192,000 lbs), .763 (112,000 lbs)
13	.630 (20,000 ft), .283 (40,000 ft)
15	.287
16	.032
17	.573
18	.108
19	.500

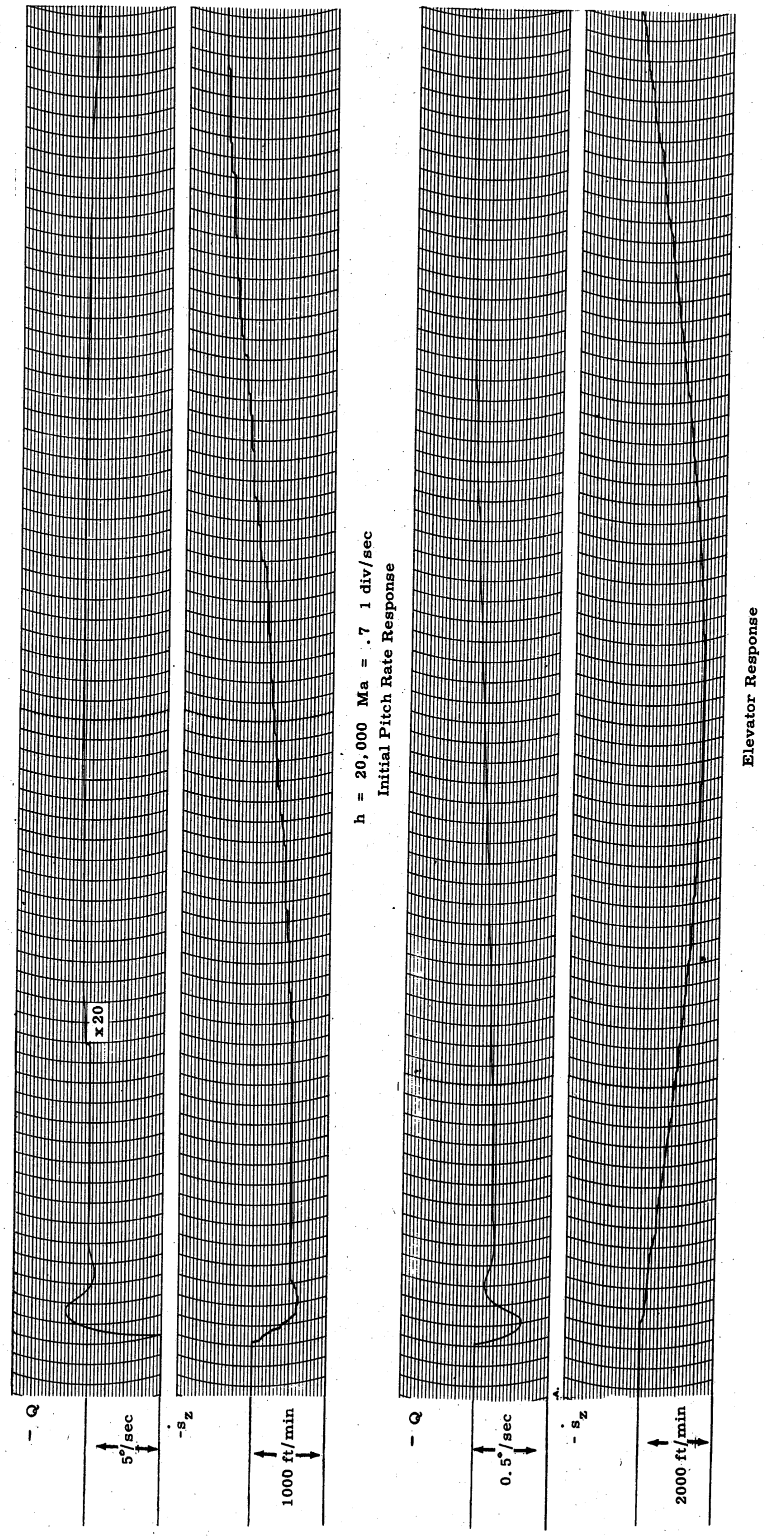

h = 20,000 Ma = .7 1 div/sec
Initial Pitch Rate Response
Elevator Response
x20
- Q
5°/sec
$-\dot{s}_z$
1000 ft/min
- Q
0.5°/sec
$-\dot{s}_z$
2000 ft/min

Figure 7

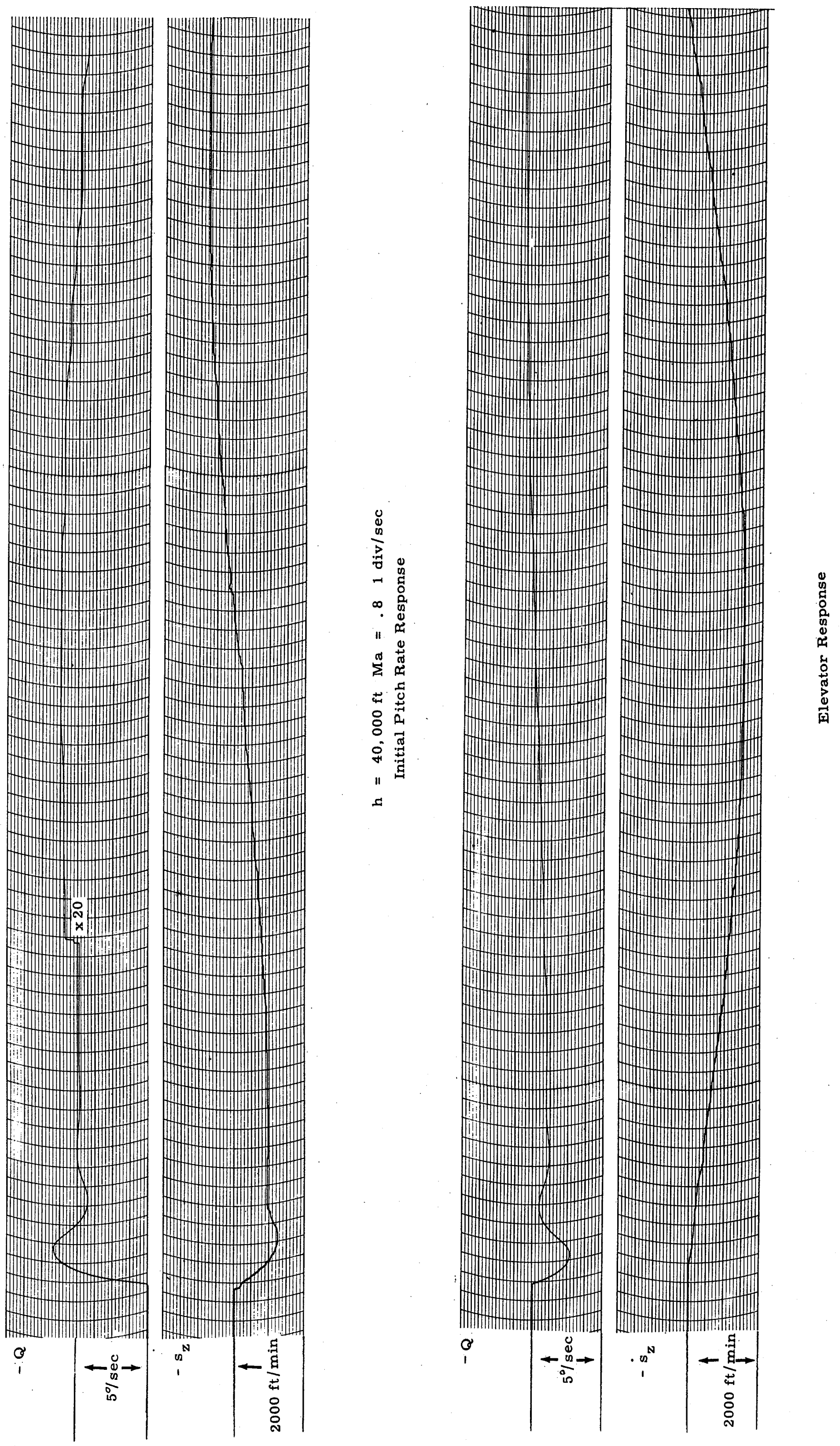

- Q
5°/sec
- s_z
2000 ft/min
x 20
h = 40,000 ft Ma = .8 1 div/sec
Initial Pitch Rate Response
- Q
5°/sec
- $\dot{s}_z$
2000 ft/min
Elevator Response

Figure 8

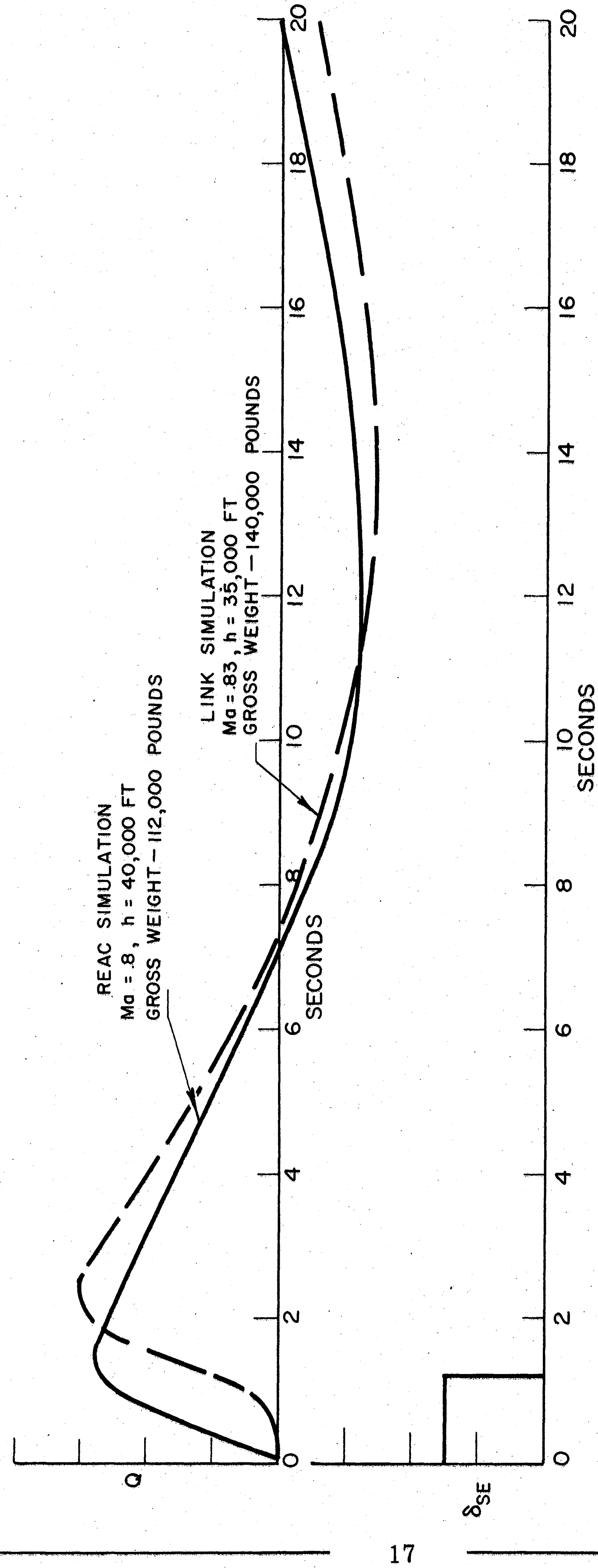


Fig. 9.

Pitch Rate Response to Impulse of Elevator

B-47 simulation were: 1. accuracy of steady-state solution; 2. ease of trimming to level flight or constant rate of climb; 3. duplication of short term dynamic response; 4. duplication of long term phugoid oscillations of small magnitude (something not achieved in any present ac simulation); 5. good resolution even for smallest motions encountered in normal flight.

The study also indicates that high resolution one-turn servo-multiplying potentiometers are adequate for the bulk of operations. This would allow use of simply, easy to maintain, high speed servos. Several critical trigonometric resolutions might require careful treatment to obtain high accuracy in small-motion simulation.

Finally, it should be pointed out that these results were obtained without component trimming, lead relocation or shielding, phase shifting, or other critical adjustments often required of large ac computers.