THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING

Department of Aeronautical and Astronautical Engineering

QUARTERLY PROGRESS REPORT

for April, May, June 1962

by
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1. Introduction

This report describes work accomplished in the period of April through June 1962 in the contractor's research program on various aspects of space vehicle booster control systems.

The study of recovery regions of linear systems with limited control effort described herein is by doctoral student Joseph L. LeMay under the supervision of Professor Elmer G. Gilbert.

2. Regions of Recoverability of Linear Systems with Input Constraints

This study is concerned with systems described by forced linear differential equations with prescribed input constraints. Previous progress reports have described results concerned entirely with the set of states which can be driven to the origin--the maximum region of recoverability, $RRM(t, t_o)$. Herein are described results concerned with other defined point sets related in some way to $RRM(t, t_o)$.

The set of states which can be reached from the origin is denoted by $REM(t, t_o)$. The set $RM(t, t_o)$ is the set of states which can be maintained in (t, t_o); i.e., the set of states for which the phase velocity can be forced to zero in the entire interval (t, t_o). Finally, $RC(t, t_o)$ denotes the region of controllability--the set of states any two of which can be "connected" in (t, t_o). For the constant case, variable-dependence simplifications occur as indicated notationally: $REM(T)$, RM, $RC(T)$. The variable T is elapsed time. For a constant system, the set RM is time invariant.

General properties of the above sets have been derived. For example, all are compact and convex if the control input set is compact and convex.

For a constant system, the set RM has been completely described. The set $RC(T)$ can be found from (or bounded by) regions of recoverability of the same and/or related systems. Results concerning $REM(t, t_o)$ exist elsewhere, primarily in the works of Roxin.

Properties of the above sets for simultaneous constraints on the input and input velocity are being investigated. Determination of the regions of recoverability for other sub-optimal control laws is being made. Delineation of the above sets for the specific example of the yaw (pitch) control of a space vehicle booster is in progress.

3. Computation of Optimum Controls

Under the direction of Professor Elmer G. Gilbert, professional degree student Robert L. Berg began work on the computation of optimum controls.

The problem considered is the minimum time regulation of a linear system subject to an amplitude constraint on the control input. The computational method is that of L. Neustadt (Journal of Mathematical Analysis and Applications, v. 1, no. 3, 4, Dec. 1960). Since it involves iterative solution of differential equations the analog computer seemed best suited and is being used. A modification of the computing method (derived by Elmer G. Gilbert) resulted in a reduction of the amount of equipment needed.

Actual computer results prove the practicability of the procedure for second order systems. In addition, analytic work on the stability of the iterative process provides a guide to choosing a suitable rate of convergence. Preliminary work on a third order system shows that there may be serious limitations with regard to computer accuracy and the number of iterations required. Details will appear in a report which is presently under preparation.

4. Stability Analysis of Elastic and Slosh Modes by the Method of Kryloff and Bogoliuboff

Under the guidance of Professor E. O. Gilbert, Capt. Guy H. Risley has completed work on a professional degree thesis which applies the method of Kryloff and Bogoliuboff to the stability analysis of booster elastic and slosh modes. The method, which entails certain averaging approximations, greatly simplifies stability calculations and gives analytical insight to the complicated couplings between "rigid body" modes and the highly oscillatory modes characteristic of elasticity and slosh. Now in printing, the thesis will be forwarded shortly.

To illustrate the basic concept of the method consider a typical elastic normal mode coordinate x described by the differential equation

$$\ddot{x} + c\omega\dot{x} + \omega^2 x = k\delta.$$

ω is the modal frequency, c is the damping coefficient assumed very small, and $k\delta$ is the forcing introduced by thrust vector deflection. Introducing $x = A \sin(\omega t + \phi)$ and requiring that $A(t)$ and $\phi(t)$ be such that $\dot{x} = \omega A \cos(\omega t + \phi)$, it is easy to show that

$$\dot{A} = -cA \cos^2 \omega t + \frac{k}{\omega} \delta \cos \omega t$$

$$\dot{\phi} = - c \sin \omega t \cos \omega t + \frac{k}{A\omega} \delta \sin \omega t$$

are equivalent to the above equation. Since the mode amplitude A and phase ϕ change very slowly, the average value of the terms on the right may be used with little error. Thus from the first equation

$$\dot{A} \approx - \frac{1}{2} cA + \text{avg} \frac{k}{\omega} \delta \cos \omega t$$

By expressing δ in terms of all the missile modes, the last term, upon averaging, becomes proportional to A. The proportionality constant α is a rather simple function of the control parameters and indicates the stability of the elastic mode. Thus if $\alpha - \frac{1}{2}c > 0$, the mode is unstable. All slosh and elastic modes are handled in a similar manner.

The details and certain ramifications of the method are discussed in the professional degree thesis. In one case, the analysis is verified by comparison with a much more complicated Nyquist criterion analysis. Another analysis concerned with slosh of oxidizer and propellant illustrates a case where the averaging technique is probably not wholly valid. Extensions of the theory to handle this important case, where two frequencies are very close together, are under way.

Statement of Man-Hours Expended
and
Summary of Expenditures
for April, May, June 1962

	April 1962	May 1962	June 1962	Three Month Period
Man-Hours Expended				
Faculty Participants	74	52	37	163
Graduate Students	88	88	80	256
Total	162	140	117	419
Salaries and Wages				
Faculty Participants	$669.40	$456.00	$292.00	$1,417.40
Graduate Students	330.00	330.00	300.00	960.00
Total	$999.40	$786.00	$592.00	$2,377.40
Overhead	519.75	495.77	316.50	1,332.02
Material and Supplies	36.60	1.35	8.55	46.50
Reports	40.09	205.53	41.00	286.62
Travel	0.00	0.00	0.00	0.00
Totals	$1,595.84	$1,488.65	$958.05	$4,042.54