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A STUDY OF LUNAR THERMAL EMISSION

by

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SUMMARY

A theoretical analysis of thermal radio emission from the moon is carried out and the available observational data reviewed. Values for the electromagnetic parameters of the moon's surface based on the results of radar investigations are used, thus enabling determination of the thermal parameters of the surface through a comparison between theoretical and experimental results. The thermal values obtained are found to be compatible with the character of the moon's surface layer suggested by radar experiments as well as by polarimetric and infrared measurements.

I

INTRODUCTION

The primary purpose of this paper is an attempt to correlate the available data on the moon's thermal emission with the properties of the lunar surface as inferred from studies of its radar reflection characteristics.

Since the original publication by Dicke and Beringer in 1946 [1], the development of increasingly sensitive receivers at microwave frequencies, as well as renewed interest in the physics of the lunar surface, has led a number of experimentors to turn their attention to the thermal radio emission from the moon. Such experiments can supply information about the lunar surface unobtainable by optical means and, in particular, enables the ratio of thermal to electromagnetic absorption in the surface layers to be calculated.

On the other hand, Senior and Siegel's original paper presented at the Paris Symposium on Radio Astronomy and since refined [2], indicated that radar reflection data from the moon contains considerable information about the electromagnetic characteristics of its surface.

It is natural then to attempt to tie in results of "passive" thermal observations with "active" radar experiments, and the picture which emerges is indeed consistent with both. In this manner one arrives at a determination of the thermal parameters involved which, in conjunction with the electromagnetic parameters deduced from the radar data, leads to somewhat more definite conclusions about the properties and nature of the lunar surface material.

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Section II is based on Senior and Siegel's paper [2] and is devoted to the electromagnetic parameters as derived from the radar experiments. Sections III and IV establish the theoretical connection between the thermal and electromagnetic parameters and the radiation characteristics observable in practice. Section V reviews the experimental data which are then compared to the theoretical results in Section VI. The conclusions reached are discussed in Section VII.

II

THE ELECTROMAGNETIC PARAMETERS

Radar studies of the moon in recent years have provided information about the electromagnetic parameters of the lunar surface. It has been shown (see [2]) that the shape of the echo can be explained if the leading part of the return corresponds to a specular reflection from a relatively small area in the center of the disc. The magnitude of this first peak then leads to a determination of the power reflection coefficient $|R|^2$ of this specular area. There are reasons to believe that the value so obtained is valid not only for the key scattering centers, i. e. the specular area or areas, but for the whole surface of the moon (under certain assumptions the diffuse part of the return yields the same value for $|R|^2$). If we postulate that the complex permittivity of the surface material is not wavelength dependent, implying that the conductivity is proportional to the frequency, then $|R|^2$ is constant. Indeed, the data obtained by observers working at various frequencies would support such an assumption, and when their results (see [2]) are averaged, it is found that the power reflection coefficient is approximately 5×10^{-4} . Since the R here refers to normal incidence, we have

$$R = \frac{1 - \sqrt{\epsilon' + i\epsilon''}}{1 + \sqrt{\epsilon' + i\epsilon''}},$$

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where ϵ' and ϵ'' are the real and imaginary part of the complex relative permittivity respectively, and we have taken the magnetic permeability μ to be that of free space. The equation $|R|^2 = 5 \times 10^{-4}$ defines a curve in the (ϵ', ϵ'') plane which has been plotted in Figure 1. Any point on this locus represents a possible value for the complex permittivity (i. e. values for ϵ' and ϵ'') which will satisfy the radar results. It should be noted that since we have ignored the frequency dependence of the measured reflection coefficients, ϵ' and ϵ'' are not determined separately as they were in [2], but only related via the loci in Figure 1. On the other hand, the fact that $\epsilon' > 1$ and $\epsilon'' > 0$ means that only the portion of the curve drawn as a solid line has to be considered.

From the figure it is evident that $\tan \delta = \epsilon'' / \epsilon'$ is exceedingly small.

The absorption coefficient for electromagnetic radiation is therefore given (see [15]) by:

$$\alpha = \frac{s}{2} \sqrt{\frac{\mu}{\epsilon' \epsilon_0}}$$

where the conductivity $s = 2\pi c_0 \epsilon'' \epsilon_0 / \lambda$; c_0 is the speed of light in vacuo, ϵ_0 the permittivity of free space, and λ the wavelength under consideration. Since $\mu = \mu_0$ and $c_0 = (\epsilon_0 \mu_0)^{-1/2}$, it follows that

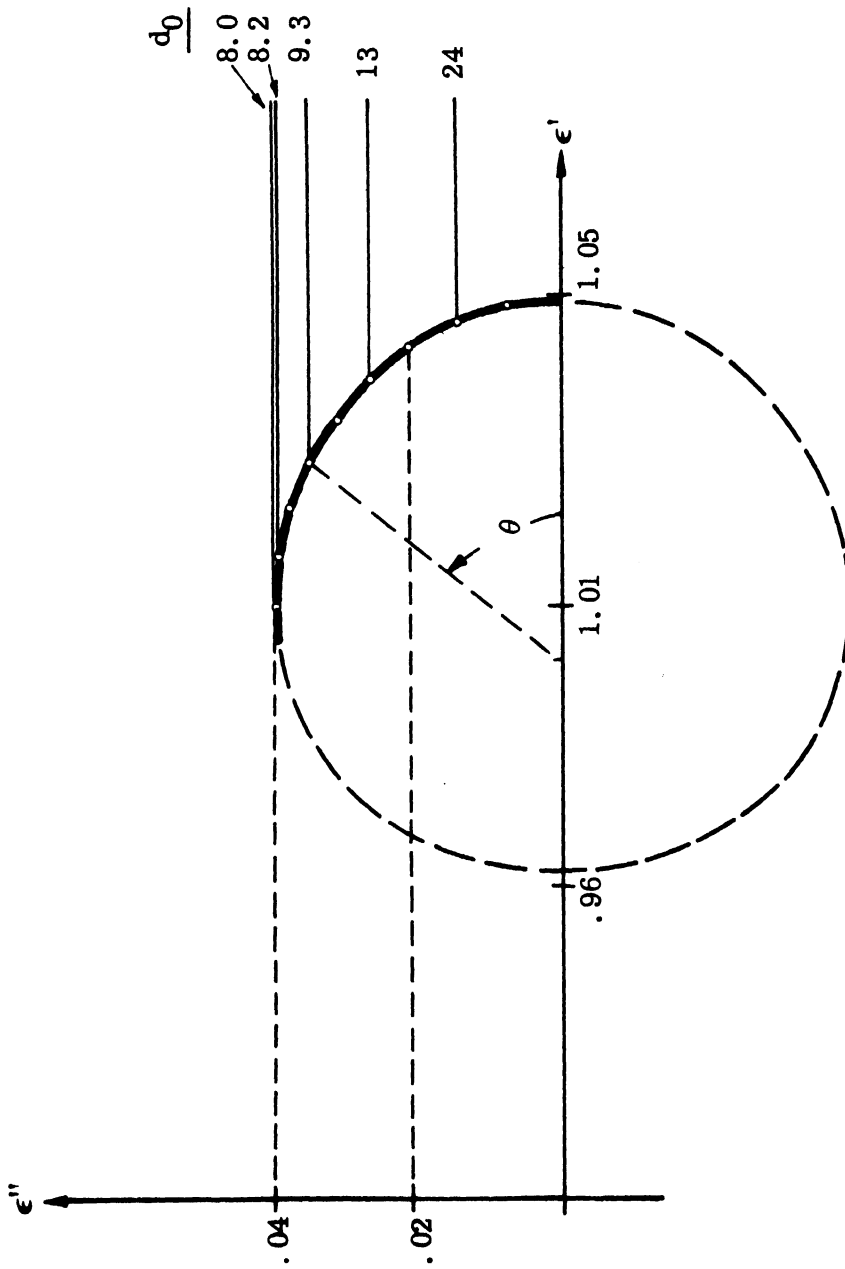


FIG. 1. VALUES OF ϵ' AND ϵ'' CORRESPONDING TO $R^2 = 5 \times 10^{-4}$.

The horizontal lines are portions of parabolas corresponding to the values of the relative skin depth shown on the right.

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$$\alpha = \frac{\epsilon''}{\sqrt{\epsilon'}} \frac{\pi}{\lambda}. \quad (1)$$

This can be rewritten as

$$\epsilon' = (\pi d_0 \epsilon'')^2, \quad (2)$$

where $d_0 = (\alpha \lambda)^{-1}$ is the relative skin depth and is dimensionless; the depth of penetration at a wavelength λ is then given by $d = d_0 \lambda$. Equation (2) defines a family of parabolas in the (ϵ', ϵ'') plane which has been plotted in Figure 1 for various values of the parameter d_0 . The following table gives a selection of values of ϵ' and d_0 for those $\tan \delta$ and ϵ' (where θ has been taken in 10^0 increments) which are consistent with radar data.

Table I

$\tan \delta \times 10^2$	ϵ'	$\epsilon'' \times 10^2$	d_0
.67	1.050	.7	47
1.31	1.048	1.4	24
1.91	1.045	2.0	16
2.42	1.041	2.6	13
2.96	1.036	3.1	11
3.34	1.030	3.5	9.3
3.67	1.024	3.8	8.6
3.87	1.017	3.9	8.2
3.96	1.010	4.0	8.0

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In passing it may be remarked that most substances occurring naturally on earth have relative permittivities at least as great as the largest value shown above, and for this reason the subsequent analysis is concentrated upon the first few lines in Table I.

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III

THE MOON'S THERMAL REGIME

The intensity and variation with time of the radio emission from the moon is connected with the thermal regime, the thermal parameters and the electromagnetic parameters of its surface layer, and to interpret the observational data it is necessary to establish the theoretical connection between these factors and the radiation characteristics observable in practice. This is best done for a small element of the moon's surface. On the other hand, the poor resolution associated with the use of relatively long wavelengths means that the observed antenna temperatures are averages over a large part of the lunar disc. This complicates the comparison between the calculated and measured temperatures. It follows that any attempt at a detailed consideration of the lunar surface from the point of view of thermal radiation must await more precise experiments, and the conclusions reached here therefore concern only the average characteristics of the surface material.

In contrast to the optical range of the spectrum where the moon acts as a reflector of sunlight, its behavior at infra-red and radio wavelengths is similar to that of a black body, and the amount of solar energy directly reflected is negligible. The portion of the lunar surface which is exposed to solar radiation changes continuously with time, and can be regarded as moving around the moon in 29.53 days, the period, P , of a synodic revolution. The heat generated at the surface

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by the absorption of energy is partly reradiated while the rest penetrates into the interior in the form of a strongly attenuated heat wave. Since there is no atmosphere to act as a thermal regulator, changes in the intensity of incoming radiation are followed rapidly by corresponding changes in the surface temperature.

The quantitative treatment of this problem requires the solution of the heat conduction equation for boundary conditions at the surface determined both by the amount of incident energy and the Stefan-Boltzmann law. The thermal parameters are the thermal conductivity, K , and the volumetric specific heat, c , and appear in the solution only in the combination $(Kc)^{-1/2}$.

The optical eclipse observations were first reduced by Epstein [3] and then, through a more refined analysis which also took into account the temperature variation during a lunation (lunar month), by Wesselink [4], Jaeger and Harper [5], Lettau [6] and Jaeger [7]. In all cases a value was arrived at for $(Kc)^{-1/2}$ of order 1000 (with CGS units, calories and degrees centigrade), under the assumption that the parameters involved were constant. This led to an agreement between theory and experiment which was good but not perfect. Indeed, all writers have recognized that it could be improved if the conductivity K increased either with depth or temperature. Muncey [8] has studied a model where K and c are proportional to the absolute temperature, while Jaeger [9] has investigated Piddington and Minnett's model consisting of a few millimetres of dust with $(Kc)^{-1/2} = 1000$ over a substratum with $(Kc)^{-1/2} = 120$.

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Although these models have their merits, each is hardly more than a convenient analytical picture and as Bracewell pointed out [10], the actual surface of the moon probably consists of more than one type of structure. In the absence of knowledge about the distribution and likely characteristics of these structures it may be better to retain the simpler and aesthetically more pleasing homogeneous model until such time as the precision of observations demands one which incorporates the more detailed features of the surface. We will have occasion later to suggest such a use of Piddington and Minnett's two-layer model.

Of importance for the discussion of microwave results is the actual distribution of the surface temperature. We have reproduced (Figure 2) Wesselink's theoretical curve corresponding to the homogeneous model with $(Kc)^{-1/2} = 920$, together with experimental values of Pettit and Nicholson [11] and Sinton [12]. In order to make use of this data, it is necessary to postulate a law for the temperature descent in the direction of the lunar poles. In agreement with most writers, a $\cos \psi$ law is assumed, where ψ is the selenocentric latitude. This is consistent with the results of infra-red observations when these have been reduced to take into account the directional effects in the radiating properties of the lunar surface. The surface temperature of a point located by its selenocentric longitude ϕ and latitude ψ can then be expressed as the Fourier series

$$T_s(\phi, \psi, t) = \cos \psi \sum_{n=0}^{\infty} T_n \cos(n\omega t - n\phi) \quad (3)$$

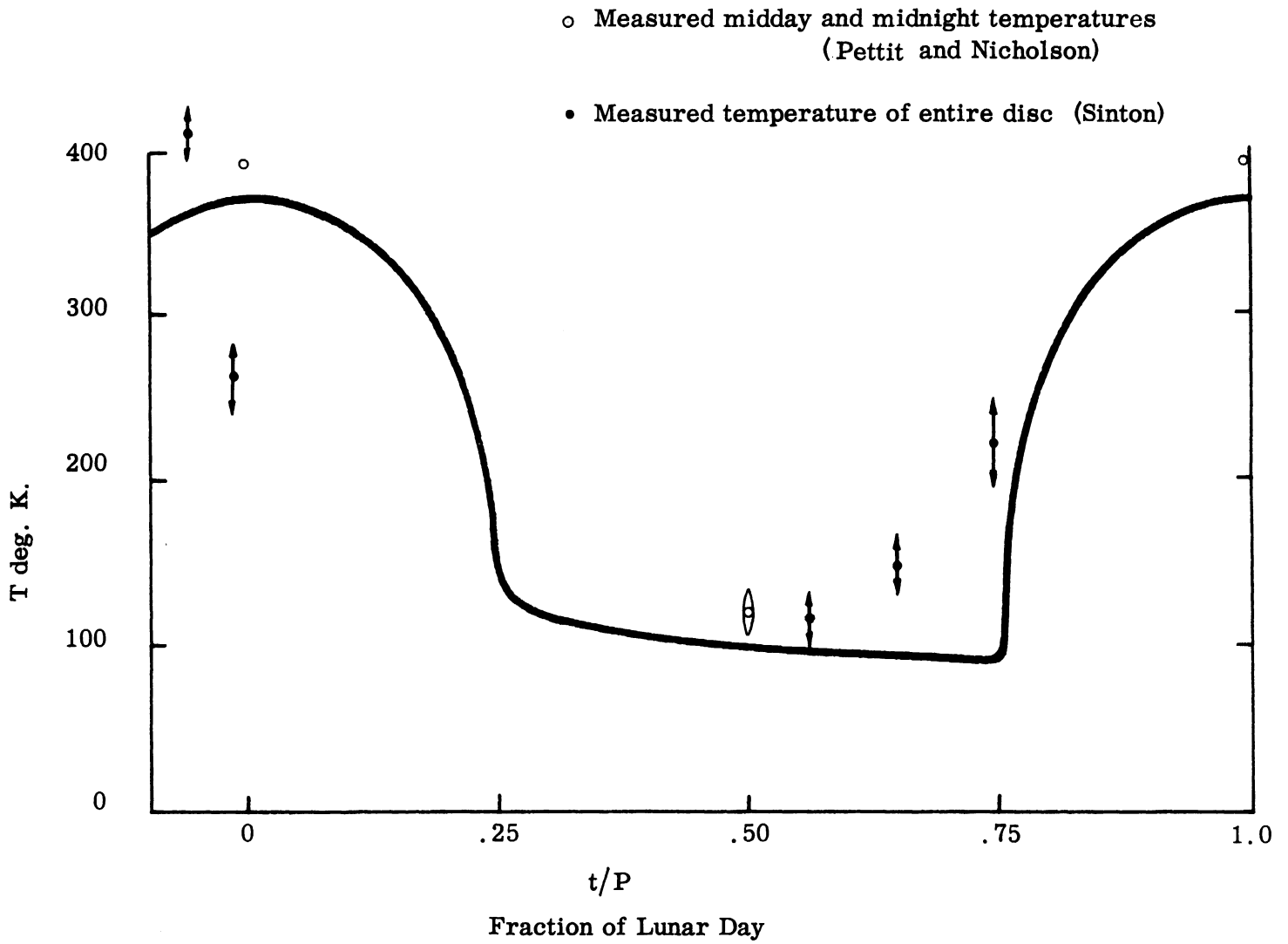


FIG. 2 CALCULATED SURFACE TEMPERATURE FOR A POINT NEAR THE CENTER OF THE LUNAR DISC.

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where $\omega = 2\pi/P = 2.46 \times 10^{-6}$ radians/sec is the angular frequency of the apparent revolution of the sun about the moon.

For the constant component, T_0 , of the surface temperature, a value of 220 deg. K is assumed. This is higher than the value found by numerical integration of the area under the curve in Figure 2, but is somewhat less than that indicated by the midnight temperature measured long ago by Pettit and Nicholson and recently confirmed by Sinton. The chosen value is therefore a compromise, and this is also true of the fundamental component, T_1 , of the variable part of the surface temperature for which the value 140 deg. K is assumed. It will be noted that if the harmonics in (3) corresponding to $n > 1$ are not negligible, the surface temperature will not be in phase with the lunar phase angle (i.e. the angle between the sun, the moon and the observer), but will lag to some extent. Although Pettit and Nicholson failed to detect such an effect, this is not surprising since the lag would only be pronounced at night, and the temperatures are then too low to be measured with any accuracy; in contrast, the daylight temperatures will be almost in phase with the incident radiation whether the harmonics are significant or not.

The variation of surface temperature will also cause periodically varying temperatures in the deeper layers as well. If x represents depth below the surface, the relation between the temperature $T(x)$ and the surface temperature at a point directly above as given by (3) is expressed mathematically by the Fourier series

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$$T_x(\phi, \psi, t) = \cos \psi \left\{ T_o + \sum_{n=1}^{\infty} T_n e^{-\sqrt{n}\beta x} \cos(n\omega t - \sqrt{n}\beta x - n\phi) \right\} \quad (4)$$

where $\beta = (2K/\omega c)^{-1/2}$ is the heat attenuation coefficient. Both phase lag and amplitude reduction are much more pronounced for the harmonics and consequently, with increasing depth, the temperature variation is ultimately provided by the fundamental component ($n=1$) alone. At a depth of $2\pi/\beta$, even the fundamental wave is reduced to less than 0.2 % of its surface amplitude, so that the temperature there is effectively constant and equal to $\cos \psi T_o$.

In the remainder of this discussion we shall consider the fundamental component alone, since the precision of the observations does not warrant the inclusion of any of the higher harmonics.

IV

THEORETICAL CONSIDERATIONS

The interpretation of radiometric measurements is more complicated than that of infra-red observations. This results from the fact that, since the upper layers of the moon's surface are partially transparent to electromagnetic radiation, the observed temperature is made up of emissions from all depths, suitably attenuated. The effective temperature, T_e , of an element, ds , of the moon's surface, defined as the temperature of a black body giving off the same intensity of radiation as ds , is derived in, for example, [13] and [14].

It can be shown (see [14]) that the effect of surface scattering is negligible due to the statistical cancellation of directivity effects when using beams of significant width (as is the case for all radiometers) and T_e is therefore given by the expression

$$T_e = \{1 - |R|^2(\theta)\} \alpha \sec \theta' \int_0^{\infty} T(x) e^{-\alpha \sec \theta' x} dx \quad (5)$$

where α is the electromagnetic absorption coefficient and $|R|^2$ the power reflection coefficient for the wavelength under consideration; θ and θ' are the angles between the normal and the axis of the elementary cones which confine the radiation through ds above and below the surface respectively, and are related by the law of refraction

$$\cos \theta' = \epsilon^{-1/2} (\epsilon - \sin^2 \theta)^{1/2},$$

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where ϵ is the dielectric constant of the material, and θ is related to the surface coordinates of ds by the equation $\cos \theta = \cos \phi \cos \psi$.

Substituting (4) into (5) and carrying out the integration, we obtain the effective temperature T_e of a small element of surface:

$$T_e(\phi, \psi, t) = \{1 - |R|^2(\theta)\} \cos \psi \left\{ T_o + \frac{T_1 \cos(\omega t - \phi - \phi_o)}{(1 + 2\gamma \cos \theta' + 2\gamma^2 \cos^2 \theta')^{1/2}} \right\} \quad (7)$$

where

$$\phi_o = \tan^{-1} \left(\frac{\cos \theta'}{1 + \cos \theta'} \right),$$

and

$$\gamma = \beta/\alpha.$$

The parameter γ is the ratio of thermal to electromagnetic absorption and indicates the relative importance of the variable part of the effective temperature.

Now if $P(X, Y)$ represents the power polar diagram of the radiometer's antenna normalized to unity, the observed temperature is

$$T_a = \frac{G_o}{4\pi} \int_{\Omega_o} T_e(\phi, \psi) P(X, Y) d\Omega \quad (8)$$

where G_o is the maximum antenna gain and Ω_o is the solid angle subtended by the moon. If T_e were independent of the surface coordinates, i. e. if the distribution of radiation temperature were given by a single constant value T_c all over the lunar disc, we would then have

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$$T_a = \frac{G_o}{4\pi} T_c \int_{\Omega_o} P(X, Y) d\Omega. \quad (9)$$

Choosing T_c such that (9) is equal to (8) we obtain the mean effective temperature

$$T_c = \frac{\int_{\Omega_o} T_e(\phi, \psi) P(X, Y) d\Omega}{\int_{\Omega_o} P(X, Y) d\Omega}. \quad (10)$$

It is of interest to consider the two extreme cases where the angular diameter of the moon is either very small or very large with respect to the beam width. In the first instance, $P(X, Y)$ can be taken to be unity over the range of integration so that from (10)

$$T_c = \frac{1}{\Omega_o} \int_{\Omega_o} T_e(\phi, \psi) d\Omega. \quad (11)$$

In the second case, the temperature will be that of the center of the disc, given by (7) with $\phi = \psi = \theta = 0$:

$$T_{\text{equator}} = \{1 - R^2(0)\} \left\{ T_o + \frac{T_1 \cos(\omega t - \phi_o)}{(1 + 2 + 2^2)^{1/2}} \right\}. \quad (12)$$

For the wide beam antenna, since

$$d\Omega = \frac{r_o^2}{r^2} \cos^2 \psi \cos \phi d\phi d\psi, \Omega_o = \frac{\pi r_o^2}{r^2}$$

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where r_0 is the radius of, and r the distance to, the moon, substitution of

(7) into (11) yields

$$T_{\text{average}} = \frac{T_0}{\pi} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \{1 - |R|^2(\theta)\} \cos^3 \psi \cos \phi \, d\phi \, d\psi$$

$$+ \frac{T_1}{\pi} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \{1 - |R|^2(\theta)\} \frac{\cos(\omega t - \phi - \phi_0) \cos^3 \psi \cos \phi}{(1 + 2\gamma \cos \theta' + 2\gamma^2 \cos^2 \theta')^{1/2}} \, d\phi \, d\psi \quad (13)$$

Equations (12) and (13) give the radiation temperature for the two limiting cases considered. This temperature has both a constant part, T_{co} , and a variable part, T_{cv} , the latter exhibiting both an amplitude reduction and a phase lag with respect to the variable part of the surface temperature. It can be seen that the effect of integrating over the disc is to reduce the constant part of the temperature by approximately 15% from its value at the center if the emissivity does not vary appreciably across the disc. The ratio T_{cv}/T_1 , denoted by A_v , and the phase lag are also reduced when using a wide beam antenna; the exact reduction, however, is connected with the dielectric constant ϵ of the surface.

Using the value of ϵ' given in Table 1, it is possible to evaluate the integral expression for T_{average} given in equation (13). For this purpose we require the reflection coefficient for angles of incidence different from normal. Its value depends on the polarization of energy and is given, to a sufficient degree

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of approximation, by the expression (see [15])

$$|R|^2(\theta) = \frac{1}{2} \left[\left(\frac{\epsilon' \cos \theta - \sqrt{\epsilon' - \sin^2 \theta}}{\epsilon' \cos \theta + \sqrt{\epsilon' - \sin^2 \theta}} \right)^2 + \left(\frac{\cos \theta - \sqrt{\epsilon' - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon' - \sin^2 \theta}} \right)^2 \right]$$

Using $\epsilon' = 1.05$, the maximum variation of the emissivity throughout the disc is easily computed. It is found to be nearly unity and practically constant almost out to the edge where it drops sharply to zero. There should therefore be a marked limb darkening in a ring approximately 1 min. of arc wide.

From the first term in the right hand side of (13) it follows that the mean value of the constant component of the microwave temperature should be about 190 deg. K. Since its value at the center is 220 deg. K., it is to be expected that the observed constant component will vary between these two values in accordance with the beam width used.

From the second term in the right hand side of (13), the dependence on γ of the ratio A_v of the variable part of the radiation temperature, T_{cv} , to that of the surface temperature, T_1 , can be found by numerical integration. The results have been plotted in Figure 3, together with those for the center of the disc. Here again the experimental value of A_v will depend on the beam width, the exact curve to be used in each case lying between the mean, integrated over the disc curve, and that which corresponds to the center of the disc.

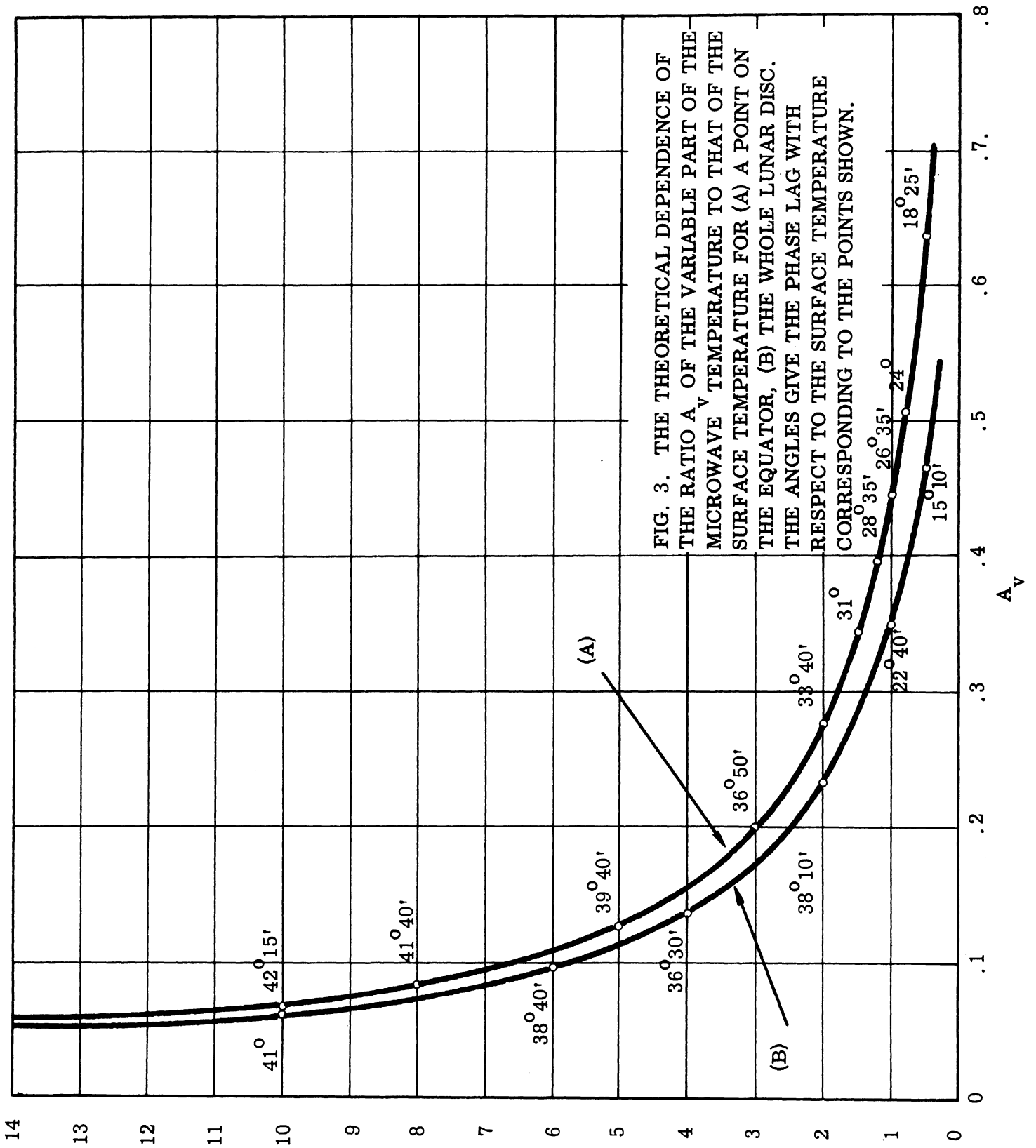


FIG. 3. THE THEORETICAL DEPENDENCE OF THE RATIO A_v OF THE VARIABLE PART OF THE MICROWAVE v TEMPERATURE TO THAT OF THE SURFACE TEMPERATURE FOR (A) A POINT ON THE EQUATOR, (B) THE WHOLE LUNAR DISC. THE ANGLES GIVE THE PHASE LAG WITH RESPECT TO THE SURFACE TEMPERATURE CORRESPONDING TO THE POINTS SHOWN.

V

RESULTS OF THE OBSERVATIONS

The radiation temperatures as deduced by various experimenters vary within wide bounds, and this variation is, perhaps, due to the differing methods used in the reduction of the data. For this reason, care must be taken when comparing the quoted values since the observations were carried out with a variety of beamwidths, making the reduction of mean temperature to equatorial temperature a frequent cause of disagreement.

Before tabulating the experimental results, some remarks of a general nature may be in order. The absolute error as given by the observers comes mainly from inaccuracies connected with the equipment (antenna calibration, etc.) and is itself an estimate; it is in the average of order $\pm 10 - 15 \%$. The relative error, a measure of the sensitivity of the reception, is in general around $\pm 5 \%$. We are, however, primarily interested in the ratio T_{cv}/T_{co} ; we should expect the measured values for this ratio to be more consistent and reliable, since it depends only on the relative error, than the measured values of either component alone.

As far as the phase lag of the radiation temperature is concerned, it will be seen, by comparing Table II and Figure 3, that the observed lag is almost always larger than theory predicts under the assumptions made. Although this

certainly indicates that the model under study is but a simplified picture, it is felt that phase lag considerations should not be given undue weight and, for the following reasons, do not justify a rejection of the homogeneous model: First of all, the determination of the phase lag is subject to a large inaccuracy which, because of the probable shape of the actual microwave temperature curve (see [16], [9]), always leads to an overestimate rather than the other way around. Secondly, as we have already seen, no matter how small the flow of heat into the interior of the moon, it will combine with the reradiated energy to give a resultant having a definite lag, especially during the lunar night; and while the theoretical microwave temperatures are referred in phase to the surface temperature, the experimental ones are referred to the lunar phase angle. This will also tend to increase the "measured" phase lag. Finally, a large phase lag can be explained by a consideration of the more detailed features of the surface, if in a number of places the ratio of thermal to electromagnetic absorption (γ) decreases with depth, either in a continuous manner (both conductivities increasing with density for instance), or discontinuously (as in the two-layer model). We will have more to say about this in Section VII

The following table shows the values of T_{co} , T_{cv} and the phase lag as found at various wavelengths. Whenever possible the estimated errors are also given.

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Table II - Experimental Results

	λ (cm.)	T_{co}	T_{cv} (deg. K)	Phase lag	Absolute Error %	Relative Error %
(i)	.43	220	60			
(ii)	.80	197	32(>39.5)	30°	±10	
(iii)	.86	225	45	40°	±15	
(iv)	1.25	216	36.4	45°	±12	
(v)	1.36	224	36	40°	±10-15	±5
(vi)	2.2	220	20		±10	
(vii)	3.2	183	< 13		±20	±7
(viii)	10	315				
(ix)	20.5	250.2	< 5		±12	±2
(x)	21	245				
(xi)	33	208				
(xii)	75	185				

(i) was derived from Coates' observations made with a 10-ft. paraboloid [17]. The beamwidth of 6.7 min. of arc permitted a detailed resolution. Since the results were stated for three days only - fortunately well distributed - the values given above involve a large amount of extrapolation.

(ii) was obtained by Salomonovich [18] with a 2-meter parabolic reflector and a beamwidth not much smaller than the moon's angular diameter. A very recent communication by the same author [19] dealing with observations made at the same wavelength with a 22-meter radiotelescope indicates that the value of T_{cv} is more than 20% of T_{co} ; the implied value of T_{cv} is shown in parentheses. The beamwidth of only 2 min. of arc permitted thermal mapping and detection of marked limb darkening near the edge of the disc in accordance with our theoretical results showing angle dependency in emissivity of the surface.

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(iii) was derived from Gibson's measurements using a 10-ft. paraboloid [16] with a beamwidth of 12 min. of arc. A curve is given which we have corrected to reduce to main lobe temperature as suggested in [20] for a stray factor of 0.2.

(iv) are Piddington and Minnett's [13] results for the average temperatures reduced slightly because of the incorrect value of the reflection coefficient used in the treatment of the data. The beamwidth was $3/8$ of a degree. The value given for the phase lag is quite high and necessitated the two-layer model by way of explanation.

(v) as given by Troitskii et al. [21] working with a 4-meter paraboloid having a beamwidth comparable to the moon's angular diameter.

(vi) was derived from observations made by Grebenkemper [22] with a 50-ft. parabolic reflector. The observations were recorded on five days only, but corresponding to fairly well-distributed lunar phases. The temperatures have been increased by 10% to take into account a spill-over factor of 0.1 suggested by the observer.

(vii) is from a report by Troitskii and Zelinskaya [23]. No phase dependence was observed within the relative error of $\pm 7\%$, whence the maximum value for T_{cv} .

(viii) is given by Akabane [24] who also found at first a phase dependence of the temperature. It seems, however, that the variation of the moon's apparent

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diameter had not been taken into account, so that in all probability no phase dependence was present. Akabane himself has indicated [25] that his value for the constant component should not be relied upon.

(ix) was obtained by Mezger and Strassl [20] from observations with a 25-meter radiotelescope. No phase dependence was observed within $\pm 2\%$.

(x) (xi) and (xii) are cited in [26]. No phase dependence of the temperature was found.

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VI

COMPARISON BETWEEN THEORY AND EXPERIMENT

If the emissivity is not wavelength dependent as is implied in our assumption about the complex permittivity of the lunar surface material, and if there is no constant temperature gradient normal to the surface of the moon (i. e. no radioactive or other internal heat source), T_{CO} should be constant, and its value should lie somewhere between 220 and 190 deg. K, the higher the more narrow the beamwidth in accordance with our theoretical results. If we average the results obtained at different wavelengths (excluding Akabane's for the reasons stated above), the value 215 deg. K is found and this is within the range. No conclusion, however, can be drawn about a possible intrinsic lunar heat flux since we have averaged values integrated over varying portions of the disc of the moon. Moreover, the precision of the observations is not sufficient to warrant further investigation of this matter.

Normalizing the data so as to obtain $T_{CO} = 215$ deg. K in all cases, we arrive, by simple proportion, at values T_{cv}^* for the variable component which can be compared with one another. The following table shows the results of normalizing the variable component in this manner. Also shown are the corresponding ratios A_v (of T_{cv}^* to T_1), and the values of γ which can then be deduced from Figure 3, the beamwidth being duly taken into account.

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Table III - Normalized Data

	T_{cv}^*	A_v	γ
(i)	58.8	.42	.9
(ii)	> 43.2 (a)	> .309	< 1.5
(iii)	43.1	.308	1.5
(iv)	36.4	.26	2
(v)	34.7	.248	2.1
(vi)	19.6	.14	4
(vii)	< 15.3	< .109	> 5.2
(ix)	< 4.3	< .031	> 20

Using (1) and the definition $\Upsilon = \beta/\alpha$ we obtain

$$\gamma = \beta d_0 \lambda \quad (14)$$

If (14) is now fitted to the data, it is found that $\beta d_0 = 1.7$. With this value, it is seen from Figure 4 that the expression for γ as a function of the wavelength satisfies the data within the margin of experimental errors. An interesting point is that the phase dependence of the temperature at 3.2 cm seems to have been just under detection. If the experiment were repeated with a relative error of $\pm 5\%$ or less, the varying component should appear. This, or any detection of a variation with phase at longer wavelengths which could be predicted on the basis of Figures 3 and 4 would lend support to our assumptions.

(a) This corresponds to the most recent observations by Salomonovich.

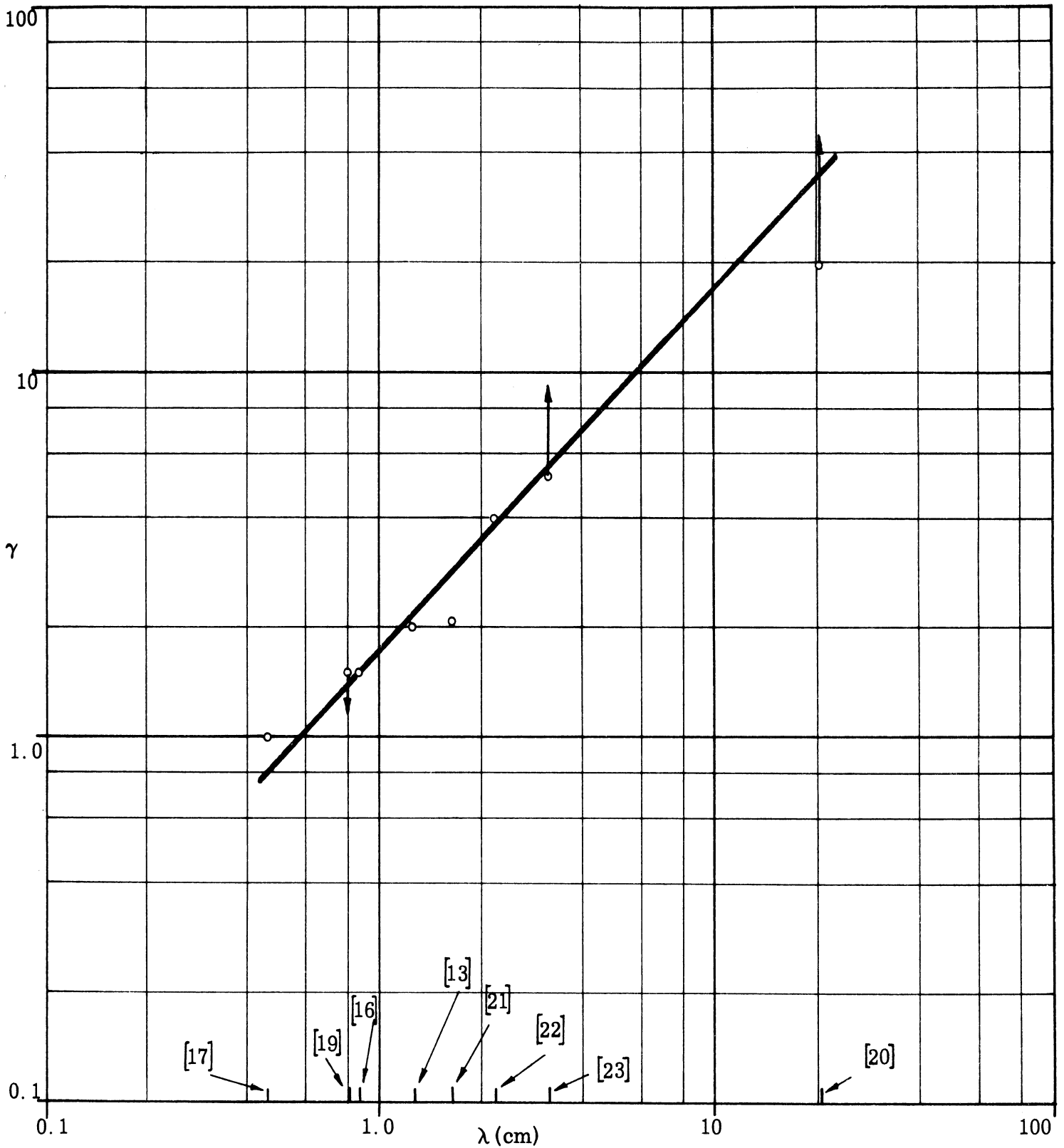


FIG. 4. COMPARISON BETWEEN THEORY AND EXPERIMENT

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Knowing the value of d_0 for different ϵ' we can obtain β . Taking only the range of values of ϵ' regarded as more probable, we have:

ϵ'	$1/\beta$ (cm)
1.05	27.6
1.048	14.1
1.045	9.4
1.041	7.6

It will be recalled that $2\pi/\beta$, the so-called heat wavelength, represents the depth of penetration of the heat wave. According to the above, it is therefore of order 1 meter or more. This is higher than the values estimated by Wesselink for the homogeneous model and by Jaeger for the two-layer model (14.5 and 5 cm respectively).

We have seen that optical observations yield $(Kc)^{-1/2} = 920$, and since

$$\beta = \left(\frac{2K}{\omega c}\right)^{-1/2} = 1.1 \times 10^{-3} \left(\frac{K}{c}\right)^{-1/2},$$

the above values of β imply:

$K(\text{cal. sec.}^{-1} \text{cm.}^{-1} \text{deg.})$	$c(\text{cal. deg.}^{-1} \text{cm.}^{-3})$
3.3×10^{-5}	.036
1.7×10^{-5}	.070
1.1×10^{-5}	.105
$.9 \times 10^{-5}$.130

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VII

DISCUSSION OF THE RESULTS

In view of the radar results quoted in Section II it is apparent that the electromagnetic conductivity and permittivity of the moon's surface material may have been overestimated by previous workers who based their assumptions on comparisons with earth rocks. For this reason the values now obtained for K and c differ by about one order of magnitude from those found before: K is larger and c is smaller.

In spite of this, the value of the thermal conductivity is still consistent with a loose material such as dust in vacuo. The low value of the volumetric specific heat almost certainly implies an equally low value for the density, and this also supports the hypothesis of a loosely packed material. Still further support is provided independently by the manner in which light is reflected at the surface of the moon. The variation of moonlight intensity with phase angle can be matched in the laboratory by a model consisting of a surface coated with very small pits as proposed by Schoenberg [27]. On the other hand, polarimetric studies by Lyot [28] and Wright [29] have shown that the characteristic form of the polarization-phase curve can only be duplicated by using a finely ground or ash-like material: the strong negative branch of the polarization curve characterizes a substance made up of juxtaposed opaque granules. Dollfus [30]

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points out that this allows one to affirm the absence of quartz or limestone in marble formation in the moon's surface layer. Another independent verification is furnished by the high emissivity of the surface in the infrared range (see [11]) which is incompatible with the low emissivity of silicates. Although it is difficult on the basis of the above to ascertain the nature of the lunar surface material, its vesicular structure appears well established.

Following Wesselink, we can derive an upper bound for the average dimensions of granules and interstices in that material by studying an imaginary powder where conduction of heat occurs mainly by radiative transfer. It is found in this way that the sum of an average granule and interstice is at most 1 or 2 mm. , since larger sizes would imply a value of the conductivity in excess of those obtained. It can be noted that unless the actual dimensions are less than this, there will be a significant dependence of the thermal conductivity on the temperature.

Whether such a loosely packed material would, under the conditions prevailing on the moon, exhibit values of the electrical conductivity and permittivity as low as what was found remains open to investigation. Experiments in the laboratory can possibly throw some light on this question.

As far as the assumption of homogeneity is concerned, the following qualitative remarks may be pertinent. We have mentioned in Section V that the observations at 4.3 and 8.0 mm. were sufficiently precise to permit detailed

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resolution. The maps obtained by Coates [17] and Salomonovich [19] show definitely that the maria are in general warmer, and respond more rapidly to changes in the amount of insolation than the highlands or lighter parts of the moon's surface. If we try to modify our average results to account for this, it is seen that the ratio of thermal to electromagnetic absorption, γ , should be larger than the average for the highlands, and smaller for the maria; however, because of the greater effective surface area possessed by the maria if bottom of craters and other dark stretches are included, it is to be expected that for them the difference from the average would not be as great as for the highlands. Since no part of the moon's surface is believed to have high electrical conductivity, the results obtained for the maria would still be consistent with a dust, but now, for the highlands, we would have to assign values to the thermal conductivity below those obtained in the average. So that even though the material which forms the maria would still not be of the nature of a hard solid, the material which constitutes the highlands would have to be even more tenuous. For reasons which are more or less obvious, this picture is unsatisfactory.

The possibility that the difference in thermal behavior between maria and highlands is due to their difference in surface roughness has been investigated and found equally unsatisfactory. The next simplest alternative is to refer back to Piddington and Minnett's two-layer model. If we assume indeed that the highlands

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consist of a relatively hard substratum covered by a thin layer of the material which can be found in the maria, then, because of the temperature drop through that layer which is at the same time practically transparent to microwave radiation, the highlands will heat up and cool off more slowly than the maria as is required by the observations.

VIII

CONCLUSION

A set of values indicative of the order of magnitude of the average electromagnetic and thermal parameters of the moon's surface layer has been determined. These include the electrical conductivity and permittivity, the thermal conductivity and the volumetric specific heat. The values obtained are consistent with optical, infrared, radio and radar observations. They are characteristics of a material in a somewhat degenerate state, possibly radiation damaged, with a low density (probably increasing with depth) indicating a loose structure. The latest observations suggest that this material, while filling up the maria, bottom of craters and other low grounds, covers the highlands with a relatively very thin layer.

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September 1960

Reference 19, correct to read, Amenitskii, N. A., et al, Astron. Zhur. 37 ,
(1), 185 (1960).

Reference 21, correct to read, Zelinskaya, M. R., et al, Astron. Zhur. 36 ,
(4), 643 (1959).

Reference 24, correct to read, Proc. Japan Acad. Sci. 31 , (3), 161; and
Tokyo Astron. Obs. Rep. No. 115 (1955).

Reference 26, correct to read, Seeger, C. L., et al, J. Astrophys. , 126 ,
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Reference 27, correct to read, Handbuch der Astrophysik , 2 , 76 (1929).

Reference 28, correct to read, Ann. Obs. Paris-Meudon, 7 , 1 (1929).