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Technical Report

TRANSPARENT CATEGORIES AND CATEGORIES OF TRANSITION SYSTEMS

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Title: "Transparent Categories and Categories of Transition Systems"
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Background: The Logic of Computers Group of the Communication Sciences Department of The University of Michigan is investigating the application of logic and mathematics to the design of computing automata. The application of the techniques and concepts of abstract algebra to automata forms a part of this investigation.

Condensed Report Contents: Several recent results in automata theory give evidence of the importance of homomorphisms in the study of transition systems and automata. It is natural therefore to inquire how much information can be retrieved from the algebra of homomorphism compositions with respect to transition systems. The natural mathematical framework for the discussion of this problem is categorical algebra.

We define a category $\mathbb{A}_W$ of the transition systems with input W, where W is any arbitrary fixed monoid, and with arbitrary sets of states. A preliminary study of $\mathbb{A}_W$ (Give'on 1964) shows that one can reconstruct the internal structure of any transition system from the way homomorphisms (i.e., the morphisms of $\mathbb{A}_W$) behave around it.

In this paper we show that $\mathbb{A}_W$ has a generator, M_W (which is W operating on itself as a transition system) and that there exists a functor $\mathrm{Mor} : \mathbb{A}_W \longrightarrow \mathbb{A}_W$ naturally equivalent to the identity functor of $\mathbb{A}_W$ which factors through $\mathrm{Hom}_{\mathbb{A}_W}(M_W,-)$.

A general exposition of the nature of properties which are retrievable from the "morphism-behavior" in an arbitrary category is presented so that it provides a rigorous general basis for studying "retrievable" properties and categories in which every structural property of objects and morphisms is "retrievable."

Finally, we prove that for a very broad class of input monoids, which includes all the types of input-monoids encountered in automata theory, the categories $\mathbb{A}_W$ are <u>transparent</u>. That is, anything which can be said about the structure of transition systems with input W, can be said by referring to their homomorphisms only. In particular, all the automorphisms of $\mathbb{A}_W$, for this type of W, are naturally equivalent to the identity functor of $\mathbb{A}_W$.

<u>For further information</u>: The complete report is available in the major Navy technical libraries and can be obtained from the Defense Documentation Center. A few copies are available for distribution by the author.

1. INTRODUCTION

Several recent results in automata theory (in particular, Hartmanis & Stearns 1964, Zeiger 1964) give evidence of the importance of homomorphisms in the study of transition systems and automata. It is natural therefore to inquire how much information can be retrieved from the algebra of homomorphism compositions with respect to transition systems. The natural mathematical framework for the discussion of this problem is categorical algebra.

We define a category $\mathbb{A}_W$ of the transition systems with input W, where W is any arbitrary fixed monoid, and with arbitrary sets of states. A preliminary study of $\mathbb{A}_W$ (Give'on 1964) shows that one can reconstruct the internal structure of any transition system from the way homomorphisms (i.e., the morphisms of $\mathbb{A}_W$) behave around it.

In this paper we show that $\mathbb{A}_W$ has a generator, M_W (which is W operating on itself as a transition system) and that there exists a functor Mor : $\mathbb{A}_W \longrightarrow \mathbb{A}_W$ naturally equivalent to the identity functor of $\mathbb{A}_W$ which factors through $\mathrm{Hom}_{\mathbb{A}_W}(M_W,-)$.

A general exposition of the nature of properties which are retrievable from the "morphism-behavior" in an arbitrary category is presented so that it provides a rigorous general basis for studying "retrievable" properties and categories in which every structural property of objects and morphisms is "retrievable."

Finally, we prove that for a very broad class of input monoids, which includes all the types of input-monoids encountered in automata theory, the categories $\mathbb{A}_W$ are <u>transparent</u>. That is, anything which can be said about the structure of transition systems with input W, can be said by referring to their homomorphisms only. In particular, all the automorphisms of $\mathbb{A}_W$, for this type of W, are naturally equivalent to the identity functor of $\mathbb{A}_W$.

Some elementary acquaintance with categorical algebra is needed. In particular, we shall make use of the following notions:

(i) <u>Category</u>, it <u>objects</u> and its <u>morphisms.</u>

(ii) <u>Epic</u>, <u>monic</u>, and <u>invertible</u> morphisms versus <u>surjective</u>, <u>injective</u>, and <u>bijective</u> functions.

(iii) <u>Initial</u> and <u>terminal</u> objects.

(iv) <u>Functors</u>, <u>natural transformations</u> and <u>natural equivalences</u> of functors.

(v) <u>Embedding</u> functors, <u>automorphism</u> functors, and <u>adjoint</u> functors.

The reader who is not familiar with these notions is referred to the literature (Kan 1958, Freyd 1964, and MacLane 1965). Additional issues of categorical algebra with reference to automata theory are discussed in (Give'on 1965).

2. CATEGORIES OF TRANSITION SYSTEMS

2.1 Let W be a fixed monoid, We denote by $\mathbb{A}_W$ the category specified as follows.

The objects of $\mathbb{A}_W$ are <u>transition-systems</u> with input W. That is, systems of the form

$$A = (S(A) \times W \xrightarrow{\lambda_A} S(A))$$

where:

(i) $S(A)$ is any set, the <u>set of states</u> of A;

(ii) $\lambda_A : S(A) \times W \longrightarrow S(A)$ is a function, the <u>transition function</u> of A, with the following properties (we write $s \cdot \omega$ for $\lambda_A(s,\omega)$):

(iii) $s \cdot 1_W = s$ for all $s \varepsilon S(A)$, where 1_W is the identity element of W;

(iv) $s \cdot (\omega_1 \omega_2) = (s \cdot \omega_1) \cdot \omega_2$ for all $s \varepsilon S(A)$ and all $\omega_1, \omega_2 \varepsilon W$.

The morphisms of $\mathbb{A}_W$ are of the form

$$A \xrightarrow{f} B$$

where $f : S(A) \longrightarrow S(B)$ is a function satisfying $f(s \cdot \omega) = f(s) \cdot \omega$ for all $s \varepsilon S(A)$ and all $\omega \varepsilon W$. (Note that $s \cdot \omega$ on the left hand of this equation refers to the transition function of A, while $f(s) \cdot \omega$ refers to the transtion function of B.)

The composition of the morphisms of $\mathbb{A}_W$ is determined in an obvious manner by the composition of the functions which underly the morphisms. That is, $(C \xrightarrow{g} D)(A \xrightarrow{f} B)$ is defined <u>only when $B = C$</u> and then it is equal to $A \xrightarrow{gf} D$.

2.2 As in many other "natural" categories of mathematical systems, we have a <u>forgetful</u> functor $S : \mathbb{A}_W \longrightarrow \mathbb{S}$ from $\mathbb{A}_W$ to $\mathbb{S}$ the category of sets where $S(A)$ is tne set of states of A and $S(A \xrightarrow{f} B) = (f : S(A) \longrightarrow S(B))$.

Note that $\mathbb{A}_W$ contains, among its objects, an <u>empty</u> object to be denoted by ϕ_A. Here we adopt the useful convention that for any set T there exists a unique function which is injective (i.e., one-one into) from ϕ, the empty set, into T. Thus, the transition function of ϕ_W is this "empty" function : $\phi \times W \longrightarrow \phi$ $(\phi \times W = \phi)$ and for any object A of $\mathbb{A}_W$ there exists a unique morphism $\phi_W \xrightarrow{\sigma_A} A$ which is determined by the "empty"

$\phi = S(\phi_W) \longrightarrow S(A)$.

The forgetful functor $S : \mathbb{A}_W \longrightarrow \mathbf{S}$ has an adjoint (cf. Kan 1958, MacLane 1965), the functor $Fr : \mathbf{S} \longrightarrow \mathbb{A}_W$, which assigns to each set T, an object $Fr(T)$ of $\mathbb{A}_W$ which is free on $T \subseteq S(Fr(T))$.

The functor $Fr : \mathbf{S} \longrightarrow \mathbb{A}_W$ can be specified as follows. For any set T, $Fr(T)$ is the transition system defined by:

$$S(Fr(T)) = T \times W ,$$

$$(t, \omega_1) \cdot \omega_2 = (t, \omega_1 \omega_2) \quad .$$

For any function $f : T_1 \longrightarrow T_2$ there exists a unique morphism

$Fr(T_1) \xrightarrow{Fr(f)} Fr(T_2)$ such that for any $t \in T_1$: $[Fr(f)](t, 1_W) = (f(t), 1_W)$.

Hence $Fr(T)$ is "free on $T \times \{1_W\}$". We identify the elements of $T \times \{1_W\}$ with the elements of T : $t \equiv (t, 1_W)$.

If T_1 and T_2 are sets which have the same cardinality, then $Fr(T_1)$ and $Fr(T_2)$ are isomorphic (i.e., there exists an invertible morphism $Fr(T_1) \longrightarrow Fr(T_2)$ of $\mathbb{A}_W$. In particular, if T is a single-element set then we denote $Fr(T)$ by M_W.

M_W serves a very important role in $\mathbb{A}_W$ as we shall see later. Note that M_W may be defined as W operating on itself. That is,

$$S(M_W) = W,$$

$$\omega_1 \cdot \omega_2 = \omega_1 \omega_2 \quad .$$

2.3 $\mathbb{A}_W$ shares with the obelian "natural" categories, e.g., of groups or of modules, (cf. Freyd 1964) the property that the monic (respectively, the epic, and the invertible) morphisms are precisely those morphisms of $\mathbb{A}_W$ whose underlying functions are injective (respectively, surjective and bijective). The arguments that establish these facts are similar to the arguments employed in the category of groups for the same end.

The existence of the forgetful functor $S : \mathbb{A}_W \longrightarrow \mathbf{S}$ implies that a morphism $A \xrightarrow{f} B$ is invertible in $\mathbb{A}_W$ iff f is bijective. Since S is an embedding functor, every morphism of $\mathbb{A}_W$ whose underlying function is injective (respectively, surjective) must be monic (respectively, epic).

In order to prove the converse (for monic and epic morphisms of $\mathbb{A}_W$) we need some additional observations about $\mathbb{A}_W$. These observations will be incorporated in the proofs of the following lemmata.

2.3.1 LEMMA: If $A \xrightarrow{e} B$ is an epic morphism of $\mathbb{A}_W$ then $e : S(A) \longrightarrow S(B)$ is surjective.

Proof: The image of $e : S(A) \longrightarrow S(B)$ is a subset $e(S(A))$ of $S(B)$ such that for any $\omega \varepsilon W$ and any $s \in e(S(A))$, $s \cdot \omega \in e(S(A))$. Hence $e(S(A))$ is a transition system $e(A)$ which is a sub-system of B.

We define a new object $B/e(A)$ of $\mathbb{A}_W$ by:

$S(B/e(A)) = (S(B) - e(S(A))) \cup \{s_*\}$ where $s_* \notin S(B)$;

the transition function of $B/e(A)$ is the same as of B except for the cases where $s \cdot \omega \varepsilon e(S(A))$; in these cases we set $s \cdot \omega = s_*$, and for all $\omega \varepsilon W$ we set $s_* \cdot \omega = s_*$.

Obviously, $B/e(A)$ is formed from B by contracting $e(S(A))$ to a single state s_*. This contraction takes the form of a canonical morphism $B \xrightarrow{q_e} B/e(A)$, where $q_e : S(B) \longrightarrow S(B/e(A))$ is identical on $S(B) - e(S(A))$ and it maps all of $e(S(A))$ onto s_*.

In addition to $B \xrightarrow{q_e} B/e(A)$, we have another morphism $B \xrightarrow{z} B/e(A)$ which maps all of $e(S(A))$ onto s_*. Clearly e is surjective iff $q_e = z$.

Obviously $ze = q_e e$, since both map all of $S(A)$ onto s_*. But e is epic and therefore $ze = q_e e$ implies $q_e = z$.

2.3.2 <u>LEMMA:</u> If $A \xrightarrow{j} B$ is a monic morphism of $\mathbb{A}_W$ then $j : S(A) \longrightarrow S(B)$ is injective.

<u>Proof:</u> Assume that for $s_1, s_2 \varepsilon\ S(A)$ we have $j(s_1) = j(s_2)$. We define two morphisms $M_W \xrightarrow{f_1} A$ and $M_W \xrightarrow{f_2} A$ by $f_1(1_W) = s_1$, and $f_2(1_W) = s_2$. Obviously, $jf_1 = jf_2$, and since j is monic, it follows that $f_1 = f_2$; i.e., $s_1 = s_2$.

2.4 For any object A of $\mathbb{A}_W$ any any subset $T \subseteq S(A)$, we define $A(T)$, _the subsystem of A generated by T,_ as follows:

$$S(A(T)) = T\cdot W = \{t\cdot \omega : t\varepsilon T \text{ and } \omega\varepsilon W\},$$

$$(t\cdot\omega_1)\cdot\omega_2 = t\cdot(\omega_1\omega_2).$$

A subset T of S(A) is said to _generate_ A iff A(T) = A; i.e., iff $T\cdot W = S(A)$. In particular A is said to be _monogenic_ iff A is generated by a single-element subset of S(A).

For example, M_W is monogenic since $\{1_W\}$ generates M_W (Obviously for any $\omega\varepsilon W$: $1_W\cdot\omega = \omega$). More generally, M_W is generated by {u} iff there exists $v \varepsilon W$ such that $uv = 1_W$.

Note that an object A of $\mathbb{A}_W$ is monogenic iff for any $T \subseteq S(A)$ which generates A there exists $t\varepsilon T$ such that {t} generates A.

2.4.1 _LEMMA:_ An object A of $\mathbb{A}_W$ is monogenic iff for any family $\{A_j\}$ of subsystems of A indexed by a set J, $\cup\{S(A_j) : j\varepsilon J\} = S(A)$ implies $S(A_j) = S(A)$ for some $j\varepsilon J$.

Proof: Assume that A is monogenic and generated by $\{s_o\}$. If $US(A_j) = S(A)$ then $s_o \varepsilon S(A_j)$ for some $j \varepsilon J$, and $S(A_j) = S(A)$.

Assume that for any family $\{A_j\}$ indexed by a set J, $US(A_j) = S(A)$ implies $S(A_j) = S(A)$ for some $j \varepsilon J$. Define the family $\{A_s\}$ for all $s \varepsilon S(A)$, where A_s is the subsystem of A generated by $\{s\}$. Obviously $US(A_s) = S(A)$ and therefore there exists $s_o \varepsilon S(A)$ for which $S(A_{s_o}) = S(A)$. Hence A is generated by $\{s_o\}$.

2.4.2 COROLLARY: For any monogenic object A of $\mathbb{A}_W$ and any automorphism $F : \mathbb{A}_W \longrightarrow \mathbb{A}_W$ of $\mathbb{A}_W$, F(A) is also a monogenic object of $\mathbb{A}_W$.

Proof: We recall that an automorphism F of $\mathbb{A}_W$ is a functor $F : \mathbb{A}_W \longrightarrow \mathbb{A}_W$ for which there exists a functor $G : \mathbb{A}_W \longrightarrow \mathbb{A}_W$ such that both $F \circ G$ and $G \circ F$ are equal to the identity functor of $\mathbb{A}_W$.

The families of subsystems of A are represented faithfully by the families of monic morphisms of $\mathbb{A}_W$ with range A. Given a set J of monic morphisms $A_j \xrightarrow{j} A$ we define a category $\mathcal{J}$ (which is a subcategory of $\mathbb{A}_W$) whose objects are all the monic morphisms

$B \xrightarrow{b} A$ such that for any $j \varepsilon J$ there exists a monic $A_j \xrightarrow{b_J} B$ with $bb_j = j$. The morphisms of $\mathbb{J}$ are of the form

$$(B_1 \xrightarrow{b_1} A) \xrightarrow{f} (B_2 \xrightarrow{b_2} A)$$

where $B_1 \xrightarrow{f} B_2$ is a morphism of $\mathbb{A}_W$ with $b_2 f = b_1$.

For any set J of monic morphisms of A_W with range A, the categrory $\mathbb{J}$ has an initial object U(J), which is unique up to an isomorphism of $\mathbb{J}$ (which is an equivalence of monic morphisms in $\mathbb{A}_W$ (cf. Freyd 1964, MacLane 1965)). U(J) is a monic morphism of A_W with range A and whose image is precisely the union of the images of the morphisms in J.

We can rephrase now Lemma 2.4.1 : An object A of $\mathbb{A}_W$ is monogenic iff for any set J of monic morphisms of $\mathbb{A}_W$ with range A, if U(J) is an invertible morphism of $\mathbb{A}_W$ (i.e., an isomorphism) then there is a $j \varepsilon J$ which is invertible.

Since this characterization of the monogenic objects in $\mathbb{A}_W$ is preserved under the automorphisms of $\mathbb{A}_W$ the proof follows.

2.5 In the proof of Cor. 2.4.2 we have shown that the property of being a monogenic object of $\mathbb{A}_W$, which was defined originally by "looking inside A," is in fact definable by means of general properties of morphisms in categories. Knowing the way morphisms behave around an object A is sufficient in order to determine whether A contains a state from which all the rest of the states of A are accessible. In other words, the property of being a monogenic object in $\mathbb{A}_W$ is categorical. In Chapter 4 we shall present a rigorous explication of this notion. The properties of M_W, that we shall derive in the next chapter, will yield the result that all properties of objects of $\mathbb{A}_W$ (which are invariant under isomorphisms in $\mathbb{A}_W$) are categorical (provided that W belong to a very broad class of monoids). That is, if W satisfies some weak conditions, then all the properties of the transition systems with input W can be derived from the categorical-algebra study of $\mathbb{A}_W$.

3. A STUDY OF M_W

3.1 LEMMA: An object A of $\mathbb{A}_W$ is monogenic iff there exists an epic morphism $M_W \xrightarrow{e} A$.

Proof: Assume that A is monogenic and generated by $\{s_o\}$. Define a morphism $M_W \xrightarrow{f_{s_o}} A$ by $f_{s_o}(1_W) = s_o$ (recall that M_W is free on 1_W). Obviously, $f_{s_o} : W \longrightarrow S(A)$ is surjective and therefore $M_W \xrightarrow{f_{s_o}} A$ is epic.

On the other hand, if $M_W \xrightarrow{e} A$ is epic then A is generated by $e(1_W)$ since $e(\omega) = e(1_W)\cdot\omega$.

3.2 We define a functor $H_{M_W} : \mathbb{A}_W \longrightarrow \mathbf{S}$ by:

$$H_{M_W}(A) = \mathrm{Hom}_{\mathbb{A}_W}(M_W, A)$$

$$H_{M_W}(A \xrightarrow{f} B) = (\ell_f : \mathrm{Hom}_{\mathbb{A}_W}(M_W A) \longrightarrow \mathrm{Hom}_{\mathbb{A}_W}(M_W, B))$$

where $$\ell_f(M_W \xrightarrow{g} A) = (M_W \xrightarrow{fg} B) .$$

We define a transformation of functors $\rho: S \longrightarrow H_{M_W}$ as follows:

For any object A of $\mathbb{A}_W$, $\rho(A) : S(A) \longrightarrow H_{M_W}(A)$ is given by

$[\rho(A)](s) : W \longrightarrow S(A) : \omega \longrightarrow s\cdot\omega$.

In other words, $[\rho(A)](s)$ is the morphism $M_W \xrightarrow{f_s} A$ which is determined by $f_s(1_W) = s$.

The function $\rho(A) : S(A) \longrightarrow H_{M_W}(A)$ is bijective. It is injective since $f_{s_1} = f_{s_2}$ implies $f_{s_1}(1_W) = f_{s_2}(1_W)$. It is surjective since for any morphism $M_W \xrightarrow{g} A$ we have $f_{g(1_W)} = g$.

Furthermore, for any morphism $A \xrightarrow{g} B$ of $\mathbb{A}_W$, and for any $s \in S(A)$ we have

$$g\, f_s = f_{g(s)} \quad ,$$

where $f_{g(s)} = [\rho(B)](g(s))$.

For we clearly have

$$(gf_s)(\omega) = g(s\cdot\omega) = g(s)\cdot\omega = f_{g(s)}(\omega).$$

From this follows directly, that for any morphism $A \xrightarrow{g} B$ of $\mathbb{A}_W$, the following diagram is commutative.

$$\begin{array}{ccc} S(A) & \xrightarrow{g} & S(B) \\ \rho(A) \downarrow & & \downarrow \rho(B) \\ H_{M_W}(A) & \xrightarrow[H_{M_W}(g)]{} & H_{M_W}(B) \end{array}$$

Thus we have proved:

3.2.1 PROPOSITION: The transformation $\rho : S \longrightarrow H_{M_W}$ is a natural equivalence of functors.

3.2.2 The pair (M_W, ρ^{-1}) is a representation of the forgetful functor $S : \mathbb{A}_W \longrightarrow \mathbb{S}$ (cf. MacLane 1965).

3.2.3. Since $S : \mathbb{A}_W \longrightarrow \mathbb{S}$ is an embedding functor (i.e., one-one on the morphisms) it follows that H_{M_W} is also an embedding and therefore M_W is a generator of $\mathbb{A}_W$ (cf. Freyd 1964).

3.2.4 COROLLARY: M_W is a projective object of $\mathbb{A}_W$. (An object P of a category $\mathbb{C}$ is projective iff for any morphism $P \xrightarrow{g} B$ and any epic morphism $A \xrightarrow{e} B$ of $\mathbb{C}$ there exists a morphism $P \xrightarrow{f} A$ of $\mathbb{C}$ for which the following diagram is commutative:

$$\begin{array}{ccc} & & P \\ & \overset{f}{\swarrow} & \downarrow g \\ A & \xrightarrow[e]{} & B \end{array} \quad .)$$

Proof: It is sufficient (and necessary) to show that if $A \xrightarrow{e} B$ is an epic morphism of $\mathbb{A}_W$ then $H_{M_W}(e) : H_{M_W}(A) \longrightarrow H_{M_W}(B)$ is surjective. From the commutative diagram for $\rho : S \longrightarrow H_{M_W}$ we derive $H_{M_W}(e) = \rho(B)e(\rho(A))^{-1}$. Hence $H_{M_W}(e)$ is surjective.

3.3.3 PROPOSITION: The bijection $\rho(M_W) : W \longrightarrow H_{M_W}(M_W)$ determines an isomorphism of monoids

$$W \xrightarrow{R} \mathrm{End}_{\mathbb{A}_W}(M_W)$$

where $\mathrm{End}_{\mathbb{A}_W}(M_W)$ is the monoid of the morphisms $M_W \longrightarrow M_W$ of $\mathbb{A}_W$ with respect to the composition of morphisms in $\mathbb{A}_W$.

Proof: Since $f_\omega(\omega') = \omega\omega'$, it follows that $f_{\omega_1} f_{\omega_2} = f_{\omega_1\omega_2}$.

3.4 From Prop. 3.3 it follows that for any object A of $\mathbb{A}_W$, the set $H_{M_W}(A)$ enjoys a structure of a transition system with input W by combining $H_{M_W}(A)$ with $W \xrightarrow{R} \mathrm{End}_{\mathbb{A}_W}(M_W)$.

Formally, we define a functor $\mathrm{Mor} : \mathbb{A}_W \longrightarrow \mathbb{A}_W$, where for any object A of $\mathbb{A}_W$ we define Mor(A) by:

$$S(\mathrm{Mor}(A)) = H_{M_W}(A),$$

$$f_s \cdot \omega = f_s f_\omega = f_{s\cdot\omega} \text{ for any } M_W \xrightarrow{f_s} A \text{ and } \omega \varepsilon W.$$

For any morphism $A \xrightarrow{g} B$ we define $\mathrm{Mor}(A) \xrightarrow{\mathrm{Mor}(g)} \mathrm{Mor}(B)$ by $\mathrm{Mor}(g) = H_{M_W}(g)$.

An immediate verfication shows that Mor(A) is an object of $\mathbb{A}_W$, and that $H_{M_W}(g)$ determines in fact a morphism of $\mathbb{A}_W$. Furthermore, it follows directly from the fact that H_{M_W} is a functor that $\mathrm{Mor} : \mathbb{A}_W \longrightarrow \mathbb{A}_W$

is also a functor. Likewise, the transformation $\rho: S \longrightarrow H_{M_W}$ determines directly a transformation $\rho_{\mathbb{A}_W} : I \longrightarrow Mor$ from the identity functor of $\mathbb{A}_W$ to Mor, and we have:

3.4.1 <u>THEOREM:</u> The transformation $\rho_{\mathbb{A}_W} : I \longrightarrow Mor$ is a natural equivalence of functors.

3.4.2 Intuitively speaking, the functor Mor constructs the "internal structure" of any object A of $\mathbb{A}_W$ from a part of the category $\mathbb{A}_W$ which lies around M_W and between M_W and A. Hence it is intuitively clear, that if M_W can be recognized in $\mathbb{A}_W$ (up to an isomorphism) by means of some categorical predicate, then the "internal structure" of any object can be reconstructed "categorically," and therefore any property of the transition systems with input W can be determined "categorically" as well.

3.5 LEMMA: If W is a unit-commutative monoid (i.e., if $uv = 1_W$ in W then $vu = 1_W$) or a finite monoid then every epic morphism $A \xrightarrow{e} M_W$ of $\mathbb{A}_W$, where A is monogenic, is an isomorphism.

Proof: If W is a finite monoid then the cardinality of the set of states of any monogenic transition system with input W cannot exceed the cardinality of W. Hence $e : S(A) \longrightarrow W$ must be bijective.

If W is a unit-commutative monoid and A is generated by $\{s_o\}$, then $\{e(s_o)\}$ must generate M_W, that is, $e(s_o)v = 1_W$ for some $v \in W$, and therefore $ve(s_o) = 1_W$.

Assume that $e(s_o \cdot \omega_1) = e(s_o \cdot \omega_2)$ for some $\omega_1, \omega_2 \in W$, then we have $\omega_1 = ve(s_o)\omega_1 = ve(s_o \cdot \omega_1) = ve(s_o \cdot \omega_2) = ve(s_o)\omega_2 = \omega_2$, and therefore $s \cdot \omega_1 = s \cdot \omega_2$, which shows that e is also injective.

3.5.1 COROLLARY: If W is a unit-commutative monoid or a finite monoid, then for any automorphism F of $\mathbb{A}_W$, $F(M_W)$ is isomorphic to M_W.

Proof: From lemma 3.5 it follows that an object M of $\mathbb{A}_W$ is isomorphic to M_W iff

(i) M is monogenic, and

(ii) for any monogenic object A of $\mathbb{A}_W$ there exists an epic morphism $M \xrightarrow{e} A$ of $\mathbb{A}_W$.

Since these properties of morphisms and objects of $\mathbb{A}_W$ are preserved under the automorphisms of $\mathbb{A}_W$, the corollary follows.

3.5.2 Note that the class of unit-commutative monoids is broad enough to cover all the classes of monoids which are employed in automata theory. For example, the left-cancellative and the right-cancellative monoids are all unit-commutative. Hence the free monoids and the groups are unit-commutative. Note also that the cartesian products of unit-commutative monoids are unit-commutative, and therefore we can apply our results to multi-input transition systems as well.

4. CATEGORICAL PREDICATES AND TRANSPARENT CATEGORIES

4.1 A subcategory $\mathbb{D}$ of $\mathbb{C}$ is said to be <u>very full</u> iff for any morphism h of $\mathbb{D}$ and for any morphisms f and g of $\mathbb{C}$ such that $fg = h$, $fh = g$ or $hf = g$ holds in $\mathbb{C}$ it follows that f and g belong to $\mathbb{D}$.

A functor $T : \mathbb{D} \longrightarrow \mathbb{C}$ is said to be a <u>very full embedding</u> iff T is an embedding and the image of T is a very full subcategory of $\mathbb{C}$.

Let $\mathbb{D}$ be a category and D a class of morphisms of $\mathbb{D}$, we denote by $(\mathbb{D},D,\mathbb{C})$ the class of all the images of the morphisms in D under any very full embedding $T: \mathbb{D} \longrightarrow \mathbb{C}$. That is, $f \in (\mathbb{D},D,\mathbb{C})$ iff there are a morphism $d \in D$ and a very full embedding $T : \mathbb{D} \longrightarrow \mathbb{C}$ such that $f = T(d)$.

A class K of morphisms of $\mathbb{C}$ is said to be <u>categorical (in $\mathbb{C}$)</u> iff there is a category $\mathbb{D}$ such that $K = (\mathbb{D},D,\mathbb{C})$ for some class D of morphisms of $\mathbb{D}$.

4.2 <u>PROPOSITION:</u> A class K is categorical in $\mathbb{C}$ iff it is closed under all the automorphisms of $\mathbb{C}$.

Proof: Since for any very full embedding functor $T : \mathbb{D} \longrightarrow \mathbb{C}$ and for any automorphism F of $\mathbb{C}$, $F \circ T : \mathbb{D} \longrightarrow \mathbb{C}$ is also a very full embedding, it follows that every categorical class in $\mathbb{C}$ is closed under all the automorphisms of $\mathbb{C}$.

On the other hand, let K be a class of morphisms of $\mathbb{C}$ which is closed under all automorphisms of $\mathbb{C}$. Denote by $\mathbb{D}(K)$ the minimal very full subcategory of $\mathbb{C}$ which includes K, then $K = (\mathbb{D}(K), K, \mathbb{C})$.

In order to see this, let $T : \mathbb{D}(K) \longrightarrow \mathbb{C}$ be any very full embedding and define $F_T : \mathbb{C} \longrightarrow \mathbb{C}$ by

$$F_T(f) = \begin{cases} T(f) & \text{if } f \in \mathbb{D}(K), \\ f & \text{otherwise.} \end{cases}$$

Since $\mathbb{D}(K)$ is a very full subcategory of $\mathbb{C}$, F_T maps $\mathbb{D}(K)$ into itself, and because T is a very full embedding, F_T maps $\mathbb{D}(K)$ onto itself in an injective manner. Furthermore, F_T must be a functor and it has an inverse, hence it is an automorphism of $\mathbb{C}$.

Now, since K is closed under automorphisms it follows that $(\mathbb{D}(K), K, \mathbb{C}) \subseteq K$, and since clearly $K \subseteq (\mathbb{D}(K), K, \mathbb{C})$ we have the

the desired equality.

4.2.1 <u>COROLLARY:</u> A class K is categorical in $\mathbb{C}$ iff $K = (\mathbb{D}, D, \mathbb{C})$ for some very full subcategory $\mathbb{D}$ of $\mathbb{C}$.

4.2.2 <u>COROLLARY:</u> Let $\mathbb{D}$ be any category and D a class of some morphisms of $\mathbb{D}$, then the class of all values of the morphisms in D under all embedding functors $\mathbb{D} \longrightarrow \mathbb{C}$ is categorical in $\mathbb{C}$.

4.2.3 <u>COROLLARY:</u> The class of all values of the morphisms in D under all functors $\mathbb{D} \longrightarrow \mathbb{C}$ is categorical in $\mathbb{C}$.

4.2.4 Note that we cannot dispense with the requirement of employing very full embeddings in the definition of the categorical classes in any arbitrary category. For example in the category $\mathbb{N}$ of natural numbers where the morphisms represent the natural partial order of natural numbers, every set of morphisms is categorical.

However, the categorical classes achieved by means of 4.2.2 or 4.2.3 are always infinite or empty.

4.3.1 A class of morphisms of $\mathbb{C}$ is said to be natural iff it is closed under all those automorphisms of $\mathbb{C}$ which are naturally equivalent to $I_{\mathbb{C}}$, the identity functor of $\mathbb{C}$.

Obviously, by Prop. 4.2 we have that every categorical class is natural. Note that a class of identity morphisms of $\mathbb{C}$ which is closed under the isomorphisms within $\mathbb{C}$ is always natural.

4.3.2 A category $\mathbb{C}$ is said to be transparent if all the natural classes in $\mathbb{C}$ are categorical.

4.4 Obviously, if all the automorphisms of $\mathbb{C}$ are naturally equivalent (i.e., to $I_{\mathbb{C}}$) then $\mathbb{C}$ is transparent.

Let us call a category $\mathbb{C}$ autotrivial iff all the automorphisms of $\mathbb{C}$ are naturally equivalent.

It is not known whether all transparent categories are autotrivial. All "natural" categories of mathematical systems that are known to be transparent are in fact autotrivial as well.

The equivalence between the notion of transparent categories and that of autotrivial categories, in a special case, takes the form of the following problem in group theory:

Do all groups whose automorphisms are all (conjugate) class preserving have only inner automorphisms?

Any example of a group all of whose automorphisms are class preserving and which has an outer automorphism, yields a transparent category (with a single object and all its morphisms are invertible and in one-one correspondence with the elements of the group) which is not autotrivial.

5. THE DISTINGUISHABILITY OF M_W AND THE TRANSPARENCE OF $\mathbb{A}_W$.

5.1 The features exhibited by M_W in $\mathbb{A}_W$ are quite common in "natural" categories. As we shall see presently, they provide a reduction of the autotriviality of categories to the categoricity of certain classes of identity morphisms.

An object M of a category $\mathbb{C}$ is said to be a <u>generator</u> of $\mathbb{C}$ iff $H_M : \mathbb{C} \longrightarrow \mathbb{S}$ (where $H_M(A) = \mathrm{Hom}_{\mathbb{C}}(M,A)$) is an embedding. In this case, the values of H_M form a subcategory $H_M(\mathbb{C})$ of $\mathbb{S}$. In particular, M is said to be a <u>faithful</u> generator of $\mathbb{C}$ iff there exists a functor $R_M : H_M(\mathbb{C}) \longrightarrow \mathbb{C}$ such that $R_M \circ H_M : \mathbb{C} \longrightarrow \mathbb{C}$ is naturally equivalent to $I_{\mathbb{C}}$.

5.1.1 <u>EXAMPLES:</u> The additive group Z of integers is a faithful generator of both the category of obelian groups and the category of all groups (Freyd 1964). The single-element set U is a faithful generator of $\mathbb{S}$ the category of sets; in fact H_U is already naturally equivalent to the identity functor of $\mathbb{S}$.

From our results in Chapter 3 we know that M_W is a faithful generator of $\mathbb{A}_W$.

5.2 LEMMA: For any object A of $\mathbb{C}$ and any automorphism F of $\mathbb{C}$ with inverse G, $H_A \circ F$ is naturally equivalent to $H_{G(A)}$

Proof: We shall prove a stronger result; namely, for any two objects A and B of $\mathbb{C}$ and any automorphism F of $\mathbb{C}$ with inverse G, there exists a bijection

$$\Phi(A,B) : \mathrm{Hom}_{\mathbb{C}}(G(A),B) \longrightarrow \mathrm{Hom}_{\mathbb{C}}(A,F(B))$$

which is natural in both A and B.

Put differently, F and its inverse are adjoint. By Kan's characterization of adjoint functors (Kan 1958, MacLane 1965) the following is a proof that F and G are adjoint (to each other!):

Let us denote by e_C the identity morphism of an arbitrary object C of $\mathbb{C}$.

(i) Every morphism $A \xrightarrow{f} F(B)$ can be factored as $f = F(h) \cdot e_A$

for some $G(A) \xrightarrow{h} B$ (i.e., $h = G(f)$).

(ii) If $F(h_1) \cdot e_A = F(h_2) \cdot e_A$ then obviously $h_1 = h_2$.

5.3 Let M be a faithful generator of $\mathbb{C}$. From Lemma 5.2 we know that $H_M \circ F$ is naturally equivalent to $H_{G(M)}$ for any automorphism F of $\mathbb{C}$ with inverse G. Hence F, which is naturally equivalent to $R_M \circ H_M \circ F$, is naturally equivalent to $R_M \circ H_{G(M)}$. If $G(M)$ is isomorphic to M, then H_M is naturally equivalent to $H_{G(M)}$, and therefore F, which is naturally equivalent to $R_M \circ H_{G(M)}$, is naturally equivalent to $I_{\mathbb{C}}$.

5.3.1 An object A of $\mathbb{C}$ is said to be <u>distinguishable</u> (<u>in</u> <u>$\mathbb{C}$</u>) iff for any automorphism F of $\mathbb{C}$, $F(A)$ is isomorphic to A. Put differently, A is distinguishable in $\mathbb{C}$ iff the natural class of all identity morphisms of the objects of $\mathbb{C}$, which are isomorphic to A in $\mathbb{C}$, is categorical.

Thus we have proved:

5.3.2 <u>THEOREM:</u> A category $\mathbb{C}$ with a faithful generator M is autotrivial iff M is distinguishable in $\mathbb{C}$.

5.3.3 <u>COROLLARY:</u> A category $\mathbb{C}$ with a faithful generator is autotrivial iff it is transparent.

5.4 Since M_W is a faithful generator of $\mathbb{A}_W$, $\mathbb{A}_W$ is transparent iff it is autotrivial. Furthermore $\mathbb{A}_W$ is autotrivial iff M_W is distinguishable in $\mathbb{A}_W$. By 3.5.1 we know that if W is finite or unit-commutative there M_W is distinguishable in $\mathbb{A}_W$. Thus we have:

5.4.1 <u>THEOREM:</u> If W is a finite monoid or a unit commutative monoid then $\mathbb{A}_W$ is autotrivial (and therefore transparent).

6. DISCUSSION AND OPEN PROBLEMS

6.1 Our result as expressed by Theorem 5.4.1 implies that, for a very broad class of monoids, the categorical study of a domain of all transition systems with input monoid of this class, is equivalent _in principle_ to the "complete" study (or the "inside" study) of these systems. However, only experience may show us that in fact there is a psychological advantage to the categorical approach in the study of these systems

6.2 If G is a group then $\mathbb{A}_G$ is the category of all _representations of G as operating on sets._ Since every group is in particular a unit-commutative monoid, we have that the categorical study of the representations of a fixed arbitrary group G is sufficient _in principle_ for producing all the algebraic properties of the representations of G.

6.3 Our results so far, give rise to some general problems that deserve attention. For example, is it true that for any monoid W, M_W is distinguishable in $\mathbb{A}_W$?

More generally, what additional properties on faithful generators of categories, if any at all, are necessary in order to insure that they are distinguishable? In particular, is it true that every projective faithful generator is distinguishable?

6.4 Important and much more interesting categories of transition systems are those of <u>finite-state</u> transition systems (i.e., transition systems whose sets of states are finite). If the input monoid W is also finite then our results remain valid since M_W is also finite. If however W is infinite, as it is the case in the ordinary theory of finite automata (where the input monoids are finitely generated free monoids) then M_W is no longer applicable.

6.5 Another interesting restriction of $\mathbb{A}_W$ is to <u>abelian</u> transition systems. A transition system A is said to be abelian iff $s \cdot \omega_1 \omega_2 = s \cdot \omega_2 \omega_1$ holds for all $s \in S(A)$ and $\omega_1, \omega_2 \in W$.

For any arbitrary monoid W there exists a homomorphism of monoids $W \xrightarrow{ab} W^{ab}$ where W^{ab} is an abelian monoid with the following universality property : any homomorphism of W into an abelian monoid factors uniquely through ab. The direct construction of W^{ab} must be evident.

Denote by M_W^{ab} the following object of $\mathbb{A}_W$:

$$S(M_W^{ab}) = W^{ab},$$

$$ab(\omega_1) \cdot \omega_2 = ab(\omega_1 \omega_2) .$$

Obviously, M_W^{ab} is abelian. If we denote by $\mathbb{A}_W^{ab}$ the full subcategory of $\mathbb{A}_W$ of abelian objects, one can easily follow the example of M_W in $\mathbb{A}_W$ and show that M_W^{ab} is a faithful generator of $\mathbb{A}_W^{ab}$. Furthermore, for any arbitrary monoid W, M_W^{ab} is distinguishable by the same properties which distinguish M_W in $\mathbb{A}_W$ in the case of unit-commutative input monoid W (cf. 3.5.1), hence for any arbitrary monoid W, $\mathbb{A}_W^{ab}$ is transparent and autotrivial.

An equivalent proof of these properties of $\mathbb{A}_W^{ab}$ follows directly from the fact that $\mathbb{A}_W^{ab}$ is a category isomorphic to $\mathbb{A}_W ab$.

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13. ABSTRACT

This paper presents a study of the homomorphisms of transition systems, which are the main tool employed in recent studies in algebraic automata theory The problem of determining the extent of the information that can be retrieved from the algebra of these homomorphisms is defined and discussed. Naturally, this study is allied with categorical algebra. A general result which is applicable to many "natural" categories of mathematical systems is derived. This result implies the completeness of the categorical algebra study of transition systems with a fixed input monoid, provided that the input monoid belongs to a very broad class of monoids which includes all the types of monoids which are employed in automata theory.

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