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# THE APPLICATION OF A HIGH SPEED COMPUTER TO THE DEFINITION AND SOLUTION OF THE VEHICULAR TRAFFIC PROBLEM

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## Introduction

The implications of the title of this paper will be justified as follows: Operations researchers have a fairly well developed and agreed upon methodology. This methodology stresses the use of mathematical models which are central to both the statement of the problem and evaluation of its proposed solution. These models are classifiable according to whether they are analytic or numerical and whether they are deterministic or stochastic. Each of these types of models has been applied by some investigator to the vehicular traffic problem. The numerical stochastic type is best used with the aid of a large scale digital computer. This has been done notably by Gerlough at the University of California at Los Angeles and by the present author and others at the University of Michigan. By the latter, the methodology of the operations researcher, referred to previously, has been applied in detail. The computer, then, furnishes a model by means of which the entire definition and solution of the problem may be attacked.

In what follows, each of the parts of this argument is covered in some detail. Further, a comparison is made of the analytic model with the numerical one on one hand, and the use of trial and error as opposed to the use of a model at all on the other hand. Further, on the basis of the work at Michigan, a guide to the selection of particular computers for this type of problem is derived.

## Operations Research and the Model

It is true, as Bridgeman[1] says, that "there are as many scientific methods as there are individual scientists." But there are similarities among these methods. However permissible it may be for the individual scientist, working alone, to avoid discussion of his methods with his fellows, the team worker can afford no such hermitic independence. The operations researcher and the system engineer are frequently team workers. They can and do describe the steps of their method of problem attack. In a large measure they have reached some agreement on the methods to be used.

My notes show the following ordering (with no source) of the steps used by the operations researcher.

". . . a background study of the general nature of the problem and the people and things which impinge on it; choice of one or more measures of effectiveness; set up of a mathematical model to represent the system under study; gathering of data; application of logical techniques to the solution of the model; search for solutions to the problem in the light of the measures of effectiveness; and if appropriate evaluation of the resulting changes."

The position of the mathematical model in this description is central; and with good reason.

To emphasize this last point and to provide the ground work on which to base the attack on the specific vehicular traffic problem, these steps are expanded somewhat in the following paragraphs.[2] Of course each operations researcher will want to modify these statements to his own thinking; which just underlines the truth of Bridgeman's statement.

Figure 1 presents a diagrammatic representation of the relationships among the steps in the operations researcher's methodology. The statement of the problem is made up of several parts:

(a) An examination of the environment including establishment of the jargon to be used in describing the problem and the people and things which are associated with the problem.

(b) The viewpoint from which the problem is to be attacked. It is trivially obvious that in a game between two opponents the problem is the same for both but the solution is quite different because of the differing viewpoint of each. In the problems with which we are concerned, the viewpoint of pedestrian and driver differ. There are other viewpoints involved.

(c) The choice of a measure of effectiveness with which solutions to the problem are to be evaluated.

(d) A statement of the permissible areas of solution. (This is what is usually implied in the remark that the statement of the problem is 80 percent of it solution. The areas of permissible solutions, when clearly defined, will largely determine the nature of the mathematical model.)

The mathematical model is the attempt to restate the problem quantitatively. It implies a certain functional form, and a knowledge of the constants which must be inserted into the resulting functional form. Neither of the latter two objects is available without experiment and this produces naturally the third box of the diagram labeled design and analysis of experiment. The implication of the appearance of analysis prior to the conduct of experiment is that the method of analysis should be predetermined. After the experiment has been well defined, its conduct leads to measurements. The resulting estimates of the functional forms of the relationships governing the variables involved, and of the numerical values of the constants in these forms, are fed back to make the model complete.

It is through the model that the statement of the problem is connected with its solution. The actual problem solution is the process of devising alternatives. This part of the problem requires ingenuity and innovation. Having selected alternatives, it is necessary to evaluate them as proposed solutions for the problem. It is here that the process connects up again with the mathematical model. Presumably, after an evaluation has been made, a choice can also be made. This choice implies an implementation of the proposed solution which in turn affects the mathematical model which then starts the process over again.

Thus the model is central to the methodology of the operations researcher. The model offers understanding of the process; it makes possible the evaluation of the effects of a change of one variable on all other variables;[3] it provides a skeleton for the intuitive kind of thinking that leads to the choices of alternatives; and finally, when solutions are suggested, it provides a means of evaluating them.

## Classification of Models

We turn now to an examination of the various types of mathematical models which may be used. Figure 2 shows a two-by-two-fold dichotomy of mathematical models in use at the present time. These models are either analytical or numerical, deterministic or stochastic. The engineer and scientist both have occasion to use all of the types represented here. For example, $F = Ma$ as shown in the analytical deterministic example expresses a relationship among F, M, and a, such that for any values assigned to two, the third is completely determined. No deviation from this relationship is permitted. In the analytic stochastic case, the example, that of the Poisson distribution, expresses the statistical statement that on the average a certain number of events will occur in a given interval of time (or alternatively, a number of outcomes in a given region of space). It says nothing about any particular occurrence. Both of these have the characteristic that one must know the process underlying the phenomenon being described in some detail. The result is an expression in closed form, mathematically. In the case of the numerical model it is not necessary to know the mathematical relationship which describes the result of the process. As shown in the figure, for numerical integration as used by, say, the chemical engineer, the integral of the function is obtained by dividing the function into a set of rectangles with small enough base to make the error reasonably small, and adding up the resulting areas. The function so integrated may even be empirical in nature. All that is required is a numerical representation of the function in question.

The numerical integration is still completely deterministic. The model represents a one-to-one relationship between the function and its integral. For a given base line increment, the same result will always be obtained. Consider the lower right box in the figure. Here the function is again represented diagrammatically. However, in this case the scale is so chosen that the entire portion of interest of the function to be integrated lies inside the unit square. We can obtain an estimate of the integral of this function in the following fashion: From a table of random numbers lying in the interval 0 to 1 choose pairs of such numbers repeatedly. Plot a point in Figure 4 with each pair of random numbers as coordinates. Do this repeatedly until a large number of points have been plotted. Then count the number of points falling between the graph of the function and the x-axis. The ratio of this number to the total represents an estimate of the fraction which the integral is of unity. Thus a stochastic process has been used to examine a perfectly definite value of the integral. Of course, this could have been done much more quickly with a planimeter. However, in the case of five or six variables the method may have some utility. It is used here only for illustrative purposes.

The examples of the various classes of mathematical models have been chosen to illustrate the way they are used by engineers and physicists. The operations researcher (and the system engineer) has a version of each of these types. Like the engineer he has, in the past, leaned on the analytical approach, but unlike the engineer, he has stressed the stochastic model. And this is as it should be since the amount of uncertainty in the problems with which he deals is greater than in most cases dealt with by the engineer. Neither has put much emphasis on the numerical stochastic model, its usefulness being limited by available computational speeds. This paper emphasizes the numerical stochastic model used with a digital computer for the investigation of traffic. Before going into this matter, however, we examine models of the various classes as they have been used in traffic.

## Traffic Models

A simple example of an analytic deterministic model for traffic is that developed by Professor R. B. Morrison of the Aeronautical Engineering Department of the University of Michigan (unpublished notes). In this model we are interested in car spacing on an open road, single lane, and its relationship to speed. Let $\rho$ be the density of cars per mile. Then

$$\rho = \frac{5280}{\ell + L}$$

where $\ell$ is the length of a car and L is a fixed spacing between cars, both in feet. The number of cars passing a point per hour will be given by

$$m = \rho v$$

where _v_ is the speed of the line of cars in miles per hour and _m_ is in cars per hour.

Suppose that each driver follows a rule which makes him keep a distance L behind the car in front, such that, with a braking deceleration a he is just able to stop his car before hitting the one in front. Then

$$\frac{v^2}{2a} = L$$

Substituting for L in the expression for m we obtain,

$$m = \frac{5280}{\frac{v^2}{2a} + \ell} v$$

Now a maximum flow will be obtained if v is chosen such that the derivative of m with respect to v is zero. The execution of this step yields

$$v = \sqrt{2a\ell}$$

and consequently, $\ell = L$.

From this extremely simple analytical model we are able to see that, under the rules, the best spacing for maximum flow is obtained when a car length is kept between cars. Further, the "capacity" of a road under this rule may be estimated and the effect of braking deceleration on the mass flow of cars is available. This model has been extended by Professor Morrison to include the "wave"velocity of the starting action introduced when a light turns suddenly green, and each car accelerates as soon as a reaction time has elapsed. Further extensions of this type of approach are contained in Lighthill's[4,5] work.

We illustrate next an analytic stochastic model. It is taken from Kendall.[6] In this model we are interested in the flow of traffic in a single lane through a toll booth on a bridge. If $C_o$ represents a car which has just been served and $C_1$ the next car, we employ the following notation:

$T_1$ = time between instant after $C_1$ service starts and the instant after $C_1$ leaves booth.

$r_1$ = number of arrivals to the waiting line, if any, in time $T_1$.

$n_o$ = number of cars on the waiting line the instant after $C_o$ leaves.

$n_1$ = number of cars on the waiting line the instant after $C_1$ leaves.

m = average number of cars arriving per unit time ($1/m$ then equals average time between arrivals).

$\lambda$ = average number of services per unit time ($1/\lambda$ therefore = average time per service).

$\rho$ = $m/\lambda$ (assumed less than 1).

In case there is a waiting line when $C_o$ ends his service, the instant after he leaves will coincide with the instant after $C_1$ begins service. If no line waits, these times will be distinct.

If $\delta$ is taken as 1 when $n_o = 0$, and 0 when $n_o$ does not = 0, we have

$$n_1 = n_o + r_1 - 1 + \delta,$$

$$E(\delta) = \text{probability of finding line empty,}$$

$$= 1 - \rho$$

independently of probability distributions of arrivals or services.

In similar fashion, with some further manipulation, Kendall shows that

$$E(n) = \rho + \frac{\rho^2 + m^2 \sigma_T^2}{2(1 - \rho)}$$

if the car arrival distribution is Poisson. If further, the service time distribution is exponential with mean $1/\lambda$ then with w = waiting time and T = service time,

$$E(w) = \frac{\rho^2 + m^2 \sigma_T^2}{2m(1 - \rho)}$$

$$\frac{E(w)}{E(T)} = \frac{\rho\,(1 + \lambda^2 \sigma_T^2)}{2(1 - \rho)}$$

Kendall points out that the ratio of the expected waiting time to the expected service time is in a sense a measure of the goodness of the service process. If this is accepted, then in case the arrival distribution is exponentially distributed (i.e., services end at essentially a random time), this ratio becomes $\rho/(1 - \rho)$. If one is able to iron out his services so that a constant time is taken for each (i.e., the service time variance is zero), then the ratio is cut in half and becomes $\rho/2(1 - \rho)$. Thus one may, by means of such a model, examine various conditions and investigate changes in the model. The rigidity even for such an unsophisticated model is obvious. Only one lane is being handled, the introduction of various distributions requires that they be mathematically tractable, and the measure of effectiveness must be built into the mathematical expression so that it may be examined.

Further models of this type have been built by Newell[7] of Brown University and others. Considering the channel described as a traffic light instead of a toll gate, Newell has added to the model a red and green cycle which is controllable. Perusal of his paper shows the complication of handling even such a minor bit of realism as the introduction of the lighting cycle.

As in other branches of science and engineering, in traffic it is possible to introduce a <u>numerical deterministic</u> model that is sometimes useful. In connection with the determination of pathways for new highways, it has been a practice in the traffic engineering business to conduct what is known as an origin and destination survey. At the points of interest for measuring, the traffic engineer halts cars and asks of the driver his origin, destination and proposed route. These are plotted out on a map and depending upon the density of pathways over the various routes, the availability of land, the various political facets of the problem (and other considerations no doubt), choices are made of routes for new highways. I believe that recently, in Chicago, a group doing an origin-destination study used a digital computer to plot out the results of the study. A computer can, of course, be used to investigate various arrangements of routes to see what the resulting traffic burden would be assuming that there are no changes in driver desire. (This latter assumption is as usual an extremely shaky one.)

Finally we come to the <u>numerical stochastic</u> model to which we will give detailed attention. Such models were first investigated, relative to traffic, at the University of Michigan by Goode, Polmar and Wright[8] and at the University of California at Los Angeles by Gerlough.[9]

Gerlough investigated the problem of representing the flow of cars on a freeway with a digital computer. His model represented the two-lane freeway linear geometry by registers in the SWAC computer. A one corresponded to a car at a particular position, a zero to an empty space. The same geographical position was represented in several registers each of which corresponded to a particular speed of actual motion and a desired speed on the part of the driver. An algebra of car motion (multiplication, extraction and addition) was developed to make the model "run". The movement of any car depended on its position relative to cars in front, at its side, its actual speed, and its driver's desired speed. The model provides a method of examining capacities under various actual and desired speed distributions.

The model created by Goode, Polmar and Wright was intended as a first step in the creation of an overall simulated model for investigation on a digital computer. Although considerations concerned representation of an entire city, only enough effort was available to represent a single intersection of a simple type. The idea behind the model was to allow for the representation of individual cars in the model, the movement of these cars to be determined according to the conditions surrounding the car at any particular moment. Provision was made for the origination of cars at points entering lanes approaching the intersection in question, and for a stochastic machine for determination of the direction of travel of any particular car entering the intersection (i.e., right, straight ahead, or left). At the outset two possible methods of handling the problem presented themselves: in one, a number is recorded in the digital computer representing the car's present position, its direction of motion, its speed, its desired destination, etc., etc. Then by operations on this number, the car is made to "move" in the model. This would be done for every car at the end of small time intervals. An easier mode, but perhaps not quite as flexible, is to break up the geographical model into positions which may be occupied by cars. This model would then be scanned for every small instant of time (finite intervals) to determine whether or not each geographical position in the model was occupied by a car. If the position were occupied, then the car would be handled according to the conditions surrounding it at the moment and the rules created to govern such conditions. To make a car move forward, the present position would be emptied and the next position filled. The latter method was chosen as the simpler and therefore more readily demonstrable approach.

A representation of this model is shown in Figure 3. Each of the positions in the lane approaching the intersection are represented by x's. Positions in the right turn are represented by φ blocks; straight ahead positions are represented by o blocks. The left turn is represented by □ blocks. It is immediately evident that the flexibility of the model is limited by the fact that speeds are determined by the choices of the lengths of the intervals. All cars traveling through a given position travel at the same speed. This would not be so in the first suggested method but this speed assumption is tenable for a first demonstration.

It merely remains now to choose a logically consistent set of operations on these positions to produce a model of the movement of cars through an intersection. Although no great detail can be given here, it is worth indicating some of the notions. Every approach to the intersection, north, east, south and west, has a similar layout of points to that shown in the figure. Each point represents a block 11 feet long. Since cars were assumed to be 18 feet long, any car arrested with its front bumper at the initial point of one block would stretch back into the block following for a distance of 7 feet. This would leave 4 feet between it and the front bumper of a car immediately behind. Provision was made for storage in the computer of the following information: whether or not a car block was occupied--this was done by a register of the digital computer being filled with one's and zero's according to whether or not a car was or was not present in the corresponding bit-block arrangement; whether the light was presently green or red in a given direction and the amount of time it had been so; information for the generation of a Poisson distributed origin of cars at a point some 20 car positions back from the intersection in each direction; a similar random number routine associated with turns at the intersection.

In operation, each position in the model was examined to see whether a car was present. If there was one present, it was moved in accordance with the conditions around it at the moment. These conditions were determined by a set of logical rules. For example, a car which was behind a car not moving could not move to within fewer than four feet of the car in front. However, it could move only to within 26 feet (2 blocks plus 4 feet) of a moving car. If the car were at X 1 (that is, just entering the intersection), a set of logical rules and random numbers would determine whether it moved into the intersection and in which direction it would turn. Similarly, in each time interval, the decision as to whether or not to generate a car was determined by a set of random numbers. This was done for each point of origin of cars. Provision was made for the cars to backlog into a point outside the entering lane. With all of these rules, it was possible to vary the rate of arrival of traffic, the cycling of the light, the proportion of the light cycle which was red, and the fraction of cars which would make a turn in each direction at the intersection.

With such a model it becomes possible to create more and more complicated situations by making the output of one intersection, which in the present model merely disappears, the input to another intersection as shown in Figure 4. This situation has been worked on sporadically during the past year. A progress report is being given in another section.[10]

The outcome of running this model is shown in Figure 5. The measure of the effectiveness chosen was the average delay per car. Other measures of effectiveness could be used equally well. For example, the number of cars in and out of a region, or the number of cars through an intersection per unit time, or the length of time from one point in a city to another are other possibilities. The computer calculates these measures equally well from the same model. The figures in the graphs show a reasonable agreement with real intersections as measured by traffic engineers. However, since no particular intersection was in mind at the time this model was created, no real comparison is possible.

Of major importance in connection with such a model is the length of time required by the computer to represent a given situation and the amount of fast storage taken by the machine. Because of the mode of representation used here, the amount of storage required on MIDAC, the digital computer used at the University of Michigan, was relatively limited. However, the time taken for the computation was 3.2 x real time. This means that if ten minutes of traffic flow was simulated for this particular model, the computer took 32 minutes to make the necessary calculations to "run" the model and to determine the measure of effectiveness; in this case, average delay.

Estimates of Computer Capability

This model was a relatively simple one. It made no provision for a parking regime, two-lane traffic in one direction, more intersections, etc. There are a whole host of extensions of this model which are possible and it then becomes important to consider whether a digital computer can continue to handle in a reasonable time the important problems of calculation which are set up thereby. To investigate this problem, Figure 6 shows a rough plot of the speeds of the various computer classes in terms of multiplications per second. The actual position of any particular computer should be taken with a grain of salt. Sometimes the multiplication speed is not defined unless the particular multiplication is explicitly stated. In other cases the source of information was not completely accurate. The purpose is not to compare the computers so much as to get an idea of capabilities of computers in general for this type of problem. The figure shows a plot of multiplication time versus words of fast storage. The number of words of fast storage are the only things of interest since to go outside of the computer to a subsidiary mode of storage as, for example, cards or punched tape or in the case of a high-speed computer to a drum takes an excessive amount of time. Time as has been noted is of the essence in such a problem.

The size of a practical problem on the computer may be estimated as follows: In the model discussed above some 25 words (registers) of information storage will be required to keep account of cars, condition of light, etc. In addition there were some 150 words of instruction, but in any computer these may be handled in various ways and in connection with the problem we are interested in here, extensions of intersections, routines, etc., do not increase the required storage of instructions rapidly since the routines are repeated. Thus we need consider only storage of information. To store several lanes would probably double the number of required words to 50. To store parking and pedestrians would probably triple to 75. To allow speed changes might make this 100 as a requirement per intersection. As for time, the MIDAC required 3.2 times real time as noted above. When extending the problem we may use a faster computer either to go faster, or to handle a larger problem. Therefore interpolation on the figure must be used as required for the particular extension at hand. Some of the resulting points are marked off in the diagram.

It is noteworthy that each piece of information that is used in the computer, as for example the position of a particular car, is used in the same order each time the model is traversed. This makes it possible to consider relatively slow methods of storage, such as magnetic drums, and to store in such a fashion that as a piece of information is required it comes up on the drum in position to be read. It has been estimated that an ordinary drum machine such as the Datatron or the 650 may be made to look like four-microsecond access time if the programming is done carefully with regard to the machine itself. Therefore, the above picture is not quite a complete statement of the situation. However, as may be seen from the diagram, a fairly large model may be handled in terms of presently available computers. Newer and faster computers now being designed will extend these capabilities even further.

## Critique of the Use of a Numerical Stochastic Digital Computer Model for an Operations Research Problem -- the Traffic Problem

We are now in a position to turn back to the points we made about the methodology of Operations Research and examine the facility offered us by the computer in connection with this methodology. We do this with relation to the traffic problem as an example.

There are many viewpoints possible in the traffic problem. The pedestrian and the driver, to name only two, require opposing solutions for the same problem, and as a consequence the solution itself must result in compromise. The driver is a complex of freight traffic (slow, many stops), cabs (short hauls), tourists, commuters, etc. In addition to the pedestrian and the driver there are the public officials to whom the traffic authorities report. In each of these cases the difference in viewpoint leads to a difference in the choice of measure of effectiveness. The driver is interested in traveling quickly, or perhaps with the least amount of psychological delay. (The delay at a stop sign is known to be shorter than the delay at a signalized intersection. But drivers are more impatient with stop signs than with signalized intersections since in the former case they necessarily must stay on the qui vive all during the time they are stopped.) The pedestrian is concerned with safe crossing in a reasonable time and with not too great a distance to walk to get to the crossing. The public official is concerned with a minimum amount of public outcry and perhaps public praise. The Fire Department must assure itself of a route through the city and of a parking place at the point of the fire. The merchant wants a fair amount of transient traffic and a reasonable parking situation in the center of the city. It is clear that many of these viewpoints are conflicting and the examination of conditions under which each is satisfied in the actual traffic situation is prohibitive. On the other hand with the computer model not only may we examine conditions which satisfy each of these viewpoints and analyze the possible effect on the other viewpoints, but additional measures for any single viewpoint may be examined. For example, the satisfaction of the driver's viewpoint might be measured by either average delay, or time between any two points in the city, or the number of cars crossing a boundary of a region per unit time. With mathematical analysis each of these would require a different setup and would be totally impossible in terms of the difficulties encountered in such an analysis. In experiment (that is, by trial and error), the expense is prohibitive not to mention the time and organization factors.

The areas of permissible solution may be readily changed with the computer model. The effect of prohibiting traffic in the center of the city, a drastic solution in the real traffic case, may be reasonably well examined on the computer model. Suggestions for changes in parking regime which are radical, and which in the actual traffic case demand years of discussion and planning and overcoming of public objections, may in the computer case be reasonably easily tried out. When New York City, for example, changed its avenues in Manhattan to one way, the radical nature of the solution created great public outcry even though a long period of education was undertaken prior to the initiation. If it had turned out unsuccessfully, the public outcry would have been very great. Trying out such solutions on the computer is perfectly feasible. Analysis on the other hand for problems of this type again appears impossible. With some ingenuity a broad attack may be made in analytical fashion. However, very little confidence may be placed in the result because of the necessity for the reduction in the number of important factors considered which always accompanies such analyses.

To go on with considerations of the methodology of operations research, we examine what is necessary in connection with the design, analysis, and conduct of experiments. Of course the computer model does not obviate the need for experiment. However it reduces drastically the amount of required experiment. In the case of trial and error, the experiment must involve large regions (for example the so-called cordon count and the origin and destination study) and in addition must be repeated many times to insure any confidence in the result. The creation of a model on the computer limits the experiments to a local determination of constants (for example driving habits at a particular point, fraction of right and left turns, etc.) and because of the limited number of variables which are being measured, allows for a reduction in the number of experiments required to achieve a given credibility. The underlying reason for the large difference between the requirements for experiment in the case of actual traffic and those in the case of the computer model, is that in the former the physical relationships between the parts (that is, the functional forms of the model) are being determined as well as the values of the constants involved. In the case of the computer model the functional form of the overall traffic process is obtained by representing the functional form of the process in the small, i.e., at each intersection or in each lane. The parts are then joined together in the computer. Thus the experiment is confined (as it is in physics) to the measurement of characteristics or constants which are implicit in the model. This is contrary to the need for experiment in economics, for example, where the functional form is itself an unknown factor. Thus the computer model makes the study process more closely akin to the physicist's methods than the social scientist's, in that it provides a theory.

Analysis also requires relatively few experiments to determine values with a given level of confidence; however, the determination of the mathematical form to cover situations which are realistic, and which cover a large geographical domain, is formidable.

In a sense the computer model serves as an experiment itself. The knowledge of functional form at a point is used to connect those points up to obtain a model of the overall process. The values of constants locally are measured. The whole is then run on the computer repeatedly with stochastic variables introduced at points where they occur in the real process. Each of these runs may in effect be considered an experiment. This leads to another advantage which this type of approach has over the actual measurement of conditions in the real traffic process. The latter are difficult to make reproducible. Under what are seemingly the same conditions, different results are obtained for variables being measured. This results from the stochastic nature of the process making the reproducibility poor. Not only is the process itself stochastic, but the organization of the collection of data is itself a difficult task. In the handling of data by different investigators, the failure to carry out instructions and the lack of understanding of the need for the measurements result in difficulties in connection with reproducibility. The computer on the other hand is under complete control and allows a reproducible set of experiments.

On the computer, the number of runs which must be performed can be determined sequentially. Those experienced with the conduct of experiments recognize the difficulty of an analysis which predetermines, in any complicated case, the required number of runs to assure a given precision or accuracy. However, in experiments in which the number of runs does not have to be predetermined, one can watch the process and observe the stability of the statistical process "set in". On the computer if more runs are needed, they can be accomplished at a reasonable cost. In the real traffic process, discovering that not enough runs have been made and that the experiment had to be done over would be a catastrophe. (Of course this actually happens, but is disregarded.)

The measurement of the actual process is, of course, more realistic than what one can achieve on a computer. The computer model in turn is more realistic than the model achievable through analysis; a good deal more so. However, one should recognize that realism is not an objective of simulation any more than "truth" is an objective of the physicist who creates a model of particle structure. One is after a model which when operated under conditions similar to those in the real case, puts out the same numbers for the observed quantities. Thus, instead of looking for as much realism as one can get, one is really searching for as little realism as he can get away with. Only so much realism is required as will produce in the model the same amount of average delay as in the real traffic process. Similarly for other measures of effectiveness. I believe it will be found that relatively simple computer models will adequately represent the real traffic process. On the other hand, analysis gives a distorted picture of the actual traffic process. Therefore, inferences from analysis are not as useful as from the computer model.

This is not to say that the use of analysis in connection with the computer model is not desireable. Of course, the computer model may be considerably reduced in size by the careful introduction of analytical techniques at each of the permissible points in the computer model. It is conceivable that instead of dealing with individual cars, a set of four functions of four variables might be introduced in connection with a given intersection model to provide the outputs in a given length of time from each of the four directions. In this way, the time taken to simulate the flow in a particular intersection would be drastically reduced.

Thus far in the application of the methodology to the traffic problem with the computer, we have been dealing with the "statement of the problem" side of the question. It is just as useful to introduce the computer for the evaluation of alternative solutions. In fact, this discussion has been implicit in the earlier one of statement of the problem. Not much time needs be spent therefore on the solution side of the picture. Alternatives, no matter how drastic, may at reasonable cost be examined by means of the computer model. Evaluation of these alternatives is immediately available directly in terms of the measures of effectiveness. If the solutions suggested yield evaluations which make them look good, they may be introduced into the traffic process.

On occasion, the introduction of a proposed solution into the real traffic process will make requisite a new formulation of the model in the computer. Thus, if one had been simulating traffic in New Jersey prior to the introduction of the turnpike, he might have assumed (as those in charge did) that the volumes of traffic which would be encountered would be the same and that the flow of traffic along the turnpike would lessen the flow of traffic along other roads. This turned out to be not the case. Actually traffic increased, presumably due to the ease with which one could now travel south from New York, to such an extent that traffic remained essentially as heavy on the other roads as before, but the turnpike encountered great traffic. As in all methods of science, the computer reflects no more than one puts into it. Future users of simulated models, or analysis and armchair planning, must still take into account estimates of the increased desire to use facilities which are provided. But if the possibility had occurred to someone, the effect of the increase could have been estimated on a computer model.

All of the factors we have discussed in comparing analysis and trial and error in the real traffic process with the use of the simulated model are summarized in the table of Figure 7.

## Conclusion

We turn now to an estimate of the uses of such a model for the future. It is clear that the computer model can be used in the solution of day-to-day problems. These problems must be approached in the same manner in which the operations researcher approaches his other problems except that the process on which most of the experiment is carried out, and observations made, is in the computer instead of in the actual mechanism being studied. Thus one may conceive of each of the large cities having built a model of their traffic situation, introducing suggested new changes into the model and observing their effect prior to their incorporation in the actual traffic process. In the case of small cities one may envision a state supported center which has computer tapes or programs which represent the cities in question. Faced with a relatively infrequent traffic problem, the smaller communities would submit their problems to a state organization which would then introduce them into the computer and predict the consequences of proposed changes. All of this assumes the handling of traffic problems on a day-to-day basis.

However, two developments are of great importance. One is the fact that we are already introducing traffic systems which measure traffic flow in the actual traffic situation on a continuing basis, and transmit the result to a central point. At the present time the logic for the control of such traffic systems is practically non-existent. Consider if you will what you would do if you were given a board with a map representing a city, and the indicated flow of traffic at every point in this city at any moment. Suppose that a traffic jam occurred at some particular point:--which light cycle would you change? What instructions would you give to traffic officers? What instructions would you broadcast to drivers?--in order to reduce the jam? This is a difficult question to answer and requires experience before theoretical notions may be introduced. The computer model offers the possiblity of conducting such experiments on a simulated basis without introducing difficulties in the real traffic process.

The second important development is that concerning the future of our cities ten, fifteen, twenty-five years in the future. It is clear that some mode of traffic control will have to be introduced akin to that which now governs the airlines. Whether cars are to be controlled on an individual basis (that is, file some kind of flight plan) or will be controlled by party line (that is, broadcasts made to the entire moving car population requiring each individual to select instructions applying to him from the instructions given to a general group) or will be controlled by broadcast control (that is, information presented, perhaps on an oscilloscope, of the situation around the driver for about 10 blocks radius, and expect him to choose a path accordingly); whether any or all of these is to be introduced as a method of control can only be solved by theory and experiment. The computer model offers a method for introducing both, and well before the actual crisis is upon us.

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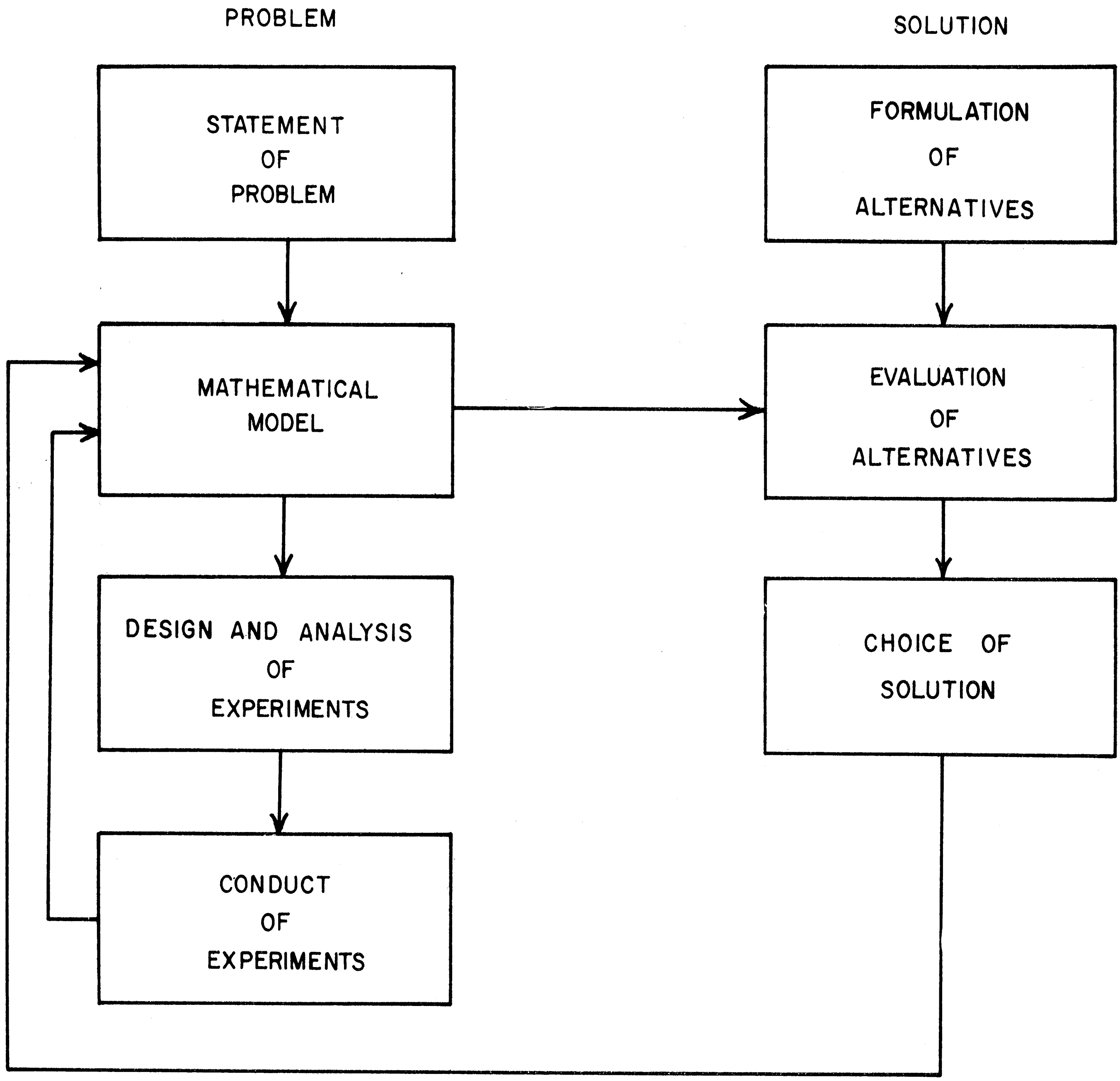


FIGURE 1. PLACE OF MODEL
IN OPERATIONS RESEARCH METHODOLOGY

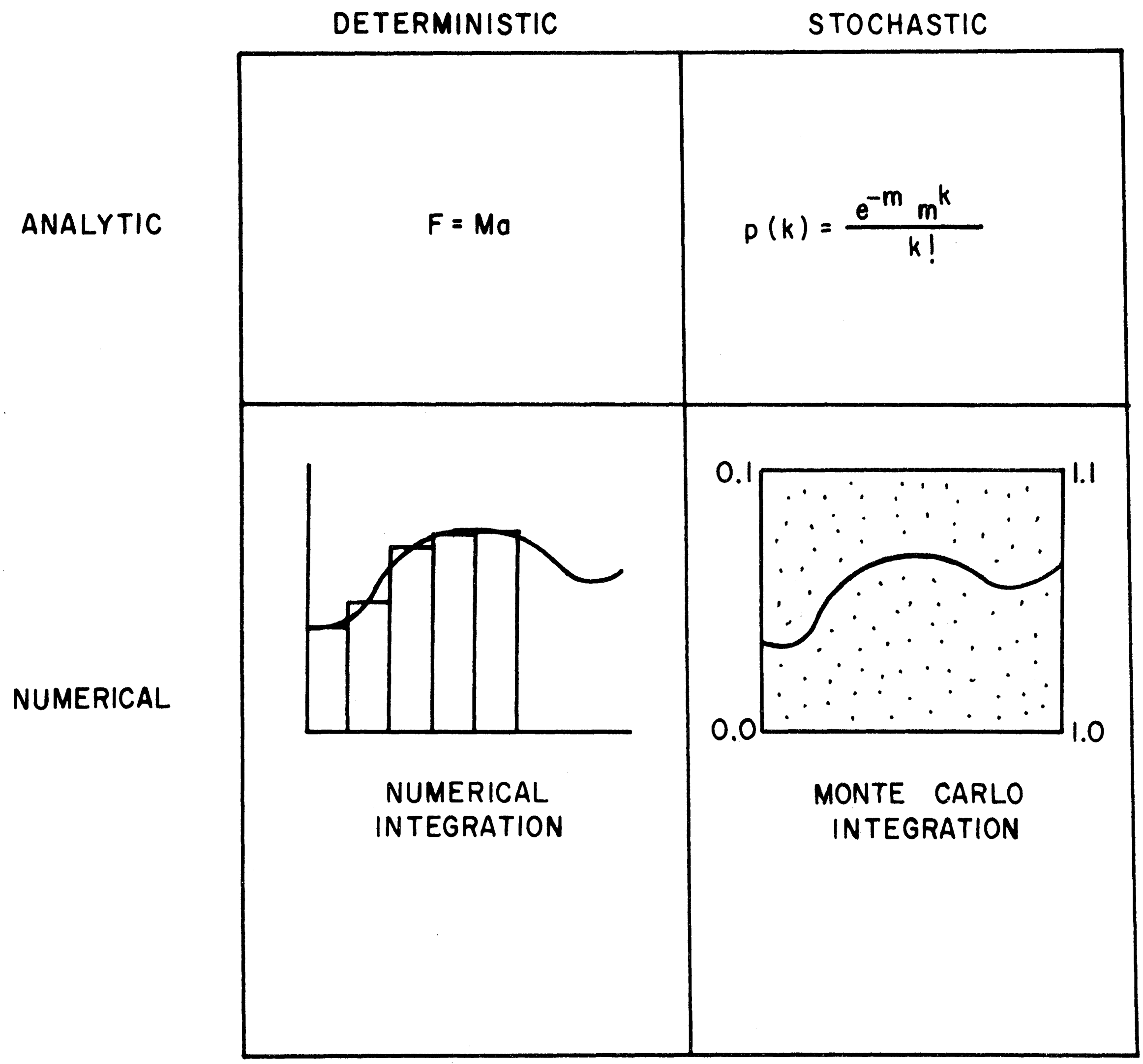


FIGURE 2. TYPES OF MATHEMATICAL MODELS

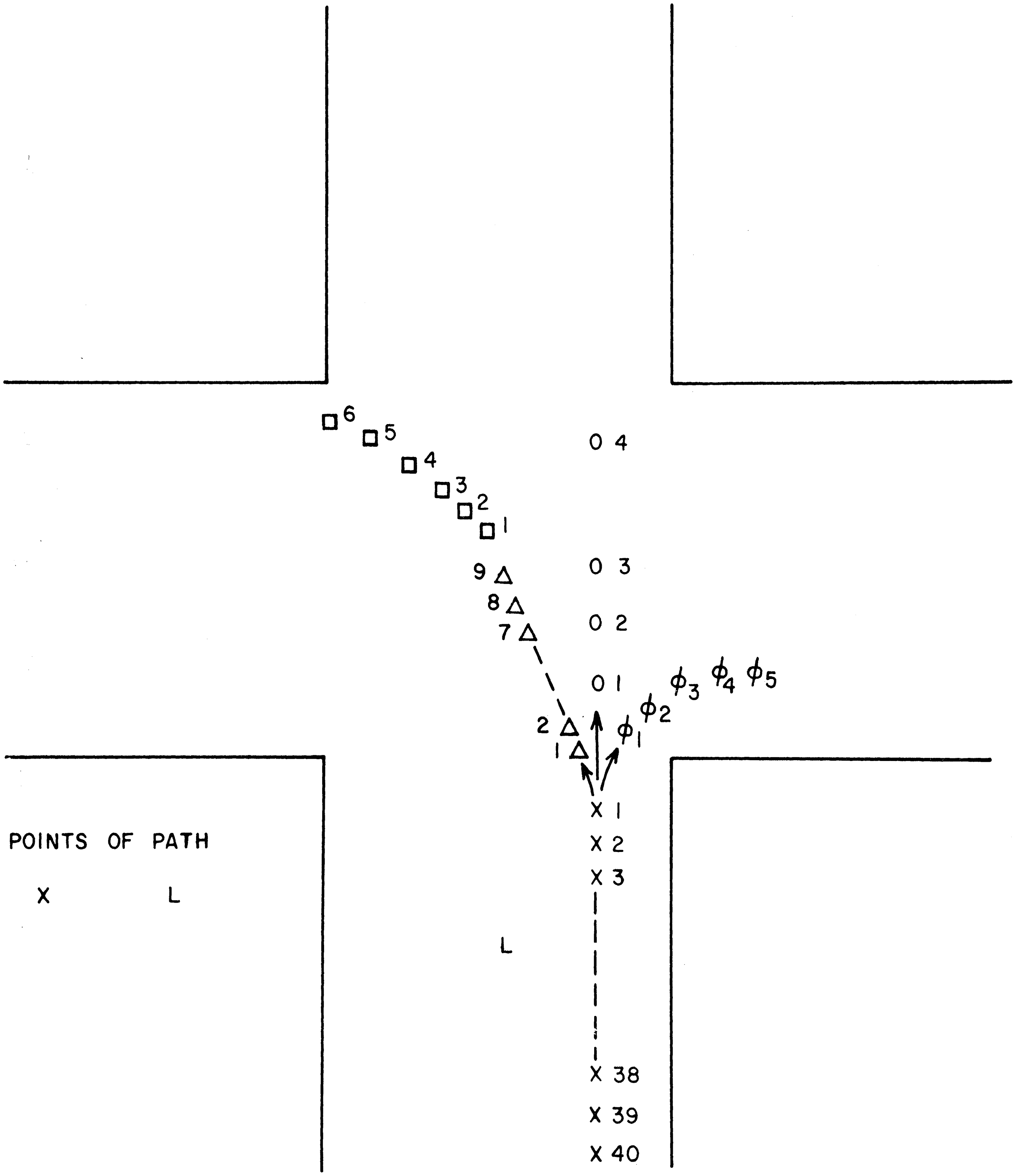


FIGURE 3. INTERSECTION

NUMERICAL STOCHASTIC MODEL

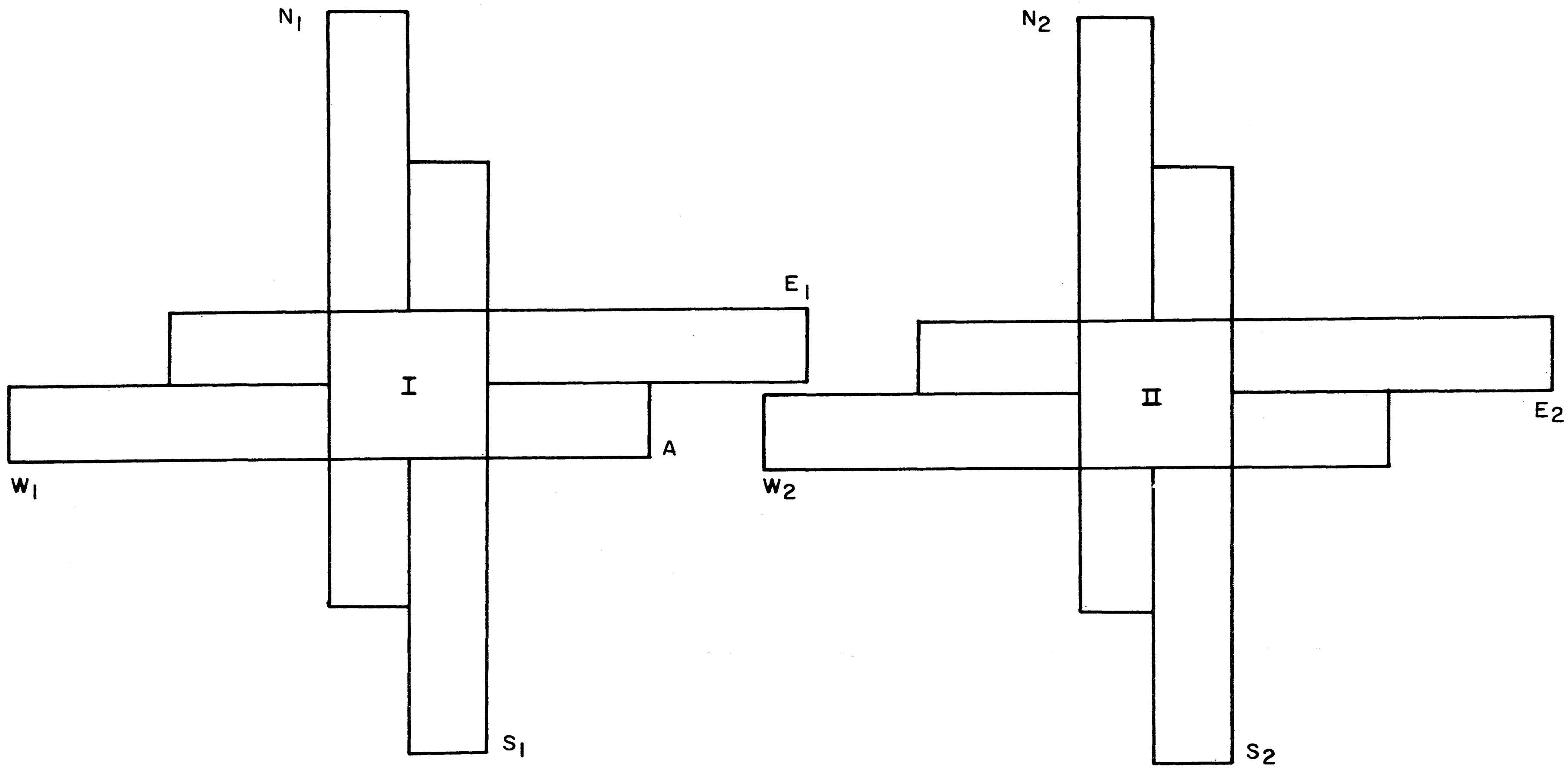


FIGURE 4.

CROSS BLOCK CONNECTION FOR SECTION

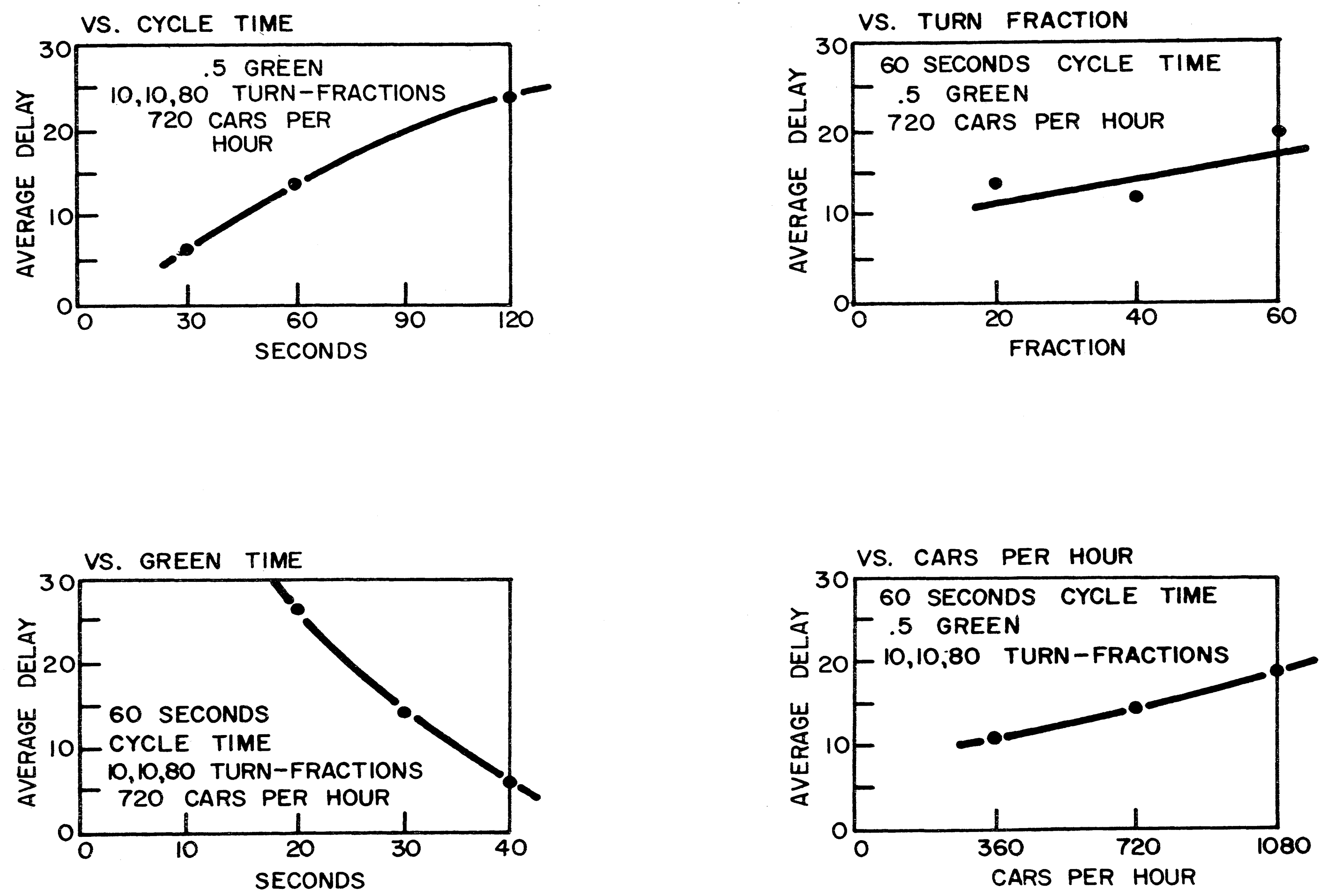


FIGURE 5. **AVERAGE DELAYS**

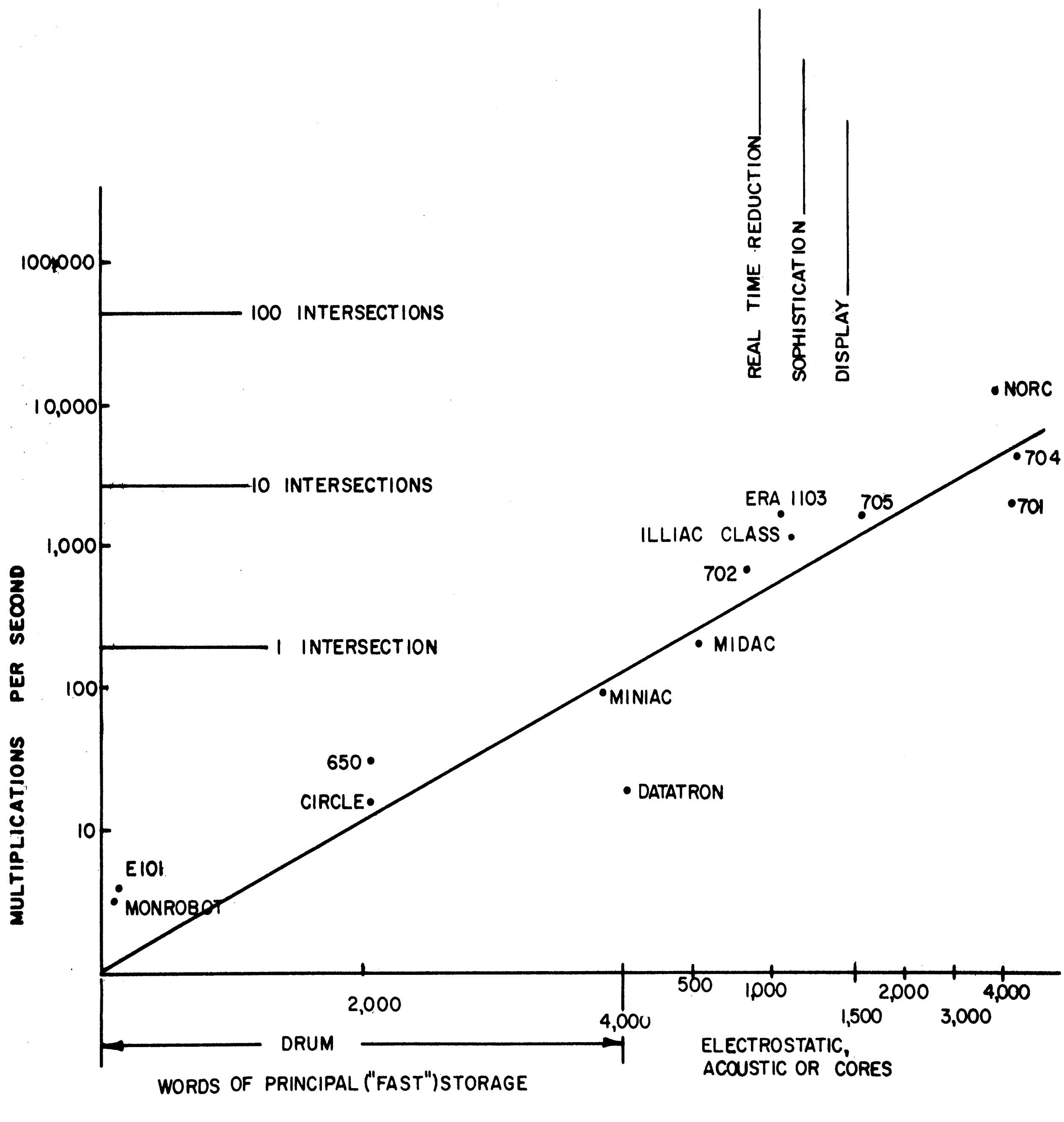


FIGURE 6.

COMPARISON OF COMPUTERS FOR TRAFFIC PROBLEM.

TABLE I

| Criterion | Analysis | Simulation | Trial |
|---|---|---|---|
| Cost | Least | Medium | Most |
| Time | Least | Medium | Most |
| Reproducibility | Most | Medium | Least |
| Realism | Least | Medium | Most |
| Generality of results (if real) | Most | Medium | Least |

Figure 7. Comparison of Analysis, Simulation, Trial and Error