ELECTRIC FIELD-ALTITUDE VARIATIONS IN VHF PROPAGATION, 
AND A SURVEY OF THE EFFECTS OF IRREGULAR TERRAIN

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By: E. C Grant

Approved by: J. A. M. Lyon

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ABSTRACT

Curves are given which relate the electric field strength at a receiver to the altitude of the receiver for radiation from a short vertical antenna located fifteen feet above a plane earth having a conductivity of $6 \times 10^{-14}$ emu and a dielectric constant of 13. The curves are for transmitter-to-receiver distances of 500 feet, 2000 feet, 1 mile, and 3 miles, and for frequencies of 100, 200, and 300 mc. The pertinent calculations and the IBM 650 computer program which was used to obtain the results are also included.

A review of recent papers pertaining to the effect of irregular terrain on received field strength is presented. Papers are reviewed which report the observed effect on transmission loss of specific terrain features such as hills, wooded areas, and soil moisture. One reviewed paper gives graphical means for determining the (theoretical) amount of diffraction loss due to hills. Also summarized is a paper by John J. Egli in which the mean and standard deviation of a large number of field strength measurements are related to the frequency, range, and receiving antenna height. A summary of his results which pertain to the variation with altitude of the vertical component of the electric field due to radiation from a vertical dipole antenna is presented. This summary is in the form of curves, from which the radiated power necessary to insure a given field strength at some desired percentage of all the receiving sites located at a fixed range and altitude can be determined. The curves given are for ranges of one and three miles, receiving antenna heights between 10 and 100 feet, and for two values of frequency (100 mc and 500 mc).
ELECTRONIC FIELD-ALTITUDE VARIATIONS IN VHF PROPAGATION,
AND A SURVEY OF THE EFFECTS OF IRREGULAR TERRAIN

1. INTRODUCTION

In certain propagation problems one is primarily concerned
with obtaining an estimate of the field strength, due to a radiating
antenna, in a region relatively close to the earth. Such problems
may be contrasted with those in which a knowledge of the nature of the
field throughout a much larger region is necessary in order to properly
predict the operation of certain equipment. In the latter instances,
information concerning field strength is usually given in the form of
antenna radiation patterns (such as the three-dimensional pattern of
a radar antenna), but this method becomes inaccurate for calculations
near the surface of the earth.

The problem that motivated this study requires an estimate
of the field strength, due to a transmitting antenna located near the
ground, in a region from ten feet to 3000 feet above the earth and for
transmitter-to-receiver distances between 500 feet and five miles. Two
types of information relating to this problem have been collected in
this memorandum. An analytical solution is presented which consists
of calculations of the field above a theoretical plane earth; a more
realistic solution for the region near the earth is then presented
which is based on results of various field-strength surveys.
2. VARIATION OF ELECTRIC FIELD STRENGTH WITH ALITUDE ABOVE A PLANE EARTH

In the frequency range of interest, 100 mc to 300 mc, the component of the energy radiated from a transmitting antenna which is represented by the surface wave is so rapidly attenuated that its contribution to the total field is usually negligible. Generally, then, it is necessary to consider only the direct and the ground-reflected transmission components. The direct wave, in the absence of any intervening obstructions, varies inversely with the distance from the transmitter, and is often referred to as the "free-space" field. The intensity of the ground-reflected wave, and thus the total field, will vary with the angle of incidence, the frequency, and the conductivity and dielectric constant of the earth.

Specifically, if the transmitting antenna is a short vertical doublet the magnitude of the vertical component of the space wave (i.e., the vector sum of the direct wave and the ground-reflected wave) at any point above the line of sight with respect to the transmitting antenna is given by the following relation:

\[ |E_y| = \frac{E_0}{d} \cos^3 \psi_1 \left[ 1 + \left( \frac{\cos^3 \psi_2}{\cos^3 \psi_1} \right)^2 + 2 \frac{\cos^3 \psi_2}{\cos^3 \psi_1} R \cos (\rho-\theta) \right]^{\frac{1}{2}}, \quad (2-1) \]

where: 

- \(E_0\) is the field strength at unit distance in the equatorial plane of the transmitting antenna;

- \(R\) and \(\rho\) are the magnitude and phase angle, respectively, of the reflection coefficient of the earth, as given below;

- \(\psi_1\) and \(\psi_2\) are as shown in Fig. 1;

- \(d\) is the horizontal distance between antennas; and

---

1 The derivation of Eq. (2-1) is given in Appendix I. See also: Ref. 12, p. 691.
$\theta$ is the phase difference between the direct wave and the ground-reflected wave.

Fig. 1 The geometry involved in Eq. (1)

$\psi_1$, $\psi_2$, and $\theta$ are given by the following relations:

$$\tan \psi_1 = \frac{h_1 - h_2}{d}, \quad (2-2)$$

$$\tan \psi_2 = \frac{h_1 + h_2}{d}, \quad (2-3)$$

$$\theta = 2\pi \frac{L_2 - L_1}{\lambda}, \quad (2-4)$$

where: $h_1$ and $h_2$ are the distances of the transmitting and receiving antennas, respectively, above ground;

$\lambda$ = wave length;

$$L_1 = \sqrt{d^2 + (h_1 - h_2)^2} \quad (2-5)$$

= path length of the direct wave;

$$L_2 = \sqrt{d^2 + (h_1 + h_2)^2} \quad (2-6)$$

= path-length of the ground-reflected wave.
If \( L_1 \) and \( L_2 \) are expressed in feet and \( \lambda \) in meters, then

\[
\theta = 1.915 \frac{L_2 - L_1}{\lambda}.
\]  

(2-7)

For the short, vertical, doublet transmitting antenna the free-space field at one kilometer from the transmitter is \( E_o = 150 \sqrt{P} \text{ mV/m} \) (Ref. 12, p. 676), where \( P \) is the radiated power in kilowatts. With \( P = 400 \text{ watts} \), \( E_o = 31.126 \times 10^{14} \text{ mV/m} \) at one foot. (Although in the derivations of \( E_4 \) (2-1) and the value of \( E_o \) the transmitting antenna is assumed to be a short vertical doublet, a negligible error is introduced if the transmitter is, instead, a half-wave dipole.)

For vertically-polarized plane waves the reflection coefficient of the earth is given by (Ref. 12, p. 699)

\[
R = \frac{\varepsilon' \sin \psi_2 \left( -\sqrt{\varepsilon' - \cos^2 \psi_2} \right)}{\varepsilon' \sin \psi_2 \left( +\sqrt{\varepsilon' - \cos^2 \psi_2} \right)},
\]  

(2-8)

where:

\( \psi_2 = \) angle of incidence (Fig. 1);

\( \varepsilon' = \varepsilon - j 6 \sigma \lambda (10)^{12} \);

\( \varepsilon = \) dielectric constant of the earth;

\( \sigma = \) conductivity of the earth (emu);

\( \lambda = \) wave length (meters);

\( j = \sqrt{-1} \).

The calculations were carried out for a transmitting antenna located fifteen feet above the ground and radiating a power of 400 watts. The dielectric constant and conductivity of the earth along the path of propagation were assumed to have the following values: \( \varepsilon = 13, \sigma = 6(10)^{-14} \text{ emu} \). These values are typical of average soil (Ref. 12, p. 709).
An IBM 650 was used to calculate the field strength given by Eq. (2-1) at a sufficient number of points to enable smooth curves to be drawn showing the relation between field strength at the receiver and altitude of the receiver up to 3000 feet. This was done for each of three frequencies at four different values of range: 500 feet, 2000 feet, 1 mile, and 3 miles.

In order to write a program for finding the reflection coefficient, $R \rho$, separate expressions must be given for $R$ and $\rho$. These are as follows:

$$ R = \left( \frac{A^2 + B^2}{C^2 + D^2} \right)^{\frac{1}{2}}, \text{ and} $$

$$ \rho = \tan^{-1} \left( \frac{D}{C} \right) + \tan^{-1} \left( \frac{-B}{A} \right); $$

where:

$$ A = \epsilon \sin \psi_2 - V \cos \eta; $$

$$ B = \beta \sin \psi_2 - V \sin \eta; $$

$$ C = \epsilon \sin \psi_2 + V \cos \eta; $$

$$ D = \beta \sin \psi_2 + V \sin \eta; $$

$$ \beta = 6 \sigma \lambda (10)^{12}; $$

$$ V = [(\epsilon - \cos^2 \psi_2) + \beta^2]^{\frac{1}{2}}; \text{ and} $$

$$ \eta = \frac{1}{2} \tan^{-1} \left( \frac{\beta}{\epsilon - \cos^2 \psi_2} \right). $$

With $\epsilon = 1 \frac{\mu}{\text{em}}$ and $\sigma = 6(10)^{-14} \text{emu}$, Eqs. (2-15), (2-16), and (2-17) become

$$ \beta = 0.36 \lambda, $$

\text{See Appendix III for the derivation of Eq. (2-9) and (2-10).}
\[ V = \left( (13 - \cos^2 \psi_2)^2 + 0.1296 \lambda^2 \right)^{\frac{1}{4}} \]

\[ \equiv (13 - \cos^2 \psi_2)^{\frac{1}{2}} \]  \hspace{1cm} (2-19)

for the range of frequencies of interest, and

\[ \eta = \frac{1}{2} \tan^{-1} \left( \frac{0.36 \lambda}{13 - \cos^2 \psi_2} \right) . \]  \hspace{1cm} (2-20)

The complete program for performing the computations on the IBM 650 computer is given in Appendix III. It is written in the GAT language, which is used by the University of Michigan Statistical Computing Laboratory and by the Willow Run Laboratories Computing Center. Altitude of the receiving antenna and wave length are written as subscripted variables, and the data must be arranged so that the program repeats for every desired value of range. The results are given in graphical form in Figs. 2 through 9. For values of radiated power other than 400 watts the values of field strength obtained from the curves must be multiplied by \( \sqrt{\frac{P}{20}} \), where \( P \) is the radiated power in watts.

3. THE EFFECT OF IRREGULAR TERRAIN AND VEGETATION ON RECEIVED FIELD STRENGTH

3.1 Diffraction Loss Due to Hills

The direct wave is affected most seriously by the shadowing effect of hills or other obstructions in the line of sight between the transmitter and receiver. The loss in signal strength due to an obstructing hill or ridge can be approximated by diffraction theory. The Fresnel-Kirchoff method for calculating the diffraction by a straight-edge yields formulas for the diffracted field; and these results can be
FIG. 2 VARIATION OF ELECTRIC FIELD STRENGTH, AS RECEIVED ON A SHORT VERTICAL ANTENNA, WITH ALTITUDE ABOVE A PLANE EARTH

RANGE = 500'
TRANS. POWER = 400 W
HEIGHT OF TRANSMITTING ANTENNA (VERT. DOUBLE) ABOVE GROUND = 15'
EARTH CONSTANTS:
\[ \varepsilon = 13 \]
\[ \sigma = 6 \times 10^5 \text{EMU} \]
FIG. 4
VARIATION OF ELECTRIC FIELD STRENGTH, AS RECEIVED ON A SHORT VERTICAL ANTENNA, WITH ALTITUDE ABOVE A PLANE EARTH

RANGE: 500'
TRANS. POWER: 400W
HEIGHT OF TRANSMITTING ANTENNA (VERT. DOUBLET) ABOVE GROUND: 15'
EARTH CONSTANTS:
\[ \varepsilon = 13 \]
\[ \sigma = 6 \times 10^5 \text{ EMU} \]
FIG. 7 VARIATION OF ELECTRIC FIELD STRENGTH, AS RECEIVED ON A SHORT VERTICAL ANTENNA, WITH ALTITUDE ABOVE A PLANE EARTH

RANGE = 2000'
TRANS. POWER = 400 W
HEIGHT OF TRANSMITTING ANTENNA (VERT. DOUBLE) ABOVE GROUND = 15'

EARTH CONSTANTS:
\( \varepsilon = 13 \)
\( \sigma = 6.107 \times 10^{-3} \text{emu} \)
FIG. 8 VARIATION OF ELECTRIC FIELD STRENGTH, AS RECEIVED ON A SHORT VERTICAL ANTENNA, WITH ALTITUDE ABOVE A PLANE EARTH

- Range: 1 M
- Trans. Power: 400 W
- Height of Transmitting Antenna: (Vert. Double) Above Ground 15'

Earth Constants:
- \( \epsilon = 3 \)
- \( \sigma = 6(10)^{10} \) EMU
used to obtain an approximation\(^1\) for the field in the shadow zone of a ridge. According to Burrows and Attwood (Ref. 1) the conditions which must be fulfilled in order to obtain reliable results are:

1. the distances from transmitter and receiver to the ridge must be large compared to the height of the ridge above a straight line connecting the transmitting and receiving antennas; (2) this height must be large compared to the wavelength; and (3) the horizontal extension of the ridge must be large compared to the height. If these conditions are satisfied the ratio of the field strength at the receiver in the presence of the ridge to that which would occur in the absence of the ridge is given approximately by the following relation (Ref. 1, p. 463):

\[
\frac{E}{E_0} = \frac{1 + j}{2} \left\{ \frac{1}{2} + \int_0^v \cos \left(\frac{\pi}{2} u^2\right) du - j \left[ \frac{1}{2} + \int_0^v \sin \left(\frac{\pi}{2} u^2\right) du \right] \right\}
\]

where:

\[
v = \hat{h} \sqrt{\frac{2}{\lambda}} \left( \frac{1}{d_1} + \frac{1}{d_2} \right);
\]

\[
h_0 = \frac{d_1 h_2 + d_2 h_1}{d_1 + d_2} - h;
\]

\(d_1 = \text{distance from transmitter to obstacle;}
\)

\(d_2 = \text{distance from receiver to obstacle;}
\)

\(h_1 = \text{height of transmitting antenna above a given reference level;}
\)

\(h_2 = \text{height of receiving antenna above the same reference level;}
\)

---

\(^1\) According to at least one source the error involved in the approximation may be quite large, as will be mentioned later.
h = height of the obstacle above the reference level. The sign of \( h_o \) will be negative, and the negative sign should also be chosen for \( v \).

If the profile of the ridge is very broad compared to its height, or if it contains more than one peak, the value of \( h \) should be the effective height as illustrated in Fig. 10.

![Fig. 10. Effective Height of a Broad Ridge.](image)

Burrows and Attwood make the following statement (Ref. 1, p. 466):

Experience shows that so long as the profile of the ridge is reasonably compact and its surface reasonably rough, the diffraction formula will give the magnitude of the field behind the ridge to within a few decibels.

On page 467 of Ref. 1 the ratio \( \left| \frac{E}{E_0} \right| \) is plotted in decibels as a function of the quantity \( \frac{h_o}{\sqrt{\lambda d}} \), assuming that the ground between the transmitter and receiver is sufficiently rough so that all ground reflection may be neglected. Figures 11 and 12 show how the ratio \( \left| \frac{E}{E_0} \right| \) varies as a function of distance from transmitter to receiver for the following conditions:

<table>
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<th>Fig. 12</th>
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<tr>
<td>Height of ridge (feet)</td>
<td>45</td>
</tr>
<tr>
<td>Height of transmitting antenna (feet)</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Fig. 11</td>
</tr>
<tr>
<td>--------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Height of receiving antenna (feet)</td>
<td>6</td>
</tr>
<tr>
<td>Frequency (mc)</td>
<td>300</td>
</tr>
<tr>
<td>$\frac{d_1}{d_2}$ or $\frac{d_2}{d_1}$</td>
<td>0.1, 1</td>
</tr>
</tbody>
</table>

When the ground near the transmitter or receiver is so smooth that the reflected wave cannot be neglected, the diffraction problem becomes more complicated. It can be solved by the method of images, i.e., by assuming that the reflected wave on the transmitter side of the obstacle issues from an image transmitter and that the reflected wave on the receiver side is incident upon an image receiver. The total field at the receiver may be written

$$ E = E_1 + E_2 + E_3 + E_4, $$

where each term on the right-hand side is given by an equation having the form of Eq. (3-1). $E_1$ corresponds to the radiation in the absence of reflection, $E_2$ to the radiation from the image transmitter to the receiver, $E_3$ to the radiation from the transmitter to the image receiver, and $E_4$ to the radiation from one image to the other. These four terms differ in the value of $v$ assigned to each of them, the effective height, $h_o$, computed by Eq. (3-3) and the path lengths being different in each case. The reflection coefficient of the earth must also be considered in computing $E_2$, $E_3$, and $E_4$.

A simplified method for computing diffraction loss in the shadow region is presented in Refs. 2 and 3. This method can be applied to the cases of reflection on either or both sides of the obstacle or of reflections on neither side. The obstacle can be at any location along the path. It is claimed that when the diffraction parameter, $v$, is greater than 1, the accuracy [with respect to results obtained from using
FIG. 12 FIELD IN SHADOW BEHIND A DIFFRACTING RIDGE

DB BELOW FREE SPACE FIELD STRENGTH

\[ \frac{d_1}{d_2} = 0.1 \]

\[ \frac{d_1}{d_2} = 1 \]

HEIGHT OF TRANS. ANTENNA = 20 FT.
HEIGHT OF REC. ANTENNA = 20 FT.
HEIGHT OF RIDGE = 100 FT.
FREQUENCY = 300 MC.

DISTANCE FROM TRANSMITTER TO RECEIVER (FEET)
Eq. (3-1)] is within two decibels.

An experimental investigation of the diffraction of radio waves by a dominating ridge, conducted by the Radio Physics Laboratory of Ottawa (Ref. 4), indicates that the diffraction loss due to such a ridge may be considerably greater than the value predicted by the Fresnel-Kirchoff knife-edge theory, the discrepancies increasing with increasing frequency. The distance between the transmitter and receiver in this experiment was 17.5 miles, the ridge being about six miles from the receiver. The top of the ridge was 600 feet above a line joining the transmitter and receiver. On one side of the ridge was gently-rolling farm land, on the other side the terrain was rugged and heavily wooded. The experimental and theoretical values of the diffraction loss obtained are given in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Frequency (mc)</th>
<th>Diffraction Loss (db)</th>
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<tr>
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<td>Experimental</td>
<td>Theoretical</td>
</tr>
<tr>
<td>173</td>
<td>27-31</td>
<td>(31-36)¹</td>
</tr>
<tr>
<td>493</td>
<td>39-45</td>
<td>41-47</td>
</tr>
</tbody>
</table>

The ranges of experimental values shown are the minimum and maximum values obtained for the range of receiver-antenna heights investigated (5 feet to 110 feet). There did not appear to be a strong dependence of diffraction loss on the transmitting-antenna height, which was about 72 feet. Seasonal variations in the terrain and foliage also appeared to have only a small effect.

¹ It was noted by the authors that the significance of these values is "conjectural" due to the substantial lateral-multipath transmission present in this case.
3.2 Loss Due to Trees and Other Vegetation

Trees also form very effective obstacles for high-frequency radio waves. A single tree may cause a drop in signal strength of several decibels; however, the attenuation is less for horizontal polarization than for vertical polarization at frequencies below about 500 mc. At higher frequencies the polarization is not an important factor. It has been reported by the M.I.T. Radiation Laboratory (Ref. 8) that 150 feet of typical New England woods with medium-size trees and summer foliage caused a decrease in the received signal of 6 db for a horizontally polarized, and 21 db for a vertically polarized, 200-mc signal. With vertical polarization there were large variations of field intensity within a small area, due to reflections from nearby trees. Moving a receiver from a line-of-sight position on one side of a wooded hill to the side away from the transmitter produced 45 db and 60 db drops in signal for horizontal and vertical polarization, respectively. The horizontal distance between the two points was 2400 feet, and the height of the hill was 200 feet.

Jansky and Bailey report that transmission at 116 mc through a large and rather dense forest (Croatan National Forest near New Bern, North Carolina) on level ground showed no adverse effect for horizontally-polarized signals, but for vertical polarization the signal level was about 18 db below that obtained over open ground. Measurements were made at ranges of 1, 2½, and 6 miles with both antennas located 19 feet above the ground.

1 Ref. 1: p. 480.

2 Ref. 5: Part 2, pp. 7, 8 and Figs. 26, 27.
In an experiment conducted by RCA (Ref. 6) measurements of the attenuation of field strength through 500 feet of woods on level ground were made at a frequency of 500 mc. The transmitting and receiving antennas were six feet and seven feet, respectively, above the ground. Results indicated a loss of 17 to 19 db in the summer as compared with transmission over open ground. There was no appreciable difference between vertical and horizontal polarization. In the winter the attenuation was 15 db with vertical polarization and 12 db with horizontal polarization. At 250 mc the attenuation through the same section of woods in winter was 14 db with vertical and 10 db with horizontal polarization. Transmission of 500-mc signals over low scrub-pines, as compared with that over sandy ground, showed a reduction in signal of 6 to 8 db due to vegetation. The antennas were 8½ feet above ground, and the height of the undergrowth was 5 to 6 feet. "This indicates that vegetation causes the ground-reflected ray to be reflected from a level considerably above that of the ground rather than being absorbed by the vegetation" (Ref. 6, p. 100).

The Department of Electrical Engineering at the University of Tennessee (Ref. 7) measured the field strength of a 410-mc signal after passing through several types of brush. Measurements were made with horizontal, vertical, and circular polarization. The average attenuation caused by 130 feet of brush and undergrowth with autumn foliage was 4.2 db with respect to transmission over open ground. Values obtained over a different path -- 120 feet of small trees and rather dense undergrowth with summer foliage -- gave 8.2 db as the average attenuation. The effect of polarization was not considered to be significant. The height of the transmitting and receiving antennas in both cases was
about five feet.

3.3 Effect of Ground Moisture

Investigations conducted by the Research Institute for National Defense in Stockholm, Sweden (Ref. 9) indicate that vhf ground-wave propagation at low heights and short distances is influenced considerably by ground moisture. Field strength recordings of vertically-polarized vhf waves over smooth ground showed variations up to 14 db over the same paths at different seasons. The measured field strengths correspond to values of the earth's dielectric constant from about three to thirty.

Investigations have shown that the value of the effective dielectric constant ($\varepsilon$) is determined mainly by the water content of the ground and is relatively independent of the type of ground. Very dry earth has effective $\varepsilon$ - values between 2.5 and 3, and for wet earth $\varepsilon$ is approximately proportional to the percentage water content, $w$ per cent. Therefore, in vhf field strength calculations the empirical relation

$$\varepsilon = 0.78w + 2.5$$

may be used (Ref. 9, p. 171).

Propagation measurements at 50 mc showed that a covering of snow or ice can cause a considerable decrease in the dielectric constant, the amount of change depending on the original value of $\varepsilon$.

3.4 Summary

In many cases, such as with mobile communications, countermeasure systems, and other systems involving movable equipment, one may not know in advance the nature of the terrain over which transmission will be required. In such situations information concerning the
propagation loss associated with a particular hill or grove of trees is of little value in determining the amount of attenuation which should be assumed in the design of the system. One approach to this type of problem would be to analyze a large number of measurements involving propagation over a great variety of terrain in order to determine the mean and standard deviation of the distribution of the losses which are attributable to terrain and other obstructions. This would allow one to predict the probability that various amounts of attenuation would be encountered. Such information has been provided by Egli (Ref. 10) of the Army Signal Engineering Laboratories, Fort Monmouth, New Jersey. A large amount of data, collected by both the F. C. C. and commercial organizations, which characterized transmission over various types of terrain, was analyzed statistically to provide a basis for estimating the transmission loss which should be assumed to provide the desired coverage at a given frequency. In the vhf region of 50 to 250 mc approximately 1400 measurements were included in the data, and in the uhf region of 288 to 910 mc transmission over a total of 804 miles was represented. The results are expressed in terms of the percentage of locations at a specified distance from the transmitter, at which the field strength can be expected to equal or exceed a specified value. For example, if a five-mile circle with the transmitter at the center is divided into 100 equal parts, with each division represented by a point (location), and this configuration is superimposed on the statistically-derived landscape represented by the data, 75 per cent coverage would mean that at 75 of the locations the median value of the field strength in the immediate vicinity would be equal to or greater than the specified value.
Egli's results are expressed in terms of the theoretical plane-earth field strength which, for small antenna heights, is given approximately by the relation

\[ E = \frac{h_t h_r f}{95d^2} \sqrt{P_t} \]  \hspace{1cm} (3-4)

where:

\[ E = \text{field intensity in microvolts/meter}; \]
\[ h_t = \text{transmitting antenna height in feet}; \]
\[ h_r = \text{receiving antenna height in feet}; \]
\[ f = \text{transmission frequency in megacycles}; \]
\[ d = \text{distance from transmitter in miles}; \]
\[ P_t = \text{effective radiated power in watts}. \]

This equation is limited to transmission over water and flat, barren land. The measured field-strength data over irregular terrain were compared with the values given by this plane-earth equation, and the median field-strength at a given frequency was described by the median deviation from the theoretical plane-earth field-strength. This median deviation from the theoretical plane-earth field-strength, called "terrain factor" by Egli, is shown in Fig. 13. The straight line indicates an inverse frequency relationship with respect to 40 mc, the point of intersection with the theoretical plane-earth field-strength. It is interesting to note that the terrain factor was also found to be essentially independent of the distance from transmitter to receiver. \(^1\)

\(^1\) This statement was confirmed during a private conversation with Mr. Egli.
Fig. 13. Median Terrain Factor for Fixed-to-Vehicular or Mobile Service (Ref. 10, p. 1385, Fig. 1)

On the assumption that the terrain factor is independent of distance, Eq. (3-4) can be modified to give an empirical relation for the median field at the 50 percentile locations, $E_{50}$:

$$E_{50} = 40 \frac{h_t h_r}{95d^2} \sqrt{P_t} \quad (h_r > 30 \text{ ft.}) \, . \quad (3-5)$$

The theoretical received power for transmission using half-wave dipoles over plane earth is given approximately by the relation

$$P_r = 0.345 \left( \frac{h_t h_r}{d^2} \right)^2 P_t (10)^{-14}, \quad (3-6)$$

which is independent of frequency. Applying the inverse frequency variation of the median deviation from the theoretical plane-earth field gives an empirical relation for the median received power

1 This equation must be modified if $h_r$ is less than 30 feet, as will be mentioned later.
\( P_{50} = 0.345 \left( \frac{h_t h_r}{d^2} \right)^2 \left( \frac{40}{f} \right)^2 P_t \times 10^{-14}. \)  \( 3-7 \)

From the nomograph in Fig. 14 one can obtain the 50 percentile median field strength, statistically derived, and the plane-earth received power, theoretically derived, both of which are independent of frequency.

Fig. 15 gives the correction factor (in db) which must be added to the value of \( E_{50} \) when the received field strength to other than the 50 percentile locations is desired; the correction factor to the theoretical plane-earth received power imposed by irregular terrain is obtained from Fig. 16. The terrain factor for the 50 percentile locations (received power) varies inversely as the frequency squared, as given by Eq. (3-7).

If at any time the calculated field strength, as obtained from Figs. 14, 15, or 16, exceeds the free-space field (Fig. 17) the free-space value should be used.

As an example, consider the following problem (from Egli):

Transmission frequency--150 mc;
Half-wave dipole antennas;
Transmitting antenna height--100 ft;
Receiving antenna height--10 ft;
Range--10 miles;
Coverage--90 per cent of locations at 10 miles;
Required--transmitter power output.

The theoretical received power in db below one watt is found from Fig. 14 to be 119 dbw. The field strength at 50 per cent of the
FIG 14 RECEIVED POWER OVER PLANE EARTH, AND 50 PERCENT LOCATION MEDIAN FIELD STRENGTH - ONE WATT RADIATED (REF. 10, P. 1385, FIG. 2)
FIG. 15 CORRECTION CURVES FOR FIELD STRENGTH TERRAIN FACTOR (REF. 10, P.1386, FIG. 4)

FIG. 16 CORRECTION CURVES FOR RECEIVED POWER TERRAIN FACTOR (REF. 10, P.1386, FIG.6)
FIG. 17 FIELD STRENGTH AND RECEIVED POWER FOR ONE WATT RADIATED BETWEEN HALF-WAVE DIPOLES IN FREE SPACE. (REF. IO, P.1386, FIG. 5)
locations, also from Fig. 14, will be 17.5 db above one microvolt per meter, one watt radiated. Since the terrain correction-factor for the 90 percentile locations (Fig. 15) is -11.5 db, the median received field strength at the 90 percentile locations is 6 db above one microvolt per meter, one watt radiated. This value does not exceed the free-space field strength at 10 miles of 53 db above one microvolt per meter obtained from Fig. 17.

The terrain correction-factor for the received power at the 90 percentile locations is -23 db, thus the received power will be -142 dbw. The half-wave dipole to half-wave dipole path attenuation for this degree of coverage is therefore 142 db. The free-space received power is -95.4 dbw, from Fig. 17, which exceeds that for the 90 percentile locations.

The results of the experiments described in Refs. 1 to 8 are compared in Table II with values obtained by Egli's method for the same conditions of antenna height and distance. $E_p$ is the theoretical plane-earth field strength as given by Eq. (3-4) (in db above one microvolt per meter), and $E_\alpha$ is the field strength corresponding to the attenuation, $\alpha$, reported by the various authors ($E_\alpha = E_p - \alpha$). $E_{50}$ and $E_{90}$ are the 50 and 90 percentile field strengths, obtained from Figs. 14 and 15. In two of the cases involving an obstructing ridge, the theoretical value of the diffracted field ($E_d$), obtained by use of the Fresnel-Kirchoff diffraction formulas, is also given. All of the values of field strength are expressed in db above one microvolt per meter, normalized to a radiated power level of one watt. Values shown in parentheses were assumed in order to obtain results for comparison.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Freq.</th>
<th>$h_t$ (ft.)</th>
<th>$h_r$ (ft.)</th>
<th>Height of Ridge or Depth of Wooded Area (ft.)</th>
<th>$d$ (mi.)</th>
<th>$\alpha$ (db)</th>
<th>$E_P$</th>
<th>$E_\alpha$</th>
<th>$E_{50}$</th>
<th>$E_{90}$</th>
<th>$E_d$ (db above 1 μW/m)</th>
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<tr>
<td>1</td>
<td>300</td>
<td>20</td>
<td>20</td>
<td>100</td>
<td>2</td>
<td>16-19</td>
<td>50</td>
<td>34-31</td>
<td>34</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>173</td>
<td>72</td>
<td>110</td>
<td>600</td>
<td>17.5</td>
<td>27-31</td>
<td>33</td>
<td>6-2</td>
<td>21</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>493</td>
<td>72</td>
<td>110</td>
<td>600</td>
<td>17.5</td>
<td>39-41</td>
<td>43</td>
<td>4-2</td>
<td>21</td>
<td>7</td>
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<td>19</td>
<td>19</td>
<td>5280</td>
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<tr>
<td>6</td>
<td>500</td>
<td>6</td>
<td>7</td>
<td>500</td>
<td>(1)</td>
<td>12-19</td>
<td>47</td>
<td>35-28</td>
<td>31</td>
<td>17</td>
<td></td>
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<tr>
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<td>6</td>
<td>7</td>
<td>500</td>
<td>(1)</td>
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<td>41</td>
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<td>7</td>
<td>410</td>
<td>5</td>
<td>5</td>
<td>130</td>
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<tr>
<td>8</td>
<td>200</td>
<td>(6)</td>
<td>(6)</td>
<td>150</td>
<td>(1)</td>
<td>6-21</td>
<td>38</td>
<td>32-17</td>
<td>31</td>
<td>19</td>
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</tr>
<tr>
<td>8</td>
<td>200</td>
<td>(6)</td>
<td>(6)</td>
<td>Wooded Hill</td>
<td>(1)</td>
<td>45-60</td>
<td>38</td>
<td>-7-22</td>
<td>31</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE II**

COMPARISON OF RESULTS OF EXPERIMENTS DESCRIBED IN REFERENCES
It can be seen that in most of the cases the values of $E_{\alpha}$ fall between $E_{50}$ and $E_{90}$. Where this is not true, the terrain features seem to be either more or less severe than would be expected in the average case. It appears, therefore, that the results of Egli are not inconsistent with the experimental results reported by other authors.

Egli also analyzed the aforementioned data in an attempt to relate field strength at the receiver to receiving-antenna height, which was defined as the height above local terrain. The data suggested an approximately linear relationship between field strength and receiving-antenna height for antennas above the surrounding terrain features. For receiving antennas which did not clear the surrounding terrain features (between six and thirty feet), the field strength appeared to be more nearly proportional to the square root of receiving-antenna height.

Figures 18 and 19 show the variation of the 10, 50, and 90 percentile field strengths with receiving-antenna height at frequencies of 100 and 500 mc for a transmitting-antenna height of 20 feet. The curves in Fig. 18 are for a transmitter-to-receiver distance of one mile, while those in Fig. 19 are for a distance of three miles. When the values of field strength in these figures are multiplied by $\sqrt{P_t}$, where $P_t$ is the effective radiated power, these curves indicate the approximate percentage of cases in which the field strength at a certain distance above the ground can be expected to equal or exceed a specified value. From Fig. 19, for example, it is seen that at a distance of three miles and a frequency of 100 mc the field strength at a point ten

---

1 The data plotted in Figs. 18 and 19 were obtained from Figs. 14 and 15.

2 According to Egli the results of his work should be reliable for ranges as low as one mile, especially for percentiles greater than about 50.
FIG. 18 VARIATION OF MEDIAN FIELD STRENGTH WITH RECEIVING ANTENNA HEIGHT FOR 10, 50, & 90 PERCENT COVERAGE.
Fig. 19 Variation of median field strength with receiving antenna height for 10, 50, & 90 percent coverage.
feet above the earth can be expected to equal or exceed \(50\sqrt{P_t}\) \(\mu V/m\) only 10 per cent of the time, while it can be expected to equal or exceed \(15\sqrt{P_t}\) \(\mu V/m\) 50 per cent of the time.

Egli found that the values of received field-strength represented by the original data, when plotted in terms of the deviation from the theoretical plane-earth field intensity, were log-normally\(^1\) distributed with a standard deviation of 8.3 \(db\) in the vhf region (taken about a center frequency of \(127.5\) \(mc\)) and a standard deviation of 11.6 \(db\) in the uhf region (center frequency of \(510\) \(mc\)). This is reflected in Figs. 18 and 19, although it could be more clearly illustrated in a three-dimensional plot.

It should be pointed out that Eq. (3-4), on which Egli's results are based, is a simplified form of the general equation for radiation over a plane-earth, Eq. (2-1), and that this simplification is made possible by assuming that the angle of incidence of the ground-reflected wave is nearly \(90^\circ\) (glancing incidence). Therefore, in order for (3-4) to be a reasonably accurate expression for the plane-earth field strength, the following conditions must be fulfilled:

\[
\frac{\lambda d}{2h_1h_2} \gg 1 \quad (d, h_1, h_2 \text{ in feet; } \lambda \text{ in meters}), \quad (3-8)
\]

\[
d < 20 \text{ miles} \quad (3-9)
\]

Terman (Ref. 12, pp. 692-693) lists other conditions which are necessary for Eq. (3-4) to be valid, but they depend on the conductivity and dielectric constant of the earth and are less easily evaluated than (3-8).

\(^1\) A random variable, \(x\), is said to have a log-normal distribution if \(\log (x-a)\) is normally distributed. Therefore, the values of terrain factor, plotted in \(db\), will have a normal distribution.
For a particular set of parameters which were investigated \( f = 300 \text{ mc}, \epsilon = 15, \text{ and } \sigma = 3(10)^{-14} \text{ emu} \) these other conditions reduced to the following limitations:

\[
\frac{d}{h_1 + h_2} \geq 0.4. \quad (3-10)
\]

For other values of earth constants and frequency, the constant in (3-10) may be considerably greater.

4. CONCLUSIONS

Propagation experiments such as those discussed in sections 3.1, 3.2, and 3.3 give some idea of the effect of specific terrain features on the received field strength. The statistical approach of Egli is well suited to the problem of estimating the attenuation caused by irregular terrain when the system is to operate in an unknown or variable environment. If any information concerning the region in which the system is to operate is available, this should, of course, be considered in predicting path attenuation. Otherwise, Egli's method might, in many cases, lead to unduly pessimistic results. In fact, one source\(^1\) has indicated that a considerable amount of the data analyzed by Egli was obtained in rather mountainous regions.

For large values of antenna height more reliance can be placed on theoretical curves, such as those of section 2, since the effect of terrain features becomes less pronounced as the antenna height is increased.

---

APPENDIX I

DERIVATION OF EQ. (2-1)

If a vertical doublet is radiating in free space, the field intensity, \( \mathbf{E}_1 \), at a distance \( L_1 \) is proportional to the cosine of the angle of elevation, \( \psi_1 \):

\[
\mathbf{E}_1 = \frac{E_0}{L_1} \cos \psi_1 \ e^{j(\omega t - \gamma L_1)},
\]

where

\[
\gamma = \frac{2\pi}{\lambda},
\]

and \( E_0 \) is the magnitude of the field intensity at unit distance in the equatorial plane of the doublet. The vertical component of \( \mathbf{E}_1 \) is

\[
E_{y1} = \frac{E_0}{L_1} \cos^2 \psi_1 \ e^{j(\omega t - \gamma L_1)}
\]

\[
= \frac{E_0}{d} \cos^3 \psi_1 \ e^{j(\omega t - \gamma L_1)},
\]

where \( d \) = the horizontal distance to the doublet.

If the radiating doublet is above a plane earth, \( E_{y1} \) is the vertical component of the field due to the direct wave. The ground-reflected wave is modified by the reflection coefficient:

\[
E_{y2} = \frac{E_0}{d} \cos^3 \psi_2 (\text{Re}^j \rho) \ e^{j(\omega t - \gamma L_2)},
\]

where \( \text{Re}^j \rho \) = the reflection coefficient of the earth and

\( \psi_2 \) = the angle of incidence.

Since the surface wave can be neglected at the frequencies of interest, the field at any point in space above the line of sight is
the vector sum of the direct and ground-reflected components. Therefore, the vertical component of the total field is

\[ E_y = E_{y1} + E_{y2} \]

\[ = \frac{E_0}{d} \cos^3 \psi_1 e^{j(\omega t - \gamma L_1)} \left[ 1 + \frac{\cos^3 \psi_2}{\cos^3 \psi_1} \Re \left( e^{-j\gamma(L_2 - L_1)} \right) \right] \]

Dropping the propagation factor, \( e^{j(\omega t - \gamma L_1)} \), this becomes

\[ E_y = \frac{E_0}{d} \cos^3 \psi_1 \left[ 1 + \frac{\cos^3 \psi_2}{\cos^3 \psi_1} \Re \left( e^{i\theta} \right) \right], \]

where \( \theta = \gamma(L_2 - L_1) = \frac{2\pi}{\lambda} (L_2 - L_1) \).

To obtain the magnitude of \( E_y \) let

\[ K = \frac{\cos^3 \psi_2}{\cos^3 \psi_1} R \]

and \( \phi = \rho - \theta \).

Since \( 1 + Ke^{j\phi} = 1 + K \cos \phi + jK \sin \phi \)

and \( |1 + Ke^{j\phi}| = \left[ (1 + K \cos \phi)^2 + (K \sin \phi)^2 \right]^\frac{1}{2} \]

\[ = \left[ 1 + K^2 + 2K \cos \phi \right]^\frac{1}{2} \]

\[ = \left[ 1 + \left( \frac{\cos^3 \psi_2}{\cos^3 \psi_1} R \right)^2 + 2 \frac{\cos^3 \psi_2}{\cos^3 \psi_1} R \cos (\rho - \theta) \right]^\frac{1}{2}, \]

then

\[ |E_y| = \frac{E_0}{d} \cos^3 \psi_1 \left[ 1 + \left( \frac{\cos^3 \psi_2}{\cos^3 \psi_1} R \right)^2 + 2 \frac{\cos^3 \psi_2}{\cos^3 \psi_1} R \cos (\rho - \theta) \right]^\frac{1}{2}. \]
APPENDIX II

DERIVATION OF SEPARATE EXPRESSIONS FOR R AND \( \rho \)

On page 4 the reflection coefficient was given as:

\[
R \angle \rho = \frac{\epsilon' \sin \psi_2 - \sqrt{\epsilon' - \cos^2 \psi_2}}{\epsilon' \sin \psi_2 + \sqrt{\epsilon' - \cos^2 \psi_2}}
\]

where \( \epsilon' = \epsilon - j60\alpha(10)^{12} \).

Letting \( \epsilon' = \epsilon - j\beta \) and expanding, this becomes

\[
R \angle \rho = \frac{(\epsilon - j\beta) \sin \psi_2 - [(\epsilon - \cos^2 \psi_2) - j\beta]^{\frac{1}{2}}}{(\epsilon - j\beta) \sin \psi_2 + [(\epsilon - \cos^2 \psi_2) - j\beta]^{\frac{1}{2}}}
\]

\[
= \frac{\epsilon \sin \psi_2 - j\beta \sin \psi_2 - \sqrt{\epsilon \sin \psi_2 - j\beta \sin \psi_2}}{\epsilon \sin \psi_2 - j\beta \sin \psi_2 + \sqrt{\epsilon \sin \psi_2 - j\beta \sin \psi_2}}
\]

\[
= \frac{\epsilon \sin \psi_2 - V \cos \eta - j(\beta \sin \psi_2 - V \sin \eta)}{\epsilon \sin \psi_2 + V \cos \eta - j(\beta \sin \psi_2 + V \sin \eta)}
\]

where \( V = [\epsilon - \cos^2 \psi_2]^2 + \beta^2 \frac{1}{4} \)

and \( \eta = \frac{\beta}{2} \tan^{-1}\left(\frac{\beta}{\epsilon - \cos^2 \psi_2}\right) \).

Let \( A = \epsilon \sin \psi_2 - V \cos \eta \),

\( B = \beta \sin \psi_2 - V \sin \eta \),

\( C = \epsilon \sin \psi_2 + V \cos \eta \),

and \( D = \beta \sin \psi_2 + V \sin \eta \);
then \[ R = \left(\frac{A^2 + B^2}{c^2 + D^2}\right)^{\frac{1}{2}} \]

and \[ \rho = \tan^{-1}\left(\frac{-B}{A}\right) - \tan^{-1}\left(\frac{-D}{C}\right) \]

\[ = \tan^{-1}\left(\frac{D}{C}\right) + \tan^{-1}\left(\frac{-B}{A}\right), \]

since \( C > 0 \) and \( D > 0 \).
APPENDIX III

IBM 650 PROGRAM FOR EVALUATION OF EQ. (2-1)

Storage assignment:

\[ \begin{align*}
X_0 &= \beta \\
X_1 &= \psi_1 \\
X_2 &= \psi_2 \\
X_3 &= L_1 \\
X_4 &= L_2 \\
C_i &= h_i, \quad i = 1, \ldots, n \\
D_j &= \lambda_j, \quad j = 1, \ldots, m \\
I_0 &= i \\
J_0 &= j \\
I_1 &= n \\
J_1 &= m
\end{align*} \]

\[ \begin{align*}
Y_0 &= \left(1 - \cos^2 \psi_2 \right) \\
Y_1 &= E \\
Y_2 &= \theta \\
Y_3 &= V \\
Y_4 &= \eta \\
Y_5 &= R \\
Y_6 &= \rho \\
D_5 &= d
\end{align*} \]

\[ \begin{align*}
Z_0 &= \cos \psi_1 \\
Z_1 &= \cos \psi_2 \\
Z_2 &= \sin \psi_2 \\
Z_3 &= \sin \eta \\
Z_4 &= \cos \eta \\
Z_5 &= A \\
Z_6 &= B \\
Z_7 &= C \\
Z_8 &= D
\end{align*} \]
Program:

600 USED IN SUBROUTINES
10 IS HIGHEST STATEMENT NUMBER
DIMENSION C(150) D(5) I(5) N
J(5) X(5) Y(10) Z(10)

1 READ

2 6,J0,J1,J2,
   X0=.36*DJ0

3 6,I0,I1,I, I2,
   X1=ARTAN((15.-C10)/D5)
   X2=ARTAN((15.+C10)/D5)
   X3=(D5P2+(15.-C10)P2)P.5
   X4=(D5P2+(15.+C10)P2)P.5
   Z0=COS.(X1)
   Z1=COS.(X2)
   Z2=SIN.(X2)
   Y0=13.-Z1P2
   Y2=1.915*(X4-X3)/DJ0
   Y3=Y0P.5
   Y4=.5*ARTAN.(X0/Y0)
   Z3=SIN.(Y4)
   Z4=COS.(Y4)
Z5=13.*Z2-Y3*Z4
Z6=X0*Z2-Y3*Z3
Z7=13.*Z2+Y3*Z4
Z8=X0*Z2+Y3*Z3
Y5=((Z5P2+Z6P2)/(Z7P2+Z8P2)) N
P.5
Y6=ARTAN.(Z8/Z7)+ARTAN.(-Z6/ N
Z5)
GO TO 5 IF Z5 W 0
Y6=Y6+3.1416
5
Y1=31.126E4/D5*Z0P3*(1+((Z1/ N
Z0)P3*Y5)P2+2*(Z1/Z0)P3*Y5* N
COS.(Y6-Y2))P.5
6
TDJO TD5 TCIO TY1 TZ2
GO TO 1
END
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