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SIGNAL DETECTION AS A FUNCTION OF SIGNAL INTENSITY AND DURATION

Technical Report No. 42

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ABSTRACT

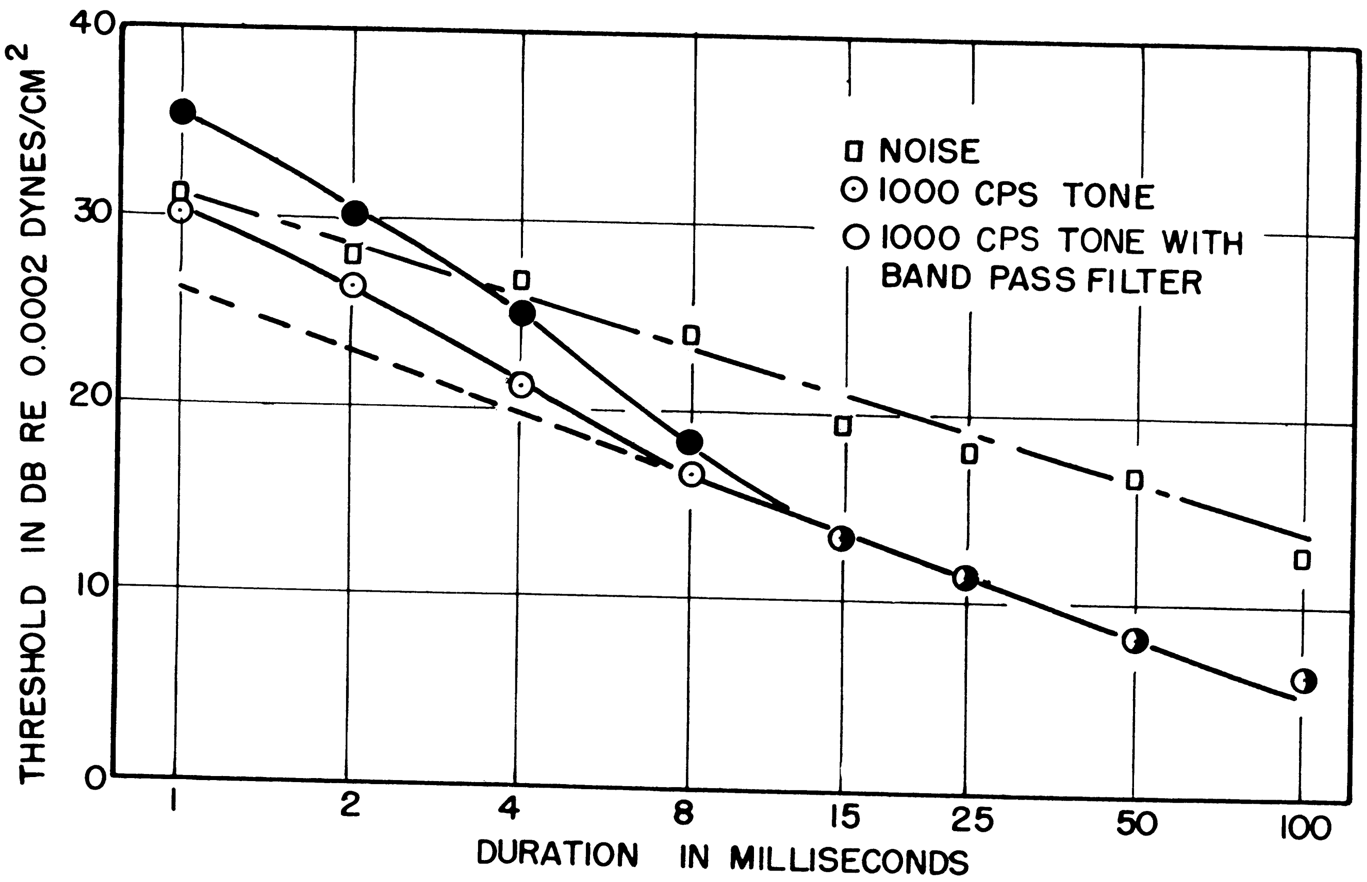
The object of this study was to determine how signal amplitude and duration effect the detectability of a pure tone partially masked by random noise. The study attempted to determine the surface of detectability in a space in which signal duration and amplitude are considered two dimensions in this space. To accomplish this task three experiments were conducted employing the same observers in each experiment. In the first experiment signal duration was held constant while amplitude was varied. In the second experiment signal energy was held constant while various pairs of values for signal duration and amplitude were tested. Finally signal amplitude was held constant while signal duration was varied. A three parameter equation was determined which provided a reasonable fit to this surface of detectability in the plane of signal amplitude and duration. The equations are consistent with the data of previous research in this area. Finally, a comparison of the results and the predictions generated by a simple filter model is discussed.

SIGNAL DETECTION AS A FUNCTION OF SIGNAL INTENSITY AND DURATION

I. INTRODUCTION

In developing a theory or model of how a human observer detects auditory signals, many parameters of the signal may be varied in a way which will provide information as to how the mechanism operates. Until recently, comparatively little research has been devoted to a study of one of these parameters, i.e., the duration of the signal. All of the recent studies in this area have had a common approach which was to determine the duration and intensity of the signal necessary for the observer to report he heard the tone. Looking at these studies in another way one might say the research was designed to determine the signal intensity and duration necessary to produce a constant detectability. In essence, then, these studies varied signal intensity and duration with detectability held constant. The results of such studies are usually reported in terms of the familiar time-intensity graph. An example of such is shown in Figure 1, which is taken from Garner.[1]

The study reported here employed slightly different procedures which, in a sense, supplement the data previously gathered. The major change in this study was to allow detectability to vary. Thus this study could be considered an attempt to determine the surface of detectability in the space determined by signal duration, intensity and detectability. To accomplish this purpose three experiments were conducted using the same observers throughout.



40
30
20
10
0
THRESHOLD IN DB RE 0.0002 DYNES/CM²
1
2
4
8
15
25
50
100
DURATION IN MILLISECONDS
NOISE
1000 CPS TONE
1000 CPS TONE WITH BAND PASS FILTER

FIG. 1

Consider the three dimensions shown in Figure 2. In the first experiment a signal duration was selected and held constant while the signal intensity was varied. The influence of signal intensity upon detectability was thus determined. Three values of signal duration were studied in this fashion. In the second experiment the product of signal duration and intensity was held constant, that is, signal energy was held constant. The detectability was then determined for the various pairs of values of signal intensity and duration necessary to maintain a constant energy signal. In the third experiment the signal intensity was held constant while the duration was varied. The effect of signal duration on detectability with intensity held constant was thus determined. It was hoped that a set of equations could be developed which would fit all of the data obtained in this manner.

1.1 Previous Research

Hughes[2] presented some data using pure tones (using six different durations, ranging from 63 to 739 milliseconds) and employing five frequencies. He obtained the intensity of the signal necessary for the subject to report he heard the signal for each duration under absolute threshold conditions. He suggested the following equation closely fitted his data.

$$I/I_o = b + a/t \qquad (1)$$

where I is intensity, I_o is a constant intensity, b and a are constants with t signifying signal duration.

If this equation holds as duration approaches infinity, then $b = I_\infty/I_o$, where I_∞ is the intensity used with an extremely long signal. Substituting this value in Eq 1 we find

$$t(I - I_\infty) = aI_o \qquad \text{a constant} \qquad (2)$$

Hughes points out that this is of the same form as the chronaxie equations of Lapicque.

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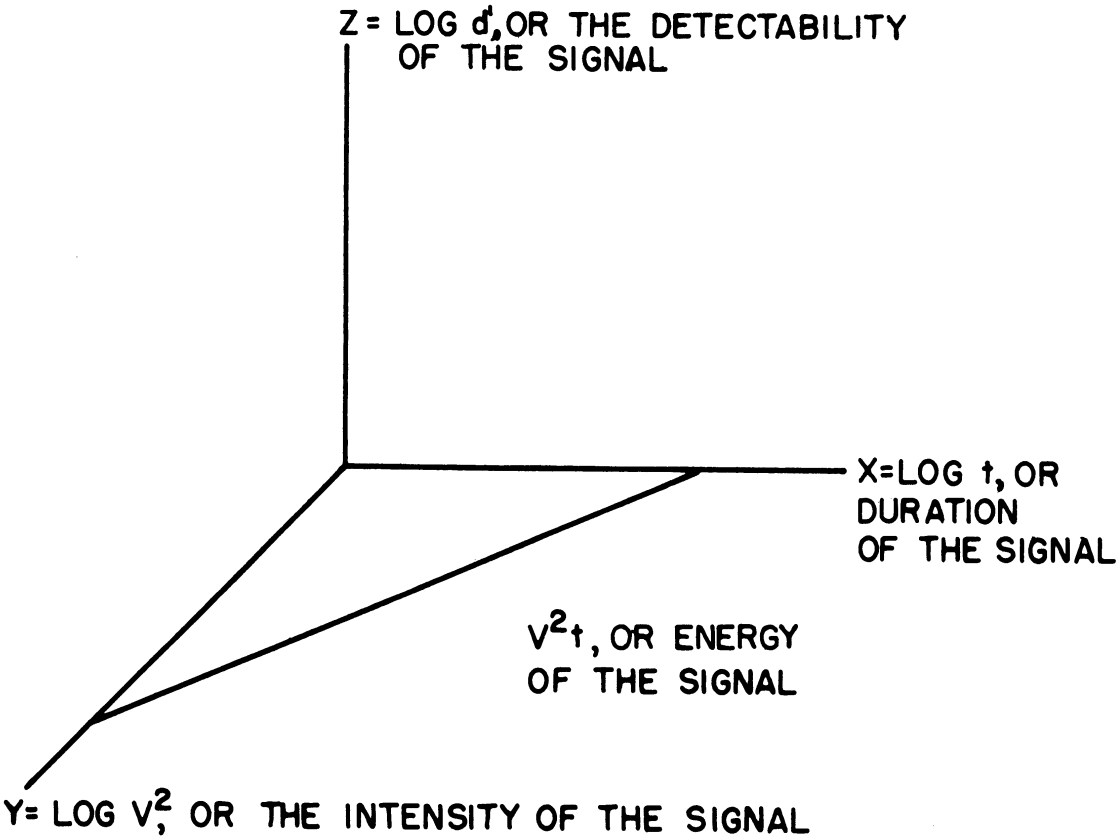

FIG. 2

Investigating the masked tonal threshold, Garner and Miller[3] employed eight durations ranging from 12.5 milliseconds to 2000 milliseconds. They used four frequencies and six noise levels ranging from 20 to 110 db sound pressure levels. They presented what Licklider[4] has called the "diverted input hypothesis". This hypothesis maintains that a certain portion of the stimulus intensity, I_o, is not an effective stimulus for the ear. All the stimulus energy above this value is integrated linearly with time.

Mathematically this hypothesis can be expressed as follows:

$$t(I - I_\infty) = C \tag{3}$$

where I_∞ is the value of intensity necessary for the detection of an extremely long signal, and C is a constant.

This equation, which is identical with Hughes' equation, also fits with reasonable accuracy the data presented by Garner and Miller. In their paper they go on to point out that this data is inconsistent with Croziers' statistical theory, although the latter theory requires one more degree of freedom.

Thus for the range of durations from about 12.5 milliseconds to 2000 milliseconds it is apparent that Eq. 1 provides a fairly good approximation to the form of the data.

Garner, in a paper concerned with very short duration, has extended knowledge in this area. Using pure tones with durations ranging from 1 millisecond to 100 milliseconds he obtained the data presented in Figure 1. Considering the power spectrum of short tones he advanced the hypothesis: "The rate of temporal integration of energy in the ear is dependent on the width of the frequency band of the energy to be integrated." The superiority of the pure tone signal to gated noise until very brief durations are reached supports this hypothesis since at very short durations the power spectrum of the gated sinewave closely resembles that of the noise power spectrum.

If this hypothesis is correct then the concept of critical bands[5] may explain time and intensity results. This suggestion will be pursued further in a later section of this paper.

1.2 Mechanisms Responsible for Time-Intensity Results

Although partial explanations have been suggested for some of the previous results, no consistent model has been suggested to explain the results over the entire range of durations investigated.

In particular the diverted input hypothesis for very short duration (Eq. 1) would be closely approximated by Eq. (4).

$$It = C \tag{4}$$

As Garner's work demonstrates, this does not hold. This is presumably explained by the spectrum characteristics of very short gated sine waves, as discussed above. Nevertheless, explanations as to why the diverted input hypothesis should approximate the data have not been advanced, nor have suggestions been made as to why some minimal intensity should not be an effective stimulus for the ear.

2. THE MEASURE OF DETECTABILITY

The major problem to be met in the study reported here is that of determining a measure of detectability since the detectability of the signal will be allowed to vary rather than being held constant. Certainly there are several reasonable ways to accomplish this task. There are, however, certain properties of the measure which would seem desirable to everyone. Consider a test situation where the signal is presented in one of four time intervals. The observer's job is to specify the time interval in which he thinks the signal occurred. He is forced to choose one of the four intervals. This situation will be called a forced-choice situation, and since there are four alternatives it is specifically a four-alternative forced-choice test. It appears obvious that the more correct identifications by the observer can make of a particular signal in this situation the more detectable this signal is. This is certainly one necessary property of the measure. For reasons of generality it would also be advantageous to obtain a measure of detectability which would yield approximately the same number for some fixed situation whether two, four, or more alternatives are used in a forced-choice test. Also, the measure should be applicable to a test situation where the subject gives a yes-no response. That is, the measure should be such that comparison is possible over different test situations. The only measure which seems to accomplish this task is the use of d' as a measure of detectability as suggested by Tanner and Swets.[6] Research by Tanner, Swets and Green[7] has demonstrated that the inferred measure, d', has approximately the same value both in a four-alternative forced-choice and in yes-no test situations where auditory signals were employed. Further research is at present in progress on this problem.

2.1 The Definition of d'

In order to define the measure d' it will be necessary to review briefly the general concept of how the observer detects any signal. Since a more complete description of this hypothesis will be available elsewhere the following will treat only the bare essentials of this concept.

It is assumed that the observer may be considered a stationary, fixed mechanism for any particular fixed set of signal and noise parameters. It is further assumed that a reasonable model is a Fourier band-limited device.

Since the device is Fourier band-limited, any input to such a device may be expressed as a series of constants corresponding to the multipliers of the various frequency terms. Thus, each input may be represented as a point in an n-dimensional space. Now each point $\lambda(y)$ can be represented as a real number by simply taking the likelihood ratio of this point. Likelihood ratio of a point $\lambda(y)$ is simply the ratio of the probability that this point arose from signal-plus-noise to the probability that this point arose from noise alone.

$$\ell\left[\lambda(y)\right] = \frac{P_{S+N}\left[\lambda(y)\right]}{P_N\left[\lambda(y)\right]} \tag{5}$$

Hence, each observation over some time interval (i.e., each $\lambda(y)$ in the n-dimensional space) may be collapsed to a one dimensional axis, $\ell\left[\lambda(y)\right]$. We may also consider the probability density function of $\ell\left[\lambda(y)\right]$ when only noise is present, and, likewise, the probability density function of the likelihood ratio $\ell\left[\lambda(y)\right]$ when signal-plus-noise is present.

Now it is assumed that there is some monotonic transformation which will yield a variable such that the density function of this variable is Gaussian when both signal and noise are present. Since the $\ell\left[\lambda(y)\right]$ numbers are all greater when the signal is present, this distribution will have a higher mean than the distribution when noise alone is present. Further, it is assumed that the standard deviations for both distributions are equal. Then it is fairly simple to transform both distributions so that they have unit variance and the noise distribution has a zero mean. In this case the mean of the signal distribution is designated as d'. The final result of these transformations is shown in Figure 3. Here the density functions resulting from noise ans signal-plus-noise is shown versus the transformation of $\ell\left[\lambda(y)\right]$.

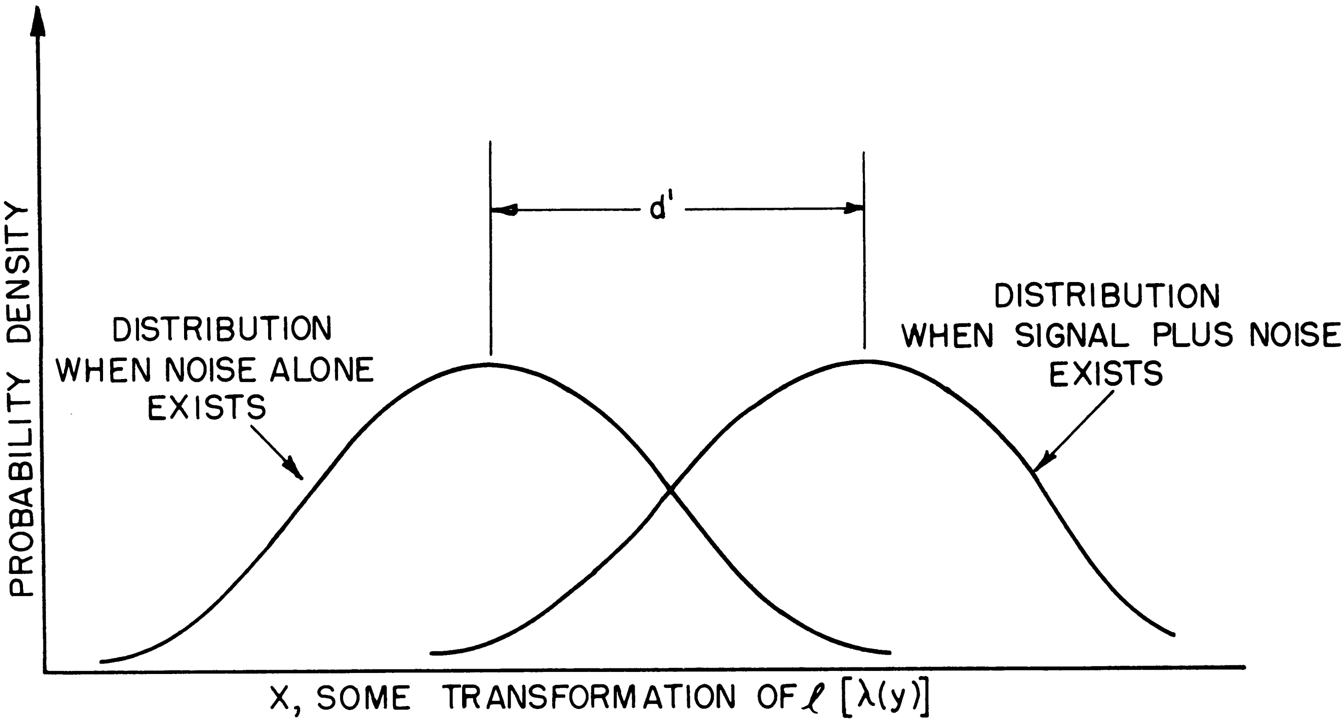
PROBABILITY DENSITY
d'
DISTRIBUTION WHEN NOISE ALONE EXISTS
DISTRIBUTION WHEN SIGNAL PLUS NOISE EXISTS
X, SOME TRANSFORMATION OF ℓ $[\lambda(y)]$

FIG. 3

Now consider how the observer's behavior is interpreted in terms of this model in a four-alternative forced-choice situation. The observer is presented with four time intervals, one of which contains the signal. A reasonable basis for the decision of which interval contains the signal would be to pick the interval in which the largest $\ell\left[\rho(y)\right]$ occurs. This would imply that of the four intervals, the one chosen has a larger probability of being signal-plus-noise, as opposed to noise alone, than for any other interval. It is assumed, therefore, that the observer makes his decisions in a manner which is equivalent to picking the largest of the four likelihood ratios. If this is true then this is equivalent to picking the interval with the largest value from the transformation of $\ell\left[\lambda(y)\right]$ shown in Figure 3 (since all the transformations are assumed monotonic). The probability of this being the correct interval (that is, the interval in which the signal was in fact present) is the probability that the value associated with the signal-plus-noise distribution of Figure 3 is larger than any of the three values associated with the noise distribution. The probability of a correct response is a function of the separation of the means (d') as shown in Figure 4. Throughout this paper the obtained percentage is used as an estimate of the probability of being correct and the corresponding estimate of d' is obtained by means of Figure 4. Explicitly, the probability of being correct is given by the following formula.

$$P(c) = \int_{-\infty}^{+\infty} F(x)^3 \, g(x) \, dx \qquad (6)$$

where $F(x)$ is the noise distribution function and $g(x)$ is the density function of the signal distribution. Since both functions are assumed to be Gaussian with unit variance this probability is

$$P(c) = \int_{-\infty}^{+\infty} \Phi^{3}(x) \, \varphi(x - d')dx \qquad (7)$$

where $\Phi^{3}(x)$ may be taken as the noise distribution function and $\varphi(x-d')$ is the density function of the signal.

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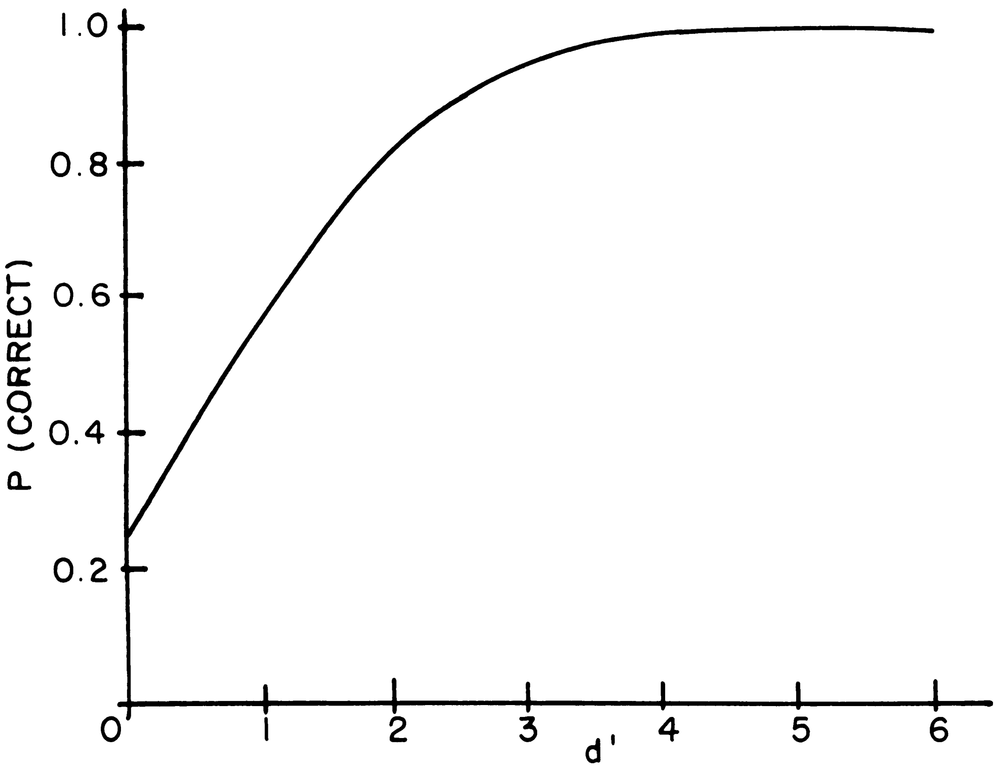


FIG. 4 P(C) AS A FUNCTION OF d'-A THEORETICAL CURVE

3. EXPERIMENTAL APPARATUS

3.1 Audio Channel

Figure 5 shows in block diagram form the relevant aspects of the audio channel. The headphones, used binaurally, in phase, were PDR-8s. The frequency of the tone was 1000 cps throughout all the experiments. White Gaussian noise, provided by a General Radio Noise Source, was used to mask the signal. The signal gate was of special design. It would gate the sine-wave only on a positive-going zero crossing and permit only an integral number of cycles to pass.

3.2 Audio Measurements

Two average-reading full-wave rectified voltmeters calibrated in sine-wave RMS were used to measure the signal and noise. Both meters showed systematic error at low scale readings; therefore, graphs were obtained using a voltage divider procedure to convert the "scale" readings to the "true" voltage. The resistance box used in the voltage divider procedure had an accuracy of .1 percent.

One meter was employed wide band, and the other read the output of a single-tuned circuit. This filter was centered at 1130 cps and had a bandpass of about 160 cps. The comparison of the two meters provided an indication of any change in the noise spectrum. No such change was noted in the course of the experiment. Some of the graphs are presented in terms of a signal voltage. To convert this to an approximate ratio of signal-to-noise power per unit bandwidth, divide the signal power by 1.78×10^{-4}. The duration calibration was accomplished by displaying the signal on a Dumont Model 329 Oscilloscope. Three complete checks of the calibration of the signal circuit were made at different times in

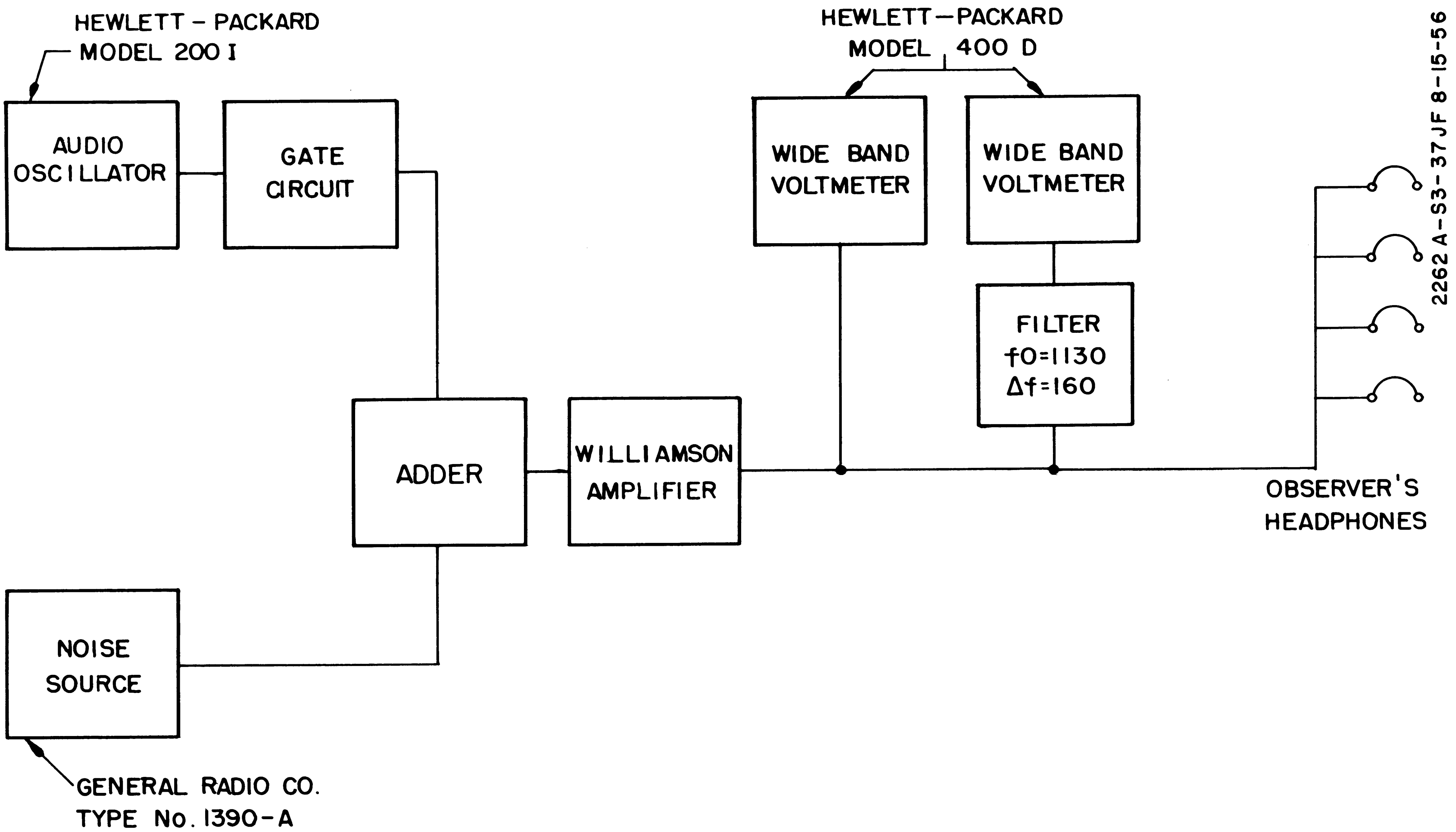


FIG. 5 BLOCK DIAGRAM OF THE SIGNAL AND NOISE CIRCUITS

the experiment and on over one-half of the durations a check was made immediately before this duration was used in the experiment. The noise level employed in all experiments was about 60 db (re .0002 dynes/cm^2 per cycle). This measurement was made at the output of the headsets, using a 6 cc coupler and a Western Electric Calibrated Condenser Microphone.

3.3 Programming of the Experiment

Throughout the experiments a four-alternative forced-choice methodology was used. Four subjects were warned that a trial was to begin by a flash of a green light. A white light blinked four times in coincidence with the beginning of the four temporal intervals in which the signal could occur. A red light then signaled the observer that a choice must be registered by pressing one of four buttons. This choice was then automatically recorded. Only one choice was possible and this choice could be recorded only during the "record" interval. At the end of the "record" interval the subject was given (by means of light) the correct answer. The next trial then began. The masking noise was present in the subjects earphones through a trial period.

The length of the signal intervals was adjusted on the duration of the signal. For the longest signal duration used (a three second signal) four seconds was allowed for each interval, making a total of sixteen seconds to complete all four intervals. For signals shorter than about one half seconds, three quarters of a second was used for each interval, making a total of about three seconds for the four intervals. All subjects agreed that the faster cycling led to less boredom in making their judgements. The experimenter on several occasions insisted on a slower speed but could find no significant differences between this procedure and a more rapid one. Thus, the experimenter yielded to the complaints of the subjects and adjusted the interval to that which the subjects judged to be a convenient speed. Of course, all intervals used were greater than the signal duration.

The programming of the experiment was accomplished electronically. A device called N. P. Psytar (Noise Programmed Psychophysical Tester and Recorder) accomplished this task. The selection of the interval in which the signal is to be presented is of considerable importance in forced-choice experiments. N. P. Psytar samples the output of a noise tube in order to determine which interval will be selected on each trial. Thus a Bernoulli (multinomial) random process governs the selection. Tests have been conducted to determine the randomness of this procedure and the null hypothesis could not be rejected.

One male and three female subjects were used throughout all of the experiments. All were students at the University of Michigan. All subjects were in their late teens or early twenties. All subjects were trained for at least two hours a day for twenty days before any data reported in this paper was taken. Three of the four had participated in a previous experiment lasting about four months.

Each cycle during which four intervals are presented and a choice obtained is defined as a trial. One hundred trials constitute a single session. Six or seven sessions were run per day. This required about seventy minutes of actual running time. The subjects took about two hours (with rests between each session) to complete the six or seven sessions. The results of the experiment were obtained in about three months.

For the longer durations (for example, the three second signal duration), only fifty trials were conducted each session. Each session, therefore, took about sixteen minutes to run. The subjects found the long duration signals very tedious and consequently were given longer rest periods.

4. RESULTS OF THE EXPERIMENTS

4.1 Experiment I

In this experiment the independent variable was the intensity, or power, of the signal. The dependent variable was the percent correct obtained by the observer. This percent correct was converted to the corresponding d' by means of Figure 4 and used as the abcissa of the graph. The parameter was the signal duration which was held constant at three values: 5, 50, and 500 milliseconds. The results of Experiment I are present in Figures 6, 7, 8, and 9. Each graph is for one of the subjects used in the experiment. Several of the points displayed on the graph were not actually obtained from Experiment I. These points were obtained in other experiments using the same observer where the physical parameters were the same as they were for this experiment but were taken at a later date. They are labeled and the experiment in which they were obtained may be determined by the key. These points were obtained one to two months later than the majority of the points shown on the graph. The high degree to which they correspond to the remaining points displays the long term stability of both equipment and observers. The line drawn through the data points is of the general form

$$d' = K\,V^2 \tag{8}$$

where K is a function of signal duration and noise level. A more complete discussion of the author's attempt to fit the data will be given later in the paper.

4.2 Experiment II

Pairs of values of signal power and duration were selected so that a constant energy signal was presented to the observer. The percent correct detection was obtained for each pair of values. The percent correct detection

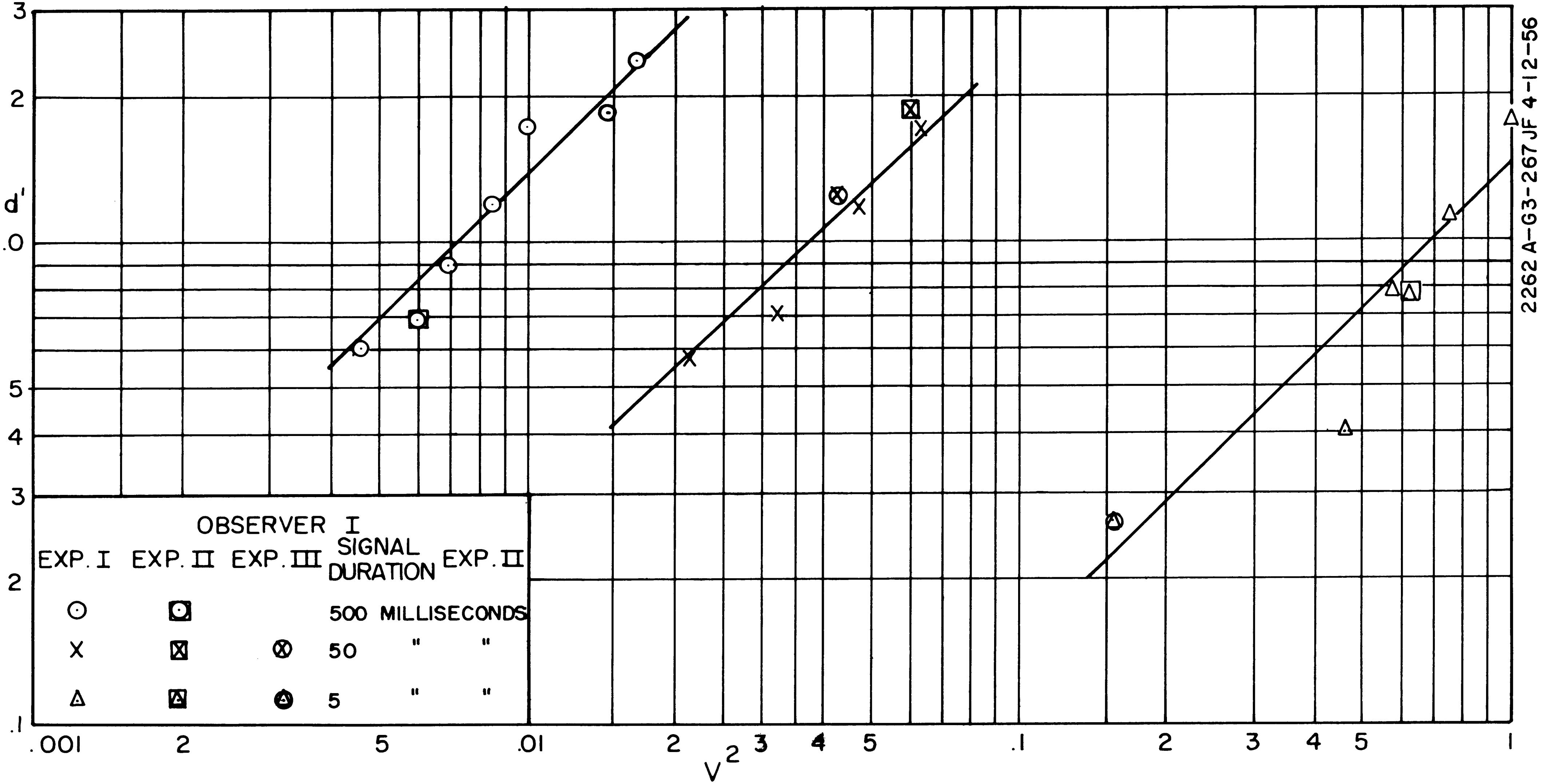

2262 A-G3-267 JF 4-12-56
d'
3
2
1.0
5
4
3
2
.1
.001
2
5
.01
2
3
4
5
.1
2
3
4
5
1
V^2
OBSERVER I
EXP. I EXP. II EXP. III SIGNAL DURATION EXP. II
500 MILLISECONDS
50 " "
5 " "

FIG. 6

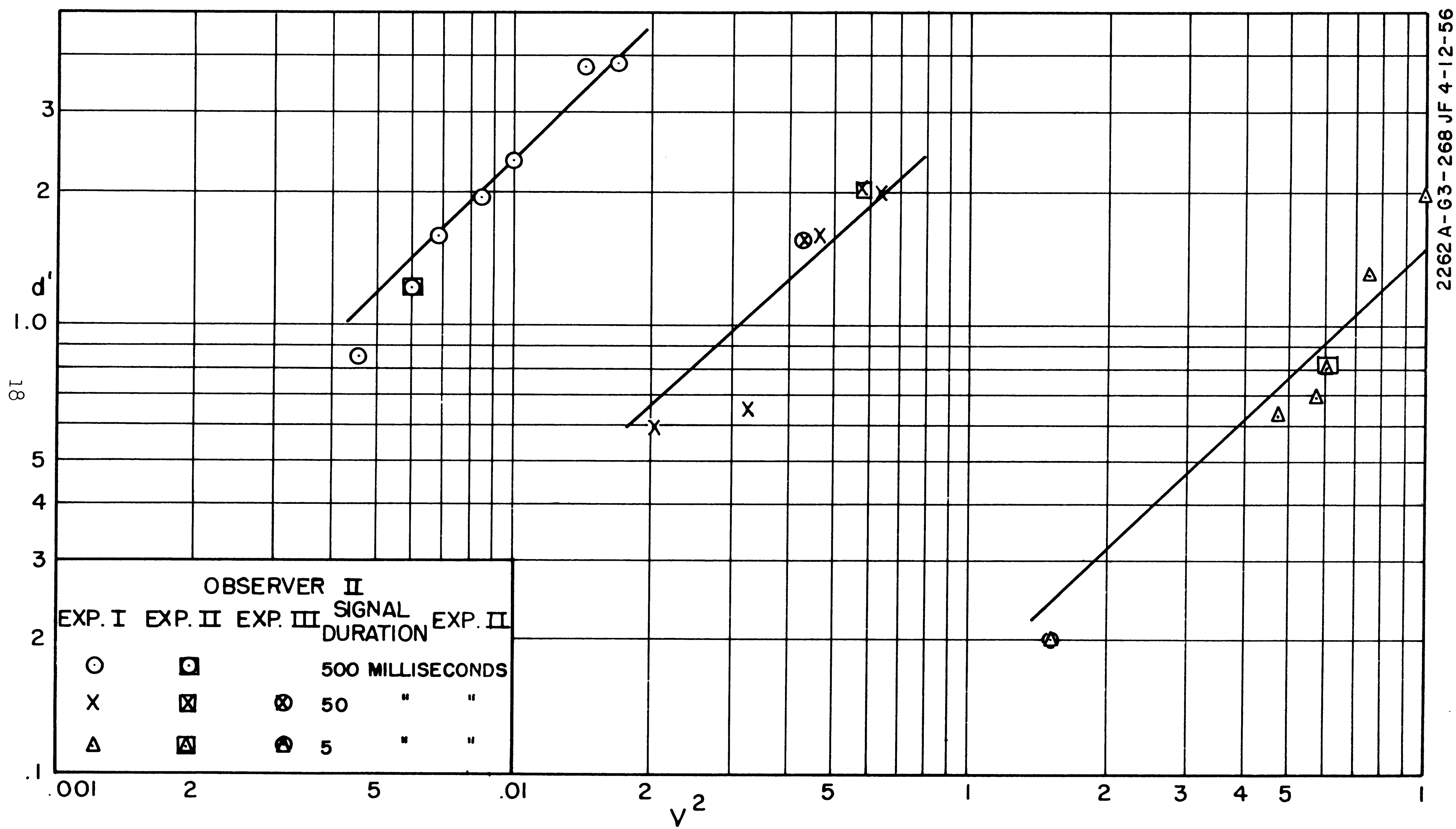

2262 A-G3-268 JF 4-12-56
d'
3
2
1.0
5
4
3
2
.1
.001
2
5
.01
2
5
1
2
3
4
5
1
V^2
OBSERVER II
EXP. I EXP. II EXP. III SIGNAL DURATION EXP. II
500 MILLISECONDS
50 " "
5 " "

FIG. 7

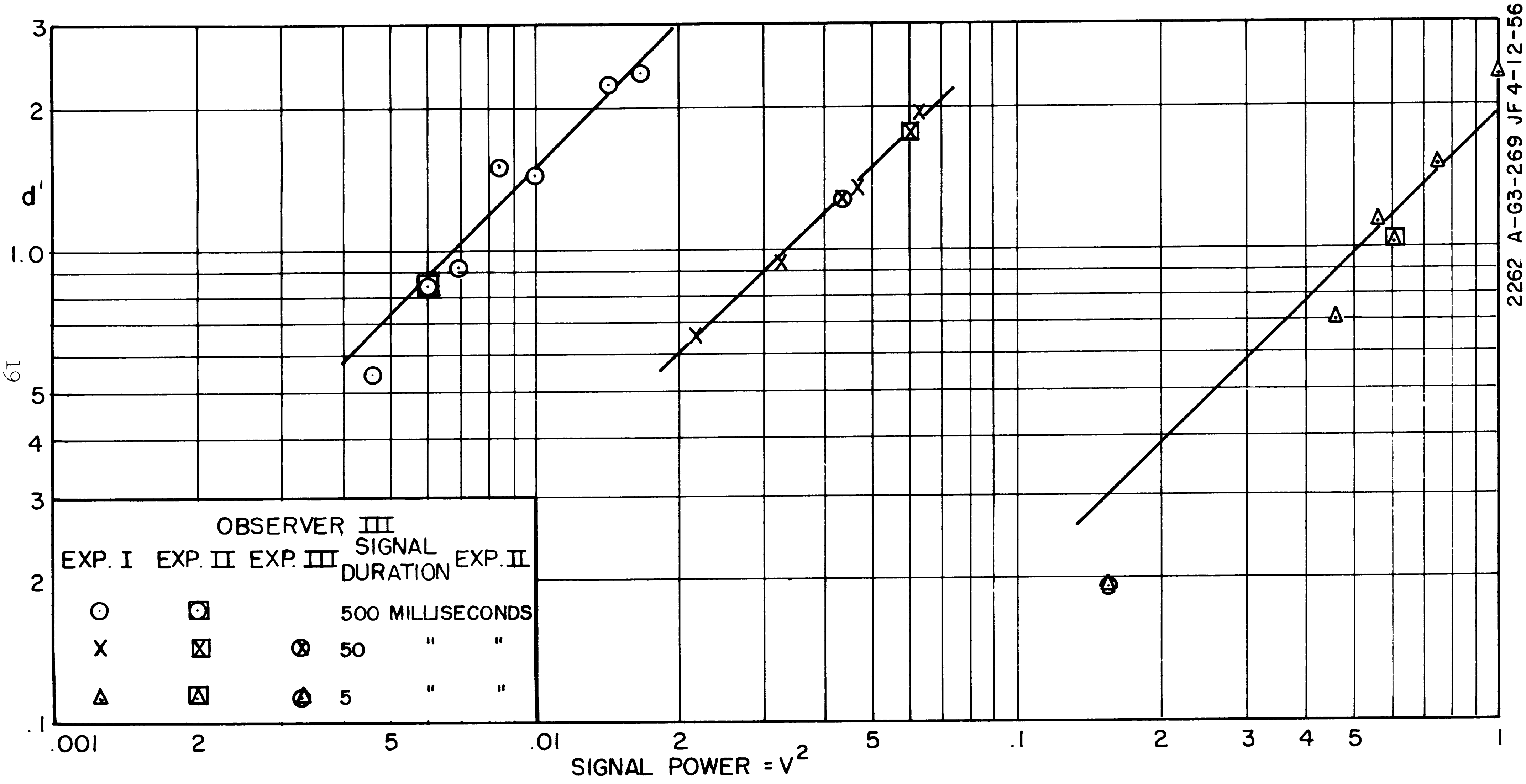

2262 A-G3-269 JF 4-12-56
d'
3
2
1.0
5
4
3
2
.1
.001
2
5
.01
2
5
.1
2
3
4
5
1
SIGNAL POWER = V²
OBSERVER III
EXP. I
EXP. II
EXP. III
SIGNAL DURATION
EXP. II
500 MILLISECONDS
50 " "
5 " "

FIG. 8

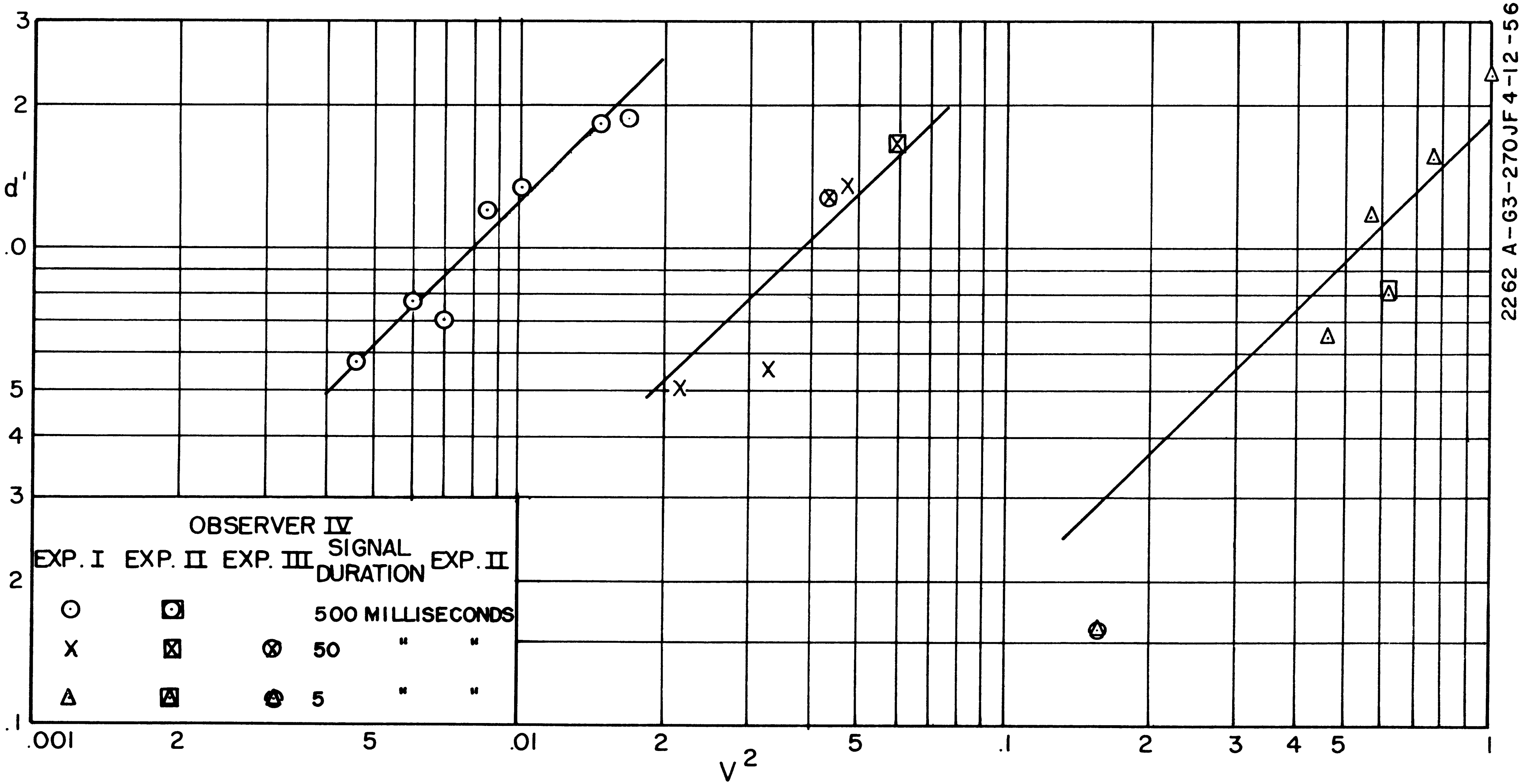

2262 A-G3-270JF 4-12-56
OBSERVER IV
EXP. I EXP. II EXP. III SIGNAL DURATION EXP. II
500 MILLISECONDS
50 " "
5 " "
d'
3
2
1.0
5
4
3
2
.1
.001
2
5
.01
2
5
.1
2
3
4
5
1
V 2

FIG. 9

was converted to d' and plotted with signal duration as the independent variable. The data obtained in Experiment II are plotted in Figures 10, 11, 12, and 13. The observers are the same as those who participated in Experiment I. The middle section of the points reflects an area where detection remains constant for a constant energy signal. This is the only region of time "where the ear is able to integrate acoustic power linearly with time", i.e., the only region where It = k. For short duration the data are fairly well approximated by a curve of the type $d' = KE\sqrt{t}$, where K is a function of the noise level and E is the signal energy. For longer durations the data are approximated by a curve of the type $d' = KE\ \frac{1}{\sqrt{t}}$, where K is a function of noise level and E is signal energy.

4.3 Experiment III

Once more the same observers were used. In this experiment the independent variable was the signal duration. The dependent variable was the percent correct detections which has been converted to d', the detectability index. The parameter was the signal voltage, or power. The results are plotted in Figures 14, 15, 16, and 17.

Three functions were fitted to the data, one for each region of signal duration. For short duration $d' = Kt^{2/3}$ where K is a function of signal power and noise level. For medium durations $d' = Kt$, where I is a function of signal power and noise level. Finally, for long duration, $d' = K\sqrt{t}$, where K is the same as before.

One rather surprising finding should be noted. This is the continued increase of detectability of the signal by extending the duration from 250 to 1000 to 3000 milliseconds while signal power was held constant. It is usually felt that for some long durations detection will be independent of signal duration and completely dependent on signal amplitude and noise level. At least for the

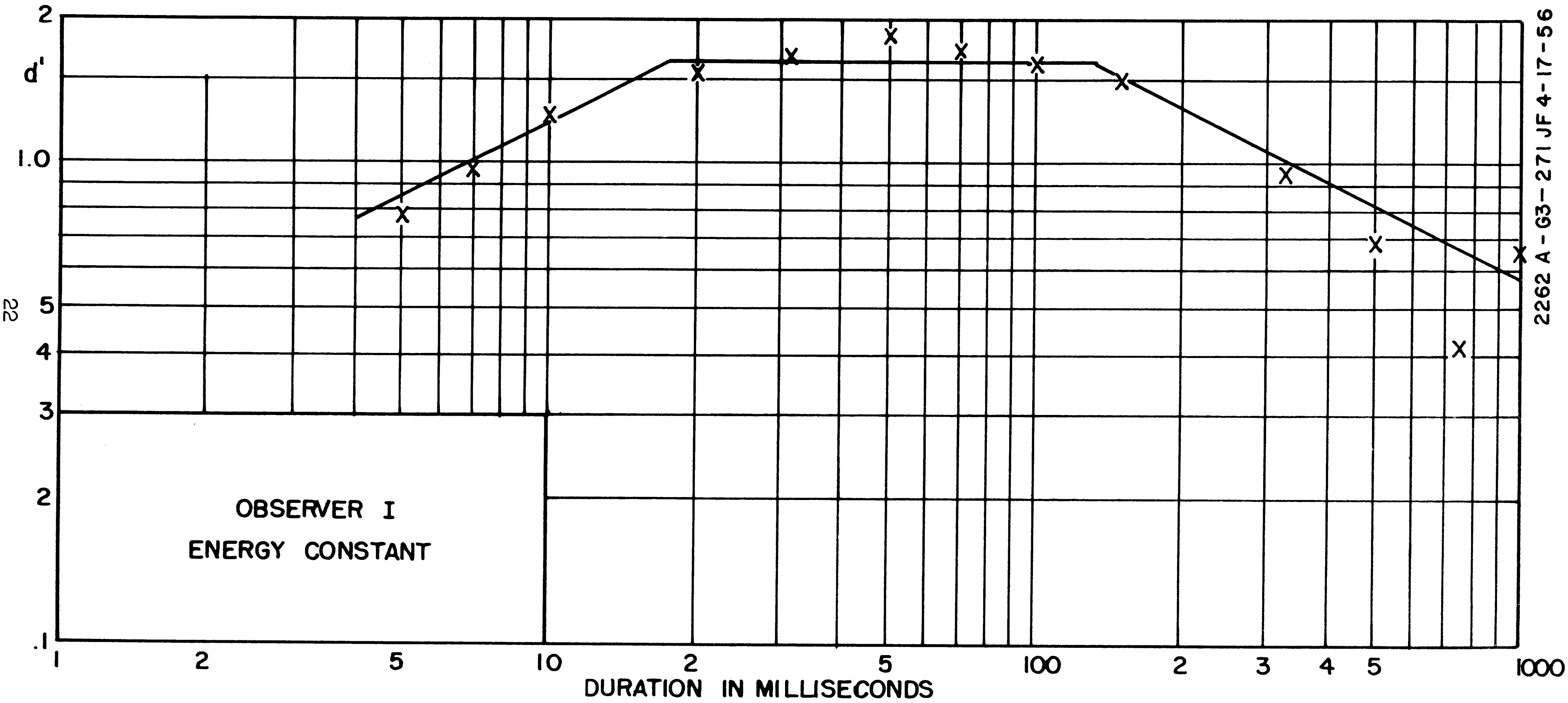
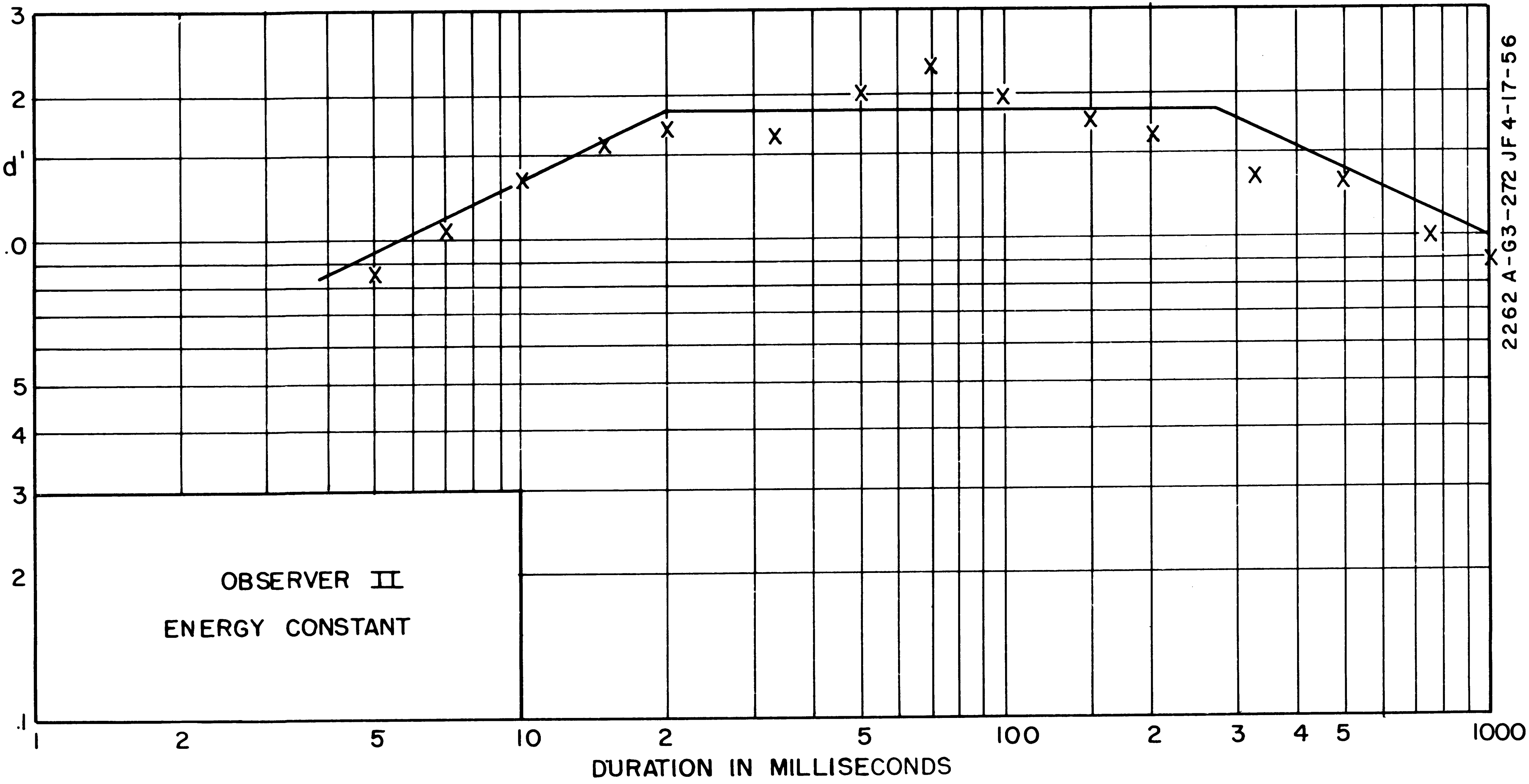

2
d'
1.0
5
4
3
2
.1
1
2
5
10
2
5
100
2
3
4
5
1000
DURATION IN MILLISECONDS
OBSERVER I
ENERGY CONSTANT
2262 A-G3-271 JF 4-17-56

FIG. 10



3
2
d'
1.0
5
4
3
2
.1
1
2
5
10
2
5
100
2
3
4
5
1000
DURATION IN MILLISECONDS
OBSERVER II
ENERGY CONSTANT
2262 A-G3-272 JF 4-17-56

FIG. 11

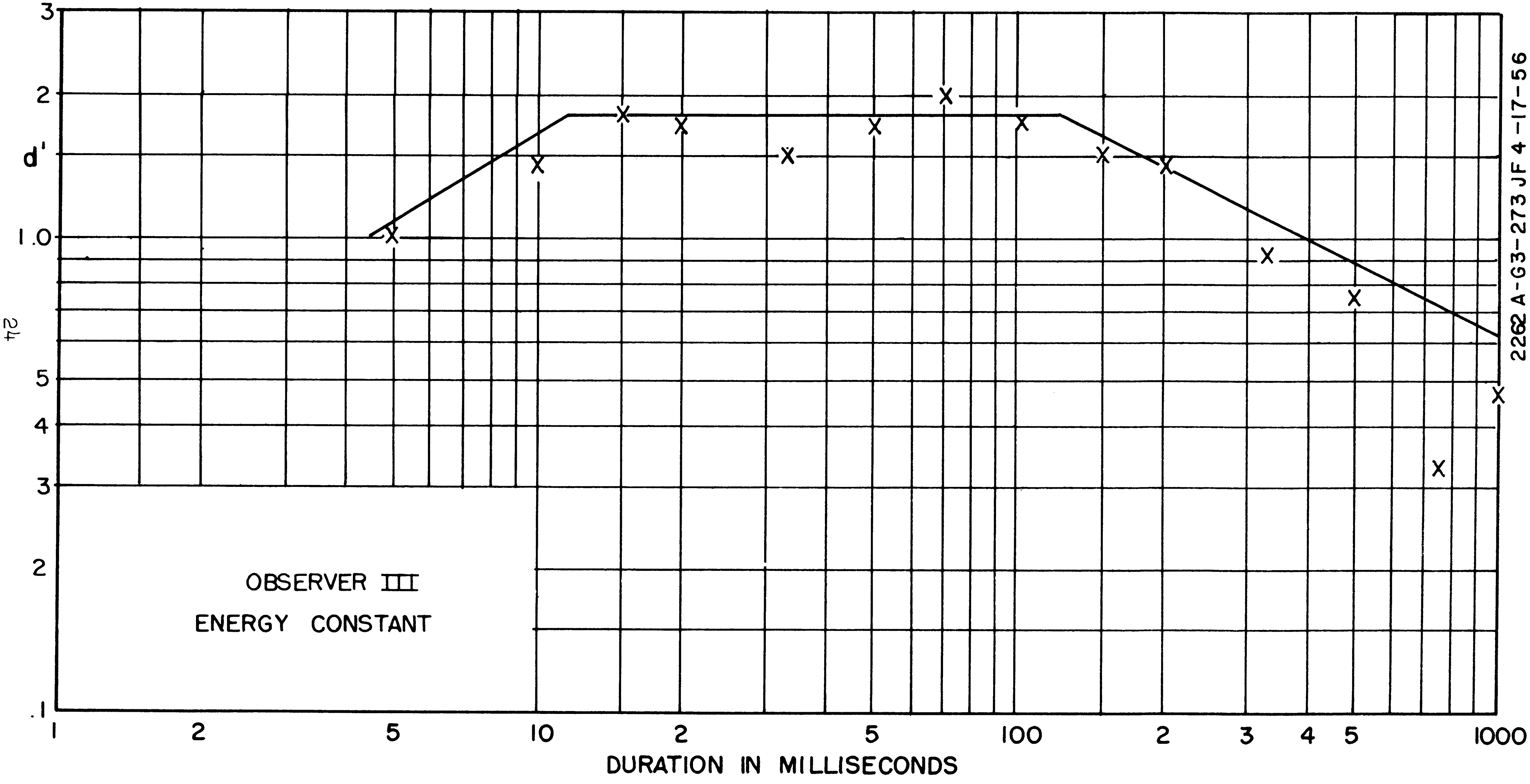

3
2
d'
1.0
5
4
3
2
.1
1
2
5
10
2
5
100
2
3
4
5
1000
DURATION IN MILLISECONDS
OBSERVER III
ENERGY CONSTANT
2262 A-G3-273 JF 4-17-56

FIG. 12

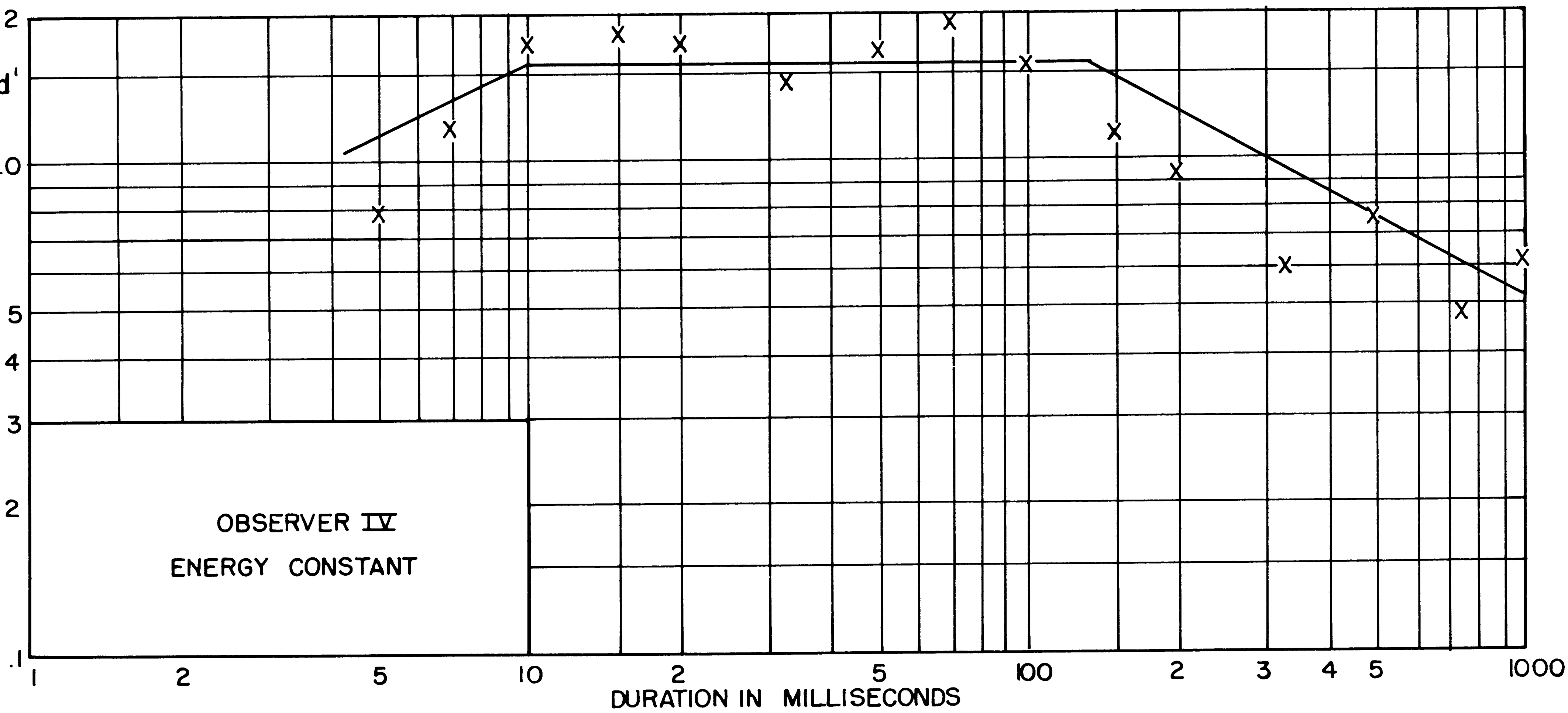


FIG. 13

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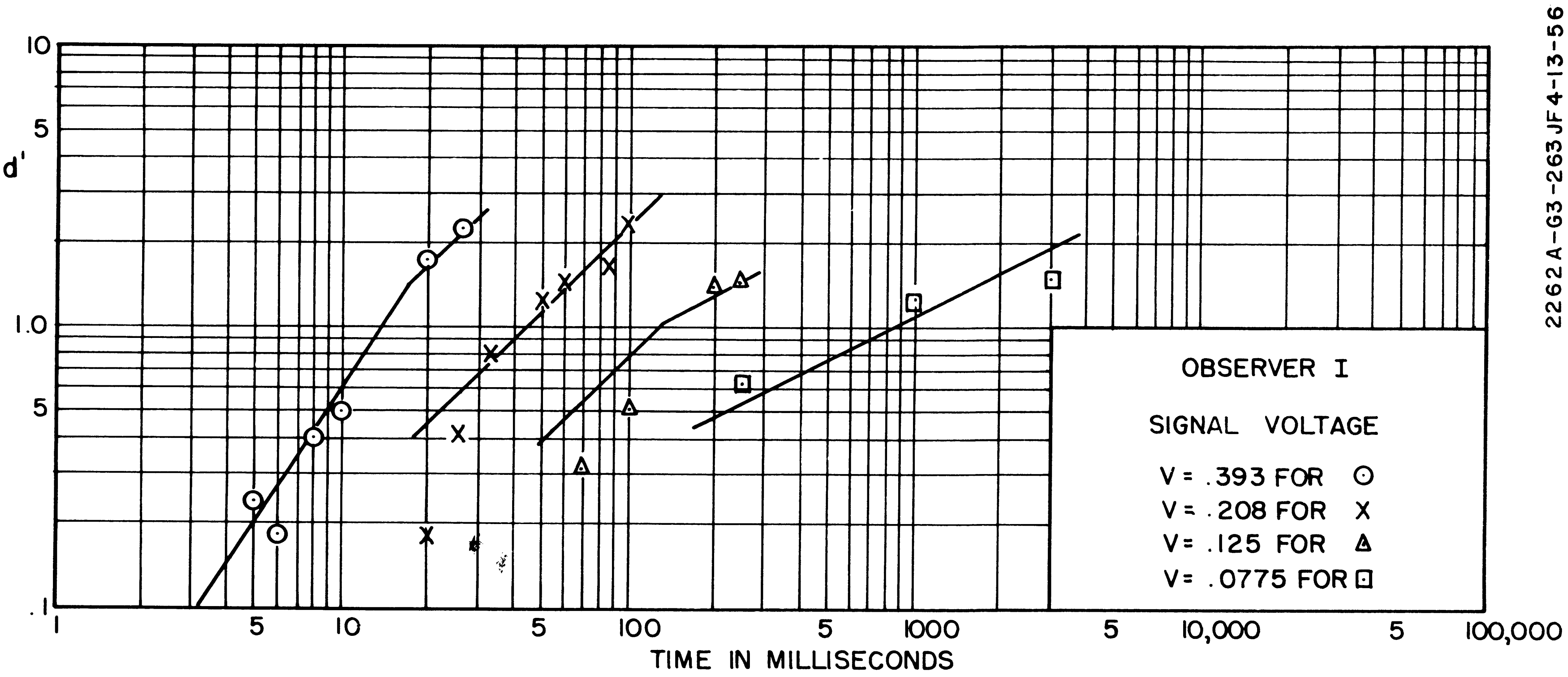

10
5
d'
1.0
5
.1
1
5
10
5
100
5
1000
5
10,000
5
100,000
TIME IN MILLISECONDS
OBSERVER I
SIGNAL VOLTAGE
V = .393 FOR ⊙
V = .208 FOR X
V = .125 FOR Δ
V = .0775 FOR ⊡
2262A-G3-263JF4-13-56

FIG. 14

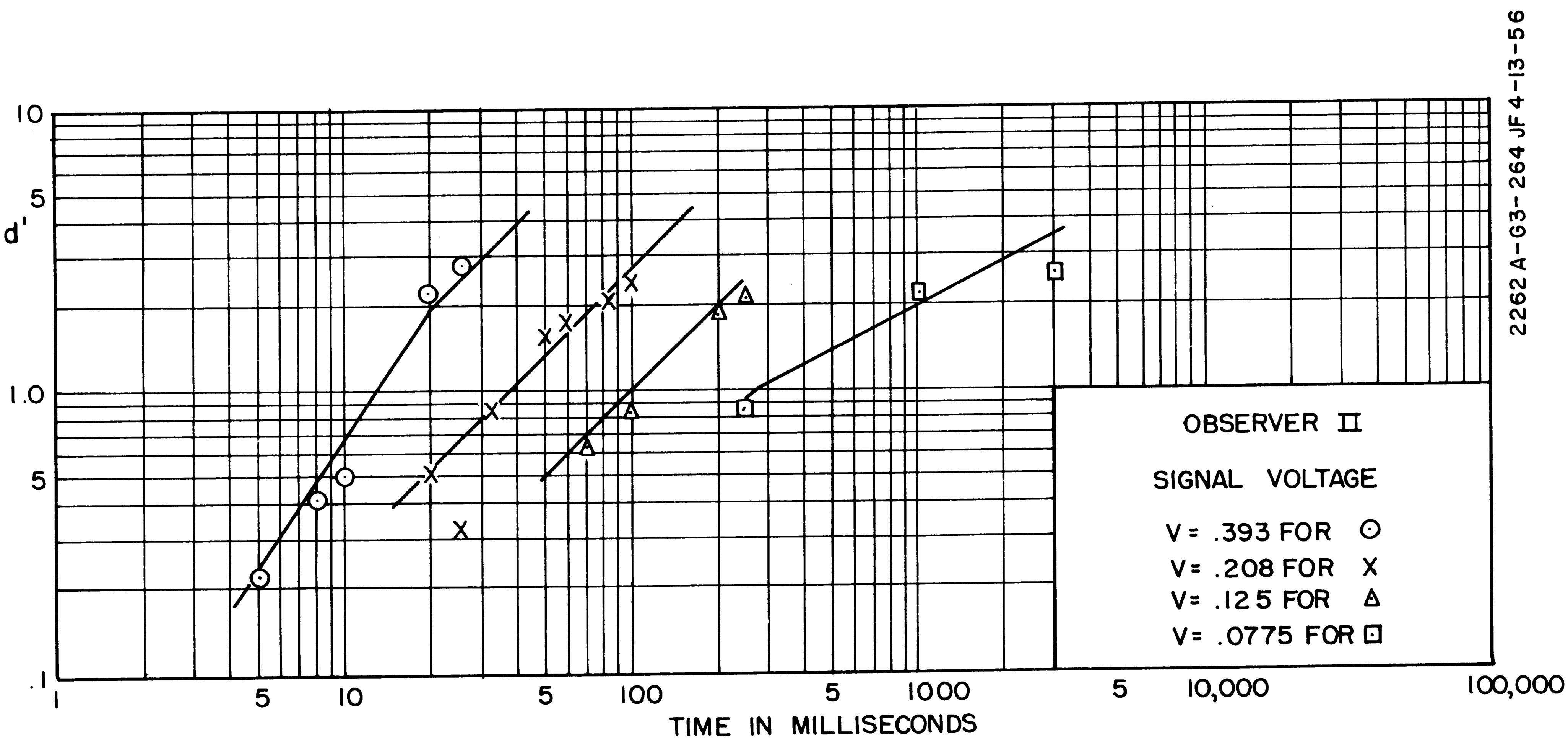
2262 A-G3-264 JF 4-13-56
10
5
d'
1.0
5
.1
1
5
10
5
100
5
1000
5
10,000
100,000
TIME IN MILLISECONDS
OBSERVER II
SIGNAL VOLTAGE
V = .393 FOR ⊙
V = .208 FOR X
V = .125 FOR Δ
V = .0775 FOR ⊡

FIG. 15

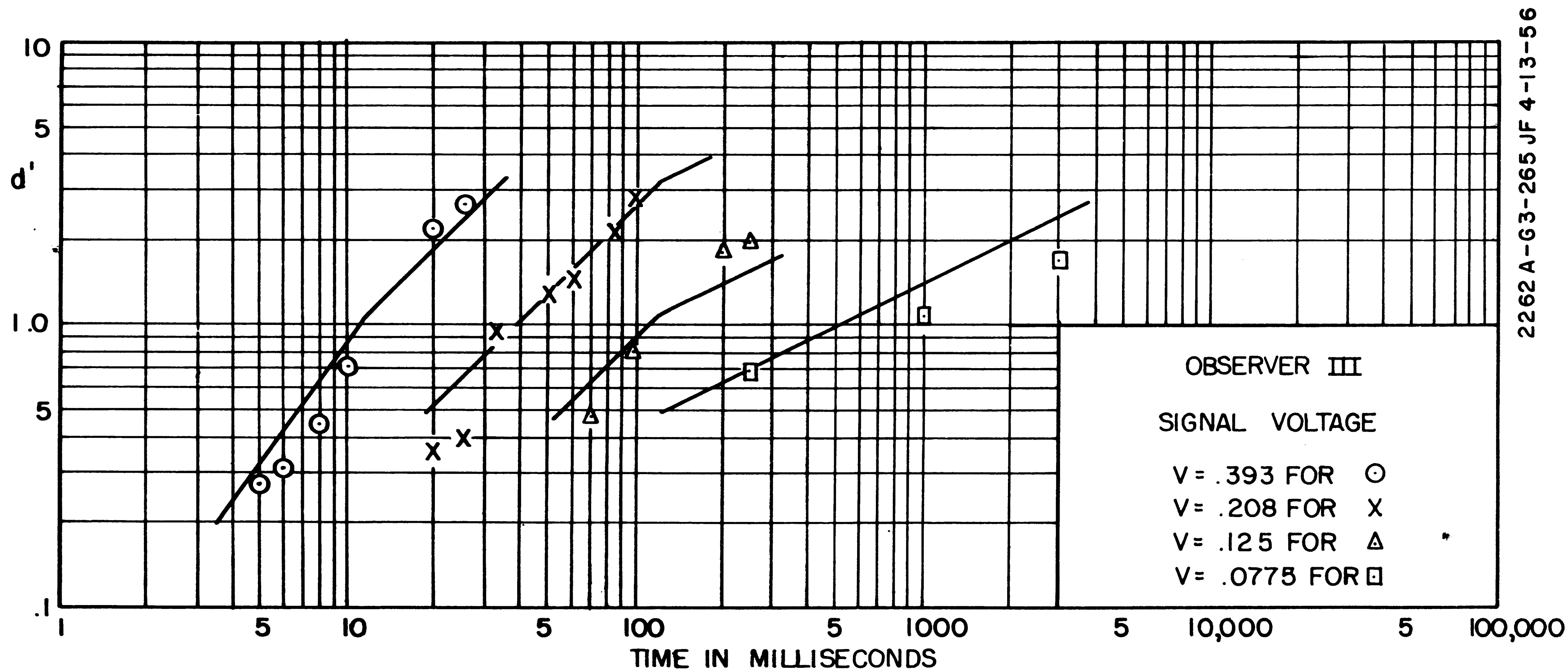
2262 A-G3-265 JF 4-13-56
10
5
d'
1.0
.5
.1
1
5
10
5
100
5
1000
5
10,000
5
100,000
TIME IN MILLISECONDS
OBSERVER III
SIGNAL VOLTAGE
V = .393 FOR ⊙
V = .208 FOR X
V = .125 FOR Δ
V = .0775 FOR ⊡

FIG. 16

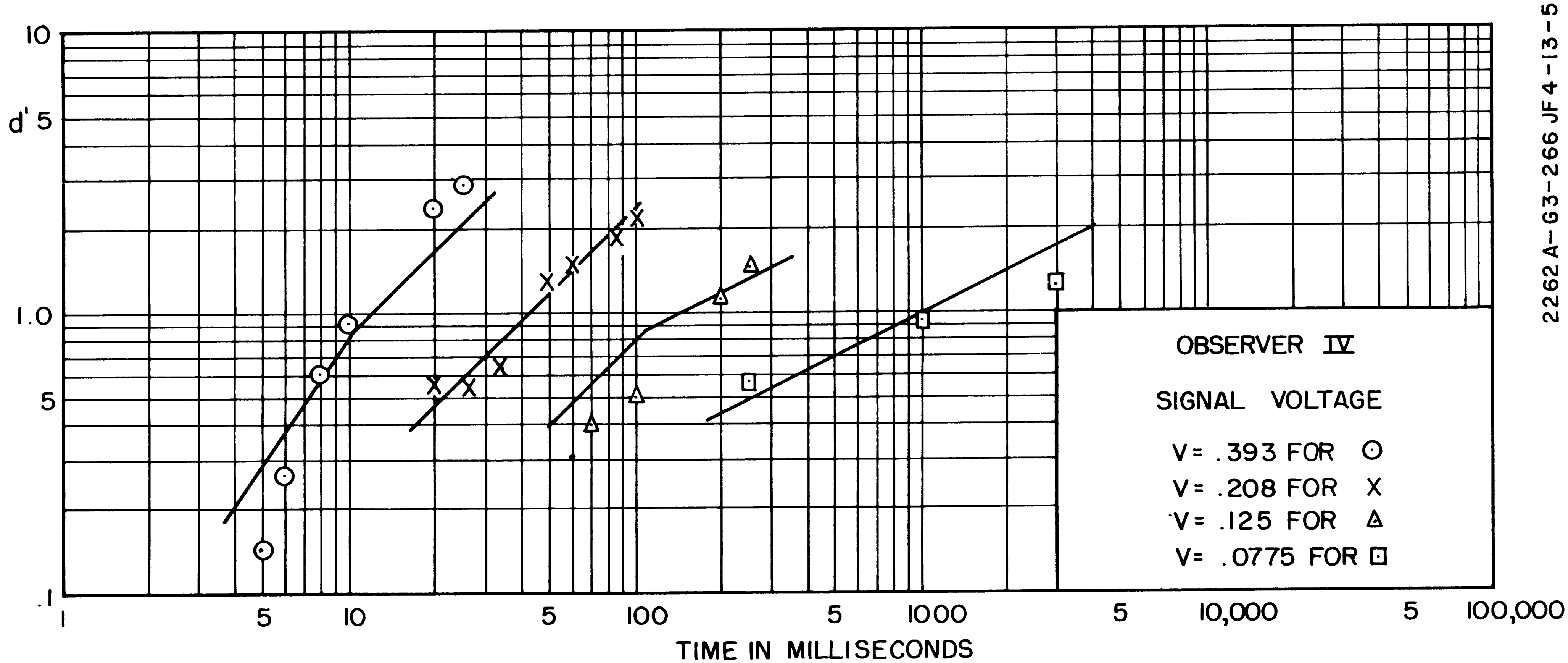
10
d' 5
1.0
5
.1
1
5
10
5
100
5
1000
5
10,000
5
100,000
TIME IN MILLISECONDS
OBSERVER IV
SIGNAL VOLTAGE
V= .393 FOR ⊙
V= .208 FOR X
V= .125 FOR △
V= .0775 FOR ⊡
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FIG. 17

durations investigated, this long duration has not been reached. For all four observers the percent correct increased as the duration was extended. The average increase from the one second to three second signal was almost ten percent in percent correct detections. Assuming this to be a binomial process, the standard deviation of the difference would be about four percent which would mean the null hypothesis (no increase) could be rejected at better than the 5 percent level.

5. EMPIRICAL FIT TO DATA

After the data had been collected the following set of equations appeared to fit the data with reasonable accuracy.

$$d' = \frac{kV^2t}{N_o}\sqrt{\frac{t}{T_1}} \qquad \text{for } t < T_1 \qquad (9)$$

$$d' = \frac{kV^2t}{N_o} \qquad \text{for } T_1 < t < T_2 \qquad (10)$$

$$d' = \frac{kV^2t}{N_o}\sqrt{\frac{T_2}{t}} \qquad \text{for } T_2 < t \qquad (11)$$

where V^2 is the signal power, t is signal duration, T_1 and T_2 are individual constants having the dimensions of time, k is some numerical constant reflecting an individual observer constant, and N_o is the noise power per unit bandwidth.

Since No was held constant through the experiment, k and N_o can be written as a new constant $C = \frac{k}{N_o}$ and the equations take the following form.

$$d' = CV^2t\sqrt{\frac{t}{T_1}} \qquad t < T_1 \qquad (12)$$

$$d' = CV^2t \qquad T_1 < t < T_2 \qquad (13)$$

$$d' = CV^2t\sqrt{\frac{T_2}{t}} \qquad T_2 < t \qquad (14)$$

Since the experiment was conducted at only one noise level and one signal frequency, k may be a function of N_o and signal frequency; hence C may not be a simple linear function of N_o. Figure 18 displays the surface of detectability as a function of signal duration and power.

5.1 Estimation of the Parameters

As can be seen from the preceeding section, the parameters C, T_1, and T_2 must be estimated in order to fit the data obtained in this study. In order to fit the data the following procedure was adopted. Using the data obtained in the first experiment it was apparent that the three signal durations fell in the three different regions of time for all of the observers; 5 milliseconds $< T_1$, 50 milliseconds between T_1 and T_2, and 500 milliseconds $\geq T_2$. Hence, considering only the data in the first experiment, all three parameters could be obtained. The 50 millisecond signal duration provided six points and a least squares technique was used to provide an estimate of C. From the 5 millisecond data, minimizing the expression,

$$\sum_{i=1}^{7} \left(d'_i - ct\sqrt{\frac{t}{T_1}}\, V^2{}_i \right)^2 \qquad (15)$$

provided a value of C $\frac{1}{\sqrt{T_1}}$. Since C had been estimated from the 50 millisecond curve, T_1 could be determined. Likewise the parameter $C\sqrt{T_2}$ could be estimated from the 500 millisecond data and then solved for T_2 using the C already estimated from the 50 millisecond experiment.

In summary, the data from the first experiment provided, by a least square technique, the estimates of the three parameters associated with each observer. Table 1 lists these parameters for each observer along with the general equations.

Table 1 shows quite sizable individual differences in the three parameters, especially those having to do with the times T_1 and T_2. The

Table 1

Parameters as Estimated from Experiment 1

	k	T_1 Milliseconds	T_2 Milliseconds
Observer 1	.0962	17.64	131.62
Observer 2	.1123	19.60	276.29
Observer 3	.1068	11.61	122.49
Observer 4	.0946	9.91	107.71

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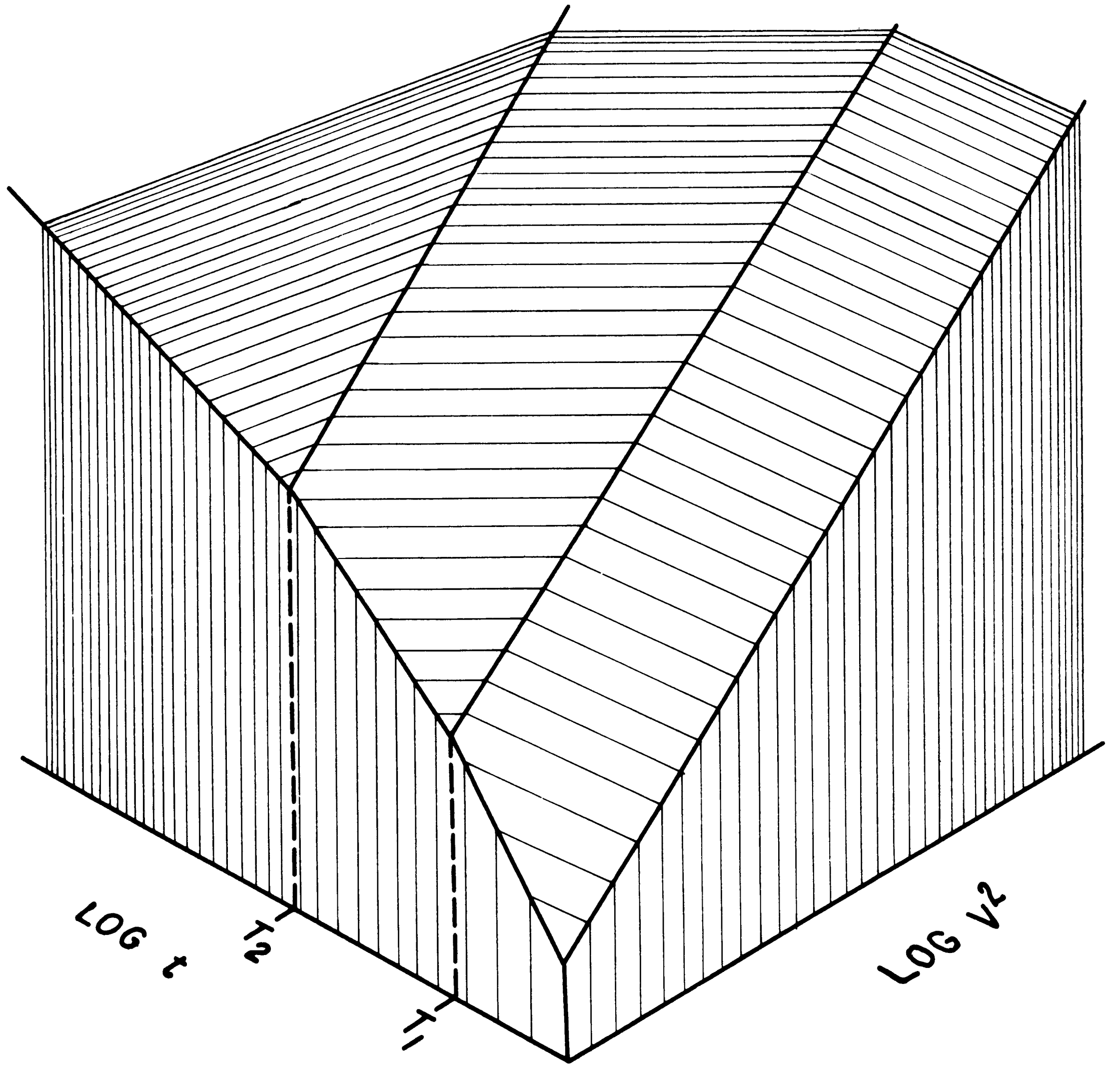

FIG. 18

PERSPECTIVE DRAWING OF THE SURFACE OF FIT TO LOG d' mean-to-sigma ratio of these four observers is about 3 for the two time parameters. Thus, to average data in experiments of this type appears hazardous. As an extreme example consider the plot in Figure 11 giving the results obtained with the second observer where signal energy is held constant. Had one assumed that T_2 was about 120 milliseconds, which is the average T_2 for the other three observers, the average error for the longer times would have been approximately 6 db. A far more serious consequence of the averaging procedure would have been to obscure the form of the data.

Having obtained the constants listed in Table 1 (as estimated from the data in the first experiment) the data obtained in the second and third experiments were "predicted". The curves shown in Figures 10-13 and Figures 14-17 are those which would be obtained using the constants estimated and the equations listed above. As can be seen, the fit is fairly good.

5.2 Test of Goodness of Fit

To measure goodness of the fit two procedures were used. The first procedure is of theoretical interest. It was assumed the percent correct obtained in each experiment was the result of a binomial process. The standard deviation could thus be estimated from the equation $\frac{pq}{n}$ where p is the probability of a correct detection, $q = 1-p$, and n is the number of observations used to estimate the point.

Since the expected d' could be obtained from the equations, this leads to some expected percent correct. The sum of the squared difference between the expected and obtained percent correct divided by the standard deviation should be distributed as chi-square under the hypothesis that there was no difference between the predicted and obtained points. The number of degrees of freedom would be the number of differences evaluated in this sum minus three, since three parameters are estimated from the data. A chi-square value was obtained from each observers data. The associated probability values were of a magnitude such that this hypothesis could easily be rejected (e.g., $p = 10^{-5}$).

A second procedure of more practical interest was to evaluate the average error in terms of the decibel error; that is, to evaluate how much one would have to change the signal power on the average to obtain a perfect fit of the data. This was computed by evaluating the average absolute deviation between the expected d' and the obtained d' divided by the expected d'. This average percent deviation was then reflected to signal power since d' and signal power are approximately linearly related (see Figures 6-9). This average error ranged from 13% to 16%. This would indicate an average error in terms of signal power of at most 0.8 db.

5.3 Comparison of the Obtained Equations and Previous Data

The comparison of the results of this study and previous data is difficult because in some other studies averaged data were presented. That is, the data were averaged over subjects. It will be assumed that the subjects were similar enough so that no untoward results are to be expected. Rewriting the equation presented previously so as to allow a direct comparison with the equation found to fit the conventional threshold procedure one finds:

$$V^2 t^{3/2} = C_1 \qquad t < T_1$$

$$\text{where} \qquad C_1 = \frac{d' \sqrt{T_1 N_o}}{k} \tag{16}$$

$$V^2 t = C_2 \qquad T_1 < t < T_2 \tag{17}$$

$$C_2 = \frac{d' \, N_o}{k}$$

$$V^2 \sqrt{t} = C_3 \qquad T_2 < t$$

$$C_3 = \frac{d' \, N_o}{k \sqrt{T_2}} \tag{18}$$

For the middle range of duration there is essentially no disagreement because the diverted input equation approximates this form at short signal durations. For the very short duration Eq 16 is precisely the form of the relation obtained in Garner's study[1]. Equation 17 predicts a decrease in signal (as time is increased) of 4.5 db per octave. Figure 1 taken from Garner shows this relation for the 1000 cps tone. The value of T_1 as obtained from Garner's data is about 10 milliseconds. This value compares favorably with the values of T obtained from the observer's in this experiment (Table 1).

For the longer duration the results obtained in this experiment indicate a 1.5 db per octave decrease in signal strength as duration is varied. About the only other study which covered the range of signal duration necessary to test this relation was done by Garner and Miller[3]. They determined signal powers necessary to obtain a constant detectability for the durations 200, 500, 1000, and 2000 milliseconds and a "very long" duration at four signal frequencies.

The decreases found were about 1 db per octave or less depending on the signal frequency. The fact that the "very long" duration did not indicate a much smaller signal power is at variance with Eq 18. The most obvious conclusion is that the present study did not extend signal duration far enough to obtain this result.

A point may be made concerning the use of extremely long duration signals. If the duration is much longer than about 1 or 2 seconds a very good procedure would be to make a decision every second or so. That is, make a decision that either the signal is present or is not present. Assume one could then remember the previous decision and simply add up the number of "yes" decisions. The results would be a binomial process with some fixed mean, depending on signal strength, and a variance which would decrease linearly with the number of decisions and hence linearly with signal duration. This is equivalent to arguing that the variance of the mean (where the mean is based on n samples) is n times better than the variance of a single sample. This argument leads to the conclusion that d' should increase at the rate of the square root of duration if signal power is held constant. Whether or not such a model is reasonable is of course an empirical question. The data obtained in this equipment are not at variance with this interpretation.

6. DISCUSSION

One of the main objectives in conducting the present experiments was to compare the results obtained from the observers with a model obtained from the work on signal detectability[8]. Specifically, the experiment conducted provided a direct comparison with the model designated "signal-known-exactly". This model supposes the receiver has an exact copy of the signal to be detected. or, equivalently, has a filter constructed on the basis of exact knowledge of the signal. It further knows the exact starting time of the signal as well as its frequency and the distribution of the masking noise. Although both from *a priori* and empirical data the model was known to be an inadequate conceptualization of the way in which the human observer behaves, no empirical comparisons have been made concerning the effects of signal duration.

Analysis from a signal-to-noise ratio standpoint has shown that the effects of signal duration and receiver bandwidth are closely linked.

The optimum procedure for detecting a signal of known duration is to adjust the receiver bandwidth to approximately the reciprocal of the signal duration. Lawson and Uhlenbeck[9] show this result for a variety of receivers. Peterson and Birdsall have also demonstrated this result[10]. The effects of mismatching the receiver bandwidth and the reciprocal of signal duration is also known, i.e., adjusting the filter either too wide or too narrow. Considering the product, bandwidth times the pulse duration, the effect of mismatching is approximately 3 db signal-to-noise ratio loss per octave mismatch, when the noise power and signal energy is held constant.

There are several ways one may understand this result. One way to consider the phenomenon is to realize that the bandwidth determines the rise time of output of the filter. If the filter is too narrow for the signal duration it acts as an integrator for the entire duration of the signal, but the signal terminates before the maximum rise of the filter has been reached. The optimum condition is obtained when the signal duration is about the reciprocal of the receiver's bandwidth; then the output of the filter reaches its maximum rise just as the signal terminates. (For a more complete discussion of this problem see Reference 8, especially Figure 8.10, page 204).

Garner[1] has suggested another way to understand the result which is obtained when the receivers bandwidth is too narrow for the signal duration. He considered the Fourier power spectrum of a short duration signal. As the signal duration decreases, if the receiver's bandwidth is fixed, more and more of the signal energy falls in a frequency range outside the receivers bandwidth. This analysis is exactly equivalent to the preceeding.

Returning to the discussion of the detection model of Peterson and Birdsall, for the case of signal-known-exactly, the following equations will hold, depending on the relation of the receiver's bandwidth and signal deviation.

$$d' = \frac{\sqrt{2}\, V \sqrt{t}}{\sqrt{N_o}} \sqrt{\frac{t}{T_1}} \quad t < T_1$$ where the receiver bandwidth (w) is fixed at $T_1 = 1/w$ and is the maximum bandwidth of the receiver (19)

$$d' = \frac{\sqrt{2}\, V \sqrt{t}}{\sqrt{N_o}} \quad T_1 < t < T_2$$ where the receiver bandwidth (w) is $w = 1/t$ and is adjusted for each signal duration. (20)

$$d' = \frac{\sqrt{2}\, V \sqrt{t}}{\sqrt{N_o}} \sqrt{\frac{T_2}{t}} < t$$ where the receiver bandwidth (w) is fixed at $T_2 = 1/w$ and is the minimum bandwidth of the receiver. (21)

Only Eq 19 is true for the signal-known-exactly ideal receiver. Equation 19 and 21 are true for less than ideal models, i.e., the ideal receiver with bandwidth mismatching.

As may be noted, the equation obtained from this model and those obtained in fitting the data differ in two important respects. For the signal-known-exactly model, d' varies linearly with signal amplitude (V). For the observer, d' varies approximately as this quantity squared (V^2). Secondly, d' varies with signal duration in a manner which is always the square root of duration greater for the human observer as compared with the model for the three ranges of signal duration.

It should be noted however, that if signal energy is held constant the relation between the behavior of the observer and that of the signal-known-exactly model is exactly the same except for an attenuation factor. If signal power and duration are varied independently, however, serious disagreement arises. That is, neglecting the attenuation term, the signal-known-exactly is decidedly different from the empirical fit by a factor of $V\sqrt{t}$.

Having accepted this conclusion there are two alternative approaches possible. The first is to change the experimental situation so that better agreement obtains between the information provided the observer and that assumed by the model. For example, the model is assumed to know the signal frequency exactly. The observer is dependent on his memory throughout a trial of 100 observations for knowledge of exact frequency information. One procedure might be to remove the noise for a short time in order to present the signal to be detected before each observation. This would presumably improve the agreement between the model's behavior and that of the observer.

A second mode of attack is to consider another model (for example, one which was not assumed to have exact frequency information) and compare this model behavior with that of the human observer. A search for a simple model which would display the same equations as those obtained in this experiment was a failure. The reasons for such a failure are instructive. If some new model is considered the detectability index (d') can be expressed as some function of the input parameters signal voltage, duration, and noise level. Rather than change Equations 19, 20, 21, it is simpler to reflect this new functional relation to the percent correct detections and maintain the same relation between d' and the physical parameters as that of a signal-known-exactly. This new relation between d' and percent correct can then be used as the measure of detectability. In this new procedure, instead of inferring a d' from some percentage correct using Figure 1, one uses some new graph which is correct for the model under consideration.

To obtain agreement between model and observer in the manner of variation with signal voltage is not difficult. A new variable which is the square root of the present inferred d' will accomplish such a result. Models which lead to such redefinition are not difficult to find. Any one of a number of uncertainties concerning the signal (e.g., frequency only partially known) would produce such results. But such a redefinition does not solve the problem since the variation between the redefined d' and signal duration is then different by a factor of $t^{1/4}$ for all but the middle range of duration.

Another way to understand the result is to remember that the form of the relation between signal-known-exactly and obtained data is exactly the same in experiment two. Experiment one and three show a discrepancy. If one somehow redefines variables so that the data obtained in experiments one and three are consistent with the model, experiment two will then be inconsistent.

The crux of the difficulty in finding a model to fit the data arises in the variation of d' and signal duration. Consider the results obtained in the third experiment. For short duration d' varies as $t^{3/2}$. This is equivalent to a 4.5 db per octave signal-to-noise ratio improvement as duration is increased for a constant detectability.

Any number of simple linear filters show 3 db per octave improvements.[9] The shape of the bandpass changes parameters in the equation but the 3 db per octave variation is the same. Thus, any simple filter model can at best give only a qualitative analysis and predict the direction of change. No simple linear filter alone seems to be an adequate model to account for the results obtained.

The present study has added some information to the somewhat fragmentary information available on the effects of signal duration. The area is still in need of considerable research. The primary need still remains; that of discovering some general mechanism or model which predicts the variation of detectability and physical parameters. The data presented in this paper and other data in this field is of sufficient consistency to invite such an attempt.

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Technical Report No. 42
ERRATA SHEET

Page 21 — Line 19, Change I to K to read "...where K is a function of signal power ...."

Page 34 — Paragraph 5.2, Line 4: The equation alluded to is

$$\sigma(\hat{p}) = \sqrt{\frac{pq}{n}}$$

Page 39 — Line 1 of 1st paragraph below equations: Should read - Only Eq 20 is true for the signal-known-exactly ideal receiver.

Page 39 — Equations 19, 20 and 21. The duration segments may be confused with the equation. For clarification:

$$d' = \frac{\sqrt{2}\ V\sqrt{t}}{\sqrt{N_o}}\sqrt{\frac{t}{T_1}}, \qquad t \leq T_1$$

$$d' = \frac{\sqrt{2}\ V\sqrt{t}}{\sqrt{N_o}}, \qquad T_1 \leq t \leq T_2$$

$$d' = \frac{\sqrt{2}\ V\sqrt{t}}{N_o}\sqrt{\frac{T_2}{t}}, \qquad T_2 \leq t$$