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A RESONANT MINIATURIZED ANTENNA

Technical Memorandum No. 45

Electronic Defense Group
Department of Electrical Engineering

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This is not a final report. Further investigation may make it desirable to have the report revised, superseded, or withdrawn.

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ABSTRACT

This report describes an antenna utilizing electric dipole radiation from inside a finite ferrite sphere. It is concluded that, with the proper materials, the physical length of such resonant radiators is reduced by a material determined factor without affecting the radiant or received energy.

OBJECTIVE

The purpose of this work is to investigate the application of solid-state devices to Signal Corps electronic countermeasures equipment.

INTRODUCTION

This report describes a means of building a device which can possess an electric aperture larger by an arbitrary amount than that calculated for an equivalent free-space radiator. This is accomplished by concentrating the incoming radiation in a ferrite medium before the signal is intercepted by the receiving antenna. Although from the viewpoint of the ferrite medium the aperture can only approach that calculated for free-space radiators, from the viewpoint of free space the aperture can be increased an arbitrary amount.

The antenna here described utilizes electric dipole radiation from a wire inside the ferrite, while the more conventional type utilizes magnetic dipole radiation from the ferrite itself.

THE FERRITE ANTENNAS

COMPARISON OF ELECTRIC AND MAGNETIC DIPOLE RADIATION

The resonant electric dipole radiator is inherently an efficient device and as such perhaps one of the most versatile tools utilized in the different techniques of handling radiation problems. A severe difficulty in the application of the dipole in portable facilities below about 100 mc is the physical size of the half-wavelength radiator. Indeed, if the frequency of interest were 1 mc, then the free-space half-wavelength radiator would, of course, be 150 m long. Since antenna elements for both transmitting and receiving intelligence are common in our present military and civilian technologies, it would be of considerable interest to find an antenna with an efficiency approaching that of the resonant dipole, but of still smaller physical dimensions at the lower frequencies. Ideally, the radiator should always be small enough to be portable.

A first step in this direction was taken with the application of high-susceptibility, uniformly wire-wound ferrite rods. Such a structure utilizes the high susceptibility to increase the radiated field. Around a ferrite rod whose axis is parallel to the Z-axis in the usual spherical coordinate system, the distant electric field is given by:

$$E_{\phi} = \frac{i\mu_o m'}{2\pi R} \frac{\omega}{c} e^{i(kR-\omega t)} \tan \Theta \sin \left(\frac{k\ell}{2} \cos \Theta \right) , \qquad (1)$$

where

$i = \sqrt{-1}$,

μ_o, the permeability of free space in MKS units, is $\frac{4\pi}{10^7}$,

m' is the magnetic dipole moment of the ferrite,

R is the distance from the radiator to the field point,

ω is the radial frequency of the oscillating wave,

$k = 2\pi/\lambda$ where λ is the wavelength,

c is the velocity of light, and

ℓ is $\frac{n\pi}{\omega} \frac{1}{\sqrt{\mu\epsilon}}$, where n is an integer, and ϵ is the permittivity of free space.

The dipole moment of the ferrite is given by:

$$m' = NIA \chi_r , \qquad (2)$$

where

N is the number of turns of wire on the rod,
I is the current passed through the turns,
A is the area of the rod, and
χ_r is the effective susceptibility.

The demagnetizing factor must be considered when computing the susceptibility. Equation (2) is valid if the moment of the ferrite rod is of constant phase throughout its length. Thus the far field is seen to be proportional to the product of the number of turns and the effective susceptibility. N can be as much as 100, χ_r as much as 1000.

An analogous equation for the radiation from an electric dipole is:

$$E_\Theta = - \frac{iI\, e^{i(kR-\omega t)}}{2\pi R} \; \frac{\cos\left(\frac{n\pi}{2}\cos\Theta\right)}{\sin\Theta} \left(\frac{\mu}{\epsilon}\right)^{1/2} . \qquad (3)$$

Comparing Equations (1) and (3), it is obvious that although it is possible to take advantage of the large $N\chi_r$ product in (1), that equation also contains a factor ω/c not present in Equation (2). Thus, regardless of the magnitude of the susceptibility of the ferrite rod, as the frequency is lowered there will be a frequency for which better results in terms of radiated energy can be obtained using an electric than a magnetic dipole radiator.

The reason for the difference is simply that electric poles exist and magnetic poles do not. The magnitude of the electric dipole therefore depends upon the integrated value of the current over the dipole, whereas the magnetic dipole depends upon the current magnitude itself. Electric quadrupole and magnetic dipole effects are thus of the same order in terms of ω/c.

ELECTRIC DIPOLE RADIATION IN A FERRITE MEDIUM

The resonant length of the dipole radiator is determined by the velocity of propagation of electromagnetic energy in the medium surrounding the radiator. Therefore if a dipole is considered immersed in an infinite medium of ferrite, the resonant length would be determined by the ferrite and would be approximately:

$$\ell = \frac{n\pi}{\omega} \frac{1}{\sqrt{\mu\epsilon}} .$$

For example, if μ and ϵ were both 10^3, the resonant length would be reduced by 10^3, as would the effective aperture, but the radiation resistance depends only upon the ratio μ/ϵ and in this case is unaffected.

To make the problem a little more practical, consider the ferrite to be no longer infinite in extent but surrounding the radiator and very much larger than the length of the radiator. In general for this case, part of the radiated energy will be reflected back from the ferrite-free space interface to the radiator, and part will be transmitted into free space. If z is the ratio of the wave impedance in the ferrite to that of free space, then the fraction of the energy reflected to the radiator is $(1-z)^2/(1+z)^2$ for normal incidence. Obviously, if the impedance of the two media are equal there will be no net reflection, and all the incident energy will be transmitted into free space.

As was the case for the infinite ferrite medium, the radiation resistance at resonance depends only upon the wave impedance, and is thus equal to that of free space for the situation considered. The aperture in the ferrite is again that fraction of the length of the linear radiator calculated for free space. However, from the point of view of radiation incident upon the ferrite, the aperture has been altered. A transmitting ferrite antenna, from the above arguments, has the same characteristics as a linear free-space radiator, a factor of $\sqrt{\mu\epsilon}$ longer physically. These radiators can be considered as two black boxes with the same characteristics. By reciprocity, these go over to receiving antennas where both boxes must exhibit the same receiving effects and thus the same effective aperture. Therefore the effective aperture has been increased by the material determined factor $\sqrt{\mu\epsilon}$ for the ferrite-surrounded radiator. This can be understood as due to the concentrating of the radiant energy in the ferrite where the smaller aperture intercepts a larger field intensity. The two effects cancel to give the same total voltage as measured by the larger free-space antenna.

CRITERIA FOR FERRITE SIZE

Although the ferrite is finite, it still is of dubious practical interest to surround a radiating element by a piece of ferrite many wavelengths long. It is therefore of interest to examine the question of just how much ferrite is enough to meet the necessary conditions for miniaturization. Since the radiant energy is not reflected and the resonant length must be determined by the energies which do return to the radiator, a measure of the effectiveness of the ferrite is the ratio of inductive energy stored in free space to that stored in the ferrite.

A precise calculation of the inductive energies around a resonant dipole radiator is difficult indeed. However, the energy surrounding a uniform dipole radiator can be calculated.

It is shown in the appendix that the effective inductance per length of radiator is given by $2W/i^2$, where W is the stored energy and i is the current, and so

$$L = \frac{1}{6\pi}\left\{\frac{2}{\epsilon_f}\left(\frac{1}{r_o} - \frac{1}{r_f}\right) + \frac{1}{3\omega^2}\left(\frac{1}{r_o^3} - \frac{1}{r_f^3}\right) + \frac{2}{\epsilon_o}\left(\frac{1}{r_f c'^2} + \frac{1}{3 r_f^3 \omega^2}\right)\right.$$

$$+ \frac{1}{\epsilon_f}\left[\frac{1}{3\omega^2}\left(\frac{1}{r_o^3} - \frac{1}{r_f^3}\right) - \frac{1}{c'^2}\left(\frac{1}{r_o} - \frac{1}{r_f}\right)\right]$$

$$\left. + \frac{\epsilon_o}{\epsilon_f}\left(-\frac{1}{c'^2 r_f} + \frac{1}{3\omega^2 r_f^3}\right) + \mu_f\left[\left(\frac{1}{r_o} - \frac{1}{r_f}\right)\right] + \frac{\mu_o}{r_f}\right\} . \qquad (4)$$

The effective inductance in the ferrite is:

$$L_f = \frac{\mu_f}{6\pi}\left\{2\left(\frac{1}{r_o} - \frac{1}{r_f}\right) + \frac{c'^2}{\omega^2}\left(\frac{1}{r_o^3} - \frac{1}{r_f^3}\right)\right\} \qquad (5)$$

and the air inductance is:

$$L_a = \frac{\mu_o}{6\pi}\left\{\frac{1}{r_f}\left(2\epsilon_{rf}\mu_{rf} - \frac{\mu_{rf}}{\epsilon_{rf}} + 1\right) + \frac{\mu_{rf}}{r_f^3}\left(\frac{2\epsilon_{rf}}{3} + \frac{1}{3\epsilon_{rf}}\right)\frac{c'^2}{\omega^2}\right\}, \qquad (6)$$

where

ϵ_f and μ_f are the permittivity and permeability of the ferrite,

ω is the radial frequency,

c' is the velocity of propagation in the ferrite, and

$\epsilon_{rf} = \frac{\epsilon_f}{\epsilon_o}$ and $\mu_{rf} = \frac{\mu_f}{\mu_o}$.

For the case where $\delta = r_o/r_f$, and $k'r_f$ and $k'r_o$ are small where $k' = \omega/c'$, the ratio of inductance in the ferrite to that in the air is approximately

$$\frac{L_f}{L_a} = \frac{3}{2\epsilon_{rf}\delta^3} . \qquad (7)$$

It is assumed that so long as this number is much greater than unity the resonant wavelength will be determined primarily by the ferrite and the conditions of the previous section will prevail.

EXPERIMENTAL VERIFICATION

A test of the theory developed depended upon the availability of ferrite material with the proper magnetic and electric characteristics in the frequency of interest as well as the proper physical dimensions. It was decided to slip a sleeve of ferrite over a wire radiator and then to measure the impedance as a function of frequency. The frequency chosen was about 20 to 40 mc. This choice was dictated by the availability of material. About the best material available was manufactured by Trans-Tech, Rockville, Md., in the form of a tube 1/4-in. OD and 1/16-in. ID. The properties given in Table I were listed by the manufacturer for TT-390.

TABLE I

LISTED CHARACTERISTICS OF TRANS-TECH FERRITE

TT-390

Frequency, mc	μ	$\tan\delta$	ϵ	$\tan\delta$
10	30	.25	15	.1
100	14	1.3	13	.03

For these conditions, $\delta=1/4$ and ϵ_{rf} is estimated as 14. Then from Equation (7) the ratio of the energy stored in the ferrite to that stored in the air is:

$$\frac{3 \times 64}{2 \times 14} \sim 6.5 \quad .$$

The following two experiments were performed. On 25 April 1957 a center-fed half-wave dipole, each arm of which was 1.75 m long, was mounted on top of the Cooley Building at North Campus. A ferrite sleeve covered the center of the radiator; two holes had been drilled from the outer to the inner diameter to accommodate the lead-in wires. The ferrite existed in three pieces; the center one was .20 m long, and on either end was an additional piece .10 m long. Two 72-ohm lead-in wires went down to the laboratory on the second floor, one the lead-in to the antenna and the other shorted for comparison. The measuring equipment was a HP 612A signal generator, a GR 916A bridge, and a HP 417A detector. The frequency where the impedance went from capacitive to inductive was about 25.5 mc.

A second experiment was performed on top of the East Engineering Building, May 29 through June 11, with a considerably shorted transmission line and with a single lead-in wire. These data were repeated and observed throughout as carefully as possible. Each radiating wire was 1.79 m long; the center piece of ferrite was cut as before except that its total length was 0.1 m. The resonant frequency with the ferrite was measured to be between 35.5 and 36.0 mc. After removal of the ferrite the same wire resonated at 40.0 mc.

INTERPRETATION

The maximum power transmitted to the load from the receiving antenna is given by $V^2/4R$ where R is the sum of radiation and loss resistance in the antenna. Since V is determined by the size of the antenna and the strength of the received signal, for optimum conditions R should be made as small as possible. Now the introduction of the ferrite will produce two effects, namely, additional losses in the material, and reflection from imperfectly matched boundaries. These phenomena will result in an effective increase in the resistance measured from the antenna terminals. Under ideal conditions the resistance of the ferrite-surrounded wire can only approach the resistance of the free-space radiator.

The material currently available has a considerable mismatch in wave impedance between ferrite and air, and an obviously large magnetic loss. These two factors combine to give a larger measured resistance than expected from the free-space radiator. The results of measurements with and without the ferrite are shown in the accompanying figure.

Because of the obvious promise of this miniaturization, it would seem to be in order to expend considerable effort towards making material with more similar permeabilities and dielectric constants with the proper geometry. The phase angle between magnetic field and moment in the material utilized was over 50°, yet measurements could be taken. In addition, the readings were necessarily observed with the antenna as a radiator, so some hysteresis loss was undoubtedly present, and thus receiving measurements would feel a smaller radiation resistance.

CONCLUSIONS

It is concluded that the technique here described presents a method, for the first time, of producing an antenna with an electric aperture larger than its physical aperture and by an arbitrary amount limited only by the availability of proper ferrite material. It is expected that the technique will be particularly useful for a receiving antenna but also utilizable for low-power transmitting antennas.

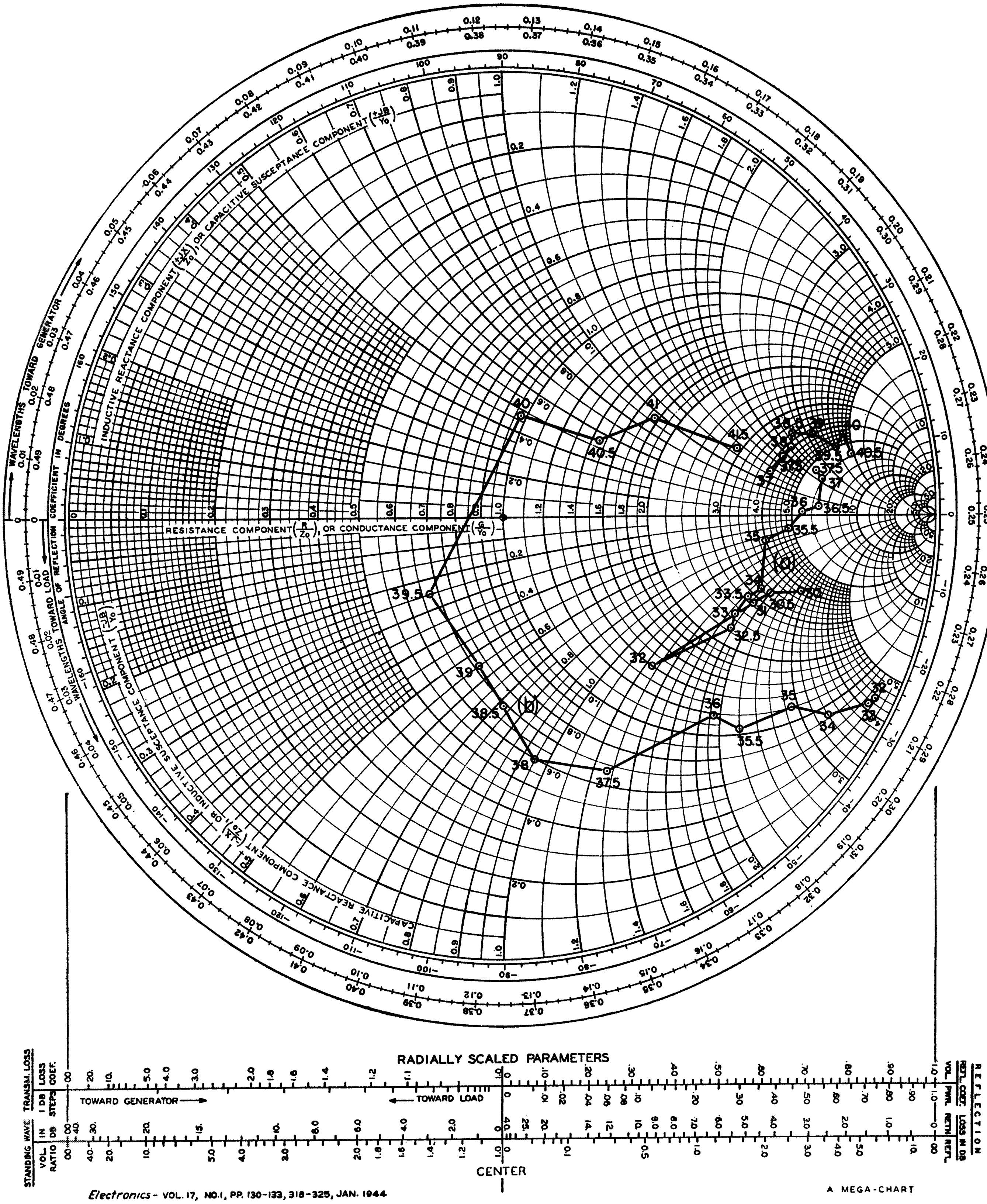
In addition to the miniaturizing effects, the ferrite itself can be shaped to control the direction of radiation flow away from the radiator.

Thus the ferrite antenna shows great promise for miniaturization, but materials must be developed before the techniques can be optimized. This method has the advantage that at the low frequencies where the physical size is the most troublesome, the ferrites can be made with the largest values of μ and ϵ, and thus the most miniaturization can be effected.

RADIATION IMPEDANCE OF A CENTER-FED HALF-WAVE DIPOLE RADIATOR;

(a) WITH A 4 INCH FERRITE TUBE;

(b) OF WIRE ALONE.



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APPENDIX

THE ENERGY STORED IN THE FERRITE AND IN THE AIR BY INDUCTIVE FIELDS

The fields in the ferrite surrounding the radiator are given by:

$$E_n = \frac{i \cos\theta \, e^{j\omega(t-r/c')}}{2\pi \, \epsilon_f}\left(\frac{1}{c' r^2} + \frac{1}{j\omega r^3}\right) \qquad \text{(A-1)}$$

$$E_\theta = \frac{i \sin\theta \, e^{j\omega(t-r/c')}}{4\pi \, \epsilon_f}\left[\frac{1}{c' r^2} + j\left(\frac{\omega}{c'^2 r^2} - \frac{1}{\omega r^3}\right)\right] \qquad \text{(A-2)}$$

$$H_\phi = \frac{i \sin\theta \, e^{j\omega(t-r/c')}}{4\pi}\left(\frac{j\omega}{c' r} + \frac{1}{r^2}\right) . \qquad \text{(A-3)}$$

The energy density ω is given by:

$$W = \frac{1}{2}\left[\epsilon \, \vec{E} \cdot \vec{E}^* + \mu \, \vec{H} \cdot \vec{H}^*\right] , \qquad \text{(A-4)}$$

where the arrows indicate vectors and the asterisks the complex conjugate. The energy density in the ferrite is:

$$W_f = \frac{i^2}{2}\left\{\frac{\cos^2\theta}{4\pi^2\epsilon_f^2}\left(\frac{1}{c'^2 r^4} + \frac{1}{\omega^2 r^6}\right)\epsilon_f + \frac{\sin^2\theta}{16\pi^2\epsilon_f^2}\left(\frac{1}{c'^2 r^4} + \frac{\omega^2}{c'^4 r^2} - \frac{2}{c'^2 r^4} + \frac{1}{\omega^2 r^6}\right)\epsilon_f + \frac{\sin^2\theta}{16\pi^2}\left(\frac{\omega^2}{c'^2 r^2} + \frac{1}{r^4}\right)\mu_f\right\} . \qquad \text{(A-5)}$$

Now the boundary conditions for normal incidence are given by:

$$\epsilon_f E_{rf} = \epsilon_o E_{ra}$$

$$E_{\theta f} = E_{\theta a}$$

$$H_{\phi f} = H_{\phi a} , \qquad \text{(A-6)}$$

so the energy density W_a is given by:

$$W_a = \frac{i^2}{2}\left\{\frac{\cos^2\theta}{4\pi^2\epsilon_o^2}\left(\frac{1}{c'^2r^4} + \frac{1}{\omega^2r^6}\right)\epsilon_o + \frac{\sin^2\theta}{16\pi^2\epsilon_f^2}\left(\frac{1}{c'^2r^4} + \frac{\omega^2}{c'^4r^2}\right.\right.$$

$$\left.\left. - \frac{2}{c'^2r^4} + \frac{1}{\omega^2r^6}\right)\epsilon_o + \frac{\sin^2\theta}{16\pi^2}\left(\frac{\omega^2}{c'^2r^2} + \frac{1}{r^4}\right)\mu_o\right\} . \qquad \text{(A-7)}$$

The total energy W is obtained by integrating (A-5) over the ferrite and (A-6) over all space. So doing:

$$W = \frac{i^2}{6\pi}\int_{r_o}^{r_f}\left\{\frac{1}{\epsilon_f}\left(\frac{1}{c'^2r^2} + \frac{1}{\omega^2r^4}\right) + \frac{1}{2\epsilon_f}\left(\frac{\omega^2}{c'^4} - \frac{1}{c'^2r^2} + \frac{1}{\omega^2r^4}\right) + \frac{\mu_f}{2}\left(\frac{\omega^2}{c'^2} + \frac{1}{r^2}\right)\right\}dr$$

$$+ \frac{i^2}{6\pi}\int_{r_f}^{\infty}\left\{\frac{1}{\epsilon_o}\left(\frac{1}{c'^2r^2} + \frac{1}{\omega^2r^4}\right) + \frac{\epsilon_o}{2\epsilon_f^2}\left(\frac{\omega^2}{c'^4} - \frac{1}{c'^2r^2} + \frac{1}{\omega^2r^4}\right) + \frac{\mu_o}{2}\left(\frac{\omega^2}{c'^2} + \frac{1}{r^2}\right)\right\}dr . \qquad \text{(A-8)}$$

The terms not containing r in the denominator represent radiated energy and will thus be ignored. The first integral gives the energy in the ferrite, the second gives the energy in the air. If $2W/i^2$ is taken as the inductance, then:

$$L_f = \frac{\mu_f}{6\pi}\left\{2\left(\frac{1}{r_o} - \frac{1}{r_f}\right) + \frac{c'^2}{\omega^2}\left(\frac{1}{r_o^3} - \frac{1}{r_f^3}\right)\right\}$$

$$\text{(A-9)}$$

$$L_a = \frac{\mu_o}{6\pi}\left\{\frac{1}{r_f}\left(2\epsilon_{rf}\mu_{rf} - \frac{\mu_{rf}}{\epsilon_{rf}} + 1\right) + \frac{1}{r_f^3}\left(\frac{2\mu_{rf}\epsilon_{rf}}{3} + \frac{\mu_{rf}}{3\epsilon_{rf}}\right)\frac{c'^2}{\omega^2}\right\} .$$

To simplify these equations set $\mu_{rf}, \epsilon_{rf} \gg 1$, $\mu_{rf} \sim \epsilon_{rf}$. Thus

$$L_f = \frac{\mu_f}{6\pi\ r_o}\left\{2(1-\delta) + \frac{1}{k'^2r_o^2}(1-\delta^3)\right\}$$

$$\text{(A-10)}$$

$$L_a = \frac{\mu_f}{3\pi}\frac{\epsilon_{rf}\delta}{r_o}\left(1 + \frac{1}{3k'^2r_f^2}\right) ,$$

and the ratio of inductances is:

$$\frac{L_f}{L_a} = \frac{1}{2\delta\epsilon_{rf}} \left[\frac{2(1-\delta) + \frac{1}{k'^2 r_o^2} (1-\delta^3)}{1 + \frac{1}{3k'^2 r_f^2}} \right] . \qquad \text{(A-11)}$$

Now the wavelength in the ferrite is presumably much larger than either r_o or r_f, so:

$$\frac{L_f}{L_a} \cong \frac{3}{2\epsilon_{rf}\delta^3} . \qquad \text{(A-12)}$$

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