STATISTICAL ANALYSIS OF VEHICLE
VIBRATION AND DYNAMIC LOAD, AND
SELECTION OF SUSPENSION DESIGN
PARAMETERS

by

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Foreword

This report represents the English rendering of work published by Mr. K.H. Guo in China in 1975. Apart from the general interest of this work, it was also useful in providing the necessary modeling background for an optimal design project at the University of Michigan. This work resulted to the article entitled "A Design Procedure for Optimization of Vehicle Suspensions", authored by X.P. Lu, H.L. Li and myself, to appear in the International Journal of Vehicle Design. Thus the present report was prepared as a basic reference for the model development used in the optimization project.

The report was made possible through the translating efforts of X.P. Lu and H.L. Li, both of whom I would like to thank.

Ann Arbor   P. Papalambros
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Abstract

A suspension design method is proposed for obtaining statistical values of vehicle vibration and dynamic load easily. The method is based on the random vibration analysis of a two-degree of freedom suspension system. Concise relationships between the RMS of vibrations, dynamic loads and corresponding level distribution, and the design parameters are given. The rational selection of damping and suspension stroke and the method for estimating speed limits on rough road are studied. Suggestions related to experimental evaluation are included.

INTRODUCTION

Roughness of road surface always stimulates the corresponding vibrations and dynamic loads of vehicles. Reducing these vibrations and dynamic loads is always a goal for automobile designers. Because of the random properties of the road surface, vibrations and dynamic loads of a vehicle will be random. For reducing the dynamic load tramsmitted to passengers and cargos, it is necessary to reduce vibrations. For improving the durability of loading parts and reduce vehicle weight, it is necessary to reduce the dynamic loads transmitting to the parts. For improving direction control of the vehicle at high speed, it is necessary to reduce the dynamic load between tires and road surface, because vibration of adhesion will cause a reduction of "dynamic cornering stiffness."

When driving on a rough road, the vehicle speed is usually decreased significantly, since the level of vehicle vibration is too high and uncomfortable for the driver, especially when heavy hitting on the suspension bumper occures. For decreasing the probability of bumper hitting, it is necessary to design the travel stroke of suspension appropriately while trying to reduce the relative displacement between sprung and unsprung masses. From the viewpoint of durability of suspension springs, this latter part is also desirable.

No procedure and calculation method appears to having been derived to-date which is convenient enough for engineering application. To some extent, suspension design is still performing

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with a rough calculation and experience. A more precise calculation method is proposed in this paper. By using this method, the statistical characteristics (such as root mean square, by which probability levels are estimated directly by a Gaussian hypothesis) of vibrations and dynamic loads will be estimated easily by explicit formulas. The optimal damping selection, estimation of speed limit on rough road and experimental evaluation are discussed.

2. STATICAL CALCULATION OF VEHICLE VIBRATION AND DYNAMIC LOAD ON ACTUAL ROAD SURFACE.

A large amount of statistical measurement data on road roughness have been acquired in several countries. All these data imply that a certain regularity exists behind the random movement. The distribution of road roughness is essentially similar to the Normal (Gaussian) distribution, and the frequency constitution of different road surfaces are very similar to each other. The power spectral density of different surfaces may be expressed by an unique formula as follows

$$S(\Omega) = A/\Omega^2 \quad (cm^2/c/m) \quad (1)$$

where, Ω: spacial frequency (circle/m)

   n: frequency index (dimensionless), representing a relative distribution of frequency components.
   For most of road surface, $n \approx 2$.

   A: coefficient, representing the intensity of the road roughness $(cm^2/c/m)$

It has been proved that, for a linear system, if the input is Gaussian random process then the output of the system must be a Gaussian random process too. For a Gaussian distributed random variable (which may represent the roughness, vibration acceleration or dynamic load), we will have a clear understanding about the distribution of x if and only if the mean value m_x and the root mean square σ_x are known. The probability density will be

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-m_x)^2}{2\sigma_x^2}} \tag{2}$$

The probability N(a) of x exceeding a certain level(s) may be obtained from integrating equation (2)

$$N(a) = 1 - \int_{-a}^{a} P(x)\,dx \tag{3}$$

The results of the integration have been made already in a form of table in many mathmatical handbooks or books on probability theory. For instance

$$|x| > a = 3\sigma_x, \qquad N(a) = 0.0027$$

$$|x| > a = 3.5\sigma_x, \qquad N(a) = 0.00047$$

$$|x| > a = 4\sigma_x, \qquad N(a) = 0.00006$$

$$|x| > a = 4.89\sigma_x, \qquad N(a) = 10^{-6}$$

If we like the probability of vehicle vibration acceleration $\ddot{z} > 1g$ to be less than 0.00006, then we should ensure $\sigma_{\ddot{z}} < 0.25g$ (the probability of $\ddot{z} > 4\sigma_{\ddot{z}}$ is 0.00006)

If we like the probability of suspension bumper hitting to be less then two point seven thousandths, then we should ensure

the RMS of suspension travel σ_y less than one third of the maximum stroke (the probability of $Y > 3\sigma_y$ is 0.27%).

If we like the probability of dynamic load P larger then static load G to be less than 4.7 ten thousandths, then we should ensure the RMS of dynamic load σ_p less 1/3.5 times G (the probability of $|P| > 3.5G$ is 0.00047) and so forth.

Thus for a random process with Gaussian distribution, a calculation problem of level distribution of vibrations and dynamic loads will become a calculation problem of RMS value of vibrations and dynamic loads on the actual road.

The RMS of a random process x can be obtained by integrating over its power spectral density $S_x(\omega)$ as following

$$\sigma_x^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega)\,d\omega \tag{4}$$

where the dimension of frequency ω is rad/sec. If a dimension of c/sec is used instead of rad/sec, let $2\pi f = \omega$, then,

$$\sigma_x^2 = \int_{-\infty}^{\infty} S'_x(f)\,df = 2\int_{0}^{\infty} S'(f)\,df \tag{5}$$

where the last equality holds due to $S'_x(f)$ being an even function.

If $S_x(\omega)$ represent a power spectral density (PSD) of the output vibration or dynamic load, then it will relate with PSD of road surface as follows:

$$S_x(\omega) = |W(j\omega)|^2 S_q(\omega) \tag{6}$$

Here $|W(j\omega)|$ is the amplitude response function which can be obtained from the transfer function W(s) by substituting

$S = j\omega, \quad (j = \sqrt{-1}).$

Since the transfer function $W(s)$ is a fraction in which the order of numerator is always less than the order of the denominator, and power spectrum densities are always an even function, i.e.

$$S_q(\omega) = \sqrt{S_q(j\omega)} \cdot \sqrt{S_q(-j\omega)}$$

with regard to equation (6), equation (4) can be expressed as

$$\sigma_x^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{G(j\omega)}{F(j\omega) \cdot F(-j\omega)} \, d\omega \tag{7}$$

Letting $S = j\omega$, we get

$$\sigma_x^2 = \frac{1}{2\pi j} \int_{-j}^{j} \frac{G(s)}{F(s)F(-s)} \, ds \tag{8}$$

where, $G(s) = b_0 s^{2n-2} + b_1{}^{2n-4} + \ldots + b_{n-1}$ is always an even function since all of power index of s are even.

$$F(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_0$$

Equation (8) represents an integration of a complex variable function. The integration path is along the whole imaginary axis. According to Jordan's theorem, we can close the integrating path by adding an infinite half circle without causing any difference of the results, and the value along the closed path will equal to the sum of all residues in the left half plane. After some maniplations, the result of equation (8) is obtained as

$$\sigma_x^2 = \frac{(-1)^n}{2a_0} \frac{\Delta 1}{\Delta} \tag{9}$$

where

$$\Delta \; = \; \begin{vmatrix} -a_1 & a_0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -a_3 & a_2 & -a_1 & a_0 & 0 & 0 & 0 & \cdots & 0 \\ -a_5 & a_4 & -a_3 & a_2 & -a_1 & a_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \\ -a_{2n-1} & - & - & - & - & - & - & - & - & - & - & (-1)^n a_n \end{vmatrix}$$

$$\Delta_1 \; = \; \begin{vmatrix} b_0 & a_0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ b_1 & a_2 & -a_1 & a_0 & 0 & 0 & 0 & \cdots & 0 \\ b_2 & a_4 & -a_3 & a_2 & -a_1 & a_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \\ b_{n-1} & - & - & - & - & - & - & - & - & - & - & (-1)^n a_n \end{vmatrix}$$

For a Gaussian variable, once σ_x is obtained, all probability distributions will be known.

3. TYPICAL ACTUAL ROAD SURFACE AND INPUT POWER SPECTRUM DENSITY

As mentioned above, random roughness of road surfaces is approximately Gaussian. The spacial PSD of road surfaces can be expressed as follows

$$S(\Omega) \; = \; \frac{A}{\Omega^2} \quad (cm/c/m) \tag{10}$$

If the vehicle speed is v, the temporal PSD will be [5]

$$S'(f) \; = \; \frac{1}{v} S(\Omega) \; = \; Av/f^2 \quad (cm^2/c/sec), \tag{11}$$

Letting $\omega = 2\pi f(rad/s)$ and noting that

$$1(cm^2/c/sec) \; = \; \frac{1}{2\pi} \; (cm^2/rad/sec),$$

the temporal PSD of road surface can be expressed as

$$S_q(\omega) = 2\pi A v \frac{1}{\omega^2} \quad (cm^2/rad/sec) \quad (12)$$

Noting further that the coefficient of right hand side of equation (12) only contains A and v and both are of first order, the input PSD (function $S_q(\omega)$) of different road surfaces are only dependent on the product Av. Increasing road roughness constant A is equivalent to increasing vehicle speed v at the same rate. Once the vibration and dynamic load of any A and v is calculated, the vibration and dynamic load for other A and v will be directly deduced by modifying a simple constant. It is known from equations (4) and (6) that if the value (Av) increases n times, the output (vibration and dynamic load) will increase n times also. In order to relate this calculation to actual road surfaces, some typical values of A are as follows:

cross country rough road   A = 1 $(cm^2 \cdot c/m)$

low quality road surface   A = 0.1   ("   )

high quality road surface   A = 0.01   ("   )

excellent high speed road   A = 0.001   ("   )

4. STATISTICAL CALCULATION OF VIBRATION ACCELERATION OF VEHICLE BODY

For most modern motor vehicles, the dynamic index of mass distribution are approximately equal to 1. So the front and rear suspension can be considered as separate systems as shown in Fig. 1. The differential equations of motion for each system are as follows

$$M\ddot{z} + k(\dot{z} - \dot{\zeta}) + c(z - \zeta) = 0$$

$$M\ddot{z} + m\ddot{\zeta} + c_k(\zeta - q) = 0 \tag{13}$$

where, z: vertical displacement of sprung mass (vehicle body)

ζ: vertical displacement of unsprung mass (wheels)

q: elevation of road surface

M: sprung mass

m: unsprung mass

c: spring constant of suspension

c_k: vertical stiffness of the tire

k: coefficient of damping

The transfer function of body acceleration z vs road input q is as follows:

$$\frac{\ddot{z}}{q} = \frac{\omega_k^2(2\varepsilon S + \omega_0^2)S^2}{S^4 + 2\varepsilon(1+\mu)S^3 + (\omega_0^2 + \mu\omega_0^2 + \omega_k^2)S^2 + 2\varepsilon\omega_k^2 S + \omega_0^2\omega_k^2} \tag{14}$$

where, $\omega_0^2 = \dfrac{c}{M}$, $\varepsilon = \dfrac{k}{2M}$, $\omega_k^2 = \dfrac{c_k}{m}$, $\mu = \dfrac{M}{m}$,

s: the Laplace variable.

From equation (12) the PSD of road surface may be written as

$$S_q(\omega) = 2\pi Av \frac{1}{j\omega} \cdot \frac{1}{-j\omega}$$

and letting $S = j\omega$, yields

$$S_q(\frac{S}{j}) = 2\pi Av \frac{1}{S} \cdot \frac{1}{-S} \tag{15}$$

Referring to equation (4), (6), (8), the RMS of body acceleration is

$$\overline{\ddot{z}^2} \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{\ddot{z}}{q}(j\omega)\right|^2 S_q(\omega)\,d\omega \;=\; \frac{2\pi Av}{2\pi j}\int_{-j\infty}^{j\infty}\frac{\ddot{z}}{S_q}(s)\cdot\frac{\ddot{z}}{-S_q}(-s)\,ds$$

$$\;=\; 2\pi Av\left[\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\frac{G(s)}{F(s)F(-s)}\,ds\right] \tag{16}$$

where

$$G(s) \;=\; b_0 s^6 + b_1 s^4 + b_2 s^2 + b_3$$

$$F(s) \;=\; a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

$$a_0 \;=\; 1,\; a_1 = 2\varepsilon(1+\mu),\; a_2 = (\omega_0^2+\mu\omega_0^2+\omega_k^2),\; a_3 = 2\varepsilon\omega_k^2,\; a_4=\omega_0^2\omega_k^2$$

$$b_0 \;=\; 0,\; b_1 = 4\varepsilon^2\omega_k^2,\; b_2 = -\omega_k^4\omega_0^4,\; b_3 \;=\; 0$$

Substituting these values into equation (9), yields

$$\overline{\ddot{z}^2} \;=\; \frac{(-1)^4}{2a_0}\cdot\frac{\Delta_1}{\Delta}\cdot 2\pi Av \tag{17}$$

$$\Delta \;=\; \begin{vmatrix} -2\varepsilon(1+\mu) & 1 & 0 & 0 \\ -2\varepsilon\omega_k^2 & (1+\mu)\omega_0^2+\omega_k^2 & -2\varepsilon(1+\mu) & (1+\mu)\omega_0^2+\omega_k^2 \\ 0 & \omega_0^2\omega_k^2 & -2\varepsilon\omega_k^2 & \omega_0^2\omega_k^2 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$\Delta_1 \;=\; \begin{vmatrix} 0 & 1 & 0 & 0 \\ 4\varepsilon^2\omega_k^4 & (1+\mu)\omega_0^2+\omega_k^2 & -2\varepsilon(1+\mu) & (1+\mu)\omega_0^2+\omega_k^2 \\ -\omega_0^4\omega_k^4 & \omega_0^2\omega_k^2 & -2\varepsilon\omega_k^2 & \omega_0^2\omega_k^2 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Expanding the determinants Δ and Δ_1, many terms are canceled and the following simple result is obtained

$$\overline{\ddot{z}^2} = 2Av\frac{4\varepsilon^2\omega_k^2+(1+\mu)\omega_0^4}{4\varepsilon\mu} = 2\pi Av(\frac{c_k}{c}\psi + \frac{1+\mu}{4\mu\psi}) \qquad (18)$$

where, $\psi = \frac{\varepsilon}{\omega_0}$ is damping constant. Once $\overline{\ddot{z}^2}$ is obtained,

$\sigma_{\ddot{z}}$ and the probability distribution are known.

From equation (18), the following observations can be made:

(1) When M and m are constant, reduction of either suspension spring rate or tire stiffness will cause a decrease of $\overline{\ddot{z}^2}$.

(2) Differentiating $\overline{\ddot{z}}$ vs μ, we get

$$\frac{d\overline{\ddot{z}^2}}{d\mu} = -\frac{\omega_0^4}{4\varepsilon\mu^2}$$ which is always less than zero. So

we know that increasing μ will cause a decrease of $\overline{\ddot{z}^2}$, i.e.

either increasing M or reducing m will cause a decrease of $\overline{\ddot{z}^2}$.

(3) If we increase the damping constant ψ, the first term of equation (18) will increase and the second term will decrease monotonically. Only a particular value of ψ relates to the minimum of $\overline{\ddot{z}^2}$. Differentiating equation (18) and letting $\frac{d\overline{\ddot{z}^2}}{d\psi} = 0$ the particular damping constant $\psi_{\ddot{z}min}$ is obtained as follows

$$\psi_{\ddot{z}min} = \frac{1}{2}\sqrt{\frac{1+\mu}{\mu}\frac{c}{c_k}} = \frac{1}{2}\sqrt{\frac{f_k}{f_s}} \qquad (19)$$

where, $f_k = \frac{(M+m)g}{c_k}$ is the static deflection of tire,

$f_s = \frac{Mg}{c}$ is the static deflection of suspension spring.

When we take $\psi = \psi_{\ddot{z}min}$, the RMS value of body acceleration is equal to

$$\overline{\ddot{z}}^2 = 2\pi Av\omega_0^3 \sqrt{\frac{1+\mu}{\mu} \frac{c_k}{c}}$$
$$= 2\pi Av\omega_0^3 \frac{1+\mu}{\mu} \sqrt{\frac{f_s}{f_k}} \qquad\qquad\Bigg\} \qquad (20)$$
$$= 2\pi Av\frac{c}{m} \sqrt{\frac{c_k}{M} \frac{1+\mu}{\mu}}$$

From equation (20) further observations are:

(4) where M and m are constant, $\overline{\ddot{z}}^2$ is proportional to $c\sqrt{c_k}$ and reducing spring constant c is more effective than reducing tire stiffness c_k for reducing $\overline{\ddot{z}}^2$

(5) The damping constant $\psi_{\ddot{z}min}$ corresponding to minimum $\overline{\ddot{z}}^2$ is proportional to $\sqrt{\frac{f_k}{f_s}}$, i.e., the softer the suspension spring, the smaller the $\psi_{\ddot{z}min}$. But the softer the tire the larger the $\psi_{\ddot{z}min}$.

Example 1: A medium truck (loaded) has the following characteristics: $\mu = \frac{\mu}{m} = 4$, $\frac{c_k}{c} = 3.52$, $f_s = 6$ cm, $f_k = 2.13$ cm, $\omega_0 = 12.78$ rad/s, $\omega_0^2 = 163$ $(\frac{rad}{s})^2$, $\omega_k^2 = 2300$ $(\frac{rad}{s})^2$. From equation (19) we calculate the damping constant for minimizing $\overline{\ddot{z}}^2$ as

$$\psi_{\ddot{z}min} = \frac{1}{2}\sqrt{\frac{f_k}{f_s}} = 0.298$$

From equation (20) the corresponding $\overline{\ddot{z}}^2$ is

$$\overline{\ddot{z}}^2_{min} = 2\pi Av\omega_0^3 \sqrt{\frac{1+\mu}{\mu} \frac{c_k}{c}} = 27300Av$$

If the truck is driven on a road of A=1 (cm^2 c/m) with a speed

36km/h, then $Av = 10 \ cm^{2 \cdot c}/s$ and

$$\sigma_{\ddot{z}} = \sqrt{\overline{\ddot{z}^2}} = 522 \ cm/s^2 = 0.533 \ g$$

From the Gaussian distribution table the probability of $|\ddot{z}| > 1g$ (when the cargo begins to lift up from the bed) is equal to 6%.

Example 2: A large-size car, (with normal load) has the following characteristics: $\mu = 6.12$, $\dfrac{c_k}{c} = 8.35$ suspension static deflection $f_s = 18 \ cm$, tire static deflection $f_k = 2.51 \ cm$, $\omega_0^2 = 54.6$, $\omega_k^2 = 2790$. The damping constant corresponding to the minimum of vertical acceleration of spring mass is now

$$\psi_{\ddot{z}min} = \frac{1}{2} \sqrt{\frac{f_k}{f_s}} = 0.187$$

The corresponding RMS value of vertical acceleration of sprung mass is

$$\overline{\ddot{z}^2} = 2\pi Av\omega_0^3 \sqrt{\frac{1+\mu}{\mu} \frac{c_k}{c}} = 7920Av$$

If the car is driven on the same road with same speed as the truck in example 1, (A=1, v=10), then $\sigma_{\ddot{z}} = 0.288g$, which is only at level of 56% of the truck in example 1. From the Gaussian distribution table, we found that probability of $|\ddot{z}| > 1g$ is equal to 0.05% which is a value of a hundredth compared with the truck mentioned in exampled 1.

As explained later, for a high speed car, from the viewpoint of direction control, it is preferred to choose a value ψ larger than ψ_{zmin}, especially for the rear suspension.

5. THE STATISTICAL CALCULATION OF SUSPENSION TRAVEL.

From equation (13), letting $z - \zeta = Y$, the differential equations becomes

$$M\ddot{Y} + k\dot{Y} + cY + M\ddot{\zeta} = 0$$
$$M\ddot{Y} + (M+m)\ddot{\zeta} + c_k\zeta = c_k q$$
(21)

From this, the transfer function of suspension travel Y vs road input q is found as

$$\frac{Y}{q}(s) = \frac{-\omega_k^2 s}{s^4 + 2\varepsilon(1+\mu)s^3 + (\omega_0^2 + \mu\omega_0^2 + \omega_k^2)s^2 + 2\varepsilon\omega_k^2 s + \omega_0^2\omega_k^2}$$
(22)

Referring to equations (4) and (6), the RMS of suspension travel will be

$$\overline{Y^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left|\frac{Y}{q}(j\omega)\right|^2 S_q(\omega)d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left|\frac{Y}{q}(j\omega)\right|^2 \frac{2\pi Av}{\omega^2}d\omega$$

Letting $\omega = \frac{s}{j}$, yields

$$\overline{Y^2} = 2\pi Av\left[\frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{G(s)}{F(s)F(-s)} ds\right]$$

where, $G(s) = b_0 s^6 + b_1 s^4 + b_2 s^2 + b_3$

$F(s) = a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$

$a_0 = 1$, $a_1 = 2\varepsilon(1+\mu)$, $a_2 = (1+\mu)\omega_0^2 + \omega_k^2$, $a_3 = 2\varepsilon\omega_k^2$, $a_4 = \omega_0^2\omega_k^2$

$b_0 = 0$, $b_1 = 0$, $b_2 = \omega_k^4$, $b_3 = 0$.

Substituting the above values into equation (9) we have

$$\overline{Y^2} = 2\pi Av\frac{(-1)^4}{2a_0} \frac{\Delta_1}{\Delta}$$
(23a)

where,

$$\Delta = \begin{vmatrix} -2\varepsilon(1+\mu) & 1 & 0 & 0 \\ -2\varepsilon\omega_k^2 & (1+\mu)\omega_0^2+\omega_k^2 & -2\varepsilon(1+\mu) & 1 \\ 0 & \omega_0^2\omega_k^2 & -2\varepsilon\omega_k^2 & (1+\mu)\omega_0^2+\omega_k^2 \\ 0 & 0 & 0 & \omega_0^2\omega_k^2 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & (1+\mu)\omega_0^2+\omega_k^2 & -2\varepsilon(1+\mu) & 1 \\ \omega_k^4 & \omega_0^2\omega_k^2 & -2\varepsilon\omega_k^2 & (1+\mu)\omega_0^2+\omega_k^2 \\ 0 & 0 & 0 & \omega_0^2\omega_k^2 \end{vmatrix}$$

Expanding Δ and Δ_1, many terms are canceled and a simple result is obtained as follows:

$$\overline{Y^2} = 2\pi A v \frac{1+\mu}{4\varepsilon\mu} \tag{24}$$

It is seen that, under a certain operating condition (A and v kept constant), the value $\overline{Y^2}$ only depends on ε and μ. When $\mu = \frac{M}{m}$ is constant, $\overline{Y^2}$ will be inversely proportional to $\varepsilon = \frac{k}{2M}$. Increasing damping ε will cause an effective approach for reducing the RMS of suspension travel.

Once $\sigma_y = \sqrt{\overline{Y^2}}$ is obtained, the probability distribution of suspension travel will be obtained also, by referring to a Gaussian distribution table.

Example 3: The same truck as in example 1, $\mu = 4$, $\varepsilon = \psi\omega_0 = 0.298 \cdot 12.78 = 3.8$ rad/s (corresponding to $\psi = \psi_{\ddot{z}min}$), driven

on the same road (A=1, v=10m/s), the bumping stroke [Y]=6 cm.

Calculate the probability of bumper hitting and Y>4cm as follows:

From equation (24)

$$\overline{Y^2} = Av \cdot \frac{\pi}{2} \cdot \frac{1+\mu}{\mu\varepsilon} = Av\frac{\pi}{2} \cdot \frac{5}{4 \cdot 3.8} = 0.516Av$$

$$\sigma_Y = \sqrt{\overline{Y^2}} = 0.719\sqrt{Av} = 2.27 \text{ cm}$$

$$\frac{[Y]}{\sigma_Y} = 2.64$$

Referring to Gaussian distribution table, the probability of $|Y|>2.64 \; \sigma_Y$ is 0.008.

6. THE SPEED LIMIT ON ROUGH ROAD

When a vehicle is driven on a rough road, it usually has to slow down to avoid unacceptable vibrations. Some particular vehicles, e.g. ambulances or trucks carrying some fragile cargo, even driven on a smoother road, require a low level of vibration, and can only be driven under a certain low speed. Thus, the speed limit on rough road is usually a significant index of vehicle ride quality.

There are two cases which the vehicle speed is restricted with no regard to the engine power,

(1) Bumping stroke [Y] is large enough, but the acceleration of body vibration $\sigma_{\ddot{z}}$ exceeds a certain level which is unacceptable for driver or passengers. In other words, in this case, the probability of $\ddot{z}$ being larger than a certain value, is too large.

(2) Bumping stroke [Y] is not large enough, while σ_y exceeds an allowable value so that the probability of bumper hitting reaches an endurance limit, because intense bumper hitting causes an unacceptable shock.

Suppose the endurance limit of RMS of vehicle vibration is $[\sigma_{\ddot{z}}]$. From equation (18), letting $\overline{\ddot{z}^2} = [\sigma_{\ddot{z}}]^2$, the first kind of speed limit (related to case (1)) is obtained as follows

$$[V_a]_1 = \frac{0.573}{A} \frac{[\sigma_{\ddot{z}}]^2}{\varepsilon \frac{\omega_k^2}{\mu} + \frac{\omega_0^4}{4\varepsilon} \frac{1+\mu}{\mu}}$$

or

$$[V_a]_1 = \frac{0.573}{A} \frac{[\sigma_{\ddot{z}}]^2}{\omega_0^3 [\frac{c_k}{c}\psi + \frac{1+\mu}{4\mu\psi}]} \qquad (25)$$

Determining $[\sigma_y]$ according to bumping stroke and acceptable bumper hitting probability, substitution into equation (24), gives the second kind of speed limit (related to case (2)) as follows

$$[V_a]_2 = \frac{2.29}{A} \frac{\varepsilon\mu}{1+\mu} [\sigma_y]^2 \qquad (26)$$

Obviously, the actual speed limit will be the smaller one of these two limit values.

Now we introduce two parameters ξ, η, defined by $[\sigma_{\ddot{z}}] = \frac{g}{\xi}$, and $[\sigma_y] = \frac{[Y]}{\eta}$. The value of ξ and η imply the probability of $|\ddot{z}| > 1g$ (cargo throwing) and $|Y| > [Y]$ (bumper hitting). For an ordinary truck or an off-road vehicle, it is chosen that $\xi = 2$, $\eta = 3$.

Example 4: The same truck as in example 1 and 3, driven on a rough road A=1(cm·c/m). To calculate the first speed limit: we substitute $\sigma_{\ddot{z}} = \frac{g}{\eta} = \frac{g}{2}$ into equation (25) and get:

$$[V_a]_1 = \frac{0.573}{A} \frac{[\sigma_{\ddot{z}}]^2}{\varepsilon\frac{\omega_k^2}{\mu} + \frac{\omega_0^4}{4\varepsilon} \cdot \frac{1+\mu}{\mu}} = \frac{0.573 \cdot (\frac{980}{2})^2}{3.8 \frac{2300}{4} + \frac{2600}{4 \cdot 3.8} \cdot \frac{5}{4}} = 31.45 \text{ km/h}$$

The second kind of speed limit is found by substituting

$[\sigma_y] = \frac{[Y]}{3} = 2 \text{(cm)}$ into equation (26) yielding,

$$[V_a]_2 = \frac{2.29}{A} \cdot \frac{\varepsilon\mu}{1+\mu} [\sigma_y]^2 = 2.29 \cdot \frac{3.8 \cdot 4}{5}[2]^2 = 27.8 \text{ kM/h}$$

It is seen that the speed limit of this truck depends on bumper hitting and its value is equal to 27.8 km/h. It is noticable that in order to increase vehicle speed on rough road, it is important to reduce the suspension spring constant c and the tire stiffness c_k and to select the damping constant properly. In the case that the bumping stroke is large enough, it is acceptable to choose $\psi = \frac{1}{2}\sqrt{\frac{f_k}{f_s}}$ (see equation (19)). However, it is seen from equation (26), that in the case when the bumping stroke is not large enough, changing c and c_k will not affect the $[V_a]_2$. In this case, the only effective way is to increase damping. As a result of increased damping, the $[V_a]_2$ will increase, but $[V_a]$ may decrease, if $\psi > \psi_{\ddot{z}min}$. So the optimum value of damping will be obtained according to the condition $[V_a]_1 = [V_a]_2$, when the condition for bumper hitting is dominant. This reasonable value of damping will be obtained by equating (25) (26), i.e.

$$\psi \;=\; \frac{\sqrt{1+\mu}}{2\omega_0\omega_k} \;\cdot\; \sqrt{\frac{[\sigma_{\ddot{z}}]^2}{[\sigma_y]^2} - \omega_0^4} \tag{27}$$

In the case that the tire stiffness c_k is constant, decreasing the spring constant c and choosing damping according to equation (27)(if $\psi > \psi_{\ddot{z}min}$) will cause an increase of speed limit. For example, in the case of example 4 ($\psi=\psi_{\ddot{z}min}$ = 0.298, [Y] = 6cm, $[\sigma_y]$ = 2 cm), according to equation (27): ψ = 0.334, $\varepsilon = \omega_0\psi$ = 4.26(1/s) and substituting into equation (26) yields $[V_a]_1 = [V_a]_2$ = 31.2 km/h. Comparing with the speed limit under $\psi = \psi_{\ddot{z}min}$ = 0.298, it is increased by 12%. However, when we decide to reduce the spring constant, besides properly choosing damping and bumping stroke, some appropriate arrangements should be made to avoid having rolling pitching and changing of body height become unacceptable.

7. DESIGN OF BUMPING STROKE.

According to the above discussion, to guarantee the desirable high speed limit, it is needed to choose different bumping stroke [Y] for different types of vehicles.

Of course, choosing the bumping stroke as large as possible may have the advantage of enhancing the second speed limit. But if the [Y] is chosen so large that the $[V_a]_2$ becomes larger than $[V_a]_1$, then increased [Y] will be no longer helpful, but rather lead to very high center of gravity height and very large spring stress. Thus, it is reasonable to choose [Y] such that $[V_a]_1 = [V_a]_2$. Equating (25) (26) yields

$$[\sigma_y] = \frac{[\sigma_{\ddot{z}}]}{\sqrt{4\varepsilon^2 \frac{\omega_k^2}{1+\mu} + \omega_0^4}}$$ (28)

Substituting $[\sigma_{\ddot{z}}] = \frac{g}{\xi}$ and $[\sigma_y] = \frac{[Y]}{\eta}$ into equation (28), gives

$$[Y] = \frac{\frac{\eta}{\xi} g}{\sqrt{4\varepsilon^2 \frac{\omega_k^2}{1+\mu} + \omega_0^4}} = \frac{\eta}{\xi} \frac{f_s}{\sqrt{4\psi^2 \frac{f_s}{f_k} + 1}}$$ (29)

As mentioned before, in most cases it is chosen $\xi = 2$, $\eta = 3$, therefore

$$[Y] = \frac{1.5f_s}{\sqrt{4\psi^2 \frac{f_s}{f_k} + 1}}$$ (30)

If the damping is chosen such that the condition of minimum $\overline{\ddot{z}^2}$ is satisfied. i.e. $\psi = \frac{1}{2}\sqrt{\frac{f_k}{f_s}}$, substituting into equation (29) we have

$$[Y] = \frac{\eta}{\xi} \frac{f}{\sqrt{2}}$$ (31)

Taking $\xi = 2$, $\eta = 3$, we find

$$[Y] = 1.06 \, f_s$$ (32)

This means that we should design the bumping stroke as large as 1.06 times the static deflection under $\psi = \frac{1}{2}\sqrt{\frac{f_k}{f_s}}$ condition. For the suspension with relatively small static deflection (e.g. $f_s \leq 6$ cm), such a design is feasible. But for softer suspensions, because of other considerations (reducing e.g. height and spring stress, proper wheel guidance etc.),

it is necessary to choose smaller [Y] and try to reduce the bumper hitting probability by means of increasing damping ψ. In this case, the [Y] is given and damping ψ will be calculated by equation (27), while accepting a certain increase of $\overline{\ddot{z}^2}$. However, if damping ψ is determined in advance for some reasons, then [Y] will be calculated by equation (30) or equation (29).

Example 5: Same truck as in example 1, suspension static deflection, f_s = 6 cm, damping ψ is chosen to meet the minimum $\overline{\ddot{z}^2}$ condition i.e. $\psi = \psi_{\ddot{z}min} = \frac{1}{2} \sqrt{\frac{f_k}{f_s}} = 0.298$.

Originally choosing [Y] = 6 cm, the second speed limit condition becomes dominant. In this case, the speed limit (in A=1 road) is $[V_a]_2$ = 27.8 $\frac{km}{h}$ (see example). If we redesign the bumping stroke according to equation (32), then the bumping stroke will be [Y] = 1.06 x 6 = 6.36 cm. From equation (25) or equation (26) we have $[V_a]_1 = [V_a]_2$ = 31.45 km/h. Comparing with original value, the speed limit is increased by 13%.

8. THE PROBABILITY DISTRIBUTION OF DYNAMIC LOAD BETWEEN TIRE AND ROAD SURFACE

The dynamic load between tire and road surface is $P = M\ddot{z} + m\ddot{\zeta}$. After solving the transfer function $\frac{\ddot{z}}{q}(s)$ and $\frac{\ddot{\zeta}}{q}(s)$, the transfer function of the relative dynamic load between tire and road surface $\frac{P}{G}$ (G is the static load) vs road input q, is obtained as follows.

$$\frac{P/G}{q}(s) = \frac{(\frac{s^2}{1+\mu} + 2\varepsilon s + \omega_0^2)s^2}{s^4 + 2\varepsilon(1+\mu)s^3 + (\omega_0^2 + \mu\omega_0^2 + \omega_k^2)s^2 + 2\varepsilon\omega_k^2 s + \omega_0^2\omega_k^2} \tag{33}$$

Referring to equation (4) and equation (6) the RMS of the relative dynamic load will be

$$\overline{\left(\frac{P}{G}\right)}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{P}{G_q}(j\omega) \right|^2 S_q(\omega) d\omega$$

Recalling $S_q(\omega) = \frac{2\pi Av}{\omega^2}$, and letting $\omega = \frac{s}{j}$ yields,

$$\overline{\left(\frac{P}{G}\right)}^2 = 2\pi Av \frac{\omega_k^4}{g^2} \left[\frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{G(s)}{F(s)F(-s)} ds \right] \tag{34}$$

where,

$$G(s) = b_0 s^6 + b_1 s^4 + b_2 s^2 + b_3$$

$$F(s) = a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

$$b_0 = \frac{-1}{(1+\mu)^2}, \quad b_1 = 4\varepsilon^2 - \frac{2\omega_0^2}{1+\mu}, \quad b_2 = -\omega_0^4, \quad b_3 = 0$$

$$a_0 = 1, \quad a_1 = 2\varepsilon(1+\mu), \quad a_2 = (1+\mu)\omega_0^2 + \omega_k^2, \quad a_3 = 2\varepsilon\omega_k^2, \quad a_4 = \omega_0^2\omega_k^2$$

Subsituting these into equation (8) and equation (9) yields

$$\overline{\left(\frac{P}{G}\right)}^2 = 2\pi Av \frac{\omega_k^4}{g^2} \left[\frac{(-1)^4}{2a_0} \frac{\Delta_1}{\Delta} \right]$$

where Δ, Δ_1 are the determinants defined by equation (9).
After expanding there determinants, many terms are cancelled and finally the following result is obtained:

$$\overline{\left(\frac{P}{G}\right)}^2 = \frac{\pi Av}{2g^2 \varepsilon \mu} \left[\left(\frac{\omega_k^2}{1+\mu} - \omega_0^2 \right)^2 + \mu\omega_0^4 + 4\varepsilon^2\omega_k^2 \right] \tag{35}$$

Notice that $f_s = \frac{g}{\omega_0^2}$, $f_k = \frac{1+\mu}{\omega_k^2} g$, $\psi = \frac{\varepsilon}{\omega_0}$, Equation (35) may be rewritten as follows:

$$(\overline{\tfrac{P}{G}})^2 = \frac{\pi Av}{2f_s^2 \omega_0 \mu \psi} [(\tfrac{f_s}{f_k} - 1)^2 + \mu + 4\psi^2 (1+\mu)\tfrac{f_s}{f_k}] \qquad (36)$$

After determining $(\tfrac{P}{G})^2$ by equation (35) or (36), the RMS value of the dynamic load will be obtained as

$$\sigma_P = G\sqrt{(\overline{\tfrac{P}{G}})^2} \qquad (37)$$

From this and the probability tables, the probability distribution of the relative dynamic load will be obtained.

Example 6: To find the probability distribution of the dynamic load between the tire and road surface for the same car as in example 2, $\mu = 6.12$, $f_s = 18$ cm, $f_k = 2.51$ cm, $\omega_0 = 7.39$ rad/s, $\psi = 0.187$ (the minimum z condition). From equation (36) we have

$$(\overline{\tfrac{P}{G}})^2 = \frac{\pi Av}{2 \cdot 1.8^2 \cdot 7.39 \cdot 6.12 \cdot 0.187}[(7.18-1)^2 + 6.12 + 4(0.187)^2 \cdot 7.12 \cdot 7.18]$$

$$= 0.0295AV$$

$$\frac{\sigma_P}{G} = 0.172\sqrt{Av}$$

If the car is driven at a speed $v = 40$m/s (144km/h) on a low quality road surface of $A=0.1$ cm$^2 \cdot$c/m, then $\frac{\sigma_P}{G} = 0.172\sqrt{Av}=0.334$. The probability distribution can be found in the available Gaussian distribution table. For example, the probability of the wheel leaving the road surface is equal to the probability of $P <-G$ or $P<-\frac{\sigma_P}{0.344}$, which is equal to 0.00185.

The damping value which relates to the minimum dynamic load between tire and road surface can be found by differentiating

equation (36) with respect to ψ. Letting $\dfrac{d}{d\psi}\left(\dfrac{\overline{P}}{G}\right)^2 = 0$, we have

$$\psi_{Pmin} = \frac{1}{2}\sqrt{\frac{(\theta_f-1)^2+\mu}{(1+\mu)\theta_f}} \tag{38}$$

where, $\theta_f = \dfrac{f_s}{f_k}$

Obviously, in general, this ψ_{Pmin} is only relating to minimum $\overline{P}^2$ but it is not necessary to be the same as $\psi_{\ddot{z}min}$ relating to minimum $\ddot{z}^2$. In most cases, ψ_{Pmin} is larger than $\psi_{\ddot{z}min}$; the larger the f_s compared with f_k, the larger the difference between ψ_{Pmin} and $\psi_{\ddot{z}min}$ will be. Some compromise may be possible from global considerations. Substituting equation (38) into equation (36), at the minimum point the RMS of tire dynamic load will be

$$\left(\frac{\overline{P}}{G}\right)^2 = \frac{2\pi A v}{f_s^2\omega_0\mu}\sqrt{[(\theta_f-1)^2+\mu](1+\mu)\theta_f} \tag{39}$$

Example 7: To find the optimum damping from the viewpoint of minimum dynamic load between tire and road surface and the corresponding RMS value and probability distribution of the dynamic load. Substituting $\theta_f = \dfrac{f_s}{f_k} = 7.18$, $\mu = 6.12$ into equation (38) yields

$$\psi_{Pmin} = \frac{1}{2}\sqrt{\frac{(\theta_f-1)^2+\mu}{(1+\mu)\theta_f}} = 0.466$$

However, $\psi_{\ddot{z}min} = 0.187$ (see example 2) which is smaller than ψ_{Pmin}. From the viewpoint of dynamic load, the damping

should be much larger than that from the viewpoint of body

vibration. When designing, it is preferable to plot out the

$\overline{\ddot{z}}^2 \sim \psi$ and $\overline{\frac{P}{G}}^2 \sim \psi$ curves, then decide a best value of damping which

may be a compromise of both requirements.

Generally a value close to $\psi_{\ddot{z}min}$ is more preferable when

ride comfort is being emphasized, and the value close to ψ_{Pmin}

is more preferable, when the road holding consideration is

dominant.

From equation (39) the RMS value of dynamic load between

tire and road surface may be obtained for $\psi = \psi_{Pmin}$ as follows

$$\overline{(\frac{P}{G})}^2 = \frac{2\pi Av}{18^2 \cdot 7.39 \cdot 6.12} \sqrt{[(6.18)^2 + 6.12] \cdot 7.12 \cdot 7.18} = 0.0204 Av$$

If $(Av) = 4 \text{ cm}^2 \cdot c/s$, then $\sigma_p = 0.286G$. Compared with example

6, the dynamic load is decreased by 17%.

9. THE PROBABILITY DISTRIBUTION OF DYNAMIC LOAD OF OTHER LOADING PARTS

The dynamic load transmitted by a connecting element 3

(Fig. 2) to a cargo or an equipment (M_1) which is a part of

the sprung mass, may be counted as an inertial force by Newton's

Second Law:

$$P_1 = M_1 \ddot{z}$$

So when $\sigma_{\ddot{z}}$ and the probability distribution of $\ddot{z}$ is obtained,

these are just the RMS and probability distribution of the

relative dynamic load $(\frac{P_1}{G_1})$ transmitted to M_1, e.g. in example

1, $\sigma_{\ddot{z}} = 0.533g$, thereby, $\sigma_{P_1} = 0.533 M_1 g$.

For the dynamic load of some unsprung parts, an extra analysis is needed.

Now imagining that the unsprung mass is divided into two parts m_1 and m_2 (Fig. 3), the force transmitted by connecting element 3 will be considered as the dynamic load between part 1 and part 2.

$$P_x = m_1 \ddot{\zeta} + M\ddot{z}$$

$$\frac{P_x}{P_0} = \frac{\ddot{z}}{(1+\mu_1)g} \left(\frac{\ddot{\zeta}}{\ddot{z}} + \mu_1\right)$$

where, $\mu_1 = \dfrac{M}{m_1}$, $P_0 = (M+m_1)g$

$$\frac{\ddot{\zeta}}{\ddot{z}} = \frac{s^2+2\varepsilon s+\omega_0^2}{2\varepsilon s+\omega_0^2}$$

$$\frac{P_x}{P_0} = \frac{\ddot{z}}{g}\omega_k^2 \frac{\dfrac{s^2}{1+\mu}+2\varepsilon s+\omega_0^2}{2\varepsilon s+\omega_0^2}$$

Referring to equation (14) yields

$$\frac{P_x/P_0}{q}(s) = \frac{\left(\dfrac{s^2}{H\mu} + 2\varepsilon s + \omega_0^2\right)s^2}{s^4+2\varepsilon(1+\mu)s^3+(\omega_0^2+\mu\omega_0^2+\omega_k^2)s^2+2\varepsilon\omega_k^2 s+\omega_0^2\omega_k^2}$$

Notice that μ_1 only appears in the numerator while μ only appears in the denominator.

From equation (4) and equation (6):

$$\overline{\left(\frac{P_x}{P_0}\right)^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{P_x}{P_0 q}(j\omega)\right|^2 S_q(\omega)d\omega \qquad (41)$$

Recalling $S_q(\omega) = 2\pi Av/\omega^2$, and $\omega = \dfrac{s}{j}$, this becomes

$$\left(\frac{\overline{P_x}}{P_0}\right)^2 = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{P_x/P_0}{q}(s), \frac{P_x/P_0}{q}(-s) \cdot S\left(\frac{s}{j}\right) ds$$

$$= 2\pi Av \frac{\omega_4^2}{g^2} \left[\frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{G(s)}{F(s)F(-s)} ds\right]$$

where,

$$G(s) = b_0 s^6 + b_1 s^4 + b_2 s^2 + b_3$$

$$F(s) = a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

$$b_0 = -\frac{1}{(1+\mu_1)^2}, \quad b_1 = 4\varepsilon^2 - \frac{2\omega_0^2}{1+\mu_1}, \quad b_2 = -\omega_0^4, \quad b_3 = 0$$

$$a_0 = 1, \quad a_1 = 2\varepsilon(1+\mu), \quad a_2 = (1+\mu)\omega_0^2 + \omega_k^2, \quad a_3 = 2\varepsilon\omega_k^2, \quad a_4 = \omega_0^2\omega_k^2$$

Comparing these with equation (33) and (34), for dynamic load between tire and road, the only difference is that in $G(s)$, μ substituted by $\mu_1 = \frac{M}{m_1}$. From equation (8) and (9) we get

$$\left(\frac{\overline{P_x}}{P_0}\right)^2 = 2\pi Av \frac{\omega_k^4}{g^2} \frac{(-1)^4}{2a_0} \frac{\Delta_1}{\Delta}$$

Expanding Δ and Δ_1, we have

$$\left(\frac{\overline{P_x}}{P_0}\right)^2 = \frac{\pi Av}{2g^2\varepsilon\mu} \left[\left(\frac{\omega_k^2}{1+\mu_1} - \omega_0^2\right)^2 + \mu\omega_0^2 + 4\varepsilon^2\omega_k^2\right]$$

$$= \frac{\pi Av}{f_s^2\omega_0\mu\psi} \left[\left(\frac{1+\mu}{1+\mu_1} \cdot \frac{f_s}{f_k} - 1\right)^2 + \mu + 4\psi^2(1+\mu)\frac{f_s}{f_k}\right] \quad (42)$$

Comparing it with equation (35) and (36), the only difference is that the first term $\frac{\omega_k^2}{1+\mu}$ is substituted by $\frac{\omega_k^2}{1+\mu_1}$ (or $\frac{f_s}{f_k}$ is substituted by $\frac{f_s}{f_k}\frac{1+\mu}{1+\mu_1}$). When $\frac{\sigma_{P_x}}{P_0} = \sqrt{\left(\frac{\overline{P_x}}{P_0}\right)^2}$ is calculated by equation (42), referring to a Gaussian distribution table, the probability of $\frac{P_x}{P_0}$ will be obtained.

Example 8: To find the probability distribution of dynamic load transmitted to rear axle for the same car in example 6. The value m_1 can be determined by subtracting the wheel weight from unsprung mass m. It is given $\frac{M}{m_1} = \mu_1 = .12$

$\mu = 6.12$, $f_s = 18$ cm, $f_k = 2.51$ cm, $\omega_0 = 7.39$ 1/s,

$\psi = \psi_{\ddot{z}min} = 0.187$.

From equation (42)

$$\left(\frac{\overline{P_x}}{P_0}\right)^2 = \frac{\pi Av}{2f_s^2 \omega_0 \mu \psi} \left[\left(\frac{1+\mu}{1+\mu_1} \cdot \frac{f_s}{f_k} - 1\right)^2 + \mu + 4\psi^2(1+\mu)\frac{f_s}{f_k}\right] = 0.01253Av$$

$$\sigma_{P_x} = 0.112\ P_0\sqrt{Av}$$

Suppose the car is driven on an intensified proving track of A=1 $cm^2 \cdot c/m$, at a speed v = 16 m/s (58km/h), $\sigma_{P_x} = 0.112\ P_0\sqrt{16} = 0.448P_0$ From this the whole probability distribution can be found by referring to a Gaussian distribution table, e.g. the probability of $P_x \geq P_0$ is 0.025.

10. INDOOR ENDURANCE TEST AND DURABILITY PREDICTION OF LOADING PARTS.

Recently, the technique of quickening the indoor endurance test is being developed. Besides the random loading endurance test by means of an expansive electro-hydraulic loading system, an effective method is the mixed circulation loading method. A stable durability index can be obtained by this loading method such that $\Sigma N\sigma_a^k$ = const., and k ≈ 6.8 (N - amount of loading circles, σ_a - stress amplitude).[8] Thereby, a durability prediction may be expected.

Usually, an eight-step loading program is adopted which is drawn up according to the RMS value σ_p of the load measured by road test. The load levels and cycles of each step for a big cycle is shown in Table 1. For quickening the test process, sometimes, the first three steps ($\frac{P}{\sigma_p}$ < 1.75), which have only little influence on the durability are neglected. Thus, a big cycle will consist of only 497 cycles instead of 4000 cycles. In this way, the testing process will be much quicker.

To date, the simplest constant amplitude endurance test is still adopted in many cases. Obviously for estimating durability, it is reasonable to determine the loading amplitude according to RMS value of actual load in operation, e.g. $3.5\sigma_x$ or $3\sigma_x$ may be selected as a loading amplitude. Hereby, the dynamic effects to durability may be included.

Using the method to calculate the RMS of dynamic loads, it is possible to estimate the durability of the loading parts. For example, after redesigning suspension system of a vehicle, the RMS value of dynamic load of the axle decreases by 10%, and since the durability index k = 6.8, $(1+0.1)^{6.8} \approx 2$, we may expect that the durability, under new suspension system will be as long as twice than that before.

From above analysis, it is noted that the RMS of dynamic loads are always proportional to $\sqrt{Av}$. Therefore the influence of road roughness and operating speed to the durability of loading parts may be estimated, e.g. if operating speed keeps constant,

the coefficient A decreases to 50%, then σ_x will decrease to

$\frac{1}{\sqrt{2}}$ of original value. Since $(\sqrt{2})^{6.8} \approx 10$, then the durability

of loading parts will decrease to $\frac{1}{10}$ of original value.

Thus the importance of improving road surface is seen.

The influence of operating speed is just the same as the

coefficient A. Improperly increasing operating speed for 20%

will cause a reduction of durability of loading part to a half.

11. EVALUATION BY STEP RESPONSE TEST

When a vehicle is driven on a certain road surface, the

probability distribution of vertical acceleration $\bar{\bar{z}}$, suspen-

sion travel Y, and dynamic load on road P or on any point P_x

can be calculated according to the method deduced in this paper.

Usually, it is necessary to make a final check by the actual

load on a proving track and to figure out the RMS or probability

distribution. Certainly, this is reliable but expansive. A

simpler experimental evaluation method is suggested as follows.

Suppose that step from road input function (with an

altitude of H) is simulated at the wheel contact point, (by

driving over a step, or given a step function by a hydraulic

actuator), and output x (may be $\ddot{z}$, Y, P, P_x etc.) is recorded

as shown in Fig. 4. There is no difficulty to calculate an

integral of

$$E = \int_0^\infty [\frac{x(t)}{H}]^2 dt \tag{43}$$

On the other hand, the value E may also be calculated by input

q(t) and transfer function W(s). The Laplace transform of

step input $q(t)$ is $\frac{H}{s}$, so that Laplace transform of output $x(t)$ will be

$$x(s) = W(s)\frac{H}{s} \tag{44}$$

According to the integral theorem of the original function [7]

$$\int_0^\infty x^2(t)dt = \frac{1}{2\pi j} \int_{c_0-j\infty}^{c_0+j\infty} x(s)x(-s)ds \tag{45}$$

Since the system is stable, there is no pole on right side of the S plane. So, we may consider the integrating path such that $c_0 = 0$, thus

$$\int_0^\infty x^2(t)dt = \frac{H^2}{2\pi j} \int_{-j\infty}^{j\infty} \frac{W(s)}{s} \cdot \frac{W(-s)}{-s} ds \tag{46}$$

Comparing equation (46) with equation (16), (23), (34), (41) yields

$$E = \frac{1}{H^2} \int_0^\infty x^2(t)dt = \frac{\sigma_x^2}{2\pi Av}$$

$$\sigma_x^2 = \frac{2\pi Av}{H^2} \int_0^\infty x^2(t)dt = 2\pi AvE \tag{47}$$

The equation (47) showing that, because the PSD is proportional to $\frac{1}{\omega^2}$, which is the same as the energy spectral density with a proportionality constant. Therefore, the RMS of any output (z, Y, P, P_x etc.) may be obtained from the square integral of the step input response. For example, from a step input test, the transient curve of $\ddot{z}$ is recorded and the related square integral is figured out as

$$E_{\ddot{z}} = \int_0^\infty [\frac{\ddot{z}}{H}]^2 dt = 0.00254 \cdot g^2 \ sec/cm^2$$

Suppose a certain road surface of $A=1 \ cm^2 \cdot c/m$ and a vehicle speed of 10m/sec are considered, the RMS value of $\ddot{z}$ will be

$$\overline{\ddot{z}^2} = 2\pi A v E_{\ddot{z}} = 0.16g^2, \quad \sigma_{\ddot{z}} = 0.4g$$

And if, meanwhile, the stress at a certain point $\tau(t)$ is recorded the related square integral is calculated as

$$E_\tau = \int_0^\infty [\frac{\tau(t)}{H}]^2 dt = 3980 \ kg^2 \cdot cm^2 \cdot sec.$$

Under the operating condition $(Av)=10 \ cm^2 \cdot c \ / sec$, the RMS of the stress will be

$$\overline{\tau^2} = 2\pi A v E_\tau = 250000 \ kg^2/cm^4$$

$$\sigma_\tau = 500 \ kg/cm^2$$

By referring to Gaussian distribution table, the probability distribution of $\ddot{z}$ and τ will be obtained. e.g. the probability of $\ddot{z} \geq 1g$ equal to 0.0124 and the probability of $\tau \geq 1500 \ kg/cm^2$ is equal to 0.0027.

It is necessary to discuss the problem of magnitude and direction of step input. Because of the different damping coefficient in different direction of the damper, it is preferable to do both upward and downward step input test and to take the average value. For a linear system, the different value of H will be no influence on the related square integral E, while a larger H will reduce the measuring error. However too large an H will cause the system to go beyond the linear

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