THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

DISSIPATION OF 'ELASTIC' WAVE ENERGY IN GRANULAR SOILS

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INTRODUCTION

Elastic wave energy is considered to be at a level which does not cause noticeable change in the soil structure. A dynamically loaded footing introduces elastic wave energy into the supporting soil. Part of this energy is returned to the footing to contribute toward its oscillatory motion and part is absorbed in the soil, thereby producing a damping to the motion of the footing. There have been many attempts in the past to determine experimentally a "damping constant" for a given oscillator-soil system. A damping constant determined from test results necessarily lumps together the internal damping within the soil and the dissipation damping associated with the geometry of the foundation-soil system. It is important to maintain the distinction between these two types of energy losses.

The dissipation of elastic wave energy from a foundation can be studied by the use of the theory of elasticity. From numerous solutions available in the literature, the problem of a rigid circular footing vibrating at the surface of a semi-infinite elastic solid was chosen to illustrate the method of evaluating dispersion damping.

The primary object of this investigation was to establish numerical values for the internal damping in selected granular materials. Laboratory tests were devised whereby the influence on damping of confining pressure, degree of saturation, amplitude of resonant oscillation, and grain characteristics could be studied for longitudinal and torsional oscillations of cylindrical samples. The primary test method utilized measurements of amplitude decay from the resonant condition after the driving power was turned off. The initial resonant amplitude at the top free end of an 11 in. long specimen fixed at the base varied from about 1×10^{-5} in. to 1×10^{-3} in. double amplitude in longitudinal oscillation and from about 1×10^{-5} to 2.5×10^{-3} radians double amplitude in torsional oscillation.

Notation. - The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in the Appendix.

REVIEW OF PREVIOUS WORK ON DAMPING IN SOILS

Dissipation of Wave Energy in Elastic Bodies.

Theoretical treatments of elastic wave energy dissipation in elastic bodies have developed from the theory given in 1904 by H. Lamb.[1] G. F. Miller and H. Pursey[2] determined analytically the distribution of energy between the compression, shear and Rayleigh waves caused by a single load, or a group of single loads, acting vertically at the surface of a semi-infinite elastic solid. When the solid has a Poisson's ratio of 0.25, they found for a single force on the free surface that 67 per cent of the energy was dissipated as a Rayleigh (surface) wave, 26 per cent as the shear wave, and 7 per cent as a compression wave. E. Reissner[3] showed that when purely torsional oscillations were applied at the surface of a semi-infinite, homogeneous, isotropic, elastic body, no surface waves (Rayleigh waves) were developed, but that all of the wave energy was directed downward into the body.

The dispersion of energy by the propagation of elastic waves outward from the source establishes a quantity which may be termed a geometrical or dispersion type of damping. The theory for oscillators resting on the surface of an elastic semi-infinite body illustrates the characteristics of this geometrical type of damping. Such theories were developed by E. Reissner,[4] T. Y. Sung,[5] and extended by T. K. Hsieh.[6]

Damping of Elastic Waves in Soils.

Elastic waves which are developed from a localized source decrease in amplitude with distance from the source. For the elastic, isotropic, homogeneous, semi-infinite body, Lamb[1] found that both the horizontal and vertical components of the Rayleigh wave amplitude diminish according to the law of annular divergence (i.e. with $r^{-1/2}$). This reduction of wave amplitude is due to geometry alone, because the assumption of an ideal elastic body precludes energy losses by internal damping within the medium.

In 1911, L. Mintrop[7] reported on a comprehensive study of energy transmitted through a stiff clay to nearby structures. He used a single impact developed by dropping a 4000 kg. ball through a distance of 14 meters and obtained readings up to 2.5 km. away using a seismograph which had a magnification of 50,000 times. He also used steady state vibrations generated by horizontal-cylinder type coal-gas engines which operated at 140-160 rpm. These engines produced unbalanced forces up to 17,000 kg. horizontally and up to 25,000 kg. vertically. Readings were made 400 m. away. In 1931, G. Bornitz[8] made similar observations in the neighborhood of a large bore, slow speed machine, and took measurements at different depths on the surface of a convenient vertical mine shaft, down to a depth of 250 m. He described the amplitude of the propagated wave as

$$X_n = X_1 \sqrt{\frac{r_1}{r_n}}\, e^{-\alpha(r_n - r_1)} \tag{1}$$

where X_n is the amplitude at distance r_n, X_1 is the amplitude at a distance r_1 and α is defined as an absorption coefficient (in the more recent literature on acoustics, the quantity α corresponds to the coefficient of attenuation, which is a measure of the decay in intensity of an elastic wave with distance). In Eq. (1) it is seen that the wave amplitude varies both as a function of the annular divergence, as noted by Lamb[1], and as a function of the absorption coefficient, α. Values of α were determined by Bornitz as 0.00001/m for the marshy soil of the Oder river flats, 0.001/m for a deposit of loamy, clayey soil, and 0.003/m for a layer of fine-grained, dense dry sand over a layer of heavy clay. These are only representative values, obtained for a particular amplitude of vibration at particular frequency.

Experimental results of the damping in various solid and granular materials have shown that damping occurs even for very small strains. Often this damping effect, or the deviation from Hooke's law, is not of practical importance, but it is of considerable interest to evaluate the quantities which influence its magnitude. The tests on steel spheres by J. Duffy and R. D. Mindlin[9] indicated that the logarithmic decrement was independent of the amplitude of vibration within the range employed. The energy dissipated per cycle varied with the square of this amplitude instead of the cube as predicted by theory. L. Knopoff and G. J. F.

MacDonald[10] found that the specific dissipation function (which contains the coefficient of attenuation, α) was independent of the frequency over a range of 10^{-2} to 10^{7} cps for solids other than ferromagnetic material. Data for Amherst sandstone show that the logarithmic decrement was independent of frequency in the dry condition, but that it depended on frequency in the moist state.

Numerous other studies have been made of parts of the problem of elastic wave damping in solid materials, granular materials, or suspensions. Papers by E. L. Hamilton,[11] W. L. Nyborg, I. Rudnick, and H. K. Schilling[12] and G. Shumway,[13] to note a few, have each treated a part of the problem by studying selected materials. A more detailed description of previous work on this problem is given by F. E. Richart, Jr., J. R. Hall, Jr., and J. Lysmer.[14]

Example of Dispersion Damping in Ideal Elastic Solids.

A dispersion type of damping results from the loss of energy by radiation of elastic waves from a source. In the example to be considered, the source of input energy is an oscillator vibrating vertically on the surface of a semi-infinite ideal elastic solid which has weight. Reissner[4] developed the theory and considered that the oscillator could be represented by a uniformly distributed pressure over the circular contact area. Sung[5] extended Reissner's theory to cover the cases for which the stress distribution was parabolic, uniform, or corresponding to that produced by a rigid base. Recently, Hsieh[6] has reworked the fundamental equations in the Reissner-Sung theory in order to place them in a form comparable to that developed for the conventional one-degree-of-freedom system with viscous damping. The following equations are a condensation of Hsieh's study.

In order to evaluate the force transmitted to the elastic body, we first consider a weightless rigid circular disk which rests on the surface of the body (Fig. 1a). The elastic body is homogeneous and isotropic, has a shear modulus, G, and a mass density of $\rho = \gamma/g$. A vertical periodic force $P = Z\, e^{i\omega t}$ acts on the disk. The vertical displacement w, given by the Reissner-Sung theory is

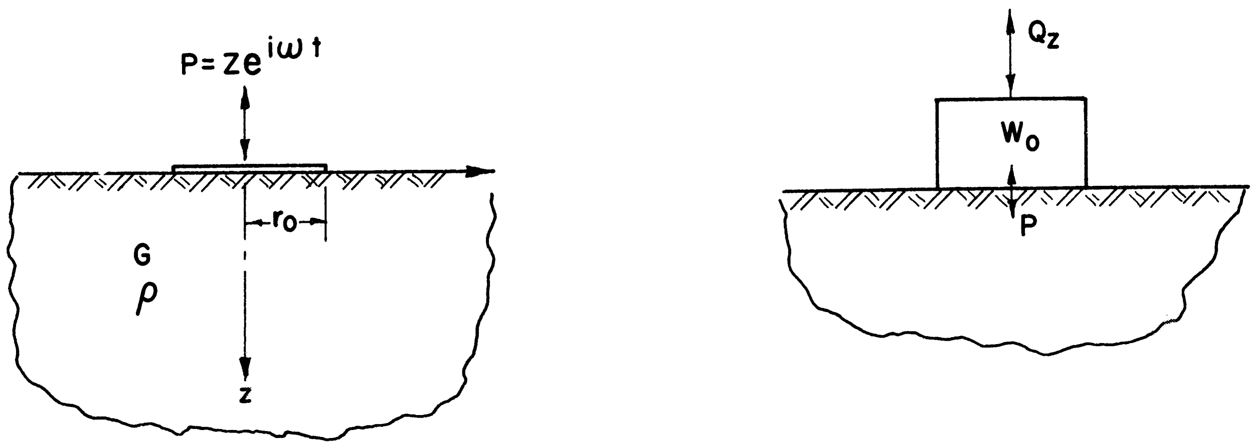
$$w = \frac{Z}{G r_0}\left[f_1 + i f_2\right] e^{i\omega t} \tag{2}$$

in which f_1 and f_2 are functions of the frequency of oscillation $(\omega/2\pi)$ and $i = \sqrt{-1}$. The time variable is t and $e = 2.71828.....$ In order to eliminate the imaginary term, Hsieh took the derivative of Eq. (2) with respect to time to obtain

$$\frac{dw}{dt} = \frac{Z\omega}{G r_0}\left[i f_1 - f_2\right] e^{i\omega t} \tag{3}$$

and by combining Eqs. (2) and (3) he determined

$$f_1 \omega w - f_2 \frac{dw}{dt} = \frac{Z\omega}{G r_0}\left[f_1^2 + f_2^2\right] e^{i\omega t} = \frac{P\omega}{G r_0}\left[f_1^2 + f_2^2\right]$$



(a) Weightless Circular Footing.

(b) Cylindrical Mass

Fig. 1. Footings Acting on a Semi-Infinite Elastic Body.

or

$$P = \frac{-Gr_o}{\omega}\left[\frac{f_2}{f_1^2+f_2^2}\right]\frac{dw}{dt} + Gr_o\left[\frac{f_2}{f_1^2+f_2^2}\right]w \qquad (4)$$

Eq. (4) indicates that the force transmitted to the elastic body is a function not only of the displacement of the disk, but also is a function of its velocity. For convenience in computations, the dimensionless frequency term, a_o from the Reissner-Sung theory,

$$a_o = \omega r_o\sqrt{\frac{\rho}{G}} = \frac{\omega r_o}{v_s} = \frac{2\pi r_o}{L_s} \qquad (5)$$

may be substituted into Eq. (4). Then by using the notation,

$$F_1 = \frac{-f_1}{f_1^2+f_2^2}\,, \qquad F_2 = \frac{f_2}{f_1^2+f_2^2} \qquad (6)$$

Eq. (4) becomes

$$P = -\frac{\sqrt{G\rho}}{a_o} r_o^2 F_2 \frac{dw}{dt} - Gr_oF_1 w \qquad (7)$$

For vertical oscillation, the functions F_1 and F_2 can be expressed approximately by the following expressions:

for Poisson's ratio, $\mu = 0$ $\quad F_1 = 4.0 - 1.2\, a_o^2$

$$F_2 = 3.4\, a_o - 0.12\, a_o^4$$

$$\mu = 1/4 \quad F_1 = 5.33 - 1.35\, a_o^2$$

$$F_2 = 4.20\, a_o - 0.12\, a_o^4 \tag{8}$$

$$\mu = 1/2 \quad F_1 = 8.0 - 2.40\, a_o^2$$

$$F_2 = 6.45\, a_o - 0.35\, a_o^4$$

(Eqs. 8 include modifications submitted by Richart in a discussion of Hsieh's paper.)

Eq. (7) can be further simplified by substituting

$$R_V = \frac{\sqrt{G\rho}}{a_o} r_o^2 F_2 \tag{9a}$$

and

$$K_V = G r_o F_1 \tag{9b}$$

to give

$$P = -R_V \frac{dw}{dt} - K_V w. \tag{10}$$

Now, if a cylindrical mass of radius r_o and weight W_o is placed on the weightless rigid disk and subjected to a vertical exciting force Q_z, (Fig. 1b), the equation of motion is

$$m_o \frac{d^2 w}{dt^2} = Q_z + P \tag{11}$$

or by substituting Eq. (10) into Eq. (11)

$$m_o \frac{d^2 w}{dt^2} + R_v \frac{dw}{dt} + K_v w = Q_z \tag{12}$$

Eq. (12) is similar to the equation for the one-degree-of-freedom system with viscous damping,

$$m \frac{d^2 x}{dt^2} + c \cdot \frac{dx}{dt} + kx = Q_x \ , \tag{13}$$

with the exception that both R_v and K_v depend on the frequency factor a_o. From Eqs. (8) it is evident that the effect of frequency on R_v and K_v is small for small values of a_o.

The magnitude of dispersion damping can be evaluated in terms comparable to conventional damping criteria if we specify as critical damping

$$R_{vc} = 2\sqrt{K_v m_o} \qquad \left(\text{from} \qquad C_{CR} = 2\sqrt{km}\ \right) \tag{14}$$

Then the damping ratio (ratio of actual to critical damping) can be evaluated from

$$\frac{R_v}{R_{vc}} = \frac{\sqrt{G\rho}\ r_o^2 F_2}{2a_o\sqrt{G r_o F_1 m_o}} \tag{15}$$

For a specific condition, for example for Poisson's ratio of 0.25, Eq. (15) becomes

$$\frac{R_v}{R_{vc}} = \frac{2.10\,(1-0.0286\,a_o^3)}{\sqrt{b}\,\sqrt{5.33-1.35\,a_o^2}} \tag{16}$$

in which

$$b = \frac{m_o}{\rho r_o^3}$$ = the dimensionless mass ratio.

Thus the damping of a particular foundation contributed by dispersion of elastic wave energy into the soil can be estimated directly by use of the dimensionless mass ratio. The value of a_o at maximum amplitude of vibration can be found from the curves given by Sung[5] or by F. E. Richart, Jr.[15].

THEORIES FOR THE EXPERIMENTAL DETERMINATION OF MATERIAL DAMPING

Determination of Logarithmic Decrement from Free Vibrations with Vicous Damping.

For a one-degree-of-freedom system with viscous damping, the general solution which corresponds to free vibrations is given by

$$X = e^{-\frac{c}{2m}t}\left[X_0 \cos\sqrt{\omega^2-\left(\frac{c}{2m}\right)^2}\, t + \frac{V_0 + \frac{c}{2m}X_0}{\sqrt{\omega^2-\left(\frac{c}{2m}\right)^2}} \sin\sqrt{\omega^2-\left(\frac{c}{2m}\right)^2}\, t\right] \tag{17}$$

in which m is the mass of the system

c is the viscous damping coefficient

X_0 is an initial displacement at time $t = 0$

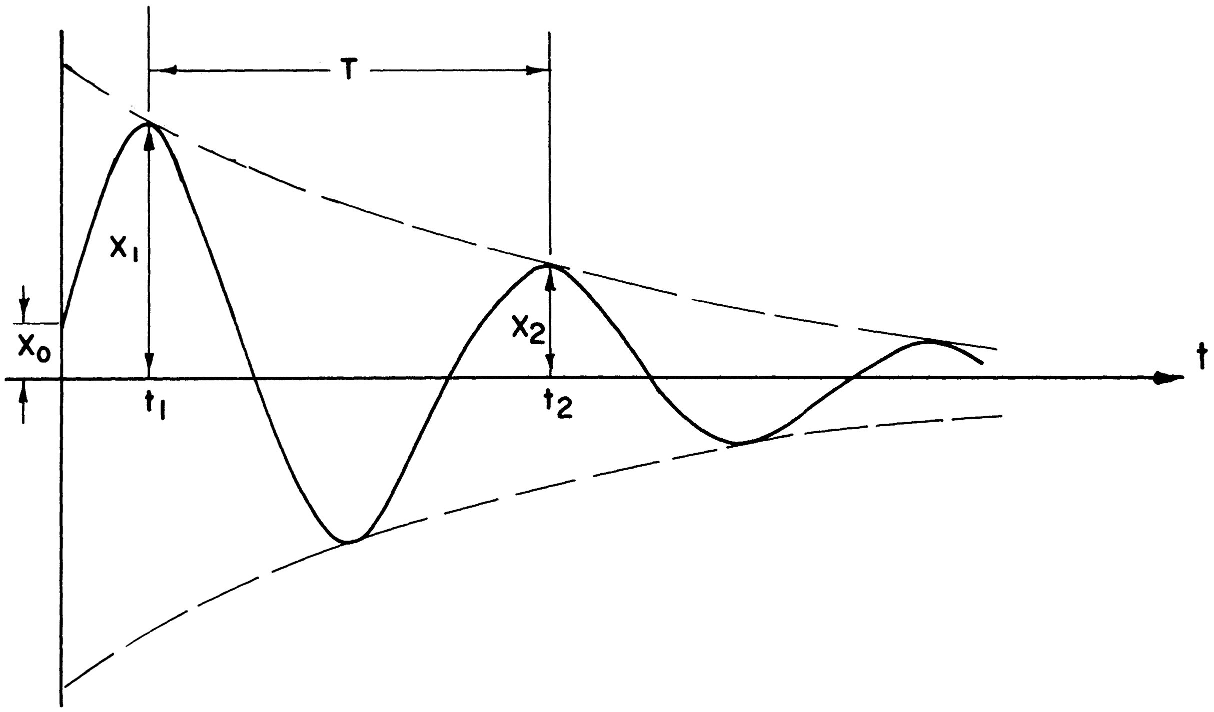
V_0 is an initial velocity at time $t = 0$

e is the base of the natural logarithms

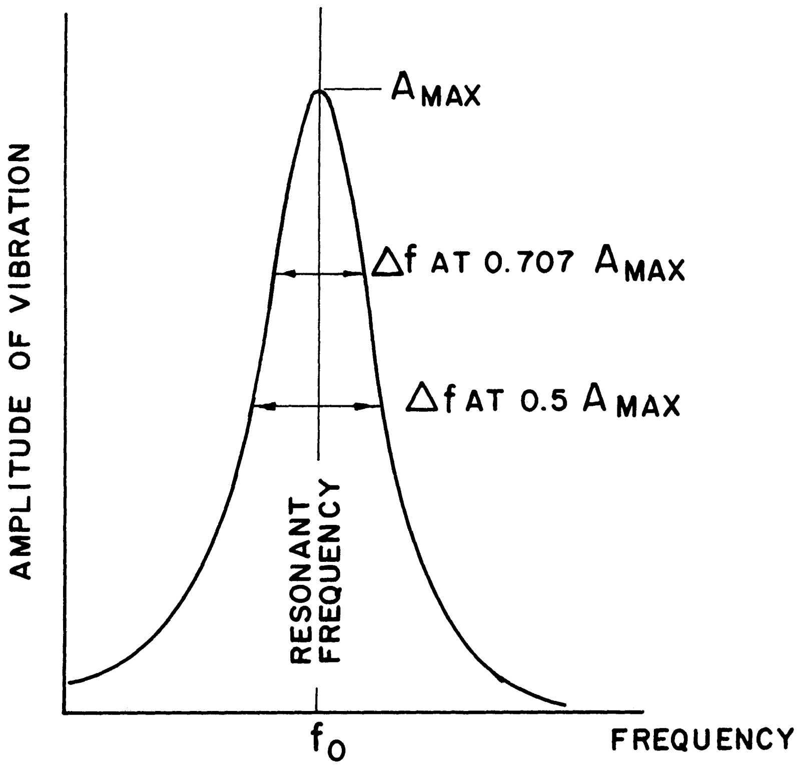
ω is the undamped natural frequency.

On a plot of displacement vs. time (Fig. 2a), Eq. (17) defines a sinusoidal type curve with decreasing amplitude. If we take advantage of the fact that the maximum point on the sine curve is very close to the point of tangency with the $e^{-\frac{c}{2m}t}$ curve, we can approximate the ratio of successive peaks, X_1 and X_2 at the time t_1 and t_2. The interval $t_2 - t_1$ represents the period, T. Thus,

$$\frac{X_2}{X_1} = \frac{e^{-\frac{c}{2m}t_2}}{e^{-\frac{c}{2m}t_1}} = e^{-\frac{c}{2m}(t_2-t_1)} = e^{-\frac{c}{2m}T} \tag{18}$$



(a) Free Vibrations With Viscous Damping.



(b) Amplitude-Frequency Curve For a Constant Force Type of Excitation

Fig. 2. Properties of a One-Degree-Of-Freedom System Used For the Determination of Damping.

The period is equal to the reciprocal of the frequency of oscillation, or

$$T = \frac{1}{f} = \frac{2\pi}{\sqrt{\omega^2 - \left(\frac{c}{2m}\right)^2}} \tag{19}$$

Then by substituting Eq. (19) into (18) and taking logarithms of both sides,

$$\ln \frac{X_1}{X_2} = \frac{c}{2m} \cdot \frac{2\pi}{\sqrt{\omega^2 - \left(\frac{c}{2m}\right)^2}} = \frac{c}{2m\omega} \cdot \frac{2\pi}{\sqrt{1 - \left(\frac{c}{2m\omega}\right)^2}}$$

or

$$\ln \frac{X_1}{X_2} = \frac{2\pi \frac{c}{C_{CR}}}{\sqrt{1 - \left(\frac{c}{C_{CR}}\right)^2}} = \delta = \quad \text{LOGARITHMIC DECREMENT.} \tag{20}$$

From Eq. (20) it is evident that for small values of C/C_{cr},

$$\delta \cong 2\pi \frac{c}{C_{CR}} .$$

Damping Determined from the Amplitude-Frequency Curve.

From computations based on forced vibrations with viscous damping for the constant force type of excitation the logarithmic decrement can be defined in terms taken directly from an experimental amplitude-frequency curve, (Fig. 2b). It depends primarily on the frequency range bounded by the curve at a specified amplitude. The general expression is

$$\delta = \pi \frac{\Delta f}{f_o} \sqrt{\frac{A^2}{A_{MAX}^2 - A^2}} \left[\frac{(f_L + f_H)}{2 f_o} \cdot \frac{1}{\left\{1 - \left(\frac{c}{C_{CR}}\right)^2\right\}} \right] \tag{21}$$

Usually the terms in the brackets are nearly equal to 1.0. For convenience, Eq. (21) is often used in one of the following forms:

$$\delta = 1.814 \frac{\Delta f_{0.5 A_{MAX}}}{f_0}$$

or

$$\delta = \pi \frac{\Delta f_{0.707 A_{MAX}}}{f_0}$$

Additional Methods for Evaluating Material Damping.

The literature covering internal damping in materials, particularly for metals, is voluminous. An indication of the type of work represented in these studies may be found in the references in the report of J. W. Jensen[16] or in the bibliography prepared by L. J. Demer[17]. The terminology and definitions used in the following paragraphs will follow those given by Jensen.

One list of the quantitative expressions used to define the internal damping of materials includes: viscosity, damping capacity, constant of internal friction, hysteretic constant, specific damping capacity, logarithmic decrement, elastic-phase constant, and coefficient of internal friction. Other terms which may be added to this list are: damping modulus, resonance-amplification factor, damping factor, specific damping energy, stress-strain phase angle, specific dissipation function, and attenuation.

Of these terms, the logarithmic decrement was described by Eq. (20) and the resonance-amplification factor and its relation to the logarithmic decrement was defined by Eq. (21).

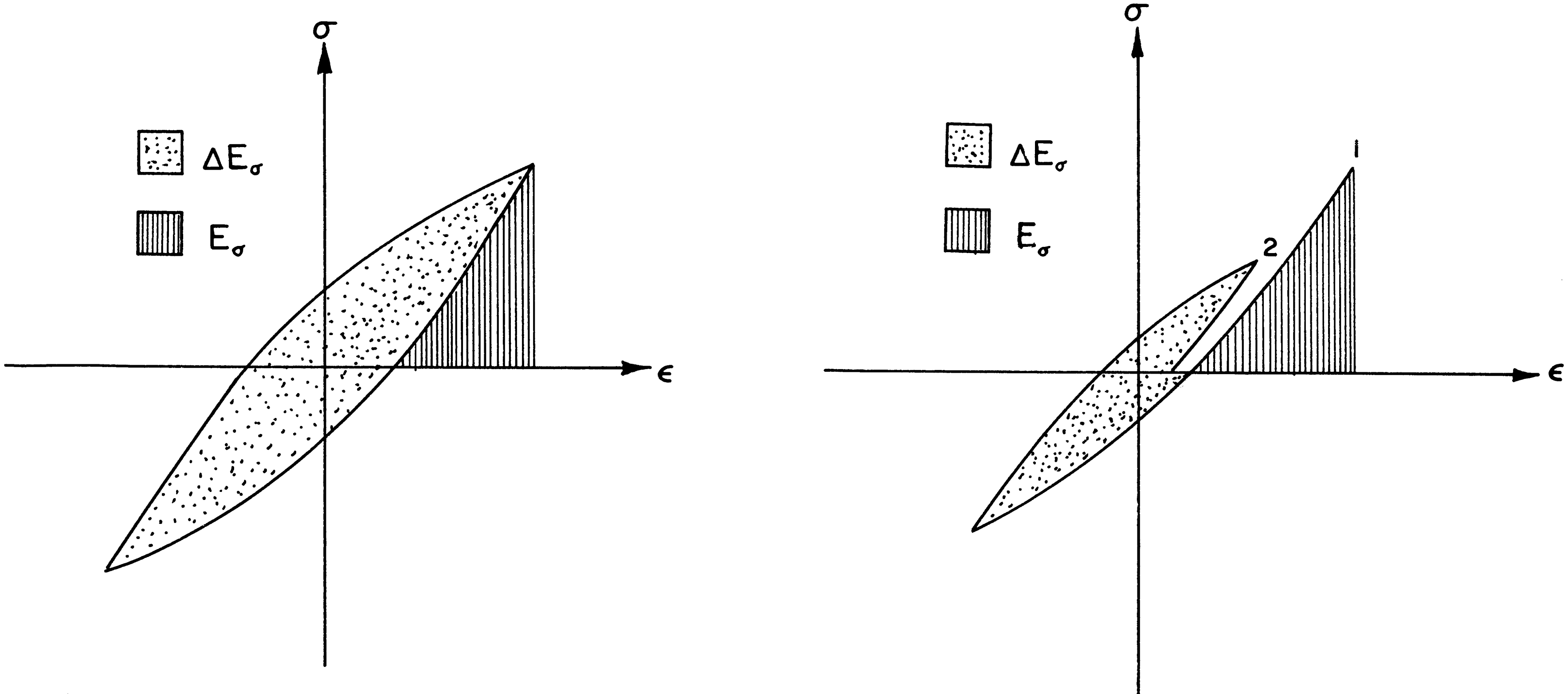
The term "specific damping capacity" has been used to indicate the ratio of the energy absorbed in one cycle to the strain energy at maximum stress. In terms of the stress-strain diagram the damping capacity represents the ratio of the area enclosed by the hysteresis loop to the area under the curve as shown in Fig. 3. For the steady-state condition as shown in Fig. 3a the damping capacity is given by

$$\psi_{ss} = \frac{\Delta E_\sigma}{E_\sigma} \tag{22}$$

The condition of decaying vibrations is represented by Fig. 3b. Point 1 corresponds to the maximum stress of a cycle which starts and ends at points 1 and 2 respectively. It is easily seen from Fig. 3 that the value of ψ depends upon whether the steady-state or the decay condition is considered. For conditions of decaying vibrations there is a relationship between logarithmic decrement, δ, and specific damping capacity, ψ_D. The strain energy for the nth cycle can be expressed by

$$E_n = k_n X_n^2$$

where X_n is the amplitude and k_n is a proportionality constant. Thus by definition



(a) Steady State Vibrations.

(b) Free Vibrations

Fig. 3. Stress-Strain Curves For a System With Hysteresis Damping.

$$\psi_D = \frac{k_n X_n^2 - k_{n+1} X_{n+1}^2}{k_n X_n^2}$$

Rearrangement of terms gives

$$\frac{k_n}{k_{n+1}} (1 - \psi_D) = \left(\frac{X_{n+1}}{X_n}\right)^2$$

from which

$$\delta = -\frac{1}{2} \ln \frac{k_n}{k_{n+1}} (1 - \psi_D) \tag{23}$$

or

$$\psi_D = 1 - \frac{k_{n+1}}{k_n} e^{-2\delta} . \tag{24}$$

It should be emphasized that there is no relationship between ψ_{ss} and δ but that for small values of δ, $\psi_{ss} \approx \psi_D$ and $k_n+1/k_n \approx 1.0$.

Often it is desirable to evaluate the decrease in amplitude of vibration as a function of distance from a source. In addition to the reduction in amplitude caused by geometrical dispersion of the wave energy there is a reduction caused by energy losses within the soil. This is designated by "attenuation," a measure of energy loss as a function of distance, and it is measured in terms of the <u>coefficient of attenuation</u>, α. The coefficient of attenuation is related to the logarithmic decrement by

$$\delta = \frac{2 \pi v \alpha}{\omega} \tag{25}$$

in which v is the phase velocity and $\omega/2\pi$ is the frequency of the propagating wave.

A variation of this attenuation constant determines the specific dissipation function, Q^{-1}. Thus the relation is

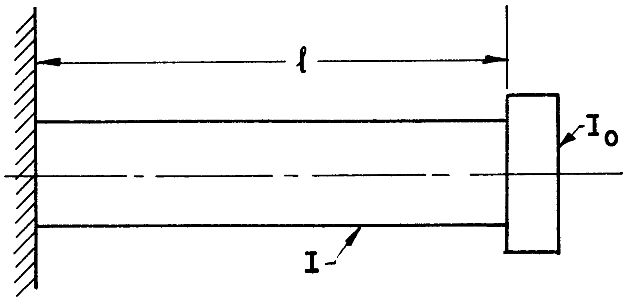
$$\frac{1}{Q} = \frac{2v\alpha}{\omega} \quad . \tag{26}$$

Theories Used in Connection with the Experimental Determination of Damping from Laboratory Specimens.

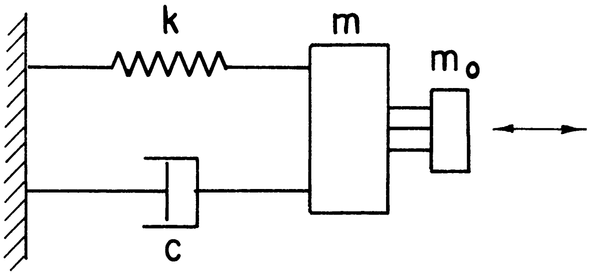
In order to vibrate a specimen some sort of a driving mechanism must be attached and another mechanism must be connected for measurement of the response. The addition of a mass to a material in which the resonant frequency is to be measured results in a slight change in the resonant frequency. The conditions of the specimen in the present research may be represented by Fig. 4a. The solution governing the natural frequency of such a system under torsional vibrations is given by

$$\omega L' \sqrt{\frac{\rho}{G}} \tan \omega L' \sqrt{\frac{\rho}{G}} = \frac{I}{I_o} \tag{27}$$

where I is the mass polar moment of inertia of the specimen and I_o is



(a) Elastic Circular Rod With a Rigid Mass Attached to the Free End.



(b) System With One Degree of Freedom.

Fig. 4. Models Representing Experimental Conditions.

the mass polar moment of inertia of the mass attached to the free end. Eq. (27) must either be solved graphically or by trial and error. It is convenient to put Eq. (27) into the form

$$\beta \tan \beta = \frac{I}{I_o} \quad . \tag{28}$$

Thus,

$$v = \frac{2\pi f L'}{\beta} \quad . \tag{29}$$

In this investigation the values obtained for damping by measurement of the decay of vibrations were corrected to compensate for the added mass of the pickup and driver. The effect of the added mass is approximated by considering a single-degree-of-freedom system as shown in Fig. 4b. The mass of the specimen is represented by M, the mass of the driver and pickup by M_o', and the spring constant is k. First consider the case without M_o'. We have the relationships

$$\delta = \frac{\pi c}{\omega m}$$

$$\omega = \sqrt{\frac{k}{m}} \quad .$$

Thus,

$$\delta = \frac{\pi c}{\sqrt{km}} \quad .$$

With the addition of M_o' we have similarly

$$\delta' = \frac{\pi c}{\sqrt{k(m+m_o')}} \quad .$$

Finally

$$\frac{\delta}{\delta'} = \sqrt{\frac{m+m_o'}{m}} \quad . \tag{30}$$

In order to use **Eq.** (30) it is necessary to convert the mass of the soil specimen into an equivalent concentrated mass. It can be shown that the equivalent concentrated mass is 0.405M. This is based on the condition that both systems have the same undamped natural frequency. Using the above approximation the corrected value of logarithmic decrement is given by

$$\delta = \delta' \sqrt{1 + \frac{m_o'}{0.405\,m}} \quad . \tag{31}$$

The same correction may be used for torsion by substituting the analogous torsional inertias.

LABORATORY TESTS OF DAMPING IN GRANULAR MATERIALS

Materials.

Four different materials were used in this investigation. Each is described below and the grain size curve for each is shown in Fig. 5.

Ottawa Sand. Standard Ottawa sand passing the No. 20 sieve and retained on the No. 30 sieve was used for most of the investigation. This is the material prepared and used by B. O. Hardin[18]. He reported a minimum void ratio of 0.50 corresponding to a unit weight of 110.5 lb/ft^3 and a maximum void ratio of 0.77 corresponding to a unit weight of 93.6 lb/ft^3.

Glass beads No. 2847. Glass beads, all of which lie between the No. 16 and No. 20 sieve, were obtained from the Prismo Safety Corporation, Huntingdon, Pennsylvania. These beads appear to be perfect spheres when examined under a microscope. They have a specific gravity of 2.50, a minimum void ratio of 0.57, and a maximum void ratio of 0.75.

Glass beads No. 1725. This material was also obtained from the Prismo Safety Corporation. Ninety-five per cent pass the No. 200 sieve and 96 per cent are retained on the No. 400 sieve. They have a high specific gravity of 4.31 resulting from the requirement of a high index of refraction for their commercial use. The minimum void ratio for this material is 0.57 and the maximum void ratio is 0.76.

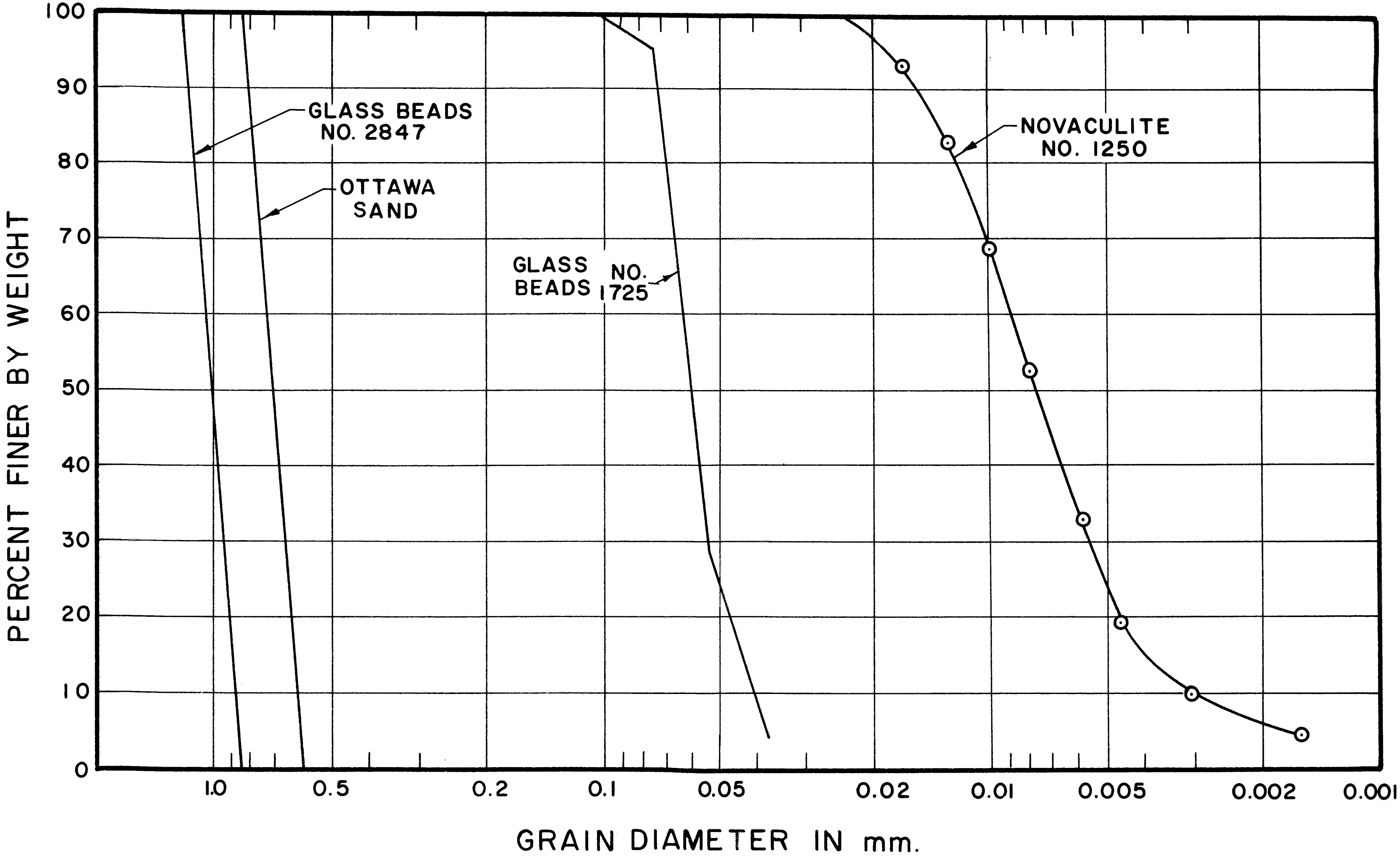


Fig. 5. Grain Size Curves for the Materials Used in the Present Research.

Novaculite No. 1250. This is a very fine quartz powder obtained from the American Graded Sand Co., 189-203 East Seventh Street, Paterson 4, New Jersey. This material was considered to be a silt as shown by the grain size curve in Fig. 5.

Summary of Tests.

Three groups of tests were run and are summarized in Table 1 and as follows:

Group I. These tests were run with Ottawa sand to obtain data on the effects of amplitude, pore fluid (air, water, and dilute glycerin), and density on damping for both torsional and compressional vibrations.

Group II. After the tests of Group I were completed, tests were run with the two sizes of glass beads in the dense condition, both dry and saturated.

Group III. A torsional vibration test to determine damping characteristics on Novaculite No. 1250 was run in the dry condition.

Equipment.

Two pieces of equipment were specially designed and built to vibrate the specimen at relatively large amplitudes in the longitudinal and torsional modes. Each was constructed so that one end of the specimen was free and the other end was fixed as shown in Figs. 6 and 7. Both pieces of equipment are basically the same except for the driver and pickup as shown in Fig. 6. The frames were made from a piece of 4 in. steel pipe

(a) Torsional Vibration Apparatus. (b) Longitudinal Vibration Apparatus.

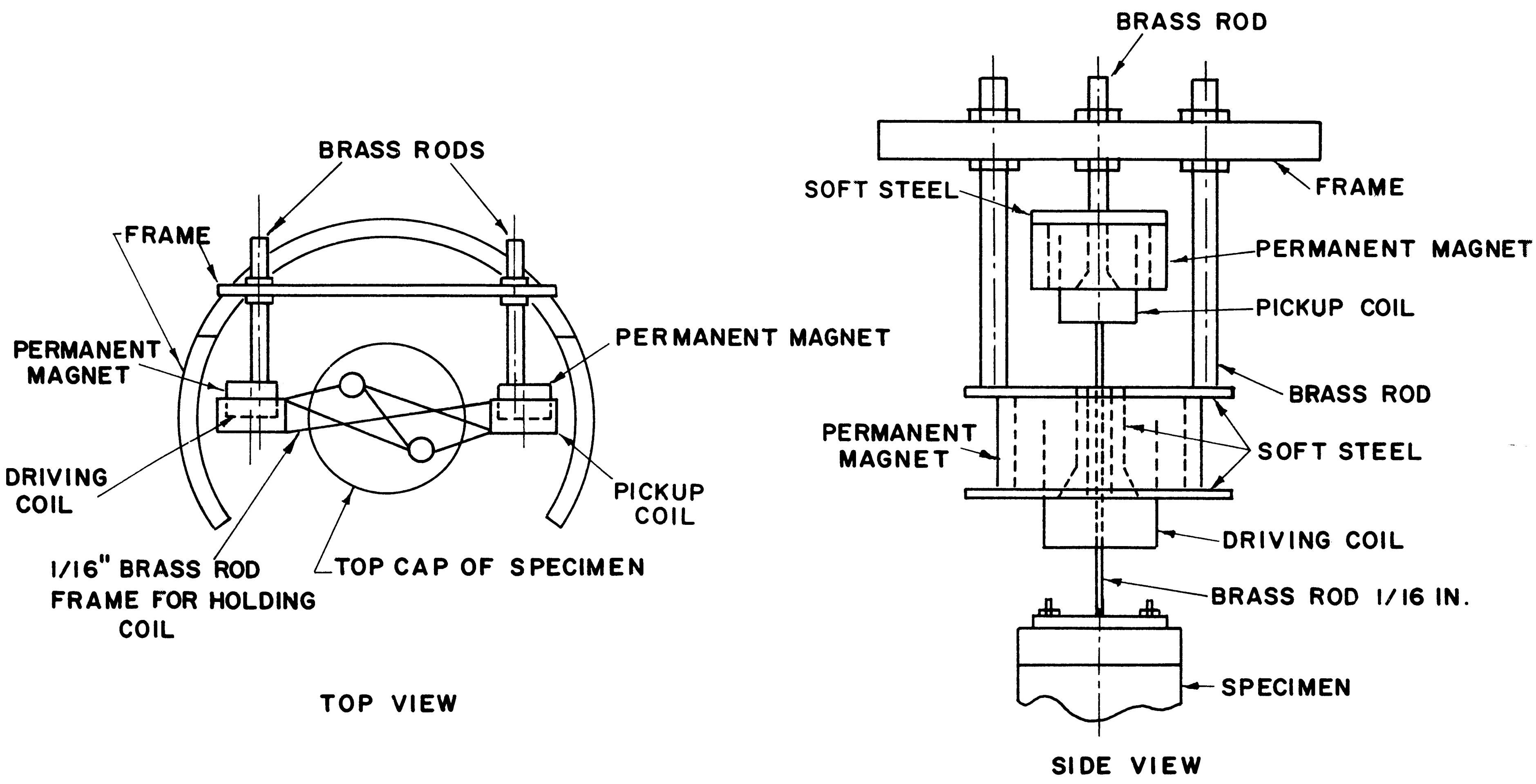
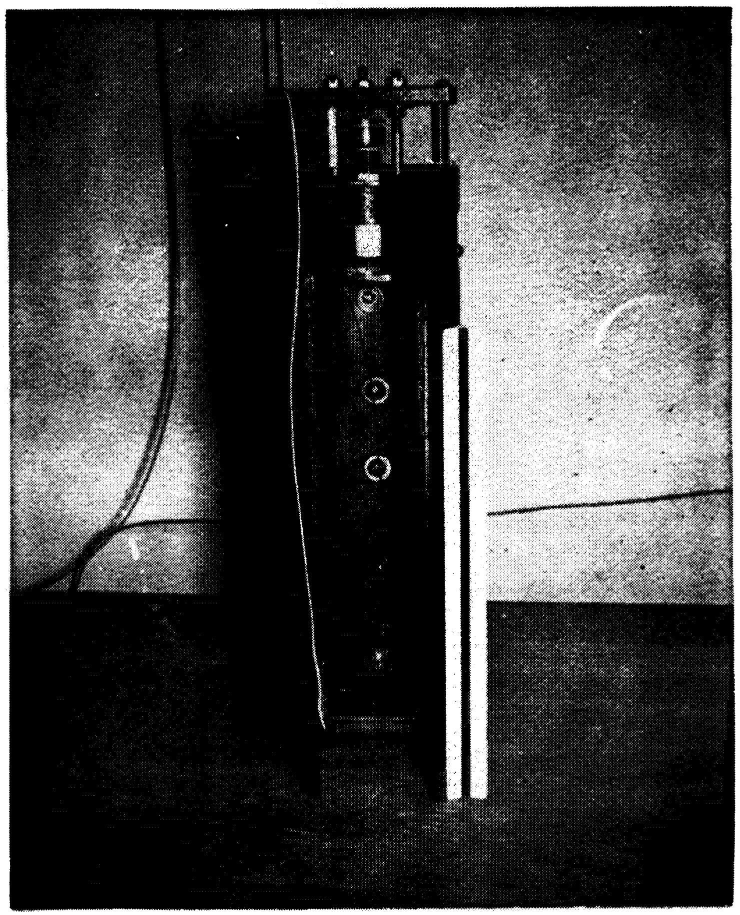
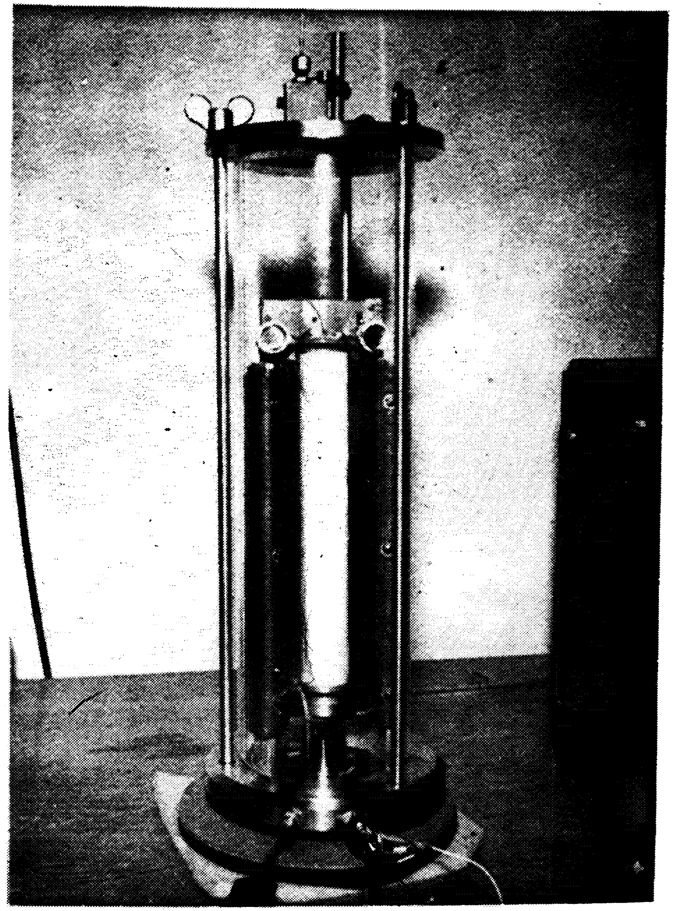


Fig. 6. Vibration Mechanisms Used in the Present Research.



Compression Apparatus



Torsion Apparatus

Figure 7. Fixed-Free Vibration Equipment.

with lead attached for added mass to give a total weight of about 30 lbs for each apparatus. This is necessary in order to reduce the movement of the "fixed" end to a negligible amount.

Power to the driving coil was supplied through an amplifier connected to an oscillator in the MB Electronics Type P11 power supply. Pickups were calibrated with an MB Electronics Model C31 calibrator and also with an MB Electronics Type 115 vibration pickup. A Tektronix Model 502 dual beam oscilloscope was used for the measurement of output from the pickups and also for monitoring the input to the driver. Decay curves for damping measurements were recorded with a Dumont Type 450 oscilloscope camera.

Test Procedures.

Special membranes were prepared for these samples by dipping a glass mandrel into liquid rubber solutions. About eight to ten dips were required for each membrane with a minimum drying time of four hours between dips. The natural rubber latex was found to be affected by absorption of water and neoprene, while unaffected by water, has poor elastic qualities. Consequently, a composite membrane made of latex (Type Vultex 1-V-10) and a layer of neoprene (Vultex 3-N-10) was prepared, then the neoprene surface was placed on the inside next to the specimen. Both liquids may be obtained from the General Latex and Chemical Co., 665 Main Street, Cambridge 39, Massachusetts.

The soil specimens were 1.59 in. in diameter and 10.8 in. long. The top cap was made of Plexiglas and the bottom cap was made of aluminum. Rubber O-rings were used to hold the membrane against the caps. Dow Corning silicone stopcock grease was used between the membrane and caps to provide a good seal.

When dealing with granular materials it is necessary to use a mold in forming the specimen. The mold was made from a piece of PVC tubing which was cut to form two halves. Tubes were placed on each half of the mold for vacuum line connections and filter paper strips were placed on the inside to allow a good distribution of the applied vacuum.

Several methods were used for placing the soil into the mold, depending upon the type of soil and the density desired. For the Ottawa sand and the glass beads the dense condition was obtained by pouring in approximately 50 cc. layers and vibrating each layer with a 1/8 in. brass rod attached to a small vibrator. This resulted in a condition close to 100 per cent relative density. The loose condition for the Ottawa sand was obtained by pouring the sand through a funnel attached to a 3/16 in. internal diameter glass rod which extended to the bottom of the mold. The rod was kept full of sand and slowly retracted from the mold allowing the sand to be deposited in a loose condition. The specimens prepared with the Novaculite No. 1250 were compacted. Since the material is very fine, a special procedure had to be followed. A vacuum was applied to the bottom pore pressure line during the compaction to prevent the material from blowing out of the mold and also as an aid to compaction. A teaspoon of material was added and pressed five time with a No. 7 rubber stopper attached to the end of a standard Proctor miniature compactor. Prior to construction of the specimen the Novaculite was dried in the oven at 220°C. for a period of several days.

After the specimen was prepared the pore pressure line was connected to a vacuum and measurements were taken for the determination of void ratio. The specimen was then placed in the vibration apparatus which fits into a triaxial cell. Air pressure in the cell was measured with a mercury manometer.

Measurements for damping were made by driving the specimen at the resonant frequency of the first mode of vibration, then cutting off the power and recording the decay with an oscilloscope camera. A typical recording of the decay of vibrations is shown in Fig. 8. The top trace of the photograph represents the motion of the end of the specimen which is the output from the pickup and the bottom trace is a measurement of the input voltage to the driving coil.

After the test was completed, measurements were again made to determine the void ratio. There was no measureable difference in most cases.

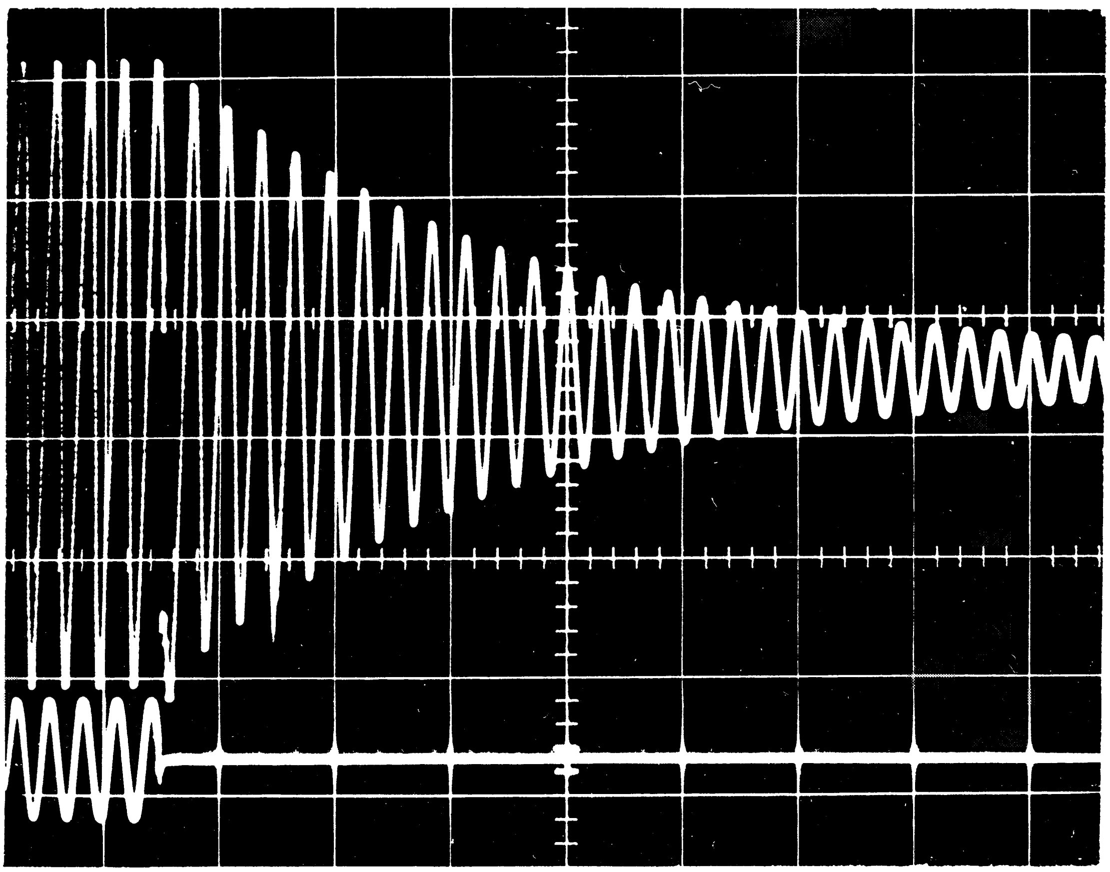


Fig. 8. Recording of the decay of vibrations.

Results of Damping.

Group I. Figures 9 through 12 show the results of logarithmic decrement for Ottawa sand obtained from the decay curves of Group I. In general the decay curves were such that the logarithm of amplitude plotted against the wave number was a straight line. The slope of this line represents the value of logarithmic decrement which is computed from the relationship given by Eq. (31). This value of logarithmic decrement was taken to be the value corresponding to an amplitude equal to the steady state amplitude at which the specimen was vibrating before the power was turned off. In cases of high amplitude and large values of δ the plot of logarithm of amplitude vs. wave number was not a straight line but a curve of decreasing slope. In this case the value of logarithmic decrement was taken as the average slope of the first several cycles of the decay curve.

Tests in Group I were to determine the variation of damping with confining pressure, amplitude of vibration, pore fluid (air, water, and dilute glycerin) and density for both torsion and compression.

Figure 9 shows the comparison of logarithmic decrement in the first mode of vibration for the dry and saturated condition of loose Ottawa sand in torsion. The same comparison is made for the longitudinal wave in the loose and dense conditions as shown in Figs. 10 and 11, respectively. Figure 12 shows the results for Ottawa sand saturated with glycerin.

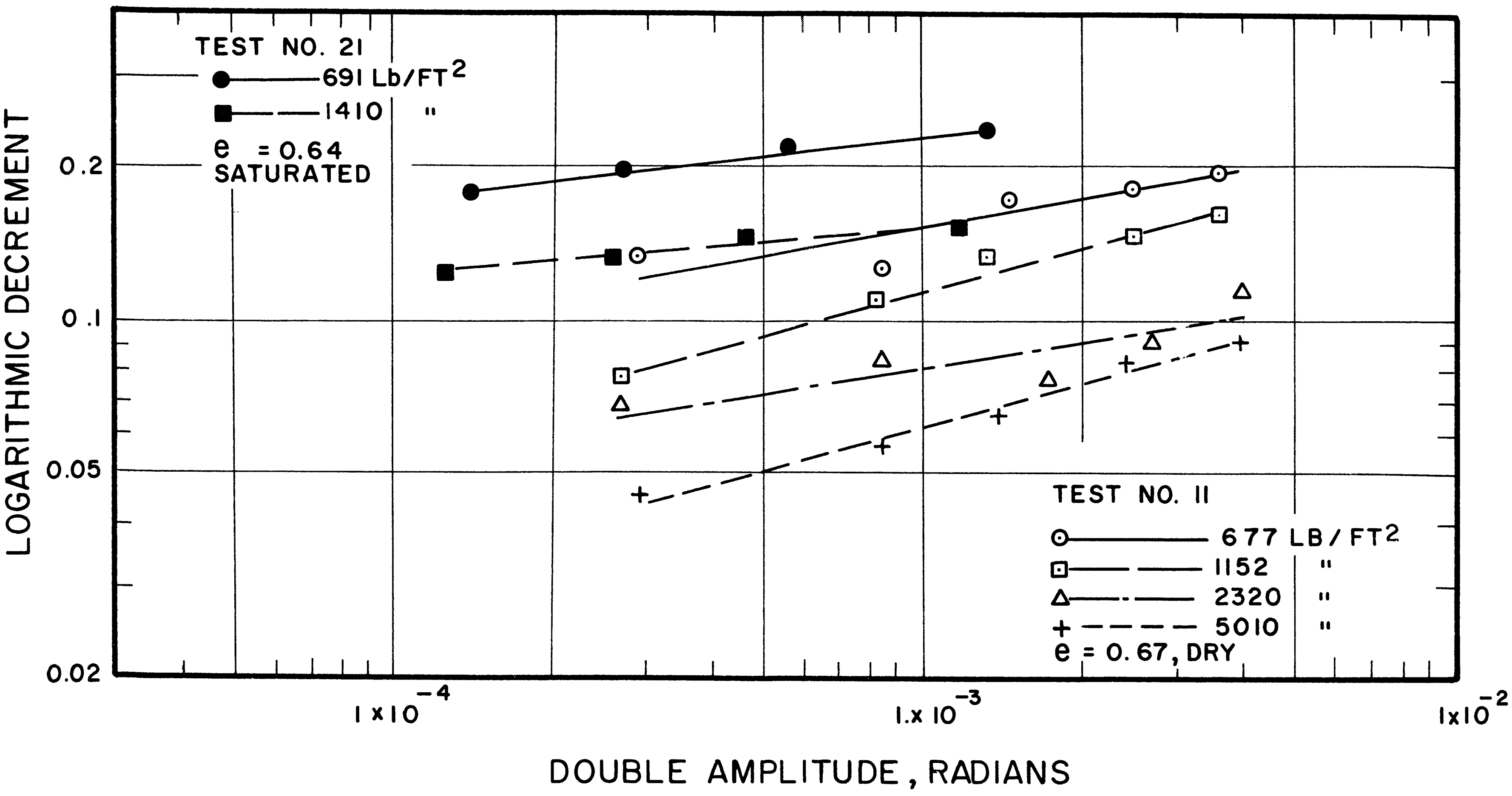


Fig. 9. Comparison of the Variation of Logarithmic Decrement With Amplitude for Dry and Saturated Ottawa Sand in Torsional Oscillation.

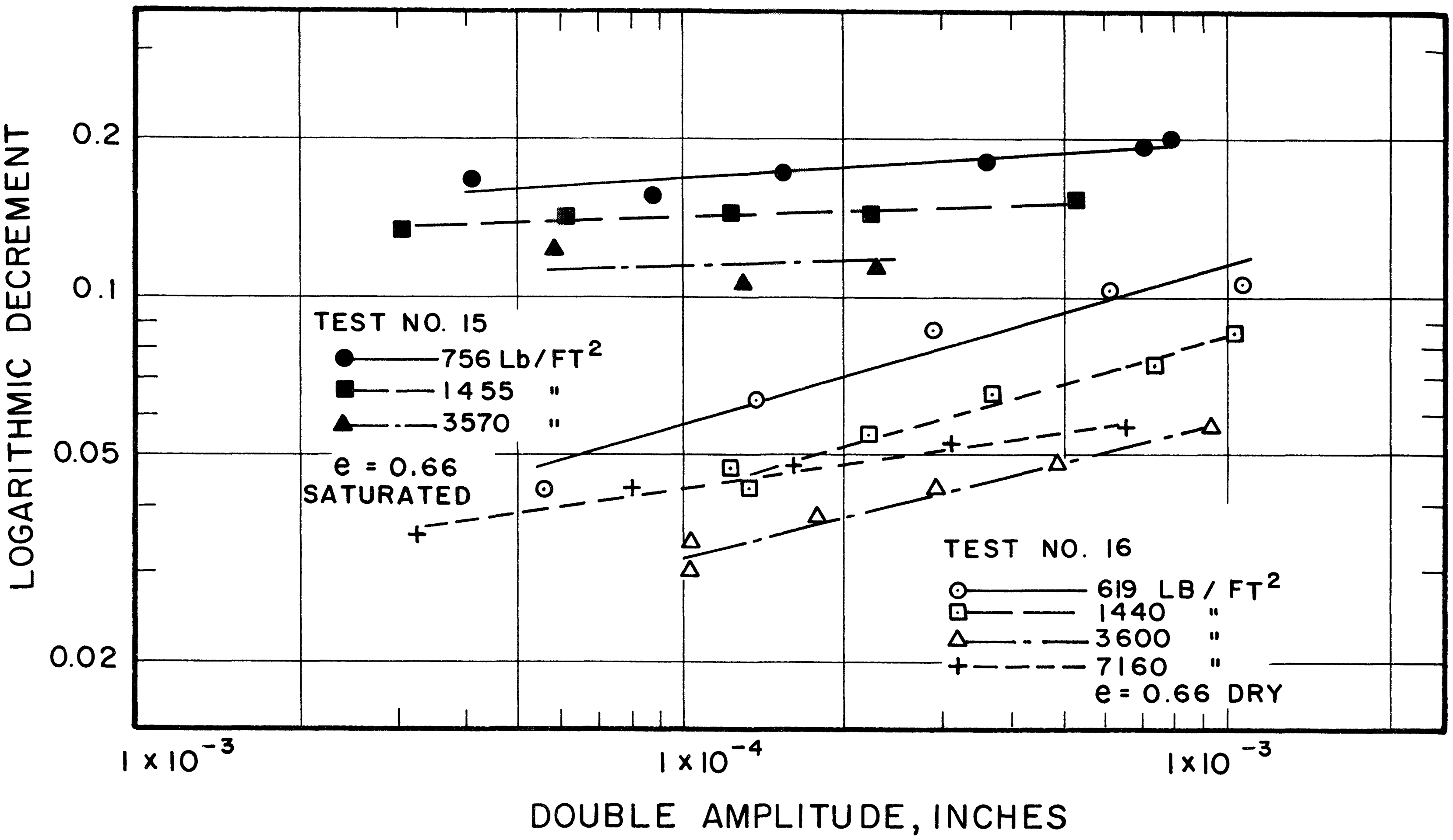


Fig. 10. Comparison of the Variation of Logarithmic Decrement with Amplitude for Dry and Saturated Ottawa Sand in Longitudinal Oscillation.

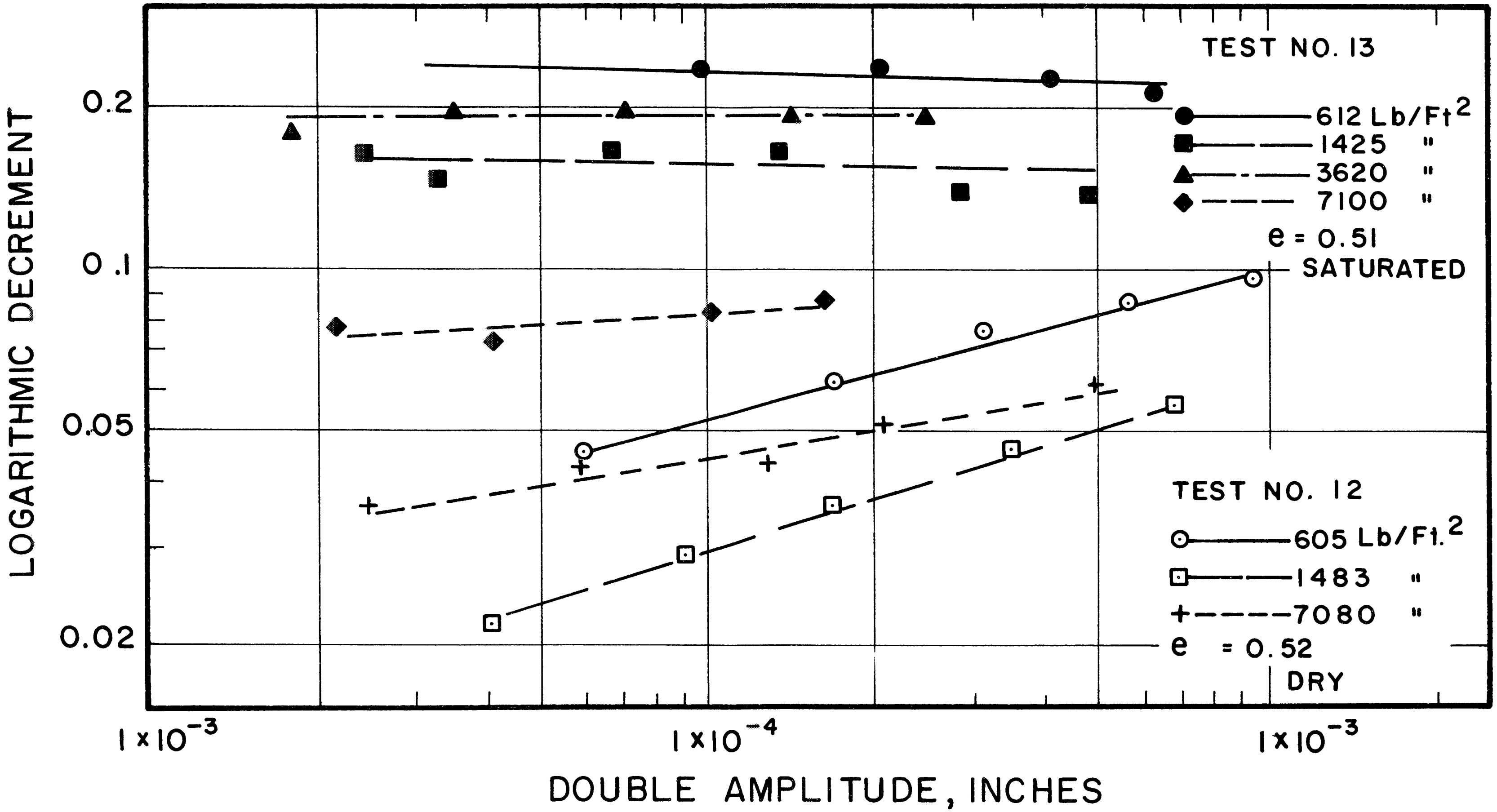


Fig. 11. Comparison of the Variation of Logarithmic Decrement with Amplitude for Dry and Saturated Ottawa Sand in Longitudinal Oscillation.

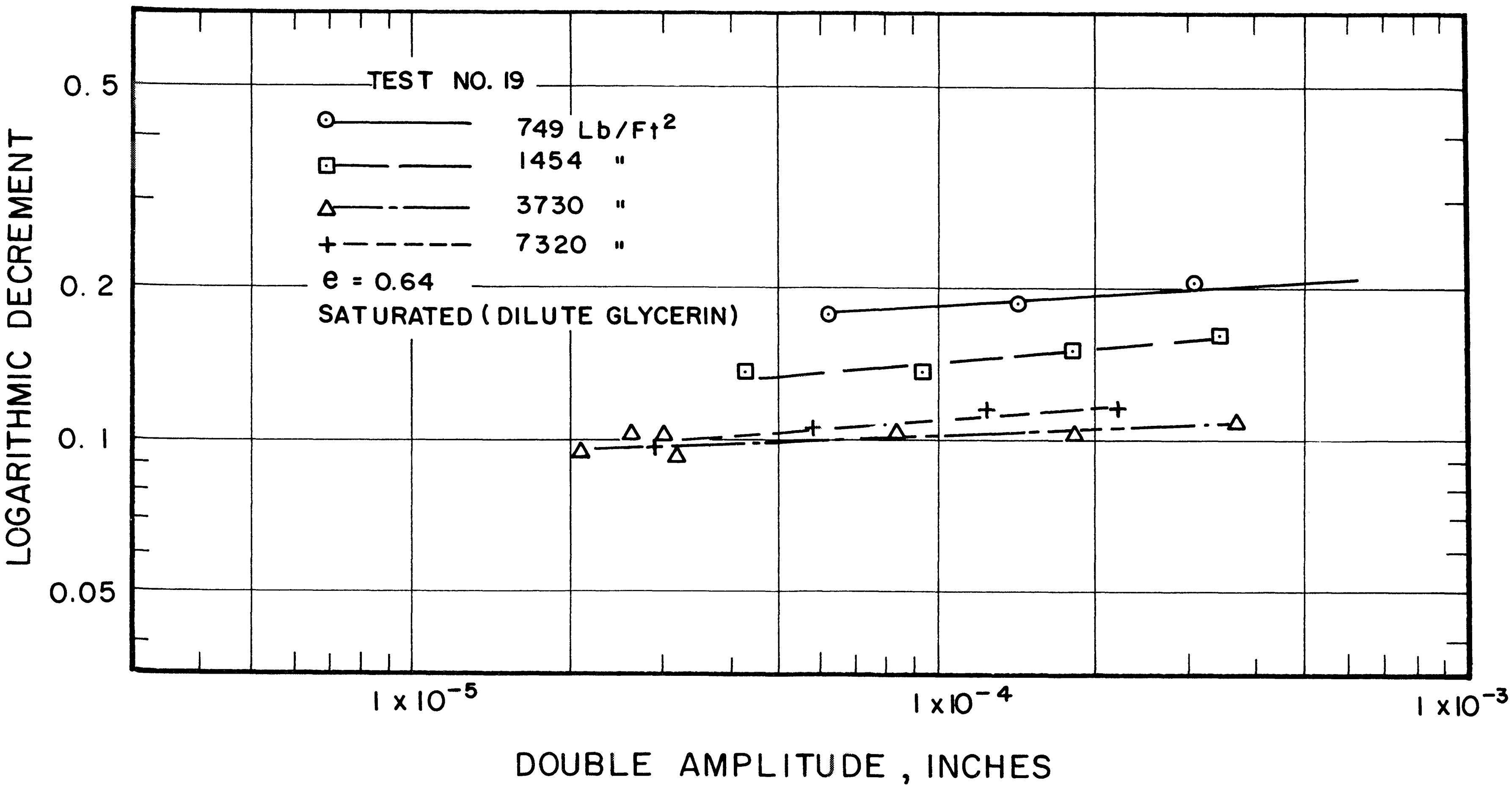


Fig. 12. -Variation of Logarithmic Decrement with Amplitude for Ottawa Sand Saturated with Dilute Glycerin in Longitudinal Oscillation.

Group II. Figures 13 through 15 show the results for damping calculated from the test results for glass beads. These were tested in the dense condition for the dry and saturated case under torsion and compression. The results are plotted showing the variation with pressure and amplitude for the first mode of vibration.

Group III. Figures 16 through 18 show the damping results in torsion for Novaculite No. 1250.

The behavior of this material is quite different than that for Ottawa sand and glass beads due to the very small grain size. The properties of this material depend upon time as well as stress history. During test No. 28, which started at a void ratio of 0.83 under a pressure of 14 lb/in^2, the specimen consolidated to a void ratio of 0.80 after having been subjected to a stress cycle with confining pressures as high as 50 lb/in^2. The stress history of test no. 28 was as follows:

1. The specimen was compacted and placed under a vacuum. Measurements were made for void ratio giving a value of 0.83.
2. The specimen was placed in the triaxial cell and a pressure of 2030 lb/ft^2 was applied. Damping measurements were made intermittently over a period of 38 hr.
3. The pressure was raised to 4100 lb/ft^2. Damping measurements were made intermittently over a period of 140 hr.

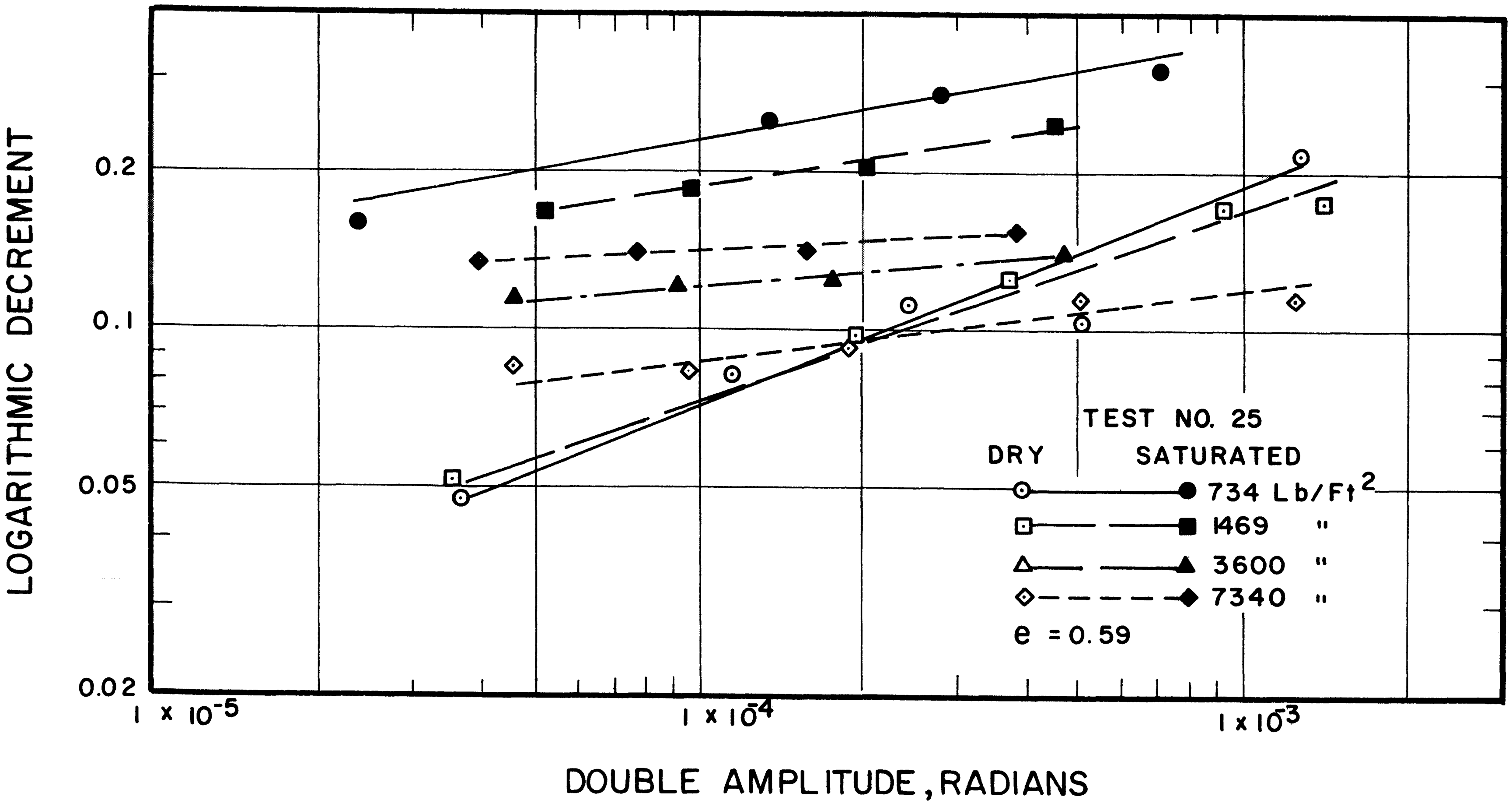


Fig. 13. Variation of Logarithmic Decrement with Amplitude for Glass Beads No. 2847 in the Dry and Saturated Condition in Torsional Oscillation.

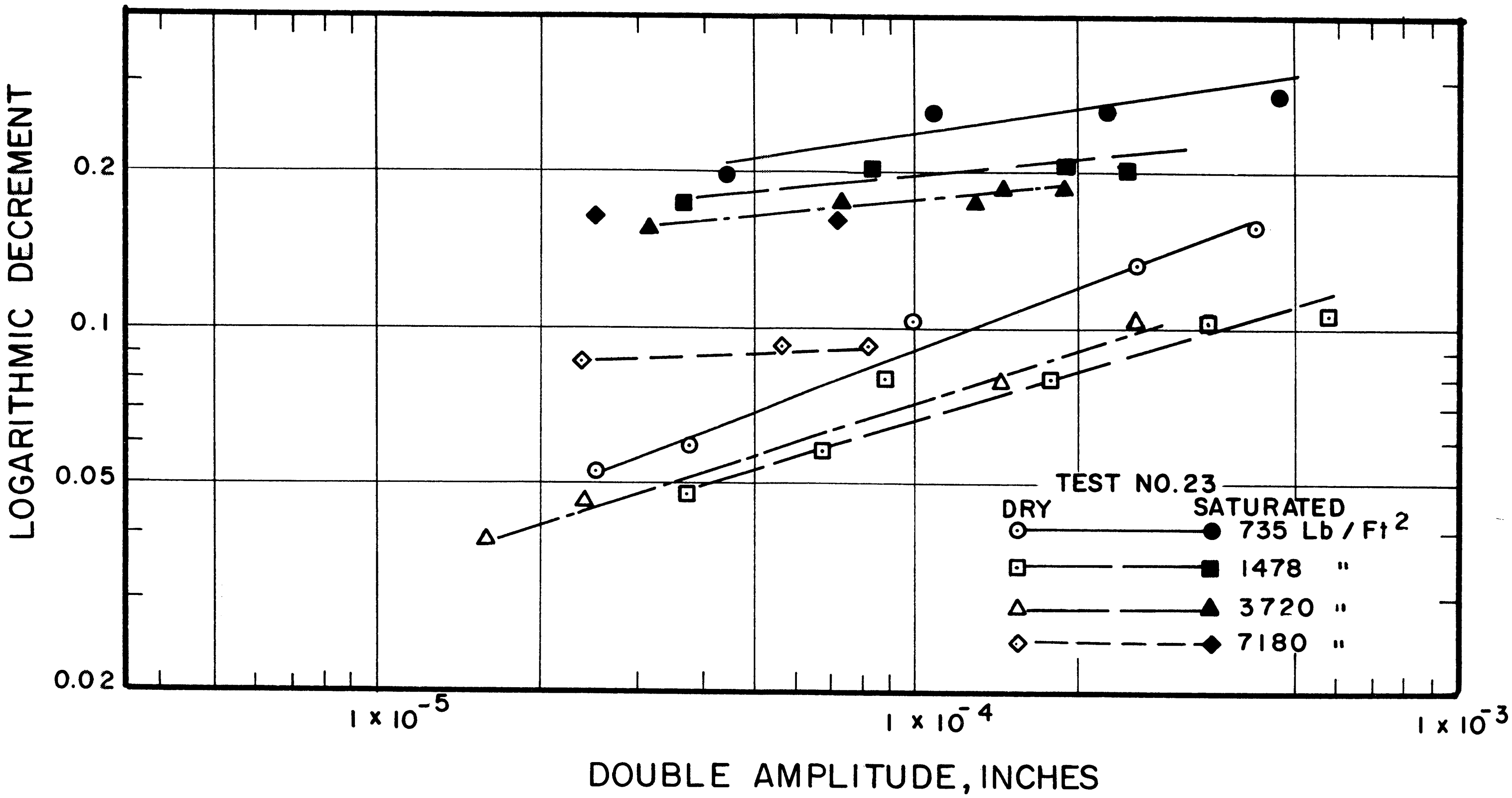


Fig. 14. -Variation of Logarithmic Decrement with Amplitude for Glass Beads No. 2847 in the Dry and Saturated Condition in Longitudinal Oscillation.

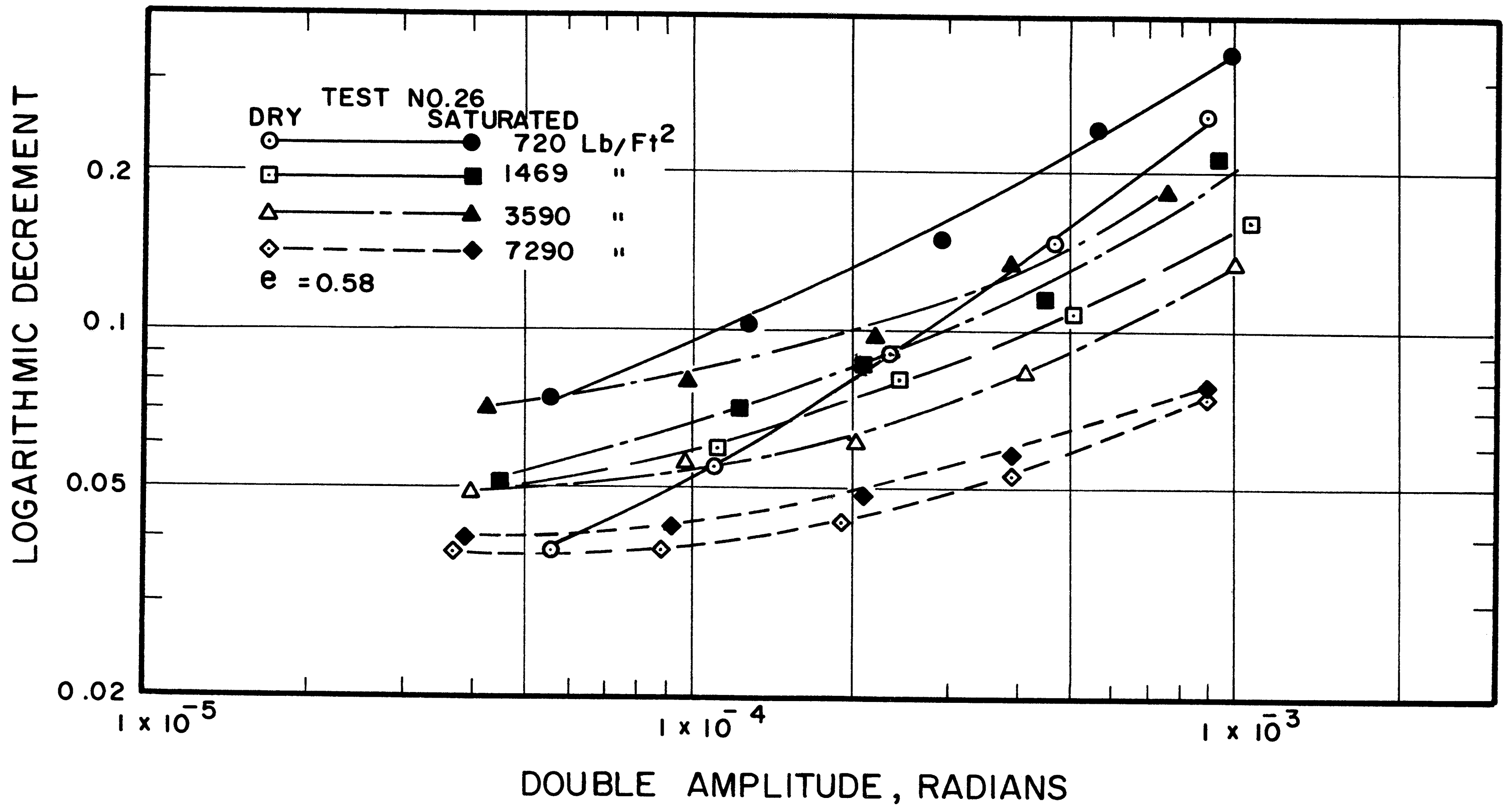


Fig. 15. -Variation of Logarithmic Decrement with Amplitude for Glass Beads No. 1725 in the Dry and Saturated Condition in Torsional Oscillation.

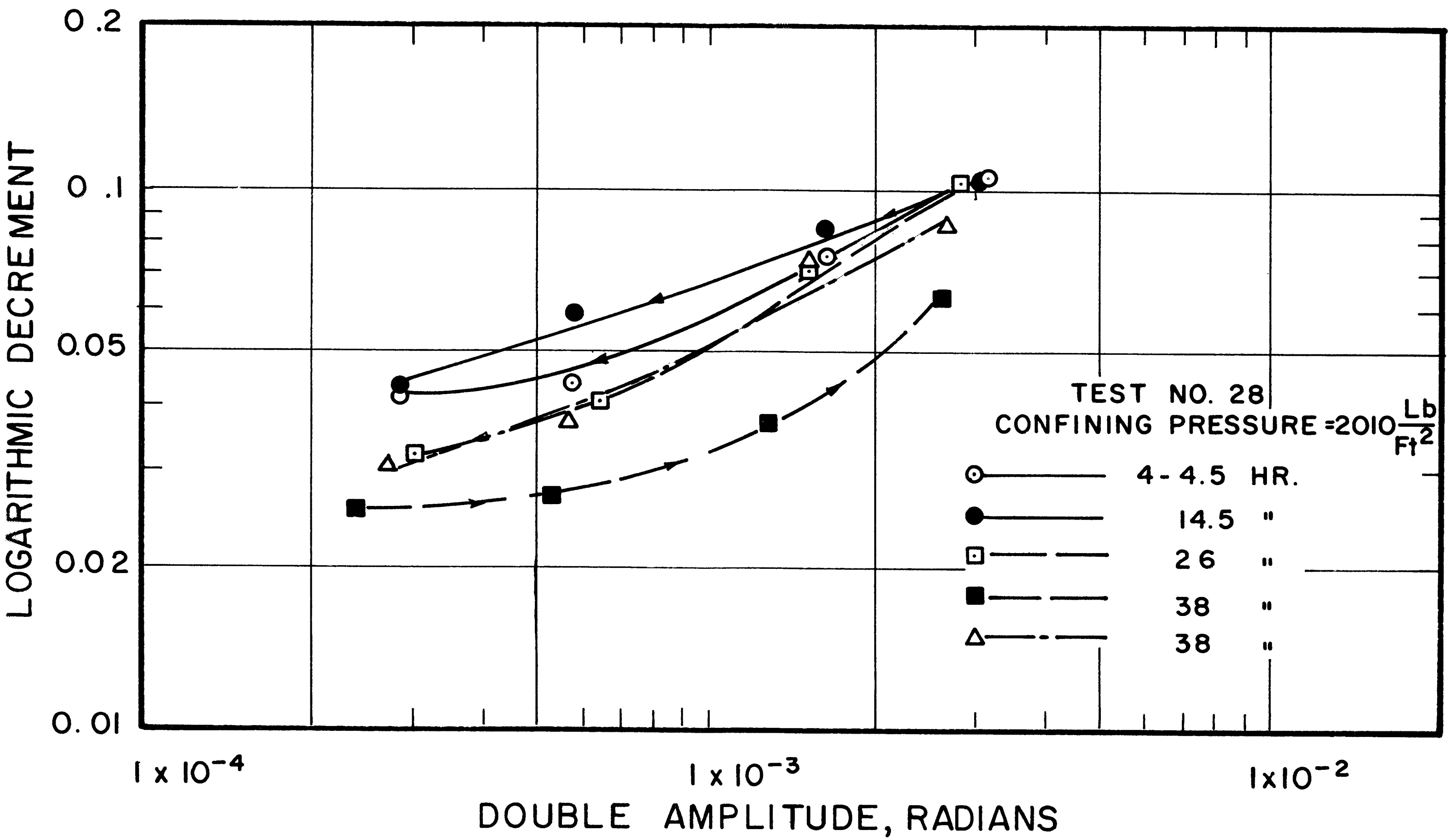


Fig. 16. Variation of Logarithmic Decrement with Amplitude for Novaculite No. 1250 Consolidated to 2010 lb./ft.^{2} in Torsional Oscillation.

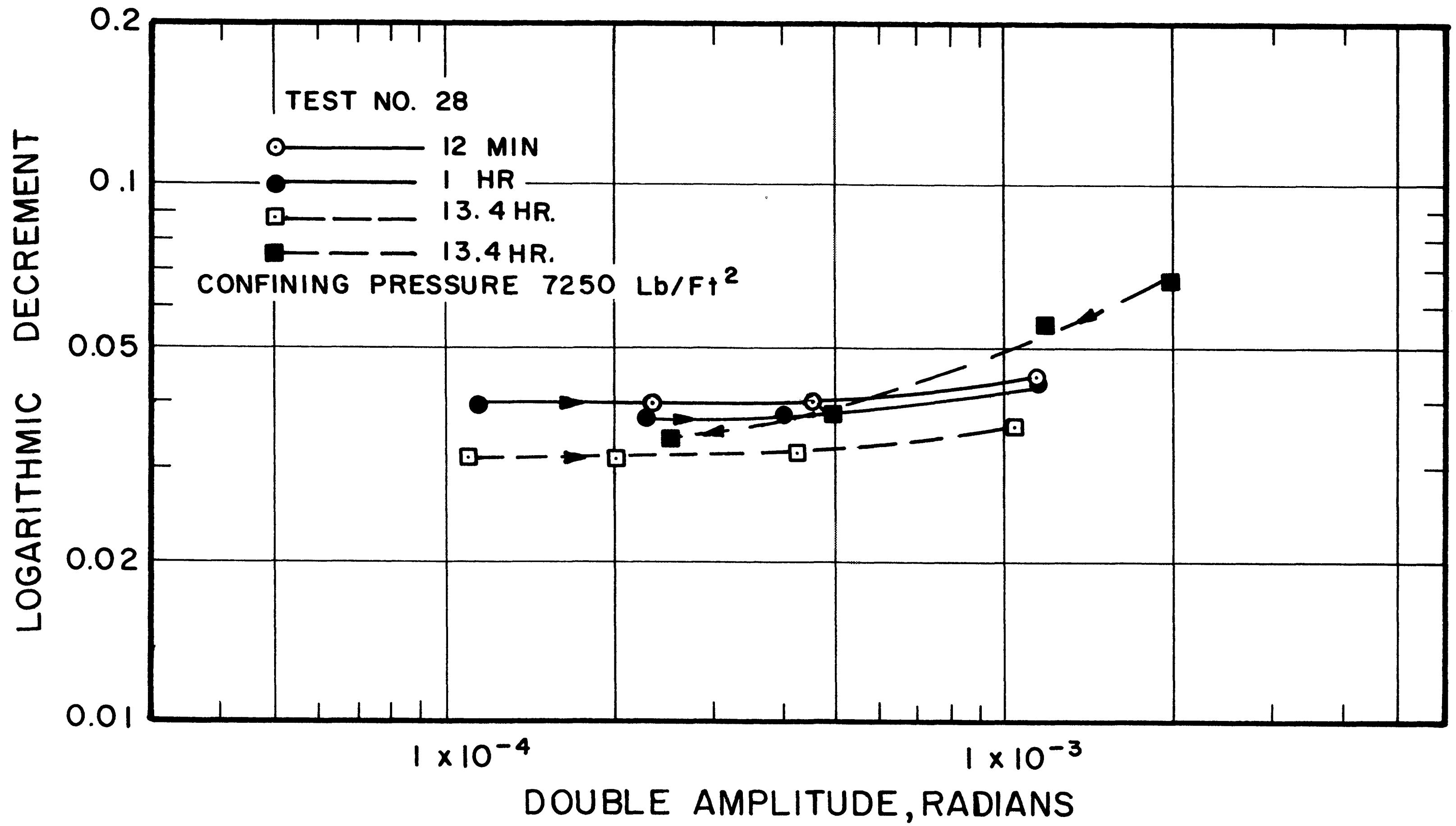


Fig. 17. Variation of Logarithmic Decrement with Amplitude for Novaculite No. 1250 Consolidated to 7250 lb./ft.^{2} in Torsional Oscillation.

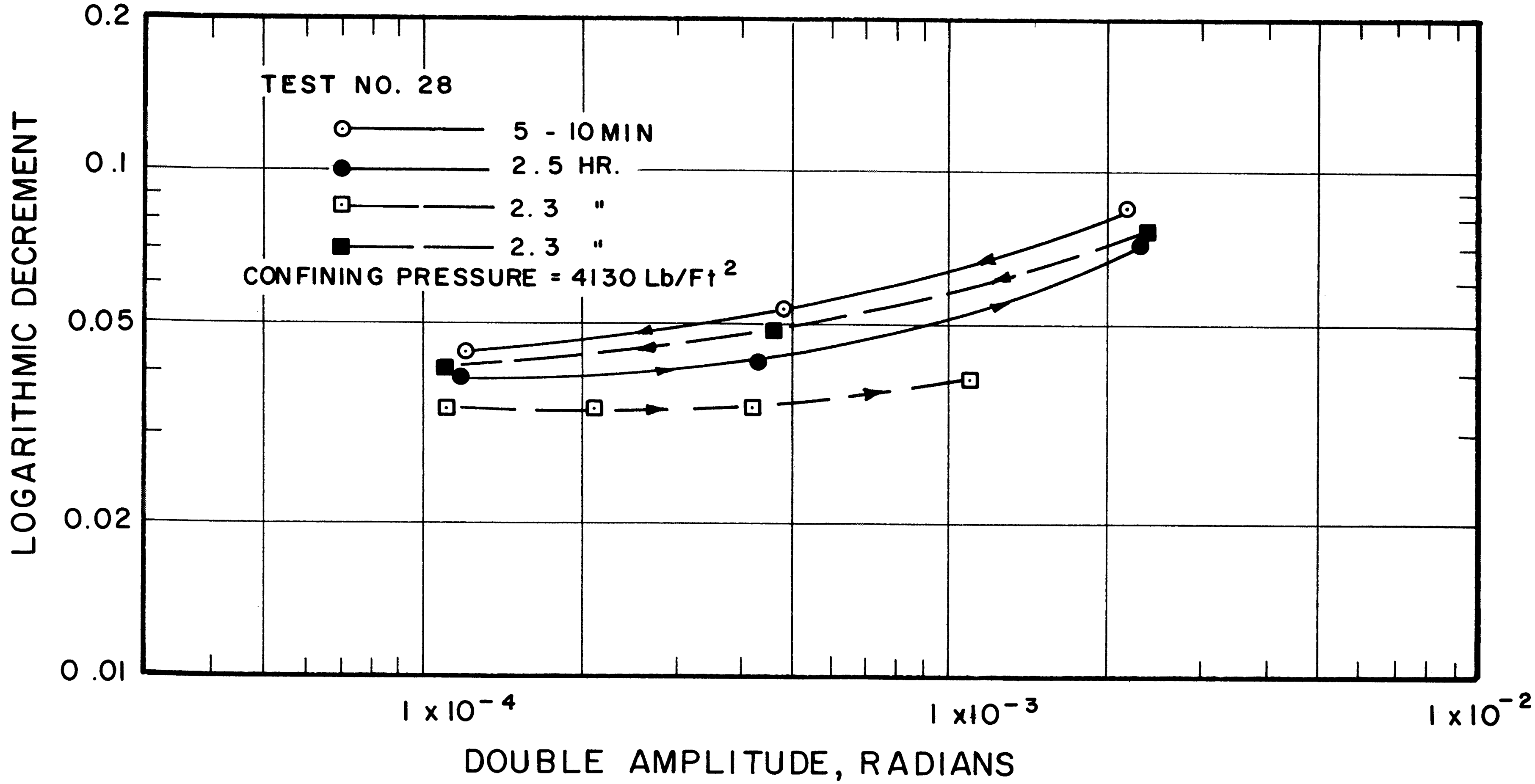


Fig. 18. Variation of Logarithmic Decrement with Amplitude for Novaculite No. 1250 After Rebounding from 7270 lb./ft.^{2} to 4130 lb./ft.^{2} in Torsional Oscillation.

4. The pressure was rebounded to 2050 lb/ft^2 and measurements of damping were made intermittently over a period of 12 hr.
5. The specimen was placed under a vacuum and measurements were made for void ratio which gave a value of 0.81.
6. The specimen was replaced into the triaxial cell and the pressure was raised to 7270 lb/ft^2. Measurements of damping were made intermittently over a period of 13 hr.
7. The pressure was reduced to 4130 lb/ft^2 and measurements of damping were made intermittently over a period of 30 hr.
8. Final measurements under a vacuum gave a value of void ratio equal to 0.80.

The Novaculite properties were not only sensitive to time and stress history but also to vibrations. During the time intervals between measurements the specimen was not vibrated. The first measurements after each time interval were made at low amplitudes of vibration. The following measurements were made at increasing amplitudes until the maximum amplitude obtainable with the equipment was reached. Since the high amplitude vibrations affect the low amplitude measurements a second set of measurements were usually taken after the specimen had been vibrating at high amplitude for a period of approximately five minutes. These two methods of measurement are indicated in the figures by arrows on each curve.

DISCUSSION OF THE RESULTS

Group I.

Effect of amplitude. In general, the logarithmic decrement for the dry Ottawa sand decreases with a decrease of amplitude of vibration. For the first mode of vibration the average variation of logarithmic decrement is with the 0.25 power of amplitude in both cases. Individual values vary from 0.16 to 0.34. When the Ottawa sand is saturated with water the variation of logarithmic decrement with amplitude is decreased. For the torsional motion the logarithmic decrement varies between the 0.0 and the 0.13 power of amplitude. The longitudinal motion shows practically no variation of logarithmic decreement with amplitude in the saturated condition. Figure 12 shows the results when dilute glycerin was used as the pore fluid. The results are very much the same as those for the water saturated condition in that there is practically no variation of logarithmic decrement with amplitude of vibration.

Effect of confining pressure. The curves generally show that the logarithmic decrement decreases as the confining pressure is increased. However, in some cases the damping increases when the confining pressure is increased. Also, since the curves of logarithmic decrement vs. amplitude are not parallel for the different confining pressures this indicates that the pressure variation depends upon the amplitude of vibration.

The inconsistency of the results is such that the only definite observation is that the damping tends to decrease with an increase of confining pressure.

Effect of density. The effect of the specimen being loose or dense is rather small. Figures 10 and 11 show the results for the longitudinal motion in the loose and dense conditions. Any differences in results between the loose and dense conditions for the dry tests are too small to be detected. The effect of amplitude is much more significant than any effects due to differences in density.

Effect of pore fluid. Figures 9, 10 and 11 compare the differences between dry and water saturated Ottawa sand. The effect of the water apparently depends upon the amplitude of vibration since the slopes for the dry condition are greater than the slopes for the saturated condition. Over the range of amplitudes measured, the water increases the logarithmic decrement by a factor of 1.5 to 4 times that for the dry condition. Comparison of Figs. 10 and 12 shows that there is practically no difference between the tests in which the specimen is saturated with water and with dilute glycerin. The glycerin solution was composed of 3 parts water to 1 part glycerin. Pure glycerin was not used because of the fact that its viscosity is so high that an unreasonable length of time is required to saturage the specimen with it.

Group II.

Effect of amplitude. The results of damping for the glass beads are shown in Figs. 13 through 15. For the glass beads No. 2847 the average variation of logarithmic decrement in the dry condition is with about the 0.38 power of amplitude. In the saturated condition the damping varies with about the 0.15 power of amplitude. The glass beads No. 1725 behave somewhat differently than the other beads or Ottawa sand. In Fig. 15 it can be seen that as the amplitude of vibration is decreased the damping becomes less dependent upon amplitude. At higher amplitudes the logarithmic decrement varies with about the 0.54 power of amplitude in the dry condition and with approximately the 0.47 power of amplitude in the saturated condition. It seems that the variation of damping with amplitude for the dry material is affected by the type of grain surface. The glass beads have a very smooth surface compared to the surface of Ottawa sand. It would be difficult to determine from the data whether the difference between the large and small glass beads is due to the size effect or the difference in density. An increase in density should tend to cause a decrease in the logarithmic decrement because of the increased mass. Comparison with the two types of beads shows that a smaller amount of damping is associated with the higher density beads.

Effect of confining pressure. As with the Ottawa sand the variations of damping with confining pressure are such that the only observation that can be made is that there is generally a decrease in damping with confining pressure.

Effect of pore fluid. The saturation of the glass beads No. 2847 has the same effect as it did in the Ottawa sand. The values for damping were increased and the variation of damping with amplitude was reduced, indicating that the amount of damping contributed by the water increases at smaller amplitudes. Figure 15 shows that for the glass beads No. 1725 the amount of damping contributed by the water is also greater at smaller amplitudes but to a much smaller extent. This difference is most likely due to the higher specific gravity of the beads in comparison to that of the water.

Group III.

Effect of amplitude. The variation of logarithmic decrement with amplitude for this material does not plot as a straight line on a log-log scale. At double amplitudes below about 3×10^{-4} rad. the logarithmic decrement does not vary much with amplitude. At higher amplitudes there is a significant variation of damping with amplitude. As the specimen was allowed to stand under a given confining pressure the damping decreased. If values of damping were then measured starting with low amplitudes and increasing the amplitudes until measurements were finally made at the highest attainable with the equipment, a curve corresponding to the lowest curve in Fig. 16 was obtained. If the specimen was then allowed to vibrate at a high amplitude over a period of approximately 5 minutes, then the curve corresponding to the triangular points was obtained. Thus, the time effect which resulted in a decrease in damping could be destroyed by vibrations of high amplitude.

Effect of confining pressure. When the confining pressure was increased after having been maintained at a steady value over a long period of time, the damping increased. After a new pressure was reached the damping started to decrease with time. If high amplitude vibrations were applied to the specimen, the decrease of damping that occurred over a period of time could be destroyed. If the values of damping are compared at low amplitudes for the different confining pressures, it can be seen that time is more significant as a variable than confining pressure. The values of damping for low amplitudes at each confining pressure are all within the same order of magnitude.

CONCLUSIONS

The conclusions obtained from this study necessarily apply to granular soils which have been subjected to several load repetitions and have reached a relatively stable condition. This corresponds to construction conditions where the soil has been pre-vibrated or pre-compacted to eliminate the disastrous settlements which may accompany the first dynamic load application on loose granular soils.

The tests on the Ottawa sand and glass beads gave results which should be typical for clean sands with rounded grains. The more important conclusions are listed below:

1. For dry Ottawa sand the logarithmic decrement varies with about the 0.25 power of amplitude. Saturated Ottawa sand shows little variation of logarithmic decrement with amplitude. Therefore, the proportion of the total damping contributed by the water apparently increases as the amplitude decreases. The variation of logarithmic decrement with amplitude for glass beads No. 2847 is with the 0.38 power for the dry condition and with the 0.15 power when saturated with water.

2. The damping determined from the decay of steady state vibrations in samples of rounded granular materials behaved like viscous damping. The values of logarithmic decrement

varied from 0.02 to about 0.20 for the various materials and test conditions used.

Dynamic tests on the Novaculite No. 1250, a very fine-grained crushed quartz, produced results which were somewhat different from those obtained from the larger grained materials. The primary difference is that the damping values obtained from laboratory tests are dependent upon the stress history and upon the time the loading has been applied. The damping decreases slightly as a particular confining pressure continues to be applied to a specimen. However, it was also found that higher amplitudes of vibration tend to destroy this time-dependent decrease in damping. Further investigations are required to evaluate the time-dependent decrease of damping and the vibrational energy required to destroy this decrease.

For comparable conditions of confining pressure and amplitude of vibrations, the Novaculate No. 1250 has a significantly lower value of logarithmic decrement than that for the Ottawa sand. This is in line with the test results for the glass beads No. 1725 which are considerably lower than for the larger diameter glass beads or Ottawa sand. However, some of the difference between the large and small glass beads is due to the difference in specific gravity as mentioned previously. Thus the value of logarithmic decrement seems to decrease as the average grain size decreases.

Finally, it should be re-emphasized that the tests described herein are concerned with the damping in granular materials which are in a stable condition. Because the deformations are recoverable when the stresses are below about 20 per cent of the failure stress, the behavior of the material in this range has been termed "elastic" although damping is definitely present. The order of magnitude of the logarithmic decrement is generally below 0.2 for the test conditions used. This defines a value of $C/C_{cr} \approx 0.03$ if a steady state response is to be considered. The damping due to dispersion of elastic waves in an ideal medium can also be estimated by procedures described in the paper. A value of R_v/R_{vc} (corresponding to C/C_{cr}) can be expressed approximately by

$$\frac{R_v}{R_{vc}} \approx \frac{0.91}{\sqrt{b}} \qquad (\text{for } \mu = 0.25)$$

where $b = M_o/\rho r_o^3$ for the oscillating footing. Thus a comparison can be made between the material damping and dispersion damping for a given foundation system.

APPENDIX. - NOTATION

a_o = dimensionless frequency factor defined by Eq. (5);

A = amplitude of vibration;

A_{MAX} = amplitude of vibration at resonant frequency;

b = dimensionless mass ratio defined by Eq. (16);

c = viscous damping coefficient;

c_{cr} = critical viscous damping coefficient defined by Eq. (14);

e = void ratio;

E_σ = strain energy at maximum stress;

E_n = strain energy available at the nth cycle of vibration;

ΔE_σ = energy absorbed in one cycle of vibration;

f = frequency of vibration;

f_o = resonant frequency for forced vibration;

f_1, f_2 = variables in the Reissner-Sung theory which are functions of the frequency of vibration;

f_L, f_H = low and high frequencys of oscillation at equal amplitudes of oscillation;

F_1, F_2 = frequency functions defined by Eqs. (6);

g = acceleration of gravity;

G = shear modulus of elasticity;

i = $\sqrt{-1}$;

I = mass polar moment of inertia of the specimen;

I_o = mass polar moment of inertia of mass at free end of the specimen;

k = spring constant;

k_n = proportionality constant;

K_v = coefficient defined by Eq. (9b);

L' = length of the specimen;

L_s = wave length of shear wave;

m = mass of specimen;

m_o' = mass attached to free end of specimen;

m_o = total static mass of footing;

P = vertical periodic force at the base of the footing;

Q = reciprocal of the specific dissipation function;

Q_z = periodic exciting force;

r = radial distance;

r_o = radius of circular footing;

R_v = coefficient defined by Eq. (9a);

R_{vc} = coefficient defined by Eq. (14);

t = time;

T = period of vibration;

v = phase velocity;

v_s = velocity of shear wave;

V_o = initial velocity of vibration;

w = vertical displacement of footing;

W_o = weight of footing;

X = amplitude of vibration;

X_n = amplitude of nth vibration;

Z = periodic force amplitude;

α = coefficient of attenuation;

β = dimensionless frequency correction factor;

γ = unit weight;

δ = logarithmic decrement;

ρ = mass density;

ψ = specific damping capacity;

μ = Poisson's ratio; and

ω = circular frequency.

TABLE 1

SUMMARY OF TESTS

Group	Test No.	Material	Void Ratio	Pore Fluid	Type
I	10	Ottawa sand	0.52	Air	Torsion
	11	Ottawa sand	0.67	Air	Torsion
	14	Ottawa sand	0.52	Water	Torsion
	21	Ottawa sand	0.64	Water	Torsion
	12	Ottawa sand	0.52	Air	Compression
	16	Ottawa sand	0.66	Air	Compression
	13	Ottawa sand	0.51	Water	Compression
	15	Ottawa sand	0.66	Water	Compression
	20	Ottawa sand	0.50	Dil. glycerin	Compression
	19	Ottawa sand	0.64	Dil. glycerin	Compression
II	25	Beads #2847	0.59	Air	Torsion
				Water	Torsion
	26	Beads #1725	0.58	Air	Torsion
				Water	Torsion
	23	Beads #2847	0.58	Air	Compression
				Water	Compression
	24	Beads #1725	0.58	Air	Compression
III	28	Novaculite	0.80 0.83	Air	Torsion

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