THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

EFFECT OF VIBRATION AMPLITUDE ON WAVE VELOCITIES IN GRANULAR MATERIALS

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INTRODUCTION

The dynamic properties of a soil can be determined from measurements of the propagation of stress waves through the soil. In order to estimate these properties as they occur in the field it is necessary to know the relative importance of the variables which may exist. Some studies have been made to determine the effects of confining pressure, relative density, or per cent saturation on the velocities of wave propagation and damping in soils, for example, the papers by K. Iida[1,2], G. Shumway[3], and B. O. Hardin and F. E. Richart[4]. However, there are several other variables which may have a significant influence on test results.

S. D. Wilson and E. A. Sibley[5] have shown comparisons between values of the constrained modulus of elasticity as determined from seismic tests, laboratory vibration tests, and static confined compression tests. The amplitudes of deformation differ by orders of magnitude in these three testing methods. The smallest deformations are associated with the seismic tests while relatively large deformations are necessary to insure accuracy of measurement in the confined compression tests. The results obtained by Wilson and Sibley show that the seismic determination gives the highest value of constrained modulus while the confined compression tests give the lowest values.

The primary object of this investigation was to determine the relative importance of the effect of amplitude of vibration on the wave velocity. The resonant column method was used to determine the velocity of the shear and longitudinal waves in a column of soil 11 inches long which was fixed at one end and free at the other. All test results are related to the amplitude at the free end of the specimen. This amplitude varied from about $2x10^{-6}$ to $1x10^{-3}$ inch double amplitude in longitudinal oscillation and from about $1x10^{-5}$ to $2.5x10^{-3}$ radians double amplitude in torsional oscillation.

THEORIES USED IN THE EXPERIMENTAL DETERMINATION OF WAVE VELOCITY

The wave velocity through a column of soil was determined from the resonant frequency of the column when excited into longitudinal or torsional oscillation. The resonant frequency is that frequency which produced the maximum amplitude of oscillation at the upper, free end of the specimen. For this type of test equipment, the resonant frequency, f, for the first mode of vibration is associated with a wave length which is four times the length of the specimen. Thus, by designating the specimen length as L' and the wave length as L, the wave velocity may be calculated from the relationship

$$v = fL = 4fL' \tag{1}$$

From the computed values of shear or longitudinal wave velocities, the shear modulus and longitudinal modulus of elasticity may be evaluated from

$$v_L = \sqrt{\frac{E}{\rho}}, \tag{2}$$

and

$$v_S = \sqrt{\frac{G}{\rho}}. \tag{3}$$

In Equations (2) and (3), $\rho = \gamma/g$ is the mass density of the material, E is the dynamic longitudinal modulus of elasticity, G is the dynamic shear modulus, v_L is the longitudinal wave velocity, and v_S is the shear wave velocity.

The specimens used in this research were fixed at the base and the driver and pickup were fastened to the top free end. Therefore a correction was applied to correct for the added mass at the end of the specimen. The solution governing the natural frequency of such a system under torsional vibrations is given by

$$\omega L'\sqrt{\frac{\rho}{G}}\ \tan \omega L'\sqrt{\frac{\rho}{G}} = \frac{I}{I_o} \qquad (4)$$

where I_o is the mass polar moment of inertia of the mass attached to the free end, I is the Mass polar moment of inertia of the specimen of length L', and ω is the circular frequency $(2\pi f)$. Equation (4) must be solved graphically or by trial and error. It is convenient to put Equation (4) into the form

$$\beta \tan \beta = \frac{I}{I_o} \qquad (5)$$

Thus,

$$v = \frac{2\pi f L'}{\beta} \qquad (6)$$

LABORATORY TESTS OF VELOCITY IN GRANULAR MATERIALS

Materials

Four different materials were used in this investigation. Each is described below and the grain size curve for each is shown in Figure 1.

Ottawa sand. Standard Ottawa sand passing the No. 20 sieve and retained on the No. 30 sieve was used for most of the investigation. This material has a minimum void ratio of 0.50 and a maximum void ratio of 0.77. It has a specific gravity of 2.67.

Glass beads No. 2847. Glass beads, all of which lie between the No. 16 and No. 20 sieve, were obtained from the Prismo Safety Corporation, Huntingdon, Pennsylvania. These beads appear to be perfect spheres when examined under a microscope. They have a specific gravity of 2.50, a minimum void ratio of 0.57 and a maximum void ratio of 0.75.

Glass beads No. 1725. This material was also obtained from the Prismo Safety Corporation. Ninety-five per cent pass the No. 200 sieve and 96 per cent are retained on the No. 400 sieve. They have a high specific gravity of 4.31 resulting from the requirement of a high index of refraction for their commercial use. The minimum void ratio for this material is 0.57 and the maximum void ratio is 0.76.

Novaculite No. 1250. This is a very find quartz powder obtained from the American Graded Sand Co., 189-203 East Seventh Street, Paterson 4, New Jersey. This material was considered to be a silt as shown by the grain size curve in Figure 1.

Equipment

Two pieces of equipment were specially designed and built to vibrate the specimen at relatively large amplitudes in the longitudinal and

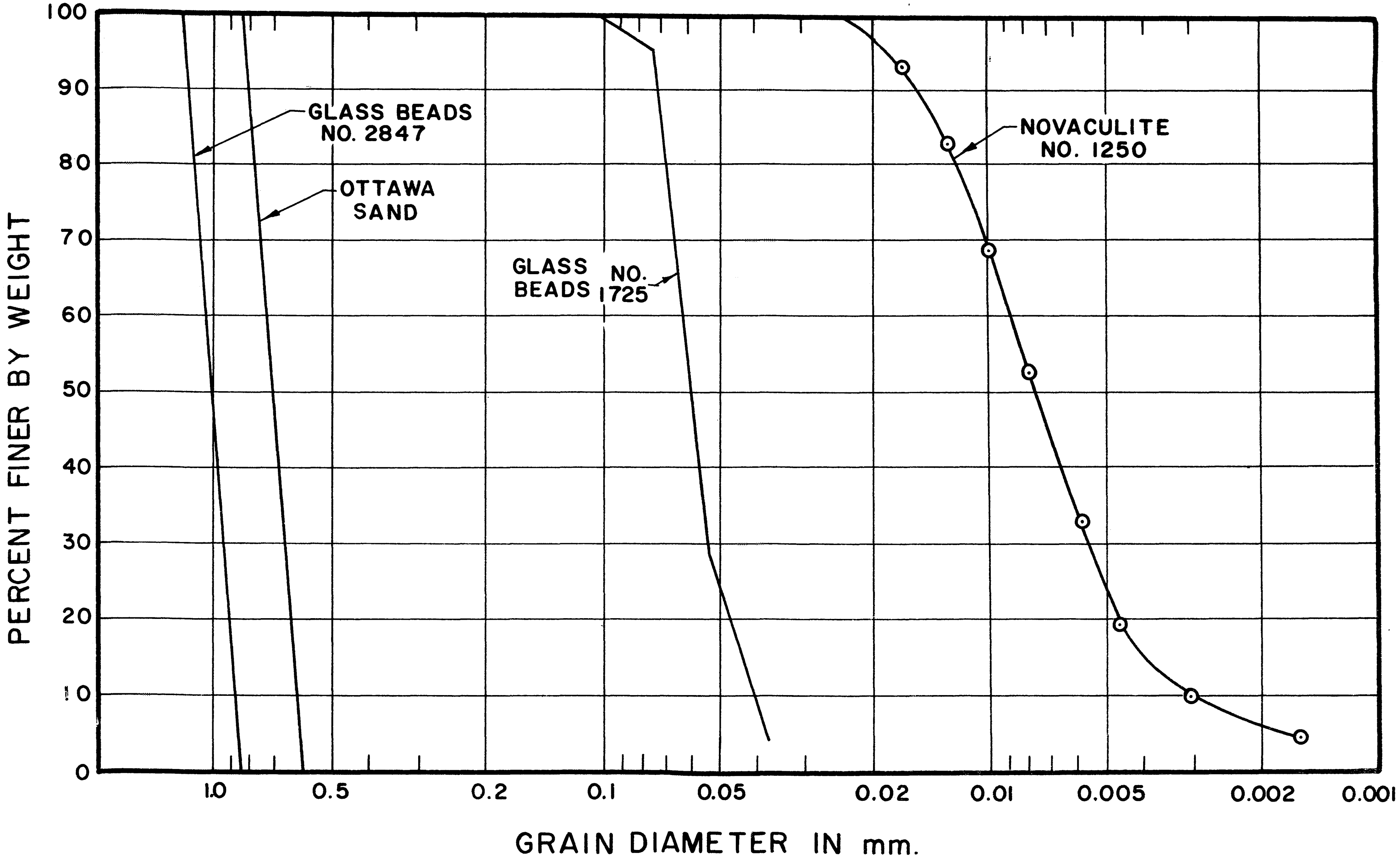


Fig. 1. Grain Size Curves for the Materials Used in the Present Research.

torsional modes. Each was constructed so that one end of the specimen was free and the other was fixed as shown in Figures 2 and 3. Both pieces of equipment are basically the same except for the driver and the pickup as shown in Figure 2. The frames were made from a piece of 4 inch steel pipe with lead attached for added mass to give a total weight of about 30 lbs. for each apparatus. This is necessary in order to reduce the movement of the "fixed" end to a negligible amount.

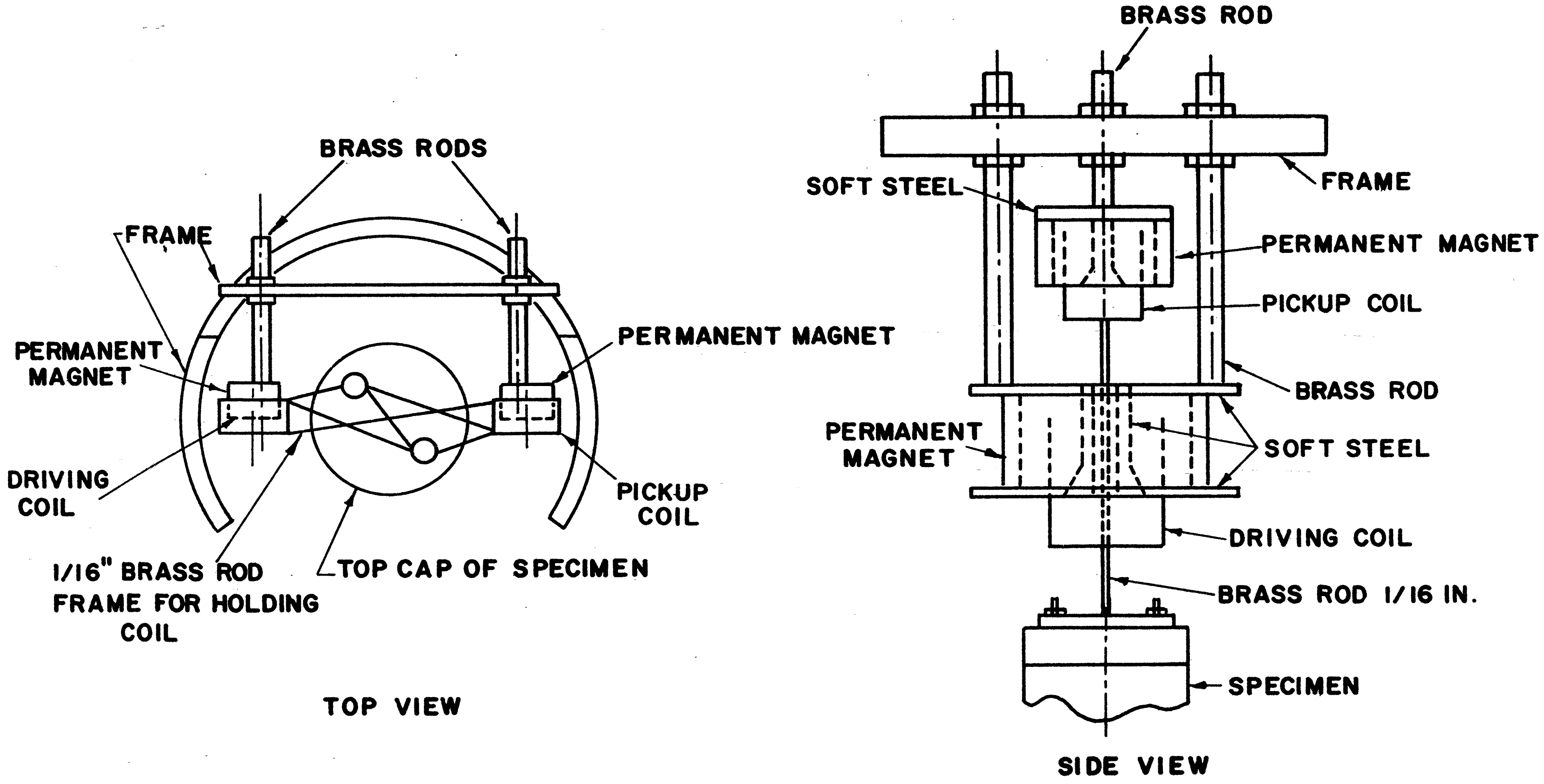
Power to the driving coil was supplied through an amplifier connected to an oscillator in the MB Electronics Type P11 power supply. Pickups were calibrated with an MB Electronics Model C31 calibrator and also with an MB Electronics Type 115 vibration pickup. A Tektronix Model 502 dual beam oscilloscope was used for the measurement of output from the pickups and also for monitoring the input to the driver. A more detailed description of the equipment and testing procedures was given by J. R. Hall[6].

Summary of Tests

Three groups of tests were run and are summarized as follows:

Group I. These tests used specimens of Ottawa sand to obtain data on the effect of amplitude on wave velocity. The test variables included the confining pressure, pore fluid (air, water, and dilute glycerin), and the relative density of the material.

Group II. After tests of Group I were completed, tests were run on samples of each of the two sizes of glass beads in the dense condition both dry and saturated.



(a) Torsional Vibration Apparatus.

(b) Longitudinal Vibration Apparatus.

Fig. 2. Vibration Mechanisms Used in the Present Research.

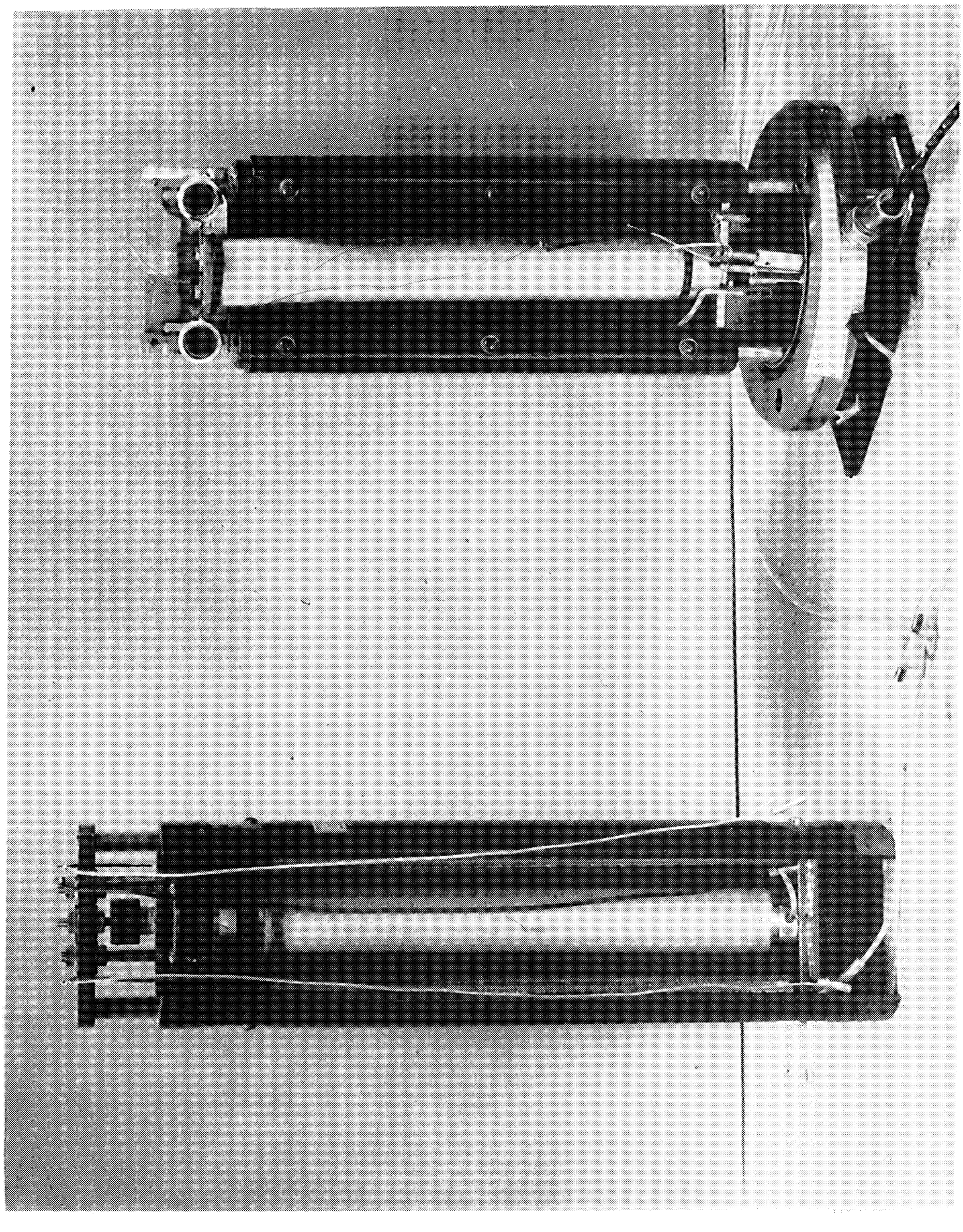


Fig. 3. Fixed-free vibration equipment.

Group III. A torsional vibration test was run to determine the variation of velocity with amplitude for a specimen of Novaculite No. 1250 in the dry condition.

Results of Velocity

Group I. Figures 4 through 6 are representative of the results for velocity in torsional and longitudinal oscillation as calculated from the tests of Group I on Ottawa sand. In these tests the variation of velocity with amplitude of vibration was determined for various confining pressures, density, and pore fluids (air, water, and dilute glycerin). The confining pressures chosen for each test were approximately 5, 10, 25, and 50 lb/in^2 and the results at each pressure are plotted in the same figure. Tests were run using the sand in both the loose and dense conditions corresponding to void ratios of approximately 0.65 and 0.51, respectively. For saturation with dilute glycerin, the mixture was 3 parts water to 1 part glycerin. Pure glycerin was not used because its viscosity is so high that an unreasonable length of time is required to saturate the specimen.

Figure 4 shows the variation of velocity with amplitude for dense Ottawa sand, dry and saturated with water, in torsional oscillation. Figure 5 shows the variation of velocity with amplitude for loose Ottawa sand, dry and saturated with water, in longitudinal oscillation. The results for the specimen in the loose and dense conditions are essentially the same except for the change in velocity due to a change in void ratio, therefore curves for torsional oscillation of loose Ottawa sand and

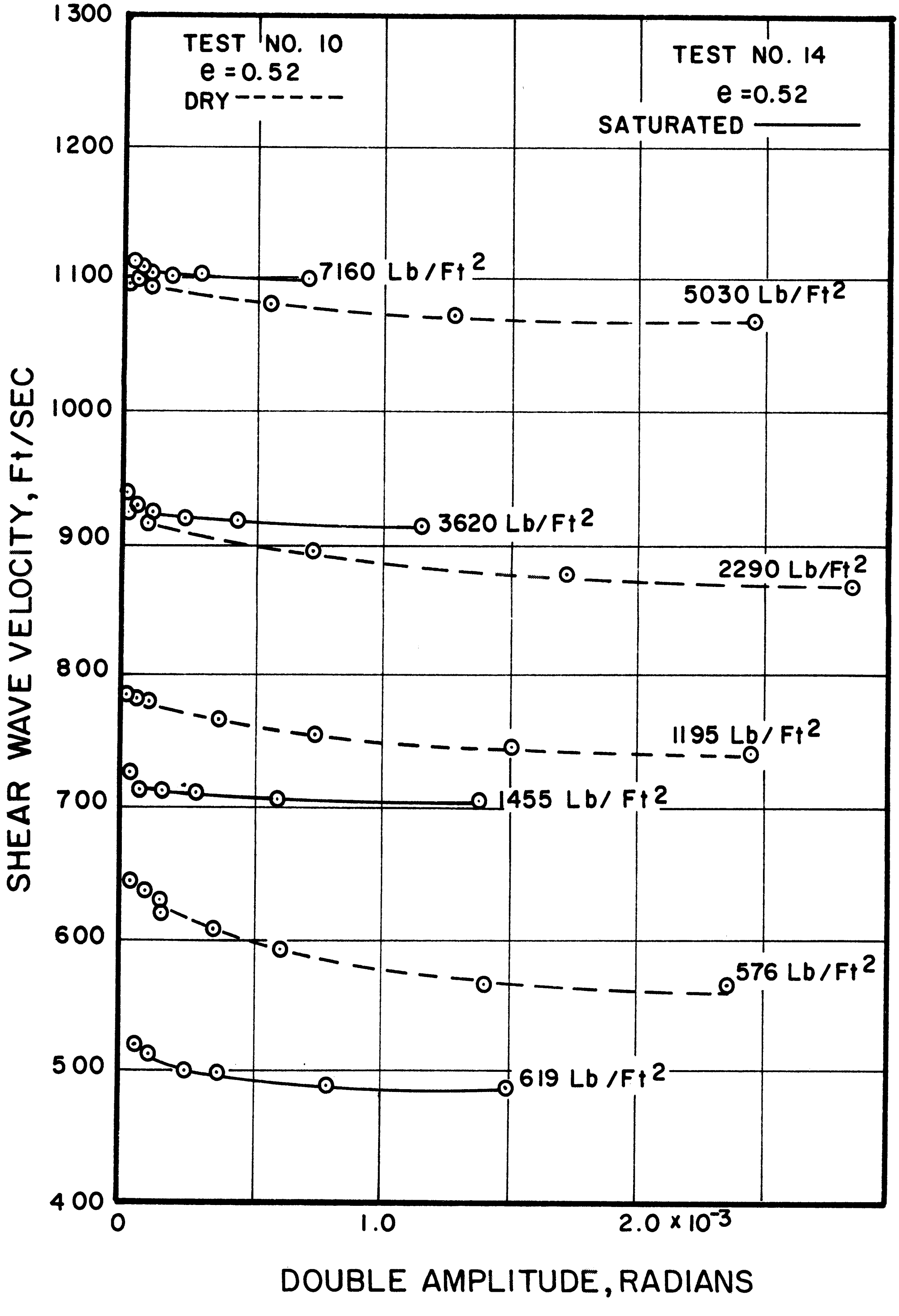


Fig. 4. Variation of Velocity with Amplitude in Torsional Oscillation for Ottawa Sand Dry and Saturated with Water.

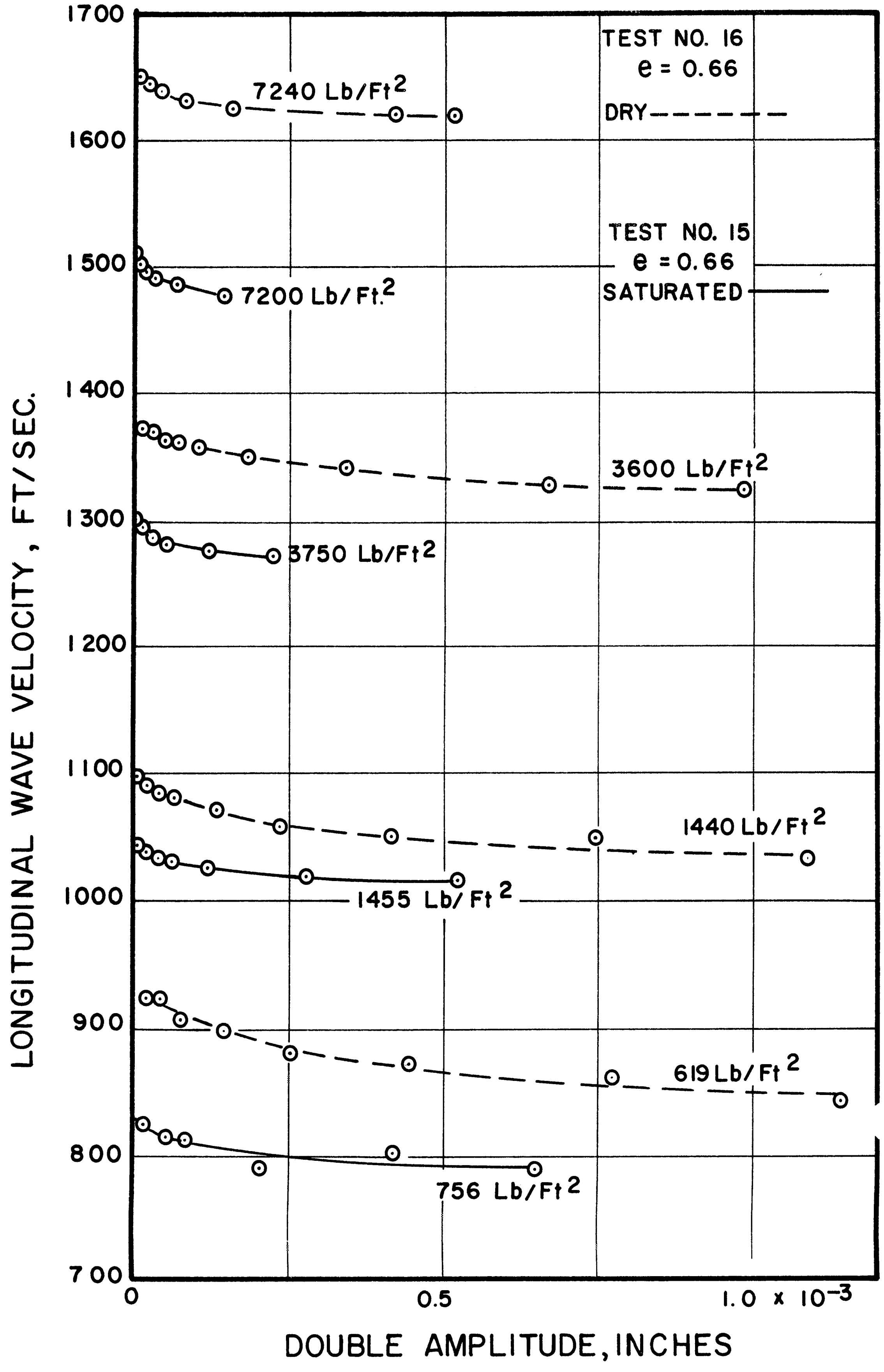


Fig. 5. Variation of Velocity with Amplitude in Longitudinal Oscillation for Ottawa Sand Dry and Saturated with Water.

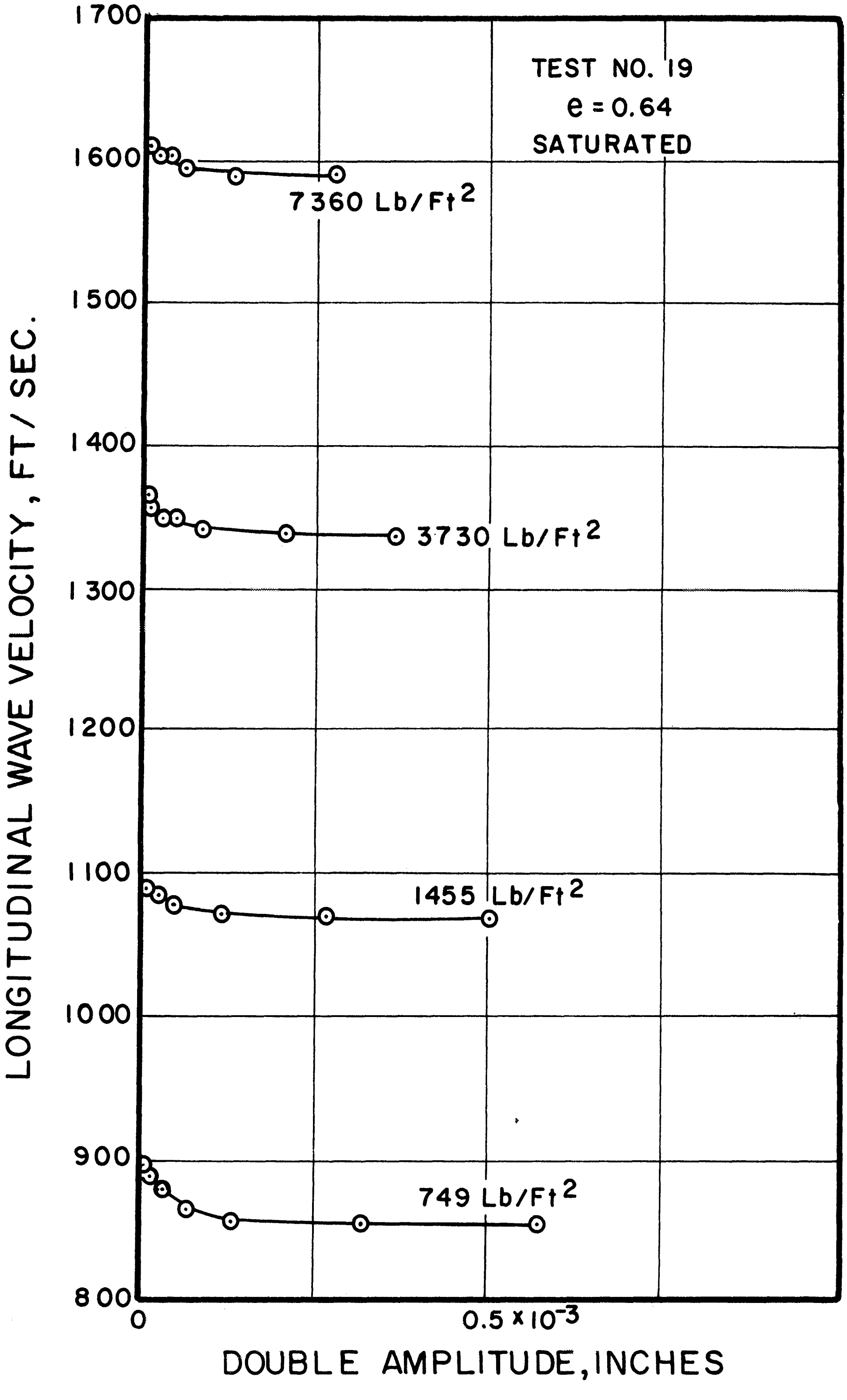


Fig. 6. Variation of Velocity with Amplitude in Longitudinal Oscillation for Ottawa Sand Saturated with Dilute Glycerin.

longitudinal oscillation of dense Ottawa sand are omitted. Figure 6 shows the variation of velocity with amplitude for specimens of loose Ottawa sand, saturated with dilute glycerin and excited into longitudinal oscillation.

Group II. Typical test results for the large size glass beads are shown in Figure 7. These curves show the results of the variation of velocity with amplitude for samples in the dense condition, both dry and saturated, in longitudinal oscillation. Figure 8 shows results of the variation of velocity with amplitude in torsional oscillation for the small size glass beads in the dense condition. The same specimens were used for both the dry and saturated tests of the glass beads.

Group III. Figures 9 and 10 show the results for the variation of wave velocity with amplitude for a specimen of crushed quartz. This specimen was formed by compaction of material which had been dried for several days at a temperature of 220°C. The behavior of the crushed quartz is quite different than that of the Ottawa sand or the glass beads because of the very small grain size. The stress wave velocities depend upon the time of load application and stress history as well as upon the other variables. During test No. 28, for which the initial void ratio was 0.83 under a pressure of 14 lb/in^2, the specimen consolidated to a void ratio of 0.80 after having been subjected to a stress cycle with confining pressures as high as 50 lb/in^2.

After each pressure was applied to the specimen, measurements of the resonant frequency were made at different time intervals. These time intervals are noted on Figures 9 and 10 and represent the total

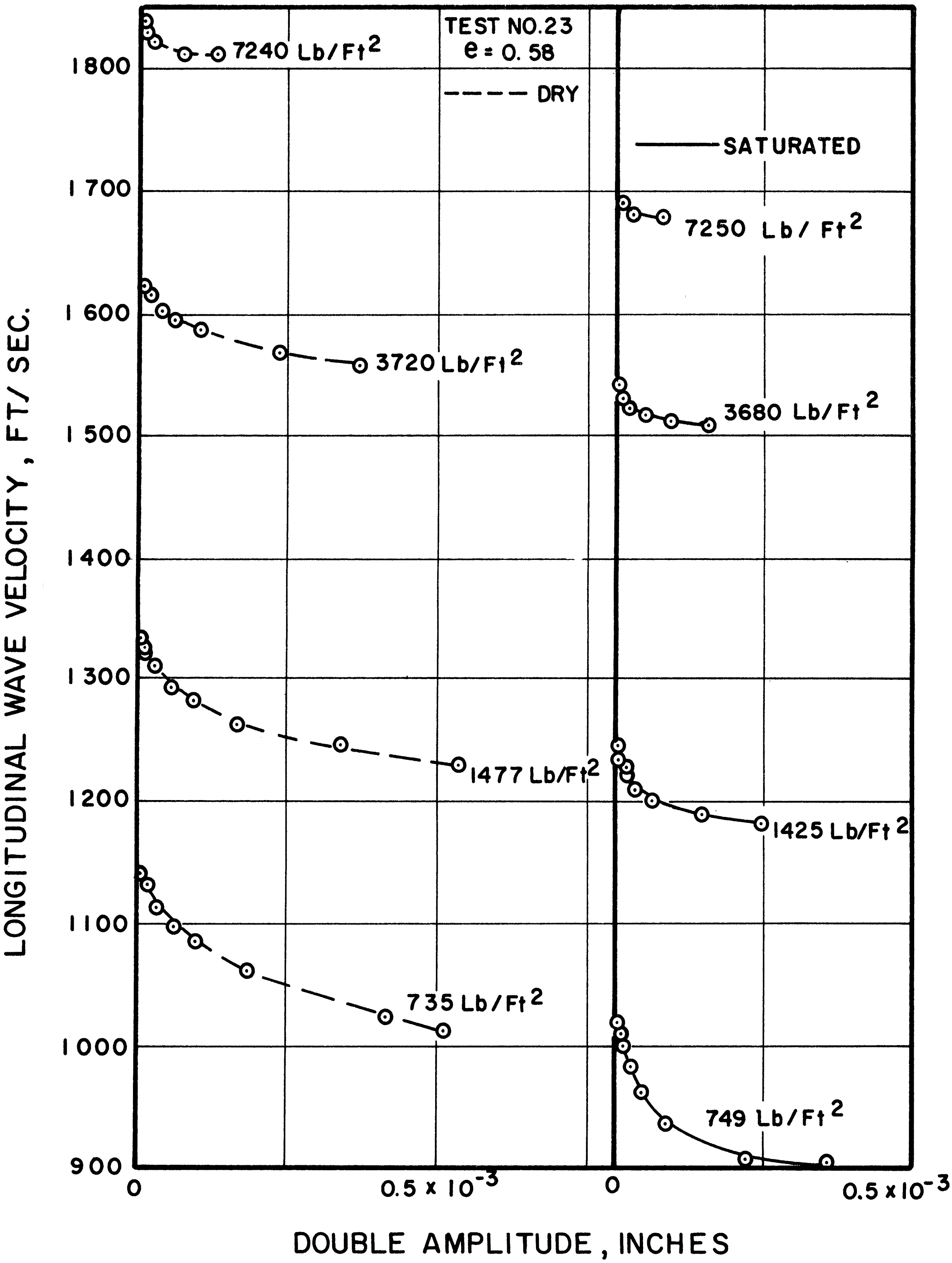


Fig. 7. Variation of Velocity with Amplitude in Longitudinal Oscillation for Glass Beads No. 2847 in the Dry and Water Saturated Condition.

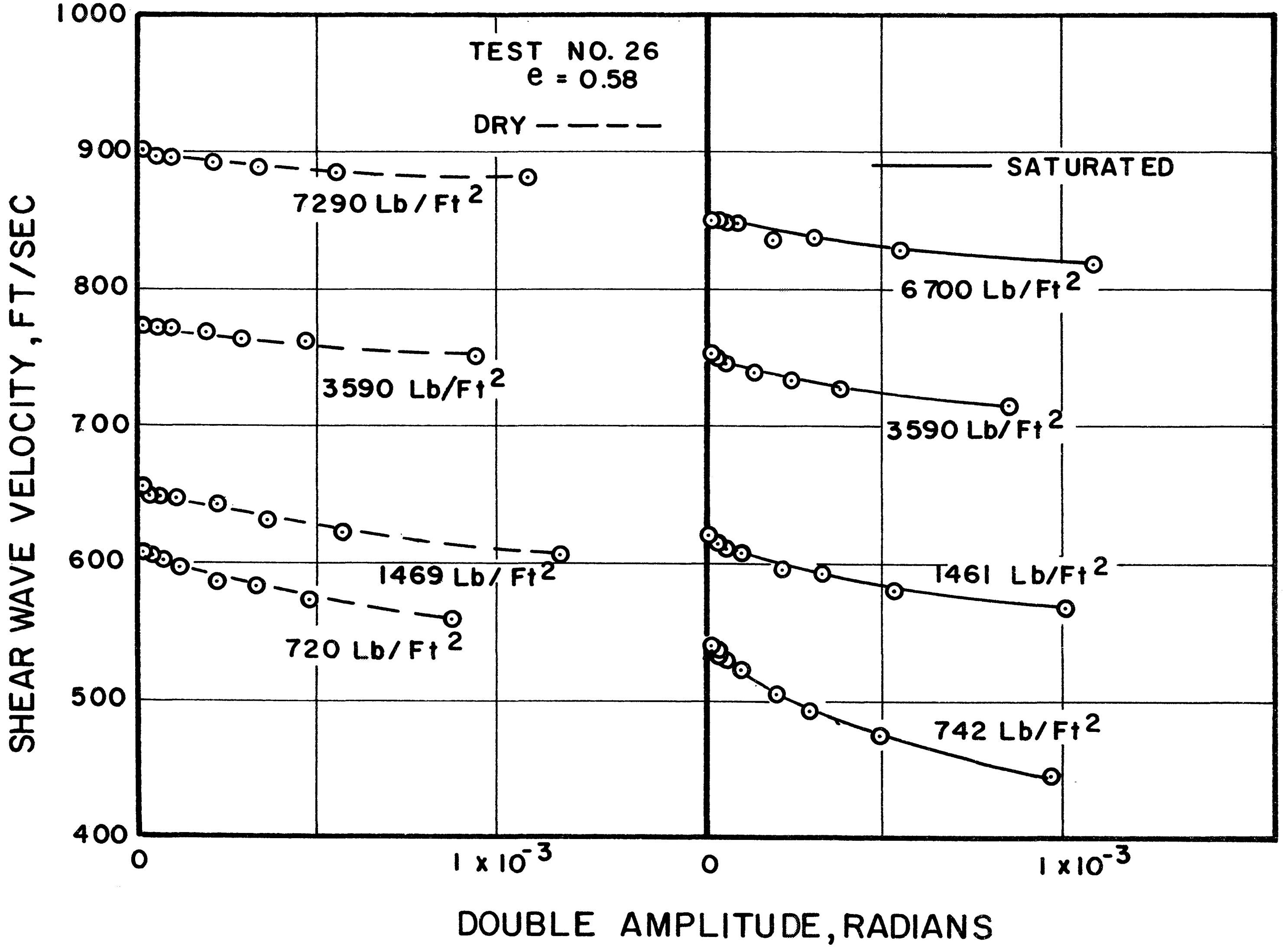


Fig. 8. Variation of Velocity with Amplitude in Torsional Oscillation for Glass Beads No. 1725 in the Dry and Water Saturated Condition.

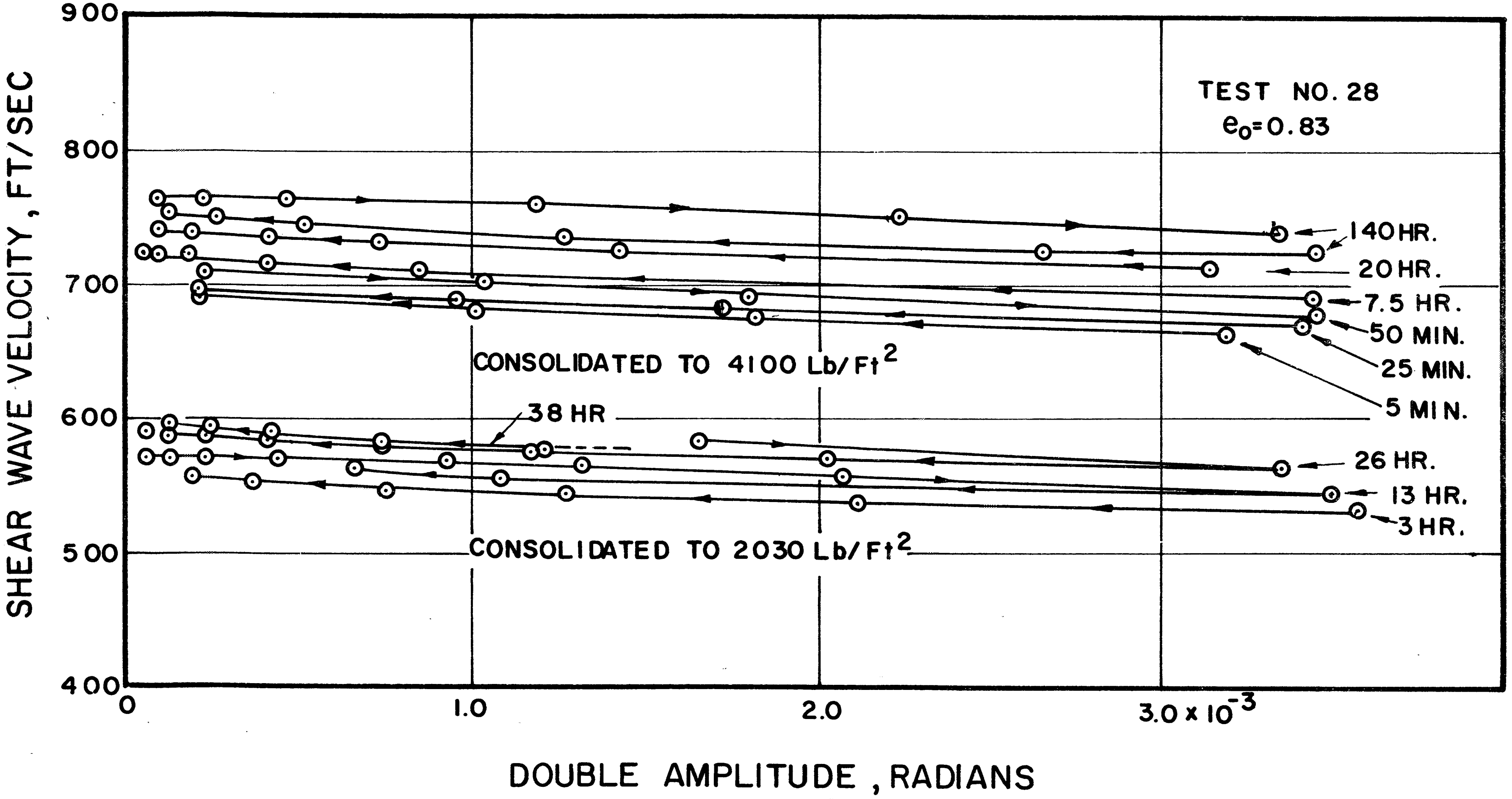


Fig. 9. Variation of Velocity with Amplitude in Torsional Oscillation for Novaculite No. 1250 Consolidated to 2030 lb/ft^2 and 4100 lb/ft^2.

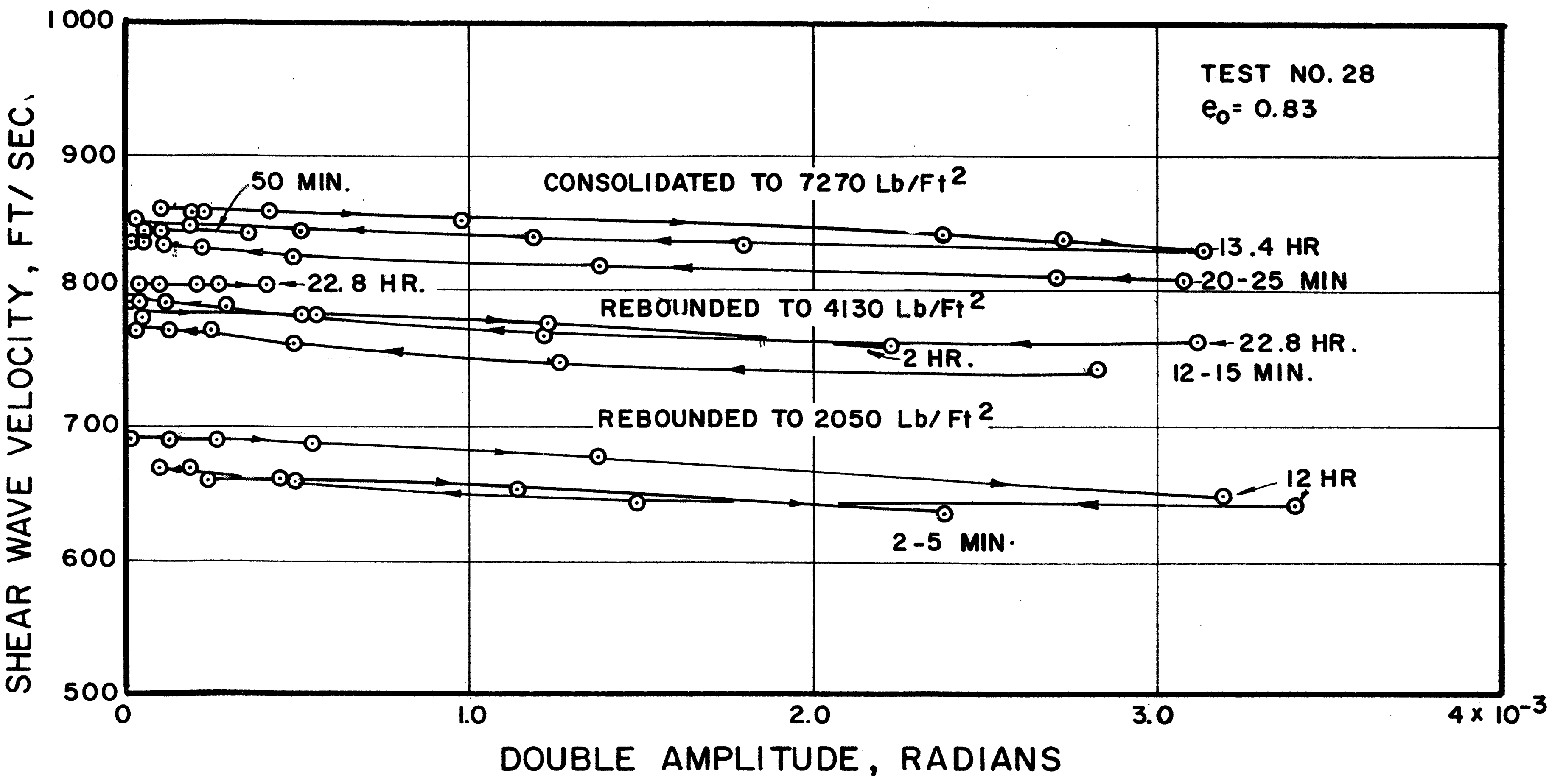


Fig. 10. Variation of Velocity with Amplitude in Torsional Oscillation for Novaculite No. 1250 Consolidated to 7270 lb/ft^2 and rebounded to 4130 lb/ft^2 and 2050 lb/ft^2.

elapsed time after the pressure was applied to the specimen. The stress history of test No. 28 was as follows:

1. The specimen was compacted, placed under a vacuum, and measurements were made for void ratio. The initial void ratio was 0.83.
2. The specimen was placed in the triaxial cell and a pressure of 2030 lb/ft^2 was applied. Velocity measurements were made intermittently over a period of 38 hrs.
3. The pressure was raised to 4100 lb/ft^2. Velocity measurements were made intermittently over a period of 140 hours.
4. The pressure was rebounded to 2050 lb/ft^2 and measurements of velocity were made intermittently over a period of 12 hours.
5. The specimen was placed under a vacuum and measurements were made for void ratio. At this time the void ratio was 0.81.
6. The specimen was replaced into the triaxial cell and the pressure was raised to 7270 lb/ft^2. Measurements of velocity were made intermittently over a period of 13 hours.
7. The pressure was reduced to 4130 lb/ft^2 and measurements of velocity were made intermittently over a period of 30 hours.
8. Final measurements under a vacuum gave a value of void ratio equal to 0.80.

During the time intervals between measurements the specimen was not vibrated. The first measurements after each time interval were made at low amplitudes of vibration. The following measurements were made at increasing amplitudes until the maximum obtainable with the equipment was reached. Since the high amplitude vibrations affect the low amplitude measurements, a second set of measurements were usually taken after the specimen had been vibrating at high amplitudes for a period of approximately five minutes. These two methods of measurement are indicated in the figures by arrows on each curve.

DISCUSSION OF THE RESULTS

Group I

Figure 4 shows the results for Ottawa sand in the dense condition under torsional oscillation. The results for this material in the loose condition were essentially the same and are not shown. The curves for the dry specimen show that the variation of velocity with amplitude is greatest at low confining pressures and low amplitudes of oscillation. The amount of velocity variation over the amplitude range measured is about 13 per cent at the low confining pressure and about 3 per cent at the high confining pressure. The results shown in Figure 5 for the longitudinal oscillations of the specimen in the dry and loose condition are very similar to those in Figure 4 for the torsional oscillations. The curves are similar in shape and the amount of variation of velocity is approximately the same order of magnitude. The results for specimens in the dense condition for longitudinal oscillations are not shown because they were essentially the same as for the loose condition. When the specimen was saturated with water the amount of variation of velocity with amplitude was decreased somewhat and most of it took place at small amplitudes. There was little variation of velocity with amplitude at the higher amplitudes of oscillation. Figure 6 shows the results for Ottawa sand saturated with dilute glycerin. These curves are practically the same as those for the same material saturated with water.

Group II

Figure 7 shows the results for the large size glass beads in the dense condition under longitudinal oscillation. For this material the effect of amplitude of vibration on the stress wave velocity is much more pronounced than that for the Ottawa sand. The saturated condition of the specimen gives a much greater variation of velocity in the low amplitude range than for the dry condition. The difference between the behavior of this material and that of Ottawa sand is probably due to the difference in the grain characteristics. The particles in both cases are rounded but the surface of the glass beads is much smoother than that of the Ottawa sand. The small size glass beads also show a larger variation of velocity with amplitude than does the Ottawa sand. The curves for the small glass beads in the saturated condition shown on Figure 8 do not flatten out as was indicated for the other test conditions. This difference could be due to the small grain size as well as the difference in the specific gravity of the particles. As mentioned before, the specific gravity of this material is 4.31 as compared to 2.50 for the large size glass beads. If the tests for the small glass beads could be carried out to higher amplitudes the results might be similar to those for the Ottawa sand and the larger size glass beads.

Group III

Figures 9 and 10 show the results for the silt-size crushed quartz. As mentioned before, the dynamic properties of this material are time dependent. Consequently, the results are shown in terms of the time elapsed after the particular confining pressure had been applied. For this material there was also a variation of wave velocity with amplitude of vibration. However, the behavior is much different than for the other materials tested. The other materials showed the maximum amount of velocity variation within the small amplitude ranges while this material shows almost no variation in the same range of amplitude. It can be seen in Figure 9 that as the material is allowed to remain under a given pressure the velocity continues to increase. However, some of this build-up can be destroyed by vibrations of larger amplitude, as shown by Figure 9. After the specimen had remained at a confining pressure of 4100 lb/ft^2 for 140 hours the variation of velocity with amplitude was as shown by the top curve. After the specimen had been vibrated at high amplitudes the lower curve for 140 hours was obtained. Figure 10 shows the results when the specimen had been consolidated to 7270 lb/ft^2, rebounded to 4130 lb/ft^2, and then to 2050 lb/ft^2. When the confining pressure was reduced from a high to a low pressure the velocity decreased, but when it was allowed to remain at the lower pressure the velocity increased. This increase can also be destroyed by high amplitude vibrations as shown by the lower of the three curves of Figure 10.

CONCLUSIONS

The conclusions obtained from this study necessarily apply to granular soils which have been subjected to several load repetitions and have reached a relatively stable condition. This corresponds to construction conditions where the soil has been pre-vibrated or pre-compacted to eliminate the disastrous settlements which may accompany the first dynamic load application on loose granular soils.

The tests on Ottawa sand and glass beads gave results which should be typical for clean sands with rounded grains. The more important conclusions are listed below:

1. Both the shear and longitudinal wave velocities decrease as the amplitude of vibration is increased. This decrease may be as much as 10 to 15 per cent as the double amplitude is increased from $1x10^{-5}$ to $2.5x10^{-3}$ radians in the torsion tests or from $2x10^{-6}$ to $1x10^{-3}$ inch in the longitudinal tests.
2. Tests have shown that the effects on wave velocities produced by changes of void ratio from the maximum to the minimum value is about 10 to 15 per cent. Thus void ratio changes and changes of amplitude influence the wave velocities by comparable amounts.

Tests on the Novaculite No. 1250, a very fine-grained crushed quartz, produced results which were somewhat different from those obtained from the larger grained materials. The primary difference is that the wave velocity values obtained from laboratory tests are dependent upon stress history and upon the time the loading has been applied. The wave velocity increases slightly as a particular confining pressure continues to be applied to the specimen. However, it was also found that the higher amplitudes of vibration tended to destroy this time-dependent increase in velocity. Further investigation is required in order to evaluate the time-dependent increase in wave velocity and the vibrational energy required to destroy this gain.

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