EXPERIMENTAL STUDY OF ELASTICA VIBRATION

M. Hammer
Rafael
P.O. Box 2250
Haifa, Israel

N. C. Perkins
Assistant Professor
Department of Mechanical Engineering and Applied Mechanics
The University of Michigan
Ann Arbor, Michigan 48109

July, 1989

Report No. UM-MEAM-89-07

EXPERIMENTAL STUDY OF ELASTIC VIBRATION

1. Introduction

This research is concerned with the design of a nonlinear suspension employing an elastica, as originally proposed by Shoup [2-4]. The following summary, from Shoup [2], states the key characteristics of his design.

Shock and vibration problems in the aerospace and transportation industries arise from many causes, such as the isolation of instruments and controls or the protection of human occupants of vehicles. The usual solution to these problems involves the use of lightly damped flexible supports. These soft supports cause the natural frequency of the suspension system to be far below the disturbing frequency. This solution is effective for the isolation of steady-state vibration; however, when these suspensions encounter shock excitation their softness often leads to damagingly large deflection. It has been pointed out that this undesirable feature is not present in suspension systems utilizing symmetrically nonlinear springs that harden. These springs become progressively stiffer when subjected to large deflection from their "operation point".

A number of ingenious ways have been developed to produce nonlinear spring devices, but unfortunately many of these are not symmetrical in behavior or are rather complex to construct. As weight, cost, and reliability requirements become more important, designers are forced to search for new ways to improve existing designs by reducing the number of moving parts in suspension systems. Shoup proposes using thin elastic strips in the form of an elastica, as the nonlinear restoring elements.

In his analysis, Shoup treats the elastica as massless spring with nonlinear characteristics. In the actual designs, however, the mass of the elastica element is often comparable to the mass of the suspended payload and the inertialess assumption remains suspect.

The purpose of this research is to propose a dynamical model for the elastica which accounts for its inertia. The object of this project is to provide experimental evidence in support of the theoretical model.

The specific goals of this project are as follows:

1. Develop a suitable test stand for studying elastica dynamics.
2. Perform an experimental modal analysis of the elastica to determine its vibration characteristics.

2. Design and Construction of Elastica Suspension Test Stand

2.1 Design Requirements

Design a test stand that allows measurement of the natural frequencies and the mode shapes of a test rod. The ends of the rod must connect to support that allow the rod to rotate, with minimum friction, in the vertical plane. One support must also be able to slide horizontally with the motion of the payload. The supports should restrict out-of-plane motion of the elastica but permit a large range of test rod lengths up to 1.9m. to be used. The measurement equipment should be selected to detect motion in the vertical plane. The design should permit the attachment of a vibration shaker for future studies.

2.2 Description of the Design

A schematic of the test stand is shown in figure 1 and a photograph of the test stand is provided in figure 2. The test rod (1) was cut to length L, and the accelerometer base (8) and the impact surface (9) were assembled on the rod. The ends of the test rod were connected to the holders (3) that freely pivot in the vertical plane about small shafts in the supports (2). The shafts were lubricated with graphite to reduce friction. The supports are mounted directly to the base at the left end of the rod (6) and to a bearing seat (2) and (4) at the right end. Two parallel ground shafts (10) are used to guide the linear bearings (5) horizontally. The ground shafts are connected to the base (1) of the test stand which consists of extruded aluminium channel having rectangular (7.6 cm. x 4.8 cm.) cross-section and length 2m. To prevent excessive deflection of the ground shafts, two clamps (7) provide additional support and may also lock the movable support at one location. The accelerometer base (8) can be adjusted to any angle to allow measuring the horizontal and vertical components of the rod acceleration. The accelerometers (12) are bonded to the accelerometer base with petro wax. The impact surface (9) provides a planer surface for hammer impacts.

Schematic drawing of Test Stand (part 1)

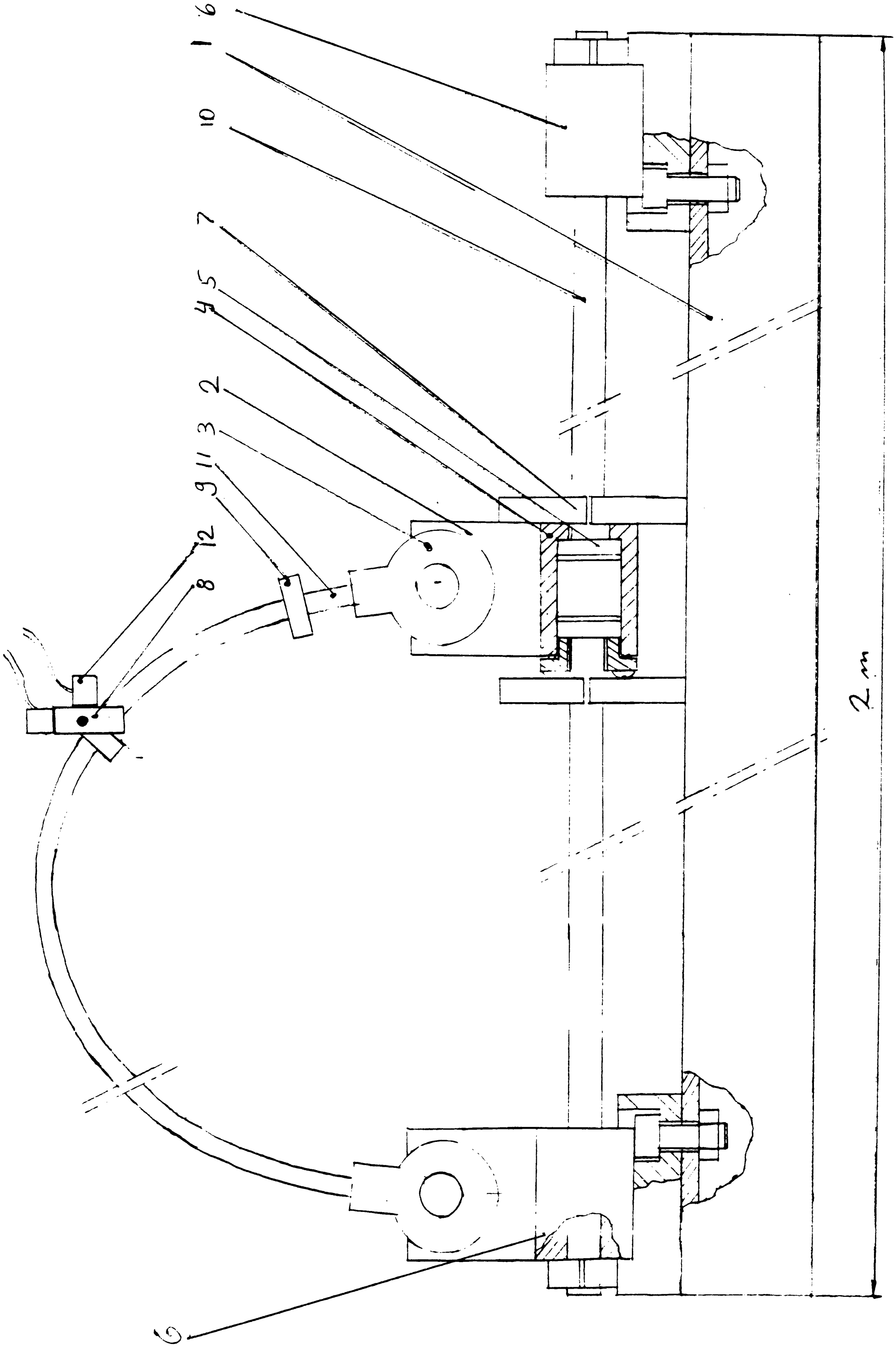


Figure 1

Parts list (part 2)

No.	Name	Quantity
1	Base	1
2	Support	2
3	Rod Holder	2
4	Bearing seat	1
5	Linear bearing	2
6	Shaft support	2
7	Clamp	2
8	Accelerometer base	1
9	Impact surface	1
10	Ground shaft	2
11	Test rod	1
12	Accelerometer	2

Table 1

Measuring equipment (part 3)

No.	Name	Quantity
1	Piezoelectric Unixial, Accelerometer-PCB-309A	2
2	Amplifier-PCB-483A08	1
3	Structural Dynamics Analyzer - Gen-Rad 2515	2
4	Piezoelectric Impact hammer-PCB-086B03 (with soft, rubber tip)	1

Table 2

Rod Material Properties (part 4)

No.

1 Material - LEXAN

2 $E = 2344$ Mpa (Young's modulus)

3 $\gamma = 1190 kg/m^3$. (specific gravity)

4 O.D. = 6.35mm.

5 $L = 1.2m$.

Table 3

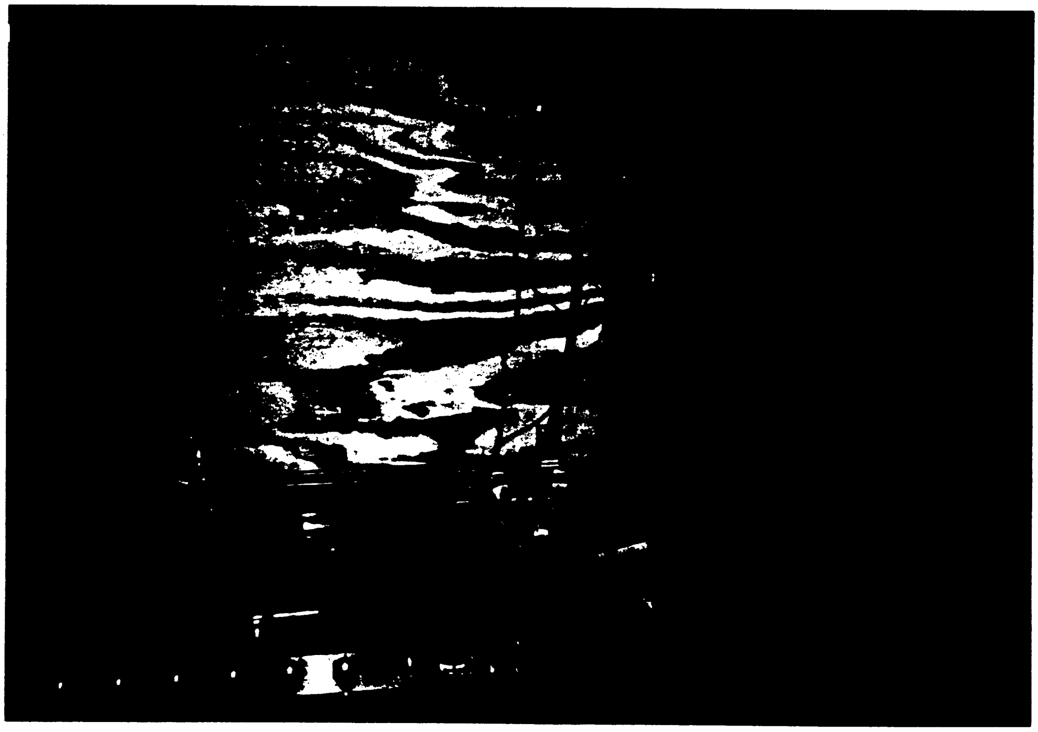


Figure 2

3. Experimental Modal Analysis

3.1 Purpose of Test

A theoretical model of elastica vibration has recently been proposed in [1]. The purpose of the present project is to validate this model by providing companion experimental results. Figure 3 depicts the theoretical natural frequencies and mode shapes as functions of the applied end-load,n. The end-load, n, is related to the support separation $|x\ (1)|$ as observed in figure 4. Both of these quantities are nondimensional and are related to dimensional quantities as described in [1].

3.2 Test Procedure and Results

The test procedure was divided into two major parts. First, the natural frequencies of the first four modes of the test rod were found, and second, the mode shapes were determined.

To find the natural frequencies of the test rod an accelerometer was bonded to the test rod in the normal direction to detect the nomal component of the acceleration. The amplified accelerometer signal was sampled by the structural dynamics analyzer and the frequency response of this signal was computed following on initial impact; see figure 6. The natural frequencies of the rod appear as local maxima in the frequency response plot. Five tests were conducted for each of nine end-load values in the range n = 10-20. The average of these test results are shown in table 6 which also shows the corresponding theoretical frequencies.

The experimental frequencies were non-dimensionalized as shown below [1]:

$$\Omega = \frac{\omega}{C} = \frac{\omega}{\sqrt{\frac{\rho L4}{EI}}} \left({}^{rad}/_{sec}\right)$$

where

Ω is the dimensional circular frequency (rad/sec.)

ω is the nondimensional circular frequency

ρ = mass/length = A_γ/g

L = length of test rod

I = area moment of inertia = $\pi d^4/64$

E = young's modulus

A = cross section area = $\pi d^2/4$

In this computation, ρ is the mass/length of the rod alone and does not include the accelerometer and base, the holders, and the impact surface.

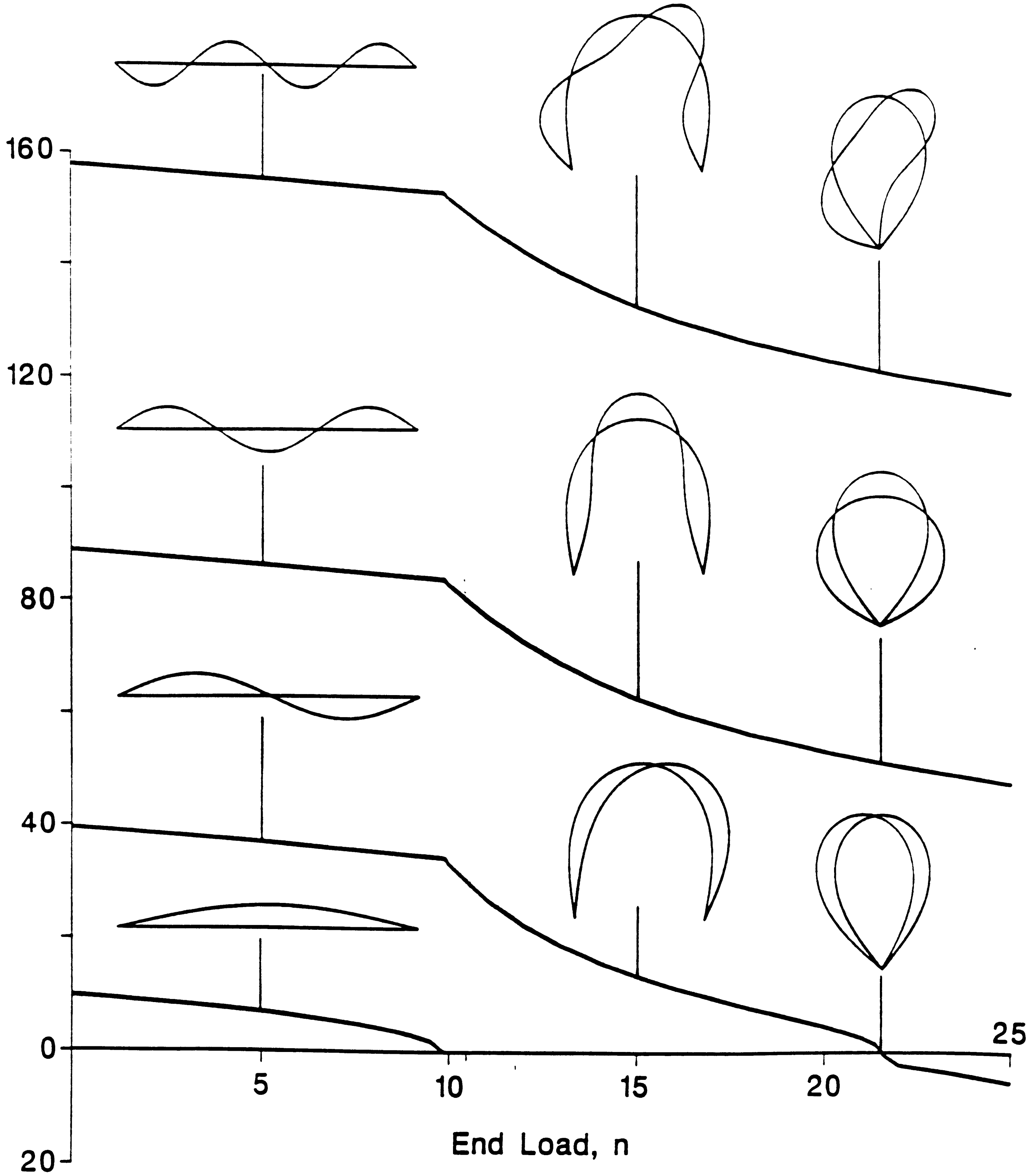


Figure 3

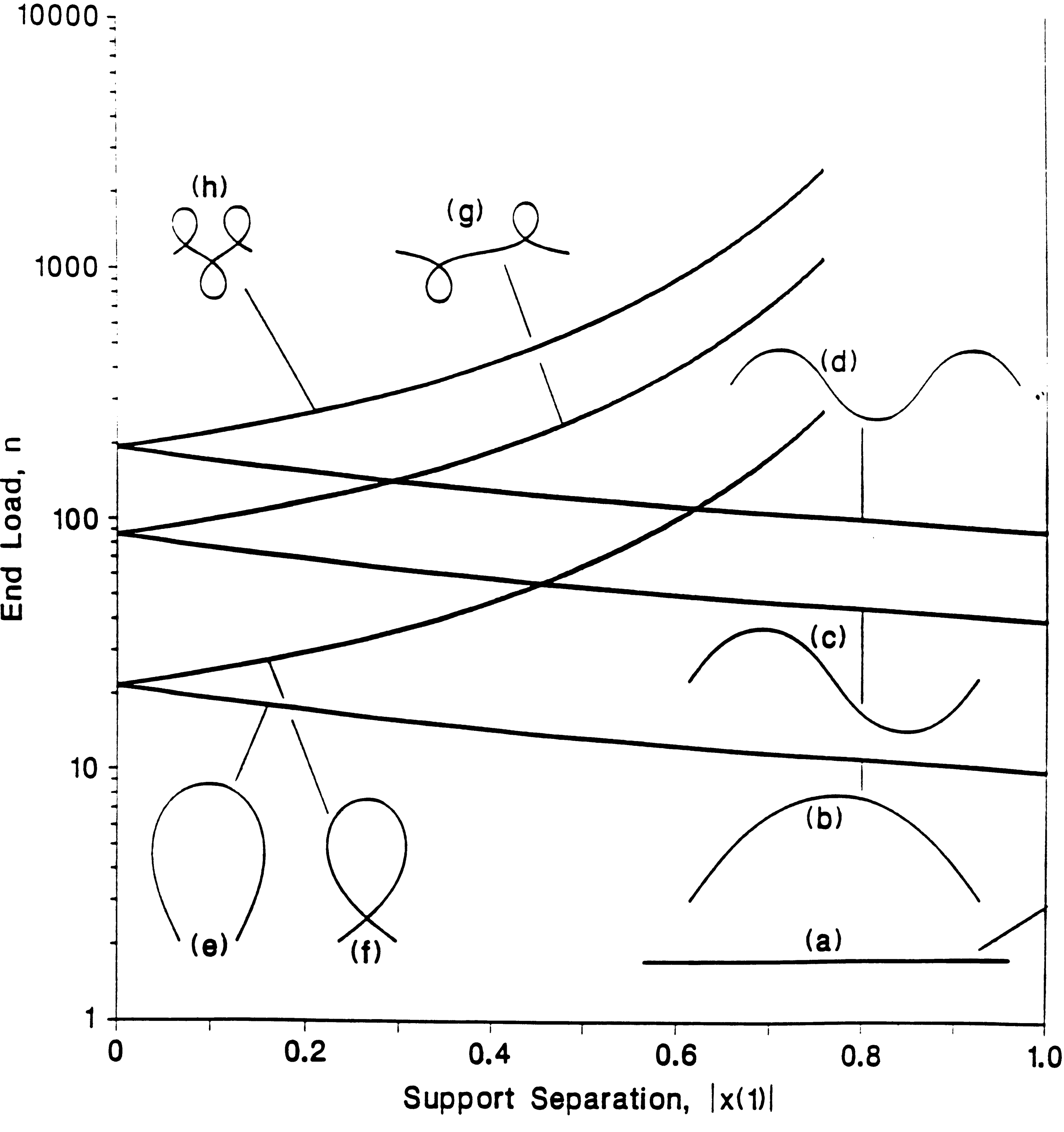

10000
1000
100
10
1
End Load, n
0
0.2
0.4
0.6
0.8
1.0
Support Separation, |x(1)|
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)

Figure 4

Every mass attached to the rod shown in figure 5 was measured and the overall mass/length was computed using different combinations of these masses; see table 4. The effect this calculation has on the agreement between measured and theoretical frequencies is shown in table 5.

The best overall agreement occurred when the mass of the accelerometer Ma, the accelerometer holder Mb, and the impact surface Mb were included.

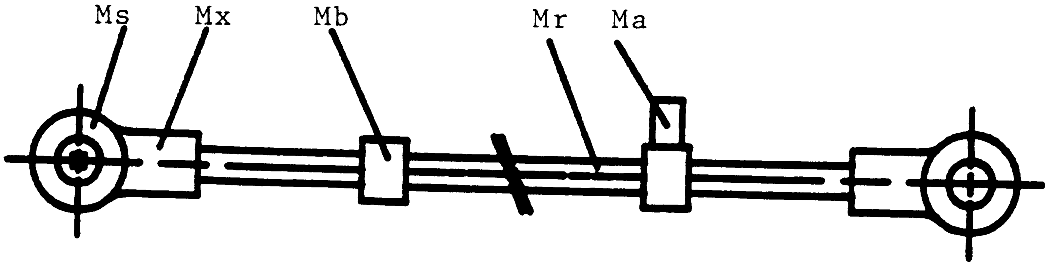


Figure 5

Ms = 5.09 gr

Mx = 2.89 gr

Mb = 1.68 gr

Mr = 44.95 gr

Ma = 1 gr

M1 = 2(Ms+Mx+Mb)+Mr+Ma

M2 = 2(Mx+Mb)+Mr+Ma

M3 = 2Mb+Mr+Ma

M4 = Mr+Ma

M5 = Mr

No.	M_i(gr)	ρ_i(kg/m)	Ci
1	65.27	0.00303	0.6479
2	55.09	0.00255	0.654
3	49.31	0.00229	0.6782
4	45.95	0.00213	0.7156
5	44.95	0.00209	0.78

Table 4

Summary of the results

The natural frequency for N = 12

Ci / Mode	C1	C2	C3	C4	C5
M1	4.27	4.6	4.81	4.95	5.5
M2	13.87	14.94	15.36	16.09	17.87
M3	27.16	29.26	30.62	31.5	34.98
M4	44.4	47.52	49.73	51.16	56.84

Table 5

N	H/L	AVERAGE FREQUENCY				
		1	2	3	4	
10	0.974	33.188	82.928	152.133	241.01	W(TH)
		29.454	78.595	124.966	227.653	W(EX)
11	0.8	26.95	77.353	146.866	253.813	
		25.721	75.893	140.263	230.415	
12	0.653	22.4	72.77	142.464	231.387	
		22.158	71.393	147.993	237.497	
13	0.53	18.873	68.929	138.717	227.567	
		18.54	67.481	143.135	237.556	
14	0.433	16	65.659	135.49	224.236	
		15.93	64.302	140.34	232.937	
15	0.349	13.58	62.841	32.672	221.305	
		13.313	61.677	134.971	228.446	
16	0.274	11.947	60.16	129.97	218.463	
		11.173	58.976	132.056	215.406	
18	0.154	7.899	56.327	126.075	214.31	
		6.982	55.175	118.429	211.358	
20	0.06	4.567	53.114	122.785	210.74	
		4.696	52.626	113.741	216.949	

C = 0.6782

Table 6

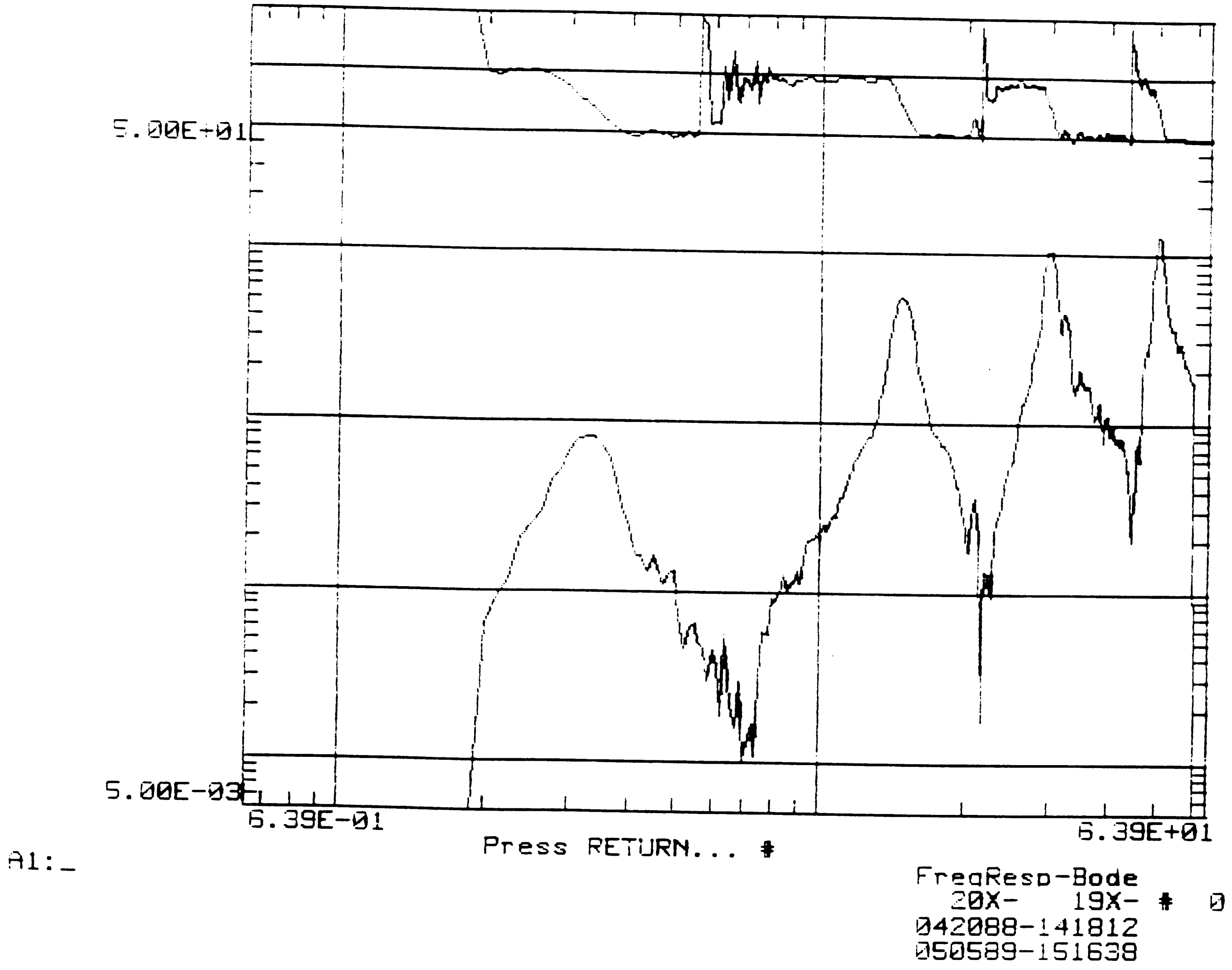

5.00E+01
5.00E-03
6.39E-01
6.39E+01
A1:_
Press RETURN... #
FreqResp-Bode
20X- 19X- # 0
042088-141812
050589-151638

Figure 6

A complete modal test was performed for a representative geometry defined by

$n = 15$, $H = 0.42$m and $H/L = 0.349$.

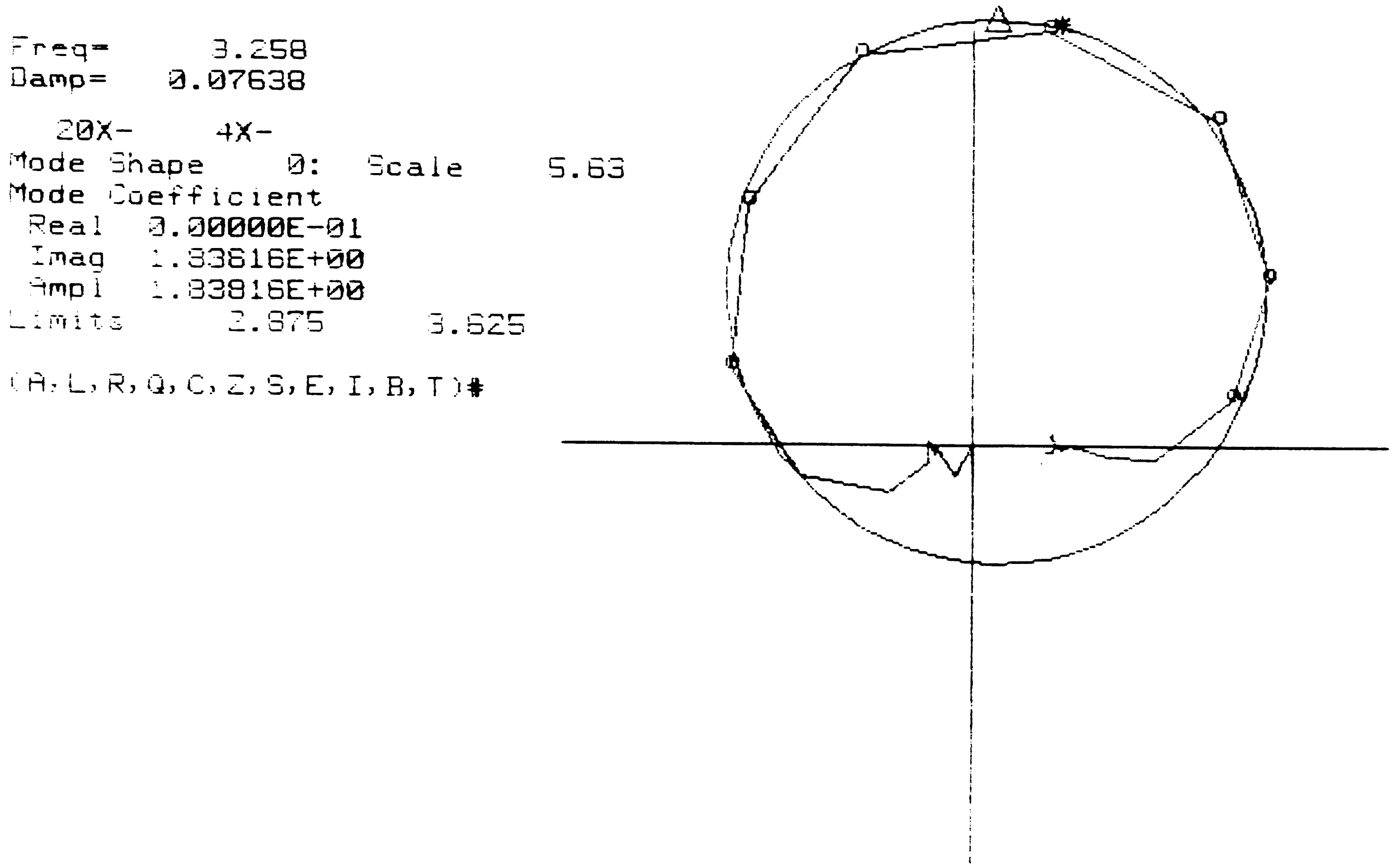
The modal test procedure was divided into three major parts. First the geometrical shape of the rod was entered into the structural dynamics program. Second, the force input and response output data were collected. And third, the modal parameters were extracted from the data using standard modal analysis procedures.

The rod was divided into 23 equally spaced points including the ends. No measurements were taken at the ends. The rod was subjected to the end-load, $n = 15$, by adjusting H such that $H/L = 0.349$. The X - Y coordinates of every test point were measured and entered into the structural dynamics program.

The impact surface was located at point 21 near the right support and the accelerometer base was placed at each point with accelerometers aligned with the X-Y directions. The accelerometer and force signals were sampled by the structural dynamic analyzer during the after the rod was struck by the hammer (see appendix 3). For each location of the accelerometer, 10 tests performed and the average frequency response was computed and stored.

The natural frequencies, damping ratios, and modal participation factors were extracted from the measured data using a single degree of freedom, circle fit routine as seen in figure 7.

The first two mode shapes determined in this way are given in figure 8, and compare very favorably with the theoretical predictions shown in figure 2.



```
Freq=      3.258
Damp=   0.07638

  20X-    4X-
Mode Shape    0:  Scale     5.63
Mode Coefficient
 Real  0.00000E-01
 Imag  1.83816E+00
 Ampl  1.83816E+00
Limits      2.875      3.625

(A,L,R,Q,C,Z,S,E,I,B,T)#
```

Figure 7

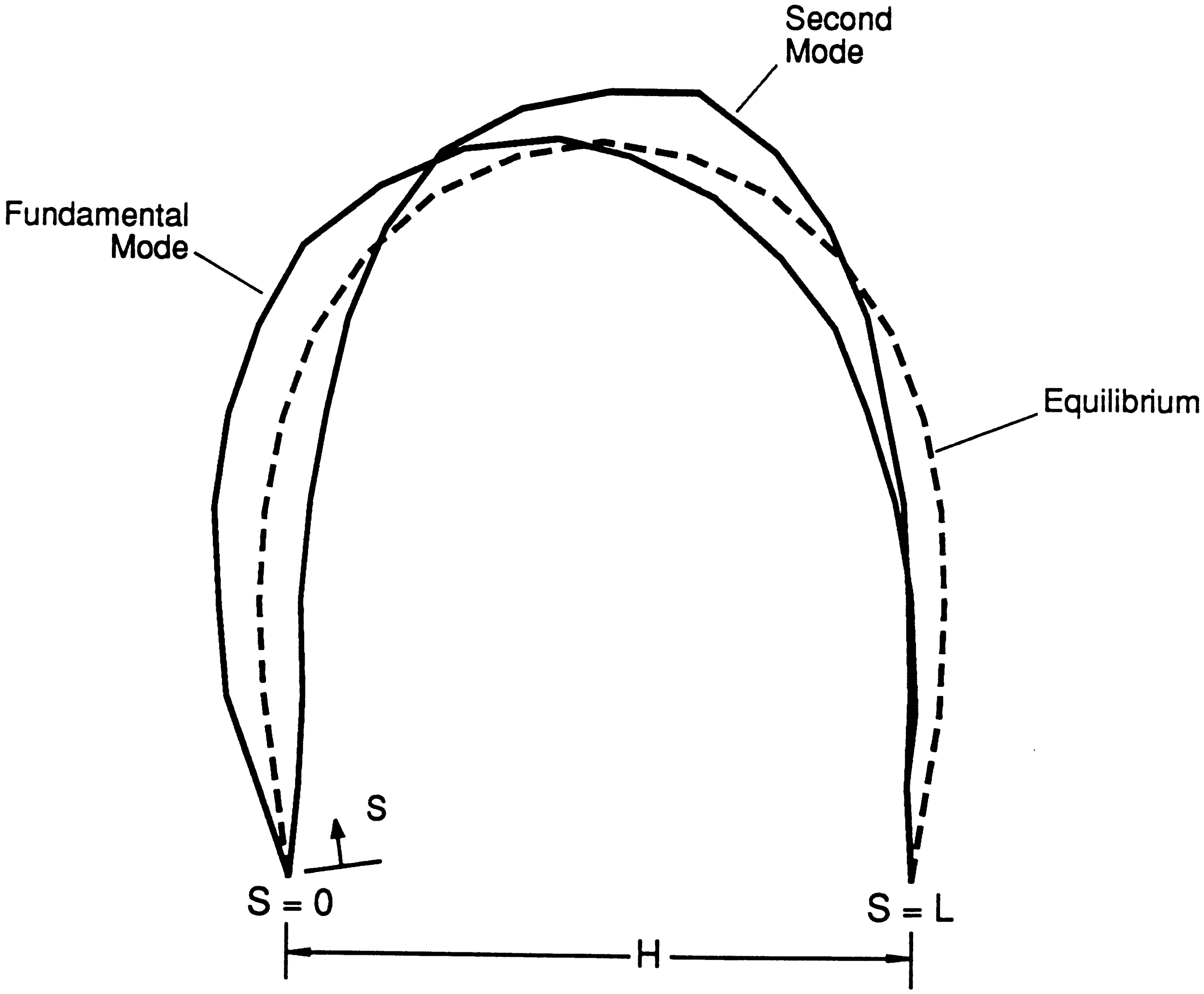


Figure 8

4. Summary and Conclusions

This project provides experimental evidence in support of a theoretical model for planar elastica vibration. A test stand was designed to permit vibration tests on wide range of elastica geometries. Results for the first four natural frequencies show that the natural frequencies predicted by the model are generally accurate to within 5% of those measured experimentally. Larger discrepancies occur when the ends of the elastica are close and these can be attributed to the influence of the rotary inertia of the supports which were not modeled. Experimental measurements of the first two vibration mode shapes for a representative goemetry are in superb agreement with the theory.

5. References

1. Perkins, N. C., 1989, "Planar Vibration of an Elastica Arch: Theory and Experiment," submitted to the ASME J. of Vibration and Acoustics.

2. Shoup, T. E., 1971, "Shock and Vibration Isolation Using a Nonlinear Elastic Suspension", AIAA Journal, Vol. 9, No. 8, pp. 1643-1645.

3. Shoup, T. E., 1972, "Experimental Investigation of a Nonlinear Elastic Suspension," AIAA Journal, Vol. 10, No. 4, pp. 559-560.

4. Shoup, T. E., 1977, "An Adjustable Spring Rate Suspension System," AIAA Journal, Vol. 15, No. 6, pp. 865-866.

Appendix 1

Measured Frequency

ROD FREQUENCY

TEST	DATE	H/L	N	C
1	2-27-89	.653	12	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	5.503762	17.87580	34.99596	56.83957
ω^2	501.99	5295.5	20296.06	53539.87

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	5.23	16.83	34.57	55.74	**
RUN #2					
	5.08	16.64	34.77	55.74	
RUN #3					
	5.23	16.64	34.77	56.39	
RUN #4					
	5.23	16.83	34.77	55.1	
RUN #5					
	5.23	16.83	34.77	55.7	
AVERAGE	21.16855	68.20345	141.3815	226.8861	w(nd)
	5.2	16.754	34.73	55.734	f(Hz)
ERROR %					
	5.519178	6.275555	0.759998	1.945076	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
2	3-1-89	.53	13	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	4.636232	16.93214	34.07549	55.90128
ω^2	356.21	4751.16	19242.44	51786.83

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	4.35	15.8	33.59	55.74	**
RUN #2					
	4.35	15.8	33.59	55.78	
RUN #3					
	4.352	15.89	33.59	55.74	
RUN #4					
	4.352	15.89	33.59	55.74	
RUN #5					
	4.352	15.8	33.59	55.74	
AVERAGE	17.71319	64.46638	136.7407	226.9431	w(nd)
	4.3512	15.836	33.59	55.748	f(Hz)
ERROR %					
	6.147933	6.473751	1.424773	0.274212	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C	
3	3-1-89	0.433	14	0.6479	
THEORETIC FREQUENCY					
MODES	1	2	3	4	
	3.931049	16.12893	33.28180	55.08299	
ω^2	256.09	4311.09	18356.48	50281.79	
MEASURED FREQUENCY					
MODES	1	2	3	4	
RUN #1					*
	3.747	15.09	33.01	54.16	**
RUN #2					
	3.747	15.09	33.01	54.79	
RUN #3					
	3.747	15.09	32.63	54.79	
RUN #4					
	3.747	15.09	33.01	54.79	
RUN #5					
	3.704	15.09	33.01	54.79	
AVERAGE	15.21856	61.42951	134.0702	222.5303	w(nd)
	3.7384	15.09	32.934	54.664	f(Hz)
ERROR %					
	4.900703	6.441436	1.045026	0.760654	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
4	3-1-89	0.349	15	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	3.337461	15.43674	32.59051	54.36296
ω^2	184.59	3949	17601.84	48975.84

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	3.117	14.49	31.53	51.72	**
RUN #2					
	3.117	14.49	31.71	54.16	
RUN #3					
	3.117	14.41	31.71	53.85	
RUN #4					
	3.153	14.49	31.71	54.16	
RUN #5					
	3.117	14.49	31.71	54.16	
AVERAGE	12.71823	58.92185	128.9409	218.2396	w(nd)
	3.1242	14.474	31.674	53.61	f(Hz)
ERROR %					
	6.389925	6.236707	2.812205	1.385061	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
5	3-1-89	0.274	16	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	2.774510	14.77807	31.92687	53.66485
ω^2	127.57	3619.19	16892.29	47726.07

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	2.622	13.84	30.99	50.84	**
RUN #2					
	2.622	13.84	30.99	50.84	
RUN #3					
	2.622	13.84	30.99	50.84	
RUN #4					
	2.622	13.84	30.99	50.26	
RUN #5					
	2.622	13.84	30.99	49.97	
AVERAGE	10.67383	56.34092	126.1564	205.7827	w(nd)
	2.622	13.84	30.99	50.55	f(Hz)
ERROR %					
	5.496855	6.347743	2.934440	5.804279	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
6	3-1-89	0.154	18	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	1.940458	13.83673	30.96995	52.64471
ω^2	62.4	3172.8	15894.86	45928.82

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	1.743	12.99	30.11	47.17	**
RUN #2					
	1.608	12.99	28.92	50.26	
RUN #3					
	1.617	12.92	27.46	47.99	
RUN #4					
	1.608	12.92	26.99	52.32	
RUN #5					
	1.617	12.92	25.48	50.26	
AVERAGE	6.670537	52.70969	113.1377	201.9154	w(nd)
	1.6386	12.948	27.792	49.6	f(Hz)
ERROR %					
	15.55605	6.422984	10.26139	5.783518	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
6a	3-1-89	0.154	18	0.654

THEORETIC FREQUENCY

MODES	1	2	3	4
	1.922325	13.70766	30.68108	52.15368
ω^2	62.3978	3172.796	15894.86	45928.82

MEASURED FREQUENCY

MODES		1	2	3	4	
RUN #1						*
		1.743	12.99	30.11	47.17	**
RUN #2						
		1.608	12.99	28.92	50.26	
RUN #3						
		1.617	12.92	27.46	47.99	
RUN #4						
		1.608	12.92	26.99	52.32	
RUN #5						
		1.617	12.92	25.48	50.26	
AVERAGE		6.733340	53.20596	114.2029	203.8164	w(nd)
		1.6386	12.948	27.792	49.6	f(Hz)
ERROR %						
		14.75951	5.541894	9.416506	4.896467	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
6b	3-1-89	0.154	18	0.6782

THEORETIC FREQUENCY

MODES	1	2	3	4
	1.853732	13.21853	29.58630	50.29270
ω^2	62.3978	3172.796	15894.86	45928.82

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	1.743	12.99	30.11	47.17	**
RUN #2					
	1.608	12.99	28.92	50.26	
RUN #3					
	1.617	12.92	27.46	47.99	
RUN #4					
	1.608	12.92	26.99	52.32	
RUN #5					
	1.617	12.92	25.48	50.26	
AVERAGE	6.982494	55.17474	118.4288	211.3583	w(nd)
	1.6386	12.948	27.792	49.6	f(Hz)
ERROR %					
	11.60535	2.046655	6.064640	1.377345	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
6c	3-1-89	0.154	18	0.7156

THEORETIC FREQUENCY

MODES	1	2	3	4
	1.756849	12.52768	28.04000	47.66421
ω^2	62.3978	3172.796	15894.86	45928.82

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	1.743	12.99	30.11	47.17	**
RUN #2					
	1.608	12.99	28.92	50.26	
RUN #3					
	1.617	12.92	27.46	47.99	
RUN #4					
	1.608	12.92	26.99	52.32	
RUN #5					
	1.617	12.92	25.48	50.26	
AVERAGE	7.367550	58.21741	124.9597	223.0138	w(nd)
	1.6386	12.948	27.792	49.6	f(Hz)
ERROR %					
	6.730744	-3.35507	0.884483	-4.06129	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
6d	3-1-89	0.154	18	0.78

THEORETIC FREQUENCY

MODES	1	2	3	4
	1.611796	11.49334	25.72491	43.72886
ω^2	62.3978	3172.796	15894.86	45928.82

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	1.743	12.99	30.11	47.17	**
RUN #2					
	1.608	12.99	28.92	50.26	
RUN #3					
	1.617	12.92	27.46	47.99	
RUN #4					
	1.608	12.92	26.99	52.32	
RUN #5					
	1.617	12.92	25.48	50.26	
AVERAGE	8.030589	63.45665	136.2053	243.0838	w(nd)
	1.6386	12.948	27.792	49.6	f(Hz)
ERROR %					
	-1.66296	-12.6564	-8.0353	-13.4262	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
7	3-1-89	0.060	20	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	1.121939	13.04739	30.16188	51.76811
ω^2	20.86	2821.13	15076.23	44412

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	1.119	12.77	27.46	51.43	**
RUN #2					
	1.119	12.19	28.92	51.43	
RUN #3					
	1.136	12.19	24.61	51.43	
RUN #4					
	1.134	12.41	28.42	51.72	
RUN #5					
	1.003	12.19	24.05	48.55	
AVERAGE	4.486919	50.27531	108.6598	207.2564	w(nd)
	1.1022	12.35	26.692	50.912	f(Hz)
ERROR %					
	1.759378	5.345066	11.50421	1.653741	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
8	3-1-89	0.80	11	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	6.620276	19.00167	36.07740	57.92678
ω^2	726.32	5983.56	21569.81	55607.66

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	5.02	17.83	31.7	50.8	**
RUN #2					
	6.62	17.63	30.1	55.1	
RUN #3					
	6.11	17.83	32.63	52.3	
RUN #4					
	6.32	17.93	34.57	54.79	
RUN #5					
	6.11	17.83	35.58	57.37	
AVERAGE	24.57180	72.50229	133.9969	220.1203	w(nd)
	6.036	17.81	32.916	54.072	f(Hz)
ERROR %					
	8.825565	6.271440	8.762843	6.654589	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

ROD FREQUENCY

TEST	DATE	H/L	N	C
9	3-1-89	0.974	10	0.6479

THEORETIC FREQUENCY

MODES	1	2	3	4
	8.152533	20.37111	37.37106	59.20329
ω^2	1101.44	6877.1	23144.44	58085.47

MEASURED FREQUENCY

MODES	1	2	3	4	
RUN #1					*
	6.93	18.46	30.63	51.72	**
RUN #2					
	6.93	18.46	26.37	53.85	
RUN #3					
	6.21	17.63	29.76	55.1	
RUN #4					
	7.91	18.67	29.76	53.85	
RUN #5					
	6.58	19	30.11	52.6	
AVERAGE	28.13789	75.08323	119.3825	217.4824	w(nd)
	6.912	18.444	29.326	53.424	f(Hz)
ERROR %					
	15.21653	9.460041	21.52752	9.761784	

NOTE:

* = MEASURED FROM FREQUENCY GRAPH

** = MEASURED FROM SPECTRUM GRAPH

Appendix 2

The error [%] between the theoretical calculation and the test results as a function of the influence of the weight of the Test Rod and the Supports.

Error [%]

N = 10

mode const.	1	2	3	4
0.6479	15.216	9.460	21.527	9.761
0.654	14.418	8.607	20.788	8.912
0.6782	11.251	5.226	17.857	5.541
0.7156	6.357	-0.0006	13.327	0.332
0.780	-2.069	-9.0001	5.527	-8.6368

Error [%]

N = 11

mode const.	1	2	3	4
0.6479	8.8255	6.271	8.762	6.654
0.654	7.9671	5.388	7.9038	5.775
0.6782	4.561	1.888	4.496	2.2289
0.7156	-0.7014	-3.522	-0.7706	-3.0992
0.780	-9.763	-12.838	-9.839	-12.377

Error [%]

N = 12

mode / const.	1	2	3	4
0.6479	5.519	6.275	0.7599	1.9456
0.654	4.6296	5.393	-0.1743	1.0218
0.6782	1.1006	1.892	-3.881	-2.640
0.7156	-4.353	-3.517	-9.609	-8.302
0.780	-13.744	-12.833	-19.473	-18.047

Error [%]

N = 13

mode const.	1	2	3	4
0.6479	6.1479	6.473	1.4247	0.2742
0.654	5.264	5.593	0.496	-0.664
0.6782	1.7588	2.099	-3.185	-4.389
0.7156	-3.658	-3.298	-8.875	-10.146
0.780	-12.987	-12.595	-18.673	-20.058

Error [%]

N = 14

mode / const.	1	2	3	4
0.6479	4.90	6.441	1.045	0.761
0.654	4.0053	5.5605	0.1133	-0.1736
0.6782	0.4532	2.066	-3.582	-3.880
0.7156	-5.0363	-3.3346	-9.2949	-9.669
0.780	-14.489	-12.6341	-19.13	-19.47

Error [%]

N = 15

mode const.	1	2	3	4
0.6479	6.389	6.236	2.8122	1.385
0.654	5.508	5.354	1.897	0.4566
0.6782	2.0121	1.8517	-1.7329	-3.2268
0.7156	-3.3915	-3.5607	-7.3431	-8.9193
0.780	-12.596	-12.88	-17.003	-18.7214

Error [%]

N = 16

mode const.	1	2	3	4
0.6479	5.4968	6.3477	2.9344	5.8042
0.654	4.6071	5.466	2.020	4.9174
0.6782	1.0772	1.968	-1.6049	1.399
0.7156	-4.378	-3.4381	-7.208	-4.0386
0.780	-13.771	-12.747	-16.856	-13.40

Error [%]

N = 18

mode / const.	1	2	3	4
0.6479	15.556	6.4229	10.2614	5.7835
0.654	14.759	5.5418	9.4165	4.8964
0.6782	4.6053	2.0466	6.0640	1.3773
0.7156	6.7307	-3.355	0.8844	-4.0612
0.780	-1.6629	-12.6564	-8.03535	-13.4262

Error [%]

N = 20

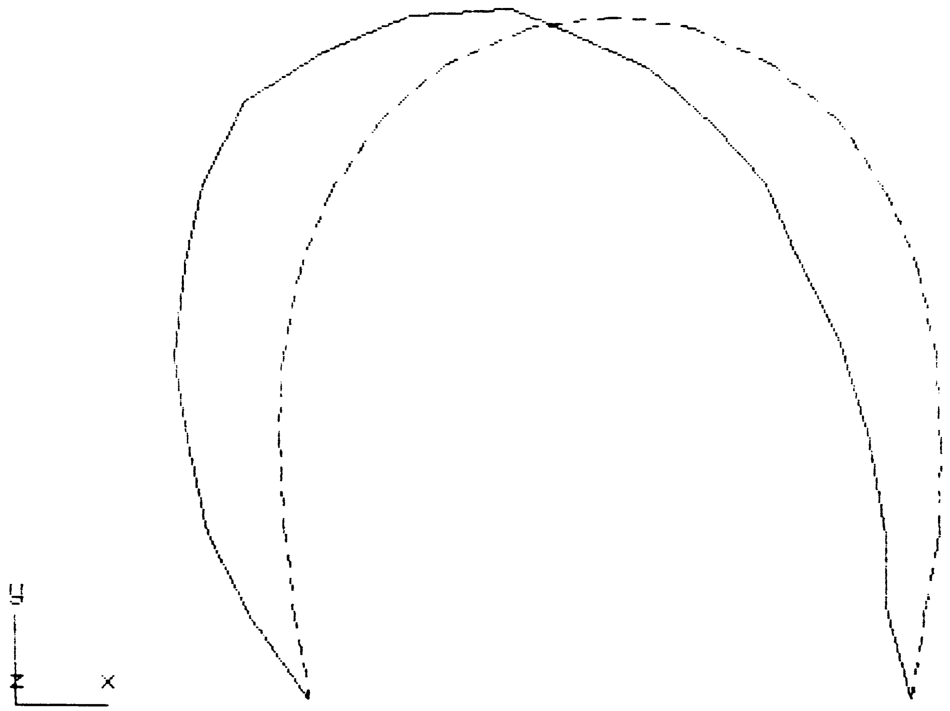
mode const.	1	2	3	4
0.6479	1.7593	5.345	11.504	1.653
0.654	0.8344	4.453	10.6710	0.7278
0.6782	-2.835	0.9283	7.3655	-2.9455
0.7156	-8.506	-4.545	2.2571	-8.6225
0.780	-18.2708	-13.954	-6.539	-18.398

Appendix 3

Mode Shape Data Aquisition Conditions

```
Data acquisition conditions:
 1  Trigger Type         2               9  Trigger Delay        0

 2  Trigger Level       15              10  Clear Freq  L     2.0000

 3  Coupling Code        0              11  Clear Freq  U     60.000

 4  Weighting Code       0              12  Minimum Freq     0.00000

 5  Ensemble Size       10              13  Overlap Facto        0

 6  Maximum Freq    64.000              14  Auxil Scale       1.0000

 7  A-A Filters     80.000              15  Reference Count      2

 8  Excitation           2              16  Response Count       2

19  Master Indent        0          **

Channel    20-Coord     21-Range     22-Scale     23-Signal     24-Bias
   1        100X+        8.0000       1.0000          4            0
   2         20X-        1.0000       1.0000          4            0
   3         20X-        2.0000       1.0000          3            0
   4         20Y+        0.5000       1.0000          3            0
   5          1X+        8.0000       1.0000          3            0
   6          1X+        8.0000       1.0000          3            0
   7          1X+        8.0000       1.0000          3            0
   8          1X+        8.0000       1.0000          3            0

Press RETURN... #
```

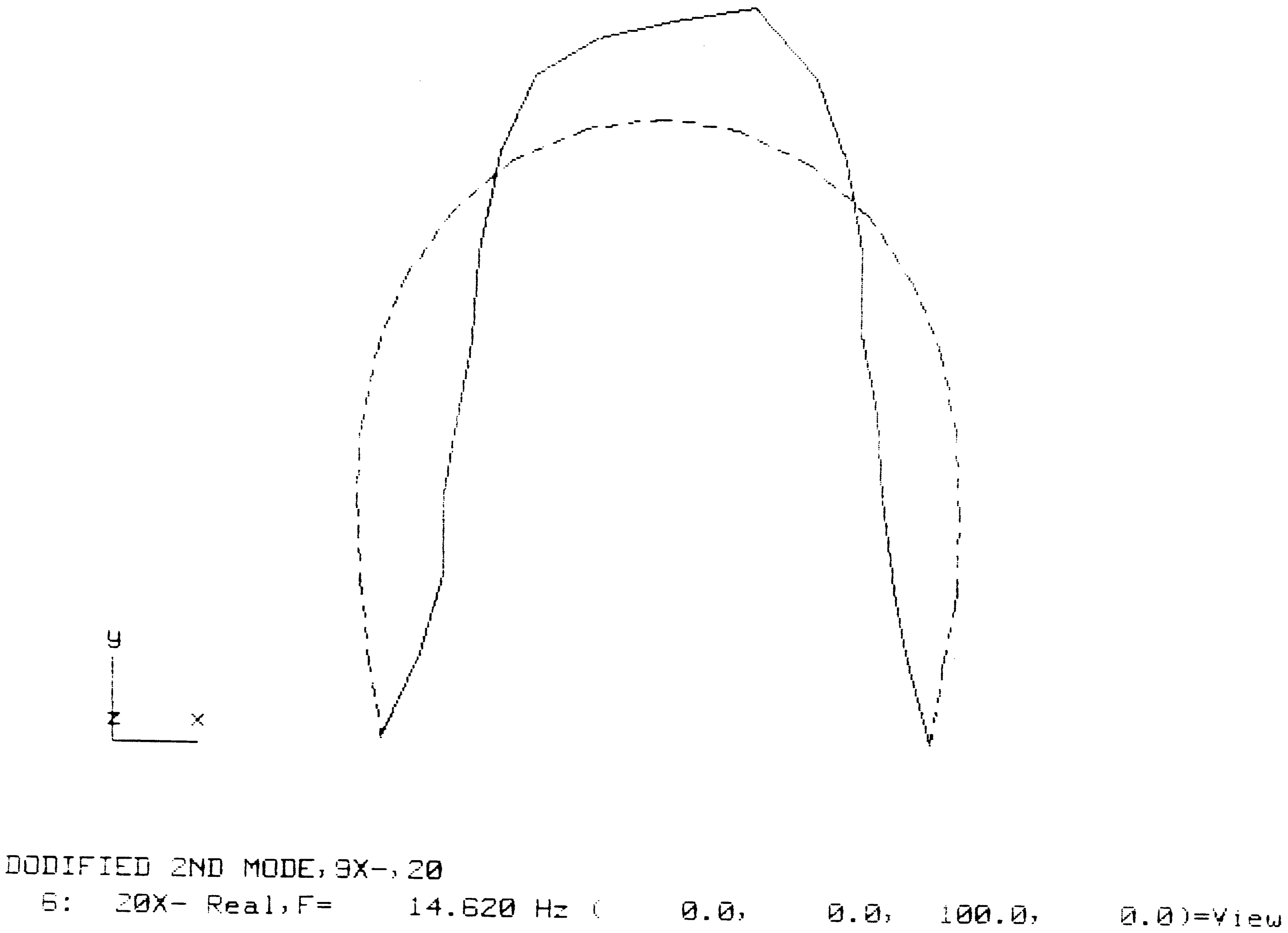
```
Modal Parameters, CONSOLIDATED

Label   Freq    Dampling    Amplitude    Phase    Ref    Res    Mode  Flags

  1      3.258  0.07638       1.483      1.571    20X-   19X-    1   0 0 0 1 1

  2     14.620  0.02088      14.48       1.758    20X-   19X-    2   0 0 0 1 1

  3     29.769  0.01973      43.83       1.596    20X-   19X-    3   0 0 0 1 1

  4     50.152  0.01300      59.86       1.490    20X-   19X-    4   0 0 0 1 1

Enter the modal parameter label number #
```



```
ALL CIR. FIT
  6:  20X- Real,F=     3.258 Hz (     0.0,     0.0,   100.0,     0.0)=View
```

First mode

Figure 7



Second mode

Figure 8