THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING

FLUID-DYNAMIC PERFORMANCE OF A CAVITATING VENTURI PART II

F. G. Hammitt
P. T. Chu
V. F. Cramer
A. Travers
C. L. Wakamo

December 1960

IP-486

ACKNOWLEDGEMENTS

The authors wish to acknowledge the financial support of the NASA in the research project upon which this report is based.

Aside from the authors, those personnel having a substantial part in the test program or analysis are: A. Ahles, Wm. Beckman, E. Brower, N. Oishi, T. Sheehan, J. Summers, and Wm. Walsh. In addition thanks are due to Mr. Edw. Rupke, Instrument Shop Supervisor, for his assistance in the design and fabrication of the components of the facility.

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NOMENCLATURE

p = pressure, p_{in} - fluid pressure at the inlet of the test section

p_{out} - fluid pressure at the outlet of the test section

p_{min} - the minimum fluid pressure measured in the test section

p_v - saturated vapor pressure of the fluid

p_{st} - static pressure of the fluid

ρ = density of the fluid

v = velocity. v_t refers to velocity of the fluid at the throat of the venturi

Re = Reynolds Number. Re_t refers to Reynolds Number at the throat of the venturi

σ Designated for two entirely different quantities. In one case, σ is designated for cavitation number, which by definition

$$\sigma = (p_{st} - p_v) \Big/ \frac{\rho v_t^2}{2g_c}$$

In other cases σ refers to variance, thus σ_σ means the varianc of cavitation number.

g_c conversion factor = $32.2 \ \frac{lbm}{lbf} \cdot \frac{ft.}{sec^2}$

FLUID-DYNAMIC PERFORMANCE OF A CAVITATING VENTURI

PART II

1.0 INTRODUCTION

A previous report[1] described the uses for cavitating venturis; i.e.: either (a) a flow measuring and/or controlling instrument, wherein the rate of flow is insensitive to downstream pressure, or (b) a research tool for the study of cavitation itself under conditions which can approximate to some extent those existing in actual fluid handling machinery. It then continued by presenting the results of experiments conducted with water in a cavitating venturi mounted in a closed loop system, which attempted to describe the general nature of the flow. These consisted of velocity profile measurements, gamma-ray void fraction measurements, and high-speed photography. In general, the point was made that the flow closely resembles supersonic flow in the diverging portion of a DeLaval nozzle, with the role of a normal shock played by a "condensation shock," or pseudo-hydraulic jump. The present report describes static pressure measurements taken in the same system under different conditions of cavitation, velocity, temperature, aeration, and test section geometry.

It is becoming increasingly well-recognized that cavitation is generally not initiated at the precise moment when the fluid static pressure becomes equal to the vapor pressure, but rather at a pressure which may depend upon absolute system dimensions, fluid velocity, type of fluid, fluid purity, temperature, gassification of the fluid, pressure-time history of the fluid, etc. The existence and importance of these various effects has been the subject of numerous recent research investigations in the cavitation field[2,3,4,5,6,7,8,9]. References 2, 3, and 4 provide an especially good summary of the literature and recent thinking on this subject. It is the purpose of this report to provide quantitative measurements of these various effects in the particular apparatus used. A third report will be issued in the near future, and will describe acoustical measurements which have been made of cavitation as a function of the various fluid parameters previously mentioned.

2.0 APPARATUS

The major equipment items and their disposition in the facility were previously described[1,10]. Those items significant to the present report will be briefly summarized for convenience. The closed loop includes approximately 20 feet of 1-1/2 inch stainless steel pipe and is powered by an overhung sump-type centrifugal pump capable of producing a head rise of about 125 feet of fluid at a flow rate of about 70 GPM. Since the transit time around the loop is quite short (~five seconds), there is the definite possibility of the reappearance, in the test section, of microscopic air bubbles liberated on the previous pass through the cavitating region. As explained in Reference 2, the restricted size of the equipment in general, and the loop in particular, is due to the desire to test high temperature liquid metals or cryogenic fluids in the same facility.

Two plexiglass venturis were used for the tests herein reported (Figures 1 and 2).* They were geometrically similar; i.e.: angles of convergence and divergence (approximately 6° diffuser included angle), and ratios of cylindrical throat length to diameter, were the same. The scale

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* Reproduced from Reference 1 for convenience

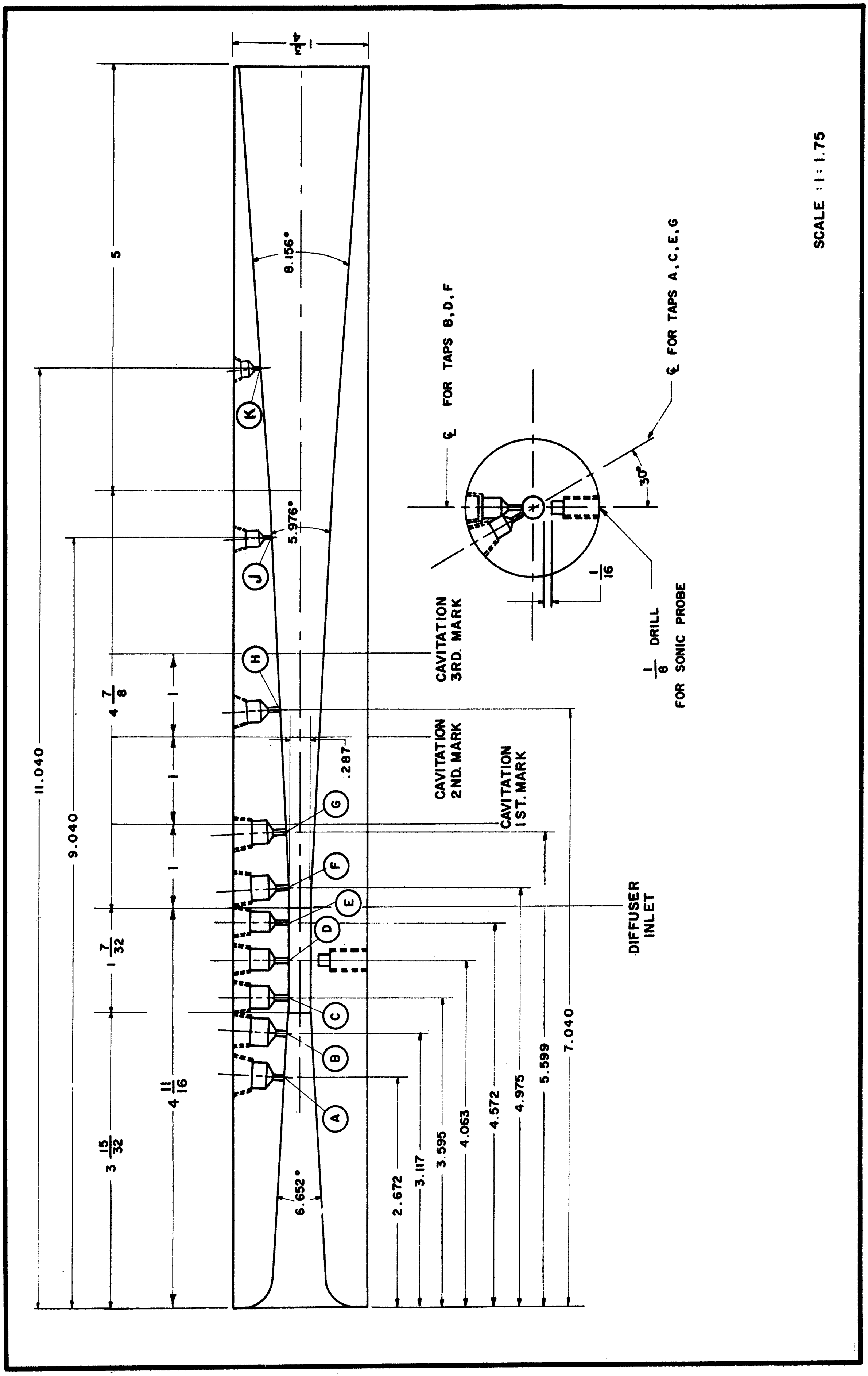

Figure 1. 1/4" Cavitating Venturi Test Section

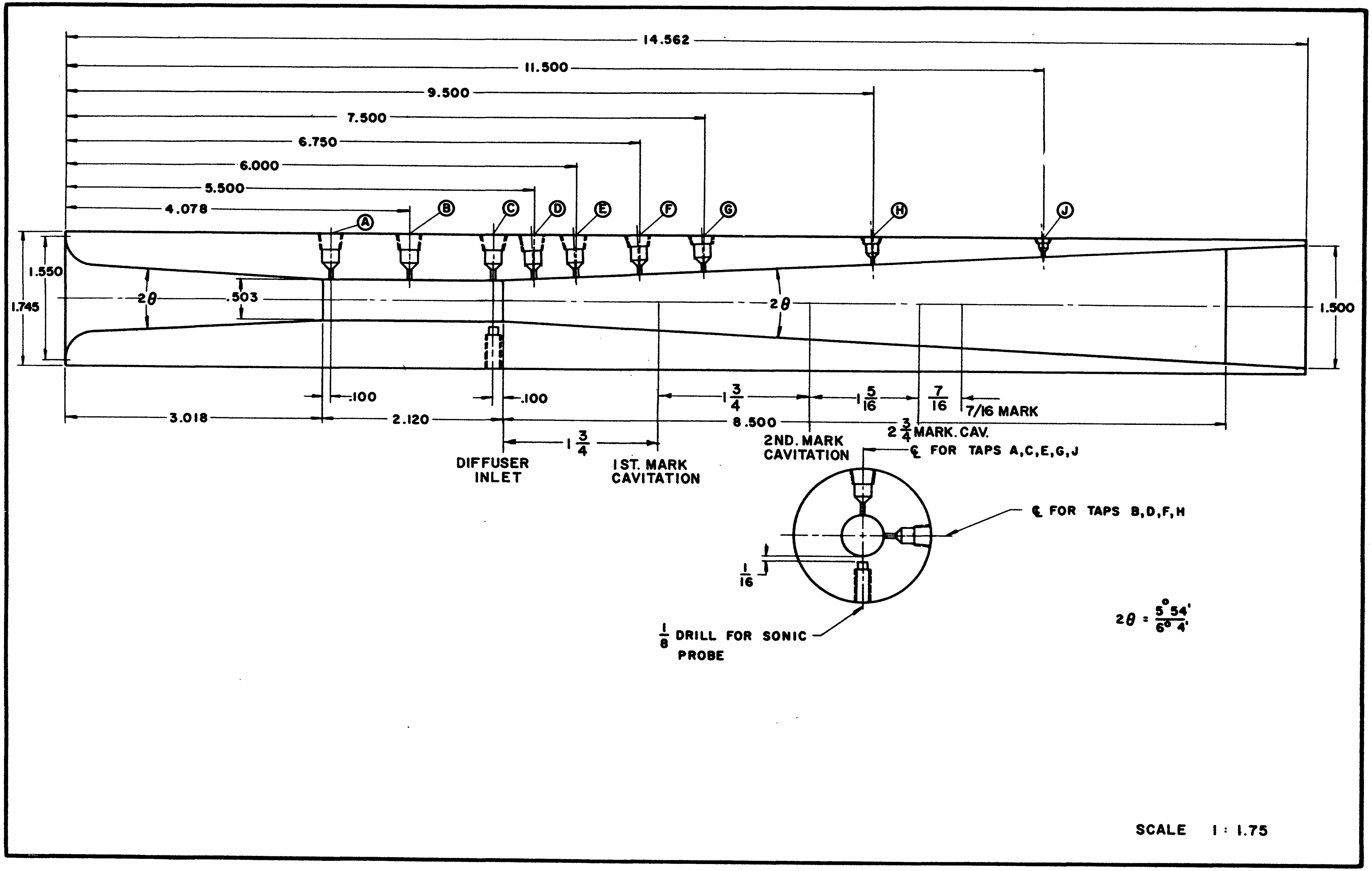

Figure 2. 1/2" Cavitating Venturi Test Section

factor between the units was about 1.75, with the throat diameter of the larger 0.503 inches, and of the smaller 0.287 inches. Since it was necessary that the overall lengths be the same so that they might be inserted interchangably into the loop, the diffuser and nozzle angles were not carried to the same diameters away from the throat in the smaller as in the larger unit. However, they were so designed that the kinetic head at the cut-off diameter was virtually negligible, so that no significant departure from similarity requirements was expected on this account.

Each venturi was equipped with pressure taps spaced along the length, the exact locations being indicated in Figures 1* and 2*. The tap size was approximately 1/16 inches and great care was taken to smooth the points of entry into the venturi.

The entire closed loop facility is shown schematically in Figure 3*, while Figure 4* is an actual photograph.

As explained in Reference 1, some control of degree of aeration could be exercised so that it was possible to maintain water with an air content of the order of one-half the saturation value at ambient temperature and one atmosphere. Air contents were measured with a Van Slyke apparatus. This was felt to be superior to the Winkler technique for oxygen determination because it measured total gas content, whereas from the Winkler method nitrogen and other constituents would have to be inferred from the oxygen measurement. It was found in preliminary tests that this was unreliable because of the depletion of oxygen with time in the loop (which was measured). It seemed unlikely that the nitrogen content should be depleted proportionately.

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* Reproduced from Reference 1 for convenience

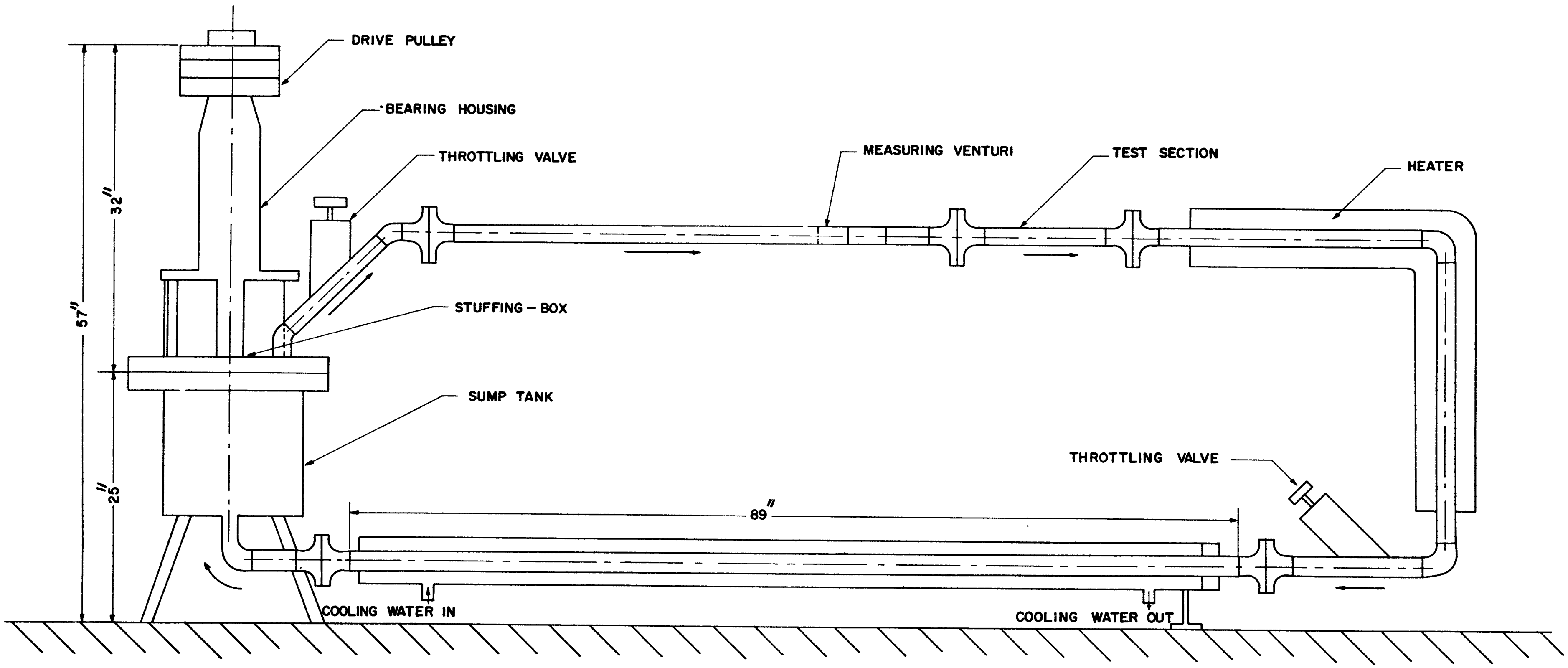


Figure 3. Sketch of Over-All Loop Layout

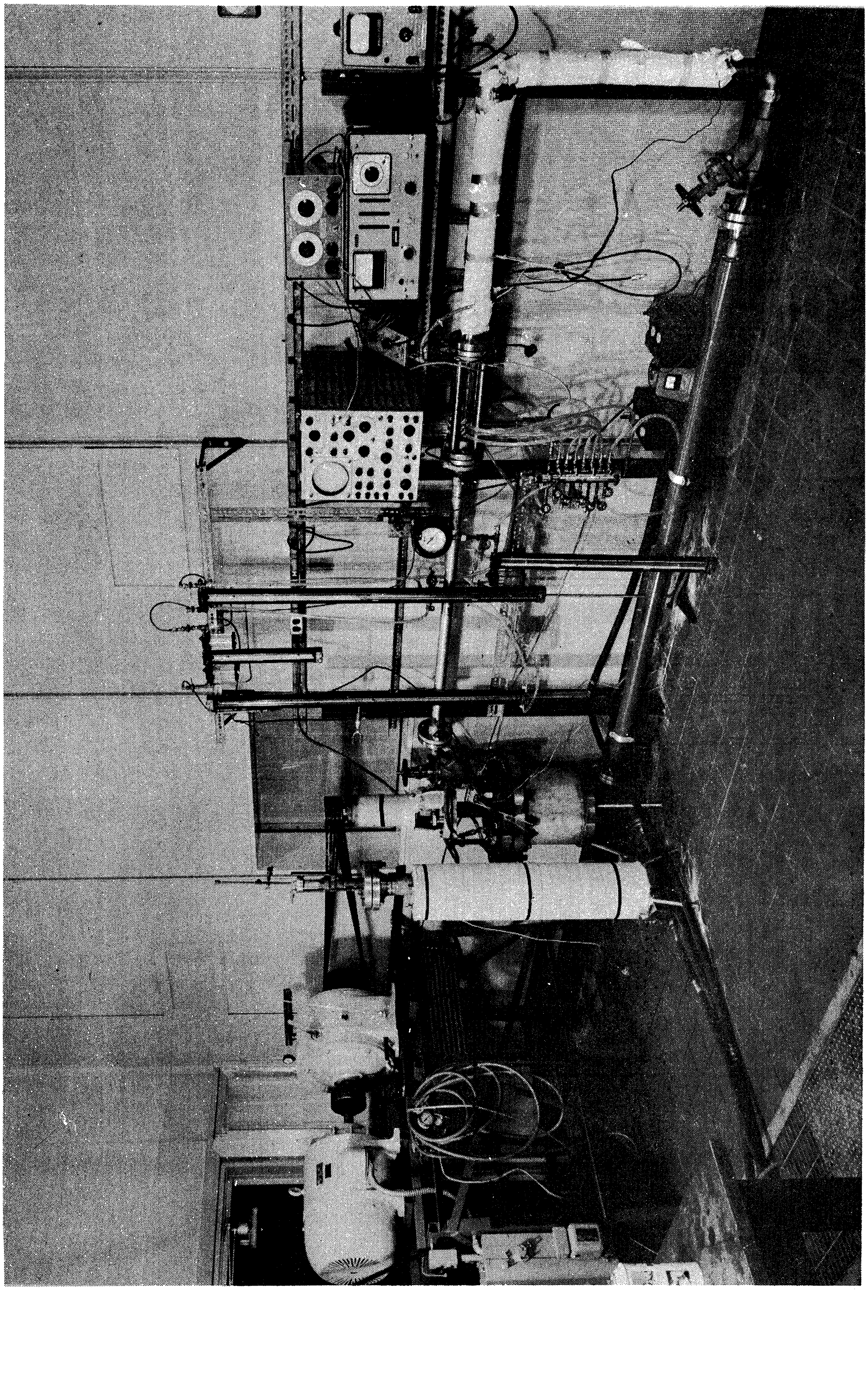


Figure 4. Photograph of Over-All Loop Layout

Static pressures at the various tap positions were measured by a mercury manometer, when in the range of plus or minus approximately one atmosphere gage, and by a calibrated bourdon gage when above this range. All pressure lines were led into a common manifold through simple on-off toggle valves. The manifold was connected through similar valves to the manometer and the bourdon gage. Hence it was not possible to read all pressures simultaneously. However, with the toggle valves, it was possible to make a reading of an individual pressure in a matter of a few seconds, so that a complete run could be finished within a minute or so. It is believed that steady-state was sufficiently well maintained over such time intervals that no significant error is introduced in this manner.

The static pressure lines subjected to vacuum were kept below the level of the test section to prevent boiling in the lines. In addition so-called "cold water" tests were run in most cases at a temperature substantially above room temperature (~90° F) so that the fluid in the lines would be cooler than the loop itself and thus would not be subjected to boiling even if the loop pressure were equal to or slightly less than vapor pressure.

The temperature of the loop was controlled by suitable use of a heater and cooler (see schematic of Figure 3). At high flow rates the energy input from the pump was more than sufficient to balance heat losses to the environment, in some cases so that it was then necessary to use the cooler to maintain steady-state. At low flow rates, the electric resistance heater, wrapped on a portion of the piping, was necessary to maintain elevated temperature (about 160 F). The maximum temperature limit is set primarily by the physical properties of the plexiglass venturis. However, it is sufficient to obtain a very great proportionate spread in water vapor pressure, and the rapidly increasing sensitivity of vapor pressure to temperature at higher temperatures would make sufficiently exact knowledge of the vapor pressure difficult to obtain.

The temperature was measured by a calibrated thermocouple probe inserted into the pump sump directly above the pump casing (Figure 5). It was felt that the mixing between the circulating fluid and the fluid in the sump was sufficiently rapid and complete to preclude the possibility of any significant difference between the temperature at this location and that in the loop. This is certainly the case if approximate steady-state exists as it did for these tests.

The exact location of a temperature measurement around the loop is not of significance, even for cases where either the heater or cooler is in operation, because energy input per pass of the fluid is not sufficient to cause a measurable temperature change.

Flow was measured by either a calibrated orifice or flow-venturi placed in the loop piping. The orifice was used for the lower flow rates associated with the small venturi; the flow-venturi for the larger flows. The flow-venturi was designed for a minimum head loss at the maximum flow rates so that the highest possible test section velocities might be achieved. Hence, its differential pressure for low flows was not sufficient for precise measurement. For this reason a sharp-edged orifice was designed and calibrated for the low flow range. The additional head loss of the orifice was permissible at low flow because the pump head is increased and loop head losses reduced.

The first manifestation of cavitation in the venturi, "sonic initiation" was determined by an acoustic pick-up, using a barium-titenate

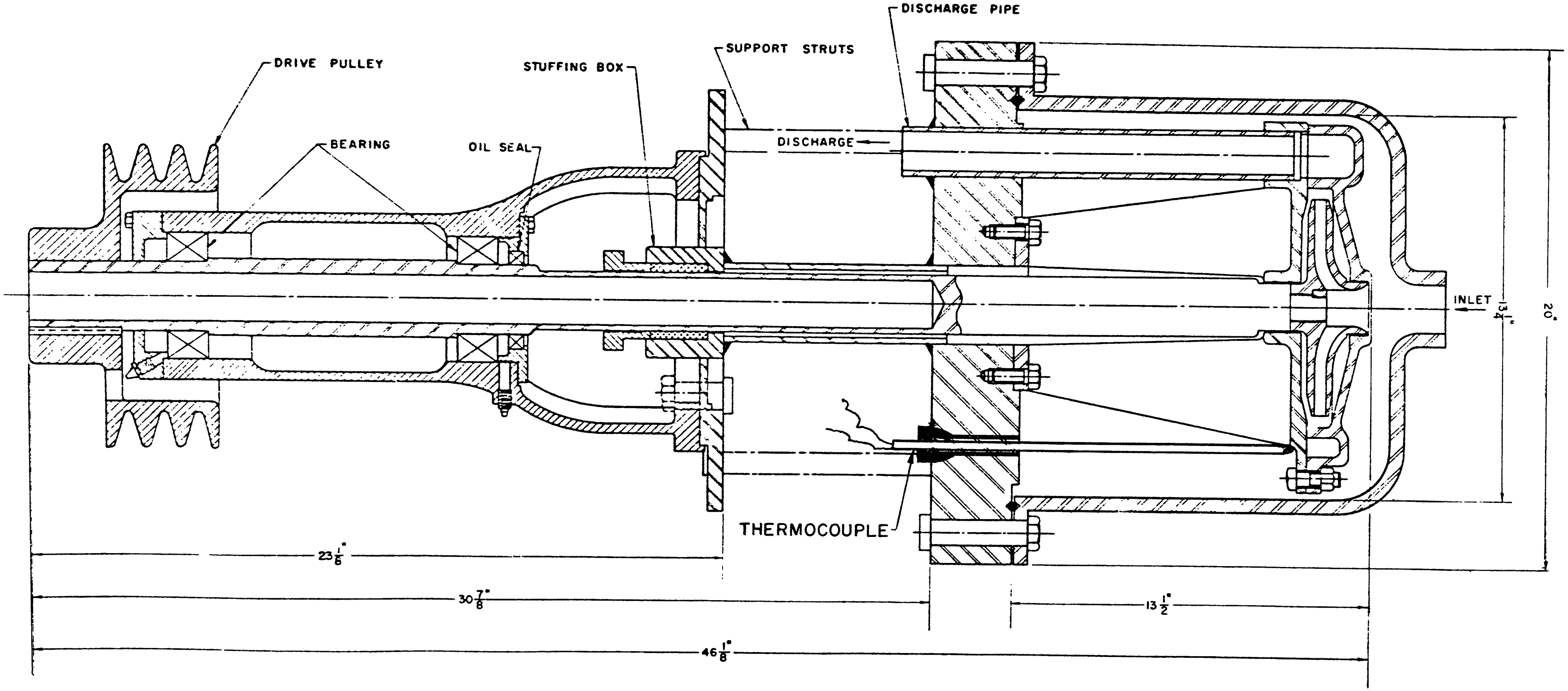


Figure 5. Sketch of Pump Cross-Section

crystal in conjunction with a low frequency filter to eliminate machinery noise, feeding an oscilloscope through suitable amplifiers. The details of the electronic circuitry will be given in a future report dealing specifically with the acoustic work. At first the pick-up was mounted externally to the test section; in later tests it was inserted into one of the pressure tap connections so that the water was in actual contact with the pick-up.

3.0 TEST PROGRAM

3.1 General Description of Tests

The tests were performed with the following parameters as independent variables: degree of cavitation, mean venturi throat velocity, fluid temperature, fluid aeration, and venturi size. The dependent variables which were measured were then the static pressures at the various axial locations along the test section. From these, the other dependent variables, i.e.: cavitation number, venturi loss coefficient, and axial pressure gradients, were computed.

The precise meaning of most of the parameters described above is clear. Those which are not in very common usage will be described below.

3.1.1 Degree of Cavitation

The "degree" of cavitation is intended to describe the geometrical configuration of the cavitating region, whereas the "intensity" describes the rate of pitting, violence of collapse, etc., depending on the velocity and other parameters. Hence, it is possible to obtain various intensities at a fixed degree and vice versa. This usage has been developed previously in the reports pertaining to this project, as well as to some extent in the literature on cavitation.

For the present tests, six degrees of cavitation have been recognized:

i) Zero Cavitation - no change in flow phenomena if pressure level is increased.

ii) Sonic Initiation - The flow condition where some acoustic manifestation, different from that for single-phase flow, first becomes evident. This was detected by a piezo-electric crystal attached to the test section with output displayed on an oscilloscope. The electronic detection so obtained corresponded closely to audible detection. The details of the electronic circuitry will be presented in a future report.

iii) Visible Initiation - The flow condition where a complete ring of cavitation first becomes visible. This occurs at the downstream end of the cylindrical throat. In the present tests some intermittent or continuous cavitation may become visible before this point, initiated by the slight discontinuity of one of the pressure taps--this was not considered in defining visible initiation.

iv) First Mark - Three marks were placed on the outside of the smaller test section, spaced one, two, and three inches respectively from the downstream end of the cylindrical throat. Similar marks were placed on the larger test section. To maintain geometric similarity the spacing was increased by the scale factor between the test sections to 1-3/4 inches. "First Mark" cavitation refers to the flow condition for which the cavitation region terminates at the first mark. The termination is not entirely sharp, becoming less so as the cavitation develops further into the diffuser. The visually determined termination region covers approximately 1/2 to 1/4 inches, depending to some extent on the other parameters. The approximate middle of this zone was judged by eye, and defined to be the termination point for the determination of cavitation degree. Repetitive tests, which will be discussed in a later section, showed a high degree of repeatability for such settings as far as the influence on the other parameters was concerned.

v) and vi) Second Mark - See explanation above.

3.1.2 Cavitation Number

As used in this report, the cavitation number is the ratio of the difference between minimum fluid static pressure in the test section and vapor pressure, to the kinetic head, corresponding to the mean throat velocity, in pressure units:

As was shown in Reference 1, the velocity profile in the throat is virtually square so that the maximum and mean velocities do not differ widely. Since the throat is cylindrical (for a length of 4.3 diameters), and the angle of convergence and divergence small, it would be expected, under the assumption of cavitation at the vapor pressure, that the cavitation number so measured would be zero. Actually this was generally not the case as will be explained in a later section.

3.1.3 Venturi Loss Coefficient

Venturi loss coefficient, as used in this report, is defined as the overall static pressure drop from inlet to exit of the venturi divided by the static pressure drop from inlet to minimum pressure point (generally near cylindrical throat discharge):

As would be expected this is a strong function of the degree of cavitation as will be explained later. It is of interest as a basic observable parameter and might well be used to indicate the degree of cavitation in a non-transparent system.

3.2 Anticipated Effects

3.2.1 Cavitation Number

According to the idealized model of cavitation, which was rather generally accepted in the past, cavitation in such a facility should occur whenever the local static pressure becomes equal to the vapor pressure. This would imply that cavitation number, for a given cavitation condition, would not be sensitive to velocity, fluid purity, type of fluid, pressure, temperature, or size of test section. It has become increasingly recognized that there are significant variations from this idealized model(2,3,4,5,6,7,8,9). It was the purpose of the tests herein described to investigate these possible variations and to measure as precisely as possible the actual cavitation numbers obtained in this facility as a function of the above variables. If possible, the cavitation numbers should be correlated with suitable non-dimensional parameters. If future meaningful results are to be obtained with fluids other than water, it is necessary to have a well-defined basis for comparison with water.

3.2.1.1 Gassification (Aeration of Fluid)

Since a pure fluid could not theoretically be caused to cavitate by the imposition of pressures near or slightly below the vapor pressure, it has been postulated by numerous investigators(2,3,6,8) that such cavitation, which is in fact observed, must be the result of local weakening of the fluid structure by discontinuities such as unwetted impurities or entrained gas particles. A plausible mechanism for the continued existence of such finite entrained gas particles in acute angle crevices of unwetted particles can be postulated even for liquids which have been maintained with total gas contents well below saturation for extended periods. Consequently it would be supposed that the gas content of the fluid is important to cavitation, and in general that, other things being equal, the cavitation number would decrease as the gas content decreased. Also, the data should become more repeatable if large and unknown quantities of entrained gas are not present.

As has been pointed out in recent studies[6,8], it would be expected that cavitation would be influenced only by that portion of the gas which is actually entrained, rather than dissolved. Typically, this may be only about one percent of the total[8]. However, aside from acoustic techniques still under development[8] only total gas can be measured. This can be accomplished either chemically, (Winkler test for oxygen for example) or mechanically, as in the Van Slyke technique, where the liquid is agitated under vacuum and the emitted gas volume measured. This latter technique has been used for the present investigation as in most other recent studies of this type. It is believed preferable to the chemical technique, since it measures the total gas content directly whereas the Winkler technique measures one constituent only. The inference that the various atmospheric gases will exist in water in the same proportions as would be predicted by Henry's Law has been found unreliable.

Although there is no way to distinguish between entrained and dissolved gas with this method, it seems generally reasonable that the entrained gas content should decrease as the total content decreases, if a reasonably long settling period before runs is allowed. This is sufficient for the present purposes: i.e., the achievement of repeatable conditions with regard to gassification which are capable of numerical specification. The precise delineation of the effect of given quantities of entrained gas would require considerably more refined equipment, and would not be justified at present. However, some approximate limits can be set.

It is not known how low a dissolved air content in a settled solution would be required to cause significant changes in cavitation behavior. There are two limiting conditions which may be mentioned.

a) Boiler feed water contains usually $\gtrsim$ 1 ppm by weight of oxygen giving an air content at most of about 15 percent of saturation at one atmosphere and hot-well temperature. This does not apparently prevent either condensate or boiler feed pump from cavitating. The percent saturation for the condensate pump may be quite high since the pressure is so low. The percent saturation at the boiler feed pump inlet is very low. However, the solution has not settled.

b) In a recent paper, Knapp(11) observed a substantial effect upon boiling temperature if the water had been pressurized to about 30 atmospheres for one hour immediately prior to the test. Assuming that the water was saturated before the test, the percent saturation during the pressurization phase was only about three percent.

It would appear from the above examples that air contents at least as low as ten percent of saturation would be necessary to cause a significant effect.

3.2.1.2 Velocity and Size Effects

Effects upon cavitation number or pressure profiles at a given cavitation number and under conditions of geometric similarity with changes in mean velocity or absolute test section dimensions are normally grouped under the heading of "scale effects". Various recent investigations have shown that such effects may be significant[3,5] although the mechanism is not clear, and the trends observed by the different investigators conflict. Such behavior probably has its origin in the nucleation process and the effects of entrained gas and impurities. It was not anticipated that the present tests would result in a fundamental explanation of the trends, but rather their delineation in this specific facility, so that in this respect future meaningful comparisons with the behavior of other fluids in the same facility could be made.

3.2.1.3 Degree of Cavitation

Let cavitation number be defined with respect to the static pressure at throat discharge. Then if it is assumed that cavitation always occurs first at this point, and whenever the static pressure reaches vapor pressure, then it would be anticipated that cavitation number would be independent of cavitation degree.

It has been pointed out previously that the second assumption is not in general true, and it is one of the objectives of the tests to discover the degree of invalidity of this assumption. The first statement appears to be valid, if it is realized that the presence of cavitation downstream of the throat discharge should not affect the nozzle efficiency or frictional losses in the throat section.

In general it would be supposed that the minimum pressure would always occur at the throat discharge, regardless of degree of cavitation. As it happens, the tests show that it actually occurs downstream of this point by an amount dependent upon degree of cavitation. In the light of this knowledge, the cavitation number has been defined in this report in terms of the minimum pressure for each degree of cavitation, and hence there is a dependence of cavitation number on cavitation degree, even if the fluid were ideal in the sense of cavitating precisely when local static pressure reaches vapor pressure.

3.2.2 Venturi Loss Coefficient

The anticipated effects upon the venturi loss coefficient of the various independent variables of the tests will be examined briefly below. The macroscopic parameter of loss coefficient is of basic importance in predicting the performance of a cavitating venturi in a flow system, and as an indication of the degree of cavitation which exists in a non-transparent system.

3.2.2.1 Degree of Cavitation

Of the various independent parameters, it seems intuitively reasonable that venturi loss coefficient should be most affected by degree of cavitation, since this represents a gross disturbance to the flow and, presumably, impairment of diffuser efficiency. Of course, it is not to be expected that nozzle efficiency should change.

The velocity profiles existing in the diffuser with various degrees of cavitation, and without cavitation, were previously reported[1]. Since the flow pattern is substantially a free central jet of liquid surrounded by vaporous regions in the cavitating area, no significant diffusing action occurs upstream of the region where cavitation terminates. There is a rapid pressure rise in the region of cavitation termination, much like a normal shock wave in a DeLaval nozzle (although spread over a greater axial distance). Since such a process is highly irreversible, it is not to be expected that the pressure increase would be as great as that which would have occurred in the absence of cavitation. For example, the non-cavitating diffuser efficiency for this unit is about 80 percent. Hence, as with a DeLaval nozzle, the pressure reduction from inlet to outlet increases as the discontinuity (normal compressible gas shock for supersonic flow or "condensation" shock for this case) is moved downstream. Assuming the ideal model of a discontinuity of zero axial thickness and perfect uniformity across the stream, it can be shown that the pressure drop across the component is very sensitive to the axial location of the discontinuity and hence should be a good indication of degree of cavitation.

3.2.2.2 Temperature

It is anticipated that a given degree of cavitation, (i.e.: axial extent of cavitating region), achieved with a given fluid but at different temperature levels, should result in different loss coefficients because of the different vapor percentages (void fractions) in the stream. Quantitative measurements of the void fractions under these conditions were reported previously, and differences in the direction theoretically anticipated were observed[1]. The theoretical reasoning is summarized below.

It has been emphasized in recent papers[12,13,14] that there should be a difference in volume formation of vapor per unit head depression for different fluids, and for the same fluid at different temperatures. This difference has been expressed in terms of a thermodynamic parameter, B, which is given in slightly different terms in the different references. However, under these assumptions it would be expected that the volume vapor formation in hot water, for example, would be considerably less than in cold water for unit head depression below the bulk saturation pressure. This is obvious considering the very great difference in vapor densities between hot and cold water, the much smaller proportionate difference in latent heat, and the large decrease in temperature depression corresponding to a given pressure depression.

It has been previously mentioned[1] that less vapor was formed for a given cavitation condition with hot (~160F) water than cold (~80F) water. It might also be expected that the loss coefficient for a given degree of cavitation, assuming a given degree implies a given gross geometrical configuration, would be less for hot than cold water, since the vapor volume would be less and the disturbance to the flow less. Similar, but more pronounced, differences might be expected to exist between cryogenic fluids at the "hot water" end of the scale, and liquid metals at the "cold water" end.

3.2.2.3 Gassification of Fluid

Some interrelation between venturi loss coefficient and gas content of the fluid might be expected if the gas content were very high, since the gas would then expand in the low pressure regions and contribute significantly to the "void" volume. However, for gas contents of the order of saturation or less, it is not believed that a significant effect could exist, since insufficient time is afforded in the low pressure region for significant quantities of gas to be liberated from the solution, and the possible quanitity of entrained gas under such conditions of total gas content is very low. No special attempts to observe effects of this type were made since it was not believed that they would be significant.

3.2.2.4 Test Section Size and Fluid Velocity (i.e.: Scale Effects)

For a given degree of cavitation, no clear-cut mechanism whereby scale effects could influence the venturi loss coefficient is evident. However, since the origin of the scale effects is not clear and probably involves the microscopic nucleation behavior of the bubbles, it cannot be stated with confidence that no such effect exists. For this reason, tests were made in this regard.

From another viewpoint, arguments for the independence of loss coefficient and test section size for a given cavitation degree rest on the assumption that ordinary dynamic and geometric scaling, as applied for single-phase fluids, is applicable to this two-phase situation. However, the formulation of suitable scaling procedures for two-phase flow does not exist at present. Various papers in the literature discuss this situation as, for example, References 5 and 8.

3.3 ACTUAL TEST RESULTS

Variations from the predictions of an idealized model of cavitating flow in a venturi which might be anticipated have been discussed in the preceding section. The actual test results which were obtained will now be discussed with special reference to those variations. In a later section those items which appear of special significance will be emphasized.

3.3.1 Axial Pressure Profiles

Knowledge of the axial pressure profiles which the fluid encounters in this facility as a consequence of the various independent parameters is essential if the results are to be at all applicable to cavitating flow in other types of components (as, for example, turbomachines). In addition, the detailed profiles are required for the determination of minimum pressure and hence cavitation number, and certain pressures are required for the calculation of loss coefficient, and nozzle and diffuser efficiency.

Figures 6 through 17 show static pressure as a function of axial position for the large and small venturis, (Figures 6 through 9 for the large test section, 10 through 13 for the small, and 14 through 17 illustrating the differences between test sections). The pressure has been normalized by dividing through by $\rho V_t^2/2g_o$, the kinetic pressure in the

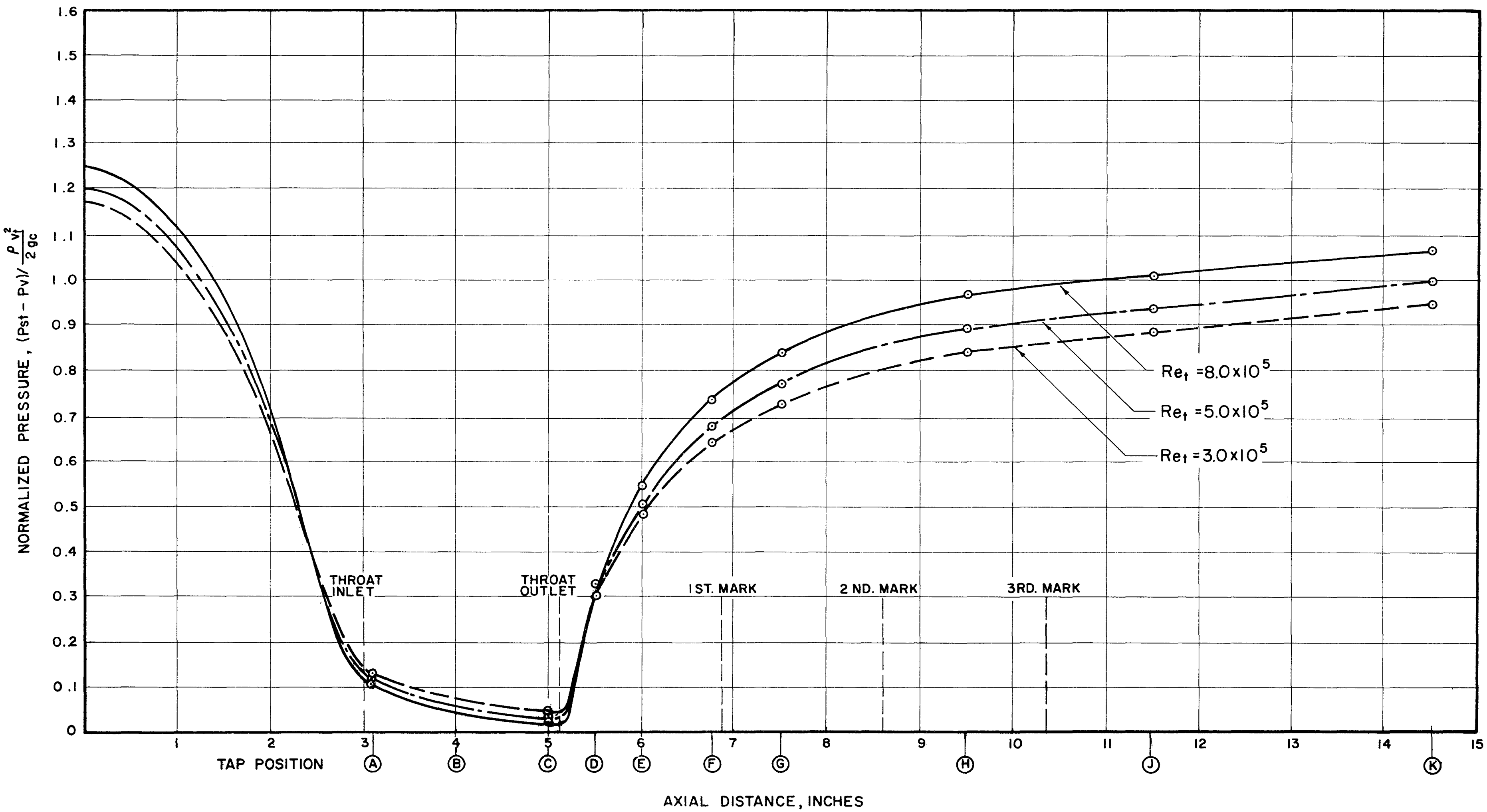


Figure 6. Large (1/2") Test Section. Sonic Initiation (or Zero Cavitation). Degassed Water (including both hot and cold water data).

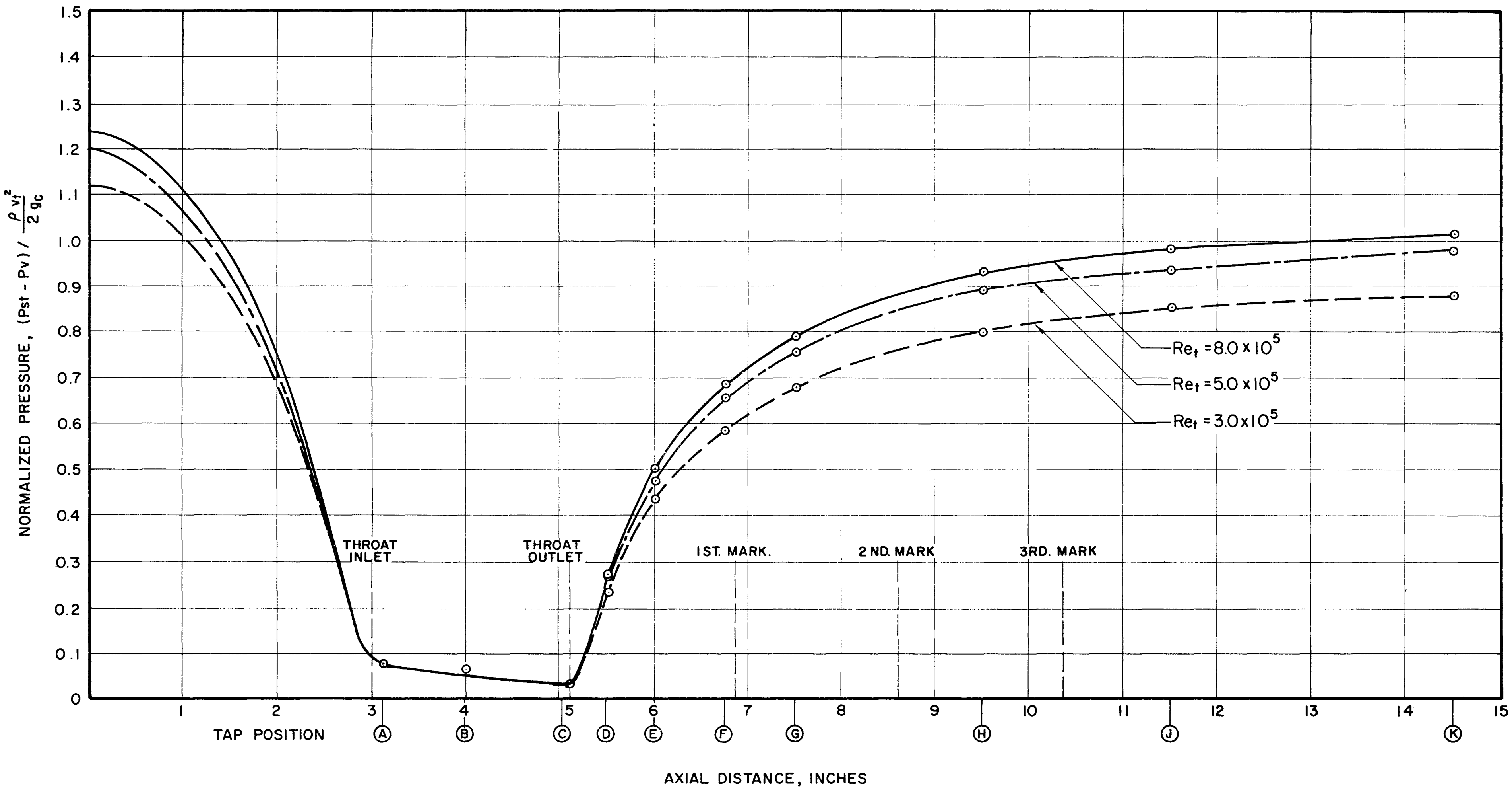


Figure 7. Large (1/2") Test Section. Visible Initiation. Degassed Water (including both hot and cold water data).

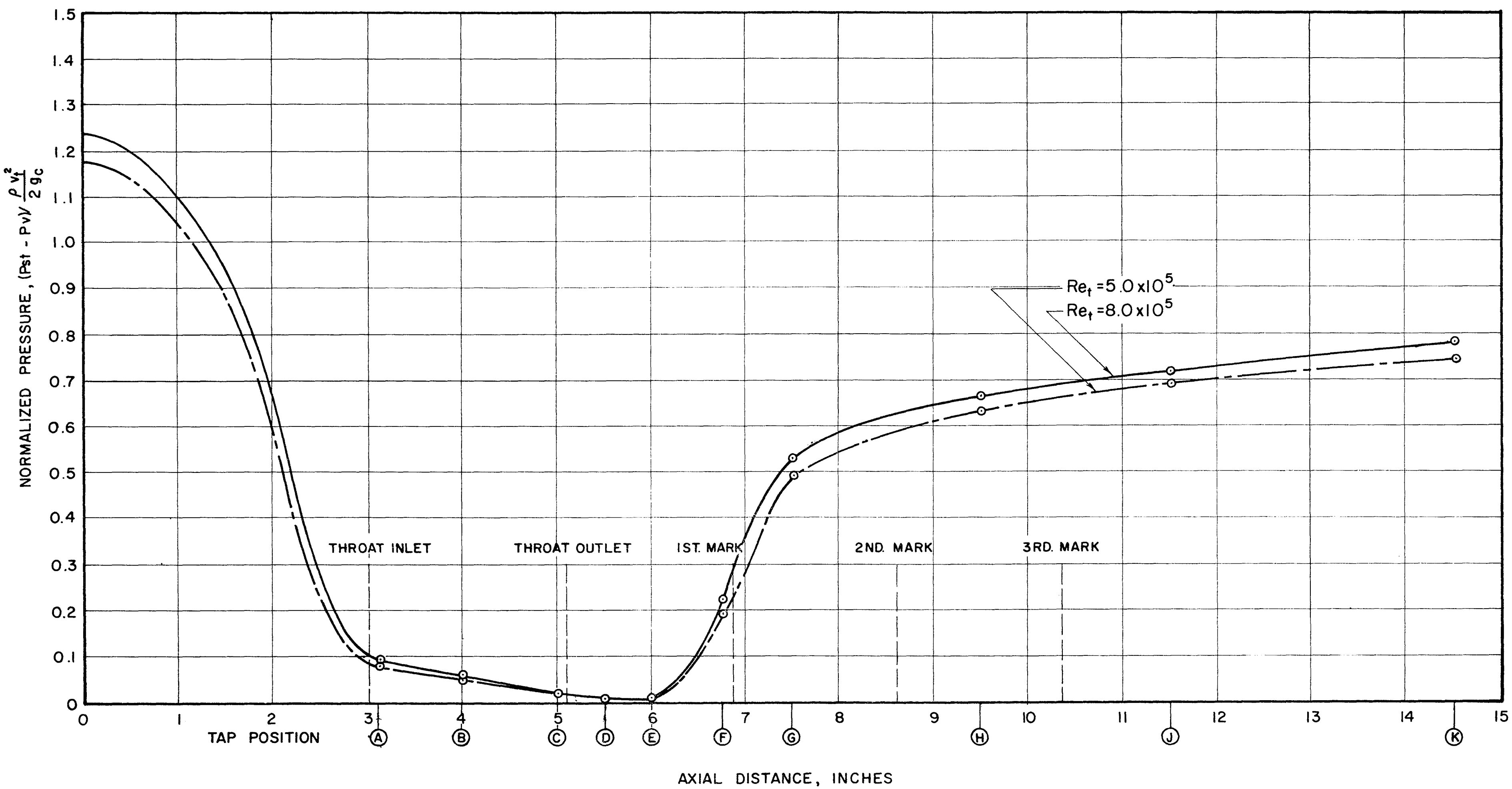


Figure 8. Large (1/2") Test Section. Cavitation Terminates at First Mark. Degassed Water (including both hot and cold water data).

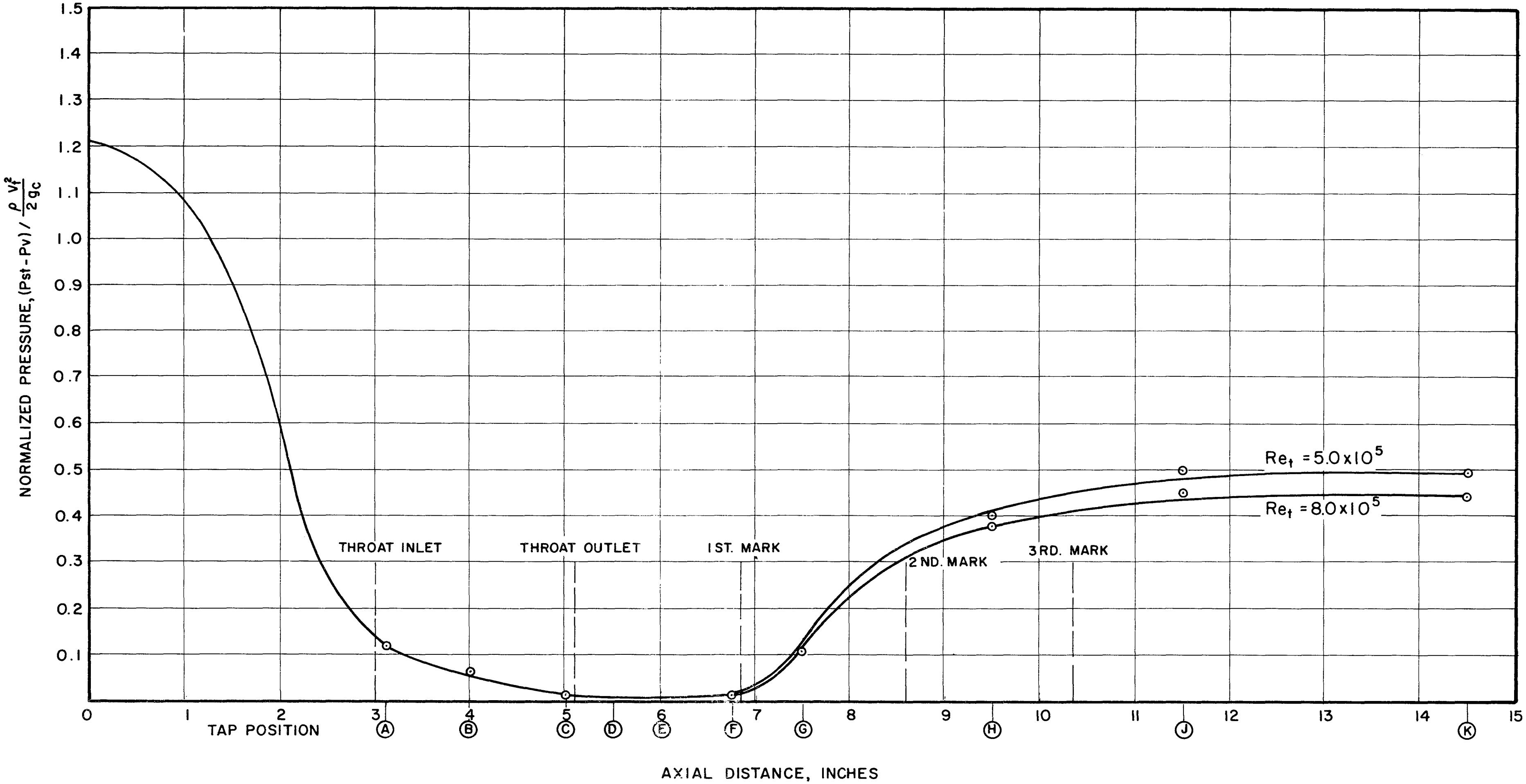


Figure 9. Large (1/2") Test Section. Cavitation Terminates at Second Mark.
Degassed Water (including both hot and cold water data).

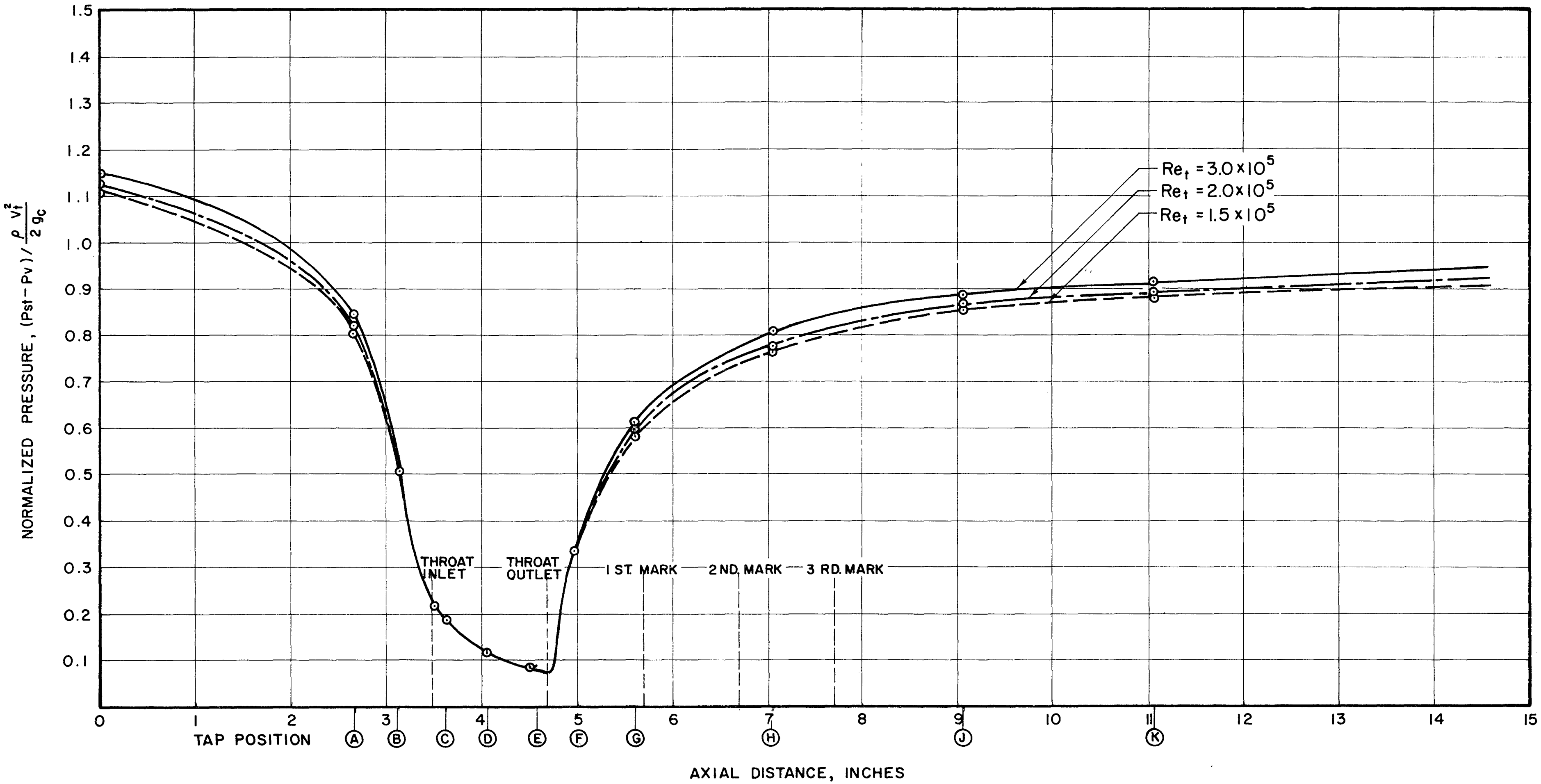


Figure 10. Small (1/4") Test Section. Sonic Initiation. Degassed Water (including both hot and cold water data).

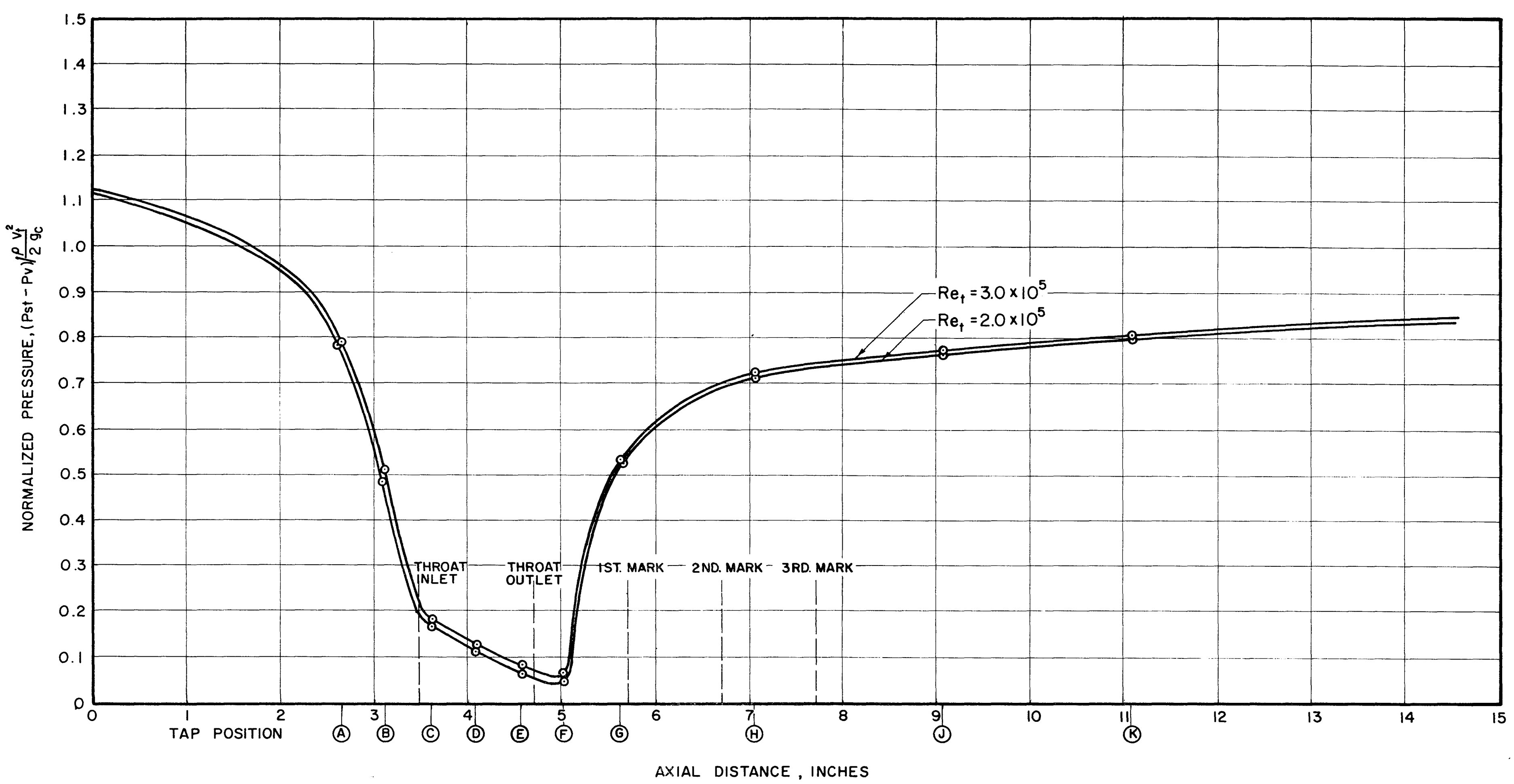


Figure 11. Small (1/4") Test Section. Visible Initiation. Degassed Water (including both hot and cold water data).

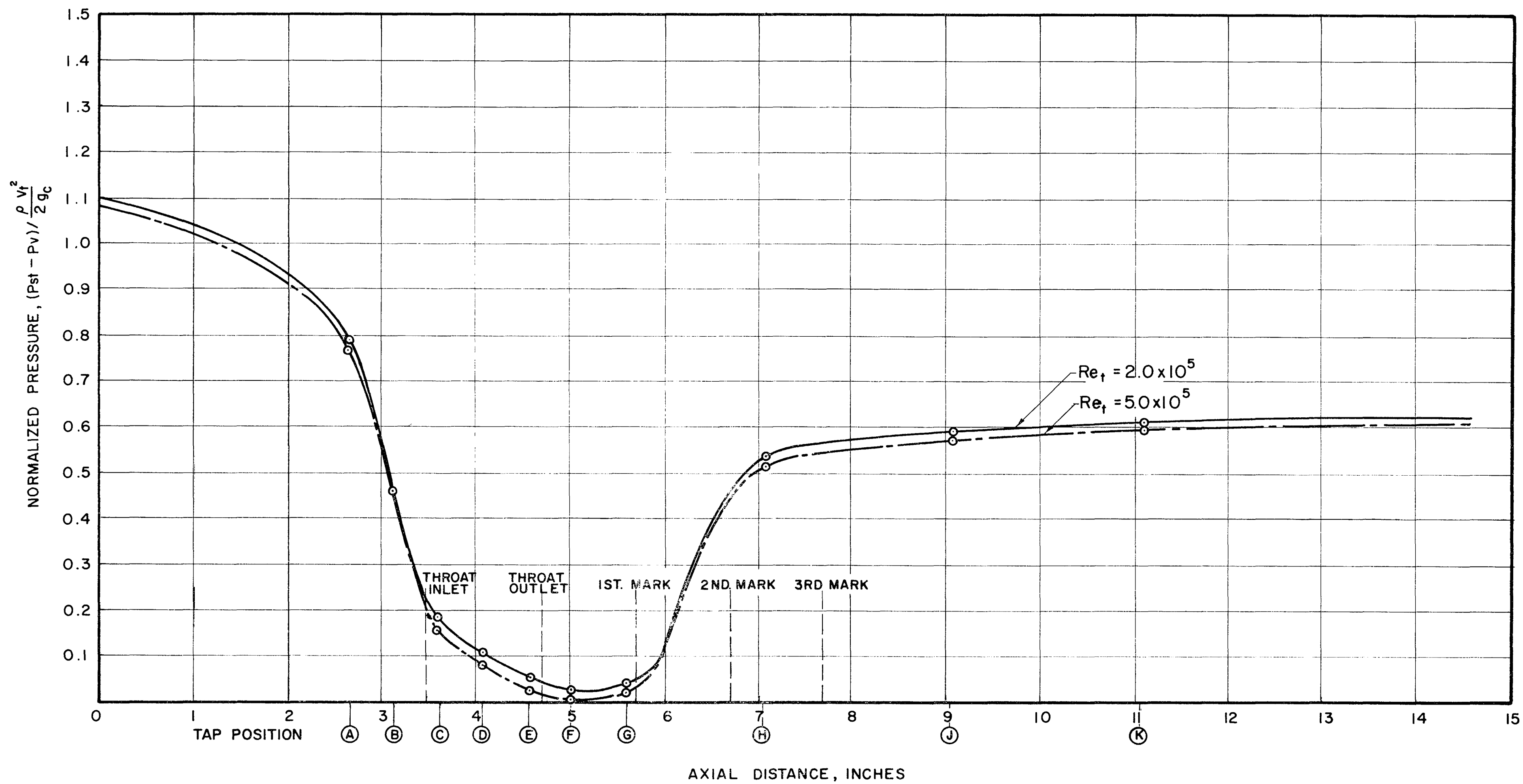


Figure 12. Small (1/4") Test Section. Cavitation Terminates at First Mark. Degassed Water (including both hot and cold water data).

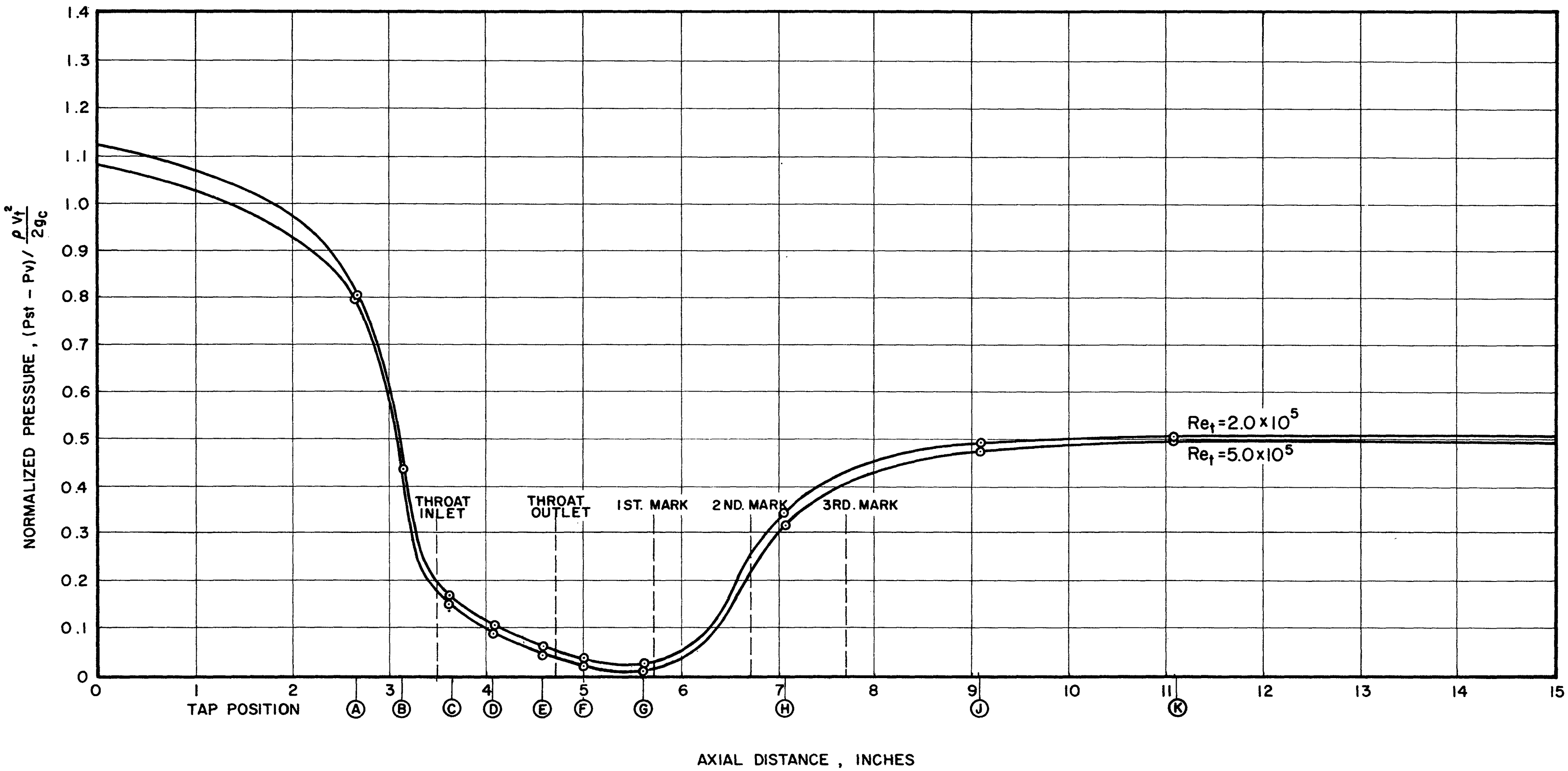


Figure 13. Small (1/4") Test Section. Cavitation Terminates at Second Mark. Degassed Water (including both hot and cold water data)

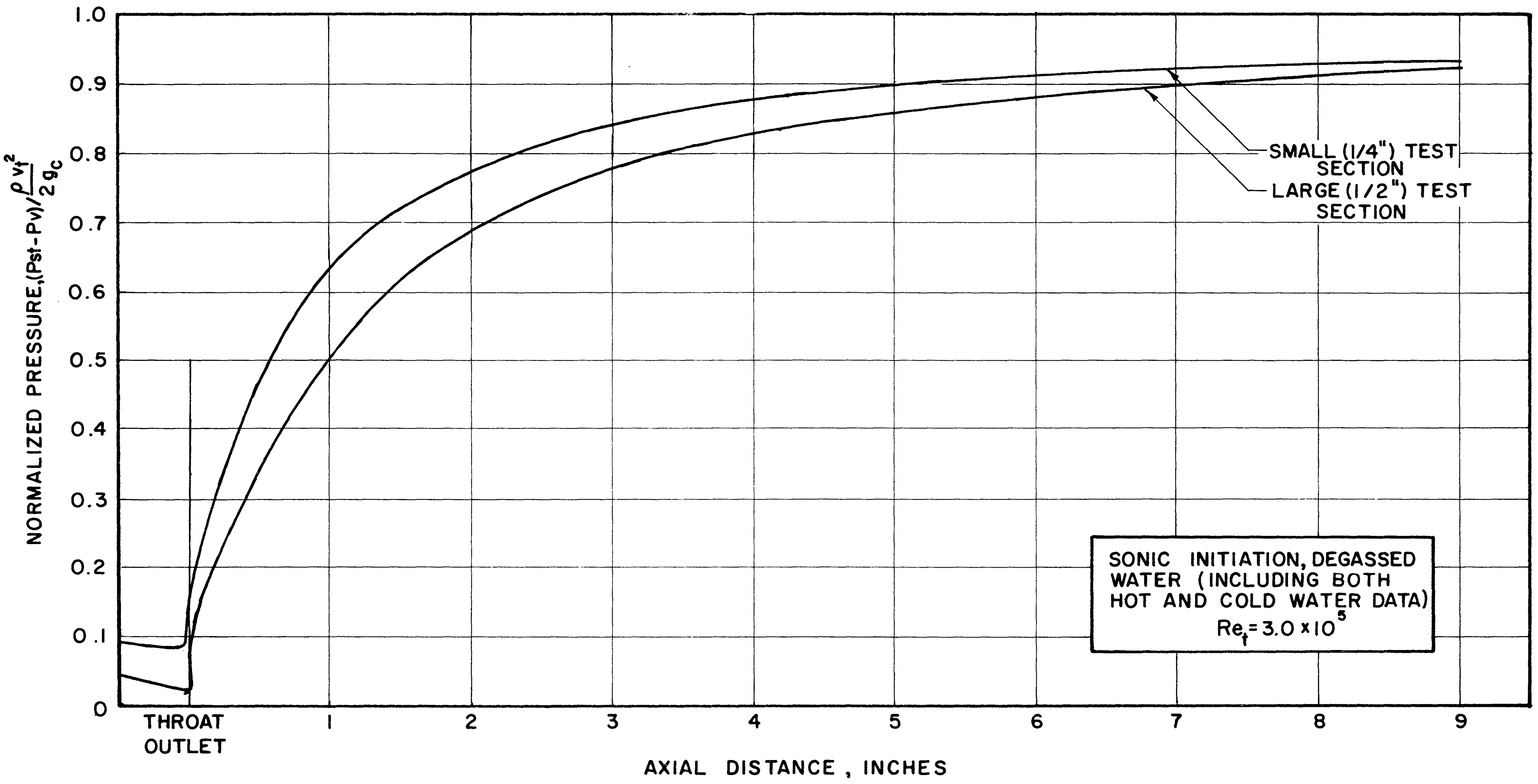


Figure 14. Comparison of Axial Pressure Profile in Different Test Sections.

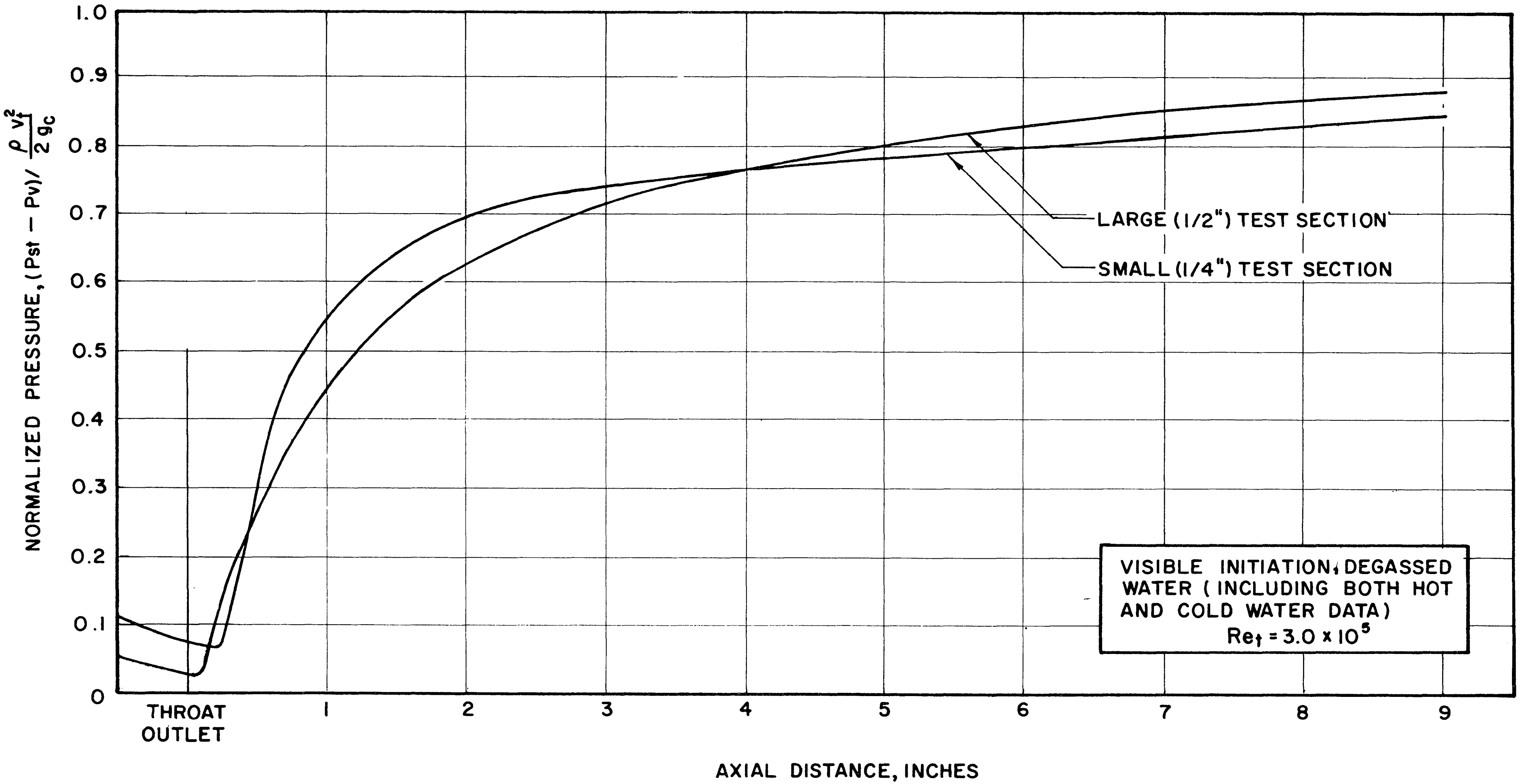


Figure 15. Comparison of Axial Pressure Profile in Different Test Sections.

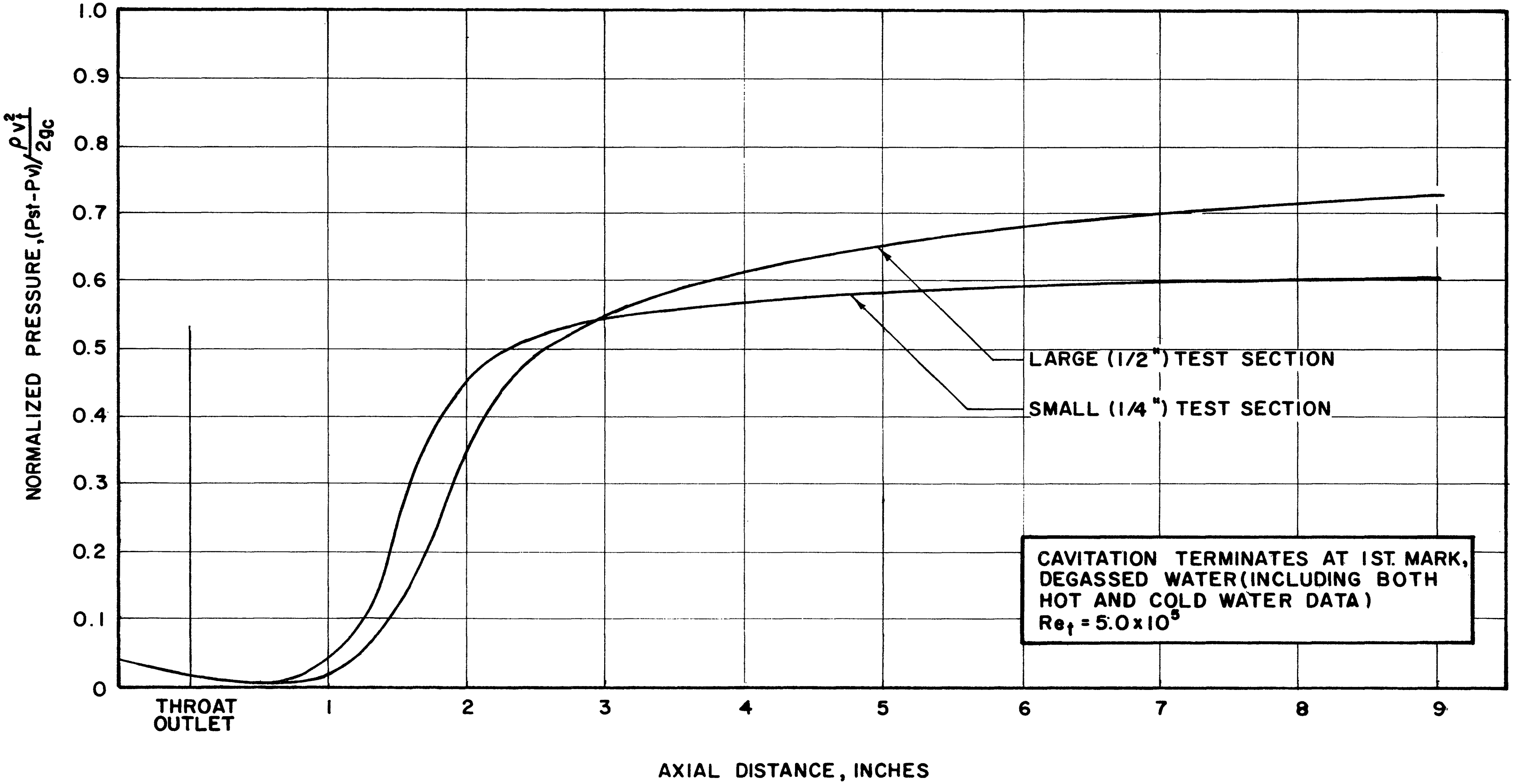


Figure 16. Comparison of Axial Pressure Profile in Different Test Sections.

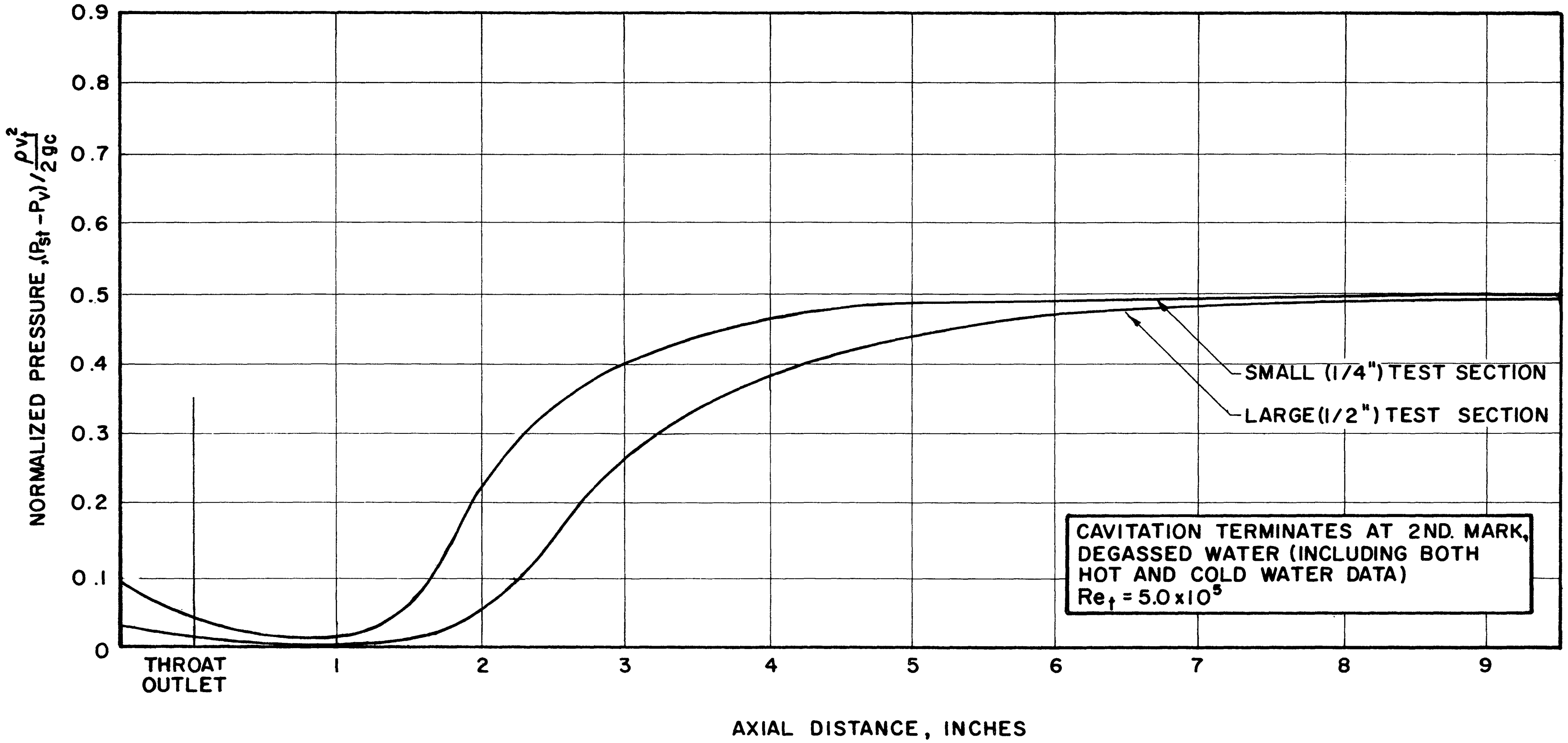


Figure 17. Comparison of Axial Pressure Profile in Different Test Sections.

throat. Curves are shown for each of the cavitation conditions for which data was obtained. Sonic initiation and zero cavitation are shown together since the very minor degree of cavitation corresponding to sonic initiation does not materially influence the flow. No differentiation is made between high and low temperature runs since no significant variation was found. For each cavitation condition and test section, curves are shown for several mean Reynolds Numbers. It is noted in general that, other things being equal, the pressure recovery (diffuser efficiency) is greater for higher Reynolds Number. This is, of course, as expected. However, the nozzle efficiency appears to be slightly less for the higher Reynolds Numbers which may be a result of experimental error.

Figures 14 through 17 show the comparison in pressure profiles between the two test sections. The horizontal axis has been shifted so that the throat discharge for each section occurs at the same axial location. Only the diffuser portion is shown. It is noted that in all cases the pressure gradient is steeper in the small test section, even for cases of well-developed cavitation. This would be expected for single-phase flow since the diffusers are geometrically similar. However, it is not self-evident that such would be the case across the semi-discontinuity of the condensation shock.

In all the curves for conditions of substantial cavitation, it is noted that the apparent region of termination which is indicated on the figures, corresponds to a sharp rise in static pressure, somewhat as would be experienced with a normal shock wave in a supersonic compressible gas. The pressure rise per linear inch is of the same order of magnitude near the throat discharge in the zero, sonic initiation, or visible initiation cavitation conditions as in the regions of collapse of cavitation in the first mark condition. However, the gradient is much less sharp for second mark condition. In the first case, the pressure rise is the result of ordinary single-phase diffusion of the throat velocity. In the second, it is the result of a condensation shock or hydraulic jump from substantially the throat velocity to a velocity consistent with the venturi diameter at that point[1]. The similarity of these gradients seems largely coincidental, since the gradient in the single-phase condition could be adjusted by a local change of diffuser angle. The curves for sonic or visible initiation, conditions wherein cavitation is insufficient to influence the overall flow pattern, indicate that the minimum pressure point occurs approximately at throat discharge, i.e., diffuser inlet. However, the curves for more developed cavitation show that the minimum pressure occurs well within the diffuser (7/8 inches for first mark cavitation, 1-1/4 inches for second mark in the large test section). This "vena contracta" effect is confirmed by measurements of the extent of the vaporous region surrounding the liquid jet[1] previously reported, which showed that the diameter of the liquid jet was less than the throat diameter in the entry portions of the diffuser under conditions of well-developed cavitation and that the jet velocity appears to be somewhat greater than throat velocity, estimated from continuity consideration. Considering the reasonably long cylindrical throat and the small diffuser angle, this effect is hard to explain. However, its existence is confirmed by the unrelated static pressure and void fraction[1] measurements.

3.3.2 Cavitation Number

According to the ideal theory, cavitation number, as previously defined for this report, should be identically zero (except for the effect of streamline curvature leaving the throat; however, this is negligible) for all test conditions, assuming no substantial roughnesses, burrs, etc. in the test section wall. The reasons for possible variations from this value have been discussed in a previous section.

Tests were performed for combinations of the following conditions and parameters using the large and small test sections (Figures 1 and 2)

i) Test Section Size: Large (1/2 inch throat) and small (1/4 inch throat)

ii) Throat Velocity: 50, 70, and 90 ft./sec.

iii) Gassification: Tap water (saturated or slightly supersaturated) or "degassed" (about 50 percent saturation)

iv) Temperature: Cold (between 50 and 90 F) and hot (between 130 and 160 F)

v) Cavitation Degree: Sonic initiation, visible initiation, cavitation to first mark, second mark, (explained in previous section).

The tests, along with resulting cavitation numbers and throat Reynolds Numbers are summarized in Table I. The results of all runs, for the different cavitation conditions, are given in Figures 18 thru 21, where cavitation number is plotted against throat Reynolds Number.

3.3.2.1 Correlation with Reynolds Number

All the experimental data has been plotted (Figures 18 through 21) in terms of cavitation number for a given degree of cavitation versus throat Reynolds Number; and a fairly good correlation was obtained. This is expecially significant since changes in Reynolds Number are a result of different temperatures (viscosity effects), test section sizes, and/or velocities.

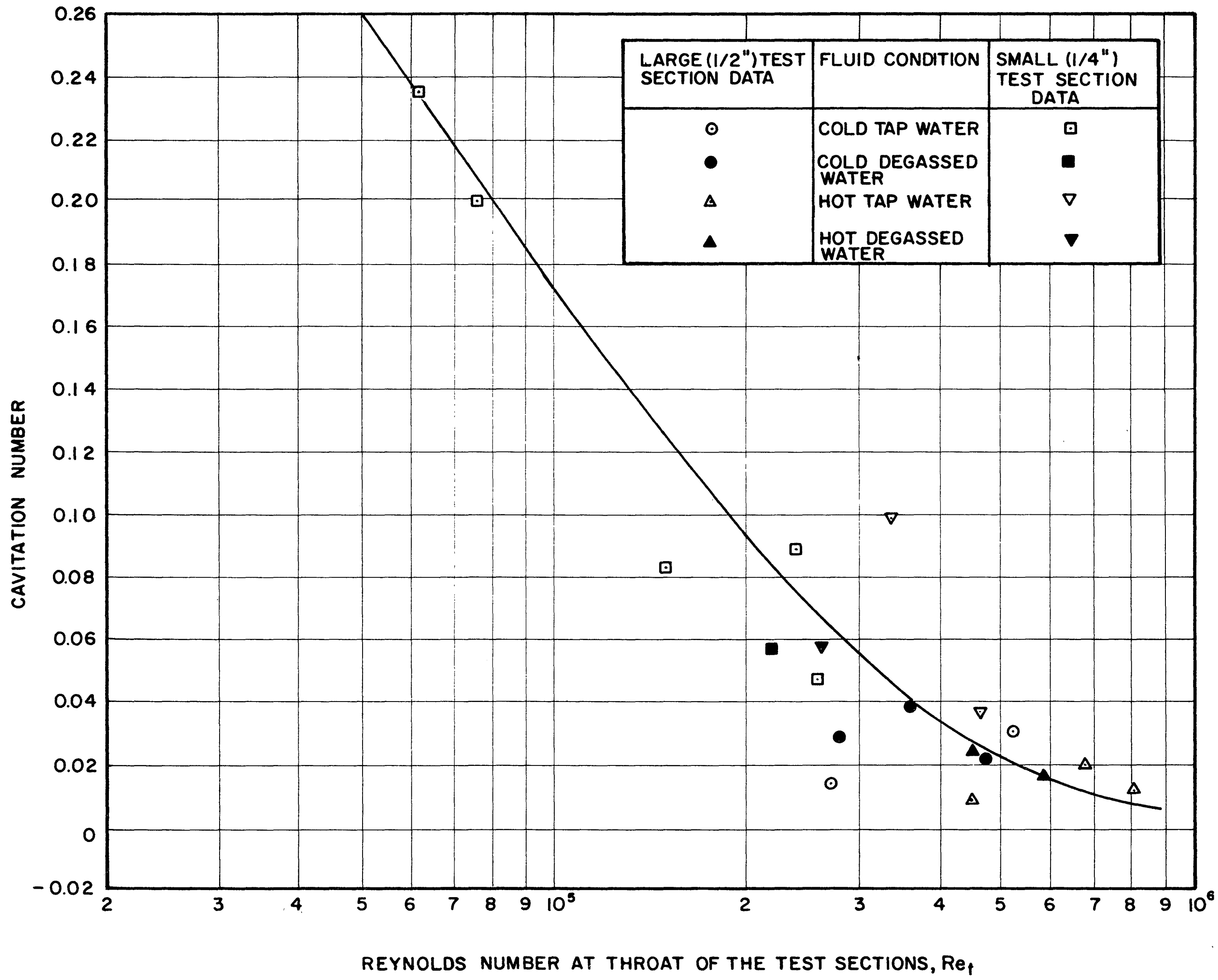


Figure 18. Reynolds Number vs. Cavitation Number, Caviatation Condition; Sonic Initiation.

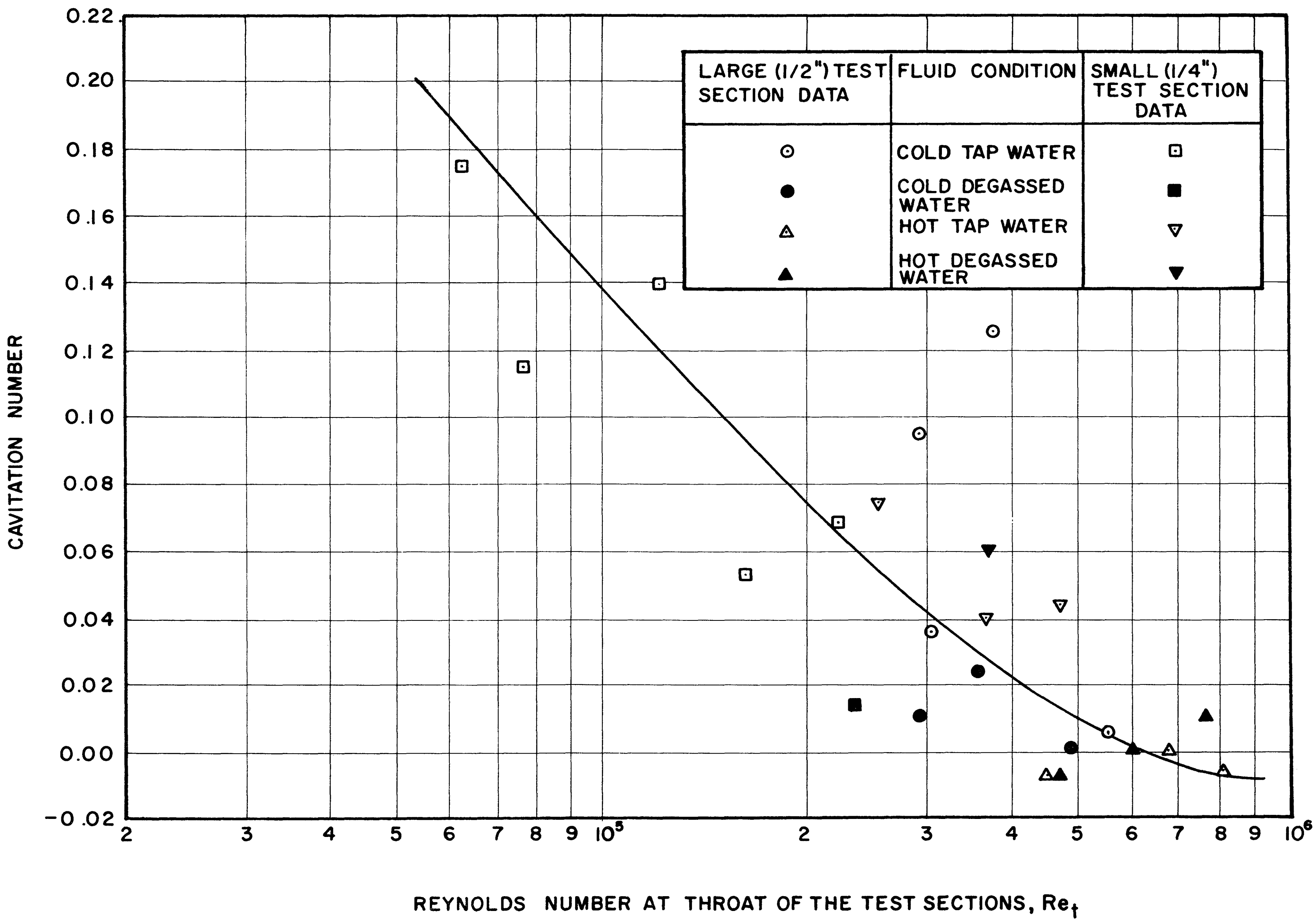


Figure 19. Reynolds Number vs. Cavitation Number, Cavitation Condition; Visible Initiation.

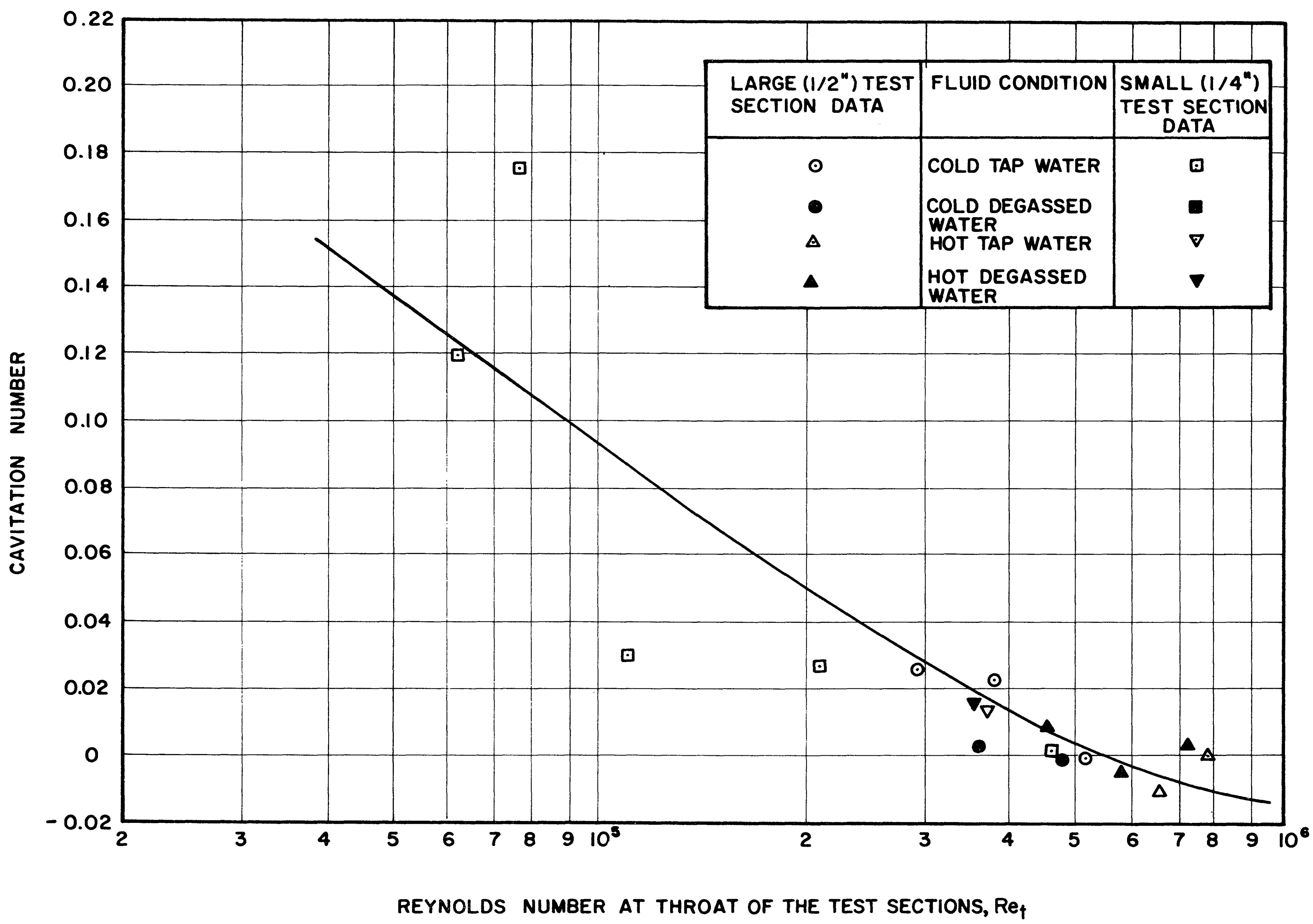


Figure 20. Reynolds Number vs. Cavitation Number, Cavitation Terminates at First Mark.

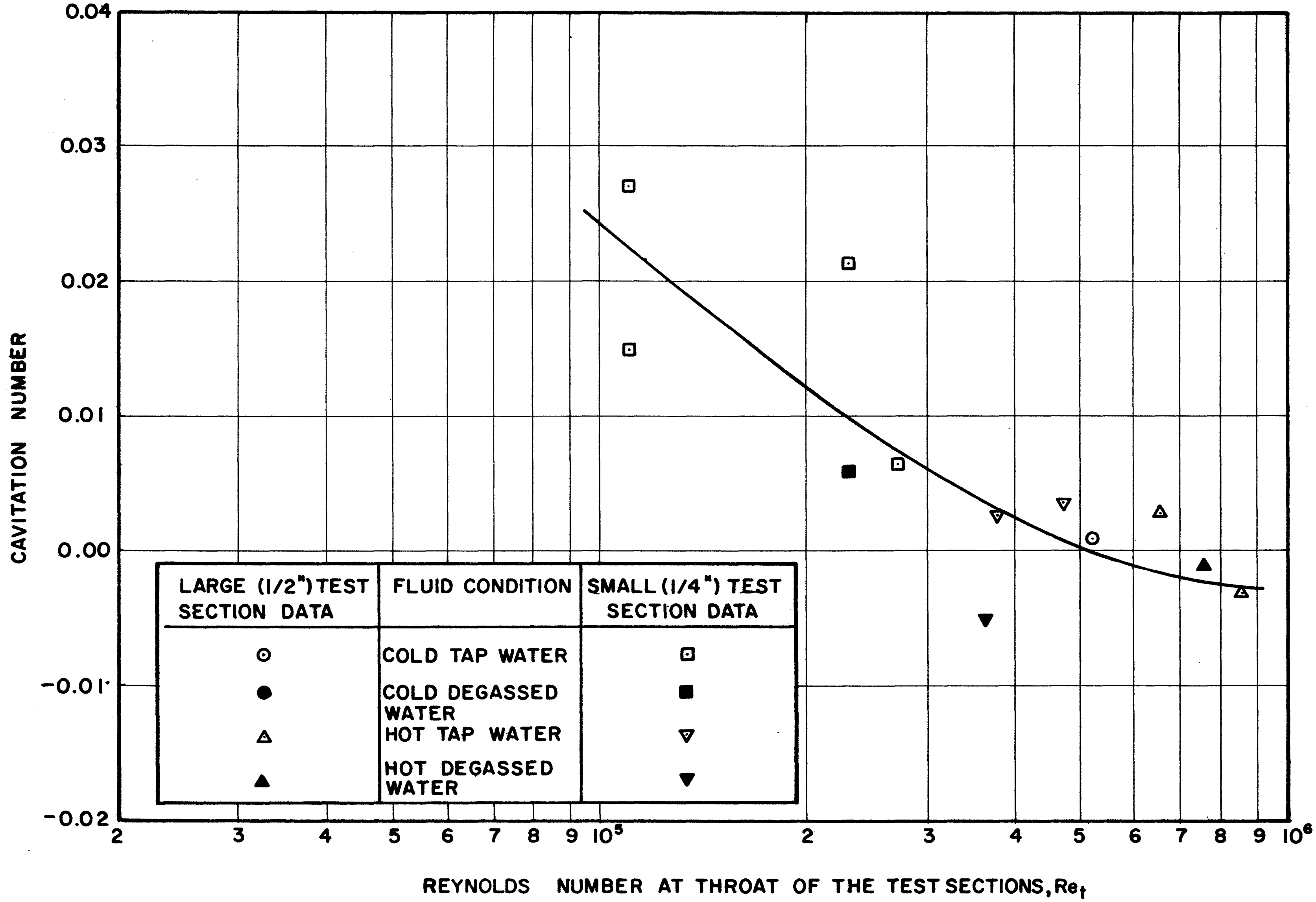


Figure 21. Reynolds Number vs. Cavitation Number, Cavitation Terminates at Second Mark.

The range of the effects overlap in that some of the high Reynolds Numbers result from low velocity but high temperature, others high velocity and low temperature, etc. A similar correlation was made in the past by Kermeen, et al[5] using data obtained from both the CIT and ORL water tunnels. However, the data from these sources did not include as wide a temperature variation (~55F to 70F). They were conflicting in that tests involving cavitation on a disc placed normal to the velocity (bluff body) correlated well with Reynolds Number even though the disc size was varied from 1/16 inches to 1-1/2 inches and velocity varied over a sizeable range, whereas tests on an ogive and on a hemisphere (smooth bodies) correlated only if size were a separate parameter. Since it has been demonstrated[5,15], that the nucleation of cavitation bubbles occurs largely adjacent to the walls in the boundary layer, it is not surprising that Reynolds Number should be a significant parameter. However, there are many effects involved in the nucleation and growth processes that are not related to Reynolds Number; hence it certainly cannot be a general á priori conclusion that it is the only scaling parameter. This fact was demonstrated by the ogive tests mentioned above. Universal scaling parameters cannot be derived until a better understanding of the growth and nucleation mechanisms is in hand.

Examination of Figures 18 through 21 indicates that for this particular system, within the range of independent variables tested, Reynolds Number does correlate the results fairly well. However, there is somewhat more scatter than can be explained on the basis of an error analysis discussed later. (Experimental error of about 0.003 in the cavitation number measurements can be justified). An examination of the individual points shows no significant trend with air content; or with velocity, size of section, or temperature beyond the Reynolds Number effect.

It does appear in some cases that there is more scatter with non-deaerated, non-settled water (fresh tap water), and that sometimes extraordinarily high values of the cavitation number are obtained. For this reason it appeared desirable to use water that was substantially undersaturated and that had been allowed to settle for a reasonable time (several hours at least). This was done for most of the tests. As pointed out in the previous discussion, it appears that air contents at least as low as ten percent of saturation must be obtained to cause a substantial effect. Since the present equipment can produce water deaerated only to about 50 percent saturation, no significant effect should be expected as long as most of the entrained air is eliminated by settling. From the viewpoint of obtaining reproducible results is is best to operate in a region where the data is not extremely sensitive to the precise gas content, as it might be for very low contents. It is not meant to imply that there are no effects upon cavitation number due to gas content in this range since such effects have been recently demonstrated[8]. However, it is likely that they are not sufficient to be of great importance in an ordinary machinery application or to be detectable in the present facility.

3.3.2.2 Detailed Results

Examination of Figures 18 through 21 shows several significant trends which will be discussed below.

i) The cavitation number depends upon degree of cavitation, decreasing as the degree of cavitation increases. As might be expected the effect is most marked for small degrees; i.e.: the difference between sonic initiation and visible initiation is much greater than between first mark and second mark, where no clear-cut difference was found. This is illustrated for several Reynolds Numbers in Figure 22 which is merely a cross-plot of Figures 18 through 21.

ii) For all cavitation conditions (see Figure 22 especially) the cavitation number decreases as the Reynolds Number increases, closely approaching the ideal theory value (i.e.: zero) for high Reynolds Number (and apparently slightly over shooting). An approach to the ideal value for high Reynolds Number is noted in the CIT and ORL results(5) for smooth bodies as an ogive and a hemisphere. However, the approach was from below with cavitation number increasing with increasing Reynolds Number. The same direction of variation is noted for the disc(5) but the cavitation number approached is considerably above the theoretical value. This is attributed to the local pressure depressions in this blunt body flow due to vortices(5). As opposed to these cases, the cavitation number approaches the ideal value from above in the present investigation. A similar condition was reported by investigators at the University of Minnesota(7). The reason for these trends in the present investigation or the others mentioned is not thoroughly understood and conflicting mechanisms can be cited. However, the purpose of the present study is to compare the behavior of different fluids in the same facility, so that knowledge limited to the trends in this facility, though not an ideal situation, will suffice for this purpose.

It is not clear from the present tests that substantially negative cavitation numbers might not be encountered at still higher Reynolds Numbers (beyond the range of the facility). Examination of Table I and Figures 18 through 22 shows that negative cavitation numbers were obtained, particularly with hot water (no discernible trend with gas content within the range of the experiments was found) and well-developed cavitation. In other words, in

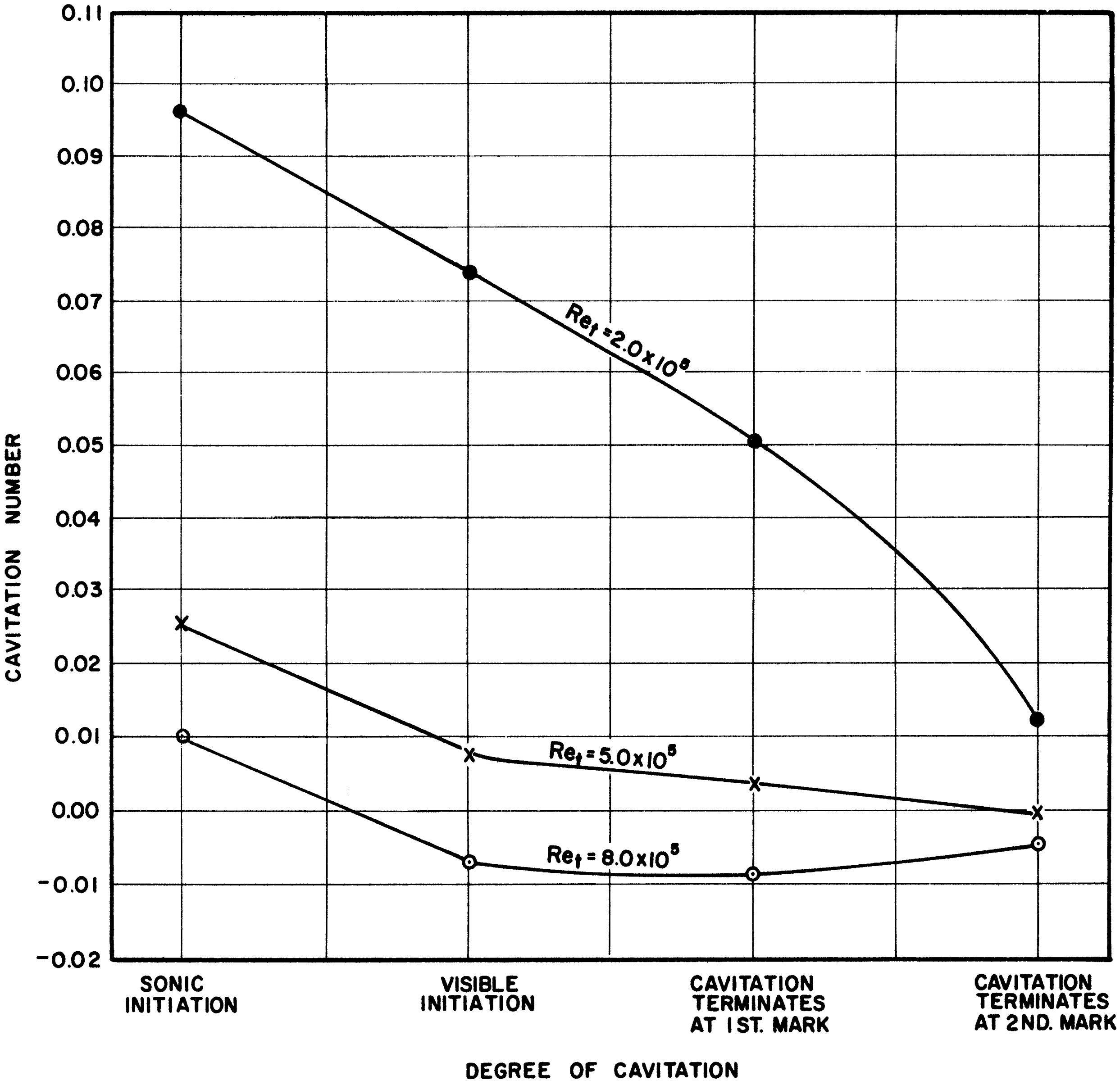


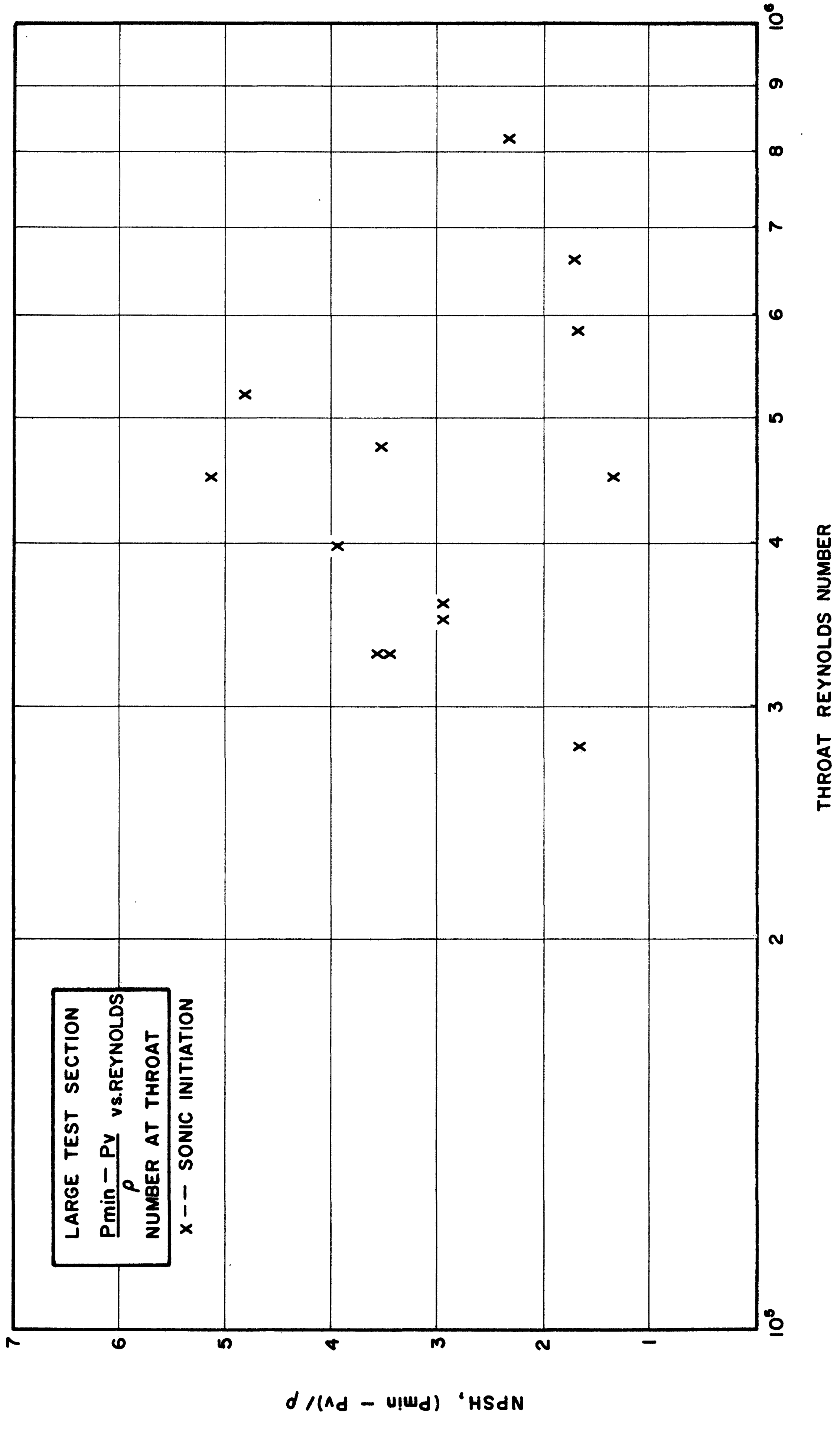
Figure 22. Cavitation Number vs. Degree of Cavitation at Different Reynolds Numbers.

these cases the static pressure at the wall, at the axial position of minimum pressure, was actually below the vapor pressure for the temperature of the fluid. However, the amount by which the cavitation number was negative is not in most cases beyond the experimental accuracy indicated by the error analysis. On the other hand the slope of the cavitation number versus Reynolds Number curves for high Reynolds Number indicates that substantially negative cavitation numbers might be obtained if higher Reynolds Numbers could have been reached.

For low Reynolds Numbers the tests indicate a substantially positive cavitation number so that the static pressure in the cavitating region was considerably above vapor pressure. This is of course especially true for sonic initiation, but to some extent even for cases of well-developed cavitation. A representative value for first mark cavitation at Reynolds Number of 10^5 is 4.84 psi. Figures 6 through 17 previously discussed also show this effect.

It might be supposed that the reduction of cavitation number with increasing Reynolds Number could be explained on the basis that net positive suction head (head above vapor pressure) was constant, and that cavitation number decreased as the kinetic head increased. Figures 23 through 25 are plotted on this basis and show that there is no correlation.

Previous investigators(5) have reported a hysteresis effect in that incipient cavitation number depended upon whether pressure were being depressed from a non-cavitating condition, or increased from a more developed cavitating condition. No such effect has been noticeable in the present facility, perhaps because of the short travel time around the loop and the absence of any resorber equipment so that a sufficiency of entrained gas nuclei is always available after any local cavitation is obtained.



LARGE TEST SECTION
$\frac{P_{min} - P_v}{\rho}$ vs. REYNOLDS
NUMBER AT THROAT
X -- SONIC INITIATION
NPSH, $(P_{min} - P_v)/\rho$
7
6
5
4
3
2
1
10^5
2
3
4
5
6
7
8
9
10^6
THROAT REYNOLDS NUMBER

Figure 23

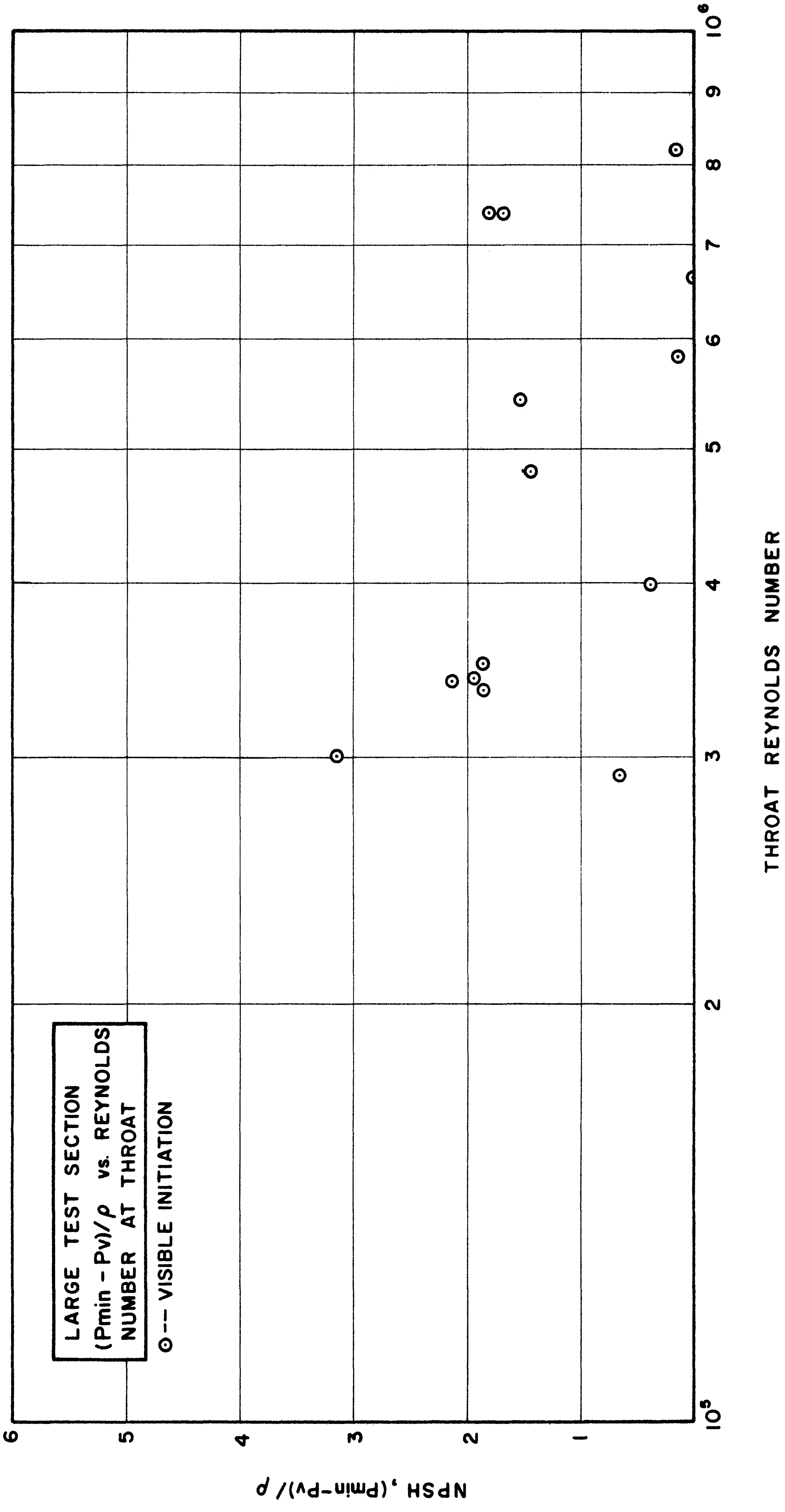

LARGE TEST SECTION
(Pmin - Pv)/ρ vs. REYNOLDS
NUMBER AT THROAT
⊙ -- VISIBLE INITIATION
NPSH ,(Pmin-Pv)/ρ
THROAT REYNOLDS NUMBER
10^5
2
3
4
5
6
7
8
9
10^6
1
2
3
4
5
6

Figure 24

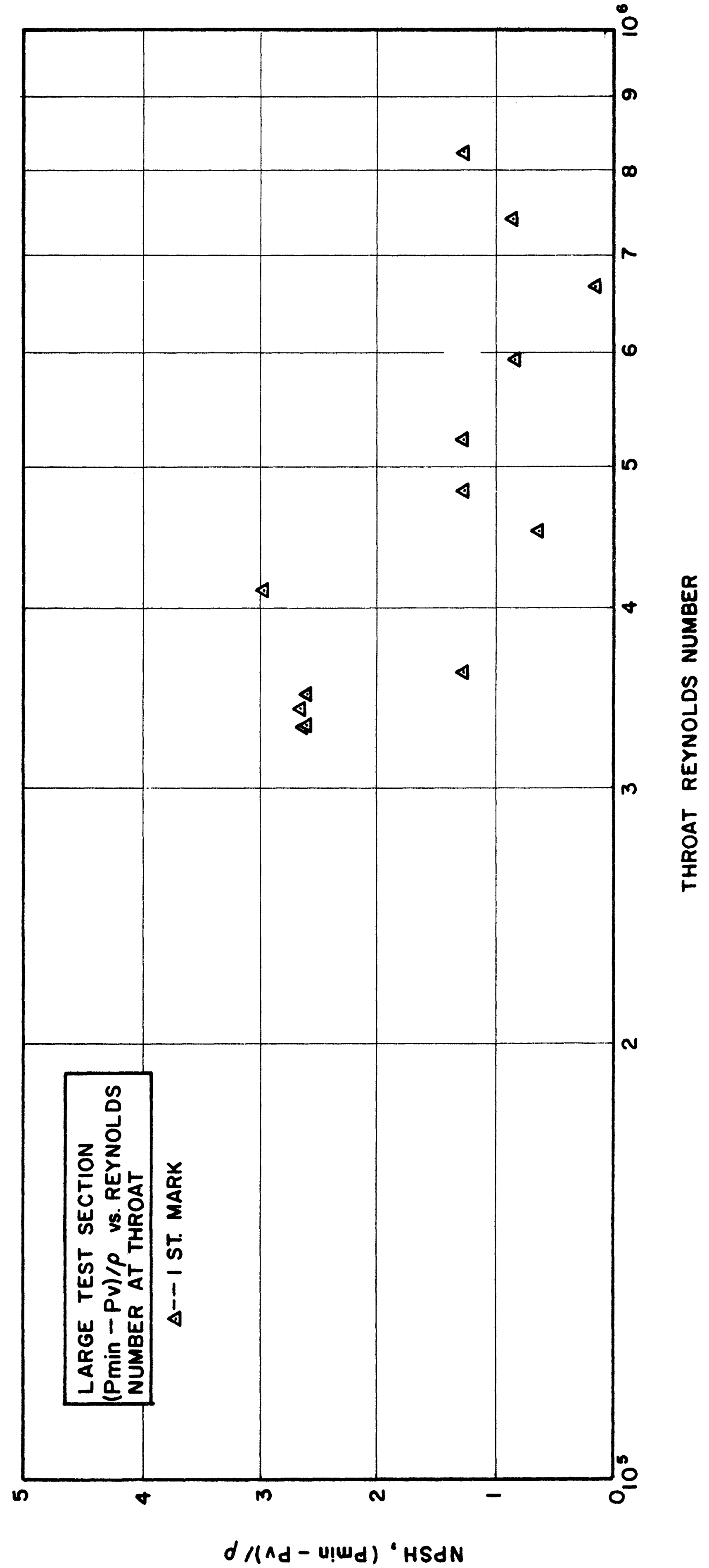

LARGE TEST SECTION
(Pmin − Pv)/ρ vs. REYNOLDS NUMBER AT THROAT
Δ-- 1 ST. MARK
NPSH, (Pmin − Pv)/ρ
0
1
2
3
4
5
10⁵
2
3
4
5
6
7
8
9
10⁶
THROAT REYNOLDS NUMBER

Figure 25

3.3.3 Venturi Loss Coefficient

As previously explained, the venturi loss coefficient is the ratio between static pressure drop from venturi inlet to discharge and static pressure drop from inlet to minimum pressure point. It is significant as a basic parameter describing the flow, and because, for the present investigation, it might be used to indicate the degree of cavitation in an opaque system.

The loss coefficient data arises from the tests previously described with regard to the cavitation number. Therefore, the combination of parameters is the same except that no significance is attached to the degree of aeration since no effect was observed. Hence this parameter is not considered in the presentation of the results. The significant variables are then:

i) Test section size: large or small (Figures 1 and 2)

ii) Throat Velocity: 50, 70, and 90 ft./sec.

iii) Temperature: Cold (50 - 90 F) or hot (130 - 160 F)

iv) Cavitation degree: Sonic initiation, visible initiation, cavitation to first mark, to second mark.

The basic results are given in Figures 26 through 30.

3.3.3.1 Correlation with Reynolds Number

For geometrically similar venturis with single-phase flow it would be expected that loss coefficient would correlate directly with Reynolds Number. For cavitating flows it seems reasonable that Reynolds Number would be an important parameter but not the only one of significance. These statements are verified by the test results.

Figure 26 shows the loss coefficient versus throat Reynolds Number for flows corresponding to sonic initiation of cavitation for the large test section and small test section. It is noted that a good correlation is

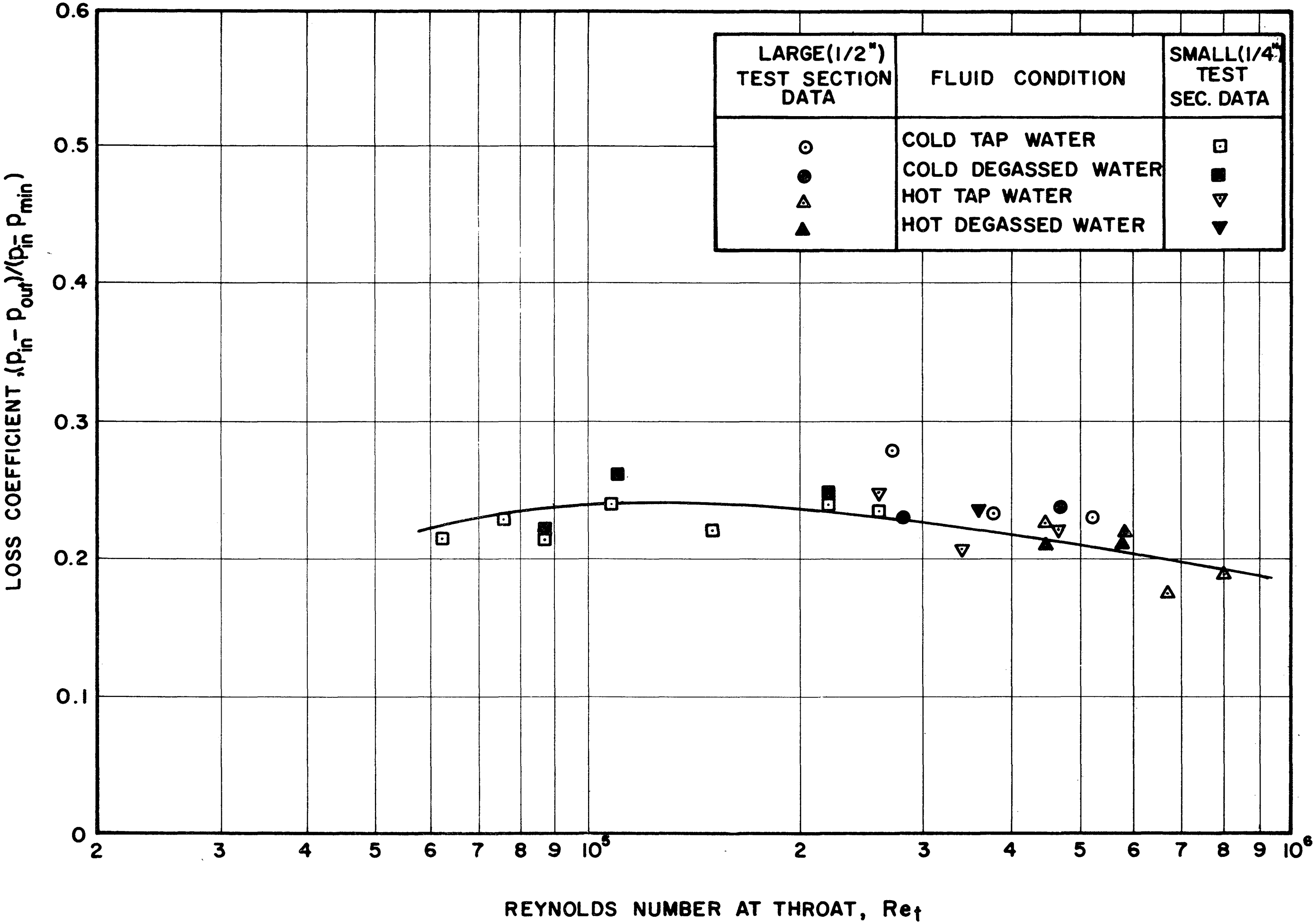


Figure 26. Loss Coefficient vs. Reynolds Number, Cavitation Condition; Sonic Initiation.

obtained in all cases, with the loss coefficient decreasing slightly as expected with increasing Reynolds Number. It is assumed that the very slight amount of cavitation corresponding to sonic initiation is not sufficient to substantially invalidate the single-phase nature of the flow or to change the overall flow parameters. This has been confirmed by tests with zero cavitation.

The corresponding curve for flows corresponding to the visible initiation of cavitation is Figure 27. Loss coefficient correlates well with throat Reynolds Number as before. However, it is noted that the loss coefficient at a given Reynolds Number in either test section is slightly higher for visible initiation than for sonic initiation (Figure 30). This would be expected since the disturbance of the flow is greater for the slightly increased cavitation. It is also noted that the correlation of points from both test sections with Reynolds Number on the same curve (Figure 27) is not as successful as for the single-phase case (sonic initiation). It appears here, and in the cases of more fully-developed cavitation, that a scaling parameter other than Reynolds Number is required.

Figures 28 and 29 are the corresponding curves for first and second mark cavitation. Examination of these figures shows a correlation with Reynolds Number exists only if a division is made between hot and cold water tests (as well as test section size, especially in the case of second mark cavitation). This will be discussed in greater detail later. In Figure 28, the differentiation appears to be more between hot and cold rather than between sizes so that a single curve for both venturis can be used. In any case, it is apparent that throat Reynolds Number alone is not a suitable scaling parameter for well-developed cavitation. Of course there is no a priori reason to assume that it should be.

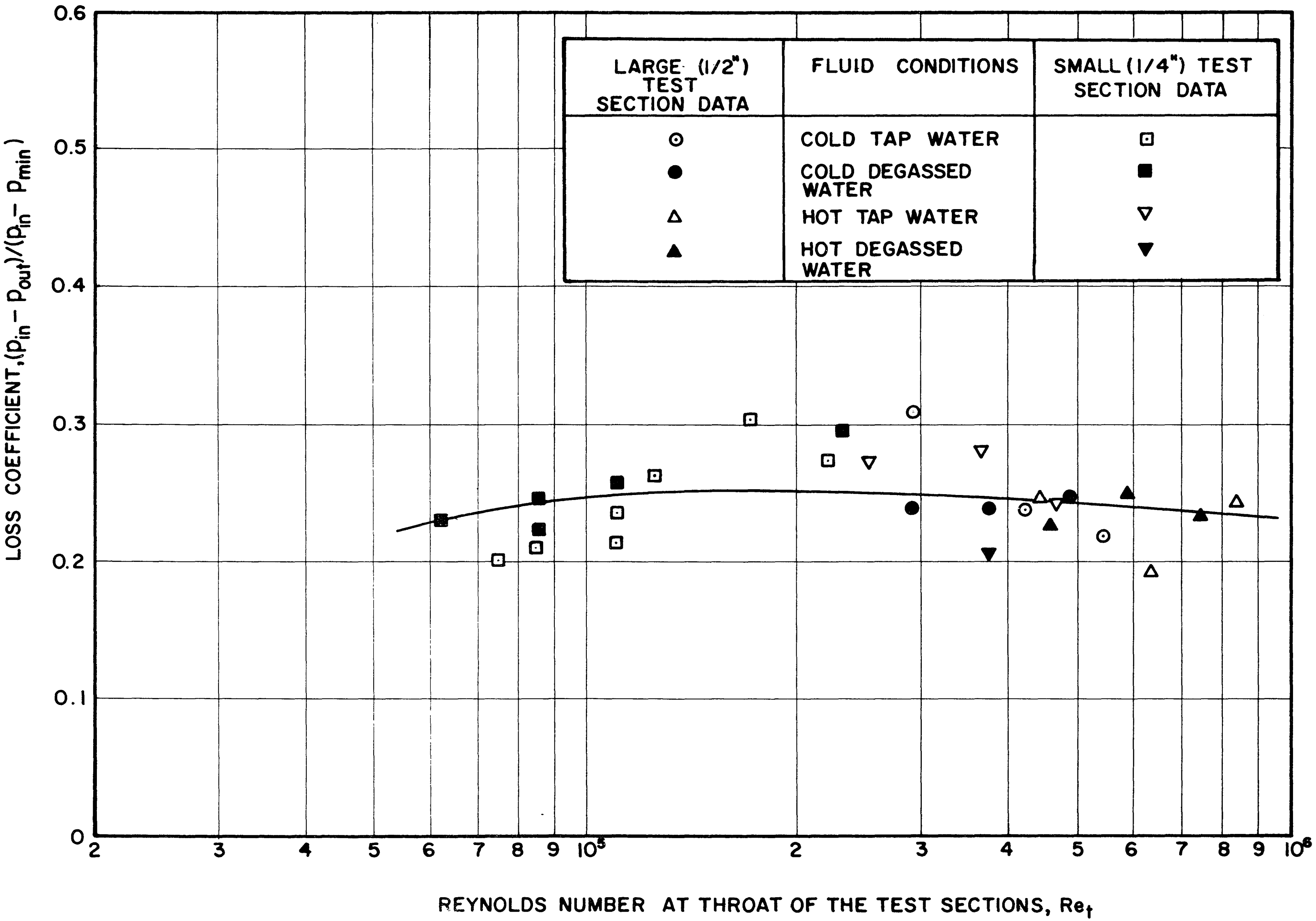


Figure 27. Loss Coefficient vs. Reynolds Number, Cavitation Condition; Visible Initiation.

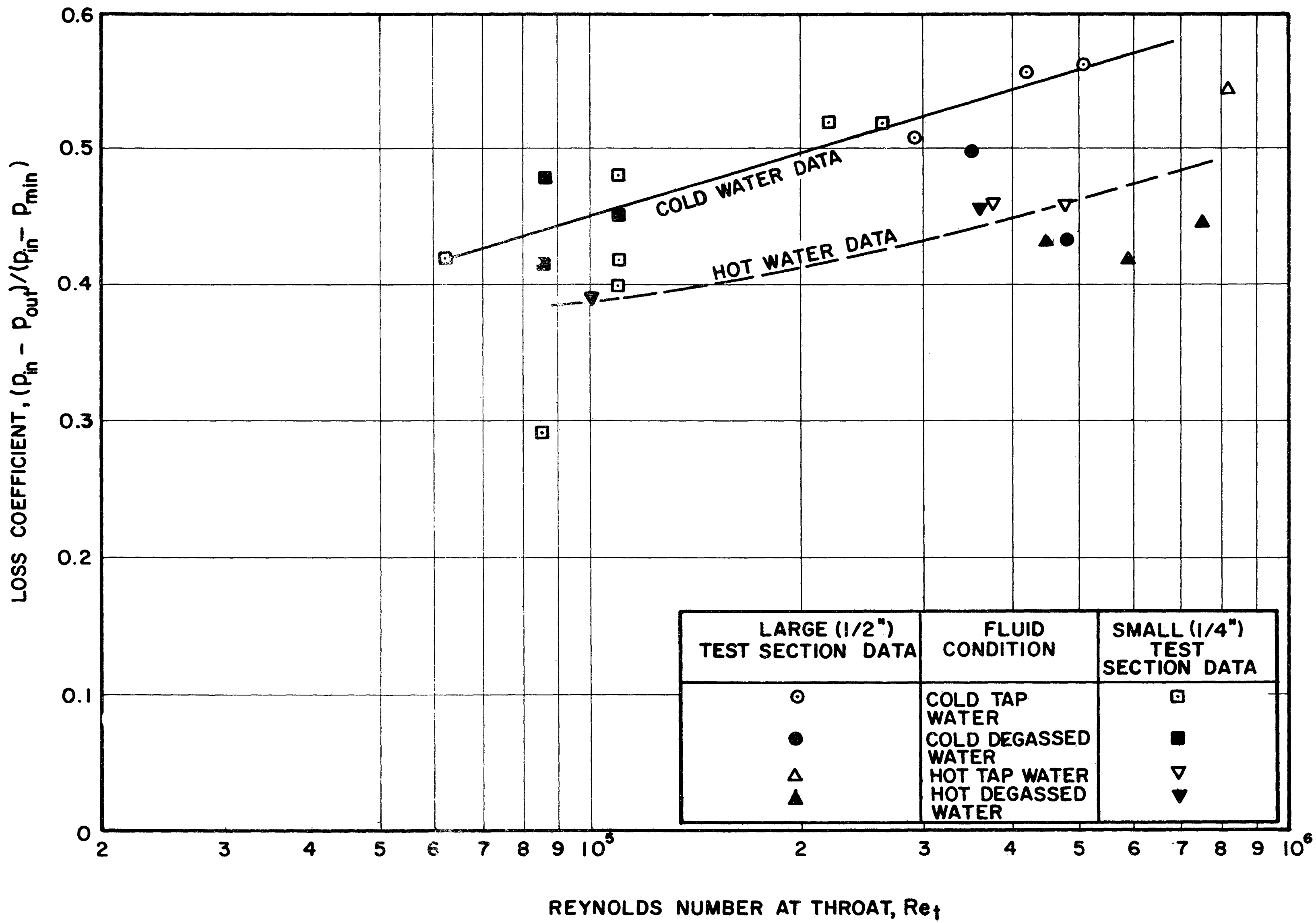


Figure 28. Loss Coefficient vs. Reynolds Number, Cavitation Terminates at First Mark.

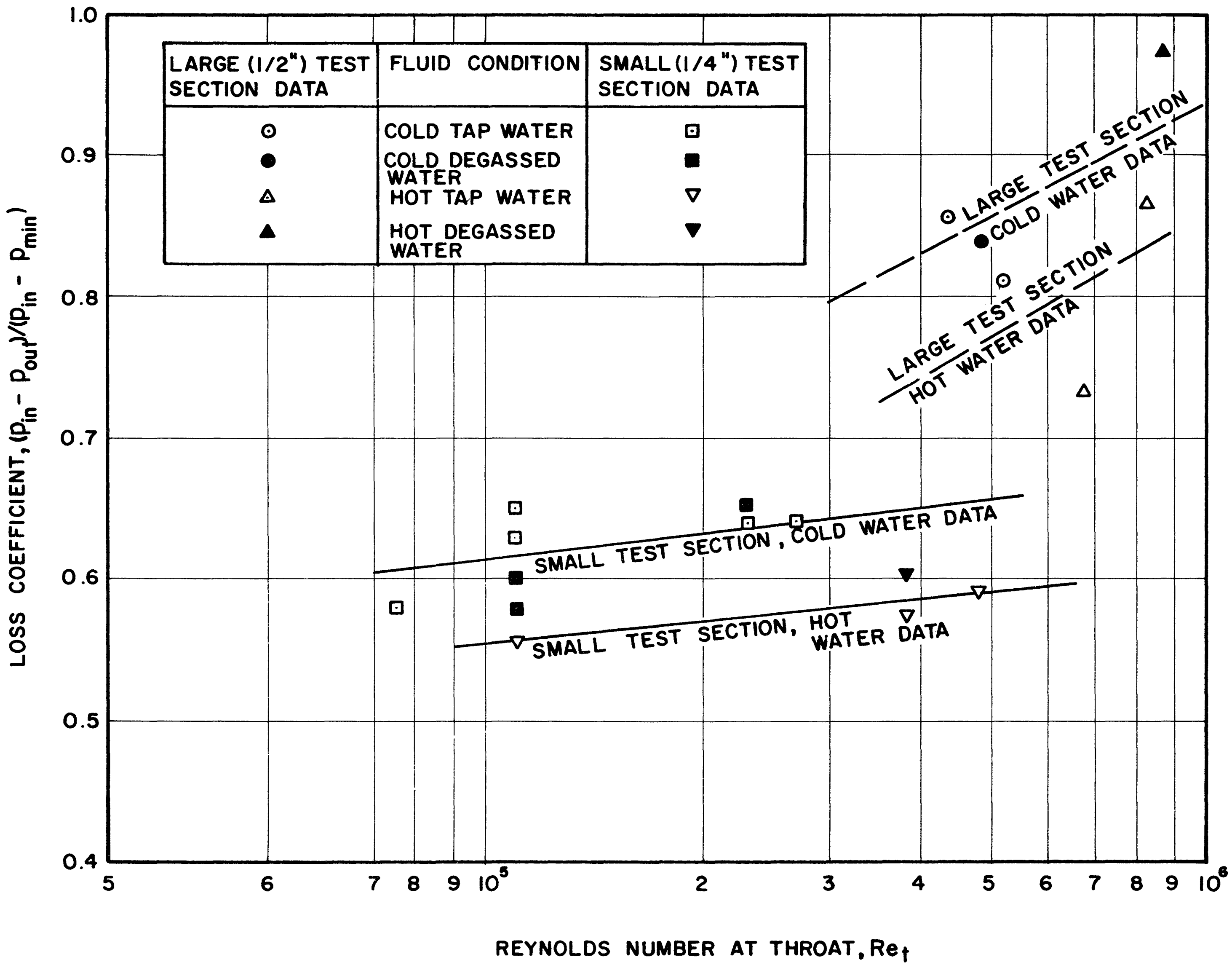


Figure 29. Loss Coefficient vs. Reynolds Number, Cavitation Terminates at Second Mark.

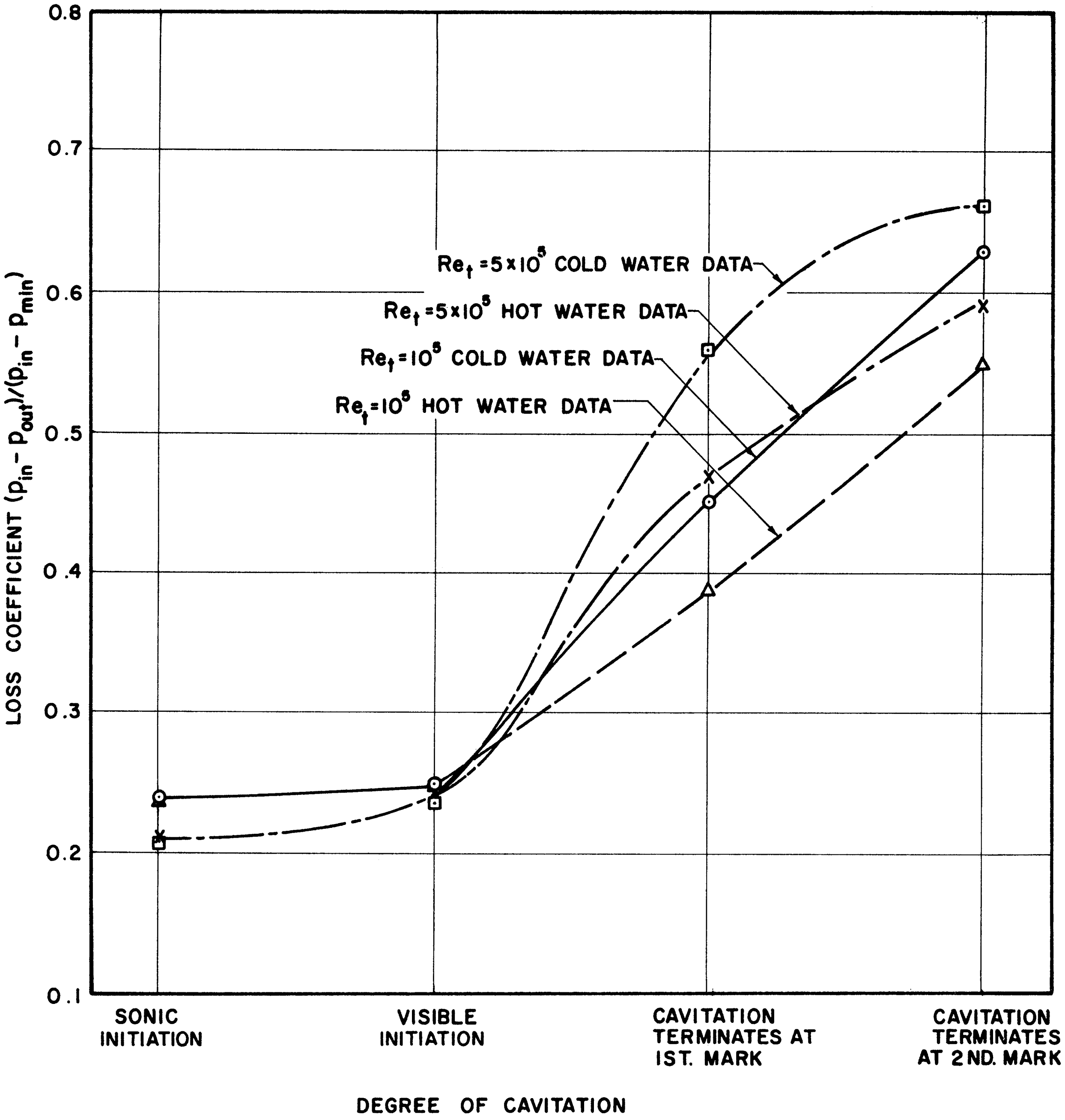


Figure 30. Cavitation Condition vs. Loss Coefficient at Various Reynolds Numbers.

It is further noted from the curves for well-developed cavitation (first and second mark), that the loss coefficient in general appears to increase rather than decrease with increasing Reynolds Number. However, the slope is not large for a single venturi for either hot or cold water. However, the trend of Figure 29 is more the result of the difference between large and small venturis, with the higher loss associated with the large section. It is again evident that geometric similarity and constant Reynolds Number does not assure similar flow patterns in such a case.

3.3.3.2 Temperature Effect

As mentioned above there is a temperature effect evident in venturi flows involving fairly well-developed cavitation (Figures 28 and 29). In all cases, the loss coefficient for a given cavitation degree (vaporous region of given proportionate extent*) at a given throat Reynolds Number is less for hot water than for cold water.

The same effect can be described from a slightly different viewpoint. If the cavitation termination point is set at a given position for cold water conditions, and if the flow rate and loop resistance (valve settings) are maintained constant but the water heated, it is noted that the termination point moves downstream. In other words, a given loss coefficient corresponds to a greater degree of cavitation with hot than with cold water.

* As explained previously, the termination of the vaporous region is not sharp. Since the lack of sharpness is presumably different for hot and cold water tests, the cavitation degrees may not be entirely similar.

This general temperature effect is in agreement with theoretical expectations as previously discussed, with observations of other investigators[13] and with the void fraction measurements previously reported[1]. It is based primarily on the much larger vapor volume formed per unit head depression with cold than with hot water.

3.3.3.3 Cavitation Condition Effect

As discussed in a previous section, it is certainly expected that loss coefficient will increase markedly with cavitation degree, and examination of the previously discussed curves shows that this is the case. The effect is shown directly in Figure 30 where loss coefficient is plotted against cavitation condition for two typical Reynolds Numbers and hot and cold water, and large and small test section. It is noted that loss coefficient is not particularly sensitive to cavitation degrees up to and beyond visible initiation, but becomes very sensitive to more fully developed cavitation. For this reason it might be used as an indication of cavitation degree for well-developed cases in future tests involving non-transparent test sections. Confirmation could be obtained by void fraction measurements[1] for well-developed cavitation; however, reliance must be placed on acoustic measurements for cavitation initiation. As will be explained in a future report, acoustic measurements may also be of considerable assistance in determining well-developed cavitation degrees.

3.4 ERROR ANALYSIS FOR CAVITATION NUMBER AND LOSS COEFFICIENT

Estimates of probable error in the computation of cavitation number and venturi loss coefficient (or non-dimensional pressure differentials which involve essentially the same thing) have been made in two ways:

i) Computation of standard deviation from four repetitive runs.

ii) Estimation and summing of individual error components based on physical considerations.

The possible sources of errors are in the following, and appear to be independent:

1) Pressure measurement

2) Velocity measurement

3) Vapor pressure measurement (actually temperature measurement)

4) Precision in setting of cavitation condition (i.e., sonic or visible initiation, first mark, etc.)

Standard Deviation Approach:

Four runs were made on a single day with a single water sample of partially deaerated water for three different cavitation conditions (sonic initiation, visible initiation, cavitation to first mark). After each run, control settings were upset so that resetting "from scratch" was necessary. In this way, it was hoped to make the errors truly random. The resulting data and standard deviations are listed in Tables II and III.

It is noted that they decrease for cavitation number as the degree of cavitation increases. The ratio of standard deviation to mean value also decreases from about ten percent to two percent. This is reasonable since the minimum fluid pressure is very insensitive to degree of cavitation once cavitation has become substantial.

The variation of the loss coefficient deviation is in the opposite direction, increasing for increased cavitation. The above mentioned ratio increases from about 2.6 per cent to 3.8 percent. This seems reasonable on the grounds that the loss coefficient is very sensitive to degree of cavitation when it is substantial, and insensitive for cavitation initiation.

The detailed calculations and tabulated data of the repetitive runs are included in the appendix.

Individual Error Components:

"Reasonable" confidence limits have been placed on the individual measurements so that it can be said that perhaps 70 percent of the readings should be within these limits (i.e., the estimated error is approximately equal to the standard deviation, see page 24 of Reference 17.)

(1) Pressure Measurements: For the cavitation number calculations, the pressure measurements are made by mercury manometer. It seems reasonable that these will not be in error by more than ± 0.2 inches of mercury (~0.1 psi).

For loss coefficient data, the pressure measurements often involved a calibrated eight inch standard quality, 0-60 psi, Bourdon gage, although sometimes the manometer was used (for low pressure cases). It is assumed that these readings should not ordinarily be in error by more than ± 1/4 psi.

(2) Velocity Measurements: Velocity measurements are made using a calibrated venturi and reading the pressure differential on a water manomenter. Errors involve: (a) conversion from flow to velocity (dimension errors), (b) venturi calibration errors (it was calibrated against a weigh-tank), (c) errors in measuring pressure differential. It is felt that the controlling error is in the pressure measurement. A reasonable value for this has been assumed at ± 0.2 inches of water, corresponding in a typical case to a velocity error of 0.1 ft/sec.

(3) Vapor Pressure: Vapor pressure is computed from temperature measurements which are read by a thermocouple located in the pump sump

directly above the pump housing (Figure 5.) The thermocouple has been calibrated several times against a precision thermometer, and no indication of EMF variation at a given temperature with time has been noted.

For "cold water" tests (90°F) there is no possibility of substantial error from vapor pressure because of its insensitivity to temperature.

For the hot water tests (~130 - 160 F), an error of ± 1/2°F corresponds to about ± 0.05 psi. It is felt that this is a reasonable error estimate.

There is no vapor pressure influence on the loss coefficient tests, and the effect of temperature variation on density for all tests is negligible.

(4) Setting of Cavitation Condition: No direct error estimate is possible on this account. It seems conceivable that a substantial error component exists, but there is no way of estimating it on physical grounds. Since its effect is incorporated in the repetitive runs used to calculate the standard deviation, it appears from an examination of these results, that this error component cannot be very large.

The errors are summed for a typical case as a square root of the sum of the squares as shown in the Appendix, and the results are given in Table IV.

In the case of cavitation number, the confidence limit so computed is about ± 0.003 giving a confidence limit of ± 8 percent for sonic initiation and 15 percent for first mark cavitation.

In the case of loss coefficient, the confidence limit is about four percent for sonic initiation and two percent for first mark cavitation.

Since the numbers derived from the error summations are the same order of magnitude as those from the standard deviation calculations based on the four repeated runs, it is felt that the order of magnitude of the error estimate is correct. Since the calculation based on repetitive runs appears conservative due to the close matching of results and the small number of runs (more should have been made), it is felt that the confidence limits in the results, corresponding to the error summation estimates are realistic (i.e., corresponding approximately also to the standard deviation as calculated).

4.0 CONCLUSIONS

Quantitative data relating to axial pressure profiles, cavitation number, and venturi loss coefficient is given for two geometrically similar cavitating venturi test sections with hot and cold water as test fluids over a range of throat velocities and total gas contents. Over the range of parameters investigated in this particular facility it has been found that cavitation number can be correlated in terms of throat Reynolds Number for a given degree of cavitation; that gas content down to the rather mild deaeration that was obtained with the available equipment makes little difference provided large quantities of entrained gas and supersaturated solutions are not used; and that cavitation number is quite insensitive to degree of cavitation for well-developed cavitation.

It has also been found, over the same range of parameters, that venturi loss coefficient is very sensitive to degree of cavitation for well-developed cavitation (increasing with the extent of the cavitating region) but not in the range of cavitation initiation; that venturi loss coefficient cannot be correlated successfully by throat Reynolds Number alone if the cavitation is well-developed, but that there are additional effects of temperature and size of test section; it does correlate successfully with throat Reynolds Number in the range of cavitation initiation.

5.0 APPENDIX

5.1 Variance Analysis of Data Obtained from Repetitive Runs

The variance denoted by σ^2 is defined as

$$\sigma^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

where $\bar{x}$ is the mean of the variable x, and n the number of observations. And σ is the standard deviation (p. 9 of Reference 17).

Table II

SUMMARY OF DATA FROM REPETITIVE RUNS

CAVITATION CONDITION	CAVITATION NUMBER - σ			
Sonic	.035	.035	.042	.042
Visible	.023	.026	.024	.023
First Mark	.020	.0205	.021	.0208
	LOSS COEFFICIENT - L.C.			
Sonic	.2438	.2348	.2318	
Visible	.241	.2317	.2442	.2378
First Mark	.4648	.4617	.4581	.4420

(1) Calculation of σ_σ for Visible Cavitation condition:

$$\bar{x}_\sigma = (.023 + .026 + .024 + .023)/4 = .024$$

$$\sigma_\sigma^{\,2} = \left[2(.024 - .023)^2 + (.026 - .024)^2 + (.024 - .024)^2\right]/3$$
$$= 2 \times 10^{-6}$$

$$\therefore \sigma_\sigma = 0.00141$$

(2) Calculation of $\sigma_{L.C.}$ for Visible Cavitation condition:

$$\bar{X}_{L.C.} = (.241 + .2317 + .2442 + .2378)/4 = .2386$$

$$\sigma^2_{L.C.} = \left[(.241 - .2386)^2 + (.2386 - .2317)^2 + (.2442 - .2386)^2 + (.2386 - .2378)^2\right]/3$$

$$\sigma^2_{L.C.} = 28.46 \times 10^{-6}$$

$$\therefore \sigma_{L.C.} = 0.0053$$

Note: Tabulated values of σ_σ and $\sigma_{L.C.}$ for all conditions can be found in Table II.

5.2 Estimation of Standard Deviation of Cavitation Number and Loss Coefficient Using Error Analysis of Independent Variables.[16,17]

5.2.1 Analysis:

Utilizing techniques of differential calculus we can derive an equation for the standard deviation of a dependent variable as a function of the standard deviations of its independent variables.

Let $z = f(x,y)$ where x and y are measured independently. To find the error in z we take the differential of z.

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \tag{1}$$

where dx and dy are the errors in x and y respectively.

Now since dx and dy are independent of each other, there certainly exists the possibility that these errors in addition to adding can also cancel one another. If we take the square root of the sum of the squares of Equation (1) it will then contain the required compensating property, (p. 26 of Reference 17). Then,

$$dz = \sqrt{\left[\frac{\partial z}{\partial x} dx\right]^2 + \left[\frac{\partial z}{\partial y} dy\right]^2} \qquad (2)$$

and now we can write the above equation in terms of standard deviation, most probable error, or any similar error quantity (p. 29 of Reference 17).

$$\sigma_z = \sqrt{\left[\frac{\partial z}{\partial x} \sigma_x\right]^2 + \left[\frac{\partial z}{\partial y} \sigma_y\right]^2} \qquad (3)$$

5.2.2 Error Analysis of Cavitation Number

The cavitation number σ is defined here as

$$\sigma = (P_{min} - P_v) \Big/ \frac{\rho V^2}{2g} \qquad (4)$$

where P_{min} is the pressure at cavitation inception, P_v is the vapor pressure, and V is the throat velocity.

Using Equation (3) we can write the equation of the standard deviation of the cavitation number as:

$$\sigma_\sigma = \sqrt{\left[\frac{\partial \sigma}{\partial P_{min}} \sigma_{P_{min}}\right]^2 + \left[\frac{\partial \sigma}{\partial P_v} \sigma_{P_v}\right]^2 + \left[\frac{\partial \sigma}{\partial V} \sigma_V\right]^2} \qquad (5)$$

Taking partial derivatives of Equation (4):

$$\frac{\partial \sigma}{\partial P_{min}} = \frac{1}{\frac{\rho V^2}{2g}};\ \frac{\partial \sigma}{\partial P_v} = -\frac{1}{\frac{\rho V^2}{2g}};\ \frac{\partial \sigma}{\partial V} = -\frac{2\sigma}{V} \qquad (6)$$

Taking conditions at a throat velocity of 70 fps and first mark cavitation as typical. We get for numerical values:

$$\frac{P_{min} - P_V}{\frac{\rho V^2}{2g}} = 0.0206 \;;\; 1\Big/\frac{\rho V^2}{2g} = \frac{1}{32.35}$$

$$\frac{\partial\sigma}{\partial P_{min}} = .03 \;;\; \frac{\partial\sigma}{\partial P_V} = .03 \;;\; \frac{\partial\sigma}{\partial V} = 0.007 \qquad (7)$$

Now estimating $\sigma_{P_{min}}$, σ_{P_V}, σ_P and σ_V (i.e.: an error value within which about 70 percent of readings will fall).

Let $\sigma_{P_{min}} = \pm\ 0.1$ Psi (For manometer readings) (8)

$\sigma_P = \pm\ 0.25$ Psi (For guage readings, not used in σ)

$\sigma_{P_V} = \pm\ 0.05$ Psi (Due to temperature reading, corresponds to 1°F error at about 130°F and about 0.5°F at 150°F)

$\sigma_V = \pm\ 0.1$ fps (Flow manometer, corresponds to 0.2 inch H_2O error in reading the flow manometer)

Substituting numerical values from Equations (7) and (8) into Equation (5) we get:

$$\sigma_\sigma = \sqrt{(.03 \times .1)^2 + (.03 \times .05)^2 + (.007 \times .1)^2} \qquad (9)$$

where the last two terms can almost be dropped since they are very small as compared with the first. However,

$$\therefore\ \sigma_\sigma = 0.0034$$

Error Analysis of Loss Coefficient:

The loss coefficient is defined as

$$\text{L.C.} = \left(P_{in} - P_{out}\right)\Big/\frac{\rho V^2}{2g} \qquad (10)$$

and proceeding as before, the equation for the standard deviation of the loss coefficient is

$$\sigma_{L.C.} = \sqrt{\left[\frac{\partial(L.C.)}{\partial P_{in}} \sigma_{P_{in}}\right]^2 + \left[\frac{\partial(L.C.)}{\partial P_{out}} \sigma_{P_{out}}\right]^2 + \left[\frac{\partial(L.C.)}{\partial V} \sigma_V\right]^2}$$

and since the variation of P_{in} and P_{out} are the same the equation can be written as

$$\sigma_{L.C.} = \sqrt{2\left[\frac{\partial(L.C.)}{\partial P} \sigma_P\right]^2 + \left[\frac{\partial(L.C.)}{\partial V} \sigma_V\right]^2} \qquad (11)$$

Now substituting appropriate numerical values from Equations (7) and (8) into Equation (11) we have

$$\sigma_{L.C.} = \sqrt{2(.03 \times .25)^2 + (.007 \times .1)^2}$$

where the second term can be dropped since it is very small as compared to the first.

$$\therefore \sigma_{L.C.} \cong 0.0106$$

TABLE I

SUMMARY OF CAVITATION NUMBER RUNS

Section	Throat Velocity		Cold Sonic σ_c	Cold Sonic Re_t	Cold Visible σ_c	Cold Visible Re_t	Cold 1st Mark σ_c	Cold 1st Mark Re_t	Cold 2nd Mark σ_c	Cold 2nd Mark Re_t	Hot Sonic σ_c	Hot Sonic Re_t	Hot Visible σ_c	Hot Visible Re_t	Hot 1st Mark σ_c	Hot 1st Mark Re_t	Hot 2nd Mark σ_c	Hot 2nd Mark Re_t
SMALL (1/4") SECTION	11	Tapwater	0.0823	1.5	0.0533	1.7	-		-		0.0587	2.6	0.074	2.5	-		-	
		Deaerated	-		-		-		-		-		-		-		-	
	14	Tapwater	0.090	2.2	0.069	2.2	0.0263	2.2	0.0214	2.3	0.100	3.4	0.04	3.6	0.015	3.7	0.0074	3.8
		Deaerated	0.0582	2.2	0.014	2.3	-		0.0058	2.3	0.006	3.6	0.060	3.6	0.065	3.6	-0.005	3.6
											0.145						-0.034	
	18	Tapwater	0.049	2.6	0.052		0.0065	2.6	0.0065	2.7	0.038	4.6	0.044	4.6	0.002	4.6	0.0003	4.7
		Deaerated	-		-		-		-		-		-		-		-	
LARGE (1/2") SECTION	33.9	Tapwater	0.16	2.7	0.095	2.9	0.026	2.9	-		0.01	4.5	-0.0056	4.4	-		-	
		Deaerated	0.03	2.8	0.011	2.9	-		-		0.027	4.5	-0.0056	4.5	0.0095	4.5	-	
	43.1	Tapwater	0.415	3.8	0.124	3.7	0.024	3.8	-		0.021	6.6	0.00	6.6	0.0005	6.6	-0.003	6.6
					0.036	3.0	0.023						0.072		-0.01			
		Deaerated	0.058	4.0	0.024	3.5	0.0206	3.6	-		0.0195	5.8	0.001	5.8	-0.003	5.9	-	
			0.0396	3.6	0.006	4.0	0.003											
	55.4	Tapwater	0.032	5.2	0.006	5.4	0.0005	5.2	-0.0007	5.2	0.014	8.1	-0.005	8.2	0.0015	8.2	-0.003	8.4
		Deaerated	0.024	4.7	0.0012	4.8	0.0002	4.8	-		0.094	5.8	0.011	7.4	-0.0025	7.4	-0.001	7.5

Note: all Re_t are multiplied by 10^{-5}

TABLE III

SUMMARY OF RESULTS OF VARIANCE ANALYSIS OF DATA FROM REPETITIVE RUNS

CAVITATION CONDITION	MEAN σ	STD.DEV. σ	STD.DEV. MEAN	MEAN L.C.	STD.DEV. L.C.	STD.DEV. MEAN
Sonic	0.0396	0.00404	10.2%	0.2368	0.0062	2.62%
Visible	0.0240	0.00141	5.88%	0.2386	0.0053	2.22%
First Mark	0.0206	0.00043	2.09%	0.4566	0.0175	3.84%

σ - Cavitation Number

L.C. - Loss Coefficient

Conditions of Repetitive Runs:

Cold deaerated water, 1/2 inch test section, 43.1 GPM

TABLE IV

SUMMARY OF RESULTS FROM INDIVIDUAL COMPONENT ANALYSIS

QUANTITY	STANDARD DEVIATION
Cavitation Number	0.0034
Loss Coefficient	0.0106

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