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NATURAL CONVECTION WITH INTERNAL HEAT GENERATION IN AQUEOUS FLUIDS AND LIQUID METALS

Preliminary Technical Report

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ABSTRACT

The general nature of the phenomenon of natural convection heat and fluid flow with particular reference to the case of a vertical tube filled with an aqueous solution in which heat is generated as a distribution of volume heat sources and removed by an external coolant is reviewed.

A method for obtaining analytical solutions to this problem is discussed. A digital computer program for implementing these solutions is reviewed.

Experimental facilities, which have been in operation at the University of Michigan under this project, designed to create the phenomenon described above where the heat is supplied as ohmic resistance in the fluid are described. The preliminary test results from these facilities are discussed. The test program which was anticipated, is reviewed.

A method for treating the overall problem, consisting of the vessel containing the heat generating fluid, the vessel walls, and the coolant is described. Various typical examples, using the preliminary data presently available are shown.

The general applicability of the theoretical and experimental work, which has so far considered only aqueous fluids, to liquid metals is examined. It appears quite possible that the analytical solutions may be applicable or that a reasonably simple modification may be utilized.

The augmentation of heat transfer due to natural convection for various fluids is examined and tabulated for a typical condition. It is found to be greatest for water, but still very substantial in this particular case for liquid metals and air.

1.0 INTRODUCTION

One phase of the nuclear engineering research activity being conducted for the Chrysler Corporation is an investigation of natural convection heat transfer in a cylindrical vertical vessel wherein the heat is generated as volumetric heat sources throughout the fluid, and removed through coolant passages around the periphery. This investigation has been undertaken because it provides a close approximation to the heat flow phenomenon which exists in a homogeneous liquid-fueled nuclear reactor wherein the core is cooled directly, rather than through the utilization of a circulating fuel system. Previous investigations, conducted for Chrysler Corporation, have indicated that:

(1) The directly-cooled "cartridge type" of homogeneous liquid fueled reactor is very suitable to light-weight compact nuclear powerplants which are of the type of interest to Chrysler, and

(2) The controlling heat flow parameter is the natural convection film coefficient. Available data is inadequate to allow a realistic design.

As reported in Reference 9, two test facilities (one water-cooled and one air-cooled) using aqueous electrolyte solutions in which heat is generated by the passage of electric current have been fabricated. These are presently in operation. Also, an analytical solution and a computing procedure designed to predict the heat flow, and local velocities and temperatures, under the applicable conditions have been devised. This solution is capable of considering arbitrary wall temperature boundary conditions and arbitrary axial distribution of heat source. A limited number of points have been computed by hand. At present a digital computer program, capable of providing a sufficient number of points to give an adequately generalized picture with varying parameter values, is in the process of compilation.

It is also necessary to consider the methods for the application of the experimental and analytical data obtained for the fluid within the test section to the overall design problem. This, of course, includes a vessel wall and an external coolant with finite resistances to heat flow.

It is the purpose of this report to consider the present status of the various related endeavors and to present that preliminary understanding of the phenomenon which has been gained to date.

2.0 GENERAL NATURE OF PHENOMENON

2.1 Comparison with Conventional Heat Exchange Apparatus

The heat exchange phenomenon as observed in a closed cell wherein heat is exchanged with the environment through the enclosing walls differs basically from that encountered with conventional heat exchange apparatus with which there is a continuous throughflow of the fluid. The configuration under present investigation consists of a vertical cylindrical vessel closed at the ends, containing a fluid with internal heat generation. Heat is removed through the containing walls to an external coolant. Under these conditions the flow pattern within the vessel is typified by an upward velocity along the centerline and a downward velocity for an annulus adjacent to the walls. Depending on the direction of flow of the external coolant, this may be considered either a counter-flow or co-flow heat exchanger. However, it differs from conventional heat exchangers in that the fluid entering into the heat exchanging annulus at the top of the vessel, ready for the downward passage along the wall, is in fact the effluent from the annulus at the bottom of the vessel. There is thus a feed-back between discharge and inlet conditions which does not exist for a conventional heat exchanger.

Consider for example, an open-ended forced convection heat exchanger, wherein heat is generated within the fluid during its passage through the apparatus. For a given rate of heat generation per unit volume within the fluid, and for a given set of conditions applying to the external coolant and the wall conductivity, the

Nusselt's number and the temperature differences which will be set up depend upon the rate of flow. However, for the case of a closed cell, wherein the fluid motion is the result of internal heat generation (through the mechanism of natural convection), if the wall conductivity and external coolant inlet temperature and flow rate is fixed, then the local velocities and temperature levels and the Nusselt's number (a suitable definition for this dimensionless number is discussed later) are uniquely fixed once the magnitude and distribution of the internal heat sources are set. There is thus a loss of two degrees of freedom when compared with an open-ended forced convection exchanger. For such a case, once the internal heat generation values and the external wall and coolant characteristics (flow rate and inlet temperature) are set, it is still necessary to adjust independently the flow rate and inlet temperature of the primary fluid in order to specify conditions uniquely. It is worth noting that for an open-ended natural convection case, for example, a cooled plate in an infinite fluid containing internal heat sources, it is possible to specify independently the strength and distribution of the heat sources <u>and</u> the temperature level at some selected point in the fluid. Hence the case of open-ended natural convection exhibits one additional degree of freedom as compared with closed-end natural convection.

2.2 Axial and Radial Temperature Gradients and Definition of Nusselt's Number

In most of the previously investigated cases of natural convection as well as in cases of forced convection, the temperature gradient of primary concern is that normal to the wall, existing between the wall and the fluid at some distance from the wall, as in the mean stream or at infinity for some of the natural convection cases. In the case of the vertical enclosed tube with internal heat generation, however, the present investigations, both experimental and analytical, have shown that in most cases (i.e., except where the tube height is small compared to the radius), the fluid temperature difference between the top and bottom of the tube is much greater than between the wall and centerline. (Nevertheless, the maximum gradient occurs adjacent to the vertical walls but persists for only a short distance). The situation is explored in greater detail in a later section of this report. It is significant in the formation of a suitable definition of Nusselt's number. It appears to the writer that from an engineering viewpoint, the temperature difference of importance is that existing between the portion of fluid which is at the maximum fluid temperature (occurring near the centerline at the top of the vessel) and that portion of the inner surface of the wall which is at the minimum temperature (at the lower end of the vessel). This is generally the minimum temperature existing in the internal system, since in most cases there is no portion of the fluid at a temperature lower than the minimum inner surface wall temperature. Nusselt's number, for the purposes of this study, has then been considered to be based upon the axial (and radial) temperature difference as described above, the tube radius, and the mean fluid thermal conductivity. Thus:

$$Nu_a = h \left(\frac{a}{k}\right) = \frac{Q}{2\pi l a \Delta T_{max}} \left(\frac{a}{k}\right) = \frac{Q}{2\pi l \Delta T_{max} k}$$

where Nu_a = Nusselt's number based on radius

a = Tube radius

h = Film coefficient

Q = Total internal heat generation rate for vessel

l = Vessel height

k = Fluid thermal conductivity

ΔT_{max} = (Maximum fluid temperature) - (minimum wall temperature of inner surface)

It is apparent that lower values for the film coefficient and the Nusselt's number will result from the consideration of this axial (plus radial) temperature difference than would be the case if, for example, the mean difference between the fluid "core" and the wall had been used (i.e., purely radial). However, if the local temperatures are known, the parameters may be expressed in terms of the mean radial temperature difference if it is desired. It is anticipated that the results of this study will make the required information available. As previously stated, if the heat source strength and distribution, vessel geometry, and coolant flow rate and entry temperature conditions are fixed, then the axial and radial temperature gradients are uniquely fixed. Thus neither can be adjusted independently.

2.3 Dependence on External Conditions

As mentioned in the previous sections, for the closed cell wherein heat is generated by internal heat sources and the flow motivated by the resulting natural convection, the flow and temperature regimes are uniquely specified once the fluid properties, strength and distribution of the heat sources, vessel geometry, and overall wall conductivity, and coolant flow rate, inlet temperature, and physical properties are specified. As far as the internal problem is concerned, the connecting link may be supposed to be the temperature distribution and level of the inner surface of the vessel wall. This temperature, which itself affects the nature of the internal fluid and heat flow phenomena, is a result of the heat input to the wall from the internal fluid and the heat removal from the wall by the coolant. The heat removal rate is of course also affected by the conductivity of the wall material, assuming the wall to have finite thickness.

The results of different types of coolant arrangements and the methods for computing the overall problem given the data which it is anticipated will result from the present investigations, is examined in greater detail in a later section of this report. For the present, it is sufficient to state that the overall problem is uniquely determined if the geometry, primary fluid, coolant and wall physical properties, coolant flow rate and inlet temperature, and strength and distribution of heat sources are fixed.

2.4 Laminar and Turbulent Flow Conditions

It is a well known phenomenon of natural convection flow that for sufficiently small Grashoff's number the flow is laminar and for large Grashoff's number it is turbulent. In the case of a cooled (or heated) plate in an infinite sea of fluid which is at rest at infinity, the flow near the leading edge of the plate (top for a cooled plate) will be laminar. However, as the flow proceeds away from the leading edge it will become turbulent at a certain distance along the plate, depending upon the physical properties of the fluid and the temperature difference between the plate and the fluid at infinity. The dimensionless grouping which relates the various parameters in a manner significant to this phenomenon is the Grashoff's number. In the case described above, the laminar-turbulent transition is found to occur at a position on the plate where the Grashoff's number based on distance from plate leading edge is approximately 10^9. (Reference 1)

For the case of the flat plate in an infinite fluid, the Nusselt's number (reference 1) is stated to vary with Grashoff's number raised to the 1/4 power for laminar flow, and to the 1/3 power for turbulent flow. For the case of the flat plate then, it would appear that the film coefficient would increase continuously with Grashoff's number, the rate of increase with turbulent flow being greater than for laminar.

It is to be expected that the laminar-turbulent transition behavior as well as the affect of the onset of turbulence may differ in the vertical closed cell from that in the classical flat plate case.

With the flat plate in an infinite fluid, the fluid contacting the leading edge of the plate is as yet undisturbed and hence with no turbulence. In the case of the closed cell the fluid commencing the downward passage along the wall has just emerged from an upward passage through the central region or core, and before that, from the previous downward passage along the wall. Thus the fluid contacting the leading edge (i.e., top of wall) is not undisturbed but may in itself be highly turbulent due to its previous history. In addition to this "feed-back" effect, there is also the effect of the counterflow within the vessel between the descending boundary layer adjacent to the wall and the ascending core. Thus the region of maximum shear is not adjacent to the wall, but is, instead, within the fluid at the juncture of boundary layer and core. Such a region is normally the location for the initiation of instabilities and may, therefore, contribute to a laminar-turbulent transition at lower Grashoff's numbers.

The preliminary experimental observations in the present study are that turbulence first occurs in the core region at the top of the vessel. As the heat input is increased, this region of turbulence spreads until presumably it would engulf eventually the major portion of the vessel. It has not yet been possible to detect the existence or lack of existence of a laminar descending boundary layer near the top when the core at the top is turbulent. Also, to the present time, it has not been possible to detect an initiation of turbulence near the bottom in the boundary layer as the boundary layer thickness increases. It is apparent however, that the lower third or quarter of the tube is relatively stagnant and hence laminar. Thus, at the present time, it appears that the initiation of turbulence for the vertical closed cell is not in the boundary layer as it increases in thickness, but rather at the opposite end of the vessel (i.e., the top) in the central portion of the fluid. No quantitative measurements regarding magnitude of Grashoff's number for transition are as yet available.

The overall effect of turbulence on the heat flow is composed of two counter-acting trends. One effect of the onset of turbulence is the enhancement of the viscosity effect by the addition of so-called "eddy viscosity", thus increasing the resistence to fluid flow. Such an effect would presumably be of major importance in the region of maximum fluid shear between the ascending and descending streams. Another effect of turbulence is the similar increase in the effective thermal conductivity due to the additional effect of physical mixing or "eddy diffusivity of heat". Thus, the effective heat fluxes would be increased by the latter and decreased by the former. An analytical study by Lighthill (Reference 2) for a closed vessel somewhat similar to the present case has shown a large decrease in Nusselt's number in the region of transition between laminar and turbulent flow regimes with an eventual increase as Grashoff's number is further increased to values considerably in excess of those realized by laminar flow. No experimental information has as yet emerged from the present investigation to shed light on this matter.

2.5 General Nature of Flow Phenomenon

As has been mentioned in the previous sections, the general nature of the flow phenomenon in a vertical tube, externally cooled, with heat generated by a distribution of internal sources, appears, as the result of the preliminary experimental observations conducted for the present investigation, to be a single system composed of an ascending core and a descending boundary layer adjacent to the walls. Although it may be theoretically possible, particularly in cases where the tube radius is large compared to the height, for the flow to be subdivided into various discrete cells of ascending and descending fluid (in a manner analogous to G. I. Taylor's rotating cylinder problem), no such phenomenon has been observed either in the preliminary experiments for the present study or in somewhat similar experiments (fluid with internal heat generation between parallel vertical plates) conducted at Oak Ridge National Laboratory. (Reference 3).

The fluid motion may be categorized as of the boundary-layer type because the radial extent of the descending layer adjacent to the wall is generally small compared to the tube radius. It has been shown analytically by Lighthill (Reference 2) that the analogy to the boundary layer (as evidenced by a heated or cooled plate in an infinite fluid) becomes more exacting as the Grashoff's number is increased. Actually the parameter used is the product of the Grashoff's number based on radius and Prandtl number (i.e., Rayleigh number) divided by the ratio of tube length to radius. In Lighthill's terminology, this is t_o. For small t_o, it is shown that the boundary layer thickness will become significant compared to the tube radius so that eventually the boundary layer approximations become inapplicable and it becomes necessary to consider the tube as a whole. For very small t_o, (perhaps the result of a large Grashoff's number but a very large length to radius ratio), the flow becomes fully developed and the velocity and temperature profiles (not the magnitudes) are no longer a function of the axial position. Physically this would be the case of an infinitely long tube (or one with vanishingly small radius). In this case, if streamline (laminar) flow is assumed, there are no end effects and no interchange of fluid in the radial direction. Thus, so far as heat flow is concerned, it is simply the case of static conductivity. With turbulent flow this is not true since there will be interchange of fluid between layers at differing radii. This particular case, for internal heat generation, is treated analytically in reference 4.

In physical cases wherein the heat flux per unit volume is high and the length to radius ratio of the passage is not infinite, the boundary layer type of solution applies. The present study has been concerned wholly with this case, since the volume heat flux for a power reactor is extremely high as applied to the present context giving large Grashoff's numbers.

The preliminary investigations of the present study have shown that in the applicable range of t_o, the flow is indeed of the boundary layer type. Exacting temperature measurements and semi-quantitative velocity measurements have shown that:

(1) the descending velocity is limited to a region close to the tube wall

(2) the radial temperature profile descends sharply to the wall temperature in the region of descending velocity and remains quite constant with radius across the inner portion of the vessel.

(3) the temperature of the core increases continuously in an axial direction towards the top of the vessel. However, this increase is much more marked in the lower portion of the vessel where the flow is relatively stagnant, and is concentrated very sharply in perhaps the lower 1/10th of the vessel.

The sharpness of the axial temperature gradient near the bottom of the vessel is easily explained on physical grounds. The axial temperature rise toward the top is of course a result of the internal heat generation. For a fluid such as water with a relatively high Prandtl number, in a case where the Grashoff's number is high, the effect of the thermal conductivity of the fluid in the interior regions where the temperature gradients are not large is almost negligible. Therefore, the heat generated in the fluid core is removed only when the fluid has completed its transport through the core and entered the boundary layer where heat is removed to the wall by virtue of the large radial temperature gradient existing in this portion. Thus the temperature increase in various portions of the core is in proportion to the length of time which the fluid remains in that location, and thus, roughly, is inversely proportional to the core velocity. For this reason the axial gradient is greatest near the bottom where the fluid motion is small.

Although the fluid temperature difference between top and bottom is much greater in most cases than that from wall to centerline, the boundary-layer assumptions may still be applied since the maximum gradient is in all regions much greater in the radial than in the axial direction. (A possible exception in some cases is the axial gradient immediately adjacent to the bottom end wall).

3.0 ANALYTICAL SOLUTION TO PROBLEM

3.1 General Description of Method

3.1.1 Existing Solution

An analytical solution, extending the work of Lighthill (Reference 2) has been devised for the case of natural convection laminar flow in a vertical cylindrical vessel wherein the heat is generated as the result of heat sources, arbitrarily distributed in the axial direction (uniform radial distribution), and removed through cooled walls. A computing procedure has also been devised, a limited number of hand computations completed, and a high-speed digital computer (IBM-650) program written. The solution is limited to those cases for which the boundary-layer assumptions (i.e., gradients normal to the wall are large compared to the gradients for the same quantities parallel to the wall) are applicable. As previously stated, the flow regimes are such that these assumptions seem applicable to those cases of interest from the viewpoint of homogeneous nuclear power reactors.

Lighthill's work considered a vertical tube with closed ends, heated through the walls and connected at the bottom to an infinite reservoir of cooled fluid. The practical case under consideration was that of turbine blades cooled by internal liquid passages which were connected at the hub to a cooling system. The role of gravity in creating a natural convection flow was played instead of by centrifugal acceleration. The most significant differences from the case which is the subject of the present investigation are the lack of internal heat generation and variable wall temperature. Because of the non-existence of internal heat sources, there is no axial temperature gradient in the core. Also by Lighthill's assumption, there are no variations of wall temperature. However, both of these variations exist in the present case.

The method of Lighthill's analysis follows to some extent the procedure outlined by Squire (p. 641 of Reference 1), wherein velocity and temperature profiles are assumed across a boundary layer to match the physical conditions at the wall and at the inner terminus of the boundary layer both with respect to velocity and temperature and their first derivatives. (It is assumed that velocity and temperature boundary layers terminate at equal radii. This assumption seems justified in that the velocities are the result of the temperature variations). However, no attempt is made to match the second derivatives. The Navier-Stokes equations are then written with the boundary layer assumptions. The assumed temperature and velocity profiles are substituted. In the cases of both Squire and Lighthill, it was then possible to integrate and obtain a direct solution. A further assumption, applying in particular to fluids with relatively large Prandtl number, was made by Lighthill in the neglecting of inertial compared to buoyancy and viscous forces. In addition, the assumption of negligible dissipation energy contribution was made as is usual. Lighthill's derivations are examined in greater detail in Reference 5.

In general, Lighthill's method is one of satisfying the conservation of mass, momentum, and energy for each radial cross-section under the simplifying assumptions listed above. Squire, in his analysis, was concerned with the case of the flat plate in an infinite fluid (this is merely an approximation for a case for which an exact solution exists). Hence the velocity at infinity was zero. Lighthill, investigating the case of a tube with closed-ends, considered a velocity in the core of a direction opposite to that of the boundary layer. The velocity and temperature in the core are assumed to be uniform (the temperature actually constant and equal to the infinite reservoir temperature). With the assumption of such a velocity, it is possible to satisfy conservation of mass, whereas otherwise it would not be. It is possible to express the velocity and temperature as functions only of the radius at which the boundary layer terminates. This terminating radius is allowed to vary with axial position so that velocities and the boundary layer temperatures are functions of the axial position (not the core temperature).

3.1.2 Extensions of Existing Solutions to Present Case

As previously mentioned, one of the outstanding characteristics of the case of the closed-tube with internal heat generation as shown by the experimental work presently in progress, is the predominently large temperature increase from bottom to top of the fluid in the central portion of the vessel (i.e., along the centerline). Another is the fact of variable wall temperature which exists in general with any practicable method of wall cooling. Any realistic analytical solution must consider these conditions.

Consequently, a procedure has been devised, following in general the method of Lighthill, but allowing for a variation with x (axial position) of core and wall temperatures. This is described in some detail in Reference 6. (Wall temperature variation was not considered in Reference 6, although a rather simple addition to the calculation considering this effect has been subsequently added). In general, the basic assumptions of Lighthill (relatively large Prandtl number so that inertial forces are negligible compared with viscous and buoyant forces, and also boundary-layer assumptions) were employed. Again, temperature and velocity profiles across the tube were assumed to match the physical conditions (first derivatives but not second) at wall and at the inner terminus of the boundary layer. Again the velocity and temperature boundary layers were assumed to terminate at the same radial position. As in Lighthill's analysis, the velocities and temperatures were expressed as functions of the boundary layer terminating radius and the core temperature. However, unlike Lighthill, the core temperature was assumed to be a function of **x**.

The core temperature variation was described under the assumption of negligible thermal conduction either axially or radially in the core. An order of magnitude analysis showed that this was a reasonable assumption. The core was assumed to originate from the boundary layer effluent so that its temperature at the lower end was fixed by the mixed-mean boundary layer temperature. The temperature rise was considered as the result of the heat capacity of the fluid, the rate of heat generation and the axial velocity.

Under these conditions and assumptions, the tube was considered as divided into a series of discs of small height compared to radius. For each of these discs, the conditions of conservation of mass, heat (including wall conduction, difference in axial heat convection, and heat generation for each disc), and momentum were set up. Also the relation for core temperature, derived as previously explained, is utilized. The final result is two simultaneous ordinary, non-linear differential equations involving the core temperature and the radius of the inner edge of the boundary layer. These are written as approximate difference relations for each disc. All the quantities are expressed in non-dimensional form. A non-dimensional heat source term, q_v, is introduced and t_o, from Lighthill's terminology, is used as the maximum non-dimensionalized temperature difference between the fluid at the tube centerline and the temperature of the inner surface of the wall. The details of the derivation and the resulting equations are given in Reference 6. It is shown that Nusselt's number is a unique function of either t_o or q_v for a given wall temperature distribution. The results so expressed are applicable to all fluids (with Prandtl number of the order of unity) and to all tube length to radius ratios if flow is laminar and in boundary layer regime.

3.1.3 Computing Procedure

The computing procedure which has been utilized involves a double iteration. The start is made from the top disc. A value for the maximum fluid to wall temperature difference, t_o, is assumed, depending upon the q_V on which the particular calculation is to be based, and also upon the wall temperature boundary condition (i.e., constant temperature, linear temperature decrease, parabolic temperature decrease, etc.). With this assumed value for t_o, it is possible by a few iterations to find the boundary layer thickness at the lower edge of the first disc. It is then necessary to proceed with the second disc, using the same t_o value and equating the boundary layer thickness at the lower edge of the first disc with that at the upper edge of the second disc. In this way, the calculation proceeds to the bottom of the tube, at which point the boundary layer thickness should be identically zero. This is a result of the physical condition of zero net heat convection for the entire tube, or, stated differently, the total wall conduction must equal the total heat source. In general, the calculation will show a boundary layer thickness at the lower end of the tube which is either negative or positive finite. It is then possible to predict the proper direction for the adjustment of t_o, and the calculation is repeated for a revised t_o assumption. It is found that linear extrapolation of the proper value for both boundary layer thicknesses and t_o is quite accurate so that convergence is obtained fairly quickly. It is obvious from the above description that the method is flexible to the extent of allowing the consideration of different heat source terms and wall temperatures for each disc. It is, therefore, possible to obtain results for uniform or arbitrarily varied distributions of wall temperature and/or heat source (axial distribution only).

3.1.4 Hand Calculations to Date

At the present time hand calculations have been completed for the case of a constant wall temperature and uniform heat source distribution. The resulting curves are shown in Figures 1 and 2. In Figure 1, Nusselt's number is plotted against the non-dimensional heat source, q_v. It could also have been plotted against t_o and the result would be similar. Figure 2 is a plot of q_v against t_o. Actually they are algebraically related according to the equation

$$Nu_a = \frac{q_v}{2t_o}$$

Also shown in these figures is the curve derived by Lighthill wherein the heat source is absent. Although there is actually no q_v in Lighthill's work, he does show a curve of t_o vs Nu_a. q_v, which has no direct physical significance in this case,*

*Heat generation (or heat sink) could be considered as concentrated at the bottom (for heat generation) in a differentially thin disc. Then q_v as plotted would be a mean value averaged over the tube, although the heat generation is concentrated in a single location.

was computed from the relation with Nusselt's number given above so that the Lighthill curve could be compared with the curve derived for internal heat generation.

It is noted that the anticipated Nusselt's number for a given q_V or t_o is considerably lower for the case of internal heat generation than for the case where the fluid is connected to a reservoir at fixed temperature (Lighthill case). This is a result of the large axial temperature gradient which exists in the former case and which is completely absent in the latter. If the Nusselt's number for the internal heat source case were computed upon the mean temperature difference from tube centerline to wall, then the two curves would roughly coincide.

An additional point is shown on the curves for a case wherein a linear drop of wall temperature from top to bottom was assumed (heat source uniform). This particular case is characterized by the fact that the sum of the temperature difference between tube centerline and wall at the top and between wall top and bottom is 5.4 times the difference between tube centerline and wall at the top. Additional points have not been calculated, but it is noted that the Nusselt's number for a given q_V is still further reduced by the drop in wall temperature. It appears that the more severe is this drop, the more severe is the loss in Nusselt's number. Also, it seems likely that a non-linear drop in wall temperature (i.e., concentrated near the bottom) would be even more severe than the linear case.

An additional curve, Figure 3, shows the maximum, non-dimensionalized boundary layer velocity as a function of q_V for the constant wall temperature and linear wall temperature in internal heat generation cases.

3.1.5 Digital Computer Program

The calculation procedure described in the preceding section is lengthy and involves a great amount of repetitive effort. Consequently, it was decided that an IBM-650 digital computer program would be useful and would allow a much broader scope to the calculations than would be practicable by hand. Such a program has been in the process of compilation for some weeks and is presently nearing completion.

As an initial step it was endeavored to write a program simply for the basic iteration for the boundary layer thickness, considering only an individual disc. This was accomplished on the basis of a fixed increment iteration. Since the fixed increment appeared to involve excessive machine time, it was altered to converge by a linear extrapolation procedure. A working program to accomplish the calculation on this basis for an individual disc has been compiled and utilized.

The second step has been the addition of a mechanism to provide the required transfer from disc to disc and finally the iteration on t_o to complete the calculation for constant wall temperature and a given q_V. It was felt that such a program would be useful in a more accurate checking of the hand calculations for uniform q_V, and would allow the extension to various q_V distributions to determine the effect of non-uniform heat source. This program also appears to be nearly perfected, although it has not yet received sufficient utilization to determine its practicality. It is anticipated that the final step in the digital computer program will be the incorporation into the program of the refinements necessary for the consideration of a varying wall temperature. These refinements do not appear to be excessive.

When the calculations are completed, it is hoped to have a family of curves showing the relation between non-dimensional heat source and Nusselt's number for the case of internal heat generation in a vertical tube for various wall temperature and/or heat source distributions for laminar flow. It is then hoped to show experimentally the general validity of these curves and in what manner and to what degree the turbulent flow cases differ from the laminar. No analytical approach to the turbulent case is anticipated. Since the laminar-turbulent transition presumably depends upon Grashoff's number, it is expected that this transition will occur at different points of the non-dimensionalized curves, depending on the length to radius ratio of the tube.

The analytical calculations, if shown to be realistic, will allow extrapolation of the results to higher and lower power ranges, to differing geometries and to fluids with differing physical properties. Also, they will allow the prediction of the local temperature and velocity at any desired point and perhaps indicate those areas in which thermal stress problems, problems of slurry precipitation, problems of possible erosion or mass transport, etc., may logically be expected. If it happens that the correlation between theoretical and analytical results is not good, it may be possible to devise some form of empirical correction to be applied to the analytical results.

At the present time no meaningful comparison of analytical and experimental results is possible.

It is noted from an examination of the slope of the q_V vs t_o plots in Figure 2 that $q_V \alpha t_o^{1.2}$. Thus, since $Nu_a = 2q_V/t_o$, $Nu_a \alpha t_o^{1.2} \alpha q_V^{1-1/1.2} = q_V^{.167}$. Therefore, $h \alpha t^{.2} \alpha q_V^{.167}$. A result of this sort, modified if necessary to accommodate the experimental data, and written as an equality with a suitable constant should result from the present study. For ordinary free convection (flat plate in infinite fluid) h is supposed to be proportional to $t^{5/4}$ for laminar flow and $t^{4/3}$ for turbulent (Reference 1).

4.0 EXPERIMENTAL PROGRESS AND ANTICIPATED TEST PROGRAM

4.1 Experimental Facilities

At the present time, two separate test facilities are in operation. Each consists of a vertical glass tube, containing an aqueous electrolyte solution in which heat is generated by the passage of electrical current through the solution. The heat is removed in each case through the tube wall. The smaller facility is cooled by natural convection with the atmosphere and hence is capable of only small heat fluxes (approximately 10,000 BTU/cu.ft.-hr). The larger is cooled by a forced convection water jacket so that the maximum heat flux is of the order of 100,000 BTU/cu.ft.-hr. or about 1 KW/liter. The dimensions of the smaller facility are 2-5/8" diameter x 18" length and of the larger 4" diameter x 24" length so that the discrepancy between the attainable values of q_V is even greater than that between the heat fluxes. In each facility it is possible to measure temperatures in the fluid at any desired point by a traversing thermocouple arrangement. It is also possible to introduce dye at any desired point and time the flow, and thus estimate the velocity. It is also possible in this manner to detect the nature of the flow; i.e., direction of velocity, laminar or turbulent, etc.

4.2 Test Facility Operation to the Present

Each facility has been operated for a substantial number of hours to the present time and considerable data has been taken. However, these preliminary runs have been largely concerned with the testing of the equipment and the experimental procedure so that at present the data is insufficient to allow any statement regarding the degree of correlation with the analytical program.

Overall heat balance conditions have been carefully checked so that the eventual disposition of the electrical heat input has been determined. Hence, it is known that no unsuspected phenomenon (chemical change in the solution, for example) is accounting for a substantial portion of the heat input.

It is believed that operation of both the air-cooled and water-cooled facilities is desirable to provide a greater range of heat inputs (and q_v) and also of wall temperature distribution. The temperature distributions differ because of the difference in cooling methods. The air-cooled facility provides a fairly close approach to the condition of constant wall heat flux, since the heat transfer from glass to air across a relatively large temperature difference (which is substantially unaffected by the comparatively small temperature differences in the fluid) is controlling. As explained in Section 5, a somewhat higher wall heat flux at the bottom is actually required. As will be explained in Section 5, the forced-convection water-cooled wall gives a case intermediate between constant wall temperature and constant heat flux. The exact form of the distribution depends on the coolant flow rate with the constant temperature case being approached more closely for large flow rates. A variation beyond the variation in flow rate is possible, since the direction of coolant flow may be reversed, providing either counter- or co-flow.

The effects of these various combinations is discussed in greater detail in Section 5. As previously pointed out, an analytical calculation for any desired wall temperature distribution can be effected with the computer program.

A further variation which it is planned to employ will be the variation of test section length. This will allow the attainment of a given q_v value under differing conditions of length to radius ratio, and should give a broader scope to the experimental results. This variation can be accomplished easily with either facility.

Aside from the general checking of equipment, test procedures, and overall heat balance, several significant results have emerged from the test program to date.

(i) It has been determined that the general nature of the flow is of the boundary layer type and is composed of a single large vortex occupying the entire tube.

(ii) It has been determined (by local voltage measurements) that the heat source is approximately uniform.

(iii) It has been tentatively determined that the first appearance of turbulence is in the core region at the top. It has been noted that the resulting mildly turbulent condition results in a slow temperature oscillation in this region so that steady-state is not achieved. Temperatures in the region of laminar flow do appear to be steady within the sensitivity of the instrumentation. It is expected that more vigorous turbulence anticipated for higher heat fluxes will result in a statistical steady-state as far as the rather slow response of the instrumentation is concerned.

(iv) It has been determined that there is no large scale chemical changes in the solution due to the a.c. current. The water-cooled facility has been operated in excess of 50 hours with no noticeable change in appearance or electrical conductivity.

4.3 Anticipated Test Program

It is anticipated that certain values of q_V, covering the full range of the equipment, will be selected and runs made at these values for differing length to radius ratios and wall temperature distributions as effected by the cooling arrangement. For the water-cooled rig, for example, a series of nominal q_V values will be selected. These may be attained under varying conditions of power input by simply varying the coolant flow rate over the full range available and also by reversing the direction of the coolant flow. It may also be possible to reach the same q_V with test sections of reduced length and in some cases with the air-cooled facility. It is anticipated that perhaps 6 or 7 different q_V values will be used. Constant q_V is used because it is physically easier to obtain than constant t_o, and also because the computing program as presently constituted is based on the original selection of q_V.

As previously discussed, it is hoped to compute a family of q_V vs Nu curves for differing wall conditions and also differing heat source distributions for laminar flow. For any desired type of wall or heat source distribution it will be possible to interpolate between these curves and hence locate the consistent Nusselt's number. Auxiliary curves will be provided to estimate the fluid velocities, temperatures, and boundary layer thickness. It will thus be possible to obtain the degree of correlation between experimental and analytical results, and presumably to plot empirical curves showing the effect of turbulence. Preliminary examples of these methods are discussed in Section 5.

5.0 TREATMENT OF OVERALL PROBLEM

5.1 General Discussion

As has been emphasized repeatedly in the previous discussions, the nature of the fluid and heat flow phenomena in the internal tube depend upon the nature of the cooling process. The connecting link, from the viewpoint of the calculation, is the temperature distribution on the inner surface of the tube wall. Stated differently, the overall design problem for say a homogeneous nuclear power reactor vessel must consider the heat transfer parameters which apply to the coolant and the dividing wall between coolant and fuel solution, as well as the natural convective phenomena within the vessel, since the nature of the latter depends upon the former. Because of the complexity of these inter-relations, it appeared necessary to consider the overall problem and develop methods for the necessary computations based upon the data anticipated from the present study.

Although such experimental data is not presently available, it is possible to use the results of the previously described hand computations to illustrate the methods which may be employed and the trends which may be expected from differing wall coolant conditions. Several such examples applying to conditions for both the air-cooled and water-cooled facilities are considered below.

5.2 Specific Examples

5.2.1 Water-Cooled Facility - Infinite Coolant Flow

5.2.1.1 Generalized Procedure

For all the examples which follow, the only available information relating to the internal fluid and heat flow is that from the previously discussed hand calculations. As stated, these have considered only the case of constant wall temperature and a single case with a linear (decreasing toward the bottom) wall temperature distribution of a particular slope. However, the methods to be used could be applied much more adequately if the additional information expected to result from the present investigation were available.

From the cases which have been calculated, it is possible to compute the wall conduction heat flux, the velocity, the boundary layer thickness, and the local temperatures. Several of these parameters are shown in Figure 4. Of particular interest in the present connection is the wall conduction heat flux. It is noted from an examination of Figure 4 that the axial distribution of the wall heat flux, for the case of constant wall temperature, is not affected appreciably by the heat source term (which is the parameter for these curves). In all cases it is much greater near the top of the tube (greater by a factor of 3 or 4 than at the lower end). It is also noted that the particular linear wall temperature case which was computed gives an approximately constant wall heat flux with axial position. Since no further information is available at present, it is assumed that this distribution also is not significantly affected by the heat source term.

The determination of the overall solution for specified coolant conditions is one of trial and error. Suppose, for example, that we assume the constant wall temperature case. For a given physical condition, we can then compute the heat flux through the wall (from Figure 4) for each axial increment of wall surface area. Then for specified coolant conditions we can compute the coolant temperature rise for each axial increment. Suppose we assume that at one end or the other, the wall heat flux as determined by the internal conditions (i.e., the assumption of constant wall temperature, for example) is matched by the heat flow into the coolant. We can then compute the change in temperature of the coolant as it passes this particular axial increment, and hence the temperature difference available to cause heat flow for the next axial increment. If the heat flow into the coolant based on this temperature difference does not match that required by the internal conditions, then the assumed wall temperature distribution cannot be correct. Nevertheless, it is possible to proceed in this fashion for the entire length of the tube, and then to sum and compare the heat flow demanded by the internal conditions and that allowed by the external conditions, considering the necessary temperature rise of the coolant.

A second wall temperature distribution assumption may be made, and the procedure repeated. If a family of curves for various wall temperature distributions is available (as it will be at the conclusion of this study), it will then be possible to estimate the consistent conditions for any arrangement of cooling apparatus, and thus to estimate suitable values for all the parameters of interest applying to the internal phenomenon. At the present time, the only other wall temperature assumption (other than constant temperature) for which data is available is that of the linear drop.

5.2.1.2 Specific Example

The specific example under present discussion is that of the water-cooled facility with an infinite coolant flow. A power input corresponding to a q_v value of 4×10^7 was chosen for the example. (With the assumption that wall temperature distribution does not depend on q_v, the results become independent of power level). The procedure outlined above was followed for the constant and linear wall temperature cases. Although the coolant flow is infinite and there is thus no coolant temperature rise, the result is not one of constant (inner surface) wall temperature. If the constant wall temperature case is assumed, the temperature difference between the inner surface of the wall and the coolant is constant. Thus, constant wall heat flux would be required. However, this is not available from the constant wall temperature case, but rather from that of the linearly decreasing wall temperature. As a result, the solution must lie between these cases, somewhat closer to that of constant wall temperature. Table I shows the results of the calculations, showing the difference between the flows as required by internal and external conditions. Since the difference between internal and external heat

TABLE I

WATER-COOLED FACILITY - INFINITE COOLANT FLOW

CALCULATION SUMMARY, $q_v = 4 \times 10^7$

Δx	Constant Heat Flux			Constant Wall Temp.		
	Q_{fluid}	$Q_{coolant}$	$Q_{fluid}-Q_{coolant}$	Q_{fluid}	$Q_{coolant}$	$Q_{fluid}-Q_{coolant}$
0-.2	78.4	368	-290	176.0	31	145
.2-.4	78.4	314	-236	87.0	31	56
.4-.6	78.4	240	-162	55.0	31	24
.6-.8	78.4	175	-97	43.0	31	12
.8-1.0	78.4	78.4	0	31.0	31	0
$\sum =$	392	1175	-783	392	155	+237

TABLE II

WATER-COOLED FACILITY - NON-INFINITE COOLANT FLOW, COUNTER-FLOW

CALCULATION SUMMARY - 1 GPM, $q_v = 2 \times 10^8$

Δx	Constant Heat Flux			Constant Wall Temp.		
	Q_{fluid}	$Q_{coolant}$	$Q_{fluid}-Q_{coolant}$	Q_{fluid}	$Q_{coolant}$	$Q_{fluid}-Q_{coolant}$
0-.2	392	1010	-618	882	129	753
.2-.4	392	855	-463	431	135	296
.4-.6	392	700	-308	274	142	132
.6-.8	392	547	-155	216	150	66
.8-1.0	392	392	0	157	157	0
$\sum =$	1960	3504	-1544	1960	713	+1247

flux is negative in one case and positive in the other, the solution must lie between these cases. Whether the suitable wall temperature distribution is linear or not can be determined by comparing the individual terms for the various axial increments for cases where the sums are matched. Further data than that presently available is required for such a determination. The preliminary experimental measurements of wall temperature indicate that the drop is not linear but accelerated rather sharply near the bottom.

5.2.2 Water-Cooled Facility - Non Infinite Coolant Flow

5.2.2.1 Counter-Flow (Coolant Inlet at Bottom)

For this case, a q_v of 2×10^8 was assumed and a water coolant flow rate of 1 GPM. Under the previously stated assumptions, the ratio of power level to coolant flow rate is significant, although not the absolute level of either. Under the conditions of a finite coolant flow rate it was necessary to estimate the film coefficient between the test section wall and the water coolant (film coefficient was assumed to be infinite for infinite coolant flow rate). Considering finite film coefficient and finite temperature rise of the coolant due to the heat input, the calculating procedure, considering the various axial increments individually as previously outlined, was completed. The results for the constant and linear wall temperature distributions are shown in Table II. In this case, it is noted that the solution is apparently midway between the two distributions (i.e., further removed from the constant wall temperature case than was true for infinite coolant flow rate). If the flow rate is increased to the order of 10 GPM, the temperature rise of the coolant as it passes through the facility is very small, the solution is virtually that of infinite flow rate.

5.2.2.2 Co-Flow (Coolant Inlet at Top)

For the water-cooled facility, with a q_v of 2×10^8, if the coolant flow rate is of the order of 10 GPM, it is virtually infinite and the direction of flow makes little difference (the temperature rise of the coolant as it passes through the facility is very small). However, if the flow rate is only 0.1 GPM with the same power input, then the temperature rise of the coolant is such that the higher wall heat fluxes required at the top for the case with constant inner wall surface temperature is approximately accommodated. Under these conditions, therefore, it might be anticipated that the inside surface of the wall would remain at approximately constant temperature. The calculations for this case, showing the difference between the heat flows at the various axial increments as required by internal and external conditions are listed in Table III. It is noted that although the sums are roughly matched, the individual increments are not, particularly at the top. Therefore, some refinement of the constant wall temperature condition is indicated.

5.2.2.3 Infinite Wall Conductivity

If the case of constant heat flux were applicable, a finite wall conductivity would cause a constant radial wall temperature difference, so that the assumption of infinite wall conductivity would result only in a closer approach between coolant and internal fluid temperatures, with no change in distribution. If the wall heat flux were not constant, then there would be some change in distribution. With the water cooled rig this effect is not great since the wall temperature drop is relatively small compared with the outside film drop. Therefore, the results with infinite

TABLE III

WATER-COOLED FACILITY - NON-INFINITE COOLANT FLOW, CO-FLOW

CALCULATION SUMMARY 0.1 GPM, $q_v = 2 \times 10^8$

	Constant Heat Flux			Constant Wall Temp.		
Δx	Q_{fluid}	$Q_{coolant}$	$Q_{fluid}-Q_{coolant}$	Q_{fluid}	$Q_{coolant}$	$Q_{fluid}-Q_{coolant}$
0-.2	392	1375	-983	882	740	+142
.2-.4	392	1120	-728	431	502	-69
.4-.6	392	810	-418	274	340	-66
.6-.8	392	584	-192	216	230	-24
.8-1.0	392	392	0	157	157	0
$\sum$ =	1960	4281	-2321	1960	1969	(-) 9

wall conductivity, for the water-cooled facility are similar to those assuming the actual wall value. This is even more true of the air-cooled facility where the outside film coefficient is completely controlling.

5.2.2.4 Zero Wall Conductance

For cases approaching a relatively zero wall conductance, the temperature drop across the wall will be very large compared to temperature variations either in the internal cell or in the coolant. The only consistent solution then to this case is that of constant wall heat flux.

5.2.3 Natural Convection External Cooling

For the case of a flat plate at constant temperature which is either heated or cooled in an infinite sea of fluid, it is generally accepted (Reference 1) that the local film coefficient (for laminar flow) is proportional to $\Delta T^{5/4}/L^{1/4}$ where ΔT is the temperature difference between the plate and the fluid at infinity, and L is the distance from the plate leading edge. Using this relation, it is possible to compare the wall heat fluxes which will be required by the coolant and those which will be available from the internal fluid under the different wall temperature distributions for which calculations are available. As is noted, the natural convection coolant requires smaller heat flow at the top than at the bottom (the bottom is the leading edge for the outside flow), if the wall temperature is constant. In lieu of more satisfactory information it has been assumed that the film coefficients as computed for the constant temperature flat plate are roughly applicable if the local temperature difference between the outer wall surface and the coolant are used.

A calculation for q_v of 2×10^8 was made for the water cooled rig with natural convection water cooling. For the case of constant inner wall temperature, the available temperature drop between coolant and outer wall surface decreases toward the top because of the higher wall heat flux at the top for this case. However, the ability of the natural convection coolant to accept heat at the top is reduced both because of this reduction of temperature difference, and also because of the effect of the increased distance from the leading edge (this latter is more important). Thus, the case of constant inner wall temperature is not satisfactory.

A second calculation for the constant inner wall heat flux case was made. It was found that this was more consistent, but that the solution required a heat flux which was greater at the bottom than at the top. Thus the consistent flow regime for this case would show a decrease toward the bottom of inner wall temperatures more severe than that required for constant wall heat flux. Presumably this would also apply to the air-cooled rig since the relations governing the behavior of the coolant are the same.

6.0 NATURAL CONVECTION HEAT TRANSFER WITH INTERNAL HEAT GENERATION FOR LIQUID METALS

6.1 General Applicability of Aqueous Solution

The foregoing discussions have considered only fluids with a Prandtl number (i.e., ratio of thermal to momentum diffusivities) of the order of unity. The basic derivations (references 2,5, and 6) include several assumptions which appear to limit the solution, strictly speaking, to this case. However, it seems desirable to examine these assumptions in detail to determine their degree of applicability to fluids such as the liquid metals for which the Prandtl number is very small (thermal diffusivity of the order of 100 to 1000 times the momentum diffusivity).

6.2 Basic Assumptions

The basic assumptions of major importance which were made in the previously discussed solutions were the following:

6.2.1 Boundary layer assumptions are valid.

6.2.2 The thickness of the temperature and velocity boundary layers are similar.

6.2.3 The inertial forces are small compared with the viscous and buoyant forces. As the equations were written, the inertial terms are multiplied by 1/Prandtl number. Thus, their importance increases with small Prandtl number.

6.2.4 There is negligible thermal conduction of heat between the core regions and the boundary layer, and also axially along either the core or boundary layer.

It would appear that those cases with liquid metals for which the phenomenon is of the boundary layer type would be more restricted than with aqueous fluids. Because of the much greater thermal conductivity, the temperature effect will be less localized in the radial direction. Thus, it appears that the boundary layer would be sufficiently thin (i.e., include a sufficiently small portion of the tube cross-section) only for higher heat fluxes and/or smaller L/D ratios than for the aqueous case. However, in the aqueous case, the boundary layer phenomenon appears to apply down to quite small heat fluxes. Hence, considering the very large heat fluxes which are of interest for the case of nuclear power reactors, it seems likely that the boundary layer assumptions will be applicable in most cases of interest.

At first glance it would appear that the approximate equality of thickness for temperature and velocity boundary layers would depend upon an equality of thermal and momentum diffusivity. This is indeed more or less the case for forced convection. However, with natural convection, there is a buoyant force motivating a velocity wherever there is a radial temperature difference. Hence, it would seem that the radial temperature boundary layer thickness would be extended with low Prandtl number fluids (as compared to aqueous fluids), but that this extension would cause an equal radial extension of the velocity boundary layer. Assumptions of the velocity and temperature distributions in the boundary layer differing from those used for the aqueous case may or may not be warranted. Further consideration and investigation is necessary to answer this question. The exact temperature and velocity profiles for aqueous fluids and liquid metals for laminar flow for the case of a heated (or cooled) flat plate in an infinite fluid are shown in reference 8. Perhaps these profiles may be used as a guide. Also measured profiles for forced convection in turbulent flow may be useful.

The applicability to low Prandtl number fluids of the third assumption regarding the neglect of the inertial terms can be resolved most suitably by an evaluation of these terms for specific cases. This has not been accomplished as yet.

The fourth assumption regarding the neglect of thermal conduction of heat between core and boundary layer, and in an axial direction in either, can also be resolved by an evaluation for typical cases. Such an evaluation has not been made to date. If these evaluations show that the assumptions are not valid, it may be possible to account in some appropriate manner for the neglected terms. It is believed quite likely that experimental results will show that the analysis is approximately valid even for the liquid metals, or at least that it may be adapted in some simplified manner to account for the variation. The analysis has served to illustrate the mechanism and the type of behavior to be expected as well as the suitable non-dimensional parameters which may be used to correlate the results.

6.3 Comparison of Effects of Liquid Metal and Aqueous Natural Convection

Natural convection, with any fluid, will increase the rate of heat flow for any set of given temperature conditions over that obtaining if the fluid were to be replaced by a solid with an identical thermal conductivity. The amount of this increase will depend upon many factors including, at the least, the ratio of momentum to thermal diffusivity and the temperature coefficient of volumetric expansion.

The smaller the ratio of momentum to thermal diffusivity (i.e., Prandtl number) and the smaller the temperature volumetric expansion coefficient, the smaller will be the proportionate augmentation of heat flow by natural convection. As it happens, the Prandtl number for liquid metals such as mercury and bismuth is about 1/100 that of water, whereas the temperature coefficient of expansion for the liquid metals is of the same order as that of water. However, the relative proportionate increases due to natural convection are not clear. Of course, the heat flow, with or without natural convection under given temperature conditions, is much greater for the liquid metals. Approximate calculations based on reference 8 for a flat plate at constant temperature in an infinite fluid at constant temperature, with laminar flow, showing the heat flow augmentations from natural convection (a solid slab thickness of 1.5 feet was assumed) under the same temperature conditions are listed in Table IV. It is noted that the improvement with water is greater than that with liquid metals, but that it is very substantial in all cases. The augmentation will decrease with slab thickness and with temperature difference.

TABLE IV

HEAT FLOW AUGMENTATION OVER SOLID DUE TO NATURAL CONVECTION
$\Delta T = 300°F$
Cooled Flat Plate (4 ft.long) in Infinite Fluid - Laminar Flow

Fluid	$h_{Free\ Convection\text{-}mean}$ Btu/hr-ft^2-°F	Augmentation Factor
Bismuth	340	23
Mercury	260	41
Sodium	2320	35
Water (60°F)	45.3	100
Water (572°F)	160	400
Air (60°F)	0.6	30

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APPENDIX

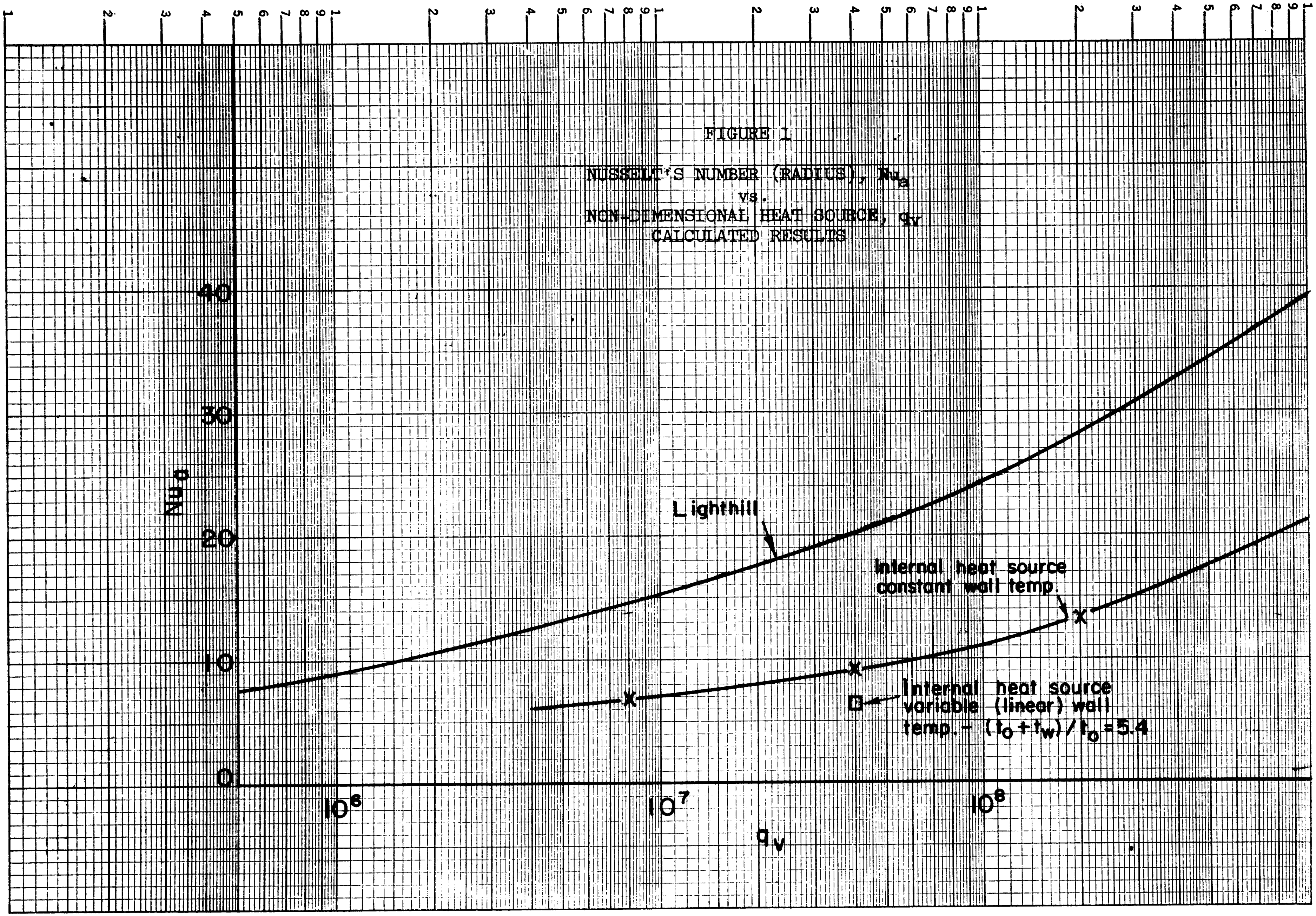

FIGURE 1

NUSSELT'S NUMBER (RADIUS), Nu_a
vs.
NON-DIMENSIONAL HEAT SOURCE, q_v
CALCULATED RESULTS

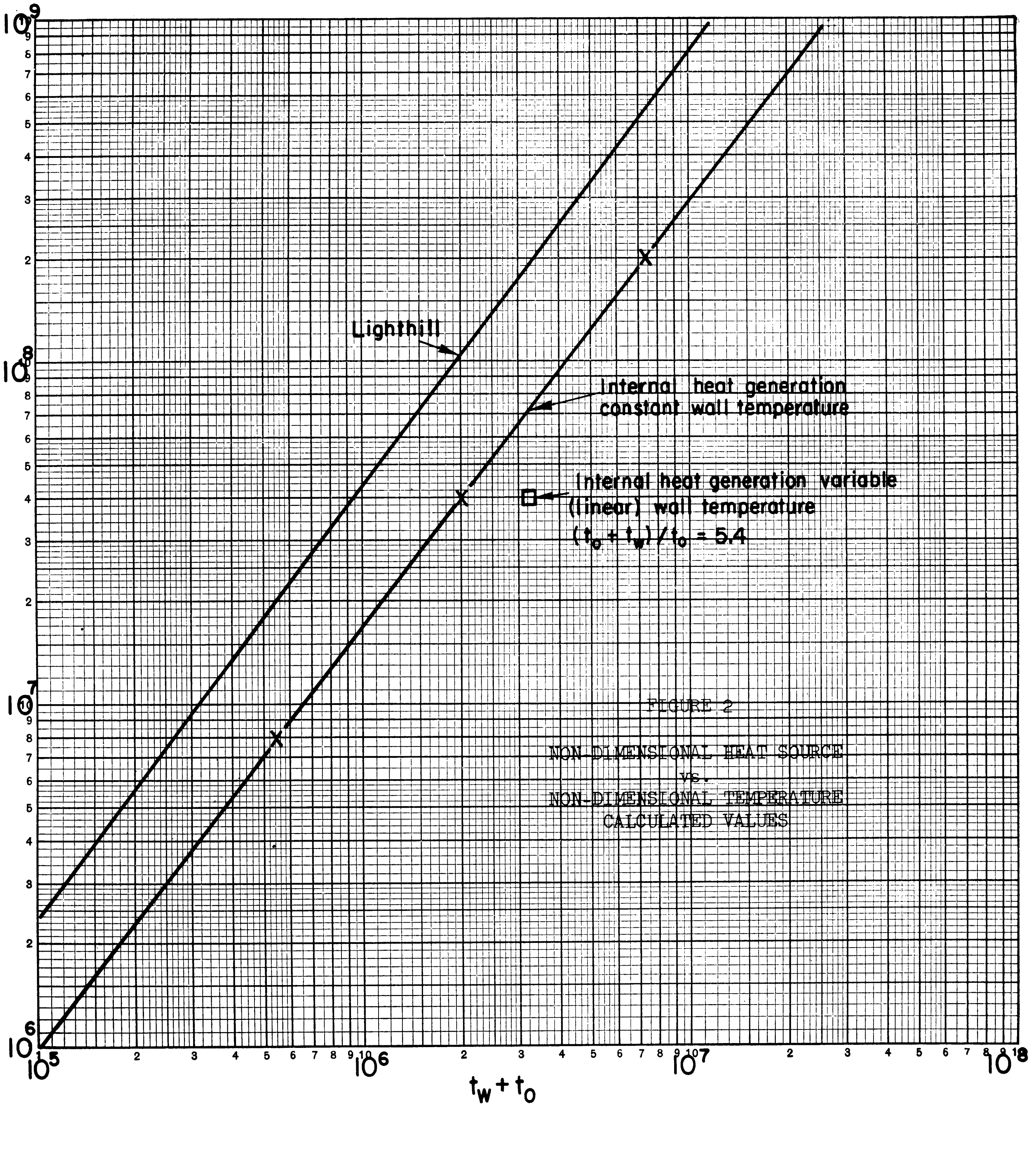

FIGURE 2

NON-DIMENSIONAL HEAT SOURCE
vs.
NON-DIMENSIONAL TEMPERATURE
CALCULATED VALUES

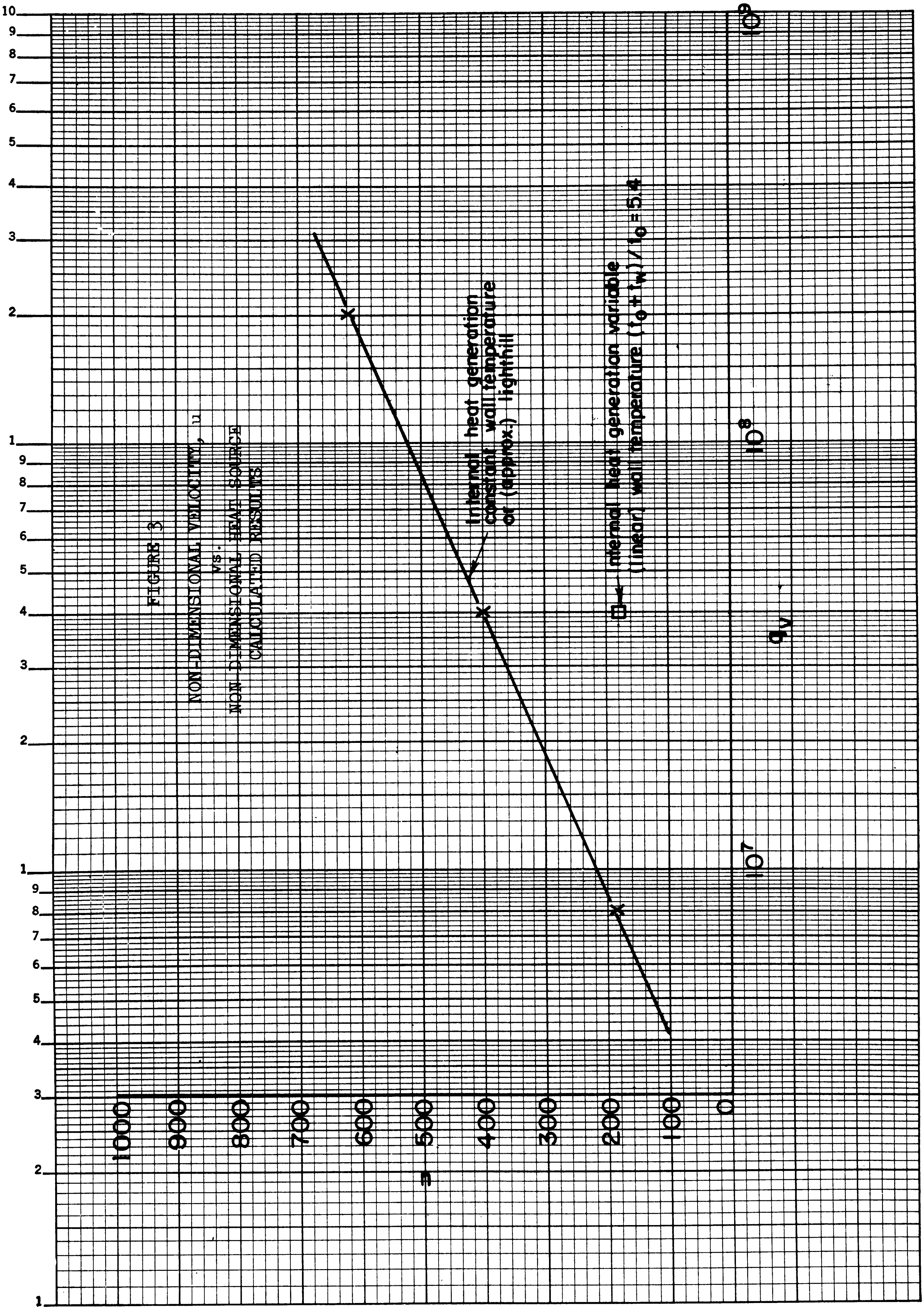

FIGURE 3
NON-DIMENSIONAL VELOCITY, u
vs.
NON-DIMENSIONAL HEAT SOURCE
CALCULATED RESULTS
Internal heat generation
constant wall temperature
or (approx.) lighthill
Internal heat generation variable
(linear) wall temperature $$(t_o + t_w)/t_o = 5.4$$
$$10^9$$
$$10^8$$
$$10^7$$
$$q_v$$
1000
900
800
700
600
500
400
300
200
100
0
u

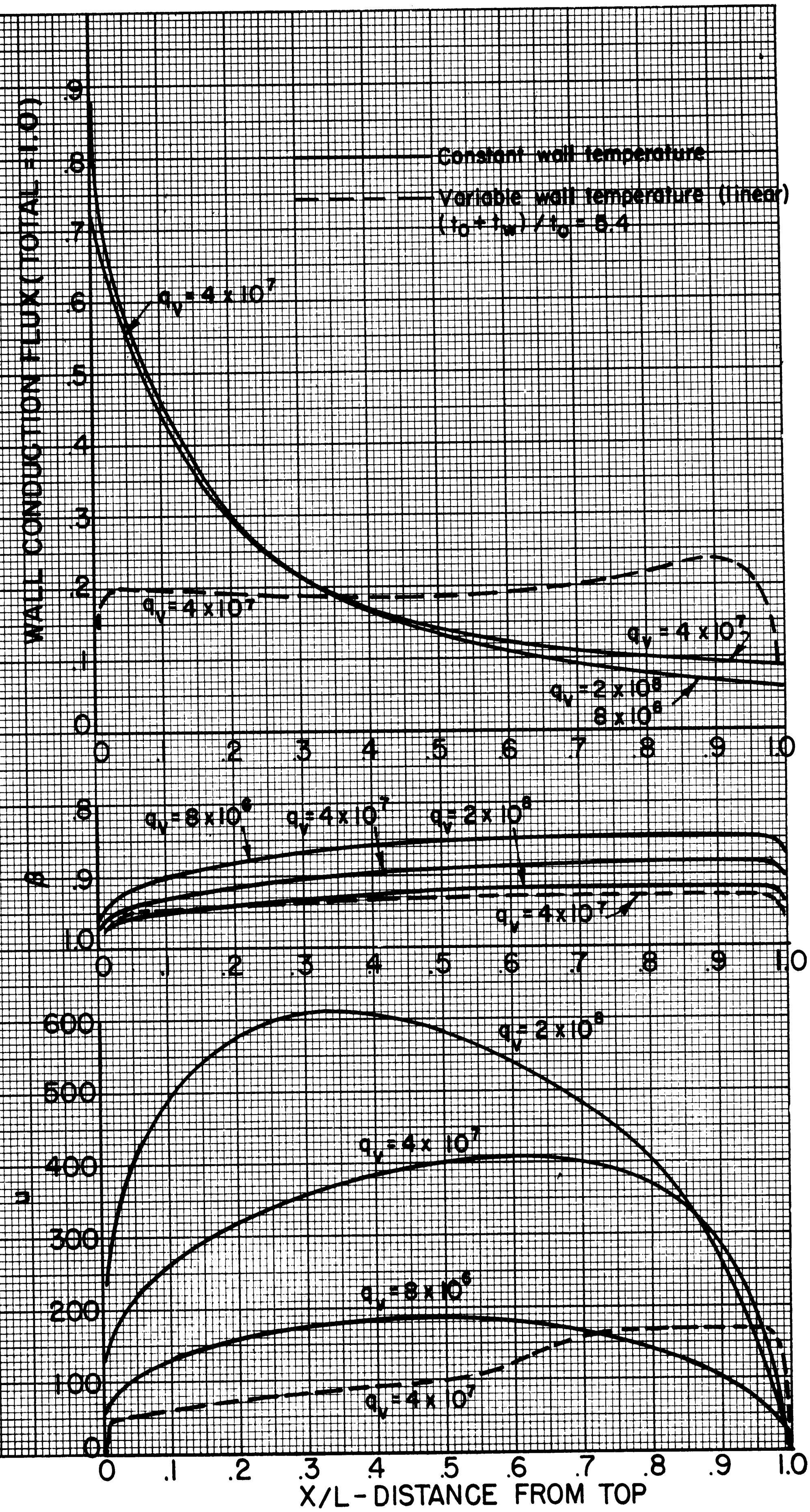

FIGURE 4

NON-DIMENSIONAL WALL HEAT FLUX, BOUNDARY LAYER THICKNESS, δ, AND VELOCITY, u

VS.

NON-DIMENSIONAL AXIAL POSITION

CALCULATED RESULTS

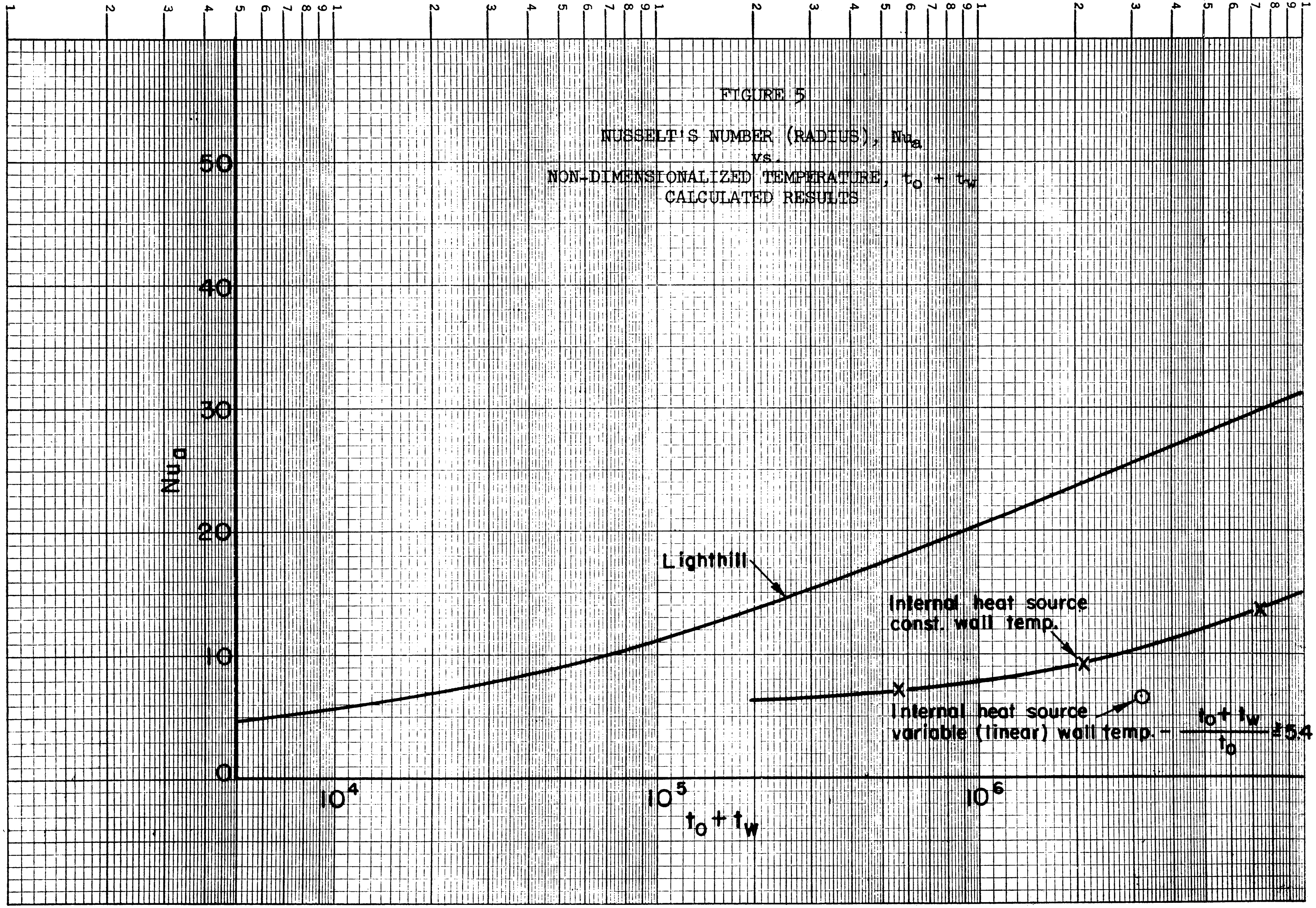

FIGURE 5

NUSSELT'S NUMBER (RADIUS), Nu_a
vs.
NON-DIMENSIONALIZED TEMPERATURE, $t_o + t_w$
CALCULATED RESULTS