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TRANSIENT INTERNAL NATURAL CONVECTION HEATING AND COOLING OF CLOSED, VERTICAL, CYLINDRICAL VESSELS

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August 1960
IP - 452

ACKNOWLEDGEMENTS

The authors wish to acknowledge the assistance of The University of Michigan Research Institute in financing the work reported herein, and of the Industry Program, Engineering College of The University in the production of the report.

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I. INTRODUCTION

The problem of estimating cooling or heating times for closed vessels containing a fluid which is at a temperature different from the external ambient temperature is very common. There are many conventional industrial applications as well as those applying to missiles and rockets (involving boil-off of liquid oxygen, cooling of nose cones, etc.). If the problem is extended to include heat generating or absorbing fluids, various problems involving nuclear reactor systems or chemical processes are also covered.

It is the purpose of this paper to present a method whereby time estimates of sufficient accuracy for most engineering purposes may be made for certain cases. A "universal" heating or cooling curve (Figure 1) for the case of no internal heat generation or absorption and uniform wall temperature is presented in terms of non-dimensionalized parameters. It is possible by similar methods to produce similar curves to cover cases of internal heat sources or sinks and/or non-uniform wall temperature distributions. However, such an extension of the method has not yet been accomplished. For physical cases the restriction of uniform wall temperature is of course particularly restrictive and could be achieved only if the heat transfer on the outside of the vessel and through the wall were negligible compared to the fluid within. Applicable cases would include liquid tanks cooled by forced convection or gas-filled tanks cooled by liquid.

A numerical example, using the curves presented, is included to illustrate the method, and show the order of magnitude of results which are obtained.

II. GENERAL APPROACH TO PROBLEM

The general approach of this paper is to view the transient heating (or cooling) phenomenon as a succession of quasi-steady-state flow regimes. In a cooling case (heat flow outward through walls), steady-state is possible only if an internal heat source is present in the fluid; in a heating case (heat flow inward through walls), an internal heat sink. In the cooling case for example, it is assumed in this analysis that the alternation of the sensible heat of the fluid replaces the internal heat source. In other words, a quasi-steady-state is assumed during which the degree of cooling is not sufficient to alter significantly the flow regime, and the heat which is transferred from the vessel through the walls is made up by a temperature reduction of the fluid, in effect replacing an internal heat source. This assumption certainly seems intuitively reasonable for such cases as the cooling of tanks to equilibrium with the surroundings wherein the rate of temperature decrease is small. Conceivably it could lose its validity in cases involving high-density heat fluxes as might be encountered for example in a nuclear reactor power surge. The assumption is reasonable within the bounds of the analysis since it has already been assumed that the inertial terms in the natural convection flow equation are negligible as compared with viscous shear terms. The steady-state analysis upon which this paper is founded, and which involved the above assumption, has been detailed in Reference 1 by the present authors. The significant features will be briefly reviewed later in the discussion and in the Appendix.

The steady-state solutions available from Reference 1 for natural convection with internal heat generation have been reduced to numerical results for different cases of internal heat source distribution and wall temperature distribution (always assuming axial symmetry). The present analysis has been limited to the cases for which tabulated results were available. However, the curves of Figures 1 and 3, appearing virtually as straight lines on a logarithmic plot, have been extended beyond the bounds of these numerical results. If desirable, it would be possible to extend the analysis to other conditions. For example, as non-uniform wall temperature, which exists in physical cases.

If an internal heat source (or sink) is to be substituted for the alteration of sensible heat of the fluid in cooling or heating, it is necessary to define the proper distribution of the heat source. (The discussion will be based upon the case of cooling. However, the application to heating involves no significant difference and the results are directly applicable).

An obvious boundary condition upon the heat source distribution is that all portions of the vessel approach the external ambient temperature simultaneously. This could be realized, for example, if it were assumed that the heat source strength were everywhere proportional to the local temperature elevation over ambient. However, numerical steady-state solutions for this case are not presently available although it might be possible to obtain them at a later date.

The approach which has been adopted in this paper is to ascertain the significance of the heat source distribution, and then, if this is not too great, use the heat source distributions which are available. If has been found, as explained in the Appendix, that the maximum uncertainty due to the specific assumption of heat source distribution upon cooling time is of the order of 20 percent if the "universal" curve of Figure 1 is used. As a base of engineering calculation, even though the uncertainty is significant, this estimation is still useful. It would be, of course, desirable to make an experimental check of the solution, but so far, this has not been accomplished.

In principle, this type of approach could be extended to cases of heating or cooling transients when an actual internal heat source or sink is present. For example, for the case of cooling in the presence of an actual internal heat source, the internal heat source hypothesized as necessary to attain the required quasi-steady-state conditions would be increased by the amount of the real heat source. The result would of course be longer cooling times and the eventual attainment of a steady-state, not in equilibrium with the surroundings, but at a temperature higher by an amount required by the actual heat source strength. The "universal" curve of Figure 1 thus applies only to the special case of no actual internal heat source or sink and uniform wall temperature. A family of similar curves for which the curve parameters would be the non-dimensionalized heat source or sink strength and wall temperature distribution would be required to deal with cases involving real heat sources or sinks.

The existence of "universal" curves, plots of non-dimensionalized mixed mean temperature differential above ambient versus non-dimensionalized time, with wall temperature distribution, and heat source or sink strength as parameters, seems intuitively obvious from the following consideration. The curve merely presents the elapsed time necessary, under given conditions described above, for the fluid to cool or heat from one mixed mean temperature condition to another. Under the assumption of a series of quasi-steady-state conditions forming the actual transient, it is obvious that the previous history or future behavior can have no influence upon this time interval. Hence, for cooling, if we consider the curve to commence with a vessel of very high mixed mean temperature (higher than that of any condition of interest), during its cooling cycle it must pass through all intervening conditions between the initial condition and ambient. For a particular case, it is possible to enter the curve at whatever point corresponds to the mixed mean temperature of the vessel considered, and leave when that temperature has reached a required value above ambient. The elapsed time between these points is not affected by the previous history of the vessel.

It is of interest to note that infinite time is actually required to acheive equilibrium with the surroundings. In a particular case it is then necessary to specify a required degree of approach to the ambient conditions to achieve a finite and meaningful solution.

III. DETAILED SOLUTION

The details of the steady-state solutions upon which the present work is based were given in Reference 1 and in the several papers upon which it was based (References 2, 3, 4, 5). Briefly, the solutions, obtained numerically with the help of a high-speed digital computer, are based upon the following major assumptions:

1) Conservation of mass, momentum, and energy on an integrated basis for planes normal to the vessel axis and for overall vessel.
2) Axial symmetry
3) Assumed temperature and velocity profiles (Figure 4) which meet known physical boundary condition and their first derivatives at vessel wall, centerline, and inner extent of boundary layer.
4) Boundary layer flow; i.e.: partial derivatives in axial direction are small compared to corresponding derivatives in radial direction.
5) Boundary layer thickness is function of axial position, being zero at top in cooling case; at bottom in heating case.

6) Laminar flow

7) Physical properties of fluid are uniform for entire vessel and are evaluated for a mean condition.

8) Prandtl's Number is of the order of unity or greater. As explained in Reference 1 and 2, this is tantamount to the assumption that inertial terms are small compared to viscous terms.

9) The vessel is fairly large or the heat source is high so that boundary layer type flow applies. The lower limit of this limit corresponds to that $t_m \gtrsim 10^2$ in Figure 1. It has been found that most physical cases of significance fall within this range. Further details on the analysis are given in the Appendix.

The computer program is so arranged that arbitrary axial distribution of heat source and wall temperature can be evaluated, and hence results for various such distributions are available over a very wide range of heat source strengths.

As detailed in Reference 1 considerable experimental data has been obtained to verify the analytical predictions. In general the trends predicted analytically have been verified (including local fluid temperatures, velocities, and wall heat flux distributions) but the observed overall temperature differential required to motivate a given rate of heat transfer has been less than that predicted by a factor of approximately 1.4 over the entire range of non-dimensional heat source strengths investigated (a range of about 10^6). It is felt that this discrepancy is at least partially a result of turbulent mixing which has been observed in most experimental runs.

The overall steady-state heat transfer results can be presented as a plot of non-dimensional temperature differential versus non-dimensional heat source strength with wall temperature distribution and heat source distribution as parameters. Since the results are presented in terms of non-dimensional parameters, they are theoretically applicable to vessels of any length to diameter ratio. Their applicability has been verified over quite a broad range of such ratios. On logarithmic coordinates the resulting curves are virtually straight lines as shown in Figure 5, and hence the relation can be expressed in an empirical form:

$$q_V = m t_{\text{℄}_o}^{n} \qquad (1)$$

where $m \cong 1.24$, and m depends on the heat source and wall temperature distribution.

Application of these steady-state solutions to the transient case involves adding a term, which is proportional to $\partial t/\partial \tau$, to the basic conservation of energy equation for differential discs normal to the centerline. τ is non-dimensional time. The detailed derivations are given in the Appendix. This term covers the contribution to the overall energy balance of heat liberation or absorption due to temperature transients. The general relation would then include a $\frac{\partial t}{\partial \tau}$ term as well as a q_V (actual internal heat source or sink term) term. In those cases for which numerical results are given in this paper, it is assumed that $q_V = 0$. However, no great complication would be involved in handling the more general case.

Assuming that transient cases are a succession of quasi-steady-state conditions, it is shown by Figure 6 (and obvious on physical grounds) that, for given distribution of heat source and wall temperature, there is a unique relation between $t_{\mathbb{C}_o}$ and q_v. Also, for each such condition, i.e.: each q_v or $t_{\mathbb{C}_o}$, the wall heat flux, local temperatures, and velocities are known from the steady-state results and are unique [1]. Consequently, there is also a unique relation between the mixed-mean temperature differential above a given datum (in non-dimensional coordinates) and the wall heat flux and hence rate of change of mixed mean temperature. For the analyses of this paper, uniform wall temperature has been assumed, and this temperature has been used for the datum. Generalization to conditions of varying wall temperature (always assuming axial symmetry) presents no over-riding complication. Evaluation of the mixed-mean temperature differential requires an integration across the radius of the assumed radial temperature profiles and then numerical integration in the axial direction (since no analytical relation is known) using the numerical data from the steady-state solutions. The time rate of change of the mixed-mean non-dimensional temperature can be written in terms of the applicable steady-state overall temperature differential using Equation (1) by substituting $\frac{\partial t_m}{\partial \tau}$ for q_v. Details are given in the Appendix.

The curves of Figures 1 and 3 are based upon the experimentally observed heat transfer rates rather than the analytical predictions, so that the heating or cooling times will represent the "best guess" currently available (i.e.: heat transfer rates for a given overall temperature differential increased by a factor of 1.4 over the theoretical calculation, but internal heat distributions are assumed from the numerical analyses).

The question of the suitable heat source distribution to be used has been discussed in a previous section. Calculations were made for extreme cases for which numerical data was available to determine the significance of this factor. The resulting curves are shown in Figure 2 and explained in the Appendix. It is noted that the curves are essentially parallel on a logarithmic plot. Cases plotted include linear distributions peaking at top or bottom and going to zero at the opposite end (triangular and inverse triangular), a sine distribution, zero at the ends and maximum at the midpoint uniform heat source, and all heat added at one end. On physical grounds, as previously mentioned, the "triangular" distribution seems most reasonable and has been used for the "Universal Curve", Figure 1. Since this is the case, it seems reasonable, again physically, that the correct solution should lie between the "disc" and "inverse triangular" curves. Figure 3 shows the cooling (or heating) curves for these cases (corrected for experimental results as mentioned above). It is noted that again they are all essentially parallel on a logarithmic plot, giving some confidence that the form of the curves is correct. The curve of Figure 1 corresponds to the "triangular" curve of Figure 3.

IV. NUMERICAL EXAMPLE

As an illustrative example, a cylindrical tank, filled with water at a mixed-mean temperature of 15°F , wherein the interior surface of the walls is held at 60°F is selected. It is desired to know how long it will take for the mixed-mean temperature of the vessel to decrease to 70°F. The vessel height is four feet and the diameter one feet. The numerical results are computed in the Appendix from both the "Universal" Curve (Figure 1) and the "disc" curve (Figure 3) which is believed to cover the range of uncertainty. The required time interval from the Universal Curve is 0.318 hours and 0.462 hours. As previously explained the "Universal" Curve represents the "best guess". The derivation from this estimate incurred by using the "disc" curve is about 20 percent.

V. CONCLUSIONS

An approximate method for the estimation of heating or cooling times of cylindrical vessels, filled with fluid, and exposed to uniform wall temperature of a value differing from the mixed-mean fluid temperature is presented. It is noted that this method can be generalized to include cases where there is internal heat generation or absorption or where non-uniform (but axially-symmetrical) wall temperature distributions exist.

VI. NOMENCLATURE

T	Temperature.
U	Vertical Velocity (parallel to x-axis).
R	Radius vector.
ℓ	Height of the cylinder .
a	Radius of the cylinder.
u	Non-dimensional velocity.
t	Non-dimensional temperature differential from the fluid to wall; in the case of constant wall temperature. $t_{\mathbb{C}}$ is the non-dimensional temperature differential from centerline to wall. Both t and $t_{\mathbb{C}}$ are functions of x; in addtion, t is also a function of r.
$x, r,$	Non-dimensional space coordinates in axial and radial directions.
Q_V	Volumetric heat generation rate.
q_V	Non-dimensional volumetric heat generation rate.
Pr	Prandtl's Number.
θ	Time.
τ	Non-dimensional time.
c_V	Specific heat at constant volume.
ν	Kinematic viscosity.
ρ	Density.
K	Thermal Conductivity.
κ	Thermal Diffusivity.
α	Volumetric thermal expansion coefficient.
m, n, C	Constants defined in the text.
γ, β, δ	Functions defined in the text.

VII. APPENDIX

Brief Review of Steady State Solution (1)

Consider a closed circular cylinder filled with fluid of Prandtl's Number of unity or greater. Within the fluid is a heat source, not necessarily homogeneous, the cylinderical wall is kept at constant temperature which is lower than the fluid temperature so that steady-state heat transfer can be established.

The heat balance equation over a circular thin disc can be written, in integral form, as:

$$\rho c_V \left[\frac{\partial}{\partial X}\int_0^a 2\pi\ UTRdR\right]\Delta X = K\left(\frac{\partial T}{\partial R}\right)_{R=a} 2\pi a\Delta X - \pi a^2\ \Delta X Q_V \qquad (2)$$

In Equation 2, the fluid properties are assumed constant throughout. The following dimensionless variables are substituted:

$$x = \frac{X}{\ell}$$

$$r = \frac{R}{a}$$

$$t = (T_{wall} - T)\ \frac{\alpha g a^4}{\nu\kappa\ell} = \frac{\alpha g a^4(\Delta T)}{\nu\kappa\ell}$$

$$u = \frac{a^2}{\kappa\ell U}$$

$$q_V = \frac{a^2}{\rho c_V}\ \frac{\alpha g a^4}{\kappa^2\nu\ell}\ Q_V$$

One then obtains the heat balance equation in dimensionless form:

$$\frac{\partial}{\partial x}\int_0^1 utrdr = \left(\frac{\partial t}{\partial r}\right)_{r=1} + \frac{q_V}{2} \qquad (3)$$

In a similar fashion, the continuity and momentum equations are obtained [1,2].

$$\int_0^1 urdr = 0 \qquad (4)$$

$$\int_0^1 rtdr + 1/2\,(t)_{r=0} + \left(\frac{\partial u}{\partial r}\right)_{r=1} = 0 \qquad (5)$$

Equation (5) has already been simplified in that the terms with the coefficient $(Pr)^{-1}$ are neglected. Order of magnitude calculations have indicated that the analysis applies only to fluids with Prandtl's Number of the order of unity or greater. [1]

An approximate solution of Equations (3), (4), and (5) is obtained by postulating temperature and velocity profiles. These are so chosen that the physical boundary conditions, and their first derivatives, are satisfied at the wall, centerline, and interface between boundary layer and core. The assumed profiles [2] for the boundary layer regime (applying to relatively large vessels or high heat source) are:

$$u = \begin{cases} -\gamma & 0 < \gamma < \beta \\ -\gamma\left[1-\left(\frac{r-\beta}{r-\beta}\right)^2 \left\{1 + \delta(r-1)\right\}\right] & \beta < \gamma < 1 \end{cases} \qquad (6)$$

$$t = \begin{cases} t(x) & 0 < \gamma < \beta \\ t(x)\left[1 - \left(\frac{r-\beta}{r-\beta}\right)^2\right] & \beta < \gamma < 1 \end{cases} \qquad (7)$$

The parameters γ, β, δ, are functions of x only. In particular, β is of special interest. Physically, it is one minus the non-dimensional "boundary layer thickness".

Equations (3), (4) and (5) coupled with Equations (6) and (7) have been programmed for a digital computer to determine the numerical values of β and other quantities of physical interest, such as wall conduction, center line temperature, etc. (1),(3),(5). A thorough study of the numerical solution reveals that the relation between heat generation and overall temperature differential can be expressed approximately as:

$$q_V = m t_{\mathfrak{C}}^{n} \tag{8}$$

Where m and n are two constants depending upon the heat source distribution. In most cases, n is about 0.24 which is approximately in agreement with natural convection on a vertical flat plate. In the latter case, the exponent is 0.25 (2).

Based on these results, transcient natural convection heat transfer has been studied.

The Equations for Non-Steady State Natural Convection:

Using the same geometry and nomenclature as in the previous section, one can consider the heat balance relation for non-steady state natural convection. For simplicity of discussion, assume there is no heat generation inside the fluid, and the vessel dimensions are such that boundary-layer type flow exists. This case applies to vessels of reasonable size and significant temperature differentials. The counterpart of Equation (1) is now of the form

$$-\frac{\partial}{\partial X}\left[\int_0^a 2\pi c_V \rho U T R dR\right]\Delta X - 2\pi a K\left(-\frac{\partial T}{\partial R}\right)_{R=a}\Delta X = \frac{\partial}{\partial \theta}\left[\int_0^a 2\pi \rho c_V T R dR\right]\Delta X \tag{9}$$

To non-dimensionalize Equation (9), one makes the same change of variables as previously. Thus

$$- \frac{\partial}{\ell \partial x} [\int_0^1 2\pi a^2 r dr c_v (T_{wall} - \frac{\nu\kappa\ell}{\alpha g a^4} t) \frac{\kappa\ell}{a^2} u\rho] \ell\Delta z$$

$$- K \cdot 2\pi a \ell\Delta x [- \frac{\partial}{a\partial r} (T_{wall} - \frac{\nu\kappa\ell}{\alpha g a^4} t)]_{r=1} = \frac{\partial}{\partial\theta}$$

$$[\int_0^1 2\pi a \; r dr \rho c_v (T_{wall} - \frac{\nu\kappa\ell}{\alpha g a^4}) \quad \ell\Delta X$$

Rearranging and simplifying:

$$\frac{\partial}{\partial x} \int_0^1 rtudr - (\frac{\partial t}{\partial r})_{r\;1} \quad - \frac{\partial}{\partial\theta} \int_0^1 \frac{a^2}{\kappa} rtdr \qquad (10)$$

To make this dimensionless, one must substitute for θ as shown below:

$$\theta = \frac{a^2}{\kappa} \tau \qquad (11)$$

and the resulting equation is

$$\frac{\partial}{\partial x} \int_0^1 rtudr = (\frac{\partial t}{\partial r})_{r=1} - \frac{\partial}{\partial\tau} \int_0^1 rtdr \qquad (12)$$

τ is then the non-dimensional time, which is defined in Equation (11).

Approximate Equations for Non-Steady State Case:

It is assumed that the rate of cooling is small as compared with velocities within the fluid so that the flow regime changes only slowly and can be represented by a series of steady states as given in the previous section. Then the following arguments can be applied.

Comparing Equation (2) with Equation (12), it is clear that the place of heat generation term in Equation (2) has been taken by the time rate of change of the enthalpy of the fluid in Equation (12). The physical interpretation has been given in the text. Thus, one may equate these two terms, since for each small step change of temperature, they represent the same physical quantity; i.e., for a disc of differential thickness in the axial direction

$$1/2\ q_v = -\frac{\partial}{\partial\tau}\int_0^1 rt\,dr \qquad (13)$$

Integrating Equation (13) with respect to x one obtains the total enthalpy change of the fluid as a function of time. Thus

$$\int_0^1 1/2\ q_v dx = -\frac{\partial}{\partial\tau}\left[\int_0^1 dx \int_0^1 rt\,dr\right] \qquad (14)$$

The left side of Equation (14), in the case of a homogeneous heat source, equals 1/2 q_v. Call the quantity in the bracket 1/2 t_m. Equation (14) can then be written as

$$q_v = -\frac{\partial t_m}{\partial\tau} \qquad (15)$$

Recall

$$q_v = m t_{\mathfrak{C}}^{\,n} \qquad (8)$$

so

$$m t_{\mathfrak{C}}^{\,n} = -\frac{\partial t_m}{\partial\tau} \qquad (16)$$

Since t_m is a function of time only, the partial differential can be replaced by an ordinary differential, and Equation (16) below can be solved if the relation between $t_{\mathfrak{C}}$ t_m can be found:

$$m t_{\mathfrak{C}}^{\,n} = \frac{dt_m}{d\tau} \qquad (17)$$

<u>Mixed Mean Temperature and Universal Cooling (or Heating) Curve</u>:

The temperature-position relation was previously mentioned. It is:

$$t = \begin{cases} t_{\mathfrak{C}} & 0 < r < \beta \\ t_{\mathfrak{C}} \left[1 - \left(\frac{r-\beta}{r-\beta}\right)^2\right] & \beta < r < 1 \end{cases} \qquad (7)$$

Therefore, for a given $t_{\mathfrak{C}}$, the temperature profile is according to Equation (7). Consequently, there is a unique mixed mean temperature of the fluid corresponding to the given condition. This temperature is obtained by integrating over the entire cylinder, i.e.,

$$t_{mean} = \frac{\int_0^1 dx \int_0^1 2\pi r t dr}{\int_0^1 dx \int_0^1 2\pi r dr} = 2 \int_0^1 dx \int_0^1 r t dr \qquad (18)$$

This shows that t_m in Equation (15) actually represents the mean temperature of the fluid. It is evaluated by substituting Equation (7) into Equation (18) to give

$$t_m = 1/6 \int \; t_{\mathfrak{C}} \, (3+2\beta+\beta^2) dx \qquad (19)$$

This is the relation required to solve Equation (17).

In integrating Equation (19), one needs the functional relation between $t_{℄}$ and β in terms of x. This is obtained by the known numerical results of the steady state solution. Since in this solution, the value of β is calculated as a function of position for each value of $t_{℄}$. To obtain t_m, one merely applies the coventional graphical integration technique to Equation (19).

Figure 2 is the result of this graphical integration. Five curves are given. The line labelled "uniform" is obtained based on the assumption that the heat source (in the steady-state calculations) is distributed uniformly within the fluid body. This is a hypothetical case since in the actual cooling (or heating) process, there is no reason to expect the rate of change of the enthalpy of the fluid to remain uniform throughout the vessel. The line labelled "Disk" represents one extreme case. This curve is calculated on the assumption that the total heat source (in the steady-state case) is concentrated into a thin circular disk at the bottom for the entire period of the heating (or cooling) process. The thickness of the disk was taken as 1 percent of the vessel height for convenience in the machine computation. This represents the case of a differentially thin heat source at one end and is physically the same case as a vessel open at that end to an infinite reservior [1]. Another extreme case is obtained by assuming an inverse triangular distribution with zero heat source at the bottom as labelled. It has been found that the ratio of t_m between these two extreme cases is 3.0.

The actual distribution of rate of change of fluid enthalpy is believed to be between these and in fact close to the "triangular" case as explained in the text. Moreover, it is found from Figure 2 that the relation between $t_{℄}$ and t_m, in general, can be expressed as

$$t_{℄} = Ct_m \tag{20}$$

(45° slope on logarithmic plot) Then, one can easily integrate Equation (17). This again, gives a family of curves which contains the postulated rate of enthalpy change distribution as parameter. The integration contant is zero as explained in the text. Figure 3 is the graphical representation of the solution of Equation (17). As a summary, the following table lists the constants involved in the equations obtained.

TABLE I

Heat Distribution	m*	n	C	Cooling Curve equation
Uniform	.921	1.24	2.5	$\tau = 1.44\ t_m^{-0.24}$
Disk	1.96	1.24	1	$\tau = 2.13\ t_m^{-0.24}$
Sine	.983	1.24	1.02	$\tau = 1.73\ t^{-0.24}$
Triangular	1.565	1.24	1.33	$\tau = 1.87\ t_m^{-0.24}$
Inverse Triangular	.623	1.24	3	$\tau = 1.72\ t_m^{-0.24}$

* Computed from experimental results rather than machine calculations.

General Limitations of Solution and a Numerical Example.

In the above sections, a detailed method for obtaining the cooling (or heating) time for a closed vessel with constant wall temperature is given. As stated in the text, the case of variable wall temperature should not introduce prohibitive complications, since the basic idea is the same. Aside from this, the present solution does not apply to the case where t_m is less than about 10^2 since the postulated boundary layer type solution loses its validity. Also, relatively long, thin vessels are required unless end sections are insulated, since no heat flow through these has been considered. As previously stated low Prandtl's Number fluids, as liquid metals, are excluded.

It is of interest and notice that the general form of the cooling curve is

$$\tau = \frac{\text{const.}}{t_m^{-0.24}} \tag{21}$$

Thus, if the non-dimensional temperature differential approaches zero, τ approaches infinity. Physically, zero non-dimensional temperature differential implies fluid in equilibrium with the surroundings. To accomplish this certainly requires infinite time.

Following is an numerical example to illustrate the method. Consider a circular cylinder 1 foot in diameter and 4 feet in height filled with water. At the beginning of the process the mixed mean temperature is 150°F. It is allowed to cool to 70°F. while the wall temperature is maintained at 60°F. It is desired to know how long this will take.

Calculation;

$T_{wall} = 60°F$

$(T_{mean})_{initial} = 150°F$ $\qquad (\Delta T)$ initial $= 150-60=90$

$(T_{mean})_{final} = 70°F$ $\qquad (\Delta T)$ final $= 70-60=10$

$$(t_m)_{initial} = \frac{\alpha g a^4 (\Delta T)_{initial}}{\nu \kappa \ell} = 7.5 \times 10^8$$

$$(t_m)_{final} = 8.33 \times 10^7$$

From Figure 1

$$\Delta\tau = 0.0228 - 0.0141 = 0.0087$$

Therefore, cooling time required $= \Delta\tau \frac{a^2}{\kappa} = 0.0087 \times \frac{1}{4} \times \frac{10^6}{1.640} \times \frac{1}{3600}$

$$= 0.369 \text{ hr.}$$

If uniform heat distribution curve (Figure 3) is used, the answer would be 0.318 hr. The answer from the "disk distribution" curve is 0.462 hr. The uncertainty is therefore ± 12 percent. It should be mentioned that above results do not apply to the case where a water tank is cooled by ambient air, since, in such a case, the controlling effect would be the heat transfer from vessel to air.

It will be noted from the previous equations that

$$\tau_1 = \frac{C_1}{K_{p_1}\left(\frac{a^4}{\ell}\,\Delta T_1\right)^{1/4}} \qquad \text{and similarly for } \tau_2 \qquad (22)$$

where C_1 is the constant defining the curve to be used on Figure 3 and K_{p_1} is a constant involving only the physical properties.

Then

$$\tau_2 - \tau_1 = \frac{C_1}{K_{p_1}}\left(\frac{\ell^{1/4}}{a}\right)\left(\frac{1}{\Delta T_2^{1/4}} - \frac{1}{\Delta T_1^{1/4}}\right) \qquad (23)$$

so

$$\theta_{1,2} = (\tau_2 - \tau)\,\frac{a^2}{K} = \frac{C_1}{K_{p_2}}\,(\ell^{1/4}a)\left(\frac{1}{\Delta T_2^{1/4}} - \frac{1}{\Delta T_1^{1/4}}\right) \qquad (24)$$

where K_{p2} is a second constant involving only physical properties.

Hence it is seen that, for a case involving a given fluid and given temperatures, the cooling or heating time is proportional to length 1/4 and radius to the first power. If steady-state conduction alone as-sumed under the same conditions, it is shown below that the time is pro-portional to a.

$$Q = [A\ K\ \text{grad}\ t]\ \theta$$

where Q is total enthalpy of fluid above a given datum

so

$$\theta = \frac{Q}{A\ K\ \text{grad}\ t}$$

For a long, thin cylinder (so axial conduction is negligible)

$$Q \propto a^2, \quad A \propto a \qquad \text{grad } t \propto 1/a$$

so

$$\theta \propto \frac{a^2}{a \cdot 1/a} \propto a^2$$

A numerical check of the time required to cool a vessel of the dimensions described in the numerical example by static conduction gives a time greater by several orders of magnitude.

BIBLIOGRAPHY

1. Hammitt, F. G., Brower, E. M. and Chu, P. T., "Free Convection Heat Transfer and Fluid Flow in Closed Vessels with Internal Heat Source", NP-9780, Office of Technical Services, Department of Commerce, Washington, D. C.

2. Lighthill, H. J., "Theoretical Consideration on Natural Convection in a Closed Circular Tube", J. Applied Mechanics and Math. March 1953.

3. Hammitt, F. G., Ph. D. Thesis. Nuclear Engr. Dept. University of Michigan, 1957. Industrial Program Report IP-259.

4. Hammett, F. G., ASME Paper No. 58-SA-30.

5. Hammett, F. G., ASME Paper No. 58-A-212.

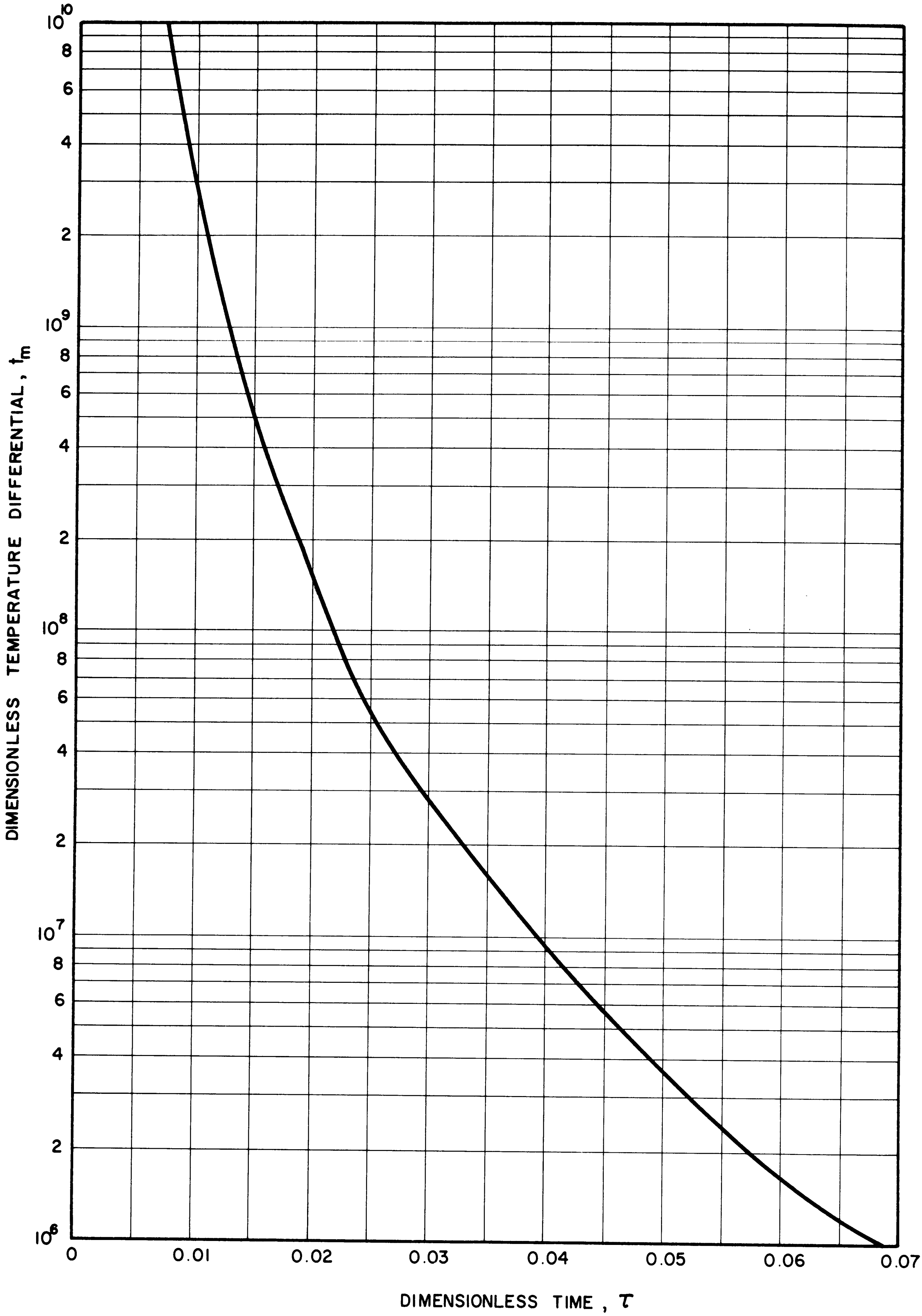

Figure 1. "Universal" Cooling Curve.

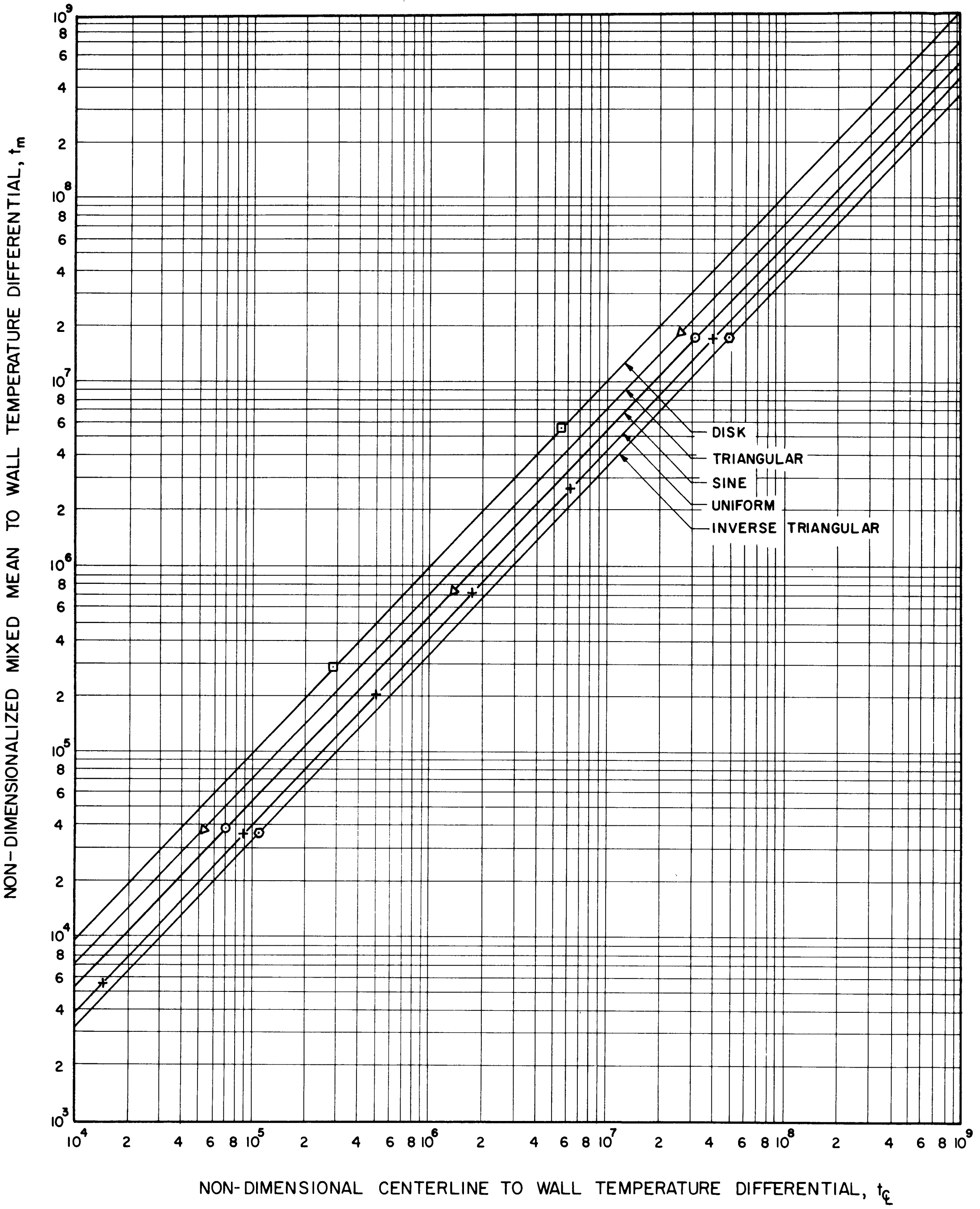

Figure 2. Non-Dimensional Mixed-Mean to Wall vs. Centerline to Wall Temperature Differential

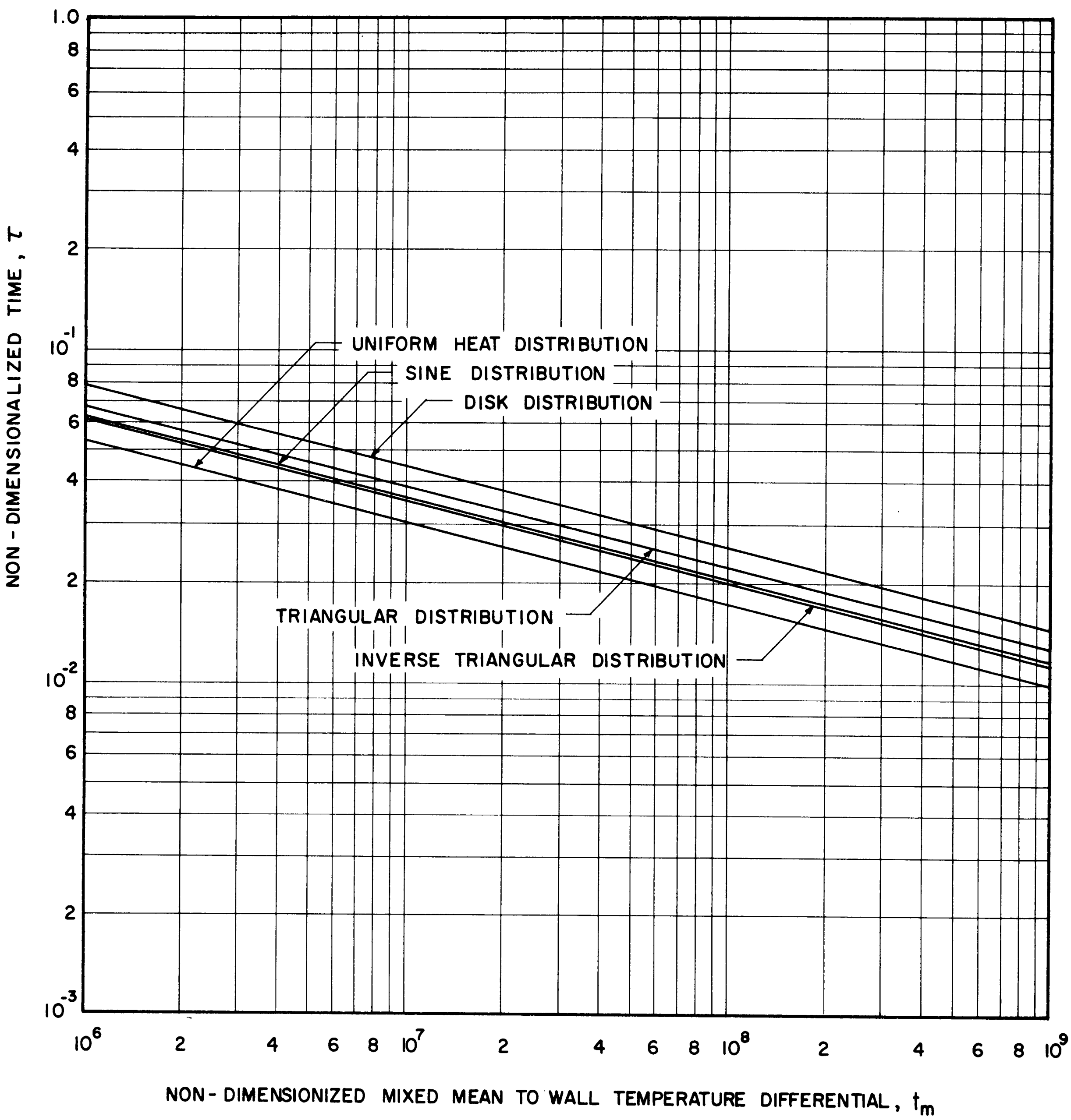

Figure 3. Non-Dimensional Time vs. Non-Dimensional Temperature Differential

TEMPERATURE AND VELOCITIES PROFILES

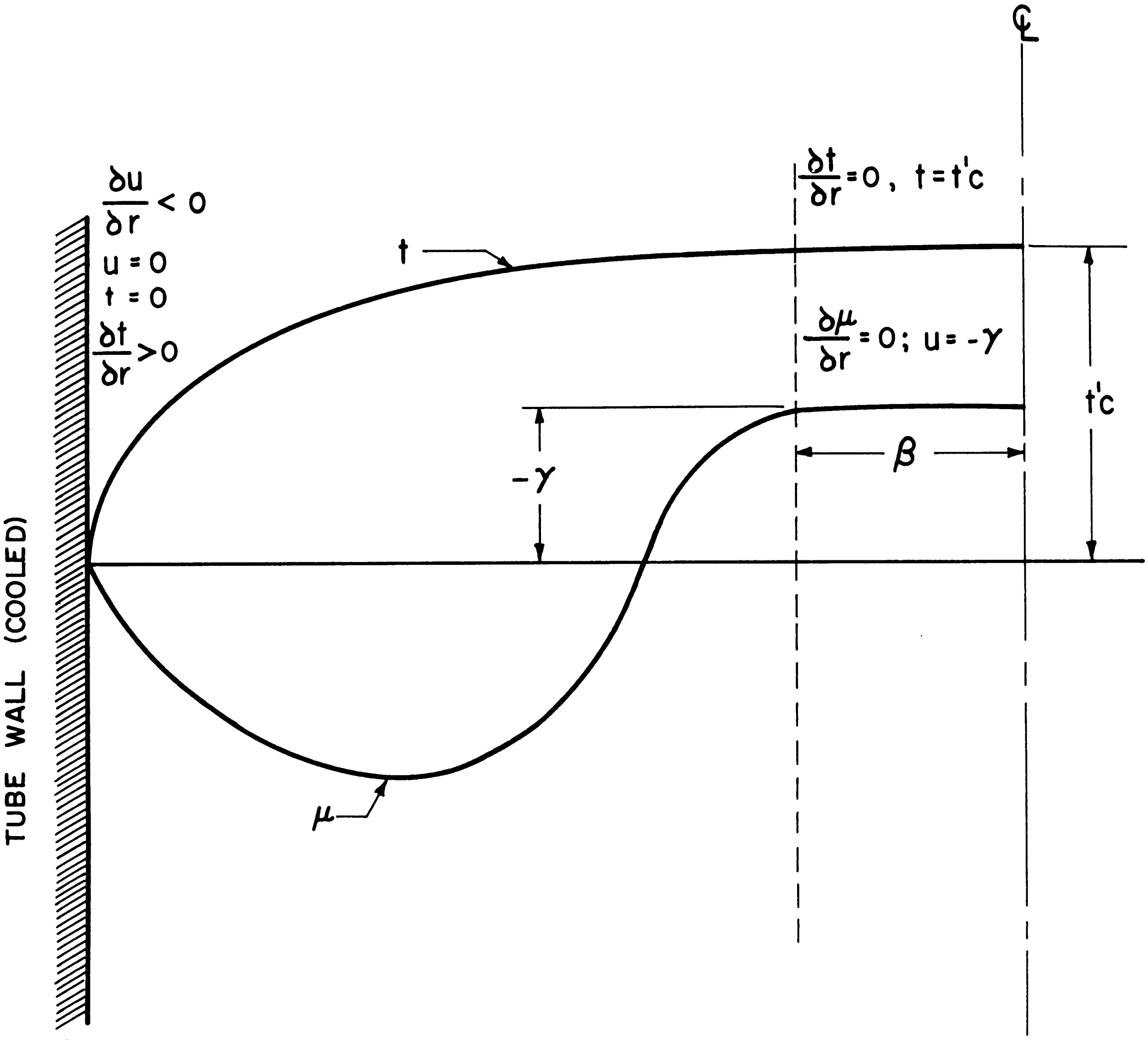

Figure 4. Assumed Temperature and Velocity Profiles for Theoretical Analysis.

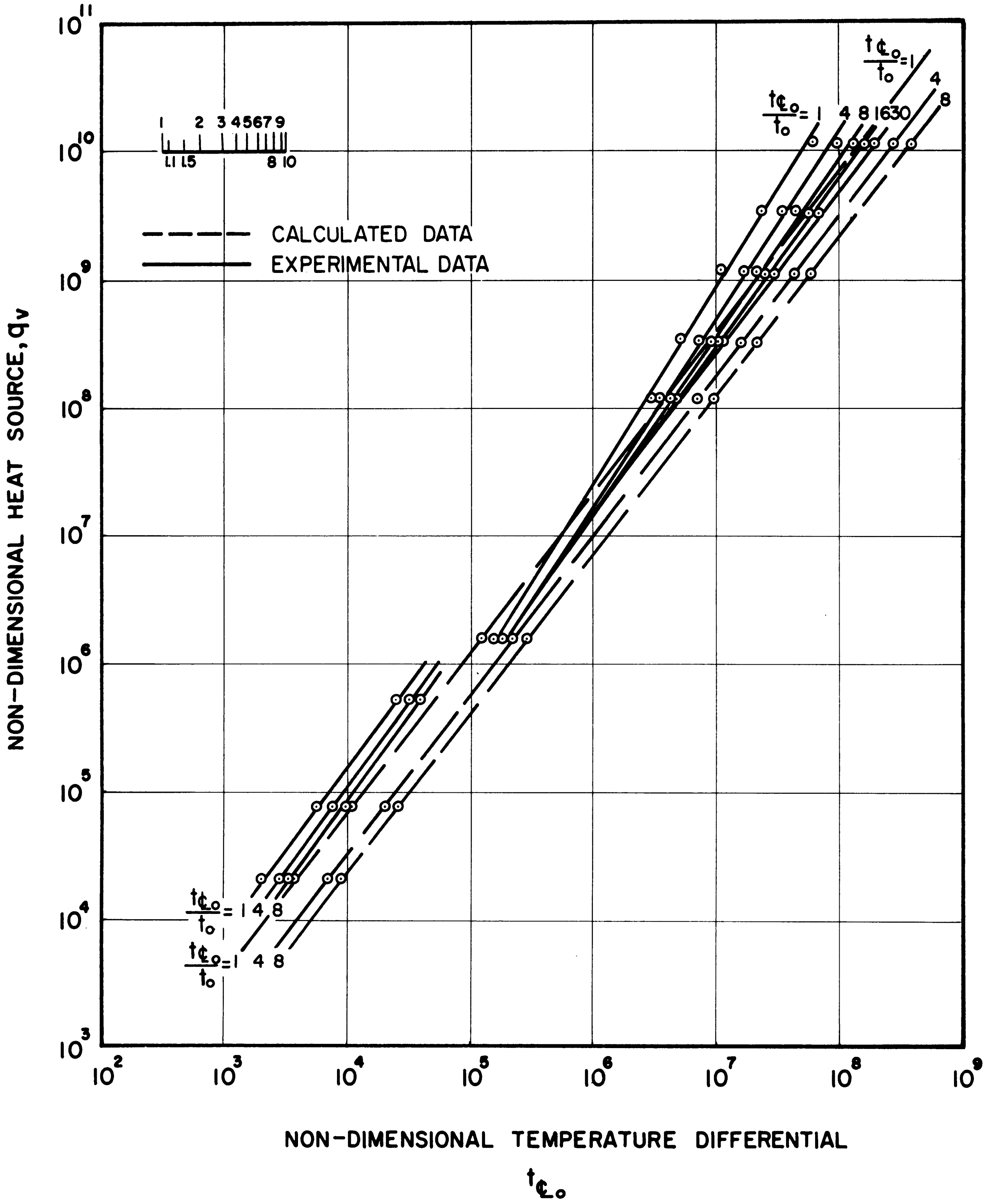

Figure 5. Non-Dimensional Heat Source, q_V, vs. Non-Dimensional Overall Temperature Differential, Experimental Data.

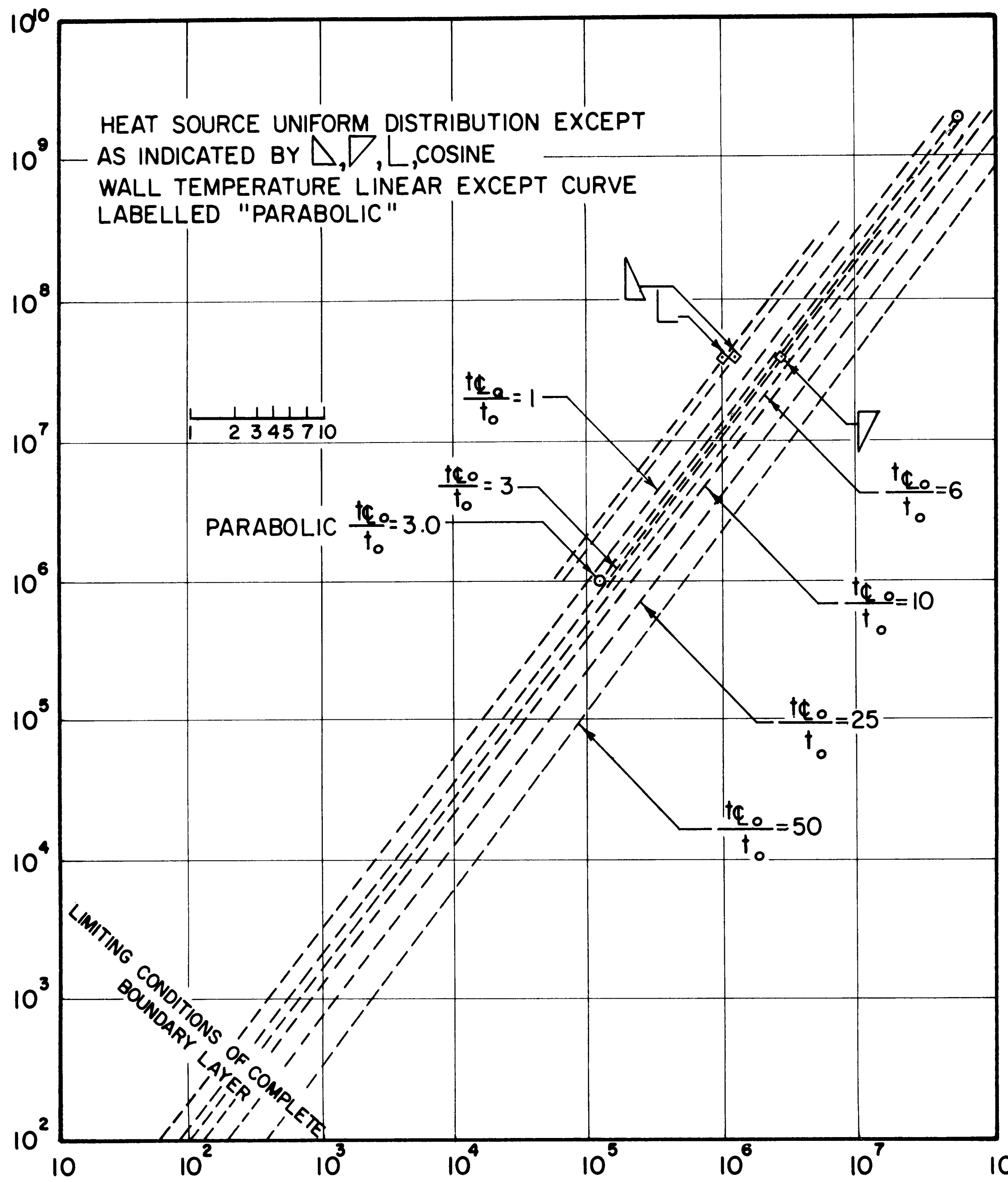

Figure 6. Non-Dimensional Heat Source vs. Overall Temperature Differential, Calculated Data.