MEASUREMENT OF FAST NEUTRON SPECTRA IN WATER AND GRAPHITE

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ABSTRACT

Detailed knowledge of the energy and spatial distributions of fast neutrons in bulk media is required in radiation shielding, radiation effects, and fast reactor physics. Although computer codes are available to calculate spatially dependent spectra with accuracies presumably limited only by uncertainties in cross sections and source spectra, few spectrum measurements which provide a good test for these calculations have been reported.

Here a relatively simple, moderate resolution (12%), differential proton-recoil, fast neutron spectrometer has been developed to measure directed neutron fluxes in the presence of high intensity fission gamma fields of \(10^7\) to \(10^8\) roentgens/hour. Good signal-to-background ratios are obtained by (1) careful shielding, (2) using silicon surface barrier detectors, (3) positioning the detectors out of the collimated beam of mixed radiation which traverses the thin polyethylene \((\text{CH}_2)\) radiator, and (4) using tandem detectors in anticoincidence. The efficiency is about \(10^{-6}\) for neutrons incident on the radiator.

Measurements of the energy spectrum from 2 to 12 Mev of normally directed neutrons which leaked from the Ford Nuclear Reactor core and penetrated 0, 20.0, 40.0, and 60.0 cm of \(\text{H}_2\text{O}\) and 0, 20.2, 40.4, and 60.6 cm of graphite are reported. The relatively high accuracy of these deep penetration measurements (±10% relative intensity and ±20% absolute intensity, ±2% energy) makes them useful for testing calculations. Comparison with detailed transport calculations shows good agreement.
CHAPTER I

INTRODUCTION

A. SIGNIFICANCE OF FAST NEUTRON SPECTRUM MEASUREMENTS IN BULK MEDIA

Knowledge of the energy and spatial distributions of fast neutrons (1 - 15 Mev) in bulk media is required in the design of radiation shielding, the study of radiation effects, and the design of fast reactors. The parallel development of high-speed digital computers with large memory capacity and multigroup computer codes based on transport and Monte-Carlo methods has made detailed calculations of fast neutron spectra feasible. With the more rigorous codes such as NIOBE, DTF-IV, and OSIR, the accuracy of the calculations is essentially limited only by uncertainties in the cross sections and source representation. Unfortunately, due to the difficulty of measuring space and energy dependent fast neutron fluxes, these calculations are usually tested only against integral flux measurements or against other calculations. Clearly the best test of a code, and the representation of the cross sections and the source is a comparison with spectrum measurements. Only by such comparisons can the accuracy of the calculations be established. This work reports a method for accurately measuring spatially dependent fast neutron spectra in bulk media. Measurements in two common reactor materials are presented and compared with detailed calculations.
B. METHODS FOR MEASURING FAST NEUTRON SPECTRA IN BULK MEDIA

Numerous reviews \(^{(1,19-25)}\) of the methods used to measure fast neutron spectra have been published. The current methods used to measure spectra \((1-15 \text{ MeV})\) in bulk media are listed in Table 1. Table 1 also gives typical values for the efficiency and resolution reported for each method and indicates which methods require beam extraction. When a beam is extracted, the directional or angular flux is measured; otherwise, the scalar flux is measured. In this section, each method is discussed briefly with emphasis on its limitations.

1. Threshold Activation Method

The threshold activation method \(^{(26-29)}\) is based on the activation of foils with activation thresholds in the energy region of interest. Its principal limitations are low efficiency and low resolution. The low efficiency results from the small size of the foils used and the low cross sections in the Mev range. The low resolution is due first to the limited number of materials with suitable activation thresholds and well-known cross sections, and secondly to the uncertainties associated with the unfolding of spectra from the activation data.

2. Time-of-Flight Method

The time-of-flight (TOF) method consists of measuring neutron flight times over a known flight path. The origin of the flight time is usually defined by using a pulsed neutron source. The application of this technique to spectrum measurements in bulk media has been made possible by
<table>
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<td>Foils</td>
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<td>Required</td>
<td>Liquid Organic Scintillator(33)</td>
<td>$10^{-8}$ - $10^{-6}$ (includes collimation)</td>
<td>10%</td>
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<td>3. Nuclear Reaction A. Li$^6$(n,α)H$^3$</td>
<td>Not Required (but used in this example)</td>
<td>Silicon Surface Barrier Detectors in Sandwich Configuration(36)</td>
<td>$10^{-7}$ - $10^{-6}$ or $10^{-9}$ - $10^{-8}$ (includes collimation)</td>
<td>21% at 3 Mev, 2% at 14 Mev</td>
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<td>B. He$^3$(n,p)H$^3$</td>
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<td>3% - 5%</td>
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<td>Not Required</td>
<td>Liquid Organic Scintillator(49)</td>
<td>$0.15$ - $0.40$</td>
<td>32% at 1 Mev, 9% at 14 Mev</td>
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<tr>
<td>B. Differential</td>
<td>Required</td>
<td>Silicon Surface Barrier Detectors in Tandem Configuration(57)</td>
<td>$3-6 \times 10^{-6}$ or $3-6 \times 10^{-9}$ (includes collimation)</td>
<td>24% at 2 Mev, 15% at 3 Mev, 12% at 12 Mev</td>
</tr>
</tbody>
</table>
the development of high intensity, nanosecond scale, pulsed neutron sources. The high intensity is required because of the long flight paths necessary for good resolution and because of the limitation imposed on the repetition rate by the increase of the pulse width along the flight path. For measurements in nonmultiplying media, (9,30,31) an electron linear accelerator (LINAC) is generally used to bombard lead or depleted uranium targets with \(\sim 30\) Mev electrons to generate bremsstrahlung. This in turn generates up to \(10^{13}\) neutrons per second with a near fission spectrum by \((\gamma,n)\) and \((\gamma,f)\) reactions. Flight paths are usually 50 to 60 meters long. For measurements in multiplying media, (9,10,32) less intense sources are required. The neutron yield obtained from a positive ion accelerator through the \(H^2(d,n)He^3\) and \(H^3(d,n)He^4\) reactions is adequate for some measurements.

With current TOF apparatus used with LINAC sources, the principal limitations on resolution are due to (1) counting statistics which require channel grouping to make statistical uncertainties less than some specified value (33) and/or (2) uncertainty in the mean time and the variance in time for neutrons to slow down and migrate from the source to the entrance of the flight path (34). According to Russell, et al., (30) this latter effect is small but nonnegligible (\(\sim 10\) ns above 0.5 Mev) in hydrogenous media, but it may lead to \(\sim 40\%\) energy resolution for measurements in a large shield of high A elements such as lead. If this is true, a 50-meter flight path is too short for these measurements.

Profio (33) points out that the main shortcomings of his TOF technique
are the uncertainties in the detector efficiency and the transient gamma background. In order to suppress the latter, a 3.14-cm thick depleted uranium plate was interposed in the flight path during measurements. The effect of the depleted uranium is indicated by spectrum measurements made with and without the filter. Addition of the filter reduced the measured 1 Mev flux, presumably after correction for attenuation by the filter, by a factor of three. The measured 10 Mev flux was reduced by 20%. These results indicate the gamma background may be causing substantial errors.

3. Nuclear Reaction Method

The nuclear reaction method uses either the $^3\text{He}^3(n,p)^2\text{H}$ reaction ($Q = +0.76$ Mev) or the $^6\text{Li}^6(n,\alpha)^3\text{He}$ reaction ($Q = +4.78$ Mev) to produce charged particles whose total energy is equal to the energy of the neutron plus the $Q$ of the reaction. This energy is measured using semiconductor detectors,$^{(35-39)}$ gas proportional counters,$^{(40,41)}$ or scintillation crystals.$^{(42,43)}$

The semiconductor detectors are arranged in a sandwich configuration with either a thin $^6\text{LiF}$ film$^{(35-37)}$ or $^3\text{He}$ gas at up to 995 psi pressure$^{(38,39)}$ between them. A sum-coincidence circuit is used to obtain signals proportional to the full energy of both reaction products and to remove noncoincident background. The positive $Q$ of both reactions also assists in gamma discrimination.

Use of the $^6\text{LiF}$ film is complicated by competing reactions and by $^6\text{Li}(n,\alpha)^3\text{He}$ reactions induced by thermal neutrons for which the reaction
cross section is very high. Similarly, the use of $^3\text{He}$ gas is complicated by competing reactions (especially elastic scattering which produces $^3\text{He}$ recoils).

4. Proton-Recoil Method

The proton-recoil method consists of measuring the energy of elastically scattered protons. The proton energy, $E_p$, is related to the neutron energy, $E$, by the relation, $E_p = E \cos^2 \psi$, where $\psi$ is the n-p scattering angle in the laboratory system. If all of the scattered protons are counted, an integral proton spectrum is obtained. This must be differentiated or unfolded to obtain the neutron spectrum.

In early applications of the integral proton-recoil method, integral proton spectra obtained with stilbene scintillation crystals were differentiated. More recently, sophisticated unfolding techniques have been developed to convert integral proton spectra obtained in stilbene scintillator crystals and liquid organic scintillators and in hydrogen filled and methane filled proportional counters to neutron spectra. Pulse shape gamma discrimination is usually used in conjunction with these detectors.

The chief limitations of the integral proton-recoil method are that the unfolding techniques require precise detector response functions in order to be accurate, and the background due to gamma interactions and neutron interactions other than single n-p scattering is difficult to determine precisely.
In contrast to the integral proton-recoil method, if only those protons scattered through $d\Psi_0$ about $\Psi_0$ are counted, a differential proton spectrum is obtained. Since this is related to the neutron spectrum through $E_p = E \cos^2 \Psi_0$, the conversion to the neutron spectrum is straightforward.

With this method, a thin hydrocarbon material, such as polyethylene ($\text{CH}_2$), is located in a neutron beam and the protons scattered into some solid angle are intercepted by a detector for energy analysis. The neutron beam is extracted from the source with a collimator system. The hydrocarbon film or proton radiator is made thin to reduce the proton energy losses in it and thus permit good energy resolution. The detectors are usually scintillator crystals\(^{(53,54)}\) or semiconductor detectors\(^{(55-57)}\) although Ciallella\(^{(58,59)}\) has reported using magnetic energy analysis. Radiators may be used in series to increase efficiency\(^{(56)}\) and detectors may be used in series to reduce background with coincidence\(^{(53)}\) or anticoincidence\(^{(57)}\) techniques. Tandem detector devices are usually called telescopes.

The spectrometer developed as part of this work is a differential proton-recoil device\(^{(57)}\). The detectors are silicon surface barrier diodes as used by Furr and Runyon\(^{(55)}\) and like Herdade, et al.,\(^{(54)}\) they are located off the collimator axis. The collimator efficiency is $10^{-3}$ and the detector efficiency varies from 3 to $6 \times 10^{-6}$ in the energy range 2 to 12 Mev. The resolution is 24% at 2 Mev, but improves to 15% at 3 Mev and to 12% at 12 Mev. For measurements from 2 to 6 Mev, a 1.25 mg/cm\(^2\) thick
polyethylene radiator is used with a 245-micron thick detector. A second
detector aligned behind the first one, is used to identify protons which
are not stopped in the first detector and, by means of an anticoincidence
arrangement, to block signals from the first detector due to these protons.
The signals must be blocked because the energy deposition in the first
detector does not represent the total proton energy. For measurements
from 4 to 12 MeV, a 2.95 mg/cm² radiator is used with a single 1040-
micron thick detector.

The principal advantages of this type of spectrometer are its low
gamma sensitivity, its well-known absolute efficiency, the straight for-
ward conversion from proton spectra to neutron spectra, and the essentially
unbounded upper energy limit (available by use of multiple thick detectors
and/or a large n-p scattering angle). The main disadvantages are its low
efficiency and the lower energy limit of about 1 MeV (due to poor resolu-
tion resulting from relatively large proton energy losses in the radiator).

C. FAST NEUTRON SPECTRUM MEASUREMENTS IN BULK MEDIA

Measurements of fast neutron spectra in bulk media which are perhaps the
best reported in the literature are summarized in Table 2. Other measure-
ments(43,73-77) were omitted because they are less suitable for testing de-
tailed calculations. Of the measurements listed in Table 2, several are more
useful than the others as tests. First, Verbinski and Bokhari's(36,37) spec-
tra (No. 4) are the most detailed measurements of reactor neutron spectra
in H₂O that have been published. However, comparison with the NIOBE and
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<th>Medium</th>
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<th>Penetration Distances</th>
<th>Calculations/Comparison</th>
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<td>1953</td>
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<td>Diff. n-p</td>
<td>1.5-10</td>
<td>H2O</td>
<td>Infinite</td>
<td>0.5, 10 &amp; 50 cm at 0°</td>
<td>None</td>
<td>(60)</td>
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<td>2</td>
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<td>1954</td>
<td>RSR-I Reactor</td>
<td>Diff. n-p</td>
<td>1.5-8.5</td>
<td>Graphite</td>
<td>Slab</td>
<td>0 &amp; 50, 7 cm at 0°</td>
<td>None</td>
<td>(61)</td>
</tr>
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<td>3</td>
<td>ORNL</td>
<td>1963</td>
<td>LINAC/Fb Target</td>
<td>TOF</td>
<td>0 - 0.1</td>
<td>LiH</td>
<td>Slab</td>
<td>0,10,20 &amp; 27.5 cm at 0°</td>
<td>NIDOE/Fair</td>
<td>(56)</td>
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<td>4</td>
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<td>1964</td>
<td>RSR-I Reactor</td>
<td>LA2(n,x)B</td>
<td>1 - 4.0</td>
<td>H2O</td>
<td>Infinite</td>
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<td>0.61,0 &amp; 91.4 cm</td>
<td>exp[-L/2(E,)]/Excellent</td>
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<td>TSR-II Reactor</td>
<td>Int. n-p</td>
<td>1.0-1.5</td>
<td>Liquid O</td>
<td>Slab</td>
<td>0.61,0 &amp; 91.4 cm</td>
<td>exp[-L/2(E,)]/Excellent</td>
<td>(49)</td>
</tr>
<tr>
<td>8</td>
<td>ORNL</td>
<td>1967</td>
<td>TSR-II Reactor</td>
<td>Int. n-p</td>
<td>0.7-1.0</td>
<td>Graphite</td>
<td>Slab</td>
<td>0.13,0 &amp; 20.5 &amp; 30.5 cm</td>
<td>exp[-L/2(E,)]/Excellent</td>
<td>(49)</td>
</tr>
<tr>
<td>9</td>
<td>ORNL</td>
<td>1967</td>
<td>TSR-II Reactor</td>
<td>Int. n-p</td>
<td>0.7-1.0</td>
<td>Pb</td>
<td>Slab</td>
<td>0 &amp; 15, 7 cm</td>
<td>exp[-L/2(E,)]/Excellent</td>
<td>(49)</td>
</tr>
<tr>
<td>10</td>
<td>GA</td>
<td>1964</td>
<td>LINAC/Target</td>
<td>TOF</td>
<td>0.15-0.75</td>
<td>Liquid H</td>
<td>Cylinder</td>
<td>0.6, 4, 11, 17, 26.7 &amp; 35.0 cm at 0°; 15°, 37°, 53° &amp; 78°</td>
<td>None</td>
<td>(63,64)</td>
</tr>
<tr>
<td>11</td>
<td>GA</td>
<td>1966</td>
<td>LINAC/Target</td>
<td>TOF</td>
<td>0.5-15</td>
<td>CH3</td>
<td>Infinite</td>
<td>0.5 &amp; 15 cm at 0°; 50 cm at 16°, 32° &amp; 60°</td>
<td>GAPS &amp; GSS/Four</td>
<td>(65,66)</td>
</tr>
<tr>
<td>12</td>
<td>GA</td>
<td>1966</td>
<td>LINAC/Target</td>
<td>TOF</td>
<td>0.5-15</td>
<td>CH3</td>
<td>Infinite</td>
<td>0.5 &amp; 15 cm at 0°; 50 cm at 16°, 32° &amp; 60°</td>
<td>GAPS &amp; GSS/Four</td>
<td>(65,66)</td>
</tr>
<tr>
<td>13</td>
<td>GA</td>
<td>1966</td>
<td>LINAC/Target</td>
<td>TOF</td>
<td>0.5-15</td>
<td>CH3</td>
<td>Infinite</td>
<td>0.20,3,35,6,30.8 &amp; 66.1 cm at 0°</td>
<td>None</td>
<td>(67)</td>
</tr>
<tr>
<td>14</td>
<td>USSN(1)</td>
<td>1963</td>
<td>Thermal Reactor</td>
<td>Int. n-p</td>
<td>1 - 10</td>
<td>CH3</td>
<td>Slab</td>
<td>0.10,20,30,60 &amp; 80 grams/cm²</td>
<td>Moments/Fair</td>
<td>(68)</td>
</tr>
<tr>
<td>15</td>
<td>USSN(1)</td>
<td>1964</td>
<td>Thermal Reactor</td>
<td>Int. n-p</td>
<td>1 - 11</td>
<td>Graphite</td>
<td>Slab</td>
<td>0.2,5,4,92,5 &amp; 125 cm</td>
<td>Moments/Fair</td>
<td>(69)</td>
</tr>
<tr>
<td>16</td>
<td>USSN(2)</td>
<td>1964</td>
<td>Thermal Reactor</td>
<td>Int. n-p</td>
<td>1 - 11</td>
<td>Pb</td>
<td>Slab</td>
<td>0.12,0,49 &amp; 74 cm</td>
<td>None</td>
<td>(69)</td>
</tr>
<tr>
<td>17</td>
<td>USSN(1)</td>
<td>1964</td>
<td>Thermal Reactor</td>
<td>Int. n-p</td>
<td>1 - 11</td>
<td>Fe</td>
<td>Slab</td>
<td>0.10,4,20,4.0 &amp; 65 cm</td>
<td>None</td>
<td>(69)</td>
</tr>
<tr>
<td>18</td>
<td>USSN(1)</td>
<td>1965</td>
<td>Thermal Reactor</td>
<td>Int. n-p</td>
<td>1 - 10.5</td>
<td>Concrete</td>
<td>Slab</td>
<td>0.10,30,60 &amp; 100 cm</td>
<td>None</td>
<td>(70)</td>
</tr>
<tr>
<td>19</td>
<td>USSN(2)</td>
<td>1964</td>
<td>Pn-Be</td>
<td>Int. n-p</td>
<td>1.5-10</td>
<td>H2O</td>
<td>Semi-Infinite</td>
<td>0.10,20,50 &amp; 40 cm</td>
<td>Moments/Fair</td>
<td>(71)</td>
</tr>
<tr>
<td>20</td>
<td>USSN(2)</td>
<td>1964</td>
<td>Pn-Be</td>
<td>Int. n-p</td>
<td>1.5-10</td>
<td>H2O</td>
<td>Slab</td>
<td>0.10,20 &amp; 50 cm</td>
<td>None</td>
<td>(71)</td>
</tr>
<tr>
<td>21</td>
<td>USSN(2)</td>
<td>1964</td>
<td>Pn-Be</td>
<td>Int. n-p</td>
<td>2 - 10.5</td>
<td>CH3</td>
<td>Slab</td>
<td>0 &amp; 14 cm</td>
<td>None</td>
<td>(72)</td>
</tr>
<tr>
<td>22</td>
<td>USSN(2)</td>
<td>1965</td>
<td>Pn-Be</td>
<td>Int. n-p</td>
<td>1.5-10.5</td>
<td>Al</td>
<td>Slab</td>
<td>0 &amp; 14 cm</td>
<td>None</td>
<td>(72)</td>
</tr>
<tr>
<td>23</td>
<td>Mich.</td>
<td>1966</td>
<td>PNR Reactor</td>
<td>Diff. n-p</td>
<td>2 - 12</td>
<td>H2O</td>
<td>Infinite</td>
<td>0.20,0.40,0 &amp; 60.0 cm at 0°</td>
<td>NIDOE/Excellent</td>
<td>(57)</td>
</tr>
<tr>
<td>24</td>
<td>Mich.</td>
<td>1966</td>
<td>PNR Reactor</td>
<td>Diff. n-p</td>
<td>2 - 12</td>
<td>Graphite</td>
<td>Slab</td>
<td>0.20,0.4,40.0 &amp; 60.0 cm at 0°</td>
<td>NIDOE/Good</td>
<td>(57)</td>
</tr>
</tbody>
</table>
DTK* calculations shows only fair agreement. Of note is a dip which appears at 3.5 Mev in the calculations and at 4 Mev in the measurements. Since this dip results from the elastic scattering resonance of oxygen at 3.5 Mev, it appears the spectrometer energy calibration was in error. Also, the NIOBE calculations show another dip at 7.3 Mev which does not appear in the measurements. In this case, it seems the NIOBE cross sections used were in error because current oxygen cross sections\(^{(78)}\) do not show a sizable resonance near 7.3 Mev.

Secondly, Verbinski's\(^{(37)}\) recently reported spectra (No. 5) of (\(\gamma,n\)) and (\(\gamma,f\)) neutrons which penetrated water in slab geometry show good agreement with NIOBE and DTK calculations. However, the comparison is somewhat biased because the measured spectra were individually normalized to the calculated spectra for convenience in comparing spectrum shapes.

Thirdly, Profio's\(^{(33)}\) spectra (No. 12) of (\(\gamma,n\)) and (\(\gamma,f\)) neutrons in a paraffin sphere (15.2 cm ID and 76.2 cm OD) were measured in very clean geometry. Even so, comparison with GAPLSN** and GGSN*** calculations shows only fair agreement.

Fourthly, Clifford, et al.'s\(^{(49)}\) recently reported spectra (Nos. 6-9) of reactor neutrons which traversed several samples were measured in narrow beam geometry. Thus, only uncollided neutrons were counted. These

---

**DTK is an early LASL version of DTF-IV.

**GAPLSN is a General Atomic version of DTF-IV.

***GGSN is a faster running version of GAPLSN which permits downscattering only.
measured spectra depend only on the spectrum of neutrons incident on the sample, the sample thickness, and the total cross section of the sample material. Comparison with calculated spectra, \( N_t(E) = N_0(E) \exp [-\Sigma_t(E)t] \), shows agreement which ranges from good to excellent depending on which of several sets of total cross sections was used. Although these measurements provide excellent tests for the total cross sections, they give no information on slowing down sources.

Thus, in view of the apparent absence of reliable measurements to test calculated spectra which include scattered neutrons, the spectrum measurements (Nos. 23 & 24) reported here should be very useful. At the deeper penetrations, they are in fact dominated by scattered neutrons. For example, the ratio of scattered neutrons to unscattered neutrons ranges from 100 at 2 Mev to 1 at 12 Mev for the neutrons which penetrated 60 cm of \( \text{H}_2\text{O} \).
CHAPTER II

SPECTROMETER DESIGN AND OPERATION

A. PRINCIPLES OF DESIGN AND OPERATION

The principal features of the proton-recoil, fast neutron spectrometer developed for these measurements are shown in Figure 1. During operation, the spectrometer is submerged in the open pool of the Ford Nuclear Reactor, the 2 megawatt research reactor at The University of Michigan. Fast neutrons from the FNR core stream through the collimator to a proton radiator. The radiator is a thin polyethylene (CH₂) film from which about one out of every 10⁶ incident neutrons elastically scatters a proton to the detector. The detector system generates a voltage pulse whose height is proportional to the energy of the incident recoil proton.

1. Energy Analysis

The energy of the recoil protons is related to the energy of the neutrons incident on the radiator by applying the laws of conservation of kinetic energy and linear momentum. Neglecting the small difference between proton mass and neutron mass, this results in the following ratio:

\[
\frac{\text{proton energy}}{\text{neutron energy}} = \cos^2 \Psi
\]

where \( \Psi \) is the n-p scattering angle in the laboratory system. This relation assumes that prior to being scattered, the protons are free and at rest. These assumptions are reasonable for hydrogen nuclei.
Figure 1. Principal features of proton-recoil fast neutron spectrometer.
held in place by ev bonds and scattered by Mev neutrons.

Since the hydrogen nuclei are scattered from within the radiator, they lose some energy before leaving it. If the remainder of their energy is deposited within the detector, then for neutrons with energy $E$ which scatter protons from the radiator through an angle $\psi$ to the detector,

$$E \cos^2 \psi = \int_{\text{Radiator}} (\frac{dE}{dx}) dx + \int_{\text{Detector}} (\frac{dE}{dx}) dx$$

This equation assumes the region between the radiator and detector is evacuated and the detector window is very thin. Because the protons are scattered through some angles in $\delta \psi$ about $\psi$, there are small variations in $\cos^2 \psi$. Also, since the protons travel various distances in the radiator before escaping it, there are variations in the proton energy losses in the radiator. Thus, it is necessary to rewrite the above equation in terms of average values in the following approximate form.

$$E \cos^2 \psi = \overline{(\frac{dE}{dx})} dx + \overline{(\frac{dE}{dx})} dx$$

(1)

To simplify the notation, define

$$\delta E_p \equiv \overline{(\frac{dE}{dx})} dx \quad \text{and} \quad E_p \equiv \overline{(\frac{dE}{dx})} dx$$

Hence the neutron energy,

$$E = \frac{\delta E_p + E_p}{\cos^2 \psi}$$

2. Energy Calibration

The energy calibration of the spectrometer is based on the above re-
lation. The proton energy deposition in the detector, $E_p$, is measured with a calibrated detector system. This calibration is described in Section II-B-5. The proton energy deposition in the radiator, $\delta E_p$, is generally much less than $E_p$. $\delta E_p$ is calculated using range-energy data (79-81) or energy loss data (53, 81) for protons traversing an average radiator thickness of $(L/2)t_R \sec \Psi$, where $t_R$ is the radiator thickness. Determination of $\sec \Psi$ and $\cos^2 \Psi$ requires a careful measurement of the radiator-detector geometry and the calculation of their average values as determined by the angular dependence of n-p scattering. Details of this calculation are in Section II-F-3.

3. Thick Detector Operation

In order for the detector response to be proportional to the incident recoil proton energy, the proton must be stopped in the depletion region. A detector thick enough to stop all incident protons is designated a "thick" detector in this connection. Since the measured neutron spectra decrease exponentially above 9 Mev, a detector thick enough to stop protons scattered by 14 Mev neutrons responds essentially as a thick detector for measurements to 12 Mev. A block diagram of the instrumentation used with this detector is shown in Figure 2.

4. Thin Detector Operation

As will be demonstrated in Section II-D-3, the use of a thin detector results in important background reduction. Of course, a thin detector does not stop all incident protons. Thus, in order to accept only those detector
Figure 2. Block diagram of instrumentation used with thick proton detector.
signals due to protons stopped in the thin detector, the two-detector, anticoincidence arrangement shown in Figure 3 is used. The energy response of the two tandem detectors is indicated in Figure 4. As these figures indicate, protons which traverse the thin detector and deposit enough energy in the second detector to exceed the discriminator level (0.5 Mev), trigger a coincidence pulse which blocks the signal from the thin detector. The dashed lines of Figure 4 show this anticoincidence arrangement should operate satisfactorily for proton energies up to 4.9 Mev. This corresponds to neutron energies up to 6.1 Mev.

The detectors are described in the next section and their instrumentation is covered in detail in Section II-C.

B. DETECTORS

Semiconductor devices were first used for nuclear radiation detection by Van Heerden\(^{(82)}\) in 1945. His bulk type detector has been succeeded by the junction type detector first reported by McKay\(^{(83)}\) in 1949. It was 1960 before extensive use of the latter detector was reported.\(^{(84,85)}\) Today surface barrier and diffused junction detectors of intrinsic and lithium-ion-drifted silicon and germanium are used widely. The detectors used with this spectrometer are intrinsic silicon surface barrier detectors.

1. Silicon Surface Barrier Detectors

These detectors were chosen because of their following characteristics: thin windows, linear response, high resolution, fast response, high stability, low gamma sensitivity, and compact size. Details of the detectors
Figure 3. Block diagram of instrumentation used with thin proton detector showing anticoincidence system that counts only protons stopped in thin detector.
Figure 4. Calculated energy response of two tandem silicon detectors to protons.
are given in Table 3. The operation of these detectors is based on the properties of the depletion region in a semiconductor diode operated with reverse bias. Electron-hole pairs generated in the depletion region by ionizing radiation are collected to provide a direct measure of the energy deposition in this region. The theory of these detectors deals primarily with the establishment of the depletion region and the generation and collection of electron-hole pairs. Although the theory is covered in the literature, (86-89) the essential ideas will be reviewed briefly here.

2. Depletion Region

When n-type and p-type crystals of the same material are placed together, electrons diffuse from the n-type into the p-type crystal and holes diffuse in the opposite direction. The region in which this diffusion occurs is characterized by a depletion of the majority charge carriers and the presence of the resulting space charges due to the bound impurity sites (donors and acceptors). Reverse biasing (plus to n-type and minus to p-type) the junction widens the depletion region. Hence, because the region is depleted of the majority charge carriers, its conductance is low and very little current flows through it. Forward biasing the junction removes the depletion region and current flows freely. This action is the basis for the semiconductor diode rectifier.

The depletion region of the semiconductor diodes is not formed by placing two crystals together. Rather, for the silicon surface barrier detector, a p-type surface layer is formed on a single n-type silicon crystal by ox-
<table>
<thead>
<tr>
<th>Detector Type</th>
<th>Thickness (mm)</th>
<th>Depletion Depth (mm)</th>
<th>Area (cm²)</th>
<th>Resistivity (ohm-cm)</th>
<th>Detector Bias (volts)</th>
<th>Capacitance (picofarads)</th>
<th>Leakage (µA)</th>
<th>5.5 Mev Alpha Resolution (keV)</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thick Detector</td>
<td>1.040</td>
<td>1.040</td>
<td>1.77</td>
<td>12,000</td>
<td>3.75</td>
<td>20</td>
<td>1.2</td>
<td>30</td>
<td>ORTEC</td>
</tr>
<tr>
<td>Thin Detector</td>
<td>0.245</td>
<td>0.245</td>
<td>1.77</td>
<td>5,700</td>
<td>55</td>
<td>80</td>
<td>0.7</td>
<td>40</td>
<td>ORTEC</td>
</tr>
<tr>
<td>(First Tandem Detector)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Second Tandem Detector | 1.0            | 0.16                 | 3.0        | 4,700                | 35                    | 200                      | 0.7           | 60                             | G. G. Sherwood        
Univ. of Mich.           |

*a* Intrinsic n-type Silicon

\[ V_{DET} = V_o + V_{SUPPLY} - I_L R_L \]

\[ R_L = 22 \text{ megohms}, \quad V_o \leq 0.5 \text{ volt} \]

*c* Measured in Evacuated Test Chamber at 22°C

*Oak Ridge Technical Enterprises Corp.*
idation. The depletion region extends into the n-type crystal. The approximate thickness of this region is found by solving Poisson's equation for constant space charge density and using the reverse bias and zero electric field as boundary conditions. This thickness is \( x = (\frac{kV}{2\pi Ne})^{1/2} \), where \( k \) is the dielectric constant of the crystal, \( V \) is the reverse bias, \( e \) is the electronic charge, and \( N \) is the charge density due to impurity sites.

If all impurities are ionized, the resistivity of the crystal is \( \rho = 1/\mu Ne \), where \( \mu \) is the majority charge carrier mobility. Thus, \( x = (\rho V k / 2\pi)^{1/2} \) shows the depletion thickness varies as the half power of the crystal resistivity and the reverse bias.

The capacitance of the depletion region is approximately that of a parallel plate capacitor of the same thickness with a silicon dielectric. Thus, the capacitance per unit area,

\[
\frac{C}{A} = \frac{k}{4\pi x} = \left(\frac{kNe}{2\pi e V}\right)^{1/2} = \left(\frac{k}{2\pi e \mu e V}\right)^{1/2}
\]

When the crystal resistivity and breakdown voltage are sufficiently high, the reverse bias can be increased until the depletion region extends from the p-type surface layer to the other side of the n-type crystal. The resulting fully depleted detector has three advantages over the partially depleted one. First, with the reverse bias somewhat higher than that required for full depletion, the thickness of the depletion region and hence the detector capacitance are insensitive to bias fluctuations. Secondly, with thin windows on both front and back, essentially all of the energy lost by charged particles traversing the detector is deposited in the depletion
region. Thirdly, the pulse rise time is reduced as the undepleted region of the detector is reduced. This is because the undepleted region contributes to the series resistance of the detector and the time constant of the detector equivalent circuit varies as this resistance. As indicated in Table 3, both the thick and thin detectors are fully depleted.

3. Electron-Hole Pair Generation

The average energy required to generate an electron-hole pair in the depletion region is predicted by Shockley\(^{(90,91)}\) to be

\[ \varepsilon = \varepsilon_R E_g + r E_R \]

where \( E_g \) is the energy gap between the valence and conduction bands, \( E_R \) is the energy of the highest frequency optical mode phonon, and \( r = (L_i/L_R) \), the ratio of the mean free path between ionizing collisions, \( L_i \), and the mean free path for phonon collisions, \( L_R \). For silicon at 300°K, \( E_g = 1.12 \) ev. With \( r = 17.5 \) and \( E_R = 0.063 \) ev, \( \varepsilon = 3.57 \) ev.

Until recently, all measurements indicated that \( \varepsilon \) was independent of the mass, charge and energy of the ionizing particle. However, careful measurements by an Italian group\(^{(92)}\) give \( \varepsilon_{\alpha} = 3.61 \pm 0.01 \) ev for the 5.49 Mev alphas of Am\(^{241}\) and \( \varepsilon_e = 3.79 \pm 0.01 \) ev for the 0.365 Mev conversion electrons of Sn\(^{113}\). These results have been substantiated by a U. S. group\(^{(93)}\) who found \( \varepsilon_{\alpha} = 3.60 \pm 0.01 \) ev for the same alphas and \( \varepsilon_e = 3.80 \pm 0.01 \) ev for the 1.05 Mev conversion electrons of Bi\(^{207}\). Both sets of results are for silicon at 300°K (\( E_g = 1.12 \) ev) and with the data extrapolated to
infinite collecting electric field to eliminate the influence of trapping and recombination. The approximately 5% difference in $\varepsilon_\alpha$ and $\varepsilon_e$ is not explained by Shockley's model. Even so, the agreement with the predicted 3.57 eV gives some credence to his model.

4. Electron-Hole Pair Collection

The collection of electron-hole pairs is impeded by two mechanisms: trapping and recombination. These phenomena usually occur at imperfection sites in the crystal. Trapping of the charge carriers refers to the immobilization of the carriers at trapping centers. If the mean carrier lifetime is on the order of or less than the amplifier clipping time, a substantial portion of the signal is lost. The loss of charge carriers by recombination is most likely to occur in the vicinity of the particle track where the ionization density is highest. This recombination takes place before the electrons and holes in the ionized column are separated by the applied electric field. Hence, recombination increases with increasing ionization density and decreasing electric field. The principal consequence of trapping and recombination is to reduce the collection efficiency and energy resolution of the detector.

The collection efficiency, $\eta = Q/Ne$, gives the ratio of the charge $Q$ on the collecting electrode to the charge generated by the release of $N$ electron-hole pairs. Here $N = E/\varepsilon$, where $E$ is the energy deposited in the depletion region and $\varepsilon$ is the mean number of ion pairs generated per unit energy deposited in this region. Most junction detectors can be biased suf-
ficiently for $\eta$ to be very nearly 100%.

5. Energy Calibration

The energy calibration of the detectors and their instrumentation is based on the use of a precision pulse generator to determine the zero energy channel and the use of the 5.49 Mev* alphas of Am$^{241}$ to fix the channel width. The pulse generator, whose output is linear, was capacity coupled to the preamplifier input and the pulse amplitudes were varied to obtain a plot of pulse height versus channel number. The intercept of this plot was the zero energy channel. The linearity of this plot demonstrated the linearity of the preamplifier, amplifier, and ADC combination.

The use of alphas to determine the channel width for protons is based on the assumption that $\eta_p/\epsilon_p = \eta_\alpha/\epsilon_\alpha$. It is generally considered that the difference between $\eta_p$ and $\eta_\alpha$ is too small to be measured with present techniques. However, a recent measurement$^{(97)}$ gives $\epsilon_p/\epsilon_\alpha \approx 0.99$. At the time of data reduction, this result was unavailable. Hence, in the absence of other data to substantiate it, this 1% is neglected. Future measurements of $\epsilon_p/\epsilon_\alpha$ for silicon may indicate the desirability of correcting the channel width for a small difference between $\epsilon_p$ and $\epsilon_\alpha$.

An approximate verification of the calibration procedure was made using the 1.05 Mev$^{(98)}$ conversion electrons of Bi$^{207}$. Two previously described measurements give $\epsilon_e/\epsilon_\alpha = 1.050 \pm 0.004$ and $1.056 \pm 0.004$. Assuming then

$^*$Am$^{241}$ alpha spectrum: 5.4870 Mev (86.0%), 5.4435 Mev (12.7%), 5.3880 Mev (1.3%), and 20 others (<0.5%).$^{(94-96)}$
that $\varepsilon_e/\varepsilon_p = 1.053$ and $\eta_e/\eta_p = 1.000$, the 1.05 Mev electrons should be counted in the channel calibrated for 1.00 Mev protons. This response was observed.

6. Leakage Currents

Another important characteristic of reverse biased junctions is the current flow in the absence of radiation. This current, called bulk current, must be sufficiently small that heating of the crystal is negligible. The bulk current has two components: the drift current due to the diffusion of minority carriers into the depletion region and the generation current due to the thermal generation of minority carriers in the depletion region.

In addition to the bulk current through the depletion region, some current flows around its edge. This edge leakage or surface leakage current depends on many factors including the design and construction of the detector, treatment and contamination of its surfaces, and ambient conditions such as temperature, humidity, and pressure. The sum of the bulk current and the edge leakage current is called the reverse current or leakage current. It must be much smaller than the signal currents resulting from radiation interactions in the depletion region if the signal-to-noise ratio is to be sufficiently high to permit good energy resolution. Leakage currents for the detectors used here are given in Table 3.

C. INSTRUMENTATION

Integrating or charge-sensitive preamplifiers are used with these junction detectors. This type of preamplifier is favored because its output
is very nearly independent of the detector capacitance. This is indicated in Figure 5 which shows the block diagram and equivalent circuit for a reverse-biased detector, charge-sensitive preamplifier, and amplifier. Here the amplifier output voltage pulse due to the collection of charge Q at the detector $V_{\text{AMP}} = A(Q/C)$, where $C = \left[ C_D + C_A + (G+1)C_F \right] / G \approx C_F$, G is the voltage gain of the preamplifier input section (with feedback capacitance $C_F$), A is the combined voltage gain of the preamplifier (after the input section) and the amplifier, $C_D$ and $C_A$ are the capacitances to ground of the detector and the preamplifier input (including stray), respectively. This equation is derived for negative G. To follow convention, (99) $|G|$ is written as G. For $G = 2000$, (i.e., $|G| = 2000$), $C_D = 80 \text{pf}$ (as for thin detector), and $C_A = 20 \text{ pf}$, $C = 1.01 C_F$ and $V = 0.99 A(Q/C_F)$.

In order to keep $C_A$ small, the preamplifiers were located within 1 ft of the detectors. This was done by putting the preamplifiers in a watertight box that was attached to the top side of the detector chamber.

Each preamplifier was connected to an amplifier in a temperature controlled instrument room through a 200 ft length of RG-71 doubly shielded, 93 ohm coaxial cable. The multichannel pulse height analyzer and anticoincidence system were located in the same instrument room.

Block diagrams of the instrumentation are given in Figures 2 and 3. Details on the components are given in Tables 4 and 5.
\[ V_{\text{AMP}} = \frac{GAQ}{C_D + C_A + (G+1)C_F} \approx \frac{A Q}{C_F} \]

Figure 5. Block diagram and equivalent circuit for reverse-biased detector, charge-sensitive preamplifier, and amplifier.
TABLE 4

PROTON ENERGY ANALYSIS INSTRUMENTATION

<table>
<thead>
<tr>
<th>Component</th>
<th>Manufacturer</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preamplifier</td>
<td>Tennelec</td>
<td>100A</td>
</tr>
<tr>
<td>Amplifier</td>
<td>Nuclear Data</td>
<td>150F</td>
</tr>
<tr>
<td>Multichannel pulse height analyzer</td>
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<td></td>
</tr>
<tr>
<td>ADC</td>
<td>Nuclear Data</td>
<td>150F</td>
</tr>
<tr>
<td>Memory</td>
<td>Nuclear Data</td>
<td>150M</td>
</tr>
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</table>

TABLE 5

ANTICOINCIDENCE INSTRUMENTATION

<table>
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<th>Component</th>
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<th>Model</th>
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<tr>
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<td>100A</td>
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<tr>
<td>Gate unit</td>
<td>Nuclear Data</td>
<td>150F</td>
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D. SIGNAL-TO-BACKGROUND RATIO

Since low signal-to-background ratios (S/B) have been the principal limitation encountered in most measurements of reactor neutron spectra, the improvement of the S/B has been one of the main objectives in the development of this spectrometer system. Here the signal refers to those counts which result from neutrons entering the source end of the collimator, travel-
ing through the collimator without being scattered, and then scattering pro-
tons from the radiator to a detector which is thick enough to stop them.
The background, as it is customarily defined, refers to all nonsignal counts.

1. Sources of Background

It is useful to consider the background as consisting of two components: nonsignal counts associated with the radiator and all other nonsignal counts. The latter component is measured by removing the radiator and is usually found to be the larger one. It is due to charged particles resulting from gamma and neutron interactions which occur within the detector and in the shielding and structural material in the vicinity of the detector. Data which indicates that gamma interactions within the detector dominate at higher energies and gamma interactions in material around the detector dom-
inate at lower energies will be presented.

The nonsignal counts associated with the radiator are due to (1) protons scattered from the radiator with such high energy that they are not stopped in the detector, (2) carbon ions scattered from the CH\textsubscript{2} radiator by neutrons, and (3) protons scattered from the radiator by neutrons which either entered the collimator through the side wall rather than through the source end or entered the collimator through the source end, but were scattered in the collimator wall before reaching the radiator. The first case is designated as a nonsignal count because the measured proton energy is less than the energy of the proton incident on the detector. The third case, although not usually designated back-
ground in the literature, is designated such here because it does not qualify as
part of the above defined signal.

2. Design Considerations.

As previously stated, most of the background counts usually result from gamma interactions within and near the detector. Hence, the spectrometer was specifically designed to reduce this source of background. The principal features are shown in Figure 1. This spectrometer has been operated with the source end of its collimator in fission gamma fields of $10^7$ to $10^8$ roentgens/hour.

The long, evacuated, conical collimator extracts a well-defined neutron beam from the reactor. It also allows the detector chamber to be far enough from the reactor core so that the pool water is an effective shield. The effectiveness of the water shielding was demonstrated by a measurement of the background count rate with the collimator removed. The measurement showed that only 10 to 20% of the background counts from 2 to 5 Mev are due to radiation which did not enter the detector chamber through the collimator.

The cylindrical steel and lead shields assist in the collimation of fission gammas. The desirability of this collimation is indicated in Figure 6. This figure shows the detector count rate without radiator as a function of the detector position above and below the collimator axis. This data was taken with only the lead shield installed. The background reduction by a factor of over a hundred along with a signal reduction of only 10% dictated the use of an off-axis detector position. The detector was positioned with its center 4.2 cm above the collimator axis and 10.2 cm
Figure 6. Background count rates for detector positions above and below collimator axis.
from the center of the radiator position. The resulting angle between the radiator-detector centerline and the collimator axis was 24°. As a result of the suggestion by Dr. F. C. Maienschein of ORNL to increase the thickness of the gamma shield, the steel shield was added. Its addition reduced the background counts from 2 to 5 Mev by 44%.

The electron filter is a 50-mil thick aluminum plate located over the front of the collimator throat to reduce the number of electrons streaming into the detector chamber. It reduces the background counts from 2 to 5 Mev by 36%. The charged particle baffle shields the detector from charged particles originating in the rear portion of the collimator throat. The beam trap reduces backscattering to the detector.

The radiator wheel is a thin brass plate with six equally spaced 2.1-cm diameter holes. They are slightly larger than the 1.9-cm diameter of the collimator throat. The wheel can be rotated and stopped so that the center of any one of the holes is aligned with the collimator axis. Various thickness polyethylene radiators can be mounted over these holes on the collimator side of the radiator wheel. For these measurements, only 1.25 mg/cm\(^2\) and 2.95 mg/cm\(^2\) thick CH\(_2\) radiators were used. A blank wheel position was aligned with the collimator axis for the measurement of the background counts not associated with the radiator.

The evacuated detector chamber is much larger than that required to house the detector, radiator wheel, and motor. The extra volume provides free space around the detector to reduce charged particle events in material near the detector.
3. Influence of Detector Thickness

The limitation imposed by a low S/B is illustrated by the data of Figure 7. The S/B for this data and for data taken with 60 cm of water between the reactor core and collimator is shown in Figure 8. The lower S/B for the latter data indicates the background is dominated by gamma interactions since water attenuates fast neutrons (the signal) more than it does gammas (the background).

The S/B for the latter data also demonstrates that reliable data (i.e., S/B > 1) cannot be taken with this radiator-detector combination through 60 cm of water in the energy range from 2 to 5 Mev. Resolution considerations prohibit the use of a thicker radiator to improve the S/B in this energy region. An alternative is to use a thinner detector. The effect of using a detector of the same area but with its thickness reduced by a factor of four is shown in Figure 9. These curves show that the ratio of background counts for thick and thin detectors is equal to the ratio of their thicknesses at high energies but is much greater at lower energies. Thus, since the detector areas are equal, the background at high energies is proportional to the detector volume. This indicates gamma interactions within the detector are the main source of background at these energies. At the lower energies, the curves indicate the background is dominated by gamma interactions in material outside the detectors.

Data taken with the thin detector is shown in Figure 10. The coincidence spectrum shows the energy lost in the thin detector by protons which are not stopped in it. The anticoincidence system discussed earlier prevents
Figure 7. Proton recoil spectrum obtained using thick (1040 microns) detector and thick (2.95 mg/cm²) radiator with collimator against reactor core.
Figure 8. Signal-to-background ratios for proton recoil spectra obtained using thick (1040 microns) detector and thick (2.95 mg/cm²) radiator.
Figure 9. Ratio of thick detector background to thin detector background.
Figure 10. Proton recoil spectra obtained using thin (245 microns) detector and thin (1.25 mg/cm²) radiator with collimator against reactor core.
these counts from being recorded. Alternately, they can be recorded separately as shown in Figure 10 and subtracted along with the counts recorded with the radiator removed. Use of the thin detector and anticoincidence system permits measurements above 2 Mev through 60 cm of water to be made with $S/B > 1$.

4. Carbon Recoils

It is shown in Appendix A that the background due to carbon recoils depends on the neutron energies of interest and on the shape of the spectrum being measured. For a hypothetical neutron spectrum which is constant from energy $4.24 \ (E_C)^\ast_{\text{min}}$ to $E_{\text{max}}$, and zero thereafter, the maximum background occurs at the lowest neutron energy of interest, $E_{\text{min}}$. For the present measurements, $E_{\text{min}} = 2 \text{ Mev}$ and $(E_C)_{\text{min}} = 1.5 \text{ Mev}$. If $E_{\text{max}}$ is 14 Mev, the maximum background due to carbon recoils is 4%, and if $E_{\text{max}}$ is 18 Mev, it is 6%. These are conservative estimates, of course, since the actual measured spectra decrease rapidly with energy above the minimum energy, $4.24 \ (E_C)_{\text{min}} = 6.36 \text{ Mev}$, required to scatter 1.5 Mev carbon recoils to the detector.

5. Collimator Effects

There are two effects which occur when the collimator walls are not black. First, some of the neutrons which exit the collimator entered it through the source end but were scattered in the collimator wall before reaching the exit. Secondly, some of the neutrons that reach the collimator exit initially entered the collimator through the side wall rather than

\*The lowest carbon recoil energy of interest is $(E_C)_{\text{min}} = (E_p)_{\text{min}} = (E_{\text{min}} \cos^2 \psi - 5E_p)$, and 4.24 is the ratio of the incident neutron energy to the carbon recoil energy.
through the source end. In Appendix E, these effects are estimated by use of a crude model to introduce 4.8% and 1.7% errors, respectively, when using the long conical collimator of this spectrometer.

Because the background due to carbon recoils and collimator effects is relatively small and the corrections are difficult to determine, no corrections were made for these measurements.

E. EFFICIENCY

The efficiency of the spectrometer can be defined either in terms of the neutron angular flux at the source end of the collimator or in terms of the neutron flux incident on the radiator. These two efficiencies, of course, differ only by the collimator efficiency.

For the first case, the total efficiency $\epsilon(E)$ is defined as the number of neutrons with energy $E$ counted per unit time per unit neutron angular flux incident on the source end of the collimator and directed about the collimator axis toward the radiator:

$$\epsilon(E) = \frac{C(E_p) dE_p}{\phi(E) dE}$$  \hspace{1cm} (2)

where:

$C(E_p) dE_p$ is the count rate in counts per sec due to protons which are scattered from the radiator to the detector by neutrons with energies in $dE$ about $E$ and which deposit energies in $dE_p$ about $E_p$ within the detector.

$\phi(E) dE$ is the number of neutrons with energies in $dE$ about $E$ per sec
per cm$^2$ per steradian incident on the source end of the collimator and directed about the collimator axis toward the radiator.

For the second case, a similar expression is used to define $\varepsilon_R(E)$, the number of neutrons with energy $E$ which are counted per unit time per unit neutron flux incident on the radiator.

$$
\varepsilon_R(E) \equiv \frac{C(E_p) dE_p}{\varphi_R(E) dE}
$$

where:

$\varphi_R(E) dE$ is the number of neutrons with energies in $dE$ about $E$ per sec per cm$^2$ directed about the collimator axis toward the radiator and incident on the radiator.

$\varepsilon(E)$ and $\varepsilon_R(E)$ are related by defining the collimator efficiency,

$$
\varepsilon_C \equiv \frac{\varepsilon(E)}{\varepsilon_R(E)}
$$

In order to simplify the evaluation of $\varepsilon_C$ and $\varepsilon_R(E)$, the following assumptions are made:

a. The radiator is positioned over the exit end of the collimator.

b. The distance from the source end of the collimator to the exit end is large compared with the largest dimension of these two apertures.

c. The distance from the radiator to the detector is large compared with the largest dimension of these two components.

d. The anisotropy of n-p scattering in the center-of-mass system is negligible for the neutron energies of interest. (This is reasonable because n-p scattering is essentially 100% s-wave scattering up to 10 Mev and
is still 99% s-wave scattering at 14 Mev.)

\(e.\) The radiator is sufficiently thin so that neutron attenuation and multiple n-p scattering are negligible.

From its definition,

\[
\varepsilon_c = \frac{\Omega_c}{L_c} = \frac{A_c}{L_c} \quad \text{in steradians}
\]

where:

- \(\Omega_c\) is the solid angle subtended at the radiator by the source end of the collimator in steradians.
- \(A_c\) is the area of the source end of the collimator in cm\(^2\).
- \(L_c\) is the distance from the source end of the collimator to the radiator end in cm.

Also from its definition,

\[
\varepsilon_R(E) = (N_R^H t_R A_R) \frac{d\sigma_{n-p}(E, \psi)}{d\Omega} \cdot S_{\Omega} \quad \text{in cm}^2
\]

where:

- \((N_R^H t_R A_R)\) is the number of hydrogen atoms in the radiator.
- \(N_R^H\) is the number of hydrogen atoms per milligram of radiator.
- \(t_R\) is the thickness of the radiator in milligrams per cm\(^2\).
- \(A_R\) is the radiator area in cm\(^2\).
- \(d\sigma_{n-p}(E, \psi)\) is the differential cross section for neutrons with energy \(E\) to elastically scatter protons through the laboratory angle \(\psi\) with respect to the direction of the incident neutron, averaged with respect to \(\psi\), in cm\(^2\) per hydrogen atom per steradian.
\[ \Omega_D = \frac{A_D}{L_D^2} \] is the solid angle subtended at the radiator by the detector in steradians.

\[ A_D \] is the area of the detector in cm\(^2\).

\[ L_D \] is the distance from the radiator to the detector in cm.

From assumption (a),

\[ \frac{d\sigma_{n-p}(E, \psi)}{dS} = \frac{\sigma_{n-p}(E) \cos \psi}{\pi} \]

where:

\[ \sigma_{n-p}(E) \] is the cross section for neutrons with energy \( E \) to elastically scatter protons in cm\(^2\) per hydrogen atom.

\[ \cos \psi \] is the cosine of the laboratory scattering angle, averaged with respect to \( \psi \).

Combining these terms,

\[ \varepsilon_R(E) = \frac{1}{\pi} \left( N_R^h t_R \right) \left( \frac{A_R A_D}{L_D^2} \right) \cos \psi \sigma_{n-p}(E) \quad \text{in} \quad \text{cm}^2 \quad (5) \]

Taking the product \( \varepsilon_C \varepsilon_R(E) \) to get the total efficiency,

\[ \varepsilon(E) = \frac{1}{\pi} \left( N_R^h t_R \right) \left( \frac{A_C A_R A_D}{L_C L_D^2} \right) \cos \psi \sigma_{n-p}(E) \quad \text{in} \quad \text{cm}^2 \text{ steradian} \quad (6) \]

An outstanding feature of these efficiency expressions is that the energy dependent term, \( \sigma_{n-p}(E) \), is very well known \((< \pm 1\%)\). Figures 11 and 12 show \( \varepsilon_R(E) \) and \( \varepsilon(E) \) as calculated using the following constants:

\[ N_R^h = 0.86 \times 10^{20} \text{ H atoms/mg} \]

\[ t_R = 1.25 \text{ and } 2.95 \text{ mg/cm}^2 \]

\[ A_C = \pi (5.1 \text{ cm})^2 = 81.7 \text{ cm}^2 \]
Figure 11. Calculated efficiency: $\varepsilon_R(E)$. 

CALCULATED EFFICIENCY: NUMBER OF NEUTRONS COUNTED PER UNIT TIME PER UNIT NEUTRON FLUX INCIDENT ON RADIATOR

$\varepsilon_R(E)$ IN CM$^2$

NEUTRON ENERGY (MEV)

2.95 MG/CM$^2$ CH$_2$ RADIATOR

1.25 MG/CM$^2$ CH$_2$ RADIATOR
\[ A_R = \pi (0.95 \text{ cm})^2 = 2.84 \text{ cm}^2 \]
\[ A_D = \pi (0.75 \text{ cm})^2 = 1.77 \text{ cm}^2 \]
\[ L_C = 315 \text{ cm} \]
\[ L_D = 10.2 \text{ cm} \]
\[ \cos \psi = \int \cos \psi f(\psi) \, d\psi = 0.91 \]
\[ \cos^2 \psi = \int \cos^2 \psi f(\psi) \, d\psi = 0.83 \]
\[ \sec \psi = \int \sec \psi f(\psi) \, d\psi = 1.10 \]

The last two quantities are used to evaluate \( E_p = E \cos^2 \psi - \delta E_p \) and
\( \delta E_p = (1/2) \int_0^{t_R} (d\delta E/dx) \, dx \neq (1/2) t_R \sec \psi \left( \frac{dE}{dx} \right) E_p \). The normalized scattering frequency, \( f(\psi) d\psi \), is derived and evaluated in Section II-F-3.

From the definitions of \( \varepsilon_R(E) \) and \( \varepsilon(E) \), the differential fluxes

\[ \varphi_R(E) = \frac{C(E_p)}{\varepsilon_R(E) dE/dE_p} \quad (7) \]
\[ \varphi(E) = \frac{C(E_p)}{\varepsilon(E) dE/dE_p} \quad (6) \]

The differential count rate, \( C(E_p) = N(E_p)/\Delta t \Delta E_p \), where \( N(E_p) \) is the number of counts accumulated during \( \Delta t \) in the channel whose width is \( \Delta E_p \) and whose midpoint is \( E_p \). To find \( dE/dE_p \), recall

\[ E = \frac{E_p + \delta E_p}{\cos \alpha \psi} \]

Thus,

\[ \frac{dE}{dE_p} = 1 + \frac{d(\delta E_p)/dE_p}{\cos \alpha \psi} \]

where

\[ \frac{d(\delta E_p)}{dE_p} = \frac{1}{2} t_R \sec \psi \frac{d}{dE_p} \left( \frac{dE}{dx} \right) E_p \]

*Detector area was measured by counting Am\(^{241}\) alphas with and without an aluminum mask (1.29 cm\(^2\) aperture) in front of detector.
Since the latter derivative is negative,

$$\frac{dE}{dE_p} \propto \left[ 1 - \frac{1}{2} \frac{\sec \theta}{\cos^2 \varphi} \frac{d}{dE_p} \left( \frac{dE}{dE_p} \right)_{E_p} \right]$$

(9)

This Jacobian was calculated and the results are shown in Figure 13.

F. ENERGY RESOLUTION

The spectrometer energy resolution is defined as $\Delta E/E$, where $\Delta E$ is the FWHM of the spectrometer response function for monoenergetic neutrons of energy $E$. Here the neutron energy $E = (E + \delta E_p)/\cos^2 \varphi$. The resolution is found using the relation

$$\left( \frac{\Delta u}{u} \right)^2 = \left( \frac{1}{u} \right)^2 \sum_i \left( \frac{\partial u}{\partial x_i} \right)^2 \left( \Delta x_i \right)^2$$

where $u = u(x_1, x_2, x_3, \ldots)$. Thus,

$$\left( \frac{\Delta E}{E} \right)^2 = \left( \frac{\Delta E_p}{E \cos^2 \varphi} \right)^2 + \left[ \frac{\Delta (\delta E_p)}{E \cos^2 \varphi} \right]^2 + \left[ \frac{\Delta (\cos^2 \varphi)}{\cos^2 \varphi} \right]^2$$

(10)

The three terms are due to variations of (1) the proton detector response, (2) the proton energy losses in the radiator, and (3) the n-p scattering angle, respectively. Each term will be discussed in order.

1. Detector Term

The detector term, $\Delta E_p/E \cos^2 \varphi$, is the smallest and is negligible at all but low energies (6% at 2 Mev, 3% at 4 Mev). Table 3 gives $\Delta E_p$ as 30 and 40 Kev for the thick and thin detectors, respectively, in an evacuated test chamber. When they are installed in the spectrometer and it is submerged in the reactor pool as it is during normal operation, $\Delta E_p$ of both
detectors increases to about 60 Kev. This increase is probably due to ground loops. Also, $\Delta E_\alpha$ increases to 140 Kev during some periods. During the same periods, a one megacycle ripple is observed on the preamplifier output. The appearance of this ripple has been found to correspond to the broadcasting time of a local, daytime radio station which transmits at 1.05 megacycles. Since spectrum measurements were taken about equally during day and night, an average $\Delta E_\alpha$ is 100 Kev. This value is used for $\Delta E_p$. As before, $\int \cos^2 \psi = \int \cos^2 \psi f(\psi) d\psi = 0.83$. Hence, $\Delta E_p / E \cos^2 \psi = 120$ Kev/E.

2. Radiator Term

The radiator term, $\Delta(\delta E_p)/E \cos^2 \psi$, is the dominant term at low energies, but becomes negligible at high energies. It can be estimated by neglecting the variations in $\psi$ and assuming all protons are scattered through some mean angle $\bar{\psi}$. Then the response function is a rectangular function whose maximum energy is $E \cos^2 \bar{\psi}$ and whose minimum energy is $E \cos^2 \psi - \int_0^{t_R \sec \bar{\psi}} (dE/dx) dx$. Thus, the width of this rectangular function is $\Delta(\delta E_p) = \int_0 (dE/dx) dx \approx t_R \sec \bar{\psi}(dE/dx)_E$. Replacing $\sec \bar{\psi}$ by $\bar{\sec} = \int \sec \psi f(\psi) d\psi = 1.10$, $\Delta(\delta E_p) = 2 \delta E_p$ and $\Delta(\delta E_p)/E \cos^2 \psi \approx 2.41 \delta E_p / E \approx 1.32 t_R \left( \frac{dE}{dx} \right)_E$.

3. Geometry Term

The last term, $\Delta(\cos^2 \psi)/\cos^2 \bar{\psi}$, is called the geometry term because it is determined primarily by the scattering angles allowed by the collimator-radiator-detector geometry. However, it also depends on the angular dependence of the n-p scattering cross section. As before, the scattering is
assumed to be isotropic in the CMOS. Also, the neutrons are assumed to be normally incident on the radiator. The small angular divergence of the long, conical collimator makes the latter assumption very good. Thus, the geometry term is determined by the radiator-detector geometry and the cosine of the laboratory scattering angle. This term is independent of neutron energy, and it is the dominant term at high energies.

\( \Delta(\cos^2 \psi) \) is the FWHM of the normalized distribution function \( f(\cos^2 \psi) \), where \( f(\cos^2 \psi) d(\cos^2 \psi) = f(\psi) d\psi \) and \( f(\psi) d\psi \) is the fraction of the counts due to protons scattered in \( d\psi \) about \( \psi \).

To find \( f(\psi) \), consider a beam of neutrons normally incident on the differential radiator area, \( dA_R \), shown in Figure 14. The resulting current of protons scattered in \( d\psi \) about \( \psi \) from \( dA_R \) to the differential detector area, \( dA_D \), is

\[
J_p(E, r, \theta, \psi) dE dA_R dA_D = J_n(E, r, \theta) dE dA_R N^R_{tr} \frac{d\tau_{n-p}(E, \psi)}{d\Omega} d\Omega_D
\]

where: \( J_n(E, r, \theta) dE dA_R \) is the number of neutrons with energies in \( dE \) about \( E \) which are normally incident on \( dA_R \) per second. \( J_p(E, r, \theta, \psi) dE dA_R d\Omega_D \) is the number of protons elastically scattered by neutrons with energies in \( dE \) about \( E \) from \( dA_R \) into \( d\Omega_D \) per second.

\[
d\Omega_D = \frac{S(r, \theta, \psi) R d\psi}{R^2} = \frac{S(r, \theta, \psi) \cos \psi d\psi}{H}
\]

Assuming

\[
\frac{d\tau_{n-p}(E, \psi)}{d\Omega} = \frac{\tau_{n-p}(E) \cos \psi}{\pi}
\]
\[ dA_D = S(r, \theta, \psi) d\rho \]

\[ dA_R = r dr d\theta \]

\[ \rho = H \tan \psi \]

**Figure 1.4.** Radiator-detector geometry used for calculation of \( f(\psi) \) and \( f(\cos^2 \psi) \).
and
\[ J_n(E, r, \theta) = J_n(E) F(r, \theta) \]

\[ J_P(E, r, \theta, \psi) dE dF dS d\psi = (N_R^h \theta_e / \pi H) \sigma_{n-p}(E) J_n(E) dE F(r, \theta) S(r, \theta, \psi) dF r \cos^2 \psi d\psi \]

Define:
\[ J_P(E, \psi) dE d\psi \equiv \frac{1}{A_R} \int_{A_R} J_P(E, r, \theta, \psi) dE d\Omega dF r \cos^2 \psi d\psi \]

\[ S(\psi) \equiv \frac{1}{A_R} \int_{A_R} F(r, \theta) S(r, \theta, \psi) dF r \cos^2 \psi d\psi \]

Substituting,
\[ J_P(E, \psi) dE d\psi = (N_R^h \theta_e \theta_e / \pi H) \sigma_{n-p}(E) J_n(E) dE S(\psi) \cos^2 \psi d\psi \]

Define:
\[ P(E, \psi) d\psi \equiv \frac{J_P(E, \psi) dE d\psi}{J_n(E) dE} \]

\[ P(E, \psi) d\psi = (N_R^h \theta_e \theta_e / \pi H) \sigma_{n-p}(E) S(\psi) \cos^2 \psi d\psi \]

But,
\[ f(\psi) d\psi \equiv \frac{P(E, \psi) d\psi}{\int P(E, \psi) d\psi} \]

\[ f(\psi) d\psi = \frac{S(\psi) \cos^2 \psi d\psi}{\int S(\psi) \cos^2 \psi d\psi} \]

\[ f(\psi) d\psi = \Xi S(\psi) \cos^2 \psi d\psi \]

where the normalization constant
\[ \Xi = \left[ \int S(\psi) \cos^2 \psi d\psi \right]^{-1} \]

To find \( f(\cos^2 \psi) \), recall \( f(\cos^2 \psi) d(\cos^2 \psi) = f(\psi) d\psi \). Since
\[ d(\cos^2 \psi) = -2 \cos \psi \sin \psi d\psi \]

\[ f(\cos^2 \psi) = -\frac{\Xi}{2} S(\psi) \cot \psi d\psi \]

The determination of \( f(\psi) \) and \( f(\cos^2 \psi) \) requires \( S(\psi) \). The function \( S(\psi) \) must be evaluated for a specific radiator-detector geometry and a
given \( F(r, \theta) \). Here it is assumed that \( F(r, \theta) = F(r) \). Thus,

\[
S(\psi) = \frac{1}{A_R} \int \int r F(r) \, r \, dr \, d\theta
\]

This integral is evaluated by the following graphical-numerical method.

Draw to scale the front view of the radiator and detector shown in Figure 14. Then draw concentric circles within the radiator that divide it into \( m \) equal areas. Next, draw radii which divide each of the \( m \) areas into \( n \) equal areas. This procedure is illustrated in Figure 15 for \( m=3 \) and \( n=12 \). Each of the resulting \( mn = 36 \) areas is equal. Due to the symmetry, only the \( 18 \) areas in the right half of the radiator are used. Arcs with radii which correspond to various values of \( \psi \) are swung from the centers of these areas to intercept the detector area. From Figure 14, the arc radius, \( \rho(\psi) = H \tan \psi \). Figure 15 shows arcs drawn from area \((m,n) = (3,5)\) for integral values of \( \psi \). The intercept of \( \rho(24^\circ) \) with the detector area is \( S(r_3, \theta_5, 24^\circ) \). A set of arcs with radii corresponding to integral values of \( \psi \) was drawn for each of the radiator areas \((m,n)\). The length of each intercept with the detector area was measured to get \( S(r_m, \theta_n, \psi) \). Using these values of \( S(r_m, \theta_n, \psi) \),

\[
S(\psi) = \left( \frac{A_R}{A_r} \right) \sum_{m,n} F(r_m) \, S(r_m, \theta_n, \psi)
\]

where \( \left( \frac{A_R}{A_r} \right) = (1/14) \).

Thus,

\[
f(\psi) = \left( \frac{3}{14} \right) \cos^2 \psi \sum_{m,n} F(r_m) \, S(r_m, \theta_n, \psi)
\]

and

\[
f(\cos^2 \psi) = -\left( \frac{3}{14} \right) \cot \psi \sum_{m,n} F(r_m) \, S(r_m, \theta_n, \psi)
\]
Figure 15. Example of construction used for calculation of $S(\psi)$. 
The latter function was evaluated and plotted. The FWHM of this function is $\Delta(\cos^2\psi)$. It was evaluated for three cases to find the influence of the radial function $F(r_m)$. The results are shown in Table 6.

**TABLE 6**

**INFLUENCE OF $F(r_m)$ ON $\Delta(\cos^2\psi)$**

<table>
<thead>
<tr>
<th>Case</th>
<th>$F(r_1)$</th>
<th>$F(r_2)$</th>
<th>$F(r_3)$</th>
<th>$\Delta(\cos^2\psi)$</th>
<th>$\frac{\Delta(\cos^2\psi)}{\cos^2\psi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
<td>0.108</td>
<td>0.130</td>
</tr>
<tr>
<td>II</td>
<td>0.500</td>
<td>0.333</td>
<td>0.167</td>
<td>0.100</td>
<td>0.120</td>
</tr>
<tr>
<td>III</td>
<td>1.000</td>
<td>0</td>
<td>0</td>
<td>0.083</td>
<td>0.100</td>
</tr>
</tbody>
</table>

The function, $f(\cos^2\psi)$ is shown in Figure 16 as calculated for case II. Due to the cylindrical collimator throat, the $F(r_m)$ used for this case is expected to be a good estimate of the actual $F(r_m)$. The results shown in the table indicate that the uncertainty in $\Delta(\cos^2\psi)$, introduced by the uncertainty in $F(r_m)$, is less than 10%. The average values of $\cos\psi$, $\cos^2\psi$, and $\sec\psi$ calculated using $f(\psi)d\psi$ are relatively insensitive to the choice of $F(r_m)$.

4. **System Resolution**

The detector, radiator, and geometry terms are combined to get the system resolution as follows.

$$\frac{\Delta E}{E} = \left[ \left( \frac{0.1\,RD}{E} \right)^2 + \left( \frac{R_{41} - SEP}{E} \right)^2 + \left( 0.1\,RO \right)^2 \right]^\frac{1}{2}$$

(11)
Figure 16. Calculated distribution function: \( f(\cos^2\psi) \).
The resolution (i.e., FWHM in per cent) is shown as a function of neutron energy in Figure 17 for the two radiator thicknesses used. Due to the better resolution at low energies, the thin radiator is used with the thin detector for neutron energies from 2 to 6 Mev. The thick radiator and thick detector are used together for neutron energies from 4 to 12 Mev.

5. Influence on Measurement

The effect of the spectrometer resolution on measured neutron spectra depends on the FWHM and shape of the resolution function and on the shape of the neutron spectra. Since the fission spectrum decreases approximately exponentially at high energies, it is of interest to know how the resolution influences the measurement of an exponential spectrum. To estimate this effect, the resolution function is assumed to be Gaussian. Thus the measured spectrum,

\[ \phi_m(E) = \int_{-\infty}^{\infty} \phi(E')P(E', E)dE' \]

where

\[ \phi(E') = \phi_0 e^{-\alpha E'} \]

\[ P(E', E) = \frac{1}{\sqrt{2\pi} \sigma(E)} e^{-\frac{(E'-E)^2}{2\sigma^2(E)}} \]

Using \( R = \Delta E/E \) and \( \sigma(E) = \Delta E/2.355 = RE/2.355 \), evaluation of the integral gives

\[ \frac{\phi_m(E)}{\phi(E)} = e^{(\frac{R\sigma(E)}{2.35})^2} \]  \hspace{1cm} (12)

For \( \alpha = (1.290 \text{ Mev})^{-1} = 0.7752 \text{ Mev}^{-1} \) and \( R = 0.12 \), \( \phi_m(E)/\phi(E) = 1.03 \) for \( E = 6 \text{ Mev} \) and \( 1.12 \) for \( E = 12 \text{ Mev} \).
Figure 17. Calculated energy resolution: $\Delta E/E$. 

2.95 mg/cm$^2$ CH$_2$ RADIATOR

1.25 mg/cm$^2$ CH$_2$ RADIATOR

FWHM (PER CENT)
The slope of the measured spectra is

$$\frac{Q'_m(E)}{Q_m(E)} = -\alpha + \left(\frac{KR}{R,355}\right)^2 E$$  \hspace{1cm} (13)

Using the same \(\alpha\) and \(R\),

$$\frac{Q'_m(E)}{Q_m(E)} = -0.765\text{ MeV}^{-1} = -\left(1.301\text{ MeV}\right)^{-1} \quad \text{for } E = 6\text{ MeV}$$

$$= -0.75165\text{ MeV}^{-1} = -\left(1.322\text{ MeV}\right)^{-1} \quad \text{for } E = 12\text{ MeV}$$

For comparison, the slope of the actual spectrum is

$$\frac{Q'(E)}{Q(E)} = -\alpha = -0.775\text{ MeV}^{-1} = -\left(1.380\text{ MeV}\right)^{-1} \quad \text{for } E > 0$$

The above ratios show the spectrometer resolution causes the measured flux to be greater than the actual flux at a given energy, and the slope of the measured spectrum to be less than that of the actual spectrum.

The influence of a change or uncertainty in the resolution on the measured slope is given by the following relation.

$$\frac{\Delta (Q'_m/Q_m)}{(Q'_m/Q_m)} = -\left[\frac{R}{(5.55/k\times R^2) - 1}\right] \frac{\Delta R}{R}$$  \hspace{1cm} (14)

Using the same \(\alpha\) and \(R\) again,

$$\frac{\Delta (Q'_m/Q_m)}{(Q'_m/Q_m)} = -0.024 \left(\frac{\Delta R}{R}\right) \quad \text{for } E = 6\text{ MeV}$$

$$= -0.039 \left(\frac{\Delta R}{R}\right) \quad \text{for } E = 12\text{ MeV}$$

Thus, a 10% change or uncertainty in the resolution results in a change or uncertainty in the measured slope in the opposite sense by less than 0.5%. 
G. ENERGY RANGE

As has been indicated, the energy range of the spectrometer is from 2 to 12 Mev. The lower limit is determined by two factors. First, the signal-to-background ratio, which increases with decreasing energy, drops below one at about 2 Mev for most of the measured spectra. Secondly, the resolution becomes poorer at lower energies. The upper limit is also determined by two factors. The low efficiency and the few neutrons with energies above 10 Mev result in poor statistics at these energies unless very long counting times are used. Also, the detector must be thicker than the range of the highest energy protons of interest.

The above considerations indicate the complex nature of the spectrometer energy range. It depends on the thickness and geometry of the radiators and detectors, and on the intensity and energy dependence of both the source and the background.
CHAPTER III
EXPERIMENTAL RESULTS

A. FORD NUCLEAR REACTOR

The spectrum measurements in water and graphite were made in the Ford Nuclear Reactor (FNR) pool using core leakage neutrons as the fast neutron source.

1. Reactor Description

   The FNR is an open pool type research reactor operated at power levels up to 2 megawatts at The University of Michigan. It is very similar to the Bulk Shielding Reactor I (BSR-I) at Oak Ridge National Laboratory. The volume fractions of the standard fuel element are 0.2% uranium (95% U²³⁵), 41.6% aluminum, and 58.2% water. The H₂O serves as moderator, coolant, reflector, and shield. The reflector system also includes a tank of D₂O on the north core face and graphite reflector elements on other faces. Different core configurations were used for the water and graphite measurements. The two configurations are shown in Figure 18.

2. Reactor Power Measurement

   The measured spectra were normalized to reactor power. Reactor power was measured by stopping the secondary coolant flow while the reactor was operating at a steady state power level and observing the rate at which the pool water temperature increased as the primary coolant water returned to the pool. The average reactor power,

\[ P = m c_p \frac{\Delta T}{\Delta t} \]

where \( \Delta T/\Delta t \) is the rate of the pool water temperature increase, \( m \) is the mass of the water in the pool and the primary cooling system, and \( c_p \) is the
FUEL ELEMENT (FUEL HEIGHT: 60 cm)
REFLECTOR ELEMENT (GRAPHITE)
FUEL ELEMENT WITH CENTRAL CHANNEL FOR SAMPLES
FUEL ELEMENT WITH CONTROL ROD IN CENTRAL CHANNEL
TUBE FOR SAMPLES
TUBE FOR DETECTOR DURING REACTOR STARTUP

Figure 18. Ford Nuclear Reactor core configurations.
specific heat of water. For the FNR, \((1/mc_p) = 8.65^\circ\text{F/megawatt hour}\). This method assumes (1) nonfission heat sources are negligible, (2) heat losses are negligible, and (3) the pool water is mixed thoroughly. These assumptions are apparently valid and the method is quite reliable and accurate \((\pm 5\%)\). (101)

B. WATER MEASUREMENTS

As indicated in Figure 18, the spectrum measurements were made on the south side of the core. The spectrometer was submerged in the reactor pool and the collimator axis was aligned with the north-south core centerline. A photograph of the spectrometer suspended above the pool is shown in Figure 19. Figure 20 shows the submerged collimator 60 cm from the south core face. The demineralized pool water ranged from 35°C to 43°C during the measurements.

1. Normalization

In order to reduce the influence of flux shifts within the core, due for instance to control rod withdrawal during xenon buildup, the spectrum measurements were normalized directly to the fast neutron leakage flux from the south core face. The spectra were then normalized to reactor power by relating the leakage flux to a single calorimetric measurement of reactor power for the same core configuration used during the spectrum measurements.

The fast neutron leakage flux was monitored by activating aluminum wires and counting the \(\text{Al}^{27} (n,\alpha) \text{Na}^{24}\) reaction* product gammas (1.37 and 2.63 MeV). (26) This reaction was used because its threshold is in the MeV range \((3.26 \text{ MeV})\) and the half life of its product is well known \((\pm 0.13\%)\) and much longer than the half lives of other reaction products present. The other reactions are the \(\text{Al}^{27} (n,p) \text{Mg}^{27}\) reaction with threshold at 1.89 MeV and the \(\text{Al}^{27}(n,\gamma) \text{Al}^{28}\) reaction. The half life of \(\text{Na}^{24}\) is 14.97 hours whereas the half lives of \(\text{Mg}^{27}\) and \(\text{Al}^{28}\) are 9.4 minutes and 2.3 minutes, respectively. Hence, the activated aluminum wires can be allowed to decay until the only significant reaction product remaining is \(\text{Na}^{24}\). The effect of \(\text{Na}^{23} (n,\gamma) \text{Na}^{24}\) reactions which result from thermal neutron capture by sodium impurities in the aluminum wires was minimized by using high purity aluminum and by covering the aluminum with 10 mil cadmium.
Figure 20. Collimator positioned 60 cm from FNIR core.
2.75 Mev) with a 3" x 3" NaI (Tl) crystal. Four wires were positioned 30 cm from the core for each activation. The wires were mounted at 90° intervals on the circumference of a ring (30-cm diameter) that was centered on the north-south core centerline. With the wires in these positions, it was found that when the collimator was against the core, the aluminum activation was approximately 33% higher than it was without the collimator present. This undesirable perturbation was negligible at the other collimator positions, but did introduce an estimated ±5% uncertainty in the normalized amplitude of the zero spectrum measurement.

2. Reactor Power

The FNR was operated at 2 megawatts for all of the measurements except those with the collimator against the core. The latter measurements were made at 0.2 and 0.5 megawatt using the thick detector and thin detector, respectively. The lower power levels were used to limit the counting rates and thus reduce pile-up distortion. This distortion was monitored by observing the width of the Am\textsuperscript{241} alpha peak. Negligible broadening ($<10\%$) was observed at the power levels used.

3. Counting Times and Statistics

The water measurements were made during a four-week period. The initial measurements were made with the collimator against the reactor core. For the measurements from 2 to 6 Mev made using the thin detector and thin radiator, three 20-minute foreground counts and three 20-minute background counts were taken in alternating sequence. After the zero spectrum was measured, measurements were made with the collimator 20, 40, and finally 60 cm from the reactor core. The measurement from 4 to 12 Mev for the 60-cm spectrum, made using the thick detector and thick radiator, required the longest counting time. Here six 10-hour foreground counts and six 5-hour background counts were taken in alternating sequence.

Due to the few number of neutrons at the higher energies, the statis-
tical uncertainties were largest at these energies. At 10 Mev the standard
deviation (sigma) of a single point varied from 3.2% to 6.0%, and at 12 Mev
it ranged from 6.5% to 12.9%.

4. Measured Spectra

A machine plot of the measured spectra of FNR leakage neutrons which
penetrated 0, 20.0, 40.0, and 60.0 cm of pool water is shown in Figure 21.
Each measured point was obtained by subtracting the normalized background
counting rate (without radiator) from the normalized foreground counting
rate (with radiator). Then, as indicated by Equation (8), the resulting
counts per second per Mev of proton energy per megawatt of reactor power
were divided by the total efficiency in cm² steradian and by the energy
Jacobian in Mev of neutron energy per Mev of proton energy. The resulting
angular flux is expressed in neutrons per second per cm² per Mev (of neu-
tron energy) per steradian (directed normally to the south core face) per
megawatt (of reactor power).

The semilog plot of Figure 21 is characterized by three principal fea-
tures. First, the FNR leakage spectrum falls off exponentially above 5 Mev
as the fission spectrum does. Secondly, the attenuation of lower energy
neutrons is much greater than that of the higher energy neutrons due pri-
marily to the higher elastic scattering cross section of hydrogen at the
lower energies. Thirdly, the progressively deepening dip in the spectra
at about 3.5 Mev is due to the elastic scattering resonance of oxygen at
that energy. The scatter of the data at the higher energies is due to
Figure 21. Neutron penetration in water.
counting statistics. The calculated spectra shown in Figure 21 are discussed in Chapter IV.

5. Experimental Uncertainties

The principal experimental uncertainties are summarized in Table 7. The uncertainties associated with the background were limited by restricting the reported data to that with signal-to-background ratios above one. Even so, with the collimator effects included, these uncertainties may be as high as $\pm 10\%$ at 2 Mev.

The influence of the spectrometer resolution was to convolute the actual spectra with the spectrometer response function, i.e., the measured spectra are averaged over the energy width of the response function. The averaging process results in loss of spectral detail and, as shown in Chapter II for an exponential spectrum like the fission spectrum above 5 Mev, the slope of the spectrum may be modified. The importance of the latter effect in determining the slope of the fission source spectrum is discussed in Chapter IV.

Excluding these resolution effects, the total relative and total absolute uncertainties in the amplitudes of the measured spectra are typically $\pm 10\%$ and $\pm 20\%$, respectively. Counting statistics increase these percentages somewhat at neutron energies above 10 Mev, and background uncertainties make them slightly higher at neutron energies near 2 Mev.

The $\pm 2\%$ uncertainty in the spectrometer energy calibration is due mainly to the uncertainties in the mean squared cosine of the n-p scatter-
TABLE 7

PRINCIPAL EXPERIMENTAL UNCERTAINTIES

A. Proton Recoil Spectra Measurement

Detector Energy Calibration: $\pm 1\%$

*Background: Avg. $\pm 3\%$, Max. $\pm 10\%$ at 2 Mev

*Counting Statistics: Avg. $\pm 3\%$, Max. $\pm 13\%$ at 12 Mev for water
   Max. $\pm 23\%$ at 12 Mev for graphite

B. Conversion from Proton Spectra to Neutron Spectra

Spectrometer Energy Calibration (includes detector): $\pm 2\%$

Efficiency: $\pm 5\%$

C. Normalization

Reactor Power Measurement: $\pm 5\%$

*Aluminum Activation: $\pm 2\%$ except $\pm 5\%$ for 0 cm water

D. Shield Geometry

*Thickness: $\pm 1.5\%$, $\pm 0.75\%$, $\pm 0.5\%$ for 20, 40, 60 cm water
   $\pm 0.5\%$ for graphite

*Density: $\pm 0.2\%$ for water
   $\pm 2\%$ for graphite

E. Spectrometer Resolution

*Denotes uncertainties associated with relative measurements.
ing angle \((\cos^2 \psi = 0.83 \pm 0.01)\) and the mean energy required for protons to generate an electron-hole pair in silicon \((e_p = e_\alpha + 1\%)\).

C. GRAPHITE MEASUREMENTS

The graphite measurements were made with graphite slabs mounted against the south core face.

1. Description of Graphite Slabs

The graphite was high density, fine grained, HLM grade (Great Lakes Carbon Corp.). Its measured density* was \(1.74 \pm 0.04\) grams/cm\(^3\). Each slab was 20.2 cm x 62.5 cm x 62.5 cm. These dimensions include the 0.635-cm thick 6061 aluminum plate which completely encased each slab to keep it dry while submerged in the FNR pool. Two views of four of the slabs fabricated for this experiment are shown in Figure 22. Figure 23 shows three of the slabs positioned against the south core face during measurements at 2 megawatts. The collimator axis was aligned with the north-south core centerline as for the water measurements. The graphite slabs were suspended from above the reactor pool. The tube attached to the top side of each slab was provided for off-gassing.**

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*The supplier specified an average density of 1.72 grams/cm\(^3\) for this grade

**Little off-gassing was observed except from the slab nearest the core for a short period after the reactor reached full power. This was probably due to thermal expansion of air within the graphite as gamma heating increased the temperature of the graphite.
Figure 22. Two views of four graphite slabs.
Figure 23. Three graphite slabs positioned against PWR core.
2. Reactor Core Modification

Unfortunately the reactivity of the graphite slabs positioned directly against the fuel elements on the south core face exceeded the maximum reactivity allowed an experiment by the USAEC license for the FNR. Thus, in order to decouple the slabs and core, a row of graphite reflector elements was added on the south core face. The resulting core configuration is shown in Figure 18. It should be noted that this configuration introduces a 2-cm water gap between the graphite reflector elements and the zero collimator position. Hence, the source spectrum for the graphite measurements differs from that of the water measurements in that the former includes the effect of an additional 10-cm penetration through the row of graphite reflector elements and the water gap. The volume fractions for the graphite reflector elements are: 85.1% graphite (1.65 grams/cm$^3$), 7.9% aluminum, and 7.0% water. In order to keep the source spectrum constant, the water gap was maintained during all graphite measurements.

3. Normalization and Reactor Power

These measurements were normalized to reactor power as in the water measurements except the aluminum wires were located differently during activation. They were positioned on a 36-cm diameter circle within the water gap. No perturbation in the wire activation due to the collimator position was observed.

Except for the measurements of the source spectrum, the reactor was operated at 2 megawatts. The source spectrum was measured at power levels
of 0.5 and 1.0 megawatt using the thick detector and thin detector, respectively.

4. Counting Times and Statistics

The graphite measurements were made during a three-week period. The counting times varied from three 20-minute foreground counts and three 10-minute background counts for the thin detector-thin radiator case with 0 slabs of graphite to five 10-hour foreground counts and five 5-hour background counts for the thick detector-thick radiator case with 3 slabs of graphite. As for the water measurements, the foreground and background counts were alternated.

At the higher neutron energies, the counting statistics were slightly poorer than those of the water data. At 10 Mev, one sigma ranged from 4.1% to 7.7%, and at 12 Mev it varied from 9.5% to 23.3%.

5. Measured Spectra and Experimental Uncertainties

A machine plot of the measured spectra of FNR leakage neutrons which penetrated 0, 1, 2, and 3 slabs of graphite is shown in Figure 24. The structure is the energy averaged effect of the elastic scattering cross section of carbon. Again, the scatter of data points at the higher energies is due to statistical fluctuations. The errors are the same as those of the water measurements except for those associated with counting statistics, aluminum activation, shield thickness and shield density. These are noted in Table 7.

The measured spectra in both water and graphite are tabulated in Appendix C.
Figure 24. Neutron penetration in graphite.
CHAPTER IV

COMPARISON WITH CALCULATIONS AND CONCLUSIONS

A. NIOBE CALCULATIONS

The calculated spectra shown in Figures 21 and 24 were obtained with
the NIOBE (Numerical Integration of Boltzmann Equation) code.* This code
was developed by Nuclear Development Corp. (now United Nuclear Corp.) be-
beginning with the work of Certeine and Mittelman\textsuperscript{(104)} in 1955 and culminat-
ing in codes for the IBM 704 in 1960\textsuperscript{(12)} and for the IBM 7090 in 1961\textsuperscript{(13)}.
NIOBE was selected because of its ability to calculate angular fluxes at
deep penetrations through multilayered configurations, its capacity for
many energy groups, and its exactness. It has two important restrictions:
only spherical configurations are allowed, and any inelastic scattering
cross sections used are treated as isotropic in the laboratory system.

1. Neutron Transport Equation

NIOBE solves the following steady state, multienergy, neutron trans-
port equation for a finite, multilayered, spherical configuration.

\[ \hat{\Omega} \cdot \nabla F(r,\mu,E) + \sum_t (r,E)F(r,\mu,E) = S(r,\mu,E) + I_{e1}(F;r,\mu,E) + I_{in}(F;r,\mu,E) \]

Where:

\[ F(r,\mu,E) \] is the angular flux, i.e., the number of neutrons with energy
E, per second, per unit energy, per steradian, which move in directions

*All NIOBE calculations were performed by G. G. Sherwood\textsuperscript{(103)} using the IBM-7094 at the Institute for Defense Analysis, Washington, D.C.
making an angle whose cosine is \( \mu \) with the radius vector crossing unit area normal to the neutron direction, at distance \( r \) measured from the origin.

\( S(r,\mu,E) \) is the number of neutrons produced by true sources per second, with energy \( E \), which move in directions making an angle whose cosine is \( \mu \) with the radius vector, per unit volume, per unit energy, per steradian, at distance \( r \).

\( I_{el}(F;r,\mu,E) \) is the number of neutrons produced per second, with energy \( E \), which move in directions making an angle whose cosine is \( \mu \) with the radius vector, per unit volume, per unit energy, per steradian, as a result of elastic scattering by all nuclei present at distance \( r \), when the angular flux is \( F(r,\mu,E) \).

\( I_{in}(F;r,\mu,E) \) is defined similarly for inelastic scattering.

\[
\mathbf{n} \cdot \mathbf{v} = \mu \frac{\partial}{\partial r} + \left( \frac{1-\mu^2}{r} \right) \frac{\partial}{\partial \mu} ; \\
\sum_t(r,E) = \sum_i N_i(r) \sigma_{ti}(E) ; \\
N_i(r) \text{ is the number of atoms per unit volume of isotope } i \text{ at distance } r. \\
\sigma_{ti}(E) \text{ is the total cross section per atom of isotope } i \text{ at energy } E.
\]

The required boundary condition is that the flux at the outer boundary of the configuration be specified. (i.e., that \( F(A,\mu,E) \) be given for \( \mu < 0 \) where \( A \) is the outer radius).
In order to reduce this equation to terms suitable for numerical integration, it is assumed that the angular dependence of \( F(r, \mu, E) \) can be represented by a finite Legendre series expansion. Accordingly,

\[
F(r, \mu, E) = \sum_{l=0}^{L} \frac{2l+1}{4\pi} F_l(r, E) P_l(\mu) \quad L \leq 12
\]

Since Legendre polynomials satisfy the orthogonality relation

\[
\frac{2l+1}{2} \int_{-1}^{1} P_l(\mu) P_m(\mu) d\mu = \delta_{lm} = \begin{cases} 
0, & l \neq m \\
1, & l = m 
\end{cases}
\]

the scalar flux terms are given by

\[
F_l(r, E) = 2\pi \int_{-1}^{1} F(r, \mu, E) P_l(\mu) d\mu
\]

The Gaussian quadrature rule is used to evaluate the above integral. This scheme requires the selection of a discrete set of \( \mu_q \), which are the zeros of the Legendre polynomial of degree \( 2Q \), where \( Q \) is an integer. This set of \( \mu_q \) are the rays of the angular mesh. The selection of discrete angular rays so as to facilitate integration is generally referred to as the discrete ordinate method.

2. Source Terms

The source terms represent the total neutron gains per second per unit volume per unit energy per steradian due to true sources such as fission neutrons and slowing down sources consisting of elastically and inelastically scattered neutrons.

The true source in the reactor core region is the fission source.
It was assumed to be separable; i.e. $S(r, \mu, E) = W(\mu)V(r)g(E)$. Also, the angular distribution was assumed to be isotropic in the laboratory system. Thus, $W(\mu) = 1/4\pi$. The radial distribution, $V(r)$, was approximated by the measured radial thermal flux distribution\(^{36}\) for the ORNL BSR-I. Subsequent NIOBE calculations were made to test the sensitivity of the spectra to this $V(r)$. A flat radial distribution produced no significant change in the core leakage spectrum. Hence, it was concluded the form of $V(r)$ used was satisfactory.

The source energy distribution was represented by a normalized Maxwellian distribution, $g(E) = (2\sqrt{\pi \beta T^3})E^{1/2}e^{-E/T}$. The Maxwellian spectrum was chosen because it agrees with measured $U^{235}$ fission spectra as well or better than Watt\(^{105}\) or Cranberg\(^{106}\) spectra\(^{107-110}\), its analytical form is simple, and it requires only one parameter, $T$. NIOBE calculations were carried out for several values of $T$ to obtain a best fit to the FNR leakage spectrum (0 cm water). The calculated spectra were sufficiently sensitive to the choice of $T$ that it could apparently be inferred to within $\pm 0.010$ Mev. The best fit was for $T=1.285$ Mev. This compares well with $T=1.30\pm 0.03$ Mev given by Terrell\(^{108}\) and the average $T=1.30$ Mev for the eight measurements reviewed by Condé and During\(^{111}\). All of these $T$ values refer to $U^{235}$ fission induced by thermal neutrons.

For the region outside the core, both the internal source distribution and the incoming flux at the outer radius were specified to be zero.

The elastic scattering source term, $I_{el}(F; r, \mu, E)$, is evaluated by representing the angular dependence of the elastic scattering cross sec-
tion in the center-of-mass system by a finite Legendre series expansion. Then the scattering integral is evaluated using a scattering model appropriate to the mass of the scattering nuclei. NIOBE includes models for hydrogeneous, infinitely heavy, and intermediate weight scatterers. The scattering model for hydrogen treats the scattering as isotropic in the center-of-mass system.

The inelastic scattering source term, \( I_{in}(F;r,\mu,E) \), is evaluated by treating the scattering as isotropic in the laboratory system. When inelastic scattering data is available, the energy distribution of the neutrons emitted per inelastic scattering at energy \( E \), \( G(E,E') \), is tabulated as a function of \( E \) and \( E' \).

In summary, the energy dependent cross section data required for each element consists of (1) the total cross section, (2) the elastic scattering cross section and Legendre coefficients, and (3) the inelastic scattering cross section and the energy distribution of inelastically scattered neutrons.

3. Energy Groups

In addition to the use of an angular mesh and a radial mesh, an energy mesh, which divides the energy region of interest into equal intervals of \( \ln E \), is used. The introduction of the energy mesh allows the transport equation to be reduced to a set of coupled equations in which the slowing down and transport terms can be treated separately. These equations are solved in two steps. First, using assumed solutions, the slow-
ing down source is calculated for each energy group. The true source is then added to the slowing down source to get the total source for each energy group. The second step is to solve the resulting set of "one velocity" transport equations which have these source terms. The solutions are then compared with the initially assumed solutions and the process is repeated until two successive iterations agree within a specified tolerance.*

Examination of the cross sections of interest showed that 4% energy intervals retained most of the differential energy detail. More importantly, 4% energy widths clearly retain more structure than is visible within the limits of the spectrometer resolution (≥ 12%). Use of 4% energy intervals required 76 energy groups for the energy range from 0.90 to 18.02 Mev. Sherwood's 76 group P7 NIOBE calculations (16 angular rays and 105 radial points) required about 1 hour on the IBM-7094 computer.

4. Geometry

For the water calculations, the FNR core was represented as a sphere of approximately equal volume (29 cm radius, 58% H₂O and 42% aluminum) and the pool water was represented as a spherical shell (80 cm thick, 100% H₂O) about this central region.

The same central region was used for the graphite calculations. Here it was surrounded by two regions: (1) a 10 cm thick shell representing the row of graphite reflector elements and the water gap (volume

---

*The convergence criterion used was 0.001.
fractions: 68.9% graphite (1.65 grams/cm$^3$), 24.7% water, and 6.4% aluminum) and (2) an 80 cm thick graphite shell (volume fractions: 93.7% graphite (1.74 grams/cm$^3$) and 6.3% aluminum).

5. Cross Sections

Cross section data was required for hydrogen, carbon, oxygen, and aluminum. For hydrogen, the total and elastic scattering cross sections are the same. The scattering model for hydrogen which treats elastic scattering as isotropic in the center-of-mass system and the theoretical total cross section curve reported in BNL-325$^{(112)}$ were used. KAPL total and elastic scattering cross sections and Legendre coefficients were used for carbon$^{(113)}$ and oxygen.$^{(114)}$ Since this data extended only up to 15 Mev, the values at 15 Mev were assumed constant to 18 Mev. Due to the unavailability of inelastic scattering data for carbon and oxygen, it was omitted in the slowing down source terms. For aluminum, $GA^{(115)}$ cross section data, including inelastic, was used.

6. Comparison With Measurements

Because the use of 76 energy groups permitted the calculations to show considerably more energy detail than the measurements, comparisons between NIOBE calculations and measured spectra were difficult to evaluate. In order to simplify the comparison, the energy dependent spectrometer response function was convoluted with the NIOBE calculations. The response function was assumed to be a Gaussian distribution function with its FWHM
given as a function of neutron energy by the curves shown in Figure 17.* The resulting spectra were normalized with the measured spectra at a single point (7.5 Mev at 0 cm) and machine plotted with them as shown in Figures 21 and 24. A quantitative comparison is given in Table 6 which shows the average and maximum ratios of the calculated flux to the measured flux.

**TABLE 6**

<table>
<thead>
<tr>
<th>Shield Thickness</th>
<th>Average Ratio</th>
<th>Maximum Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 cm H₂O</td>
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<td>0.90 at 4.0 Mev</td>
</tr>
<tr>
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<td>0.92</td>
<td>0.36 at 3.0 Mev</td>
</tr>
<tr>
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<td>0.93</td>
<td>0.80 at 2.5 Mev</td>
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<tr>
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<tr>
<td>60.6 cm Graphite</td>
<td>0.98</td>
<td>0.67 at 4.9 Mev</td>
</tr>
</tbody>
</table>

The comparison shows surprisingly good agreement, particularly with the water data, in view of the assumptions made in the NIOBE calculations.

*A smooth line between the thin radiator curve at 4 Mev and the thick radiator curve at 6 Mev was used for the 4 to 6 Mev region.*
These include (1) all regions are homogeneous and spherically symmetrical, (2) there is no inelastic scattering by carbon and oxygen in the slowing down source terms, (3) hydrogen scattering is isotropic in the center-of-mass system, (4) inelastic scattering by aluminum is isotropic in the laboratory system, and (5) carbon and oxygen cross sections and Legendre coefficients are constant from 15 Mev to 18 Mev. With regard to the graphite calculations, much better agreement was obtained by the questionable procedure of mixing cross sections and Legendre coefficients from different libraries.

In convoluting the spectrometer response function with the NIOBE calculations, an important bias is introduced. As demonstrated analytically in Chapter II, the convolution process introduces a tilt in exponential spectra. Thus, the inference of the Maxwellian T characteristic of the $^{235}$U fission spectrum from the comparison of the measured core leakage spectrum with smoothed spectra calculated for different values of T requires the response function to be sufficiently well known so as to introduce little or negligible uncertainty in finding T. The greatest uncertainty in the response function is that associated with the radial distribution function, $F(r_m)$. From Table 6, this uncertainty is estimated to be $\pm 10\%$. Equation 14 shows that a $\pm 10\%$ uncertainty in $\Delta E/E$ introduces $\leq 0.5\%$ uncertainty in the slope of smoothed spectra. Since the uncertainty in T associated with matching the measured and calculated core
leakage spectra is $\pm 0.010$ Mev, the total* uncertainty in $T$ is $\leq \pm [0.010$
Mev $+ 0.005 (1.285$ Mev$)]$ or $\leq \pm 0.016$ Mev. Most measurements\(^{108,111}\)
of this $T$ have reported uncertainties of $\pm 0.030$ or $\pm 0.040$ Mev.

B. CONCLUSIONS

A relatively simple, moderate resolution (12%), low efficiency ($10^{-6}$),

A relatively simple, moderate resolution (12%), low efficiency ($10^{-6}$), differential proton-recoil, fast neutron spectrometer has been developed
differential proton-recoil, fast neutron spectrometer has been developed
to measure directed neutron fluxes in the presence of high intensity fission gamma fields ($10^7$ to $10^8$ roentgens/hour). It has been used to measure the energy distributions (2 to 12 Mev) of normally directed neutrons
which leaked from the Ford Nuclear Reactor core and penetrated several
thicknesses of two common reactor materials, $H_2O$ and graphite.

In contrast to Clifford et al.'s\(^{49}\) recent spectrum measurements
of uncollided reactor neutrons, these measured spectra are dominated by scattered neutrons. For example, the ratio of scattered neutrons to unscattered neutrons ranges from 100 at 2 Mev to 1 at 12 Mev for neutrons which penetrated 60 cm of $H_2O$. Although measurements of uncollided neutrons test the total cross sections very well, these measurements test both the total cross sections and the slowing down sources. In contrast with other measurements dominated by scattered neutrons and compared with detailed calculations (such as Verbinski's\(^{36}\) spectra in $H_2O$ andProfio's\(^{33}\) spectra in $CH_2$), the agreement between these measured and

*This does not include effects of uncertainties in the energy calibration of the spectrometer or in the cross sections used in the NIOBE calculation of the core leakage spectrum.
calculated spectra is excellent.

In summary, because of the relatively high accuracy (±10% relative intensity and ±20% absolute intensity, ±2% energy) and the deep penetration (up to 60 cm), these measurements are the most useful that have been reported for testing detailed calculations of fast neutron spectra in bulk media.

C. SUGGESTED FUTURE WORK

Opportunities for extending the present work lie in two directions. One is in the choice of experiments and the other is in the modification and improvement of the spectrometer. However, except for modifications which give a general reduction in background (as for example might result from reducing the ID of the beam trap or reshaping the collimator throat), the important changes in the spectrometer system will probably reflect the choice of experiment. The following experiments are fairly direct extensions of the present work.

1. Spectrum measurements in H₂O at oblique angles.

2. Spectrum measurements at higher energies.¹

3. Spectrum measurements in other bulk materials (e.g. materials with relatively high and well known inelastic scattering cross sections such as iron).

4. Spectrum measurements of fission sources and (α,n) sources.

5. Measurement of angular distributions of n-p scattering using multiple detectors or a single movable detector in the detector chamber.
6. Measurements of spectrometer efficiency and resolution using \( D(d,n)\text{He}^3 \) and \( T(d,n)\text{He}^4 \) neutron sources of known intensity and energy width.

7. Collimator studies\(^{116-122} \) to measure the influence of (a) wall and liner material, (b) shape, and (c) length.
APPENDIX A

EFFECT OF CARBON RECOILS

Fast neutrons incident on CH$_2$ radiators scatter carbon ions as well as protons. Although many of the carbon ions are stopped within the radiator, those which do escape the radiator and reach the detector are indistinguishable from protons of the same energy. The necessary condition for the background counts due to carbon recoils to be negligible compared to the proton recoil counts is that

\[
\frac{C(E_C)}{C(E_P)} \ll 1 \quad \text{for} \quad (E_{\min} \cos \theta_N)^{-1} \leq E_P \leq (E_{\max} \cos \theta_N)^{-1}
\]

or

\[1.5 \text{ MeV} \leq E_P \leq 10.0 \text{ MeV}\]

where $C(E_P)$ is the count rate for protons with energy $E_P$, and $C(E_C)$ is the count rate for carbon ions with energy $E_C$.

To find the ratio of carbon recoil counts to proton recoil counts, first note that from Equations 2 and 6,

\[
C(E_P) = \varepsilon(E) \Phi(E) \frac{dE}{dE_P} = \frac{1}{4 \pi N_R^N t_P G \cos \theta N} n_p(E) \Phi(E) \frac{dE}{dE_P}
\]

(15)

where

\[G \equiv \left( \frac{A_n A_p}{L_n L_p} \right)\]

The expression for the total efficiency, $\varepsilon(E)$, was derived with the assumptions that n-p scattering is isotropic in the center-of-mass system and that all protons which are scattered toward the detector escape the radiator with some mean energy loss $\delta E_p$, where $\delta E_p \ll E_p$. Both of these assumptions are invalid for carbon ions because n-C scattering is not
isotropic in the center-of-mass system and the carbon ion energy losses in the radiator are so large that most of the ions do not escape the radiator with energies above 1.5 Mev.

The energy of the carbon ions before losing any energy is related to the energy of the incident neutrons by

$$E'_C = \frac{4A}{(A+1)^2} E' \cos^2 \psi \quad (16)$$

With $A = 12$ and $\cos^2 \psi = 0.83$, $E'_C = 0.236 E'$

An expression for $C(E'_C)$ which includes the carbon ion energy losses in the radiator is developed as follows

$$C(E'_C) dE'_C = \int_{E'_C}^{E''_C} C_0(E''_C) dE''_C P(E''_C, E'_C) dE'_C \quad (17)$$

where $C_0(E''_C) dE''_C$ is defined as $C(E'_C) dE'_C$ except that carbon ion energy losses in the radiator are assumed to be zero.

$P(E''_C, E'_C) dE'_C$ is the probability that carbon ions, which would be counted with energies in $dE''_C$ about $E''_C$ if energy losses in the radiator were zero, are counted with energies in $dE'_C$ about $E'_C$.

$E''_C = 0.236 E'_{\text{max}}$

From its definition,

$$C_0(E''_C) dE''_C = \epsilon(E') \phi(E') dE'$$

where $\epsilon(E')$ is the total efficiency for counting carbon ions with energy $E'_C = 0.236 E'$. It is evaluated as before for n-p scattering except that the carbon ions are assumed to be scattered isotropically in the laboratory system. This assumption requires that
\[
\frac{d\sigma_{n-c}(E, \Psi)}{d\Psi} = \frac{\sigma_{n-c}(E)}{4\pi}
\]

This is equivalent to replacing \((1/\pi) \sigma_{n-p}(E) \cos \Psi\) by \((1/4\pi) \sigma_{n-c}(E)\) in the previous expression for \(\epsilon(E)\) so that

\[
C_0(E'_c) dE'_c = \frac{1}{4\pi} N_R^C t_R G \sigma_{n-c}(E') \phi(E') dE'
\]

Thus,

\[
C(E_c) = \frac{1}{4\pi} N_R^C t_R G \frac{dE'}{dE_c} \int_{E'_c}^{E''_c} P(E'_c, E_c) \sigma_{n-c}(4\pi + E'_c) \phi(4\pi + E'_c) dE'_c
\]

Hence, the ratio of count rates is

\[
\frac{C(E_c)}{C(E_p)} = \frac{1}{4\pi} N_R^C t_R G \frac{dE'}{dE_p} \int_{E'_c}^{E''_c} P(E'_c, E_c) \sigma_{n-c}(4\pi + E'_c) \phi(4\pi + E'_c) dE'_c
\]

For a CH\(_2\) radiator, \((N_R^C/N_R^H) = 1/2\).

Here \(\cos \Psi = 0.91\), \((dE/dE_p) \approx (1/\cos^2 \Psi) = 1.20\), and \((dE'/dE'_p) = 4.24\).

Using these values, the ratio becomes

\[
\frac{C(E_c)}{C(E_p)} = 0.4364 \int_{E'_c}^{E''_c} P(E'_c, E_c) \sigma_{n-c}(4\pi + E'_c) \phi(4\pi + E'_c) dE'_c
\]

In order to evaluate the integral, \(\sigma_{n-c}(E)\) and \(\phi(E)\) must be specified.

For example, choose

\[
\phi(E) = \begin{cases} 
\phi(E) & \text{for } 0 < E < 4\pi + (E_c)_{\min} \\
0 & \text{for } 4\pi + (E_c)_{\min} \leq E \leq E_{\max} \\
\end{cases}
\]

where \((E_c)_{\min} = (E_p)_{\min} = (E_{\min} \cos^2 \Psi - 8E_p) = 1.5\) Mev and \(4.24 (E_c)_{\min} = 6.36\) Mev.

Since \(\sigma_{n-c}(E) = 1.5 \pm 0.8\) barns for \(6.36\) Mev \(< E < 20\) Mev, choose

\[
\sigma_{n-c}(E) = \sigma_{n-c} = 1.5\ \text{barns for } 6.36\ \text{Mev} \leq E \leq 20\ \text{Mev}
\]
For this example,

\[
\frac{C(E_c)}{C(E_p)} = 0.424 + \frac{0.7-E}{\gamma_{n-p}(E)} \int_{E_c}^{E''} P(E', E_c) dE' \]

To evaluate the integral, assume

\[ P(E_c, E_c) dE' = P(x) dx = \frac{dx}{t} \]

where

\[ x = R_{CH_2}^C(E_c') - R_{CH_2}^C(E_c) \quad \text{for} \quad 0 \leq x \leq [R_{CH_2}^C(E_c'') - R_{CH_2}^C(E_c)] \]

and

\[ t = R_{CH_2}^C(E_c'') \]

is the range of carbon ions with energy \( E_c \) in \( CH_2 \).

\[ t = t_R \sec \psi = 1.38 \text{ milligrams/cm}^2 \]

Thus,

\[ \int_{E_c}^{E''} P(E', E_c) dE' = \int_{0}^{R_{CH_2}^C(E_c'') - R_{CH_2}^C(E_c)} \frac{dx}{t} = \frac{R_{CH_2}^C(E_c'') - R_{CH_2}^C(E_c)}{t_R \sec \psi} \]

Using this result,

\[ \frac{C(E_c)}{C(E_p)} \bigg|_{E_c=E_p} = 0.525 \frac{R_{CH_2}^C(E_c'') - R_{CH_2}^C(E_c)}{\gamma_{n-p}(E)} \frac{\text{barns}}{\text{mg/cm}^2} \quad (20) \]

where \( E_c = E_p = E \cos^2 \psi - 5E_p \) and \( E_c'' = 0.236 E_{max} \).

A range-energy curve for carbon ions in \( CH_2 \) is not available. An approximate one can be obtained using the following relationship\(^{123}\).

\[ \frac{R_{CH_2}^C(E_c/12)}{R_{CH_2}^C(E_c/Mc)} = \frac{R_{CH_2}^H(E_c/12)}{R_{CH_2}^H(E_c/Mc)} \quad \text{for} \quad \left( \frac{E_c}{12} \right) = \left( \frac{E_c}{M_c} \right) \]

With \( M_H = 1 \text{ amu} \) and \( M_C = 12 \text{ amu} \),

\[ R_{CH_2}^C(E_c/12) = \frac{R_{CH_2}^H(E_H)}{R_{CH_2}^C(E_H)} \frac{R_{CH_2}^C(E_c/12)}{R_{CH_2}^C(E_c/12)} \quad \text{for} \quad \left( \frac{E_c}{12} \right) = E_H \]
where $E_H$ and $(E_C/12)$ are the energies of the hydrogen and carbon ions, respectively, expressed in MeV/amu.

A range-energy curve for protons in carbon for the proton energies of interest (0.1 to 0.5 Mev) is not available. Hence, the ratio of ranges of protons in $\text{CH}_2$ and carbon is estimated by assuming that both of the ranges can be represented by $R(E) = AE^B$ so that

$$\frac{R_{\text{CH}_2}^H(E_H)}{R_C^H(E_H)} = \frac{A_{\text{CH}_2}^H}{A_C^H} (E_H)^{B_{\text{CH}_2}^H - B_C^H}$$

The constants are evaluated using the ranges of protons in $\text{CH}_2$ and carbon at $E_H = 2$ Mev and $E_H = 4$ Mev as calculated by Barkas and Berger (81). Using this procedure, the ratio is found to vary from 0.70 to 0.75 as $E_H$ increases from 0.1 to 0.5 Mev.

Using these results and a range-energy curve (123) for carbon ions in carbon, an approximate range-energy curve for carbon ions in $\text{CH}_2$ is obtained. From this curve, which is shown in Figure 25, and the values for $Q_{n-p}(E)$ which are tabulated by Gammel (100), the ratio of count rates is calculated for $E_{\text{max}} = 14$ Mev and $E_{\text{max}} = 18$ Mev. The results are shown in Figure 26 as a function of the incident neutron energy.

It is noted that this example generally overestimates the background due to carbon recoils for the spectrum measurements reported here. Even so, as shown in Figure 26, the effect of carbon recoils is small.
Figure 25. Range of carbon ions in polyethylene.
Figure 26. Ratio of carbon recoil counts to proton recoil counts.
APPENDIX B
COLLIMATOR EFFECTS

To zeroth order approximation, the neutron beam which exits the collimator consists only of neutrons which entered the source end of the collimator and traveled to the exit end without being scattered or attenuated. Let these neutrons be designated \( F_E \). Three corrections must be considered. First, a fraction of the neutrons are absorbed or scattered by the gas in the collimator. For long, air-filled collimators this fraction can be significant. Here, it is negligible because the collimator is evacuated.

Secondly, some of the neutrons in the exiting beam entered the collimator through the source end, but were scattered in the collimator wall before reaching the exit. These neutrons are degraded in energy, although for long collimators the most probable scattering angles may be so small that the energy degradation is negligible. Even so, these scattered neutrons were not directed toward the exit when they entered the collimator so that the exiting neutrons have an additional component which is designated \( F_{EW} \).

Thirdly, some of the neutrons in the exiting beam entered the collimator initially through the wall rather than through the source end. Let this additional component be designated \( F_W \).

Both \( F_{EW} \) and \( F_W \) depend on the angular dependence of the source and on the geometry of the source and the collimator. There are no general
expressions available which can be used to evaluate these terms. However, they can be roughly estimated for long, cylindrical and long, conical collimators used with isotropic sources. Here "long" implies that the length of the collimator is much larger than its largest diameter.

Simon and Clifford(116) have estimated \( \frac{F_{EW}}{F_E} \) for the case where a point, isotropic source is located on the axis of a long, cylindrical collimator. Their calculation considers only single scattering in the collimator walls and depends on the assumptions that the scattering is isotropic in the laboratory system and that the energy degradation is negligible. Their result is

\[
\left( \frac{F_{EW}}{F_E} \right) \approx \frac{1}{2} \frac{r_s}{r_t} \left( \frac{\delta}{l} \right)
\]

(21)

where \( \delta \) is the radius of the cylinder

\( l \) is the length of the cylinder

\( \Sigma_s \) and \( \Sigma_t \) are the macroscopic scattering and total cross sections, respectively, for the wall of the cylinder.

If the collimator wall is hydrogenous, it is a good approximation to assume the scattering is isotropic in the center-of-mass system. This assumption can be incorporated into the above results by replacing \( \Sigma_s \) by \( 4 \Sigma_s \bar{\mu} \), where \( \bar{\mu} \) is the average cosine of the scattering angles associated with \( F_{EW} \). Since for long collimators, most of the neutrons which contribute to \( F_{EW} \) are scattered through relatively small angles, \( \bar{\mu} \) is approximately unity. Thus, for long, cylindrical collimators with hydrogenous walls,
\[
\left( \frac{F_W}{F_E} \right) = 2\pi \left( \frac{\Sigma_s}{\Sigma_t} \frac{\delta_s}{l} \right) = 2\pi \left( \frac{\lambda_t}{\lambda_s} \right) \left( \frac{\delta_t}{l} \right)
\]

(22)

where: \( \lambda_s = 1/\Sigma_s \) and \( \lambda_t = 1/\Sigma_t \)

Extension of the analysis of Simon and Clifford to long, conical collimators gives the same result except that

\[
\delta = \frac{1}{2} (\delta_s + \delta_e)
\]

where: \( \delta_s \) is the radius of the source end and \( \delta_e \) is the radius of the exit end

\( \left( \frac{F_W}{F_E} \right) \) can be crudely estimated for the case of a plane, isotropic source at the end of a long, cylindrical collimator. If only the uncollided neutrons are considered, it can be shown that

\[
\left( \frac{F_W}{F_E} \right) = 2 \left( \frac{\lambda_t}{l} \right)
\]

(23)

Extension of the same analysis to long, conical collimators gives

\[
\left( \frac{F_W}{F_E} \right) = 2 \left( \frac{\delta_e}{\delta_s} \right) \left( \frac{\lambda_t}{l} \right)
\]

(24)

From the above two expressions, it is noted that

(1) For long collimators, \( \left( \frac{F_W}{F_E} \right) \) depends on \( (\lambda_t/l) \), not \( (\delta/l) \);

(2) When using a long, conical collimator with a plane source, it is advantageous to use the large end of the collimator as the source end so that \( (\delta_e/\delta_s) < 1 \).

Also, it is noted that the above expressions for \( \left( \frac{F_W}{F_E} \right) \) do not include neutrons which enter the collimator through the wall and then
reenter the wall to be scattered to the collimator exit. This contribution is expected to be small compared to \( \frac{F_{EW}}{F_E} \), at least when \( \frac{\delta_e}{\delta_s} \leq 1 \).

\( \frac{F_{EW}}{F_E} \) and \( \frac{F_W}{F_E} \) can now be evaluated for the collimator used with this spectrometer. The cross section of the collimator is actually dodecagonal rather than circular, but the difference is neglected here. The wall of the collimator is 0.32 cm thick aluminum surrounded by water. Since the aluminum is very thin relative to the mean free path of MeV neutrons in aluminum, the collimator wall is represented by a water wall. Hence, these ratios are evaluated for a long, evacuated, conical collimator with water walls and the following dimensions: \( l = 305 \) cm, \( \delta_s = 5.10 \) cm, and \( \delta_e = 0.95 \) cm. For 10 MeV neutrons in water, \( \lambda_t = 9.6 \) cm and \( \lambda_s = 12.4 \) cm. Using these values,

\[
\left( \frac{F_{EW}}{F_E} \right) \approx \pi \left( \frac{\lambda_t}{\lambda_s} \right) \left( \frac{\delta_s + \delta_e}{l} \right) = 0.048 \tag{25}
\]

\[
\left( \frac{F_W}{F_E} \right) \approx 2 \left( \frac{\delta_e}{\delta_s} \right) \left( \frac{\lambda_t}{\lambda_s} \right) = 0.017 \tag{26}
\]

Thus, \( \frac{F_{EW}}{F_E} + \frac{F_W}{F_E} \approx 6.5\% \). This small percentage, although only a rough estimate, indicates the collimator effects are relatively small.
APPENDIX C

TABULATION OF MEASURED SPECTRA

The measured spectra are tabulated on the following four pages.

The neutron energy \(1.203 \times 10^2\) is read \(12.03\) Mev.

The flux \(1.9700 \times 10^9\) is read \(1.97 \times 10^8\) neutrons \(\text{sec}^{-1} \text{ cm}^{-2} \text{ Mev}^{-1}\) \(\text{steradian}^{-1} \text{ megawatt}^{-1}\). It was measured normal to the reactor core.

The phrase 0 CM WATER refers to the thickness (0 cm) of material (water) between the Ford Nuclear Reactor core and the source end of the spectrometer collimator.
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REFERENCES


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