NATIONAL ACCIDENT SAMPLING SYSTEM

TRAINING MATERIALS FOR
ACCIDENT RECONSTRUCTION

REFERENCE MANUAL
Volume I

First Edition

by

Kenneth L. Campbell
Peter Cooley
Robert L. Hess
Robert E. Scott
Christopher B. Winkler

Edited by
Kenneth L. Campbell
Thomas L. McDole

HIGHWAY SAFETY RESEARCH INSTITUTE
The University of Michigan
Ann Arbor, Michigan 48109

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16. Abstract

This Reference Manual is Volume I of the training materials for a course in Accident Reconstruction for the National Accident Sampling System. Volume II is a Student Notebook, and Volume III is an Instructor's Manual containing lesson outlines.

This manual attempts to provide a complete background for accident reconstruction in general and in the use of the CRASH2 computer program in particular. Background information on scene and vehicle examination, and basic physics and dynamics are presented prior to the material on the CRASH2 program.

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PREFACE

This Reference Manual is Volume I of the training materials developed by the staff of The University of Michigan Highway Safety Research Institute for investigators in the National Highway Traffic Safety Administration (NHTSA) National Accident Sampling System (NASS). Volume II is a Student Notebook and Volume III contains lesson outlines.

The course was conducted for senior NASS investigators in May 1979. A course evaluation was reported separately (Report No. UM-HSRI-79-81).

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1 SCENE EXAMINATION

Summary. The accident scene consists of the physical real estate or area over which the crash occurred—the roadway as well as the off-road environment relevant to the crash. The scene is best described as that portion of the roadway and off-road environment which begins when and where the vehicle enters the pre-crash phase and ends at the final rest positions of the vehicles. Included in the area are the vehicles, debris, fluid, deposits, human bodies, human matter, interior materials from vehicles, components from the vehicles, vehicle cargo, other vehicles and/or traffic as well as those portions of the roadway (signals, signs, markings, traffic control devices) involved in or relevant to the crash. The traffic control devices to be included are those which influence the traffic behavior or vehicle movement prior to impact. Thus, the scene oftentimes will involve considerable distances over which the crash vehicles moved both prior to and following impact.

The accident scene is one of the three major areas to be examined in an accident. The remaining two areas are occupant and vehicle. These three areas provide convenient categories into which the investigator may address his attention, the collection of data, and the interpretation of evidence. These basic categories have ample precedent in previous in-depth accident investigation studies conducted under contract to the National Highway Traffic Safety Administration. Human factors, vehicle factors, and environmental factors combined with the three phases of pre-crash, at-crash, and post-crash periods of the accident make up the well known nine cell matrix (see Figure 1.1) with which in-depth accident investigations have been conducted for many years.

Two of the factors, scene and vehicle, are the topics of this and a subsequent chapter. Each chapter will discuss each of the events in detail as it relates to the gathering of data relevant to the accident reconstruction process.

When conducting accident investigations sometime after the accident has occurred (not on-scene) the investigator is somewhat limited in what can be observed relevant to the accident scene as it was to the time of collision. In NASS, for example, most accident investigations are conducted twenty-four hours (or later) after the accident event. This means that the investigator must approach an old accident scene (which

CRASH PHASES

	PRE-CRASH	CRASH	POST-CRASH
HUMAN	1	2	3
VEHICLE	4	5	6
ENVIRONMENT	7	8	9

FACTOR CATEGORIES

Figure 1.1 Accident Investigation Matrix

oftentimes is marginally identified by the police accident report) and attempt to discern from examination the basic characteristics of the roadway and off-road environment at the time of the crash, and any crash evidence still remaining on and off the roadway in the crash area. The first problem is locating the boundaries of the exact crash area. Police accident reports will note key intersecting roadways, as well as the roadway upon which the accident occurred. They will also usually note the direction and distance (in tenths of one mile) from a known intersection of the roadway on which the crash occurred and the nearest intersecting roadway. The investigator should be aware that distances noted on the police report are often incorrect or lacking sufficient accuracy to enable one to accurately find the location of the crash on the roadway. Such distances noted on the police accident report are often merely readings from the patrol cruiser's odometer, and can mean the distance from the final rest position of the vehicle to the reference intersection, or the point of impact between vehicles to the

reference intersection. Many times the reference, or benchmark intersection, may be indeterminate. It could be defined as the nearest curb line of the intersection to the center of the intersecting roads. If a road is twenty feet or greater in width a sizable error may result. The most reliable and authentic information on which to base the location of the crash to be examined is tangible, solid, physical evidence of the crash related to the roadway. Examples of this type of evidence may be fractured roadside posts, gouges in the roadway, skids or scuffs on the roadway, wheel tracks off the roadway, and mounded earth where vehicles moved from impact. Fluid deposits, such as fuel, antifreeze, and oil, which are often on the roadway as a result of crash damage to the vehicle are particularly helpful. These fluid deposits oftentimes remain on the roadway for long periods of time after other physical evidence has been dispersed or moved. Most physical evidence left at the time of a crash is perishable. Even relatively deep gouges in the roadway can be filled in by wear and tear from normal traffic. It then becomes difficult for the investigator to determine whether the gouges observed resulted from the accident under examination. The perishable nature of scene evidence creates perhaps the most difficulty in conducting NASS and NASS-type accident investigations.

To confirm the crash location reported by the police there are two further sources of information available to the investigator: one is physical on-scene evidence which can clearly establish or confirm police reports. The other is information supplied by occupants of the crash vehicle(s), witnesses to the crash (when so noted on the police report), and nearby residents.

1.1 Scene Measurements

1.1.1 Measurement Benchmarks. Once the crash location has been established, the investigator must make measurements to accurately document the point of impact, trajectory of vehicles from impact to crash, objects struck, and physical features of the area. These basic data items are required as part of the crash scene documentation. In making such measurements the location of suitable benchmarks is of prime importance. Benchmarks must be easily identifiable and consist of permanent physical landmarks in the immediate area, convenient to the

investigator, and from which relevant measurements can be made in completing the scene data information. The benchmark must be permanent in the sense that it must remain in the accident area from the time the accident occurred to he time the scene investigation is concluded. Permanence is desired so as to permit one to reexamine the site, if necessary, at a later date and to repeat the same measurements taken at the time of the initial scene investigation. Permanent fixed objects make superior measurement benchmarks. Some examples of these may be utility poles, overhead luminaire standards, fire hydrants, guardrails, bridge parapets, and similar objects. Somewhat less suitable examples are manhole covers, pavement joints, catch basins, curb drain covers, trees, and sign posts. These are less suitable examples since roadways may be resurfaced (and pavement markings obscured), trees may be cut, signs may be relocated, etc. Also, culverts and similar off-road objects not directly part of the roadway make poor measurement benchmarks because they may be obscured by foliage, taken down, or covered up.

The permanence of the benchmark selected, as well as its proximity and convenience relative to the objects to be measured, is the first important consideration in examining and measuring an accident scene. The investigator should approach such measurements with the objective of developing documentation that is clear, concise, unambiguous, accurate, and sufficiently detailed so that someone unfamiliar with the accident event can duplicate the measurements with the same precision and from the same physical features.

Where no physical benchmarks exist which are permanent, nearby and convenient, one may create a reference point for measurements from the extension of curb lines between intersecting streets. Straight road edge, defined by the curb line of the street, may be visually extended as a straight line into the intersection to a point where it intersects with a similar straight line projection of an adjacent street. Another technique is to create an imaginary reference line by lining up two fixed objects and to use this line for measurement purposes. Measurements along a road edge which is straight can be made out to objects or evidence on the roadway at right angles from the road edge.

Of course the point along the road edge where the measurement into the roadway is made at right angles must be carefully located with respect to existing benchmarks. This is perhaps best illustrated in the following example. Curved skidmarks may be divided into shorter segments, and each segment rated for perpendicular distance to the straight road edge which serves as a reference line. Points along the reference line, at road edge, from which perpendicular distances out to curve segments were measured should be noted and then referenced to scene benchmarks along the road edge (see Figure 1.2).

When measuring and recording the location of important roadway or off-roadway evidence the measurements must be unambiguous. That is, a minimum of two measurements are required to identify an object or a point on the roadway. These measurements may be viewed as the loci of an arc struck from a measurement benchmark which when they intersect identify a point on the roadway. Of course, more than two measurements may be made so as to further enhance accuracy and reduce any potential for ambiguity.

Where large objects are involved, such as the entire vehicle, a human body, or large vehicle components, the investigator must identify that portion or part of the object to which the measurement is made. For example, a vehicle as seen in a plane view may be drawn as a rectangle which simulates the overhead silhouette of the vehicle on the roadway. All four corners of the vehicle on the roadway must be accurately located relative to the measurement being taken in order to draw it on an accurately scaled crash schematic.

The most convenient approach for the investigator to follow prior to starting the scene examination is to know beforehand what measurements are to be made and what benchmarks or reference lines or points associated with the roadway will be used to construct the crash schematic. Once at the crash scene and after a brief view of what is physically involved the investigator may, in the convenience and comfort of his or her vehicle, prepare a series of sketch outlines which include places for notation of all measurements desired. Items to be noted and measured could include objects, marks, structures, and similar features to be measured in relation to roadway features and benchmarks used and
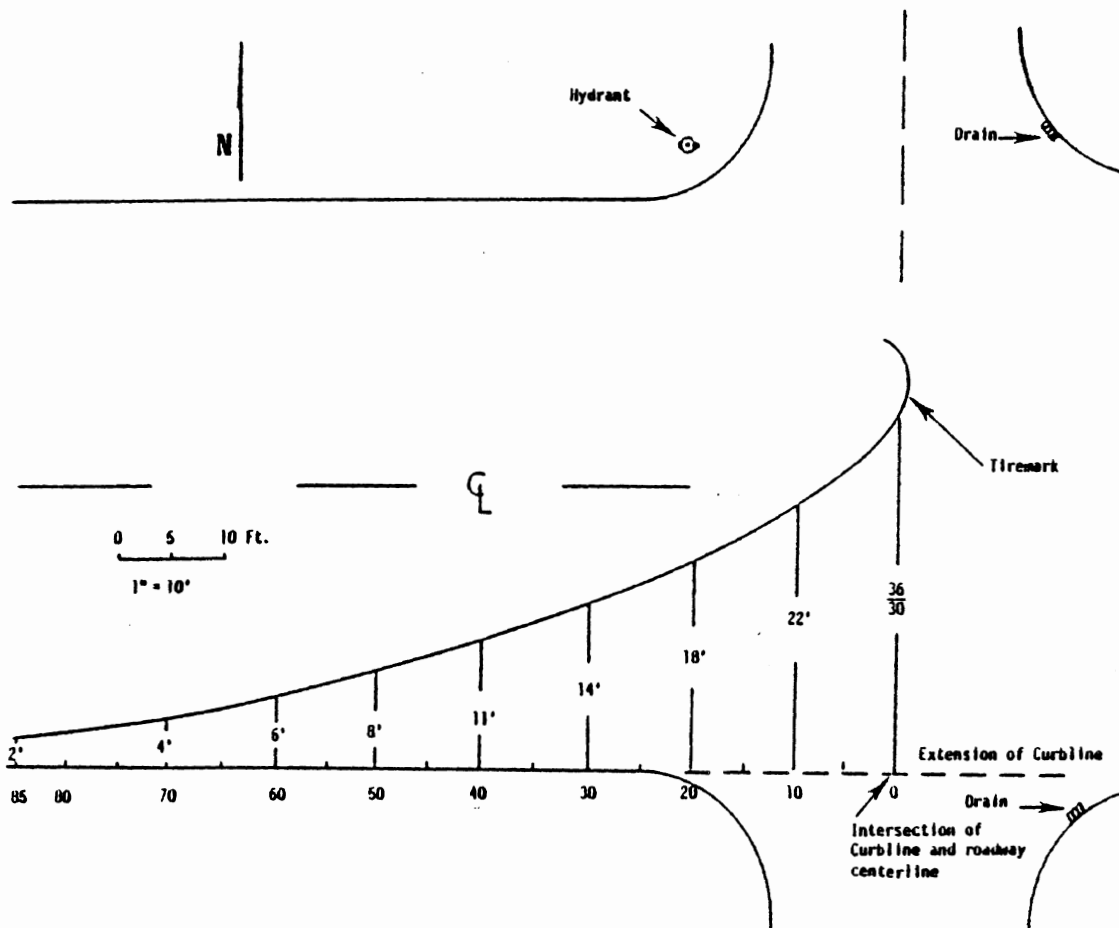
Figure 1.2 Measurements from Curb Line

incorporated in the sketch or sketches.   By preparing such detailed
sketches prior to making measurements, the investigator can be sure that
all necessary measurements as noted on the sketch will be completed with
respect to selected benchmarks, reference lines, or reference points
selected. With this procedure, it is recommended that a large general
area sketch be prepared which locates the entire roadway and all
relevant off-road features with enough distance or space so that all
facets of the crash can be included.   This should include point of
impact, paths of vehicle or vehicles from impact to rest, paths of
vehicles prior to impact, and notable roadway and off-road features
relevant to the crash.   A second sketch may include the vehicle or
vehicles with benchmarks previously located on the larger sketch from
which final rest position of the vehicle will be measured. Additional
smaller sketches as needed should be prepared so as to capture and

6 - SCENE EXAMINATION

accurately record all relevant features of the crash, each incrementally smaller than the previous sketch. These prepared sketches should include all objects with dimension lines (with arrows) between points where measurements are to be made.

1.1.2 Measurement Equipment. A tape measure and measuring wheel are perhaps the two most important pieces of equipment needed to conduct a scene examination. The tape measure may be the same one used in the vehicle measurements. This is usually a twenty foot long, one inch wide, metal retractable tape conveniently carried on the belt or in a pocket. This may be augmented by a 100 foot tape, also commonly known as a "surveyor's chain." This longer tape is also convenient for measuring chords of curved roadway features.

Roadway superelevation can be measured through use of a 4-foot carpenter's level. A 4-foot board is first laid on the roadway, lateral to the roadway edge at various places within a traffic lane, with the carpenter's level placed on top. One edge of the carpenter's level is then raised until the level bubbles indicate it is level. The height of the raised end above the companion board positioned laterally on the roadway is a measure of superelevation as seen in Figure 1.3. The height of the level end above the board, divided by the length of the level gives the tangent of the angle that the roadway makes with the horizon. This angle, the angle of superelevation, may then be easily converted to degrees by looking up the angle's tangent in a trigonometric table, or through use of a simple hand calculator which contains trigonometric functions.

Additional items helpful to the investigator are a split view range finder (which permits coarse measurements of an object's distance from the point of measurement), a compass to accurately determine the directional alignment of the roadway, and steel surveyor's pins. Sometimes a bright orange or yellow surveyor's ribbon is helpful to visually outline key measurements. The cloth ribbon may be tied to steel surveyor pins to outline various geometric shapes or merely to accent an important measurement point in photographs.

A cloth or non-metallic 100-foot tape is preferable to a metal

Hand Held
Flexible Tape

Carpenter's Level

Straight Board
Layed on
Roadway Surface

Height of
end of
level
noted

Pavement Cross Section

Horizon
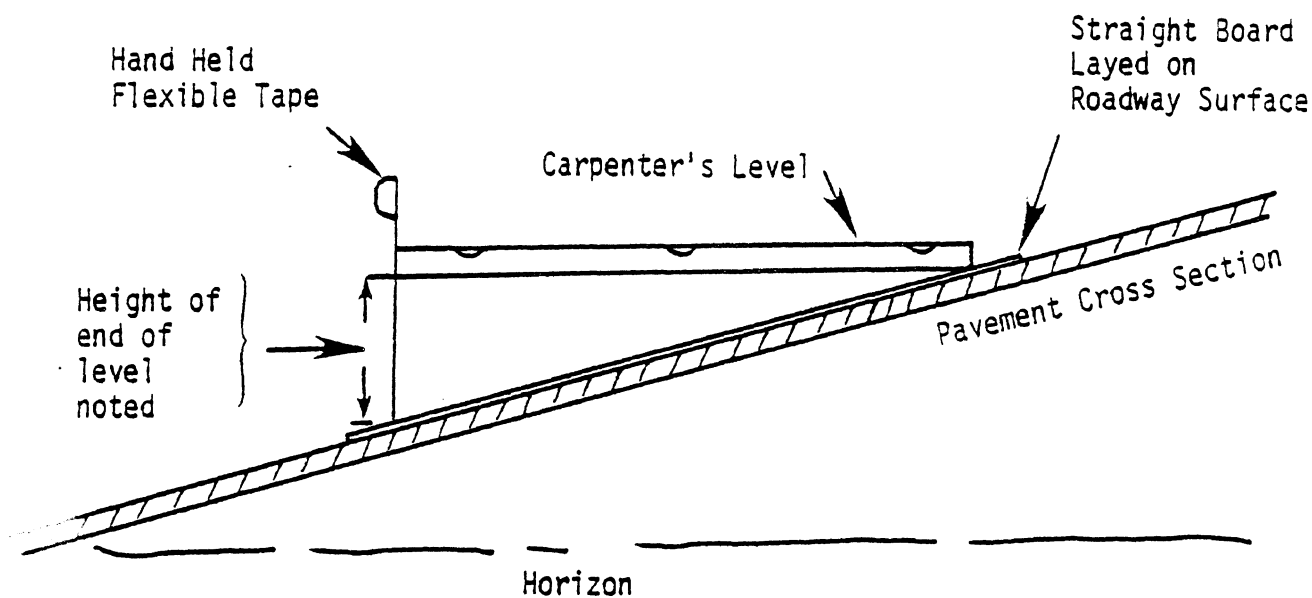
Figure 1.3 Using Carpenter's Level to Measure Superelevation

100-foot tape. Where traffic exists adjacent to where measurements are to be made, the metal tape is more easily damaged by vehicles moving over it than is a cloth tape. These tapes have been much improved in recent years through use of fiberglass or other non-organic, dimensionally stable, pliable material. However, a significant disadvantage in the use of a cloth tape is that it can be more easily blown about by the wind while measurements are being made. Where one investigator is using a long tape alone he must find a method for anchoring down the "dead end." This may be done by using a surveyor's pin inserted into an expansion joint or into the earth, or by anchoring it down with a heavy (lead) weight.

The long tape is also useful in creating right angles when using the road edge as a reference line for measurement when recording key roadside and roadway evidence. The tape may be folded in a three, four, or five foot triangle arrangement so as to create a right angle triangle

which, when aligned with the road edge, permits the investigator to measure perpendicularly from the road edge with accuracy.

Other measurement aids are colored crayons and spray paint. The latter is perhaps most convenient in that it permits contrasting type markings on or off the roadway which show up well in photographs. Where light Portland cement is involved, black spray paint should be used. With dark asphalt type pavement, either yellow or white spray paint is superior. These may be used to outline the corners of the vehicle, axle positions or wheel positions of the vehicle, intersecting curb lines on the roadway, objects, gouges or marks, as well as to create station markers at set intervals along the road edge. Here the value of the measurement is recorded on the road surface itself for convenience in making measurements as well as a reference for distance perspective in photographs.

A unique chalk useful for accenting measurement points and objects on the roadway at night is called "Codit." This material, produced by the 3M Corporation, is rather soft and depletes easily when being used, but has the unique characteristic of being highly reflective to light. Thus, marks made on a roadway using this reflective chalk and photographed with a photo flash, in the absence of street lighting, are brilliantly illuminated in photographs. This may be less relevant to NASS and NASS-type investigations which are not on-scene investigations, but are conducted after the police investigation is completed. There may, however, be instances when investigators accompany police to night crashes and later conduct a follow-up during the daylight hours when measurements can be made more accurately and conveniently. Such bright roadway markings left on-scene in nighttime crashes can also be seen in daylight and will facilitate scene measurements.

1.2 Final Rest Position

The police accident report may denote the general orientation and area of final rest of vehicles involved in the crash. Often this is by use of a sketch of the accident scene showing how the two vehicles came together, or where they were at final rest, or both. Sometimes, however, this most valuable information may be in a supplement to the police accident report (in the form of a sketch, a narrative, or a

combination of both). In many states this supplement is not considered a public document but more of a confidential investigative report which may be accompanied by interviews, police action taken, and conclusions. Where such information is made available (with the official police accident report) this supplement perhaps provides the most useful information in determining vehicle final rest positions and/or orientations on or off the roadway.

Physical evidence left on the roadway from a crash can be used to confirm the final rest position and orientation of vehicles. This may consist of scratches, gouges, fluid deposits, wheel tracks, sod mounded up by vehicle wheels slipping laterally, broken shrubs and posts, areas where sod or grass may be matted down where one side of the vehicle was in contact with the earth, and similar features. Where "leaving the roadway, loss of control" type accidents occur, the errant vehicle may "bottom-out," or contact its undercarriage on off-road objects and irregular terrain. A path of such evidence can indicate how the vehicle traveled from the roadway to final rest.

Additional information may be obtained from talking with the tow operator who removed the vehicle, from occupants of the crash vehicle(s) or from witnesses, as well as from the investigating officer. An examination of the vehicle prior to visiting the crash scene may sometimes provide clues as to the final rest location of the vehicle. Sod, weeds, or high grass may be jammed between a tire and wheel rim. Mud on the vehicle and sheet metal deformation from contact with the earth are also indications of where the vehicle traveled and came to rest. Bark fragments from a tree caught in sheet metal seams or sharp edges of the vehicle, and paint exchanges between the vehicles and fixed objects that were struck are excellent indicators of contact, path, and rest point.

Locating the final rest position of the vehicle after the vehicle has been removed is a combination of knowing where to look, confirming this with physical evidence, obtaining information from interviews, and examining the vehicle.

1.3 Roadway Examination

Marks and material on the surface of the roadway can also provide the investigator with information on where the accident occurred and sometimes how it occurred. As emphasized earlier, physical evidence which is short term and perishable can be most informative. Such physical evidence can be debris, components separated from the vehicle, roadway scratches, fluid deposits, fluid splatter, tire skidmarks, scuffs, and similar evidence. Some types of this physical evidence may persist over a long period of time, but with degradation. Some examples here are tire skids and scuffs, discoloration on the roadway from fluid spills, in the form of debris and small components from the vehicle swept off the roadway upon vehicle removal and scene cleanup.

Use characteristics of the roadway should also be noted, i.e., whether the roadway is an urban arterial, rural secondary road, or other type road classification. Many descriptive features of the roadway may be noted by the investigator that are not called for on the accident scene examination protocol. Such information can be helpful in later analysis of the crash when various descriptive roadway features are used to help explain vehicle or occupant kinematics, and how each was influenced by the roadway. Some key roadway features and potential physical evidence to look for are noted in Table 1.1.

Roadway and off-road evidence should be marked and measured. This should include vehicle locations, vehicle orientation with respect to the roadway, and point of impact. Various measuring and marking techniques may be employed to facilitate locating such evidence accurately in order to enhance photography and for later transfer to an accident schematic. Outlining discolorations on the roadway with contrasting spray paint is helpful, as is outlining scuffs or skidmarks. Key off-road features relevant to the crash, such as poles and trees, may similarly be marked. Marking with an "X" or number denoting which component or item is involved is helpful for later pictorial reference. The investigator can be resourceful in many ways in marking and noting some data. Surveying the entire crash scene while visualizing what is important, noting and recording it accurately, as well as capturing it in the photo sequence, is a good approach which keeps key items or

TABLE 1.1

CRASH SITE EVIDENCE

Crash Debris	Off-Road Damage and Miscellaneous Marks	Marks on Roadway Surface
Underbody--dirt, paint, rust, toe, etc.	Fractured roadside objects	Tire prints
Vehicle Parts	Scraped, scratched roadside objects	Skidmarks
Vehicle Fluids	Fractured guardrails/barriers	Scuffs
Cargo (liquid/solid)	Pedestrian slide marks	Scratched Surface
Road material	Holes in roadside or snow	Scraped Surface
Clothing	Piles/mounds of dirt or snow	Chips in pavement
Blood, body fluids, matter	Foot prints	Chops in pavement
--regurgitation --soft matter		Grooves in pavement
		Furrows in soft material roadways

features from being overlooked.

There are many roadway types and classification categories. The listing selected for NASS is one that was found most useful for research purposes. In general, roads are classified by their location, their service type, and how they are used in terms of traffic type and traffic characteristics. Traffic characteristics in terms of an average daily traffic (ADT) count averaged over a long period (usually one year) is an important road use descriptive characteristic. Hourly volume is similarly important data to measure roadway efficiency.

Various studies have tried to correlate accidents with such road use data, but results have been inconclusive. Some correlation has been found to exist between well defined sections of roadway and factors which result in traffic disturbance or interference. For example, in some well defined types of roadways where roadway sections are short in terms of distances between intersections and driveways with high traffic volume, accident rates are high. Long sections of similar roadway, in comparison, had lower volume and lower accident rates. Accident rates have been shown to increase with increase in average daily traffic.[1]

Where traffic lanes exist in tangent (straight) sections of highway, the investigator can describe the roadway with relative ease. However, there are many variations of actual road use dictated by roadway geometrics and traffic control devices. Some of these are acceleration lanes for merging traffic into roads from ramps, storage lanes where cars line up for turns, turn lanes to facilitate turning movements at intersections, and similar such designs. The investigator is encouraged to become familiar with various roadway features and characteristics such as roadway type, average daily traffic (ADT), and hourly volume so as to better describe their characteristics relative to crashes under study. Road edges in terms of shoulders or off-road design are also important features of the roadway relative to the study of motor vehicle accidents.

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[1]From a 1966 study of accident data provided by Ohio, Louisiana and California, summarized by Dr. Adrian Koert of Michigan State University, Highway Traffic Safety Center.

Shoulder area is that portion contiguous to the traveled portion of the roadway which serves as an off-road area for stopped vehicles under emergency conditions, as well as for the lateral support of the roadway base and surface courses. Its importance in motor vehicle crashes is that it oftentimes provides a recovery area for vehicles involved in crashes, or vehicles attempting to evade a crash. Shoulders may have detrimental effects on vehicle safety if they are not firm and dressed evenly with the roadway. Such road shoulder differentials can result in lateral forces to the vehicle which adversely affect the driver's ability to control and direct the vehicle properly. Shoulders can be made of loose material (gravel, dirt, sod) or permanent material (Portland cement, asphalt). Loose material shoulders require more maintenance than permanent material shoulders. It provides a significantly different skid factor characteristic to the vehicle. Many times such lateral forces applied to a vehicle front wheel, when the driver attempts to recover the roadway, can precipitate a crash. Noting the condition of the shoulder as well as its width, material composition, and its relationship to the roadway is an important information element relative to the examination of the scene. The investigator should not hesitate to include information beyond that called for on any given data form. This can be easily accomplished by attaching additional data or sketch sheets to the form, including notes in the margin of the field scene protocol, or by using blank backs of field forms. Such additional information can be valuable to the investigator when he/she attempts to reconstruct the crash, as well as to the data and case editor and analyst. In many analysis efforts, original copies of investigation forms and related documentation must be examined as part of a study. Shoulder width, slope, and material can be relevant considerations to the understanding of the behavior of an out of control vehicle.

Accesses to the roadway can take many forms. For divided and/or limited access highways, access is through merging traffic brought up to speed by an acceleration lane and merged into the moving traffic on the primary roadway. Roadways other than well designed limited access highways include features which present conflicts between traffic units. Such conflicts can and often do result in crashes. To minimize the

effects on such conflicts and to effect smooth flow of traffic, various traffic control devices are employed. Such devices include traffic signals, signing, and markings.

The three basic types of traffic conflicts are: (1) crossing, (2) merging, and (3) diverging. Intersections are classified as at-grade or grade separated, depending upon the treatment of crossing conflicts. The grade separated intersection, also called an interchange, limits the conflict between crossing patterns of traffic. There are a variety of at-grade intersections each having their distinct geometric characteristics and different roadway features. Some of these are presented in the sketch below (see Figure 1.4). The investigator should familiarize himself with the various intersection types commonly found in traffic so as to more accurately describe roadway features relevant to crashes.

There are two basic roadway alignments: (1) horizontal, and (2) vertical. As the name implies, horizontal alignment is either a curve or a straight section of level roadway. Vertical alignment implies grades, crests and sags in the roadway. These are minimized by being made as gradual changes of alignment in good roadway designs but can be significant crash causal related features in older roadways or on secondary roads. The vertical curve provides for the gradual transition from one grade (upgrade or downgrade) to another grade by means of a curve in the roadway which connects the two intersecting tangent sections of the roadway. When the point of intersection of these two tangents is above the road surface, the vertical curve is called a crest; when below the roadway, the vertical curve is called a sag. In addition to limiting riding comfort of vehicle occupants, changes in roadway vertical alignment can be important causal considerations in accidents because of the sight distance limitations along the vertical curve. This is illustrated in the diagram below (see Figure 1.5).

Vertical sag curves may limit vehicle operator headlight sight distance at nighttime. Headlight sight distance is the most important criteria used by highway designers in establishing vertical curves. Figure 1.6 illustrates the condition for driving the minimum length of sag vertical curve (L), in terms of the distance between the vehicle and

T TYPE

Unchannelized   Flared   Turning Roadways

Y TYPE

Unchannelized   With Turning Roadways

4-LEG INTERSECTIONS

RIGHT ANGLE

Offset   Unchannelized   Flared

SKEWED

Offset   Unchannelized   Flared

OTHERS

MULTILEG INTERSECTION   ROTARY INTERSECTION
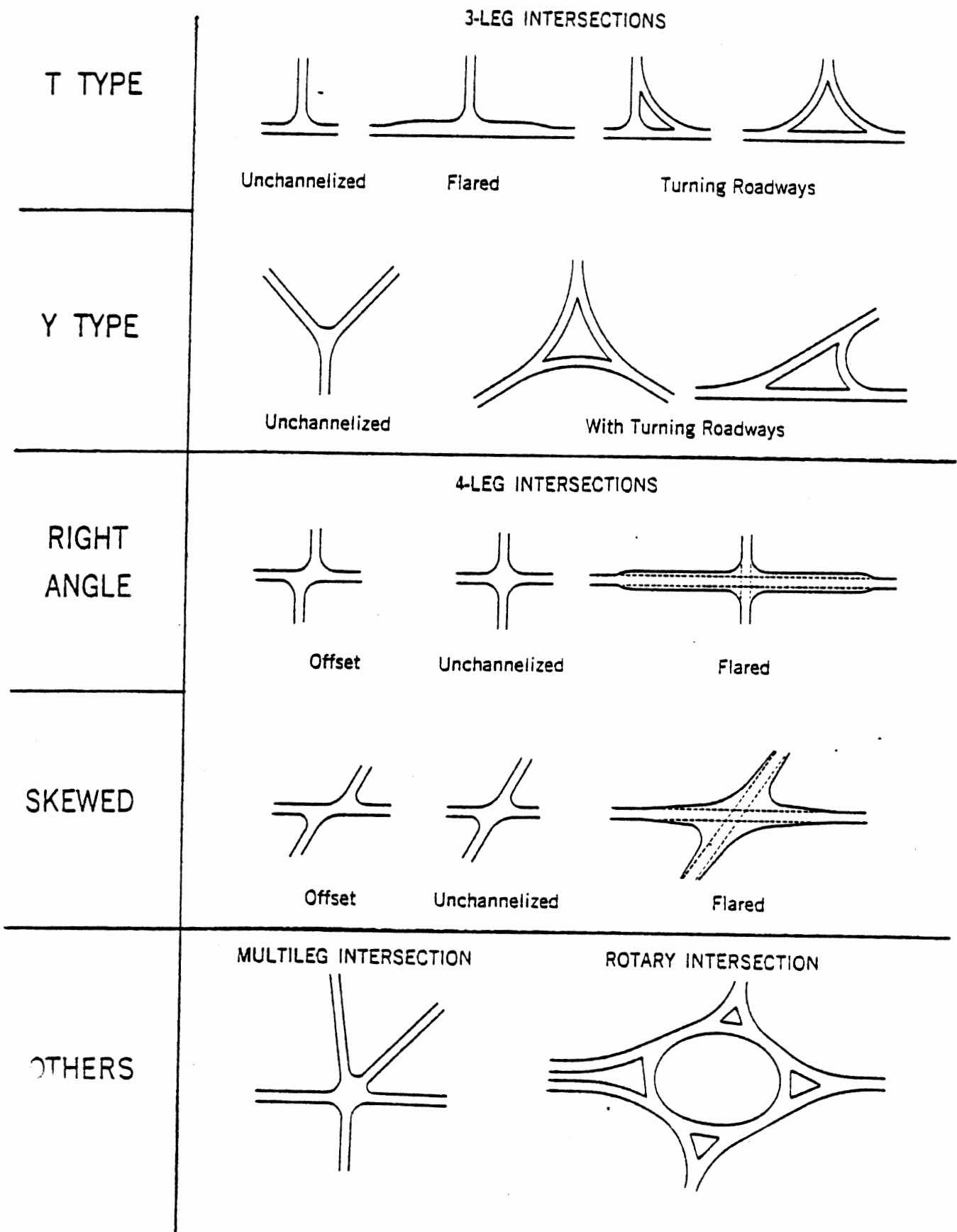
Source: Adapted from American Association of State Highway Officials, A Policy on Geometric Design of Rural Highways, American Association of State Highway Officials, 1965, p. 388.

Figure 1.4   General Types of At-Grade Intersections
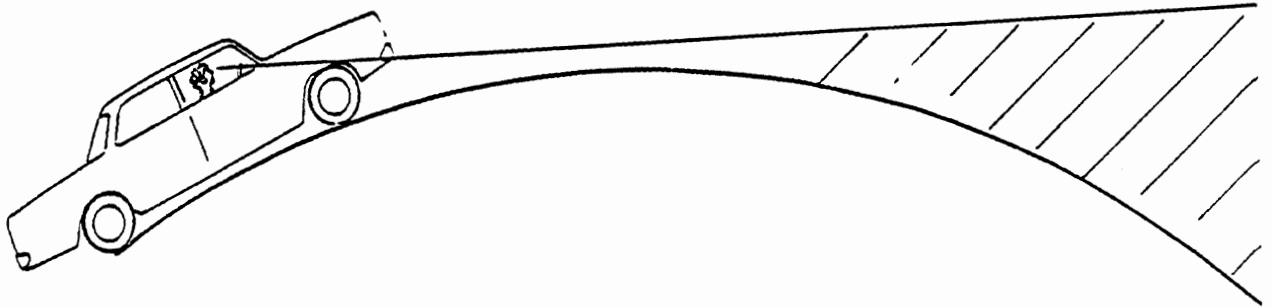
Figure 1.5 Vertical Alignment Limits Driver Vision Ahead

point where the light ray emitting from the vehicle hits the pavement surface (S). The height of the headlights above the pavement (H) shows, as does the angle of the light rays emitting from the vehicle above the horizontal (B). The equation shown in the illustration is one that is used by highway designers to determine the minimum length of the vertical sag curve (L).

Accurately documenting vertical and horizontal alignment characteristics of a roadway can be an essential part of understanding a crash. Changes in horizontal alignment in the form of roadway curves are similarly important in that they also limit riding comfort, or the speed at which the vehicle can comfortably and safely take the curve, as

$$L = \frac{AS^2}{200(H + S \tan B)}$$
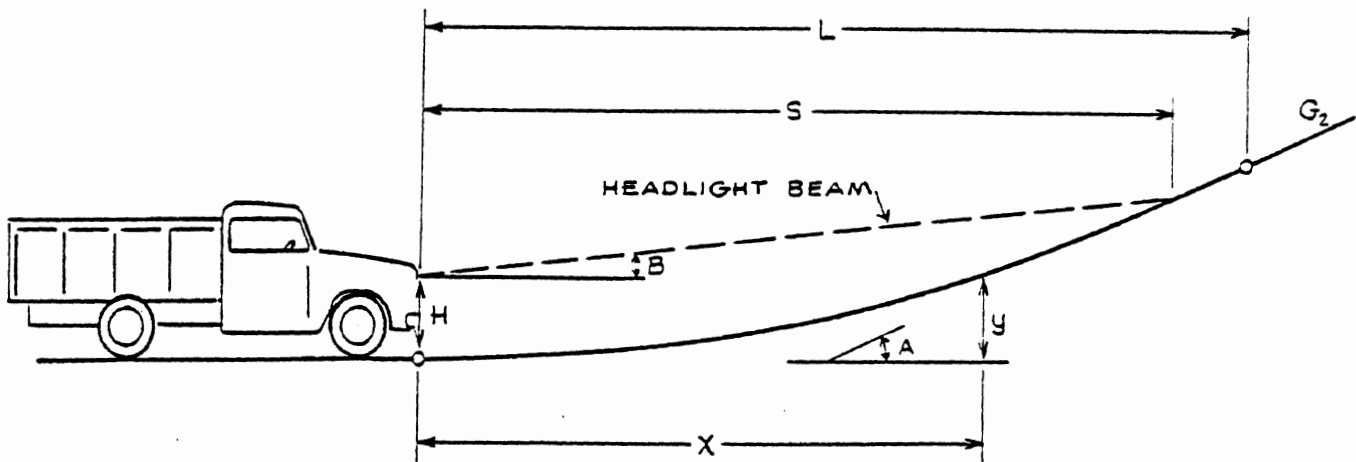
L = Length of Vertical Curve, Ft.
A = Algebraic Difference in Grades,
    Percent $(G_2 - G_1)$
S = Sight Distance, Ft.
H = Headlight Height, Ft.
B = Upward Divergence of Light Beam,
    Degrees

Figure 1.6 Vertical Curve Effects on Sight Distance

well as limiting the view of the roadway ahead at nighttime. The horizontal alignment of a roadway may be viewed as a series of straight or "tangent" sections which are joined by circular curves.

When the roadway is flat and level about a curve, the only force resisting the tendency of the vehicle to leave the roadway as a result of centrifugal force is the side friction forces developed between the tires and the pavement. Side, or lateral, friction forces to the vehicle are readily sensed by the driver. By superelevating, or banking the roadway, the vehicle then uses the force of gravity to offset the tendency of the vehicle to slide outward. Curves are superelevated primarily for speeds dictated by comfort to vehicle drivers. High

centrifugal forces when cornering also tend to cause the driver to slide across the seat of the automobile. This is an important consideration when attempting to establish whether the driver was in the driver's seat and in control of the vehicle at all times in crashes where large lateral forces to the vehicle are evident. In Northern climates, roadway superelevations are limited by icy roadway conditions which permit vehicles to slide down to the base of the superelevated roadway. Greater superelevations are most evident in Southerly climates.

Centrifugal force associated with the vehicle moving in a circular path about a curve may result in the roadway being "banked" or superelevated to increase occupant comfort and to permit the vehicle to develop sufficient lateral tire tractive forces to keep the vehicle from slipping sideways off the roadway at high speeds. One other roadway feature involved in the design of curves is the use of a section of roadway termed a transition. Here, the curve is connected to tangent sections in such a manner so as to provide for easy vehicle transition from the straight tangent section of the roadway to the curved portions of the roadway. It consists of a gradual (long radius) curve, increasing in curvature until it equals the curvature of the curve.

Measurement of roadway curvature in terms of degrees of curvature, or feet in radii, can be obtained from the local road commission, highway department, or a similar responsible roadway organization. Where such information is not available, the investigator may use his 100-foot tape to create chords about the curve and measure the offset from the curved arc portion of the roadway intercepted by the chord. This enables a calculation of the curvature over the length of the chord. The offset between the chord and roadway curved edge is called the middle ordinate, or off-set, as seen in Figure 1.7.

Such measurements provide a simple way of determining the radius of the curved roadway. This is given by the following formula:

$$R = C^2/8M + M/2$$

where:

R is the radius of the curve in feet,
C is the length of the chord in feet, and
M is the length of the middle ordinate in feet.
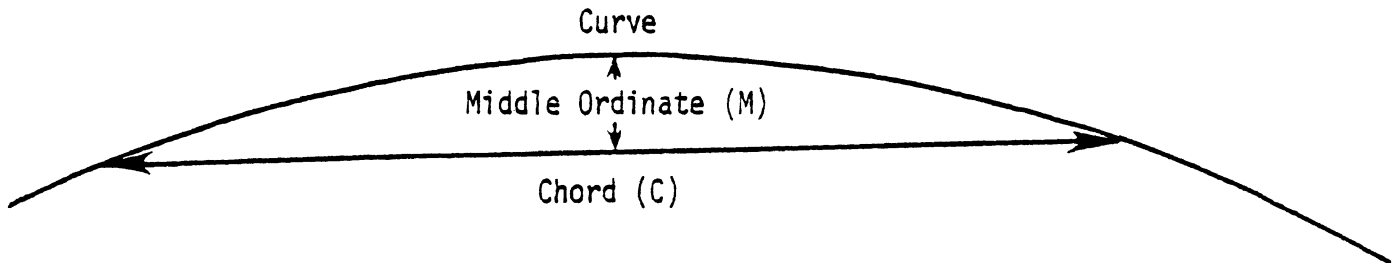
Figure 1.7   Chord Measurements of Curves

A derivation of this formula   from simple trigonometric relationships is given in Appendix A.

As previously indicated, superelevation is an important characteristic which determines a comfortable and safe speed for vehicles to traverse a curve. Knowing the roadway surface coefficient of friction permits the investigator to calculate the speed about the curve at which the vehicle's centrifugal force equals the tractive forces on the tires. Exceeding this speed permits the vehicle to "spin-out" and leave the roadway. This can be expressed by the following formula and is commonly referred to as the critical curve speed.

V =   15R u

Where R is the radius in feet and in the roadway coefficient of friction .

The vehicle will usually begin yawing, or rotating slightly, about its center of gravity while traversing a curve at the critical curve speed before it will actually leave the roadway out of control. Any marks on the roadway indicating critical cornering scuffs or yaw marks should be measured and viewed so that the actual placement of the vehicle on the roadway can be reconstructed. This permits placement of the vehicle's center of mass relative to the tracks or scuffs on the roadway. In most cases, one half of the car width must be subtracted from the radius dictated by the scuff, or cornering marks, made by the outside wheels of the vehicle. Derivation of this relationship is given in Appendix B.

The cross section of the roadway is designed to provide both a level surface for the vehicle to travel on, as well as to permit adequate drainage of water from the roadway. Most road surfaces are crowned at their center for drainage. This crowned center, in effect, results in a banking or superelevation to adjacent traffic lanes and can affect driver comfort as well as vehicle control when moving from one traffic lane to another. Oftentimes a crowned two-lane road, with a different slope to each traffic lane, is maintained with that same slope relationship around a curve. This results in one lane, usually the lane inside to the curve, having greater superelevation than the other. To an unsuspecting driver whose speed is high and control marginal when taking a curve, this could be detrimental to vehicle control. At one time in Michigan, roadways had a parabolic crown at their center which dictated the slope of each traffic lane on a two-lane roadway. When constructing the same roadway about a curve, that same parabolic crown and slope relationship was kept constant, resulting in slightly greater superelevation in one lane than on the other. This difference in superelevation was shown to affect the control of a vehicle when changing from one lane to another while traveling a curve.

When measuring superelevation, or roadway banking, the investigator should make more than one measurement so that he or she can average out any anomalies or irregularities of the roadway surface. It should never be assumed that once a slope measurement is made that the same slope is

maintained across the entire roadway. Superelevation should be measured independently for each traffic lane. A sketch of roadway cross-sections should be included at various locations across the roadway in the accident drawing, as well as along the roadway, to adequately document such characteristics when relevant to the crash. Such roadway cross-section profiles may then be identified with a position of the roadway in the plan view sketch of the roadway. Cross-sections may easily be plotted on a piece of graph paper which shows the slope of the superelevation (or banking), for each pavement section, and the slope and width of the shoulder, as well as the off-road area contiguous to the shoulder. These need not be at the same scale used in plan view drawings, but may be exaggerated in relative scale.

The surface composition of the roadway is important in determining the tractive forces that vehicle tires can develop on the pavement. It also determines characteristics of water run off, resistance to wear and smoothness of the surface. There are two basic kinds of roadway surfaces. One type of roadway surface is a permanent material surface, and the other is loose material surface. Permanent material surfaces consist of Portland cement or asphalt pavement. (Some urban streets may be constructed of stone blocks or bricks, but these are quite infrequent today.) Loose material surfaces are made up of gravel, cinders, dirt, crushed rock, and similar substances. To further characterize the roadway, the investigator should make an estimate as to the surface condition of the pavement; that is, whether it is new, with a sharp crisp looking surface, or whether the surface is traffic polished by heavy motor vehicle use. For Portland cement, the investigator should also note whether the surface is grooved or textured in some other way. For asphalt, the investigator should assess the aggregate size in the asphalt, as well as whether the surface has any evidence of bleeding or tar saturation. All these characteristic features affect surface skid characteristics.

Similar in importance to pavement surface type, is the condition of the surface. It may have been dry or wet at the time of the accident, polished clean from heavy traffic movement, or dusty and dirty. It may also have been be muddy as a result of being tracked from non-permanent

side roads or construction sites. Observing and noting these characteristics of the roadway will help in establishing an accurate coefficient of friction to be used in reconstructing the crash as well as possibly explaining other crash-related phenomena. If the roadway was wet at the time of the crash, the degree of road wetness should be determined. If there was evidence in the police report of a wet roadway, determine how wet. A very wet surface, evidencing standing water, could be indicative of vehicle hydroplaning. Hydroplaning occurs when standing water on the roadway, coupled with tire tread condition, tire pressure, vehicle weight and speed, begins to act as a lubricant between the tire and the roadway. Movement of the tire over the road surface then causes a wedge of water to form between the tire and the surface and the vehicle "planes" on the water layer. When this condition is suspected, information relating to pavement slope and texture becomes important, as does the condition of vehicle tires.

Knowledge of road characteristics is essential in accurately reconstructing a crash. Both geometric features and characteristics of the pavement surface may provide physical evidence of vehicle movements prior to the crash and after the crash. It is also essential in estimating a realistic skid factor number necessary for predicting vehicle movement on the roadway. Here the road surface skid number (drag factor or friction factor) is most usually a conservative representation of the road surface, such that a reasonable estimate of vehicle speed may be established.[2]

1.4 Traffic Controls

To ensure the safe, efficient, and timely transport of people and goods on our roadways, traffic controls are used to control traffic

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[2] Since law enforcement professionals are primarily concerned with whether a law has been violated, and if violated what citation is warranted, establishing minimum speeds is important. Speed calculations are used in the presentation of proof of vehicle speed. These calculations relate the kinetic energy of the vehicle in motion to heat energy in the wheels and tires of the vehicle in braking. Investigators conducting crash investigations for research purposes are not concerned primarily with making estimates of speed for law enforcement purposes, but they are concerned with establishing an accurate coefficient of friction for accident reconstruction.

movement and to minimize conflicts between vehicles. Roadway route numbers, street signs, geographical markers, information signs, and similar sources of information are not traffic controlled devices. Traffic control devices include traffic signs, signals, and pavement markings. They serve as warning devices, regulatory devices and information guides. They are manifested in the Manual of Uniform Traffic Control Devices[3] which has now been accepted by most states to ensure uniformity in traffic control. The investigator is referred to this manual and to his state manual (each state has one) to develop a familiarity with the use and application of the various devices on our highway system.

1.4.1 Signs. The three classifications of signs are: (1) warning, (2) advisory, and (3) regulatory. The message a sign conveys is noted in part by its shape, color, and symbol (where included). Signs must be legible and located so that they may be easily viewed by motorists.

The posting of vehicle speeds is, of course, a regulatory function and is conveyed by a rectangular sign with its major axis mounted vertically. The relationship between speed and accidents is a major consideration in establishing speed limits on roads. Speed limits can be either absolute or prima facie. The absolute speed limit subjects the driver to arrest when it is exceeded while the prima facie speed limit is somewhat more subjective and places the burden on the driver to prove that he was safely operating his vehicle. Excessive speed in conditions such as fog, rain, snow, or sleet are examples of the latter. The investigator should be cautioned here that drivers are inclined to report their speed, when asked, as being at or below the speed limit, when in fact they may have been exceeding the limit.

One relevant consideration for the investigator, is the overuse of signs. That is, where many signs exist which make demands on the motorist, the result may be to confuse and bewilder rather than to inform and direct. This clutter, or information overload, on the driver makes his driving task more difficult. The location of all signs and

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[3]Manual of Uniform Traffic Control Devices for Streets and Highways (Washington, D.C.: U. S. Department of Transportation, Federal Highway Administration, 1971).

controls relevant to the accident should be noted. The investigator is encouraged to question use and location of signs and consult the State Uniform Manual of Traffic Control Devices to assess their suitability.

1.4.2 Signals. Traffic controls, such as signals, gates, bells, and other sensory alerting devices are most often applied at intersections or major points of conflict on the roadway. Their application, timing and synchronization with larger area municipal traffic control system is the art of the traffic engineer. Your local traffic engineer (city or county) can be an important source of information in this particular area. He can provide you with the ratios of green time to red time at signals, the warrants which led to signal installation, intended modifications to signals, and the relationship of a particular signal to an overall network of traffic control devices.

The investigator should be aware of suitable locations for such signals and problems associated with signals which are not suitably located. For example, railroad at-grade crossing gates may stay down for long periods of time so that there is little relationship between conflicting railroad and motor vehicle traffic in their use. Also, overhead location of signals within an intersection can sometimes trap motorists in the intersection awaiting clearing traffic to turn. Where such anomalies exist, the investigator is encouraged to note and comment upon them as part of the scene examination protocol.

1.4.3 Markings. Pavement markings and markers are used to guide and regulate vehicles on the roadway. These are used to channel traffic into proper positions on the roadway and to separate opposing streams of traffic, to delineate no passing zones and road edges, and to establish roadway centerlines and turn lanes at intersections. They also outline pedestrian walkways across intersections and, in general, facilitate the orderly and safe movement of traffic. The condition of such markings are important when considering their effect on the motorist. Their effect is compromised when they are obscured and poorly visible to the driver. Perhaps most important is poorly maintained traffic lane striping which tends to become obscured and thus ineffective in delineating proper traffic lanes. This creates conflicts between people who are familiar with the roadway and those who are unsure and

unfamiliar with traffic movement patterns and confusion between drivers who are both unfamiliar with the roadway. The investigator should be aware of the marking condition and placement when attempting to assess all the causal factors in a crash. Some stripes or markings can be a distinct hazard to vehicular traffic. The prime example here is the use of cold and hot plastic stripes at intersections. This material is a plastic tape with an epoxy bonding used extensively for urban crosswalks. For a skidding vehicle, such as a motorcycle (which is a single track vehicle and inherently unstable when braking or skidding), the plastic stripe can contribute to the rider's inability to control his vehicle as it passes over the stripe.

The relevance of traffic controls to crashes is obvious. Faulty controls, controls improperly applied, or traffic controls not designed for existing traffic conditions are hazardous to the safe movement of motorists and pedestrians They must also be accepted by motorists and pedestrians. In addition to observing and noting all traffic controls relevant to a crash site, the investigator is encouraged to be critical and to evaluate their suitability and effectiveness.

1.5 Off-Road Features

Off-road terrain characteristics, and objects associated with off-road terrain account for a considerable portion of the damage and injury associated with loss of control crashes. A "forgiving" off-road environment is essential to mitigate the severity of these crashes. Trees, shrubs, or other vegetation often hide more hazardous fixed objects such as fences, posts, tree stumps, or culverts. The investigator should assess the general nature of the off-road terrain and note its relationship to the crash. Could the dangerous fixed object struck by the vehicle have been moved or eliminated? Is the off-road terrain and its characteristics the product of poor roadway maintenance? Could dangerous features (such as culverts) have been altered, protected, or relocated to be less hazardous to errant vehicles? Are off-road slopes too steep so that a barrier would be warranted to contain errant vehicles? Could it have been sloped differently? The investigator should ponder questions such as these when examining and evaluating off-road characteristics.

A clear roadside area is an ideal situation for vehicular safety, but sometimes impossible because of such fixed objects as drainage structures, and other permanent roadside designs. When the hazards associated with off-road terrain are greater than the hazard of a barrier installed to contain errant vehicles, a barrier is indicated. Guardrails are one such barrier type which essentially redirect errant vehicles away from off-road hazards and back on the roadway. The investigator should be familiar with various barrier structures, (e.g. "w" beam, open face, post, cable, etc.), how these barrier structures are applied, and their method of operation. The condition and maintenance of such barriers is important to the investigator. Poor maintenance may compromise the ability of the barrier to perform effectively. Rotted posts, unrepaired w-beams, improper treatment, improper lapping and fastening of guardrail sections to adjacent structures, and lack of flaring may all be adverse conditions to be considered by the investigator.

Divided and separated highways are effective for high speed, efficient, and safe movement of vehicles. These highways provide drivers with a path free of any potential conflict from oncoming traffic. However, when roadside areas serve as obstructions and are hazardous, or are in general "unforgiving" to the errant vehicle, safety is compromised. A highway design philosophy which is gaining acceptance is one termed "positive guidance." Positive guidance attempts to give the driver sufficient information about a hazard in advance, and in a form he can use to enable him to avoid a crash. Highway engineering and human factors technology are combined to create safe and efficient roadway environments. A more complete discussion of this concept is not possible in this presentation; however, the investigator should be aware of the effects of an "unforgiving" off-road environment and keep in mind that its effects can be eliminated or reduced in many ways. Where such a hazard is a primary causal consideration in a crash, it is incumbent on the investigator to identify it and comment on its involvement in the crash. The investigator should also make suggestions as to how the hazard might be reduced or eliminated.

1.6 Environmental Factors

Information on ambient conditions (such as daylight, darkness, rain, ice, snow, and fog) are most often available from the police accident report. The police accident report merely notes the presence or absence of the condition, not its extent or its relevance to the crash. For example, a "wet roadway" may be either a damp road surface or one which contained standing water. If the latter was actually the condition that existed, it might have led to hydroplaning by one vehicle, as indicated earlier. Such information is critical to the complete understanding of the conditions under which the crash occurred. An additional ambient consideration is that of glare. Knowledge of the direction of the roadway with respect to the sun, time of day at which the crash occurred and the sun angle in the sky oftentimes are clues that glare may have been a causal factor. Glare, coupled with dust-covered or smudged windshields, can exacerbate an already bad situation for the driver.

Artificial illumination in urban roadway settings is another important consideration to the investigator. Was illumination within the accident area adequate, or did it exist at all? Was illumination of the accident area important or relevant to the parties involved in the crash? Such environmental considerations could have confounding or mitigating effects on the crash.

1.7 NASS Scene Data Form: CCS

The organization and data elements provided in the NASS environmental protocol have been structured to provide the minimum information expected from the investigator in conducting an examination of the scene of a crash. However, as indicated above, the investigator is encouraged to go beyond responding to the data elements included on the data form and provide additional information which further describes the crash scene situation. This could be noting all relevant roadway features and characteristics, hazards existing at the time of the crash, and conditions under which the crash occurred. The accident sketch (or sketches) should be clear, accurate, concise and informative. More than one sketch is encouraged so that environmental conditions existing at the time of the crash can be accurately documented. Additional comments

which clarify entries in the scene protocol form should also be
included. Because of the brevity of the data elements included, the
investigator must be precise in his assessment of the situation and
record the data with utmost accuracy.

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2 VEHICLE EXAMINATION

Summary. Vehicle factors relevant to the crash, in combination with human factors and environmental factors, comprise the basic subject areas of accident investigation. The vehicle is the instrument of the crash. Thus, the investigator should expend the largest amount of energy and time on vehicle examination. Understanding the path of collision, the effects of the crash on the vehicle, and the trajectory of the occupants within are the most important tasks of the investigator.

Investigation can also provide information on the orientation of the vehicle or vehicles at crash, their movement during the impact, separation from impact, and causes of occupant injuries. The condition of the vehicle, along with other crash data, indicates how the vehicle performed in the crash.

The investigator will have some knowledge about the crash before undertaking an examination of the vehicle. This information may range from as little as how many vehicles were in a crash to the exact crash orientation between specific vehicles and how they moved during and after impact. The examination of crashed vehicles does require some knowledge about specific motor vehicles (i.e., passenger cars, vans, trucks, pick-ups, etc.), an understanding of the laws of nature (primarily how objects behave in motion), and an ability to interpret physical vehicle damage. The research-oriented investigator must go beyond these considerations.

A basic understanding of how vehicles are fabricated and the various components and subsystems which comprise the vehicle are essential for both understanding and describing how impact forces affect the vehicle, and understanding the damage resulting with those forces. A primer in auto mechanics is helpful inasmuch as it graphically presents and describes various conventional vehicle systems (braking, steering, drive train, etc.). Shop manuals for specific vehicle models are also helpful. Such information permits the investigator to become familiar with the automotive terms and component names essential to describing damage as well as how the damage occurred. No single universally accepted glossary of automotive terms and component names exists. Individual manufacturers may have different names for the same

component or areas of the vehicles. A familiarity with this background information is necessary for the understanding and interpretation of crash damage and how damage resulted, and also for accurate recording and description of one's findings.

Knowledge of the basic automotive structure is similarly important. Manufacturers' literature, textbooks, trade journal materials and shop manuals can be quite informative in areas of body design and fabrication, running equipment types and component locations. They also show fundamental differences between vehicles which influence how a vehicle is damaged and how it performs when impacted in a crash. There are many and varied sources of such information. The investigator is encouraged to become familiar with such data.

2.1 On Scene Information

In NASS crash investigations on-scene observation of the crash is rare. There may be situations where police or newspaper photographs are available which depict the crash scene before the vehicles are removed. When available, such information can help to establish the final rest positions of the crash-involved vehicles relative to road edges and other observable benchmarks. The point of impact, as well as any physical evidence resulting from the crash, may be confirmed during the post-crash scene examination. These may be such things as spilled fuel patterns or vehicle post-crash trajectory suggested by tracks, furrows, scuffs, skids, damage to roadside "furniture,"[1] and on-scene roadway markings made during the police investigation. A correlation of information included in the police investigation with the information of the crash scene examination can be helpful in revealing the way the crash occurred and the physical layout of the scene immediately after the crash.

When the opportunity presents itself, on-scene examination of crashes can provide enormous and detailed information with which to reconstruct the crash. Here the investigator must first become a good observer. The investigator must be capable of sorting out vital

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[1] Man-made objects such as barriers, signs, posts, etc., are viewed in this context as "highway furniture."

information in the form of roadway or off-road evidence, as well as noting and preserving such information during the period of confusion and disorganization immediately after the crash. This is the time when EMS personnel are handling the injured, when police are controlling traffic and on-lookers while beginning their investigation, and tow operators are eager to remove debris. This is not to imply that all accident investigators should attempt to be "on-scene" at crashes, but when the opportunity presents itself, the investigator should be capable of objective, unemotional sorting out of the information necessary to fully explain the crash. Being a good observer is the first essential attribute. Recording and preserving critical physical evidence in an orderly, unobtrusive manner is the second most important attribute.

2.2 Vehicle Identification

Identifying vehicles involved in a crash is best accomplished by using the vehicle identification number. Federal Motor Vehicle Safety Standard #115, titled "Vehicle Identification," requires that motor vehicle manufacturers install a thirteen digit and letter identification number for vehicles on the upper left section of the instrument panel, observable from outside of the vehicle through the windshield[2]. Information contained in the VIN number may vary between manufacturers, but in general it contains vehicle body style, series, name plate line, engine type and size, and may include production plant location. An attempt to standardize the VIN sequence is now under way through a notice of proposed rule making by the National Highway Traffic Safety Administration (NHTSA). Here NHTSA proposes to amend FMVSS #115 to extend its applicability to additional classes of motor vehicles, to specify VIN content and VIN format, and to simplify its use. At this writing, a newly revised VIN system is reported to be complete.

Many times, because of vehicle crash damage, the VIN tag may not be visible. In addition, there are many older model vehicles not subject to the VIN tag requirement. Here, the investigator must identify the vehicle through other means. Information may be found by examining vehicle trim, the owner's manual (often left in the glove compartment),

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[2] wherever practicable. Certain vehicles are exempted.

vehicle registration certificate (similarly often found in the glove compartment), and labels or decorative decals on the interior. Vehicle identification may also be determined by tags or plates on the vertical rearward facing door frame. The presence of four numbers embossed on a tail light level may be indicative of the model year of the vehicle. The investigator should become familiar with clues indicating identification of various vehicles.

Interpretation of the vehicle identification number can be made through the manufacturer's shop manual for the particular make/model, as well as through a small booklet published yearly by the National Automobile Theft Bureau. This publication is titled "Passenger Vehicle Identification Manual," is issued yearly, and may be obtained for a nominal fee. A similar booklet for commercial vehicle identification is published by the same organization.[3]

2.3 Tow Operator Practices and Storage

The examination of crash vehicles is normally completed on the premises of the tow operator who retrieved the vehicle from the scene of the crash. This may be a temporary storage location for the vehicle prior to disposal. Permission given by tow operators to examine vehicles on the premises is a courtesy extended to you by the tow operator. This permission should not be assumed. The investigator should approach each vehicle remembering that it is someone else's personal property and that it is of value to the owner, no matter how damaged it may appear. The investigator should always properly identify himself to the tow operator before examining the vehicle. Do not set your equipment bag equipment items on sheet metal surfaces of crashed vehicles as this may contribute to its existing damage. Always leave the vehicle in the exact same condition in which it was found. This is an unwritten rule among accident investigators, and it makes good sense. Failure to leave the vehicle in the exact condition in which it was found can confuse and mislead others who may wish to examine the vehicle

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[3]The National Automobile Theft Bureau, Passenger Identification Vehicle Manual, National Office, 390 N. Broadway, Jericho, New York 11573, (single copies are available), and Commercial Vehicle Identification Manual (same organization). Write for current price.

later on. It also indicates to the tow operator that you exhibited a lack of respect for someone else's property for which he is responsible.

The investigator must conduct himself so that he is above suspicion of any theft from a damaged vehicle. Entering the passenger compartment of a vehicle and examining the interior is an action which many question as being improper. Some construe this as trespassing on other people's property. While most tow operators do not object to the examination of the vehicle's interior, this courtesy could be suspended abruptly should even a hint of theft on the part of the investigator be suspected.

Damage to the crash vehicle which is not the result of the crash will usually result from the way the vehicle was removed and transported from the crash scene. Where vehicles are damaged to the extent that they cannot be towed with either their front or rear wheels on the pavement, portable wheeled platforms called "dollies" are employed. These elevate the vehicle off the roadway while it is in tow. Some lazy tow operators, usually with police looking the other way, will actually drag the vehicle with parts of the vehicle, other than the wheels, in contact with the roadway. Such towing methods can significantly contribute to the already existing damage.

A tow operator may create misleading evidence at the crash scene. In attempting to remove the vehicle he may create tire scuffs or skids, fluid deposits, scratches, gouges, and similar type evidence which can be confused with actual crash damage. The author is aware of one instance in which an entire 66-passenger school bus was dragged for one mile on its roof during removal. Extensive damage to the vehicle extricating injured occupants can similarly be misleading. Power "jaws" used to force open vehicle doors or sections can damage the vehicle beyond that of the crash. Deliberate dumping of fuel from the vehicle's fuel tank by emergency personnel (or tow operators) can also lead to confusion. Fuel is often emptied from a vehicle fuel tank by striking the tank with a sharp pick, axe or other similar puncturing tool so as to evacuate the fuel tank prior to removal. This sometimes occurs after the vehicle has been righted, moved onto the roadway, and attached to the towed vehicle. Fluid deposited from this position will be confusing as it has no relevance to the crash.

VEHICLE EXAMINATION - 5

Respect for, and friendship with tow operators are essential ingredients for successfully conducting crash investigations on a long term basis. The tow operator's job should be understood as should his relationship with clients and insurance companies. Friendship with the tow operator's dog is an absolute "must" relationship for the successful investigator. This may not always be achieved but should be an objective for which the investigator should constantly strive. Respect for the tow operator's dog is essential. Failure to show proper respect invariably results in some form of physical pain. The accident investigator should be unobtrusive, courteous, demand little of the tow operator's time and effort, and not interfere with tow operations while on the operator's premises.

The tow operator can be a source of valuable information regarding crashes to be investigated. Remember that the tow operator or someone in his employ was on-scene at the crash awaiting the signal from the police to remove damaged vehicles and their parts. The tow operator can be an informed observer in this process and often may be aware of facets of the crash scene that police overlook. Periodic thank you letters to the tow operator will go far toward forming a friendly and cooperative relationship.

Access to crash vehicles on the tow operator's premises can be a serious problem. Tow operators will park vehicles as close together as possible in order to make maximum efficient use of their available space. It may even be impossible to move between vehicles. This is a condition the accident investigator must accept. In some instances, friendly tow operators move vehicles as a courtesy for the investigator. Once extended, their courtesy should be accepted and not abused. Where the tow operator will elevate or reorient a vehicle for examination the investigator must be innovative in his examination of the vehicle. He must be even more innovative when the tow operator will not move or elevate the vehicle. Use of mirrors is sometimes helpful in vehicle examination. Also useful is the ability to take pictures with a camera extended out in the palm of the hand (away from the investigator) while viewing undercarriage areas at ground level.

Where situations exist at the tow operator's facility which limit

the vehicle examination, the investigator should note the situation on his vehicle examination protocol. This is helpful to the editor as well as to the analyst when questions arise regarding an incomplete examination of the vehicle.

2.4 Exterior Damage Assessment

The investigator should always have the police accident report in hand before initiating an examination of the vehicle. Prior to examining the vehicle, it is helpful to be aware of the number of occupants, as well as injuries sustained in the crash. Based on knowledge of how the occupants moved during the crash, the latter will lead the investigator to occupant contact areas and points within the vehicle.

A first step is to determine if the damage exhibited by the vehicle is consistent with your understanding of the crash in terms of crash orientation and crash dynamics. Always look for evidence of more than one impact. Many crashes which are listed as a single impact will actually involve more than one impact, e.g. a vehicle may deflect after impact into a fixed object resulting in a second impact, or vehicles may rotate abruptly and contact each other a second time. Crash damage to the vehicle is helpful in determining the object contacted. It can also help establish how the object was struck. This is most easily observed for impacts with narrow objects. In vehicle-to-vehicle impacts the investigator should look for damage to mate, or fit together in profile (between the two vehicles). This can be much like a jigsaw puzzle in which the contour of one piece must fit the contour of another piece. Many times objects or attachments on one vehicle can be made to fit corresponding damage on the other vehicle. Where vehicles have engaged during impact, that is, where they have remained together as a single object before separation, disengagement can alter the damage characteristics to each vehicle significantly. This phenomenon is characteristic of quite severe crashes where vehicles behave more as plastic bodies than elastic bodies.

The investigator should be aware of stiff and soft areas about the vehicle for various vehicle designs. In body and frame vehicles, the skeletal frame tends to accentuate the stiffness of various locations

about the vehicle. It is important here to observe whether two vehicles impacting each other engage such that the frames of each vehicle are involved—that is, such that one vehicle does not override or underride the other to a significant extent.

Ejection portals should be noted. These may be any opening in the vehicle through which a human body may pass (i.e., side glass openings, windshield openings, backglass openings, etc). Such openings need not be large. Humans may be ejected through openings which at first glance may lead one to believe human ejection is impossible. The investigator must remember that the human body is a rather pliable form which can take various shapes or postures.

Passenger compartment integrity should be thoroughly assessed. Damage which compromises the integrity of the passenger compartment through reducing its volume or opening up the compartment, should be measured and noted accurately in accordance with the proper investigation protocols. The investigator is encouraged to provide notes as part of his field data which clarify and elaborate on the various entries which relate to overall vehicle performance in the crash. Note whether the windshield has been penetrated or separated from its bonding, whether there has been pillar failure, door latch and door hardware failure, or external object intrusion.

Gross vehicle failures in a crash should also be noted. These could be in the form of a large rupture of the passenger compartment, such as separation of its roof, doors ripped off, and other gross damage characteristics which render the passenger compartment useless as a secure envelope for its occupants.

All damage should be carefully observed so as to determine whether it is direct damage, or induced as a result of impact forces elsewhere on the vehicle. Vehicle wheel movement both pre-crash and post-crash should be considered. Large abraided spots on the tire tread stock can be a clue to post-crash vehicle movement or movement off the roadway. Smooth pavement will cause rubber abrasion distinctly different from that caused by dirt or gravel road surfaces.

The determination of a correct and accurate collision damage

classification index is essential to characterizing the damage to the vehicle. Care should be taken toward establishing the principal direction of force relating to damage. This is also called the line of impact, force vector, or damage vector, and is the vector of interaction between the colliding vehicle and the object with which it impacted. As a vector it has both direction and magnitude and is the result of the crash forces each vehicle imposed upon the other during the crash. It may also be thought of as the line along which the vehicles collided. It is often established by the direction on which sheet metal or attachments to the vehicle moved or crushed as a result of crash forces. The principal direction of force is, in general, a line with respect to the longitudinal axis of the vehicle which points in the direction of which damaged parts were pushed or crushed from impact. It also indicates the direction in which one vehicle was travelling with respect to the other at that last instant before impact. Damage area classification and extent code classification should be in strict conformance to the CDC protocol which defines in detail the application of the CDC.

Fuel escaping from crashing vehicles always presents the potential for fire and greater bodily harm than may result from crash forces alone. The entire vehicle fuel system must be examined to assess its integrity during the crash. The fuel system is located throughout the undercarriage area of the vehicle. The tank is commonly in the rear with lines in front carrying fuel to the engine. Components comprising the fuel system may be ruptured or opened in various places as a result of crash damage. While these may not always be observable, it is incumbent on the investigator to attempt an assessment of whether fuel leaked as a direct result of the crash. The bulk of the vehicle's fuel is contained in the fuel tank. Approximately one liter of fuel may be distributed in the fuel lines, carburetor and fuel pump of the vehicle. Automobiles with fuel injection systems are most susceptible to fire because fuel is handled under pressure and in an atomized form, making it more susceptible to fire when ignited in a crash.

Evidence of fire in a crashed vehicle should be noted carefully. It may be observed on the vehicle's undercarriage, on the engine or

passenger compartments, or on its trunk or cargo space. Determining the ignition mechanism for fires is often not easy. The investigator should look for damage to electrical components or parts of the electrical system which may be a cause of ignition. Metal corners of the vehicle, or metal ends of damaged portions of the vehicle, which contact with the roadway may be ignition mechanisms. The object struck, and less obvious ignition mechanisms associated with the occupants in the passenger compartment interior, should also be considered.

Combustibles which burned in fires should also be noted. These may be vehicle fuel, oil, cargo, and other materials associated with the vehicle which burned. Vehicle fuel is considered an accelerant to fire, and a fuel-accelerated fire is readily distinguishable.

Damage to wheels and tires on vehicles also require careful examination. Often a driver may claim or conjecture that a tire condition, or tire failure, contributed to the crash. The investigator should be aware of the differences between tire damage which may have existed prior to a crash, damage which resulted from the crash, and damage resulting from the removal of the vehicle. Roadway tire marks often indicate tire failure prior to the crash. This is evidenced by flat tires or wobble scuffs which are irregular black tire material deposits on the roadway. These scuffs are indicative of a deflated tire being distorted between the road and the rim as it rolls flat along the pavement. Damage to tires and wheels should be noted carefully. Here a sketch of the tire with its valve stem, or serial number as a benchmark may be used to sketch and indicate damage. Knowledge of the terminology used in describing wheels and tires is most helpful here. The attached sketch shows a typical bias ply tire cross-section with descriptive nomenclature.

Blowouts or catastrophic failures of tires, which precipitate a crash, exhibit unique characteristics. They are indicated by those features of the tire which appear to have been caused by the explosive deflation of the tire. Ruptured cord fiber ends are frayed and fuzzy as if pulled apart by great force. The blowout may be the result of some incipient condition of the tire which is obscured as a result of the

GROOVE   RIB
PLY   TREAD

BELTS

SIDEWALL
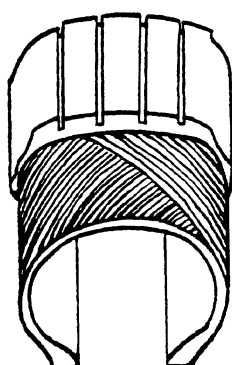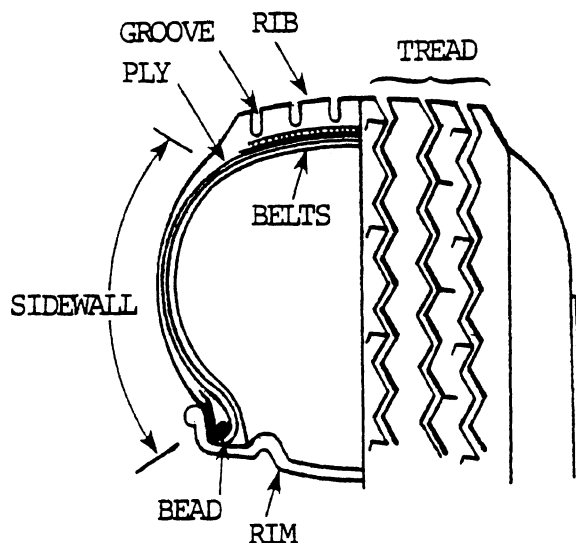
BEAD
RIM

Bias-Ply
Construction
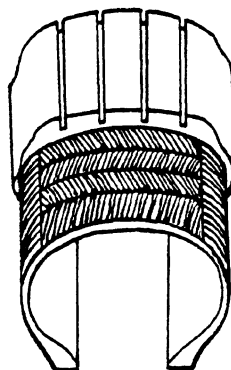
Radial-Ply
Construction
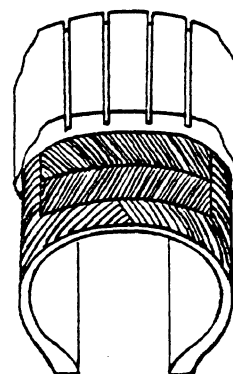
Belted-Bias
Construction

DEFINITIONS

RIM: A metal support for a tire or a tire and tube assembly upon
which the tire beads are seated.

BEAD: That part of the tire made of steel wires wrapped or reinforced
by ply cords, that is shaped to fit the rim.

SIDEWALL: That portion of the tire between tread and bead.

CORD: The strands forming the plies in the tire.

PLY: A layer of rubber-coated parallel cords.

TREAD: That portion of the tire that comes into contact with the road.

TREAD RIB: The tread section running circumferentially around the tire.

GROOVE: The space between two adjacent tread ribs.

BELT: A layer or layers made of fabric or other material located under
the tread area.

sudden severe damage from the blowout. The blowout is a sudden or explosive loss of pressure in the tire as a result of tire failure. A blowout may occur before impact, during impact, or after impact from a previous tire condition or from forces acting on the tire. Fresh cuts or slits in the tire are more readily associated with crash damage resulting from sharp metal edges coming in contact with the tire.

Damage to the wheel rim, upon which the tire is mounted, can also indicate tire damage. Tire failure can often be correlated with the same impact forces which resulted in damage to the rim. Rim "dimples" are sharp, narrow deformations to the wheel rim edge, usually resulting from contacting sharp stiff portions of other vehicles or sharp stiff off-road structures.

2.5 Vehicle Impact Orientations

While the analyst may be forced to work with a few basic crash orientations to compile usable group accident data, the investigator is aware that there are many variations to these basic crash orientations. Head-on collisions where frontal areas of both vehicles are brought into contact with one another may be offset by varying degrees and may be at various oblique angles rather than be colinear. The head-on collision between two vehicles in which damage is uniformly distributed across the entire front of each vehicle, is quite rare. Most head-on crashes involve less than 100% overlap and occur at offset angles to one another. This may be further confounded by the override or underride of one vehicle relative to another. Override occurs when the plane of the frame of one vehicle doesn't meet the plane of the frame of the other vehicle. Damage characteristics to these vehicles will dictate which vehicle overrode or underrode the other. Vehicles behave neither as solid, rigid objects nor as homogeneous and elastic objects. They behave somewhere in between, thus making predictions of how vehicles move against each other at impact, after impact, and away from each other less than precise.

Head-on collisions in which one corner of the vehicle impacts a similar corner on the other vehicle also occur. The left front corner or left front corner orientation is most common because vehicles travel on the right hand side of the roadway in the United States. These crashes

always result in rotation to one or both vehicles. Each vehicle is deflected from the point of impact towards its own side of the road while rotating counterclockwise. For right-front corner impact, rotation is clockwise. In corner to corner impacts the vehicles will also move away and in their general direction of pre-crash travel while rotating. As they separate they will thus pass forward in their same direction of travel moving past the point of impact. For oblique, or non-colinear impacts, accurately predicting a post-crash movement of vehicles is difficult. Occupant injury patterns are more predictable for head-on collisions than for other crash orientations.

Side impacts between vehicles encompass a wide variety of crash orientations and vehicle areas. The investigator should attempt to determine whether the vehicles remained engaged through part of the crash phase and where disengagement occurred. Engagement occurs where the vehicles fuse together momentarily and behave as a single body before separating and following their individual paths to final rest. Override and underride damage features on the vehicles should be noted. Vehicle sides are relatively soft compared with the front ends. Side impacts accompanied with override can result in great damage at low impact speeds. Door and pillar damage should be carefully noted, as should passenger compartment intrusion and the impact angle between the colliding vehicles. The post-impact movement between vehicles should be consistent with the principal direction of force to each vehicle and the point upon each vehicle where the force was applied. At-crash and post-crash wheel movement may be indicated by tire rubber deposits or rim scrapes on the roadway since wheels may be forced to move laterally. This leaves a scrubbing or wiping type of tire mark on the roadway. The side impact crash orientation is one in which vehicles may also come together in a second impact. After the first impact, the front portions of the vehicles may be deflected away from each other in rotation so as to bring the rear portions of the vehicles into contact with each other.

Rear-end impacts should similarly be examined closely to determine if an override/underride condition existed between the two vehicles. The tie-down mechanism for the spare tire on one vehicle can be an indirect measure of force in the impact, since the bracket which retains

the tire requires a force of particular magnitude to fail. Because of this the investigator should make an attempt to determine if the spare tire mounting was separated as a result of impact. Similarly, fuel system integrity takes on greater importance in rear-end collisions since any failure to the fuel system can result in large amounts of fuel spillage because of the proximity of the fuel tank to the vehicle's rear. Examination of the fuel tank, its mounting and attachments takes on greater importance in rear-end crashes. The investigator should note fuel tank failures or punctures, separation of the tank from its mounting, filler pipe separation from the tank, filler pipe separation or distortion as a result of being fastened to exterior sheet metal (where that applies), and fuel line separation.

One interesting type of fuel tank damage, characteristic of severe frontal impact, is hydrodynamic distortion. This occurs when the mass of fuel in the fuel tank, is large enough to result in a bending deformation to the front of the tank. The fuel continues to move forward at the pre-crash velocity of the vehicle and deforms the frontal area of the tank as the vehicle is abruptly decelerating from impact.

Impact angles between vehicles in rear-end collisions are easier to determine than in frontal type crashes. The reason for this is that the rearend of a vehicle is considerably softer and less rigid than the front end of the vehicle. In case of rear impact, the principal direction of force is more evident from damage characteristics. The investigator should note damage to the rear wheel housing areas which restrict rotation of the wheels. Where the wheels are "locked up" as a result of crash damage they will leave characteristic scrape marks and tire skidmarks on the pavement from impact to rest. Additional clues for determining the severity of rear-end impacts can be found by discussing whether the rear suspension of the vehicle was displaced forward, or the rear axle displaced forward about the differential unit. Compressive damage to the power drive line should be noted where evident. This occurs when the propeller shaft, a long tubular span underneath the vehicle which carries engine power to the rear wheels, is collapsed and distorted from force being applied to the rear of the vehicle.

Occupant injury patterns characteristic of rear-end crashes differ from those in the head-on and side impact orientations. Occupant movement when the vehicle is impacted in the rear is initially rearward with the occupant forced back into the seat and head rest. One injury that is typical of the rear-end crash is whiplash. Whiplash occurs when the occupant's head is first hyperflexed rearward and then abruptly hyperextended forward. The mass of head pivots about the neck, first rearward then recoiling back forward. The performance of the seat and head rest in rear-end impacts is important. Damage to the interior frontal areas of the vehicle (steering column, instrument panel, controls, etc.) is the result of unrestrained occupant rebound and involves considerably less force to the occupant than the force to which the occupant was subjected initially from the rearward movement at initial impact.

2.6 Rollover

The rollover crash is perhaps the most difficult of all impact configurations to reconstruct. Unlike other crash orientations, such as head-on, side or rear (which result in but a single impact), the rollover actually consists of a series of distributed impacts as the vehicle contacts the ground with its sides, top, front, rear, and corner surfaces as it rolls over. It is important to note the number of impacts to which the vehicle was subjected in rollover so as to best determine its rollover pattern and trajectory. The number of impacts, or areas on the vehicle which come into contact with the ground other than the tires, will help determine the number of rolls, or total rotation to which the vehicle was subjected. Impact areas on the vehicle also help determine what the vehicle came into contact with. Often this is not the earth but stumps, large rocks, or other irregular surface features. Rarely will a vehicle roll over and remain on the travelled portion of the roadway. In rollover configurations where the vehicle is "tripped" by a curb, or other irregular surface feature while moving forward and yawing, the upper A-pillar is usually subjected to the greatest impact forces. This can be a clue to vehicle orientation as it began rollover. Rollover may be about the lateral axis of the vehicle—that is, head over rear. It may also be about the longitudinal

axis of the vehicle, from side to top to side to wheels. It may be a combination of both. Various rollover orientations, about all three axis which define vehicle movement can make interpreting the rollover difficult. Sheet metal crush and structure deformation as a result of impacts in a rollover are informative in attempting to reconstruct the rollover. Scratches or long grooves in impact areas about the vehicle indicate how the vehicle was moving when it contacted the ground. Vehicle "tripping" mechanisms in rollover may be evident from examining the crash scene. Sod and grass forced in between the tire bead and rim may correlate well with mounds in the earth left from wheels moving laterally. Large open areas of the earth where the sod has been scooped up indicate that the vehicle may have bottomed out from fully compressing or hyperextending its suspension.

To reconstruct the rollover accurately the investigator should prepare a scale drawing of the crash site and superimpose upon it all crash evidence available from the scene examination. This can then be correlated with the crash damage to the vehicle so as to make the best fit possible between vehicle damage, roadway evidence and the vehicle's final rest position and orientation.

An assessment of fuel systems performance should similarly be made for rollover crashes. The recently revised version of Federal Motor Vehicle Safety Standard #301, "Fuel System Integrity," contains a performance test for the leakage of fuel while a vehicle is held static in a rolled over orientation. Evidence of fuel leakage may be found by examining the dust-and dirt-covered undercarriage areas of the vehicle for fuel deposits as well as examining fuel containing components.

2.7 Catastrophic Crash

Crash damage resulting from high speed, high energy, severe crashes is often so extensive as to make an assessment of the vehicle's performance in the crash impossible. In such cases crash forces are so large that many, or all, of the protective features of a vehicle fail. Crashes of this magnitude leave little from which the investigator and researcher may learn. Needless to say, there are distinct limitations in reconstruction of such crashes because of the catastrophic, or near explosive nature of the damage to the vehicle. Such vehicles may be in

two or more pieces after the crash, or crushed to the point where available occupant space is almost non-existent. The vehicle may be ripped apart to the point where occupants are subjected to great forces inside and outside the vehicle. The investigator may be confused by extensive damage from extrication of bodies and from dismemberment of the vehicle for ease in tow or transport from the accident site. He should make as many observations about crash damage as possible and if possible determine the principal direction of force to the vehicle from the initial impact. In such crashes gross damage patterns are perhaps best described by extensive photography.

2.8 Vehicle Interior Examination

The vehicle interior can provide evidence as to how occupants moved, what they contacted, and what damage resulted from this contact. Contact areas may be distinguished by tissue deposits, hair, fabric transfers, skin oil, cosmetics, evidence of jewelry worn by the occupant, imprints of occupant's features, as well as by the deformation characteristics of contacted areas. Skin oil may be discerned on the door side glass adjacent to occupant seat positions by examining the glass at an angle such that the ambient light strikes the glass so as to enhance the characteristic pattern left behind. Dust within the vehicle's interior, when it is removed or disturbed as a result of occupant contact, can also be an important clue to occupant contact. A hand-held magnifier can help the investigator observe tissue, hair, blood or fabric transfers on the interior that are invisible to the naked eye.

Occupant contact with the vehicle controls is indicated by damage to those controls. A deformed steering wheel rim, bent shift lever, or distorted dash panel control knobs indicate occupant contact. Knowledge of the injuries sustained by the occupant prior to examining the interior should be obtained wherever possible. This will assist the investigator in knowing where to look and what to look for, as well as in differentiating between damaged interior areas which were not contacted by the occupant and those that were contacted.

2.9 NASS Vehicle Form CCS

The vehicle examination field form developed by NASS permits the investigator to record key identifying features and damage features of the vehicle. These are well structured and unambiguous, so that NASS investigators in various geographic locations will provide information that is both consistent and accurate. The investigator is again encouraged to make additional notation where necessary to clarify or elaborate on entries made on structured data forms.

Of particular significance here is the addition of vehicle sketches showing the side, front, and end views of vehicles in silhouette form with notation of key damage features. Such sketches help the investigator locate and describe graphically the various damage features and characteristics for the vehicle. Identify marks on one sketch and relate them to another sketch of differing size and detail. Similarly, identifying marks can related to photographs taken of areas described by notation on the sketch. This is most helpful in accurately conveying damage features which the investigator feels are important. One suggestion is that the investigator pause and review the entire form upon its completion with a view toward determining whether someone unfamiliar with the accident could be completely apprised about the vehicle both qualitatively and quantitatively from the information presented in the field form and field notes . Additional sheets for narration and/or sketches should be included where they are felt to be helpful.

3 DATA DOCUMENTATION

Summary The art and science of investigating accidents involves acquiring data in the form of identifying numbers and codes, noting geographical features of the roadway and off-road environment, making observations of physical features and evidence, and obtaining statements from others (vehicle occupants, police, etc.) involved in the accident process. This can be a large amount of information that must be hand-organized and preserved. The ability to accurately and thoroughly document information provided by a crash is essential to the investigation of crashes. This section discusses some approaches and methods to acquire this data.

The accident investigator must first of all be a good observer. To observe means to visually search out, identify, and catalog physical details of the crash scene and damaged vehicles. Together these provide as much of an explanation of the crash under study that is available. Becoming a good observer takes practice and concentration. One should first become disciplined to think through the various elements involved in a crash in an orderly manner. This minimizes overlooking relevant evidence. The police report of the crash, and supplemental information available to the investigator, becomes the basic information on which to start the investigation. This information is oftentimes brief--such as the number of vehicles involved, basic crash orientation (head-on, side, rear, etc.) and an estimate of how many occupants were involved. From this point on, it is up to the investigator to develop whatever information is required.

NASS, NCSS, and other similar data collection projects, which examine accidents well after they occur, have well structured and detailed field data forms for the documentation of many key crash features. These include questions which identify various components (vehicle, roadway, human) of the crash and describe the consequences (damage, injuries, roadway performance, etc.) of the crash. Responses to these questions (and others) contained in the basic field data protocols (vehicle, scene, driver, etc.) are usually limited to a list of the most probable responses. This method of recording data is designed so as to be compatible with machine data storage and handling. Each response is designed so that values listed are clear, specific, and unambiguous. One word labels or responses are necessary in this system

for ease in keypunching and entering into a computer file by their appropriate code values.

Such structured field data protocol forms will not, however, tell the complete story of the crash. Only listed responses are permitted to a limited number of specific questions. Augmenting these data with information which both describes or identifies the area of the crash being investigated can be helpful as well as necessary. It can be helpful in that it could permit an analyst at some future time to review "hard copies", or the actual completed field forms, and confirm or refute certain inferences made from a subset of similar cases. It may also be necessary at times to more accurately reconstruct the crash and arrive at a more precise quantitative assessment of crash severity, such as "delta V." Perhaps the most important attribute of thoroughly documenting each case beyond the structured responses provided in the field data form, is that it permits an editor, or analyst, as well as the investigator to get a better understanding of the crash. The investigator should never leave important observations or measurements to memory.

3.1 Field Data Recording Methods

Mapping topographical features of the crash scene permits the drawing of plan view layouts of relevant items and features of the crash scene. The location of permanent, easily identifiable "benchmarks" must first be established. These may be manhole covers, utility poles, fire hydrants, curb cuts, permanent pavement markings and similar features. Benchmarks should be located so that they are convenient and near the area containing key roadway features and crash evidence. They are needed to establish the framework, or reference points, for the scene sketch.

Roadway features and crash related items can then be described with reference to such appropriate "benchmarks". For instance, the slope of a roadway shoulder can be references to a specific point on the roadway. A profile, or cross-section of the shoulder may then be drawn in a separate sketch but referenced to a line in the plan view drawing locating the specific point, and distance, where the cross-section is important. Special features may also be noted on the plan view drawing

by an arrow with explanatory narrations. An arrow may also indicate that another sketch, of expanded scale for instance, exists to better illustrate the particular features of interest.

Location of physical evidence on the sketch should be accurate and oriented in its proper position with respect to road edge, off-road benchmarks and other physical evidence. A more detailed description of preparing scene drawings is given in the chapter on Scene Examination.

A small pocket size tape recorder is a helpful aid when working alone and documenting scene evidence and features to be incorporated into the sketch or drawing. Verbal descriptions far beyond abbreviated notations on a sketch can be conveniently recorded.

3.2 Symbols for Documenting the Crash Scene

Some form of uniform use of symbols is helpful for both examining the crash scene and for later annotating field sketches. Marking the crash scene is helpful to accentuate important physical evidence such as skidmarks, fluid deposits, scratches, etc. This can be done through the use of chalk, contrasting colored crayon, or spray paint. When included in photographs, they bring the viewers attention immediately to the object or feature of interest. Marking key measurements directly on the pavement is also useful when studying the scene later from photographs. Marking of the point of impact is perhaps the one feature of most importance which should be accurately located and marked.

The same symbols used in the field may also be incorporated later in field sketches. This is sort of an outdoor sketch shorthand. Two sets of symbols are given in Figures 3.1 and 3.2. The latter contains symbols which can be made from the Northwestern Traffic Institute Accident Investigator's Template. Figures 3.3-3.7 are a set of symbols[1] developed under a NHTSA study combined with symbols used in the NHTSA NASS program that has been found to be useful when investigators are on-scene. Marking of crash scene skidmarks and tire scuffs is of particular importance. This includes identifying each wheel, the start and termination of tire marks and outlining faint

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[1] They are also contained as Appendix A of the NASS Data Collection Coding and Editing Manual, February 1979.

areas. The final rest position of crashed vehicles can be conveniently located by marking and identifying each vehicle position by a combination of marks locating all wheels and corners of the vehicle.

The investigator must use whatever symbols and markings methods felt to be superior. One other template similar to the Northwestern template that is useful for making symbols on sketches is available at nominal cost at any engineering supply store. This is Rapid Design Traffic, Symbol Template, number 130, shown in Figure 3.8.

3.3 Measurements
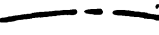
Making measurements of crash involved vehicles is a procedure fundamental to all NASS investigators. Many devices, or aids can be helpful here. Some of these are magnets to anchor string or metal measuring tape, standards or posts which fasten vertical to a freestanding base to outline corners of the vehicle, surveyor's pins, surveyor's ribbon, magnetic arrows or similar symbols, calibrated cylindrical portable tubes, mirrors and other devices. All of these can be helpful at certain times and for certain crashes. There is no hard and fast rule which dictates how measurements should be made, and what aids are most helpful. These are more like personal accessories of the investigator to be used as he, or she, finds them to be most effective.

Measurements of scene data can similarly be expedited and enhanced with various measurement aids. Intersecting road edges serve as convenient and accurately located benchmarks, i.e., where the road edge, or curb line, of intersecting streets are visually extended as tangents[2] to create a point within the intersection where the two tangent lines intersect. This point provides a convenient benchmark for measurement within and near the intersection.

Use of road edges that are straight can be helpful in making scene measurements. A straight road edge can serve as a base line from which measurements can be made to roadway features and evidence. These should be made at right angles, to the road edge out to the items or areas of interest. A carpenter's square can be used to establish a right angle,

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[2] A tangent is a straight line continuation outward from the periphery of a circle, in line with the curved portion of a circle.

Figure 3.1   UNIFORM SYMBOLS FOR SCENE MARKING

ROAD

- MARK TO SHOW BEGINNING OF REAR SKIDMARKS - ARROW SHOWS DIRECTION OF TRAVEL - NUMBER INDICATES IDENTITY OF VEHICLE INVOLVED

- MARK TO SHOW BEGINNING OF FRONT SKIDMARKS - ARROW SHOWS DIRECTION OF TRAVEL - NUMBER INDICATES IDENTITY OF VEHICLE INVOLVED

- POSITION OF REAR WHEELS AT IMPACT | NOTES END OF POST - IMPACT SKID

- POSITION OF FRONT WHEELS AT IMPACT | NOTES END OF POST - IMPACT SKID

RⁱR   - REAR WHEEL AT FINAL POSITION

RⁱF   - FRONT WHEEL AT FINAL POSITION

⊗   - POSITION OF IMPACT POINT   1 - FIRST IMPACT
2 - SECOND IMPACT

- INDICATIVE MARK FOR SCRATCHES

- INDICATIVE MARK FOR GOUGES

- INDICATIVE MARK FOR SCUFFS

- INDICATIVE MARK FOR CENTRIPETAL CURVE SCUFFS

- INDICATIVE MARK FOR ROTATING TIRE PRINT

- INDICATIVE MARK FOR PUDDLE (LIQUIDS)

- INDICATIVE MARK FOR PUDDLE WITH RUN-OFF

(INITIALS - G FOR GASOLINE; M FOR MOTOR OIL; R FOR RADIATOR COOLANT; T FOR TRANSMISSION OIL. B FOR BATTERY ACID; F FOR BRAKE FLUID; W FOR WATER; AND H FOR BLOOD TO BE INSERTED INSIDE THE CIRCLES FOR FURTHER IDENTIFICATION).

- INDICATIVE MARK FOR DEBRIS - ARROW TO SHOW DIRECTION OF FORCE

- MALE BODY (ARROW POINTING TOWARD FEET)

- FEMALE BODY (CROSS INDICATING DIRECTION OF FEET)
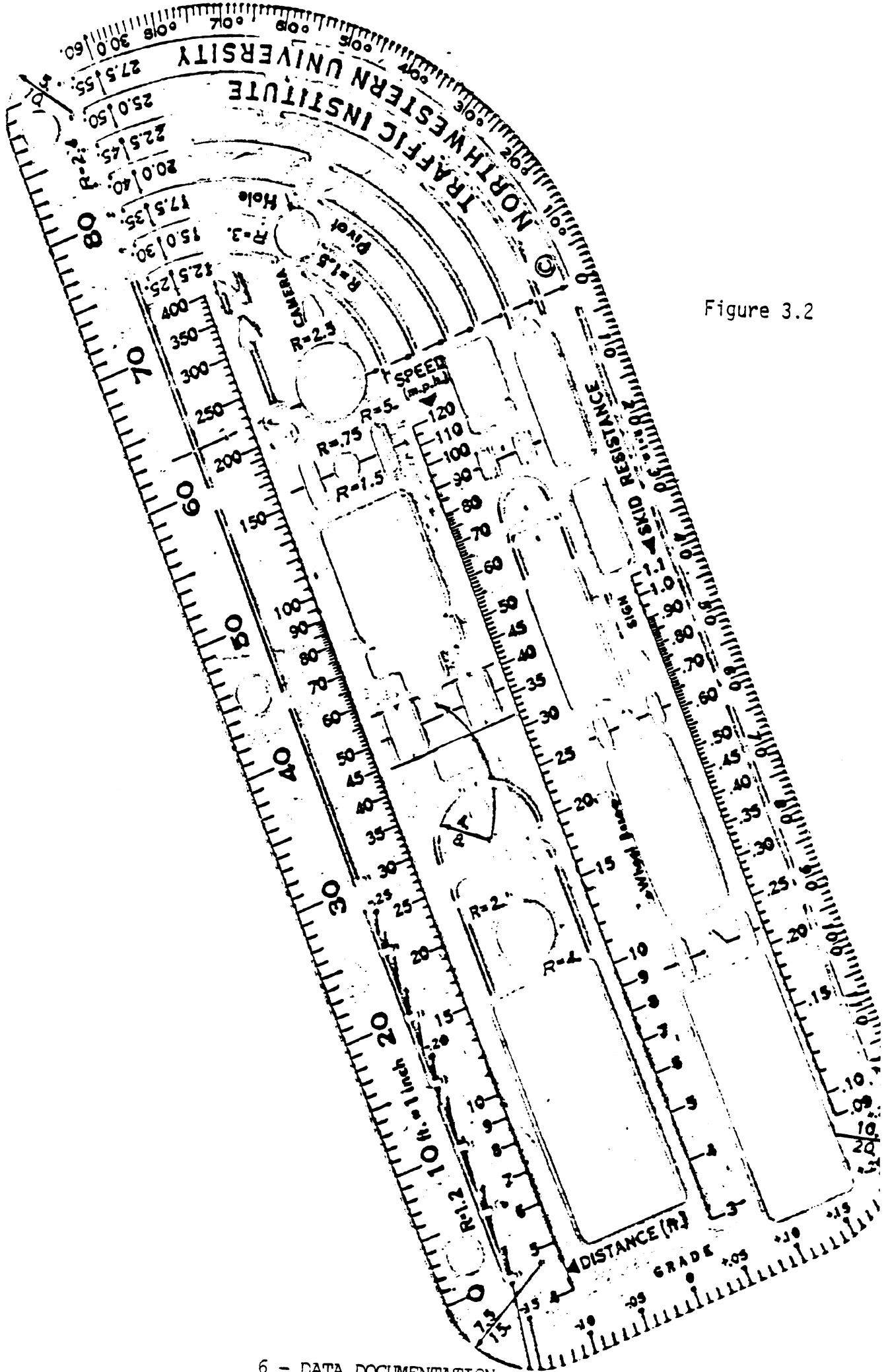
Figure 3.2

Figure 3.3   Topographical Highway & Environment Symbols

———————————————————————— Pavement edge

〰〰〰〰〰〰〰〰〰〰〰〰〰〰〰 Shoulder edge line (non-formal)

—— —— —— —— —— Shoulder edge line (formal)

—   .   ——————   . Broken center or lane lines (15' long - 25' apart)

══════   ══════ . Broken center line with No-Passing line

═══════════════════════ .Double yellow center lines

┌─────────────┬─────────────┐
│:Concrete:/Grass─┤ Raised island and Grass median
└─────────────┴─────────────┘

┌─────────────────────────┐
│/////////////////////////│ Painted median
└─────────────────────────┘

———————————————————————— Curb

┌─────────────────────────┐
│/////////////////////////│ Paved shoulders with diagonal lines
└─────────────────────────┘

➤   ➤   ⇒ Turn arrows

▭▭▭▭▭▭▭▭▭▭▭▭▭▭▭▭▭▭ Wall

▨▨▨▨▨▨▨▨▨▨▨▨▨▨▨▨▨ Bridge abutment and railing

—•——•——•——•——•——•— Guard rail

Figure 3.4

——— . ——— . ——— . ——— . ——— . ——— Fence

Railroad tracks

Embankment (arrows show "DOWN"

Shrubbery - hedges

Trees (draw trunk and perimeter of
                    foliage to approximate size

RED
GREEN   Traffic signal

RED
AMBER   Flashing light

Traffic signs back to back

Sign (indicate words or symbol:

Street light and pole (arm len
may change with scene)

o   Street light without arm

Public utility pole

Building

Fire Hydrant

Street Sign

Delineator post

All crosswalks, road surface symbols and other relevant markings should be
depicted and drawn to approximate scale on the diagram as much as possible.

Figure 3.5   UNIFORM SYMBOLS FOR ACCIDENT DIAGRAMMING

Vehicle and Pedestrian Symbols

Automobile (pre-impact or at-impact position) Exception: draw solid outline if stopped at-impact.

Automible (final rest position-showing damaged area)

Automobile (final position on its top)

Automobile (final position on its right side) (reverse for left side)

Automobile involved in the accident as as a temporary environmental factor, but not physically involved in the collision. (Non-Contact Unit)

Parked automobile not struck (give it a number if it was struck

Vehicle in which a witness was an occupant

Truck   (Panel, Van, Dump, etc.)

Truck tractor and semi-trailer

Utility trailer

Bus or streetcar

Motorcyclist; bicyclist (handlebars are curved opposite the direction of travel

Figure 3.6

SCENE ROAD MARKINGS

⊕   Point of impact

━━━━━━━━━━━━━━━━━━━━━━━━━ Skidmarks

⌣⌣⌣⌣⌣⌣⌣⌣⌣   Centripetal curve scuffs

،،،،،،،،،،،،،،،،،،،،،،،،،،،،، Tire scuff marks

▬ ▬ ▬▬▬▬▬ ▬▬ ▬ ▬ ▬▬ ▬ ▬▬▬▬▬▬ Rotating tire print

⬭ ○ ⬭   Gouges

∕ ⁄∕⁄∕ ⁄∕ ∕   Scratches

◯   Liquids (puddle and run-off)

∕ ∕ ⁄   Debris   (funnel out away from point of
∕ ∕ ⁄∕   impact to show direction of
∕ ∕ ⁄∕ ─   force)
─ ∕ ─ ═ ═
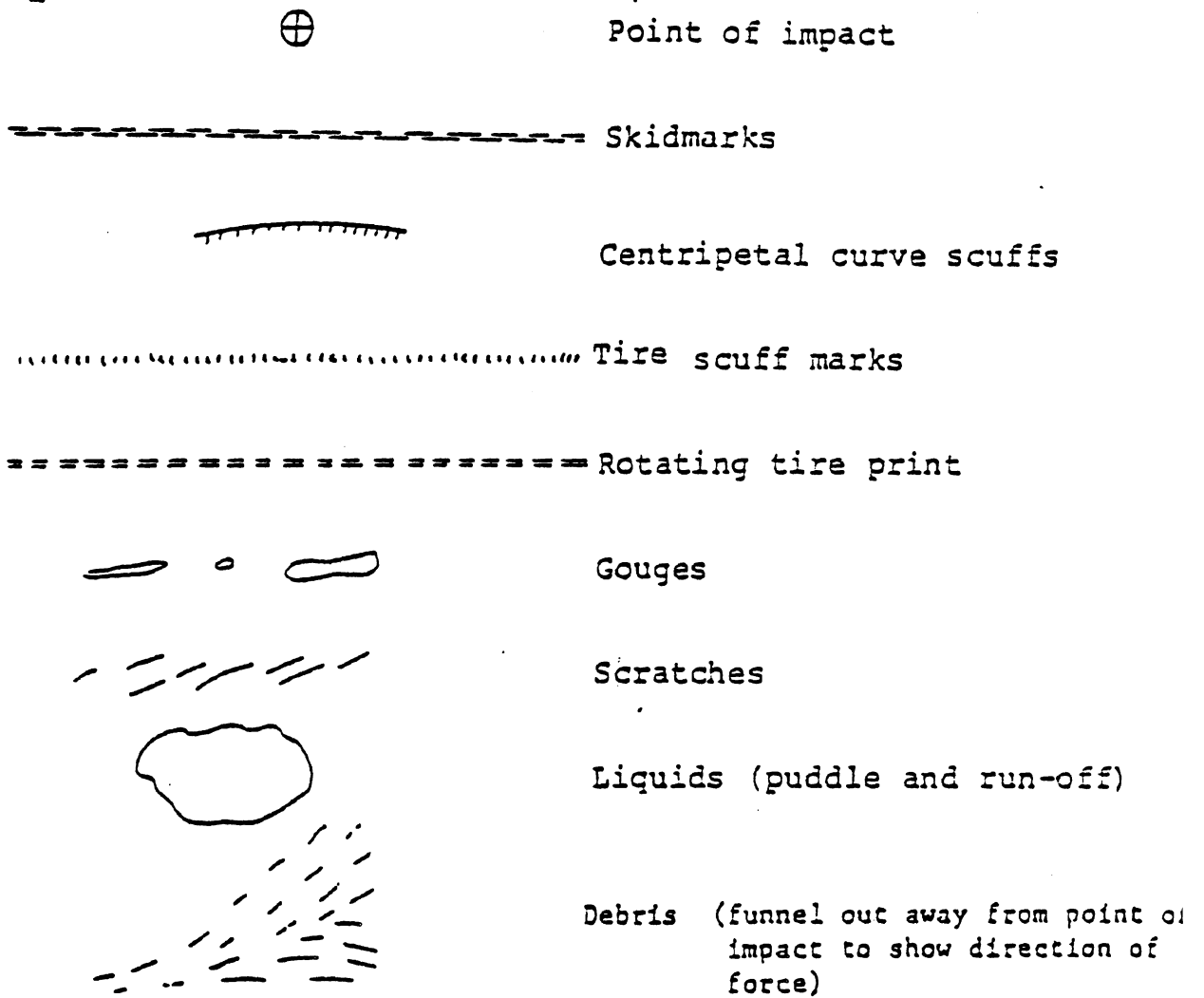
Any other accident-induced markings, components from vehicles. etc should
be shown in their approximate location and a reasonable likeness sketched
on the diagram.   However, do not clutter diagram; make an additional diagr
if necessary.

Figure 3.7
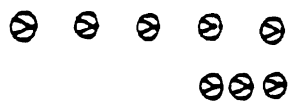
⊖ ⊖ ⊖ ⊖ ⊖   Pedestrian (pointer oriented to show
                                direction of movement and dot spacing
⊖⊖⊖   to show rate of movement; i.e., 3'
                                apart walking and 6' apart running

                        Final position of body

                        Pedestrian who witnessed accident

All symbols referring to colliding vehicles (plus Non-Contact, Witness and Parked
vehicles) are to have a broken outline if they are moving at the point in which
they are depicted; the outline should be solid if the vehicle is stopped where
depicted, or at final rest. Be careful to insure proper placement (location)
and orientation on the diagram.

Figure 3.8

12 - DATA DOCUMENTATION
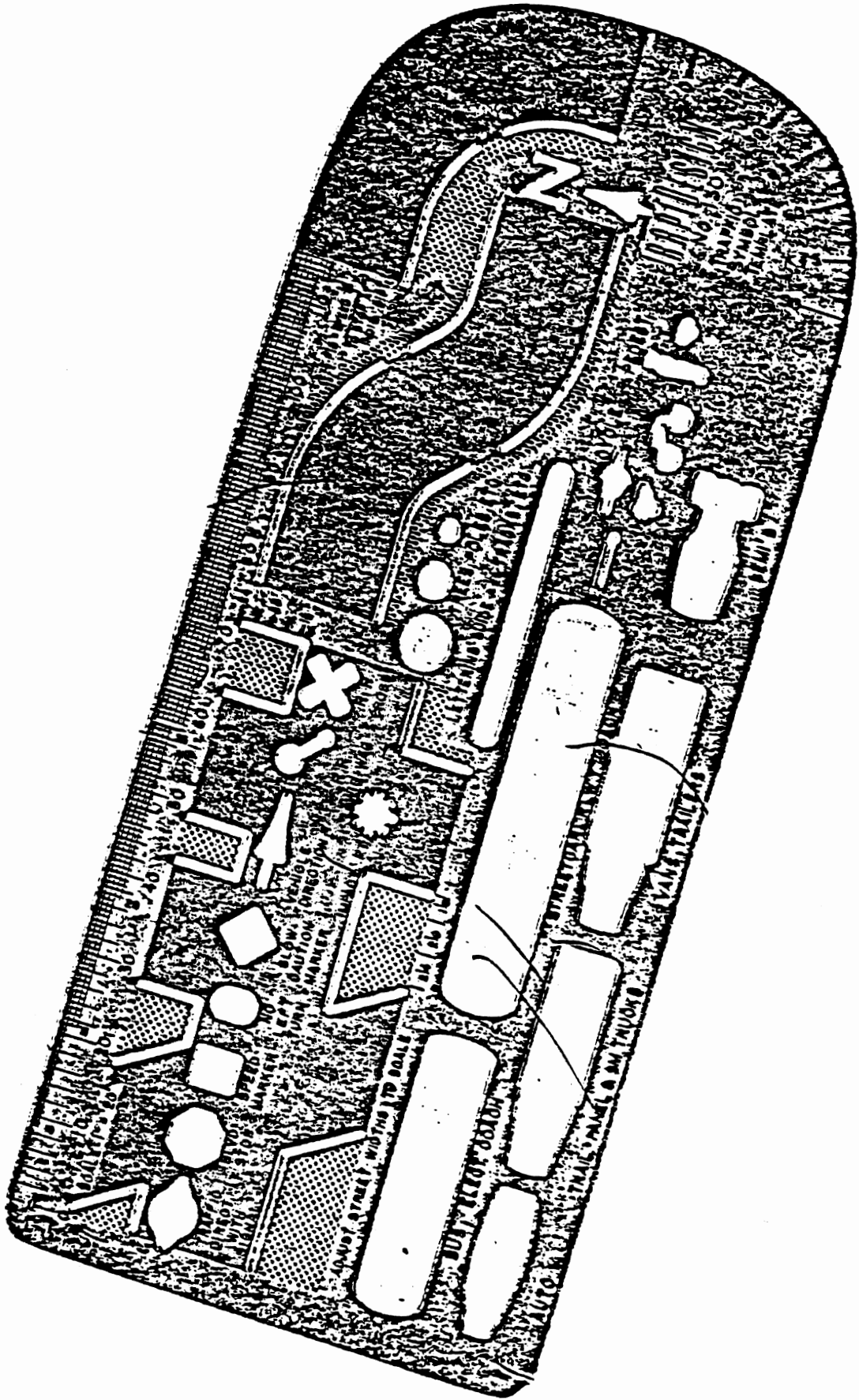
or the investigators flexible tape measure—extending and folding the metallic ribbon flexible tape to form a triangle whose sides are three feet, by four feet, by five feet, creates a right angle with which to line up measurements to the road edge.

Where a road pavement to shoulder drop-off, or differential exists it should be measured and noted on the scene sketch. Laying a coin up against a road edge, or some other convenient object of known size, then photographing it close-up against the road edge is one way of noting the information. Be careful to also note just where differentials exist along a road edge from a known benchmark. Marking the road edge at ten foot or 25 foot intervals will help in correlating close-up photographs of the road edge, with the same location in scene photographs with a wider view.

3.4   Sketch/Drawing Scale

Scene examination data forms often contain a gridded blank sheet upon which to construct sketches. Where this is inadequate use additional sheets for as many additional sketches as needed. Where distances greater than a 100 feet of roadway must be included in a single sketch to include all relevant objects and roadway features, a scale of 1 inch equal to 10 feet may be used. Various scales or graduations to permit scaled reductions of actual measurements are contained in an engineer's rule. For intersections of typical two lane, two way streets, a scale of 1 inch equal to 20 feet is convenient. For larger, expanded sections of roadway where the width of one or two traffic lanes can contain all the needed information, a scale of 1 inch to 4 feet is convenient. When information is concentrated in one small area, and this small area must be correlated with a larger area containing the entire roadway or large portions of the entire roadway, additional sketches made at a much smaller scale serve to expand upon portions of the larger sketch.

Grades and slopes can be measured as the drop from a level roadway using a straight edge level (carpenters level, or rod with level bubble, etc.) of known length and noting the height of one edge of the level, when adjusted to be level, with the roadway. This could be noted as the ratio of this height to the length of the level. For example, 3/4 would

mean 3 inches change in grade for every 4 feet of distance across the surface for 4 foot long level. By standardizing such notation, recording grade measurements became easier. This ratio, when in the same dimensional units, is also the tangent of the angle that the grade makes with the horizon, or from a level surface. Thus 3/4, or 3 inches for every 4 feet, gives a tangent of 3/48 or 0.0625. From a trigonometrical function table or hand calculator, this would indicate an angle of 0.001°. This could also be referred as a grade of 6-1/4% which is merely the tangent of the angle the grade makes with the horizon expressed as a percentage.

Keep in mind that traffic lanes vary between 9 and 12 feet width in modern roads. The interstate system has 12 foot width traffic lanes. Some older rural, low volume paved roads can have 8-9 feet wide traffic lanes. Passenger cars are nominally 18-20 feet long and 6 to 6-1/2 feet wide. Keeping these key values in mind will make it convenient in selecting a satisfactory scale for scene sketches. Use of a standard outline diagram, or silhouette, of a vehicle is helpful in graphical notating where damage is located and describing the type of damage. These are the same side, rear, front, overhead vehicle outline silhouettes a part of most vehicle field data forms. Carrying additional sheets of such vehicle outlines, representing the vehicle to be examined as closely as possible, makes it convenient to collect much additional data. Groups of these sheets, should be carried by each investigator and used as needed to graphically document vehicle characteristics of interest and vehicle damage in detail.

3.5 Photography

Taking sufficient photographs of the accident scene and accident vehicle is but a small investment in time and effort compared to the total cost of investigation. Well taken photographs are valuable in reviewing material collected on a crash, interpreting responses made in field data forms, and better understanding narration and notation in field notes. The old Chinese proverb certainly applies here; "A picture is worth a thousand words."

While an in-depth presentation of fundamentals of photography is beyond the scope of this presentation, there are a few areas dealing

with photography peculiar to accident investigation worthy of some explanation.

There have been many attempts at developing and using photography to document scene data which can also be used for extracting necessary scene measurements. This is called photogrammetry. Some elaborate stereo photographic instruments have been developed for this purpose, but in general have been too costly and complicated for every day use in the field by accident investigators. Such devices, or instrumentation, have worked out best where they have been made into an integral part of a larger mobile instrumentation center, such as a van.

There is one simple technique which is helpful when photographing a flat, level terrain which is inaccessible for free movement so as to make needed measurements. This is use of a perspective grid[3]. This consists of a rectangular grid with parallel diagonal lines located at angles of 45° within a basic rectangle as shown in Figure 3.9. Such a geometric reference device may be placed on the ground and in the center view of a photograph so that it appears within the photograph. From where it is located in photograph, the lines may be superimposed on the photograph. This may also be viewed as a "perspective template". These lines, extended from the grid in the photograph, will tend to converge in proportion to their distance in the photograph from the basic perspective grid from its place on the roadway. The known distance between lines on the perspective grid will remain constant enabling the viewer to measure locations and objects in the photograph with respect to the known distance between the grid lines superimposed on the photograph.

This technique is useful in situations where traffic or other roadway dangers prohibit the actual measuring of the roadway and roadway evidence by the investigator, while working on the roadway. The perspective grid may be located in key locations in the foreground of the area, framed by the camera, and a sequence of photos taken. Such

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[3]Perspective Grid Photography, by J.C. Crawford, Minnesota State Patrol, FBI Law Enforcement Bulletin, October 1978, and a Perspective Grid for Photographic Mapping of Evidence, by J.S. Baker, Report of Northwestern University Traffic Institute.
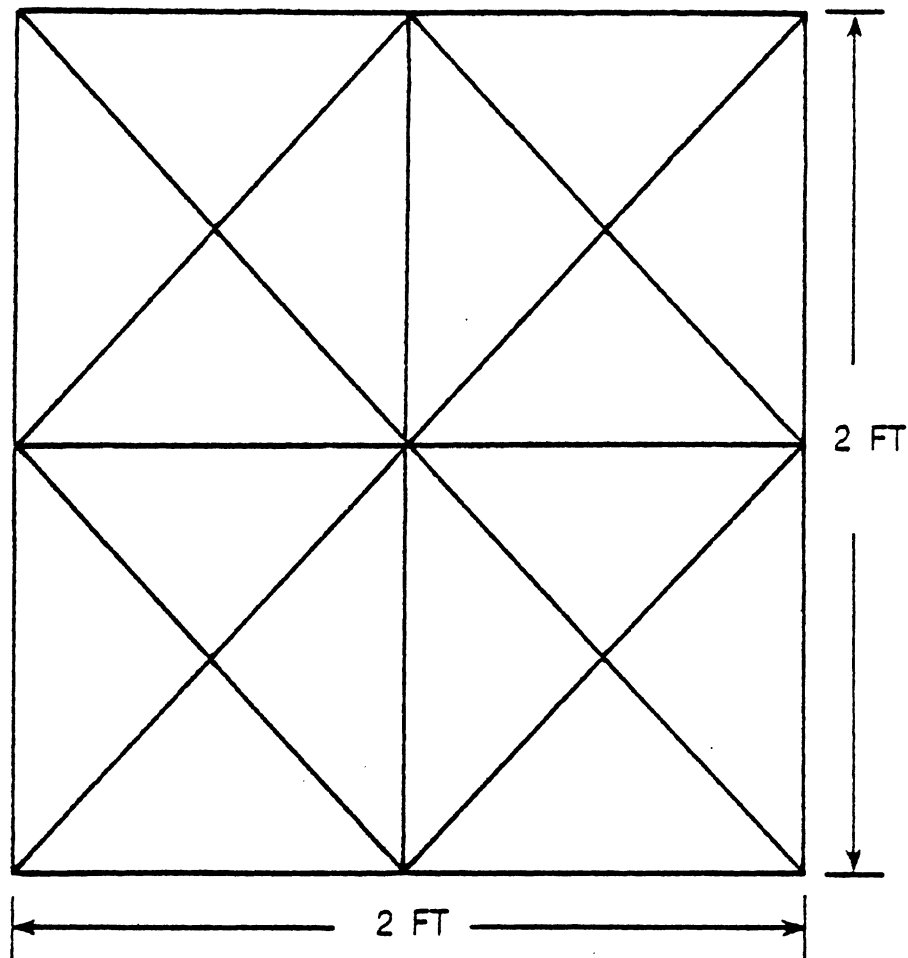
Figure 3.9 Photographic Grid

information is also helpful in confirming measurements and providing additional detail then may have been obtained through initial measurements.

Photographs in which the photographer aligns himself with features that have a known geometric form will also result in pictorial information which can be more easily interpreted. For example, taking photographs with the camera lined up with the road edge, or in alignment with two poles separated by distance, or at 45° to a right angle made by two intersecting road edges, enables the viewer of the photographs to study the pictorial information of interest with the help of known reference objects and lines located in the photograph. Placing objects or marks to accentuate features of interest on the roadway, or off-road terrain, can be helpful later when interpreting areas of interest in the

photograph, for example, where one desires to emphasize location of the point of impact, a bright traffic cone or spray painted X in a circle, or other similar marking techniques may be located at the point of impact. Key poles or posts may be identified by painting them with an X and a number so as to correlate the photographic information with similar information prepared in sketches where the items are similarly marked.

One informative way of observing the crash site is by simulating the drivers view as he approaches the conflict point on the roadway which resulted in the crash. This is by photographing the approach through the windshield of your car while following the same approach path taken by the crash vehicle. Use a wide angle lens[4] and photograph the roadway ahead through the windshield in a sequence which can be correlated with outside reference objects (mail boxes, driveways, light poles, hydrants, etc.) which the vehicle passes. This will permit the investigator to view the photographs later from a known position on the roadway. The windshield must first be clean so as not to reduce pictorial quality. Make sure that every exposure is corrected for the tinting that may be an integral part of the windshield. This method of photographing the roadway and its approach to a crash is most effective with two people in the vehicle. The driver can concentrate on the roadway ahead and handling the camera while the second person can observe traffic, note roadside reference objects and instruct the driver when the photograph should be taken. A motor driven camera makes this technique efficient.

Photographing vehicles is a special challenge. Most all investigators are by now well versed in the number of exposures required by their respective project, and the perspective (front, corner oblique, etc.) of these various exposures . Use of additional illumination in the form of a flash to fill in darken areas of the vehicles is one area where vehicle photography can be significantly improved. This is encouraged to obtain photographs with as much as small vehicle detail as possible. Photographs should be taken at drivers eye level height

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[4] A 28mm lens is preferred for such applications.

(approximately 3 and 3/4 feet above the ground) in a sequence which includes all the necessary perspective views.

Photographing vehicle interiors can be best accomplished by standardizing on two basic camera shutter speed, lens aperture and light strobe setting combinations. One to be used for light interiors, the other for dark interiors. These may require some adjustment when bright sunlight illuminates much of the interior of the vehicle being photographed, but in general will suffice for most all situations. The investigator should experiment with camera and flash settings for a typical light interior and dark interior to achieve the optimal combination for each. Once this consideration has been determined from experimentation it should be routine for interior photographs. The two setting system will ensure good interior photographs at all times and be more efficient than taking light readings and readjusting camera settings for each interior or group of interior photographs attempted.

Gross vehicle damage, that is the overall damage pattern of the vehicle, can best be shown from a combination of side and overhead views. The overhead view, taken from a high vantage point, as through use of a camera, will provide considerable information as to overall damage area and extent, principal direction of force, application of the principal force, how the force was applied by the other vehicle or object, the relationship of second impacts to the initial impact, and other information not as discernable from side views. The cooperation of a fire department high ladder truck is not necessary to obtain the overhead camera vantage for such photos. A telescopic type camera boom can be used which elevates the camera up above the center of the vehicle, so as to view looking downward. This may be telescoping sections of aluminum or fiberglass tubing. Fiberglass is preferred for the safety of the user around overhead electric lines. After the camera is visually positioned over the vehicle, the camera shutter may be released by a long air cable release, or with a remote control radio receiver attached to the camera. A wide-angle lens whose focal length is 28mm or smaller will help provide an illusion of greater height. The value of such overhead views far outweighs the in convenience of using such an aid.

Photographing human subjects with their injuries can accurately document the cosmetic effect of crash injuries. This takes considerable tack and a courteous approach. The individual to be photographed should first be advised of why the request is being made, how the photographs will be used, a description of the project under which the crash is being studied, the objectives of the project and how the data will be used, and last but most important, stressing that the subject's cooperation is entirely voluntary. Make certain to further stress, that identifying facial features (eyes, nose, center face area) will be obliterated or covered in photographic prints, when the face is involved. An appropriate consent form must be completed which contains the subjects signature, a witness signature and the date. There are many variations of such forms and one should be designed which best fits your organization and legal constraints.

Photographing subjects should only be attempted when there is pictorial information which helps to identify and show the extent of injuries. For example, minor head lacerations from contacting the windshield when photographed will vividly document the successful performance of the high penetration resistance glazing. Photographs of limbs which have bruises, lacerations, or contusions, will help document the type of injury and help establish probable areas contacted by the occupant. The investigator must always respect the wishes of the subject when a request is made, and never the question or attempt to change a refusal to cooperate in being photographed.

When photographing human subjects, the investigator should have his photographic equipment fully assembled and adjusted before a request is made. Taking photographs of human subjects should be quick and convenient to all concerned. Do not keep the subject waiting while equipment is being assembled and idle talk is required to fill in the delay. Children must never be photographed without their parents consent. Most people are more willing to be photographed themselves rather than let their children be photographed. Always respect parents wishes and do not attempt to persuade them when a positive response is lacking.

Photographs of crash scenes immediately after the crash when

occupants and the injured are on scene, must also be done unobtrusively and tactfully. Do not interfere with the work of police, EMS personnel or fire personnel. Do not take photographs which show on-scene personnel or accident human subjects in a bad light. Always identify yourself to the officer in charge before hand. If no police photographer is present accommodate the police with photographs they may not have, and be ready to take views which are suggested by them. Never place your camera in the face of an injured occupant and take pictures. This may be embarrassing or intimidating to the injured as well as add to the emotional trauma of the crash. On-scene photographs should be concentrated on large areas from different perspectives, so as to include all the crash scene information in single photograph. Close-up views may be more sensational, but have less pictorial information available with which to reconstruct the crash.

As indicated earlier this section has made no attempt to cover the fundamentals of photography. We presume that experienced investigators have mastered the fundamentals prior to receiving this instruction. For those who wish to obtain such a background, there are many books and periodicals available on the subject. For a basic primer approach to the fundamental of photography, the following two books are recommended:

1) How to Make Good Pictures by the Eastman Kodak Company.

2) This is Photography by T.H. Miller, W. Brummeitt, of the Eastman Kodak Company.

Proper care and maintenance of photographic equipment is necessary to keep such equipment in good working order at all times. Most cameras available today are manufactured for the retail amateur market. They are delicate instruments, can be misaligned easily from rough handling and may not function adequately in all environments. An accident investigator will undoubtedly subject such equipment to conditions beyond those for which the equipment was designed. Therefore, to increase camera life, constant care and periodic maintenance are essential.

4 PHYSICS AND DYNAMICS

Summary. A common analytic language has been developed to allow engineers and scientists to communicate in writing and to allow them the opportunity of modeling and experimenting by an analytic process as opposed to physical testing. In this language it is elementary to determine results which remain expressed in system variables and thus subject to optimization and prediction processes. The common language is a combination of algebra, trigonometry, geometry, and the differential/integral calculus. Elements of these areas are presented, generally in the context of their use in CRASH2, in the following sections.

4.1 Mathematical Background

4.1.1 Coordinate Systems. The most commonly used coordinate systems are the Cartesian and the Cylindrical systems. Both are three-dimensional systems by which we may describe the relative positions or locations of real, three-dimensional bodies. In addition, both may be used to describe the relative positions of points or locations in three-dimensional space. Each coordinate system has a base coordinate plane and an axis perpendicular to that plane. This axis is conventionally named the z-axis and the z-coordinate is a linear measure made parallel to the z-axis and away from the base plane in two directions, one called the positve (+) and the other called the negative (-) direction.

Within the base plane of the Cartesian system, two perpendicular axes are created, one named the x-axis and the other named the y-axis. The intersection of these two axes is a point named "the origin." The rule-of-order of the the positive ends of the three axes is the right-hand-rule:

"One axis (x-axis) in the base plane is given a positve direction by fiat. Then, if the viewer's righthand is rested upon this plane on the viewer's side of the plane, at the origin, with the fingers curved in planes parallel to the base plane and the thumb extended perpendicularly to the plane, the second (y-axis) is found at a 90 degree rotation from the x-axis in the direction pointed by the curved fingers and the third (z-axis) is pointed parallel to the thumb. The three axes are mutally perpendicular and the positive ends are found as directed by this rule."

See figure 4.1 for a sketch of the conventional right-hand-rule Cartesian coordinate system.
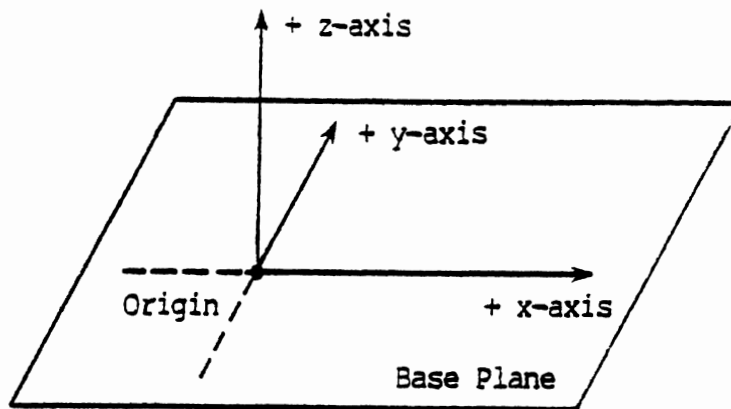
Figure 4.1 A Right-handed Cartesian Coordinate System

Within the base plane of the Cylindrical coordinate system an
origin point is arbitrarily selected and a positive ray (direction line)
is drawn in the plane from that origin. An azimuth ring is centered on
the origin with the angle measure starting at zero where the positive
ray intersects the ring and the positive direction of angle measure on
the ring being pointed by the curved fingers of the right-hand-rule
stated above. A ray projected from the origin in a desired direction in
the base plane cuts the azimuth ring at some measure. This angle
measure, which lies on the 0 to 360 degree range is one coordinate in
the base plane. The second base plane coordinate is the linear measure
along the ray mentioned. The third system coordinate is the linear
measure parallel to the z-axis defined as in the Cartesian system.

4.1.2 Coordinates. The location of a point in a body is given in
terms of a coordinate system fixed to the body. It is conventional in
automotive mechanics to locate the origin of the coordinated system at
the vehicle's center-of-mass location: The base plane is oriented
horizontally with the +z-axis pointing downward, in the direction of the
force of gravity. The x-axis of the Cartesian system and the reference
ray of the Cylindrical system are directed forward along the vehicle's
longitudinal axis. The +y-axis and the 90 degree ray of the Cartesian
and Cylindrical systems thus point transversely from the c.m. to the
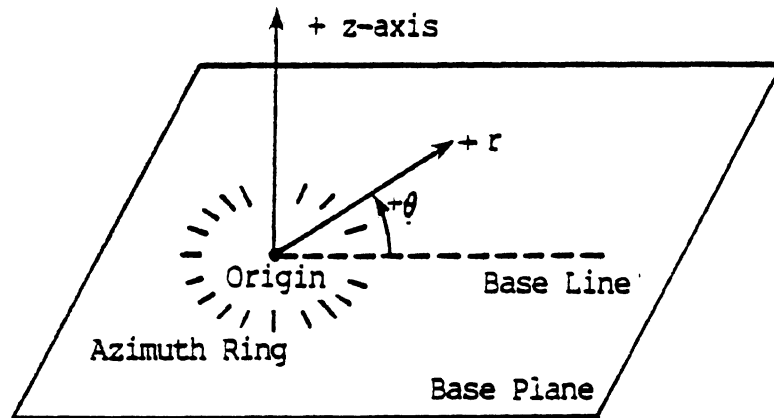
Figure 4.2 A Right-handed Cylindrical Coordinate System

right side of the vehicle.   These vehicle-fixed coordinate systems
remain right-handed as described but are oriented differently than the
ordinary mathematics texts show them. See Figure 4.3a and 4.3b in which
the two vehicle-fixed axes systems are illustratd.   (The two axes
systems exist simultaneously in any vehicle.)
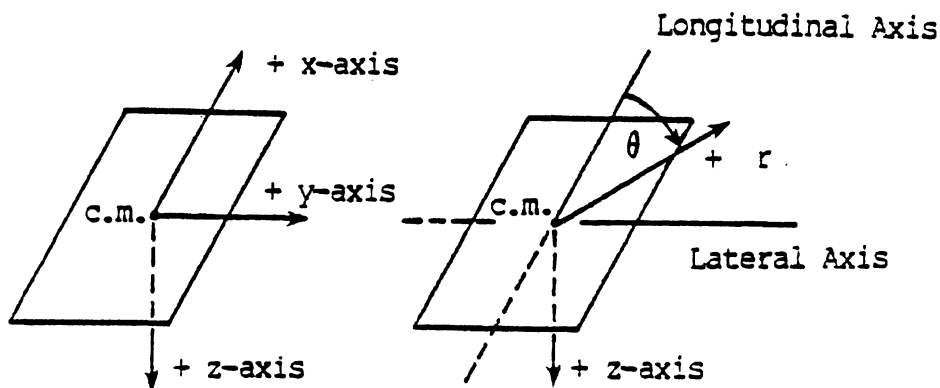
Figure 4.3 Vehicle-fixed Cartesian (a) and
Cylindrical (b) Coordinate Systems

Interestingly, in the programs CRASH2 and SMAC, vehicles are not
considered to have any height and, as a consequence, the vehicle-fixed
coordinate system base plane is also the ground plane upon which the

tires rest. This being the case, we need only be concerned with the location of points in the base plane. It is conventional for us to illustrate the two-dimensional base plane coordinate system in a true projection (plane view). Figure 4.4 presents a superposition of the Cartesian and the Cylindrical (called Polar when two-dimensional) base plane, vehicle-fixed coordinate systems along with similar ground-fixed systems.
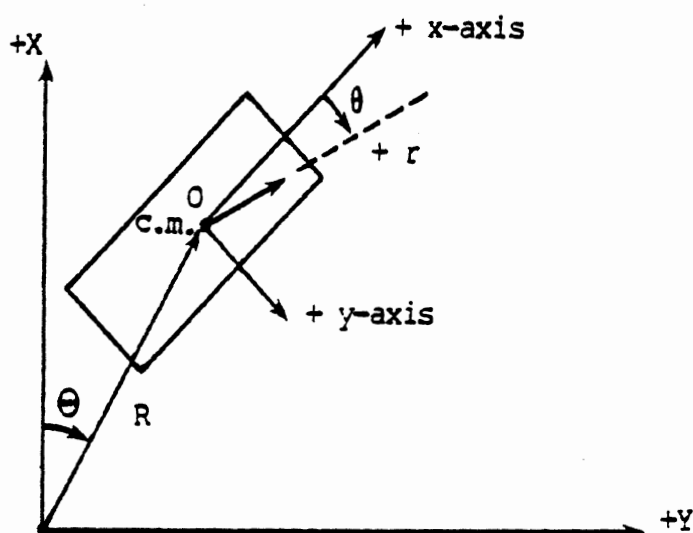
Figure 4.4 Ground-fixed and Vehicle-fixed
Two-dimensional Coordinate Systems

To locate a point in a Cartesian system one measures the perpendicular distance of the point from the y and x axes (i.e., measures the distances in directions parallel to the x and y axes, respectively. If either of these distances, from the axis to the point space is measured in the direction of the positve direction of the parallel axis, it is a positive distance, otherwise it is a negative distance. These positive or negative distances are called the coordinates of the point in the Cartesian system. The coordinates become unique instructions by which any other person could locate the same point if he knew the coordinate system description. This is the reason for adopting universal conventions—to say that the front right corner of a vehicle is at the coordinates $x = 98.8$ in. and $y = 38.5$

in. is unambiguous when we have agreed, as we have, on the location and orientation of a vehicle-fixed coordinate system.

To locate a point in a vehicle-fixed Polar coordinate system we again use two coordinate measures. The first of these is the length of the ray from the origin to the point. The second is the measure of the angle from the reference line to the ray through a point. Note that the linear measure along the ray is always positive. Then the same right front corner referred to in the last paragraph is said to have polar coordinates of r = 106.04 in. and θ = 68.7 deg. Figure 4.5 contrasts the Cartesian and polar examples.
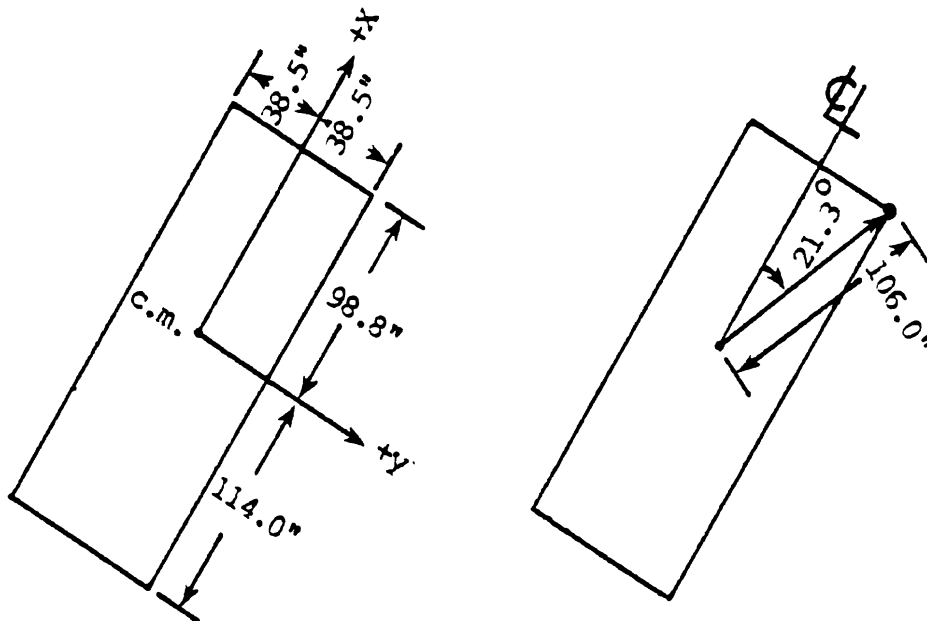
Figure 4.5   Coordinates of a Point on a Vehicle

The coordinates of the left front (LF), right rear (RR), and left rear (LR) corners of the same vehicle along with those of the right front (RF) are listed in Table 1, in each coordnate system along with the traditional corner numbers.

4.1.3 Ground-fixed System Coordinates of Points on Moving Bodies. The front right corner or the center-of-mass of a vehicle are examples of points on a moving body. In the vehicle-fixed Cartesian coordinate

Table 4.1

COORDINATES OF THE FOUR CORNERS OF A VEHICLE

Corner Designation	Cartesian Coordinate		Polar Coordinate	
	y inches	x inches	r inches	θ degrees
RF = 1	38.5	98.8	106.0	21.3
LF = 2	-38.5	98.8	106.0	338.7
RR = 3	38.5	-114.0	120.3	161.3
LR = 4	-38.5	-114.0	120.3	198.7

system the coordinates of these points, i.e., x = 98.8, y = 38.5, and x = 0, y = 0, respectively are constant or fixed—they do not vary with time. But these same two points have coordinates in the ground-fixed coordinate system and these coordinates do vary (change) with time. Figure 4.6 illustrates a plan view of the moving vehicle in the ground-fixed coordinate system and the coordinates of the c.m. point (0) and the right front corner point (P).

Examining Figure 4.6 it is clear that the c.m. point on the vehicle is conveniently located by the ground-fixed coordinates $X\phi$ and $Y\phi$. The point P of the vehicle is equally conveniently located by the coordinates XP and YP. Still, the figure illustrates that the coordinates XP and YP could be determined from the knowledge of $(X\phi,Y\phi)$, the coordinates of the origin of the vehicle-fixed axes, the angle of the vehicle-fixed x-axis from the direction of the ground-fixed x-axis, and the coordinates (X1, Y1) of the point P in the vehicle-fixed axes. In a general functional form the last sentence is written as:

$$XP = f1(X\phi,Y\phi,\theta,X1,Y1) \qquad and \qquad (4.1)$$

$$YP = f2(X\phi,Y\phi,\theta,X1,Y1).$$

The nature of the functions f1 and f2 (their structure) will be developed later.

4.1.4 Paths of Points or Plane Curves. When we agree that the

X_o = 150."
Y_o = 215.0"
θ = 30.0°
xp = 98.8"
yp = 38.5"
R = 262.2"
Θ = 55.1°
X_p = 216.3"
Y_p = 297.7"
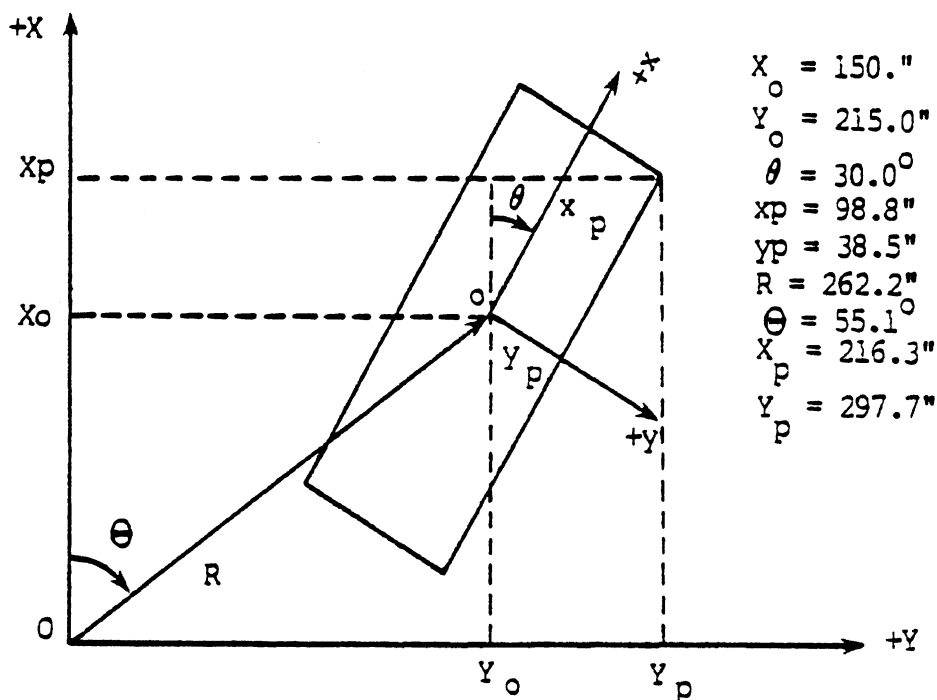
Figure 4.6 Coordinates of Typical Points on a
Moving Vehicle in Two Coordinate Systems

center-of-mass (c.m.) of a vehicle has a time varying set of coordinates $X\phi$, $Y\phi$ in the ground-fixed Cartesian system, we are agreeing that the c.m. has traveled, is traveling, and will travel along a path. A path is defined as the continuous description of past, present, and future locations of a moving point. Paths are commonly curved but sometimes are straight. We encounter both in CRASH2 and SMAC and shall now look briefly at the mathematics of plane paths, curved and straight, along with a few associated ideas.

4.1.5 **Straight Paths of Lines.** We accept that a straight line is defined as a locus of points aligned with the direction of sighting determined by two points. Given the coordinates of two points (X1, Y1) and (X2, Y2), we may predict the coordinates (X, Y) of the "typical" or "general" point on the line. The equation of prediction is written:

A*X + B*Y + C = 0   (4.2)

where A, B, and C are constants.

Sketching (see Figure 4.7) a line through "a" parallel to the x-
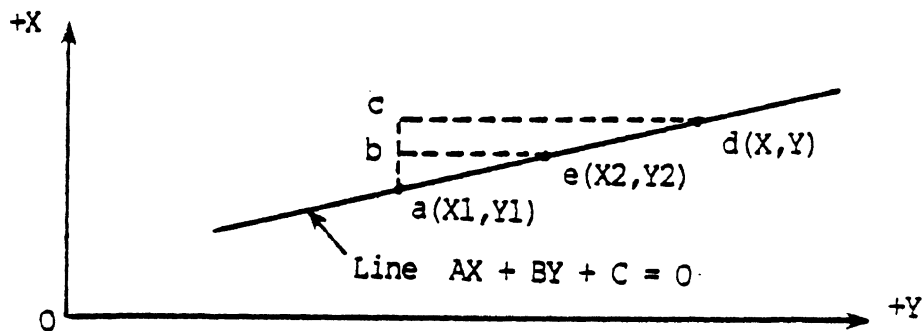
Figure 4.7 A Straight Line Defined by Two
Points and a General Point on the Line

axis and other lines through "e" and "d" parallel to the y-axis of the
:oordinate system, we see that we have created two parallel and thus
similar right triangles. The simple ratios of corresponding sides of
similar triangles are equal, thus the ratio ca/ba equals the ratio cd/
be. Writing the actual lengths into this equality we have:

$$(X-X1)/(X2-X1) = (Y-Y1)/(Y2-Y1) \hspace{3cm} (4.3)$$

or

$$X*(Y2-Y1)+Y*(X1-X2)+(Y1*X2-X1*Y2) = 0. \hspace{2cm} (4.4)$$

Equation (4.4) reduces to Equation (4.2) when

$$A = Y2-Y1$$

$$B = X1-X2 \hspace{6cm} (4.5)$$

$$C = Y1*X2-X1*Y2$$

The Equation (4.4) (or 4.2 with 4.5) may be considered a formula to
be used when the "equation-of-a-line" is needed and we are given two
points on that line. As an exercise in algebra we should recall that an
adequate equation may be developed by pure substitution into Equation
(2.4). Given the form (A*X+B*Y+C = 0) and two points on the line (X1,
Y1) and (X2, Y2) we have:

$$A*X1+B*Y1+C = 0$$

$$A*X2+B*Y2+C = 0$$

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or

$$Y2*A*X1+Y2*B*Y1+Y2 \ C = 0$$

$$Y1*A*X2+Y1*B*Y2+Y1 \ C = 0$$

---

yielding $A*(Y2*X1-Y1*X2)+C(Y2-Y1) = 0$

and similarly $X2*A*X1+X2*B*Y1+X2 \ C = 0$

$$X1*A \ X2+X1*B*Y2+X1 \ C = 0$$

---

yielding $B*(Y2*X1-Y1*X2) + C(X1-X2) = 0$

from which $A/C = (Y2-Y1)/(Y1*X2-Y2*X1)$

$$B/C = (X1-X2)/(Y1*X2-Y2*X1)$$

In this manner the logical values are found for A, B, and C:

i.e. $A = (Y2-Y1)$

$$B = (X1-X2)$$

$$C = (Y1*X2-Y2*X1)$$

4.1.6 <u>Length of a Line Segment.</u> The length of the "portion" of a line between two points is readily expressed when the two points are defined by their Cartesian coordinates. This "portion" is properly defined as a line segment, not a line since a line is unbounded in length.   In Figure 4.7 the portion of the line from "a" to "e", or vice versa, is a segment. The segment has a "projection" on the x-axis of X2-X1   and a "projection" on the Y-axis of   Y2-Y1 .   Note that "projections" of segments are considered positive, as the use of the absolute value symbols indicate. The segment length is given by:

$$\text{length of segment "ae"} = \sqrt{(X2-X1)^2+(Y2-Y1)^2} \qquad (4.6)$$

A note to field investigators is in order—the formula is derived from the study of relationships of sides and angles of plane triangles and this particular formula occurs when the angle opposite to the segment is a right angle.   (This is a reduced form of the Law of Cosines.) It follows that the investigator may use it to create a right

angle and therefore a set of Cartesian axes in the field. Let the investigator lay off a distance of 20 feet in the direction of an arbitrary x-axis, from point O to point P. Then a point Q, 20 feet from the origin O, will be 28.28 feet from point P, if the x and y axes are Cartesian. Using simple field markings and measures the investigator may set very accurate Cartesian axes in the field.

4.1.7 Distance of a Point away from a Line. CRASH2 has calculations in which the perendicular (least) distance of a point from a line must be calculated. We know the form of the equation of the line from $A*X+B*Y+c = 0$ where from (4.5) we know $A = (Y2-Y1)$, $B = (X1-X2)$, and $C = Y1*X2-X1*Y2$. From (4.6) we recognize that the line segment from $(X1, Y1)$ to $(X2, Y2)$ equals $\sqrt{A^2+B^2}$. It is not surprising to find that the distance "d" illustrated in Figure 4.8 is given by:

$$d = (A*X3+B*Y3+C) / \sqrt{A^2+B^2} \tag{4.7}$$
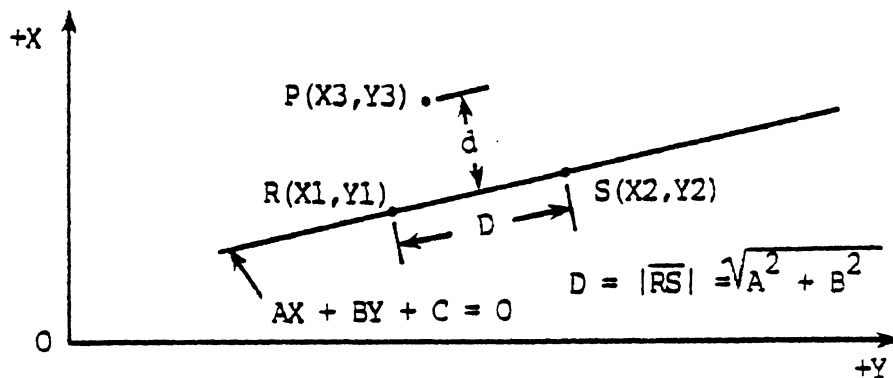
Figure 4.8 The Least Distance of a Point from a Line

It may be interesting to note that the area of the triangle RSP of Figure 4.8 is $1/2[A*X3+B*Y3+C]$. Proof of Equation (4.7) is not given here.

4.1.8 The Point of Intersection of Two Lines in a Plane. The lines defined by $(A1/C1)*X+(B1/C1)*Y+1 = 0$ and $(A2/C2)*+(B2/C2)*+1 = 0$ are said to intersect at the point $(X1, Y1)$ if the values of X1 and Y1 satisfy the equation of each line. To find the values of X1 and Y1 we symbolically substitute X1 for X and Y1 for Y in the equations which become:

$$(A1/C1)*X1+(B1/C1)*Y1 = -1 \qquad (4.8a)$$

and

$$(A2/C2)*X1+(B2/C2)*Y1 = -1 \qquad (4.8b)$$

The conventional method of solving these two linear equations (meaning the unknowns are to the first power) in two unknowns relies on the theory of determinants. The symbols

$$\begin{vmatrix} A1/C1 & B1/C1 \\ A2/C2 & B2/C2 \end{vmatrix}, \begin{vmatrix} -1 & B1/C1 \\ -1 & B2/C2 \end{vmatrix}, \text{ and } \begin{vmatrix} A1/C1 & -1 \\ A2/C2 & -1 \end{vmatrix}$$

have an obvious relationship to the constant terms of (4.8a) and (4.8b). These are second order determinates of the typical form

$$\begin{vmatrix} A_1 & A_2 \\ A_3 & A_4 \end{vmatrix}$$

The algebraic evaluation of the typical form is $A_1*A_4-A_3*A_2$. The solution of (4.8) is:

$$X1 = \frac{\begin{vmatrix} -1 & B1/C1 \\ -1 & B2/C2 \end{vmatrix}}{\begin{vmatrix} A1/C1 & B1/C1 \\ A2/C2 & B2/C2 \end{vmatrix}} \qquad Y1 = \frac{\begin{vmatrix} A1/C1 & -1 \\ A2/C2 & -1 \end{vmatrix}}{\begin{vmatrix} A1/C1 & B1/C1 \\ A2/C2 & B2/C2 \end{vmatrix}} \qquad (4.9)$$

which when algebraically evaluated gives us:

$$X1 = (B1/C1-B2/C2)/((A1/|C1)\ (B2/C2)-(A2/C2)\ (B1/C1))$$

$$Y1 = (A2/C2-A1/C1)/((A1/C1)\ (B2/C2)-(A2/C2)\ (B1/C1)) \qquad (4.10)$$

One notes that when the denominators of the Equation (4.10) are zero, the intersection is undefined and the lines are parallel.

4.1.9 <u>Perpendicular and Parallel Lines.</u> Two lines in a plane may be perpendicular, oblique, or parallel. We have noted that the value of C in the equation of a line $(A*X+B*Y+C = 0)$ has no influence on the angle of the line to either axis—its effect is on the place of intersection of the axes by the line. Thus two lines defined by:

A*X+B*Y+C1 =0   and

A*X+B*Y+C2 = 0   (4.11)

would perforce be parallel. (This is consistant with the prior comment
on the denominator for 4.10).

The point (X, Y) is on the line since it satisfies the equation A*
X+B*Y+C = 0. A nearby point with coordinates X+δX and Y+δY is on the
same line if A*(X+δX)+B (Y+δY)+C = 0. By subtraction, the condition
that the second point be on the given line is seen to be that
(A*δX+B*δY) = 0. The ratio δX/δY is seen to have the meaning of a
necessary relation between increments of x-like and y-like motion when
moving on the line. The ratio of equation (4.12) is defined as the
line's "slope."

δX/δY = -B/A   (4.12)

(Note that the reciprocal relation δY/δX = -A/B might be used)

Clearly, if two lines are parallel their slopes are equal. Thus, if we
find B1/A1 = B2/A2, the lines A1*X+B1*Y+C = 0 and A2*X+B2*Y+C2 = 0 are
parallel.
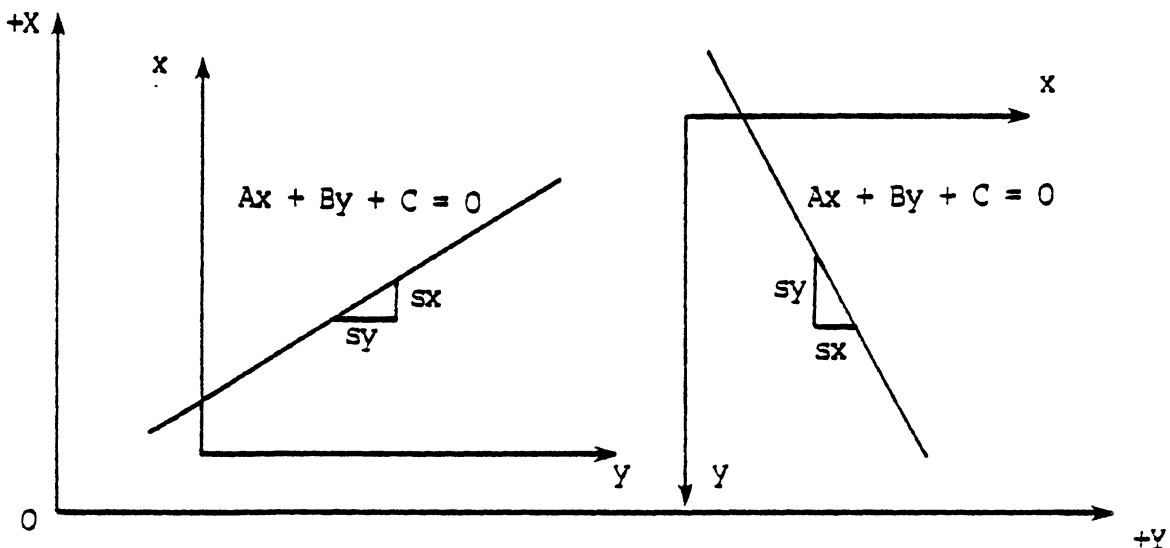
Figure 4.9 A line Fixed in a Frame with the Frame Rotation 90 Degrees

We deduce that two lines are at a 90 degree angle one to another

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when the slopes are the negative reciprocals of each other. This is seen clearly by inspecting Figure 4.9 where, in the background coordinate plane, we see that the roles of δX and δY are reversed and a negative is interjected when we examine the slopes of the lines known to be perpendicular. Two lines are known to be perpendicular when the condition.

$$B1/A1 = -A2/B2 \qquad (4.13)$$

is met, and parallel when the condition

$$B1/A1 = B2/A2 \qquad (4.14)$$

is met and are otherwise merely oblique.

4.1.10 Unit Direction Vectors. A directed, i.e., from P toward Q, line segment of unit length is called or named a unit direction vector. It is conventional to reserve the symbols $\bar{i}$ and $\bar{j}$ as the "standard" names of unit vectors parallel to the x-axis and y-axis respectively and directed toward the positive exis ends. Any other unit vector is normally desingnated by the symbol $\bar{e}$ with a subscript. For example, in the Polar coordinate system $\bar{e}_r$ would be understood to be parallel to and be directed positively along a ray of interest. (If more than one such ray was of interest, the subscript would be enlarged as $\bar{e}_{r1}$, $\bar{e}_{r2}$, etc.) In the same system, the unit direction vector $\bar{e}$ would be perpendicular to the related $\bar{e}_r$ vector and directed in the sense of increasing angle. These basic or typical unit direction vectors are illustrated in Figure 4.10.

Given a line written in the form:

$$A*X+B*Y+C = 0 \qquad (4.15)$$

then the unit vector

$$\bar{e} = \frac{-B}{\sqrt{A^2+B^2}} \bar{i} + \frac{A}{\sqrt{A^2+B^2}} \bar{j} = a\cdot\bar{i}+b\cdot\bar{j} \qquad (4.16)$$

is parallel to the line.

4.1.11 The Angle Between a Line and the X-axis. A line written in the form $A*X+B*Y+C = 0$ is at the angle $\theta = \tan^{-1}(A/-B)$ from the +x-
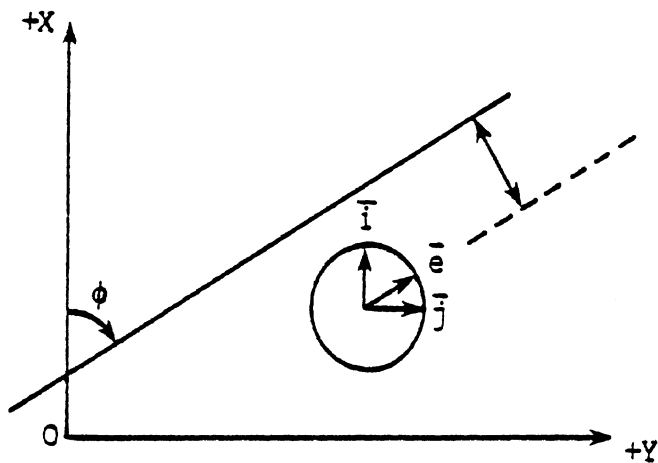
Figure 4.10   Unit Direction Vectors

axis. The reader may wish to examine a later section in which the trigonometric functions and their inverses are discussed. This angle was illustrated in Figure 4.10 where the illustrated unit direction vector $\bar{e}$ is parallel to the line and directed in the sense of motion on the line given by increasing values of x.

4.1.12 <u>The Angle Between Two Lines</u>.   Let the equations of two lines be written as:

First line   A1X+B1Y+C1 = 0

Second line   A2X+B2Y+C2 = 0

Then (without proof) we declare that the positive angle required to rotate the first line onto the direction of the second line is given by:

$$\tan \beta = (A1B2-B1A2)/(A1A2+B1B2) \tag{4.17}$$

Recall that, if the denominator is zero, Equation (4.13) is satisfied and β = 90 degrees and that, if the numerator is zero,Equation (4.14) is satisfied and β = 0 degrees.

4.2 <u>Plane Kinematics</u>

4.2.1 <u>The Position Vector of a Point</u>. In the earlier discussion of polar coordinate systems we located a point by its coordinates ϕ and R. Figure 4.12 illustrates an idealized curve representing a possible
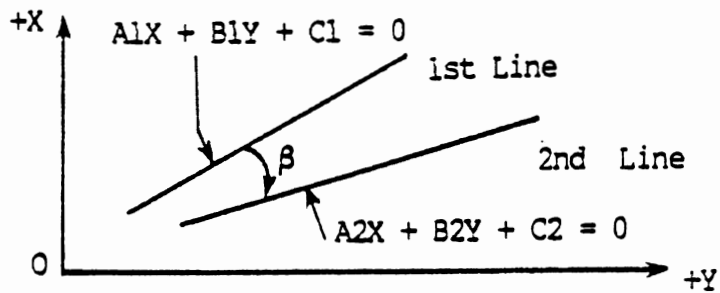
Figure 11. The Angle Between Two Lines

path of a skidding vehicle's center-of-mass point. A point P is
illustrated on the curve. Clearly, if we wish to use the ideas of
vectors, we may say that $\overline{OP}$ (the directed line segment from the origin O
to the point P) is a vector R units long. (R = Abs $(\overline{OP})$). The vector
$\overline{OP}$ is named the "position vector"—it gives the position of P uniquely
in the Polar coordinate system.
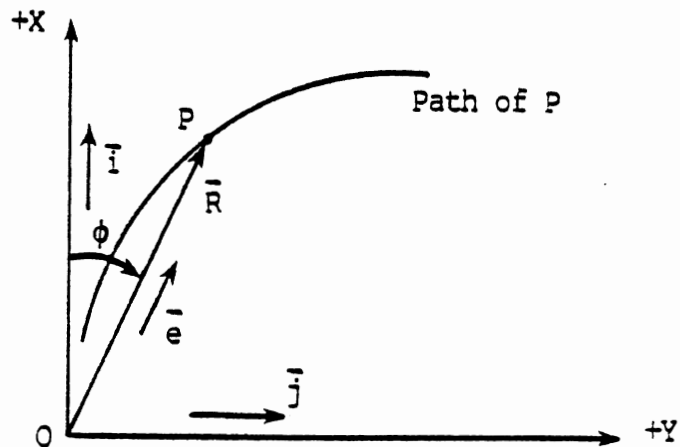
Figure 4.12. The Path of a Particle and the Particle's Position Vector

4.2.2 The Displacement Vector of a Particle. For a particle to be
"displaced" it must have moved to a new (future) position on its path as
illustrated in Figure 4.13. The particle P has moved along its path,
being at P_1 when time = t_1 and at P_2 when time = t_2. The definition of
the particle's displacement is $\overline{P_1P_2}$ is the directed line segment from

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the position P_1 to the position P_2 and is defined as the "displacement vector."
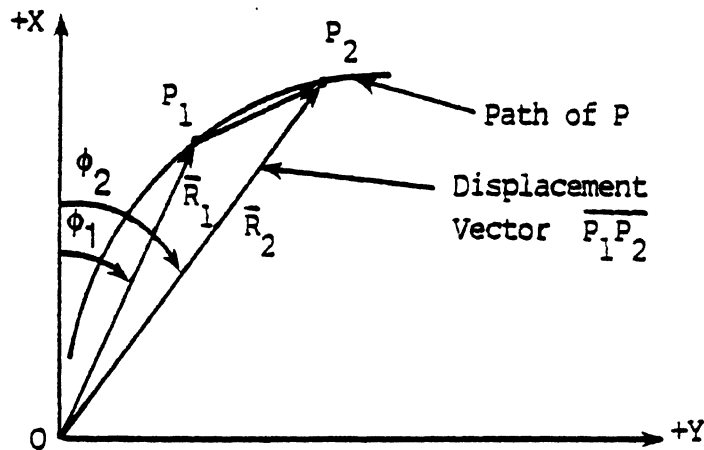
Figure 4.13. The Displacement Vector $\underline{P_1P_2}$

4.2.3 Vector Addition. Figure 4.13 illustrates that vector $\overline{OP_1}$ followed in a "tail-to-nose" manner by $\overline{P_1P_2}$ yields the same location P_2 as does the single position vector $\overline{OP_2}$. Accordingly:

$$\overline{OP_1} + \overline{P_1P_2} = \overline{OP_2} \tag{4.18}$$

This is the accepted "triangle-rule" of vector addition. Figure 4.14 is the same figure as Figure 4.13 except that "construction lines" OP_3 and P_3P_2 illustrate that triangle OP_1P_2 was one side (half) of the parallelogram $OP_1P_2P_3$ of which $\overline{OP_2}$ is the main diagonal. Since $\overline{OP_3} = \overline{P_1P_2}$ it follows that the "triangle-rule" of vector addition has an equivalent "parallelogram-rule" of vector addition.

$$\overline{OP_1} + \overline{OP_3} = \overline{OP_2} \tag{4.19}$$

4.2.4 Scalar Multiplication or Division of a Vector. Imagine that the "displacement vector" $\overline{P_1P_2}$ of Figure 4.13 is 1.0 inches long and that it lies at an angle of 80 degrees clockwise from the x-axis. Then by the rules of scalar (non-vector) multiplication, we would say that $3\overline{P_1P_2}$ was 3.0 inches long and was laying at the same 80 degree angle or that $\overline{P_1P_2}/10$. was 0.1 inches long and was also at an 80 degree angle. Examine Figure 4.15 where examples are given. We have taken only the
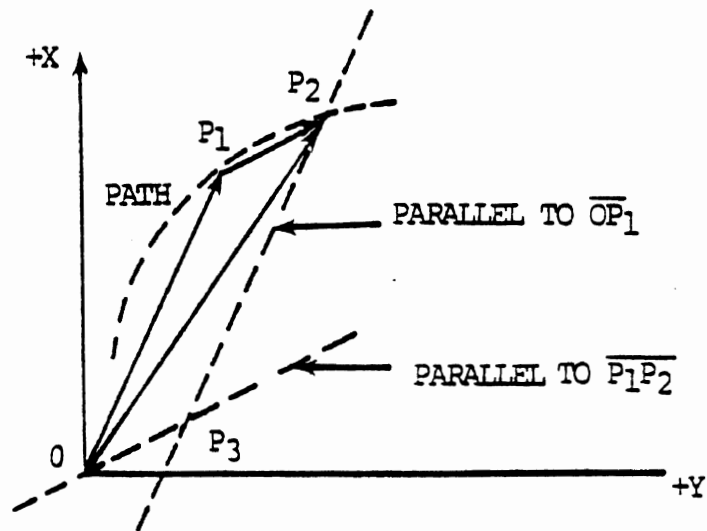
Figure 4.14. Rules of Vector Addition

liberty of moving the scaled vectors along the line-of-action of $\overline{P_1P_2}$ since the scalar multiplication has not given us the right to adopt a new line of action for the result of the operation.
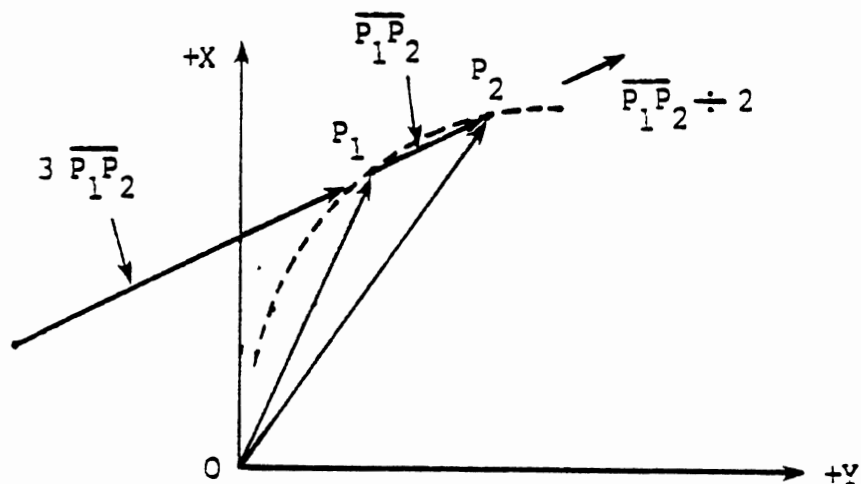
Figure 4.15. Scalar Multiplication of a Vector

The scalars 3 and 10 used in the multiplication/division example were dimensionless and did not change the units of the result from the units of $\overline{P_1P_2}$ which were inches. Not all scalar quantities are

dimensionless. For example, $T = t_2 - t_1$ is a scalar quantity (it has no inherent sense of direction) and it has the dimensions of seconds. T is the time required for the displacement of the particle $\overline{P_1 P_2}$. Dividing $\overline{P_1 P_2}$ by the scalar T produces a vector lying at 80 degrees to the +x-axis but on the line of action of $\overline{P_1 P_2}$, and has not only a different magnitude than that of $\overline{P_1 P_2}$ but also has different dimensions—inches per second.

4.2.5 <u>Average Velocity and Instantaneous Velocity of a Particle.</u>
Average velocity for the time period $T = t_2 - t_1$ during which there was a displacement vector of $\overline{P_1 P_2}$ inches is defined:

$$\text{Average velocity vector } \overline{V}_{ave} = \overline{P_1 P_2}/T \text{ inches per} \qquad (4.20)$$

The average velocity vector is parallel to the displacement vector as illustrated in Figure 4.16.
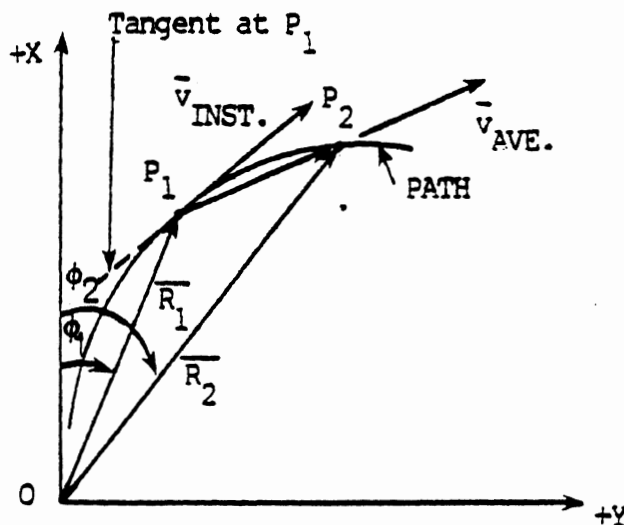
Figure 4.16. Velocity Vectors: Average, Instantaneous

By considering $\overline{OP_1}$ and $\overline{OP_2}$ to be position vectors of the moving particle at times t and t+ t where t may be arbitrarily small but not zero, it is argued that the calculation of average velocity vector becomes that of the instantaneous velocity vector. The ratio of $|\overline{P_1 P_2}|$ T is the magnitude of the (instantaneous) velocity vector, "the speed of the particle." The direction of the velocity vector $\overline{V}$ becomes the

direction of the tangent to the curved path at P. See Figure 4.16. We have that:

$$\overline{V} = \lim \delta t \rightarrow 0 (\overline{P_1 P_2}/ \delta t) \quad \text{p 433} \tag{4.21}$$

(read "the limit of $\overline{P_1 P_2}$/st as δt approaches zero).

The proper interpretation to be given to $\overline{V}$, the velocity vector of the particle at P, is "the rate of change of the position vector $\overline{OP_1}$ with respect to time." In differential calculus this world would be written:

$$\overline{V} = d(\overline{OP_1})/dt \tag{4.22}$$

We must constantly be aware that $\overline{V}$ is a vector, is tangent to the path at P, has the "sense" of the motion of the particle, and is not ordinarily either perpendicular to or parallel to $\overline{OP_1}$.

4.2.6 <u>Changes in Velocity Vectors with Time</u>. Velocity is a vector and may change its magnitude, its sense, and/or its direction. Consider the case o "rectilinear motion"—the motion of a particle on a straight path. Let this motion be on the y-axis. Imagine that the y coordinate of P is

Figure 4.17. Rectilinear Motion of a Particle

a function of time written:

$$y = 10+12t-20t^2 \tag{4.23}$$

Following an idea introduced earlier, we could compare the position vector oy at time t and at time t+δt.

$$\overline{oy} = (10+12t-20t^2)\overline{j} \quad \text{at } t = t \tag{4.24}$$

$$\overline{oy} = (10+12(t+\delta t)-20(t+\delta t)^2)\overline{j} \quad \text{at } t = t+\delta t \tag{4.25}$$

Expanding the terms of (4.25) and subtracting (4.24) we have that the

change, $\delta \overline{oy}$, in $\overline{oy}$ in time δt is:

$$\delta \overline{oy} = 12\delta t - 40t\delta t - 20(\delta t)^2 \tag{4.26}$$

Recall that we will produce a velocity vector by taking the ratio $\delta \overline{oy}$ to δt and letting δt approach zero.

$$\overline{V} = \lim_{\delta t \to 0}(12 - 40t - 20\delta t)\overline{j} = (12 - 40t)\overline{j} \text{ in/sec.} \tag{4.27}$$

The velocity $\overline{V}$ is the time rate of change of $\overline{oy}$ and has a positive magnitude for t less than 0.3 seconds and a negative magnitude for t greater than 0.3 seconds. (A negative scalar multiplying the unit direction vector $\overline{j}$ indicates a vector directed don't wait toward the -y-axis end.) Figure 4.18 depicts the displacement-time and speed-time histories of this example.
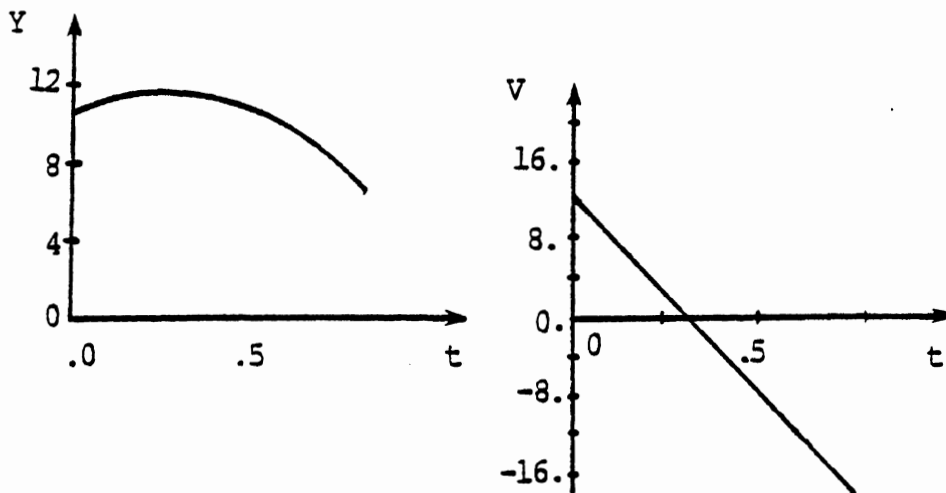
Figure 4.18. Rectilinear (Straight Line) Motion

In the case of recilinear motion the velocity vector does not change its line of action.

Consider the motion of a particle at constant speed on a circular path as depicted in Figure 4.19. Since speed is constant, the magnitude of the velocity is unchanging.

Still the velocity vector must at all times lie parallel to the tangent vector of the path or, in particular, be perpendicular to $\overline{OP}$ in this instance. Here, then, is a case of a constant magnitude, varying
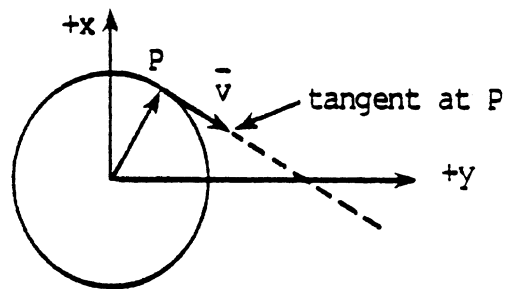
Figure 19. Circular Motion of a Particle at Constant Speed
direction velocity vector to contrast with the last case of a varying
magnitude, constant direction velocity vector.

In general, both the magnitude and the line-of-action of a velocity
vector vary with time. To answer the question of why, we must first
define two new vectors—force and momentum. To discuss the more general
cases of velocity variation we must define the acceleration vector and
also speak of the components of vectors.

4.2.7 The Hodograph and Acceleration Vectors. By drawing a very
large set of displacement vectors in the positions required to describe
the sequence of positions of a particle over a period of time, we
recognize that we are describing the particle's path as the locus of all
position vector end points. The velocity vector at any position of the
particle was shown to be tangent to the path and to be the rate of
change of the position vector with time written $\overline{V} = d\overline{r}/dt$. ($\overline{r}$ = position
vector, $\overline{V}$ = velocity vector, and t = time.) We have observed that the
velocity vector should be expected to vary in both direction and
magnitude. Thus $\overline{V}$ has a time derivative—a rate of change with time.
This time derivative is a vector quantity named acceleration.

There is a very simple means of demonstrating the direction of the
acceleration vector. This method depends upon an analogy. If, for
every position of a moving particle, the velocity vector is determined
and laid out in a diagram with its root at an origin point, then the
tips of this sequence of replotted velocity vectors describes a curve
called the hodograph. By the same reasoning used to produce the

velocity vector as tangent to the path of a particle at each position or end of a position vector, we know that the acceleration vector associated with any velocity vector is tangent to the hodograph at the tip of the velocity vector. Figure 4.20 represents the constant speed circular motion of Figure 4.19 along with the hodograph for this case. A typical acceleration vector is shown. It is observed that the acceleration vector fo this constant speed case is perpendicular to the velocity vector and is, therefore, radial to the actual path.
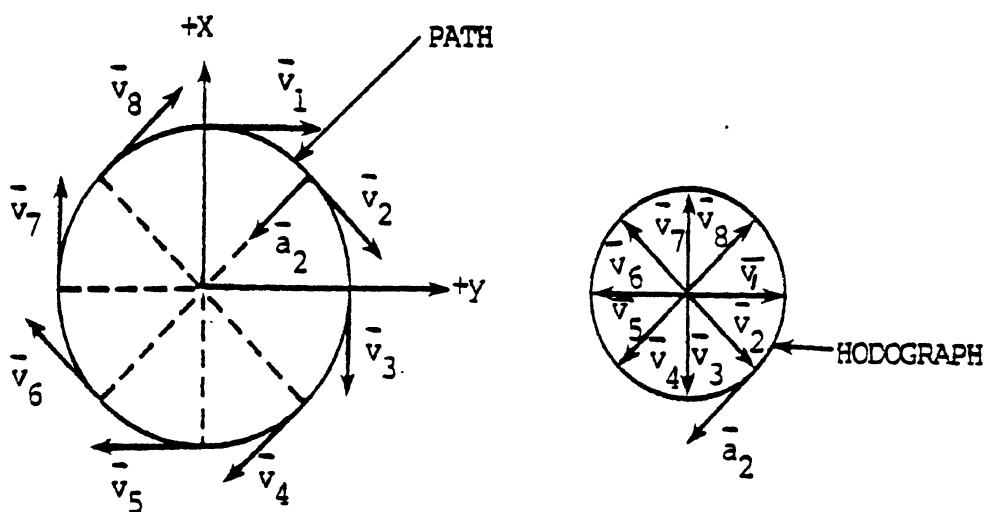
Figure 4.20. The Hodograph and the Acceleration Vector

It is clear that the analogy, i.e., the similar relationship of position vector, path, and velocity vector and the velocity vector plotted polarly, the hodograph and the acceleration vector is valid but that we have demonstrated a special case. In the case demonstrated it is logical that the acceleration vector will have a magnitude which depends upon the magnitude of the velocity vector and the angular velocity of the position vector generating the path. This angular velocity is in turn directly proportional to the magnitude of the velocity (the speed) and inversely proportional to the radius of the path. Thus the magnitude of the <u>radial</u> acceleration vector is V^2/R where V is the speed and R is the radius of the path. This component of acceleration is perpendicular to the velocity vector and is directed along the inner normal to the curve. It is named the normal

acceleration, A_n. We have:

Normal acceleration $A_n = V^2/R$ (4.28)

The normal acceleration is one component of the total acceleration vector. The second component lies along the tangent line and is named the tangential component. The normal component exists because the particle has a speed, v, on a curved path with a radius R. It exists even when the speed is constant and depends upon the directional change of the velocity vector due to the curvature of the path. The tangential component reflects the increasing or decreasing speed of the particle, i.e., it depends upon the change in the magnitude of the velocity vector with time.

Tangential acceleration component $A_t = dv/dt$ (4.29)

Figure 4.21 illustrates the two components of a particle's acceleration for the case of a general curved path. At point P the curve has a radius R as illustrated by the circle fitted to the curve. The particle's speed v is reducing which accounts for the trangential component being directed toward the negative tangent line direction. The total acceleration is the sum, i.e.

$$A = A_t \bar{e}_t + A_n \bar{e}_n = (dv/dt)\bar{e}_t + (V^2/R)\bar{e}_n$$ (4.30)

4.2.8 Definitions of Linear Momentum and Inertial Force. Figure 4.22 illustrates the general curved path of the center-of-mass point of a vehicle skidding to rest. (The curvature may be primarily a result of wheels steered or different tire road friction at the different wheel positions, or both.) The velocity vector and the acceleration vectors are illustrated at a typical position P. Recall the scalar multiplication of vectors mentioned in an earlier section. If each vector $\bar{V}$ and $\bar{a}$ were multiplied by the scalar m (mass of the vehicle), two new vectors would be produced. AS shown in the figure, these vectors have a "scaled" magnitude compared with $\bar{V}$ and a but have the same lines of action and sense. The new vectors are defined as:

Linear momentum vector $\bar{p} = m\bar{V}$ (4.31)

and

Inertial force vector $\bar{f} = m\bar{a}$ (4.32)
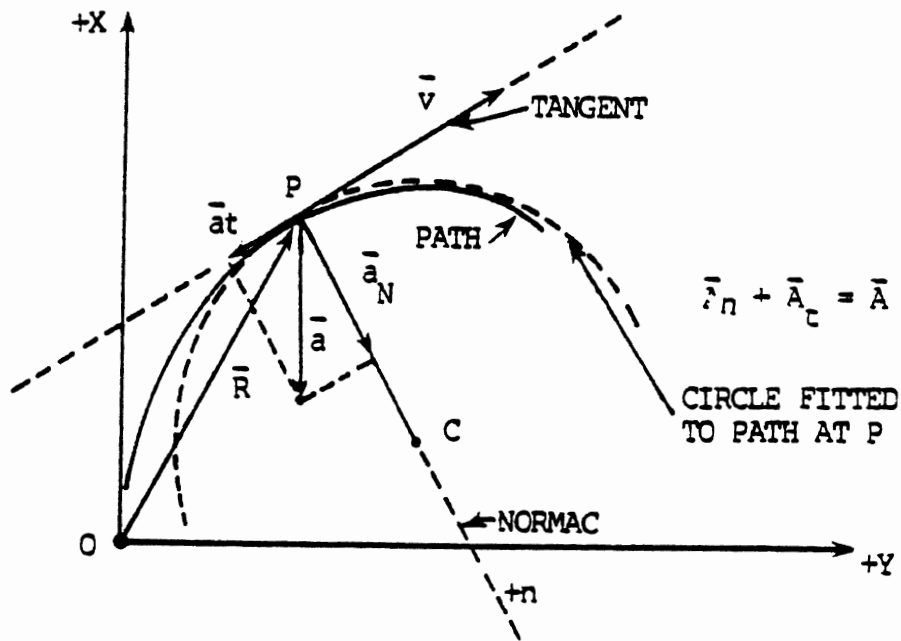
Figure 4.21   Tangential and Normal Components of Acceleration
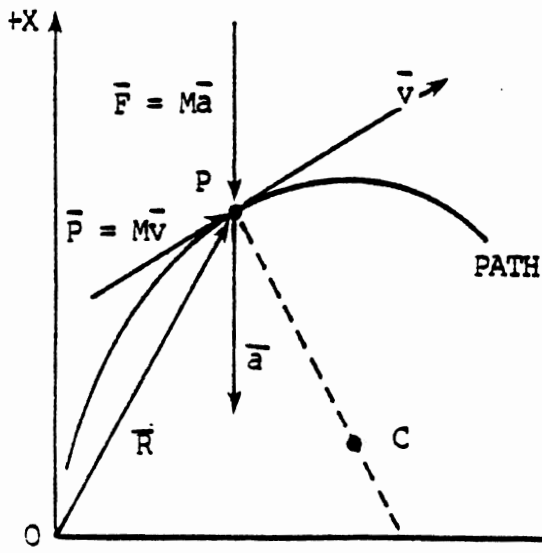
$$\bar{p} = M\bar{v} = \text{momentum vector}$$

$$\bar{a} = d\bar{v}/dt = \text{acceleration vector}$$

$$\bar{R} = \overline{OP} = \text{position vector}$$

$$\bar{v} = d\bar{R}/dt = \text{velocity vector}$$

$$\bar{F} = m\bar{a} = \text{inertial Force vector}$$

P — particle of mass m

C — center of path curvature at position P

Figure 4.22   The Linear Mementum and the Inertial Force Vectors

4.3   PLANE KINEMATICS OF A RIGID BODY

An automobile is not a particle of mass—it is a distribution of mass in a body of considerable extent. An automobile is not rigid, but for the purpose of CRASH2 it is considered to be rigid. The body, the

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axles, the suspension and wheels are fixed relative to each other in the CRASH2 dynamic analysis--there is no relative movement of these parts. The wheels are assumed to be massless and thus have no inertial resistance to rotation. The center-of-mass of the vehicle is assumed to be at the same vertical level as the bottom of the tires. Thus the CRASH2 program is a model of a thin flat rigid body moving on a flat plane parallel to the plane of the body. Contact exists at the four wheel locations between the plane of the body and the plane of the ground. This set of assumptions allows us the freedom to treat the motion of the automobile as plane motion of a rigid body.

If one wished to build a physical model of the vehicle modeled in CRASH2, he could cut out a sheet of paper in the shape of a rectangle and lay it on a horizontal table. This is illustrated in Figure 4.23.
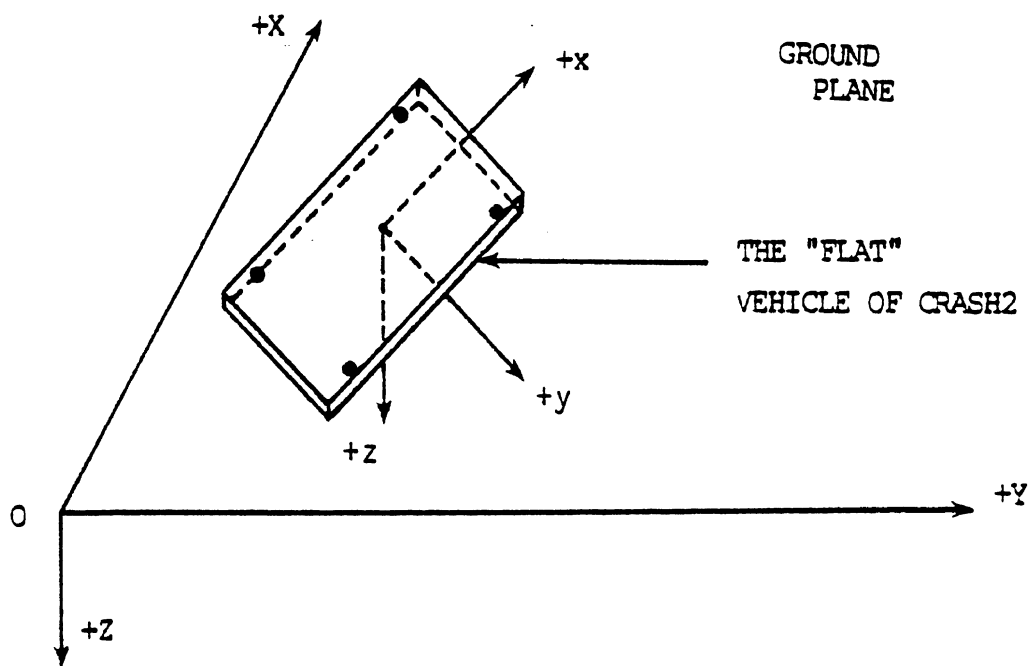
Figure 4.23 The Flat Vehicle of the CRASH2 Model

4.3.1 Angular Velocity of a Rigid Body. Marked on the rigid body model sketch of Figure 4.23 are the body-fixed axes with the origin at the center of mass point. Experience indicates that the general class of plane motion to be studied in CRASH2 involves the motion of the center-of-mass along a curved path with decreasing speed while the body rotates with a decreasing angular velocity.

Angular velocity is the name given to the rate of change of the vehicle's heading angle with respect to time. It is true that all axis, rays, or lines fixed to the vehicle have exactly the same angular velocity but it is not true that the body is rotating about its center of mass—except in unusual and brief instants. It is demonstrable that at any instant the body behaves as if it was rotating about a point in the ground plane and that as time changes the location of the instantaneous center of rotation changes. Two examples of plane rigid body motion are illustrated in Figures 4.24 and 4.25.
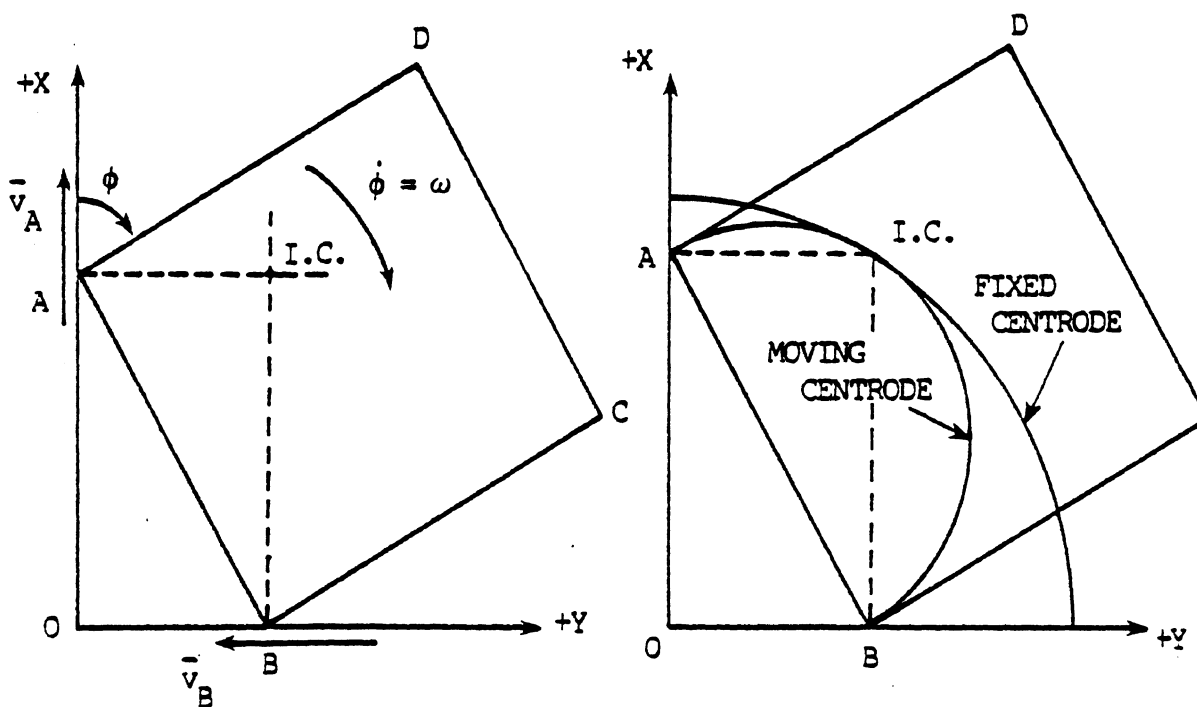
Figure 4.24   Instantaneous Center of Rotation and Centroids

4.3.2   Instantaneous Centers of Rotation.   In Figure 4.24 the x and y axis are ground-fixed. The rectangular body ABCD moves in plane motion on the ground plane in such a manner that point A remains on the x-axis and point B remains on the y-axis. At any instant there must be a point, P or i.c., on the body which has zero velocity and this point must lie on a perpendicular to the y-axis at BB. This hypothesis may be examined by noting first that, if true,

$$\overline{V}_a = \overline{V}_{ic} + \overline{V}_{a/ic} = \overline{V}_{ic} + PA*\omega\overline{i} = V_a\overline{i}$$
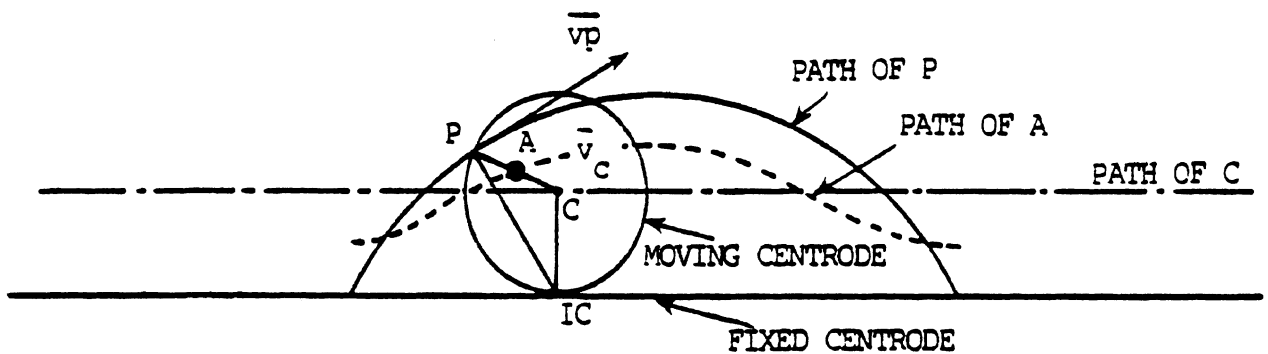
Figure 4.25 Moving and Fixed Centrodes

from which we have that $\overline{V}_{ic}$ may not have a horizontal component. In the same manner we note that, if true,

$$\overline{V}_b = \overline{V}_{ic} + \overline{V}_{b/ic} = \overline{V}_{ic} - PB^*\omega\overline{j} = -V_b\overline{j}$$

from which we have that $\overline{V}_{ic}$ may not have a vertical component. Therefore, point P or i.c. has zero velocity.

4.3.3 Centroids. Point P, the instantaneous center of rotation may be marked as a position on the moving body ABCD and also as a position on the fixed ground plane. The locus of all P's on ABCD is the semicircle APB of radius $|\overline{AB}|/2$ and is called the moving centroid. The locus of all P's on the ground plane is the quartercircle A'P'B' of radius $|\overline{AB}|$ and is called the fixed centroid. The plane motion of the body ABCD is exactly the motion that would be produced if the moving centroid rolled without slip on the fixed centroid

In Figure 4.25 a disk rolling without slip along a straight line is illustrated. The instantaneous center of rotation is at the point of non-slipping contact. The moving centroid is the circle of the disk. The fixed centroid is the straight line. The paths of three points, each of which are permanently fixed on the disk, on the ground-fixed plane are illustrated in Figure 4.25. These paths have a relationship to the skid marks of the tires of a skidding vehicle.

4.3.4 Tire Skid Marks. Figure 4.26 illustrates a hypothetical path of the center-of-mass of a skidding vehicle. Figure 4.26 also illustrates a set of skidmarks of the four tire points 1, 2, 3, and 4.
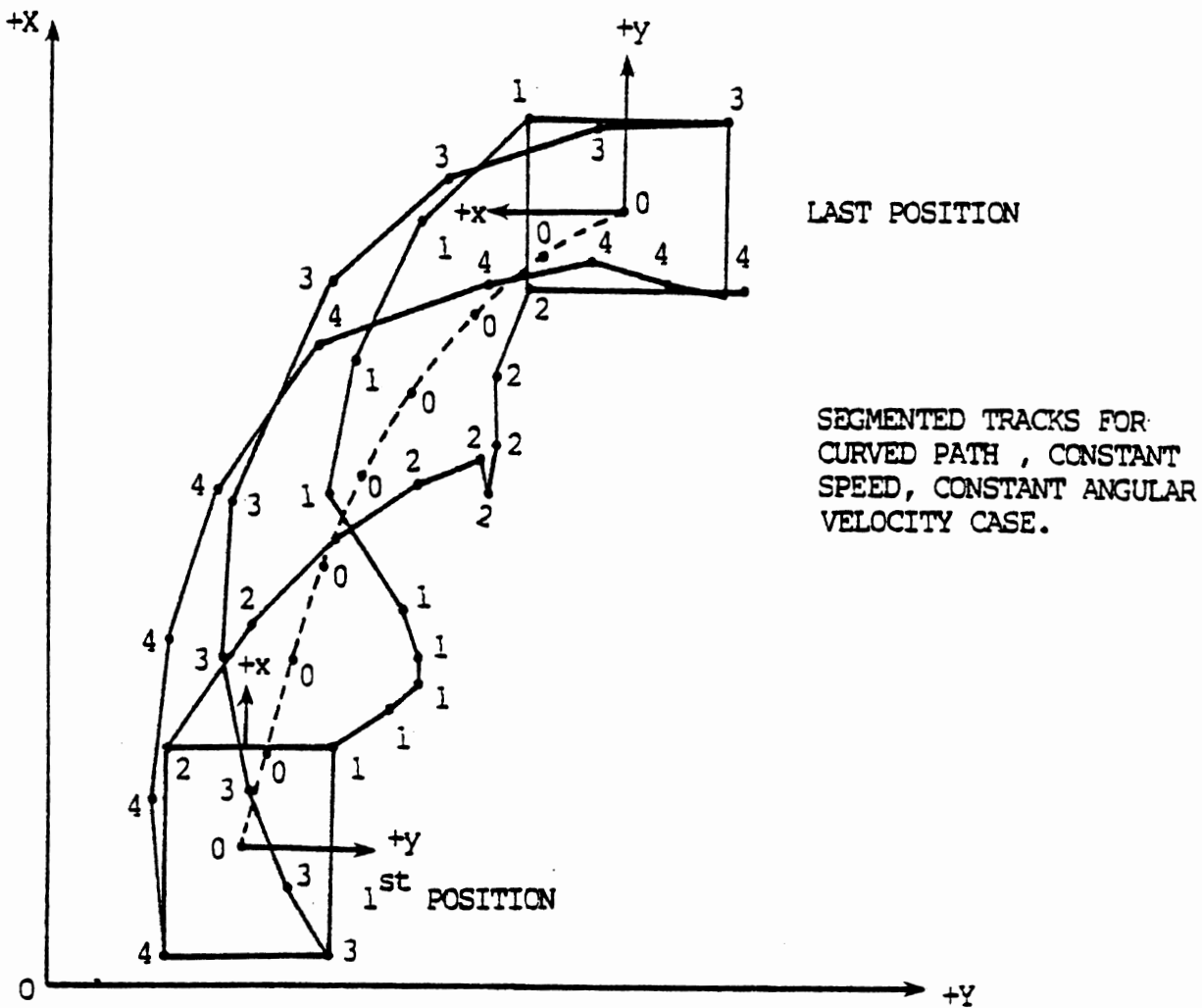
Figure 4.26   Path of a Vehicle's Center-of-Mass
and Tire Paths for a Skidding Vehicle

Tire skid marks are part of the evidence which may be found at the scene of a vehicle crash. The tire marks are undoubtably difficult to interpret and, for this reason, a computer graphics program has been prepared to produce a set of plots for study. (These plots will be provided in a revised edition.)

4.3.5 Rigid Body Rotation of a Body about a Fixed Point. Imagine a rigid body pivoted at a fixed point and moving in a plane as illustrated in Figure 4.27. The subfigures illustrate the time histories of the angular position angle and the angular velocity. The point P moves on a circular path as the angle varies with time. The velocity vector will be perpendicular to the radius $\overline{OP}$ and its magnitude will equal the product of $|\overline{OP}|$ and $\dot{\phi}$.
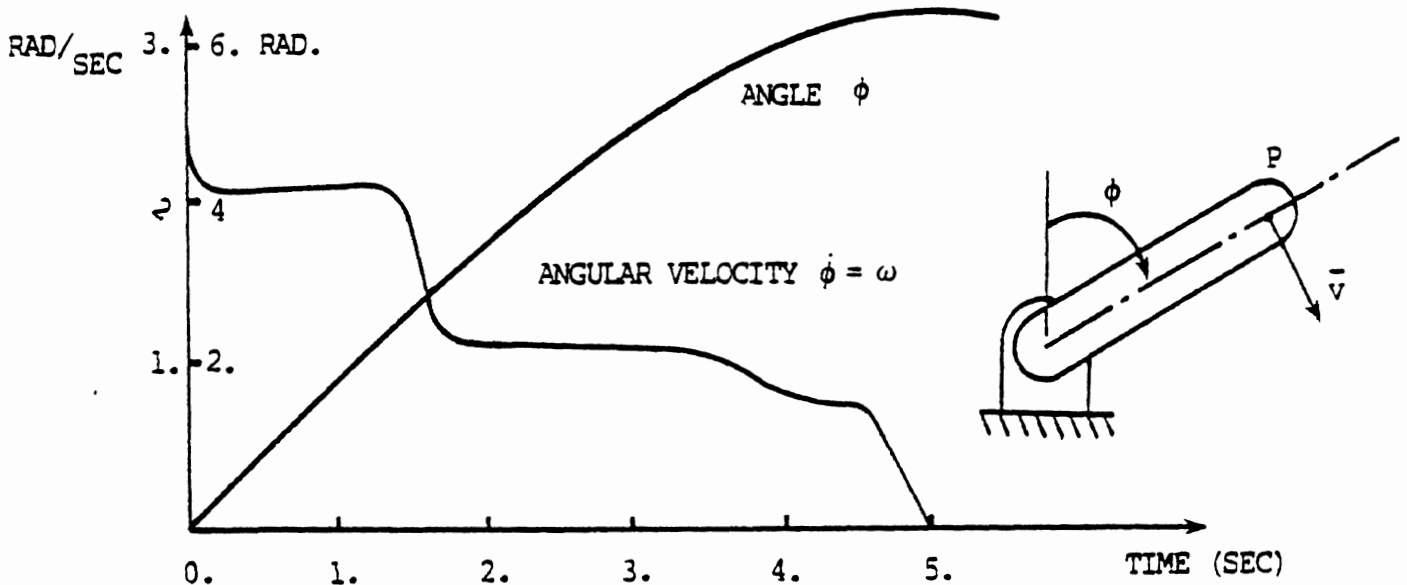
Figure 4.27 Rigid Body Rotation about a Fixed point

Imagine now an automobile (CRASH2's flat rectangular shape) rotating about a fixed vertical axis through its center-of-mass point as illustrated in Figure 4.28. The c.m. point is pictured as point 0 and the right-front tire position is pictured as P. The situation here is exactly like that in Figure 4.29 except there are four tire positions and it appears that it would be profitable not to use $\overline{V}_p$ as the velocity vector but to replace $\overline{V}_p$ by its two components parallel to the axis of the vehicle. These components are shown as $a\dot{\phi}$ and $t\dot{\phi}$ in the figure. The appeal of this method is orderliness and simplicity as seen.

Figure 4.29 enlarges on Figure 4.28 by illustrating in simple form the components of velocity of each tire position of a vehicle which is

$$\overline{VEL} = (-t\,\dot{\phi})\bar{i} + (a\,\dot{\phi})\bar{j}$$
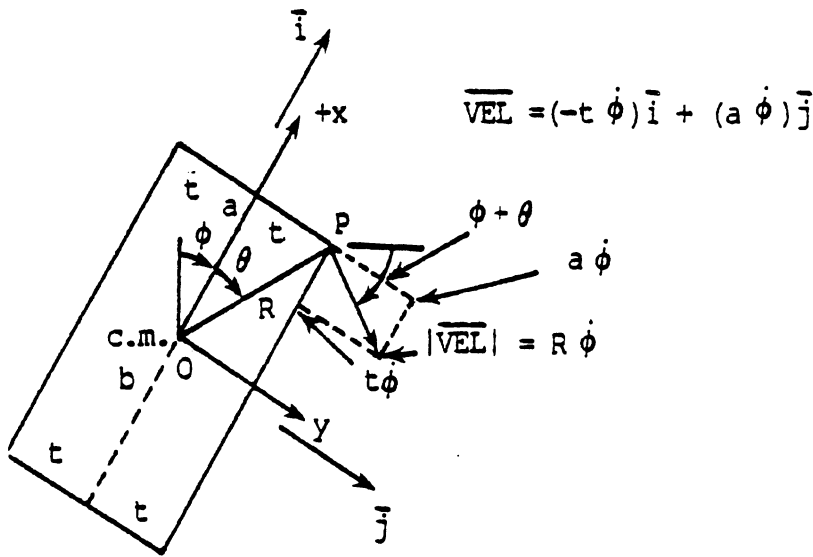
Figure 4.28 Rotation of a Vehicle about its Center-of-
Mass and the Components of Velocity of One Wheel
rotating about its c.m.

4.3.6 Total Velocity of any Point on a Body in Plane Motion. The
total velocity of any point on a body in plane motion is equal to the
total velocity of a reference point on the body plus the velocity of the
point relative to the reference point. This is merely a logical
statement of truth—it is expressable as the vector equation:

$$\overline{V}_A = \overline{V}_{REF} + \overline{V}_{A/REF} \qquad\qquad (4.33)$$

This relative motion equation may be directly applied to the vehicle by
declaring the center-of-mass point to be the reference point. Then,
since we shall have a need (later defined) for the total velocity of
each wheel, we may use the equation to our advantage. The velocity of
wheel points relative to the c.m. were depicted in Figure 4.29. Now in
Figure 4.30 we illustrate the accepted method of describing the velocity
of the center-of-mass or reference point. The total velocity is
illustrated in its components U and V which are aligned with the
vehicle-fixed axis and in its components V_x and V_y.

By using the vehicle fixed unit direction vectors we may write:
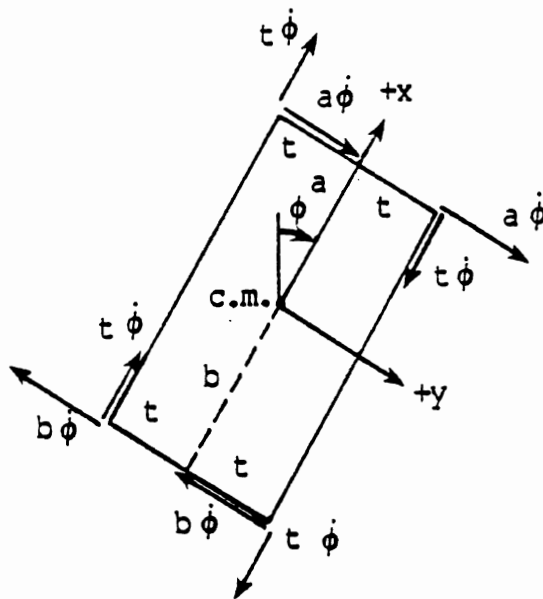
Figure 4.29 Velocity Components of the Four Wheels of
a Vehicle Rotating about its Center-of-Mass

$$\overline{V}\text{total of c.m.} \qquad = U\overline{i} + V\overline{j} \qquad\qquad (4.34)$$

$$\overline{V}\text{total of RF wheel} = (U - t\dot{\phi})\overline{i} + (V + a\dot{\phi})\overline{j} \qquad (4.35)$$

$$\overline{V}\text{total of LF wheel} = (U + t\dot{\phi})\overline{i} + (V + a\dot{\phi})\overline{j} \qquad (4.36)$$

$$\overline{V}\text{total of RR wheel} = (U - t\dot{\phi})\overline{i} + (V - b\dot{\phi})\overline{j} \qquad (4.37)$$

$$\overline{V}\text{total of LR wheel} = (U + t\dot{\phi})\overline{i} + (V - b\dot{\phi})\overline{j} \qquad (4.38)$$

The tire-road friction forces are delivered to the vehicle at the tire positions and depend upon the total velocity vector of the wheels, among other factors.

4.3.7 The Acceleration of the Vehicle's Center-of-Mass. In previous sections we defined the acceleration vector as the time rate of change of the velocity vector. We argued that a velocity vector might change either its magnitude or its direction. The first type change, magnitude change or speed change, occurs along the line of the velocity vector. The second type change, directional, occurs only if the velocity vector has an angular velocity Since the $\overline{u}$ and $\overline{V}$ components of
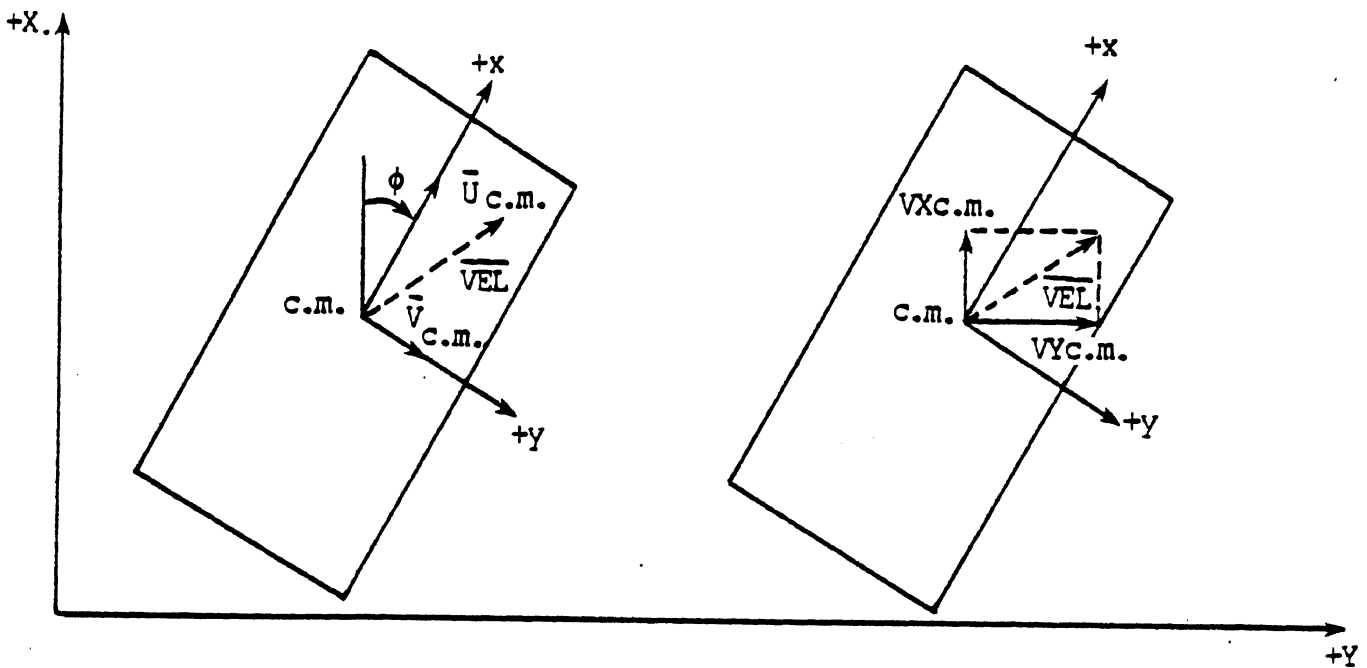
Figure 4.30 The Total Velocity of the Center of Mass Point

velocity are fixed in direction to the vehicle-fixed coordinate system, therefore these vectors have angular velocity if the vehicle does. It follows that, in component form, the acceleration of the center-of-mass is as illustrated in Figure 4.31.

Also, we write the equations here:

Longitudinal acceleration of c.m. = $(du/dt - v\dot{\phi})\bar{i}$ (4.39)

Lateral acceleration of c.m. = $(dv/dt + u\dot{\phi})\bar{j}$ (4.40)

where du/dt = rate of change of longitudinal speed
      dv/dt = rate of change of lateral speed
         u = longitudinal speed of c.m.
         v = lateral speed of c.m.
         $\dot{\phi}$= angular velocity
$\bar{i},\bar{j}$ = vehicle fixed unit direction vectors

4.3.8 Tire or Wheel Slip Angles.   The four wheel positions of the flat vehicle model will each have a different velocity vector in general. The angle of the wheel's velocity vector in the vehicle fixed
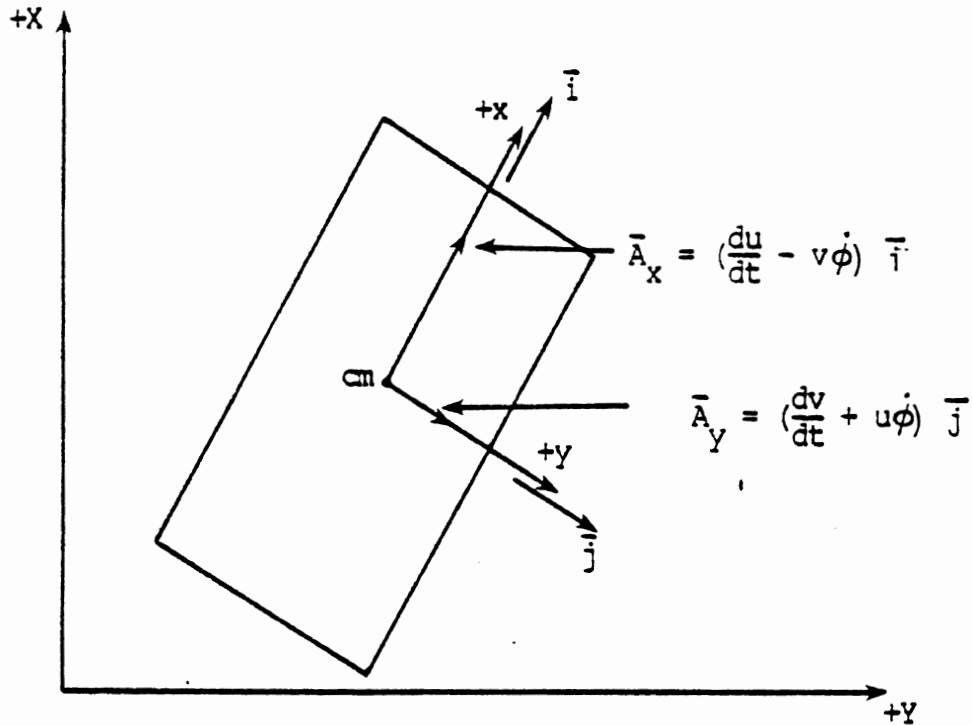
Figure 4.31 Components of Acceleration of a Vehicle's Center-of-Mass

axis system is given by the following:

$$\text{Wheel } \#1 \ (RF) = atan((v+a\dot{\phi})/(u-t\dot{\phi})) = A1 \qquad (4.40a)$$

$$\text{Wheel } \#2 \ (LF) = atan((v+a\dot{\phi})/(u+t\dot{\phi})) = A2 \qquad (4.40b)$$

$$\text{Wheel } \#3 \ (RR) = atan((v-b\dot{\phi})/(u-t\dot{\phi})) = A3 \qquad (4.40c)$$

$$\text{Wheel } \#4 \ (LR) = atan((v-b\dot{\phi})/(u+t\dot{\phi})) = A4 \qquad (4.40d)$$

Each wheel may be "steered," meaning turned at an angle away from the vehicle's + x - axis. Let these angles of steer be ST1, ST2, ST3, and ST4. The difference in the angle of the plane of the wheel and the angle of its velocity vector, i.e., (ST1-A1), etc., is named the "tire slip angle." It is an experimental observation that tires develop "side forces" which are a function of the tire slip angle. For small angles the side force applied by the road-surface to the tire, parallel to the road surface and perpendicular to the wheel plane is linearly dependent

upon the slip angle and acts in such a direction as to tend to reduce the slip angle. These forces will be exhibited in a later figure.

4.4.1 Center-of-Mass   The center-of-mass of a flat rigid body normally lies in the midplane of the body, i.e., $m = 0$. Any rigid body may be analytically subdivided into a number, N, of individual small elements or particles M_n. If a "working set" of Cartesian axes is scribed on the body then each particle M_n can be assigned a position vector $\bar{r}_n$. There is one location in the "working set" of coordinates, given by the position vector $\bar{r}_{cm}$ for which the sum of all terms $\bar{\rho}_n M_n$ equals zero, i.e., $n=N$

$$\sum_{n=1}^{} \rho_n M_n = 0 \qquad \text{where:} \quad \bar{r}_n = \bar{r}_{cm} + \bar{\rho}_n \qquad (4.41)$$

If, as illustrated in Figure 4.32, the vectors $\bar{r}_n$ and $\bar{r}_{cm}$ are rewritten in the form

$$\bar{r}_n = x_n \bar{I} + y_n \bar{J} \qquad\qquad\qquad (4.42a)$$

$$\bar{r}_{cm} = x_{cm} \bar{I} + y_{cm} \bar{J} \qquad\qquad\qquad (4.42b)$$

where $\bar{I}$ and $\bar{J}$ are unit direction vectors parallel to the "working axes."
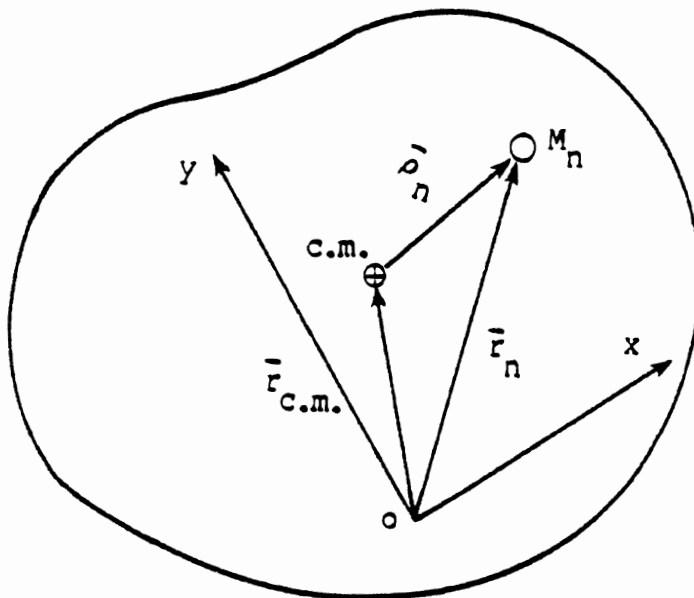
Figure 4.32 Center of Mass of a Flat Rigid Body

Multiplying Equation (41) term by term by M_n and summing the N such equations produce:

$$\Sigma \, \bar{r}_n M_n = \bar{r}_{cm} \Sigma M_n + \Sigma \bar{\rho}_n M_n \qquad (4.43)$$

The third term of Equation (43) is zero by definition leaving:

$$\bar{r}_{cm} = (\Sigma \bar{r}_n M_n)/\Sigma M_n \qquad (4.44)$$

yielding

$$X_{cm} = (\Sigma X_n M_n)/M \qquad (4.45)$$

and

$$Y_{cm} = (\Sigma Y_n M_n)/M \qquad (4.46)$$

The coordinates expressed in (4.45) and (4.46) are the coordinates of the flat body's center-of-mass.

It is clear that the calculations done above might be replaced by a physical experiment taking advantage of the gravitational force F_n applied to each M_n being proportional to M_n. As illustrated in Figure 4.33, the center-of-mass of a suspended body will lie on a vertical line through the point of suspension when the body is in static equilibrium.
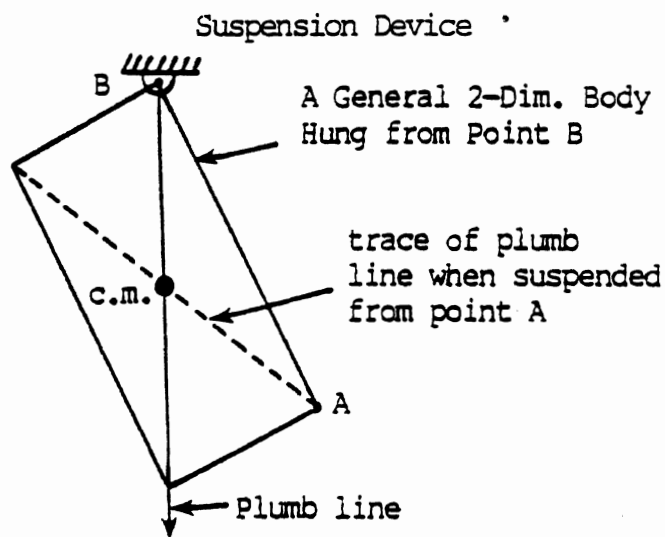
Figure 4.33 Determinging C>M> Location Experimentally

Use of two or more suspension points provides a semi-graphic method of

locating the c.m. at the intersection of the indicated lines. The c.m. is then called the center-of-gravity.

4.4.2 Newton's Axiom. Newton's axiom states that the time rate of change of the linear momentum of a particle is a vector quantity equal to the resultant force acting on that particle.

$$\bar{F} = d(\overline{mv})/dt = m\bar{a} \tag{4.47}$$

The term $m\bar{a}$ is the "inertia force" term introduced earlier. The existance of acceleration of a paticle (time rate of change of velocity) requires the existance of a real fore equal to the inertia force. In the field of mechanics, D'Alembert introduced the concept of the "reversed inertia force" which, as the name implies, is the negative or opposite to the vector $m\bar{a}$. Then Newton's axiom is rewritten as:

$$\bar{F} - m\bar{a} = 0 \tag{4.48}$$

This simple rewriting leads to the observation that the "sum of all real forces and all D'Alembert forces acting on a particle equals zero."

4.4.3 Rectilinear Motion of a Rigid Body. The real body may be considered to be a collection of particles of mass, M_1, M_2, ——, M_n. Each particle may have both external and internal forces acting upon it. Any internal force must be one of an action-reaction pair, i.e., there must be a second particle with the opposite internal force acting upon it. Since the body is rigid and moving on a straight line without rotation (rectilinear motion) all particles have the same acceleration. The sum of all expressions like (4.48) produces:

$$\Sigma\bar{F}_{ext} - \Sigma m_i \bar{a}_i = 0 \tag{4.49}$$

or

$$\bar{F}_{total} - m\bar{a} = 0 \tag{4.50}$$

which is the "equation of motion" of the body.

4.4.4 Rotation of a Rigid Body about a Fixed Axis. To derive the "equation of motion" of a rigid body rotating about a fixed axis we may again apply the D'Alembert observation that the net effect of all real forces acting on a system of mass plus all reversed inertia forces is a zero or equilibrium effect. In the case of pure rotation each particle

of mass travels on a circular path about the axis of rotation. As previously discussed under the topic of kinematics, such a particle has an acceleration vector which is conveniently decomposed into a tangential component and a radial component. These components of acceleration, multiplied by the mass of the particle, are then the basis for components of the reversed inertia force acting on the particle. The radial reversed inertia forces and the external bearing reaction force all pass through the axis of rotation and thus have no "moment" or turning effect about that axis. It follows that the equation of motion is expressed as:

$$\Sigma(\bar{r}_i x \bar{F}_i) - \Sigma[\bar{r}_i x(m_i r_i \dot{\omega}\bar{e})] = 0 \tag{4.51}$$

or

$$M - (\Sigma r^2_i m_i)\dot{\omega} = 0 \tag{4.52}$$

where M symbolizes the moment of all external
forces about the axis of rotation
$\dot{\omega}$ symbolizes the angular acceleration of the body
and $\Sigma r^2_i m_i$ symbolizes the second moment of the mass about the
center of rotation

More conventionally, Equation (4.52) is written:

$$M_o - I_o \dot{\omega} = 0 \tag{4.53}$$

The symbol I_o is named the polar moment of inertia of the mass about axis 0 and equals $\Sigma r^2_i m_i$.

4.4.5 <u>Plane Motion of a Rigid Body</u>. In the subsection on "rectilinear motion" we took advantage of the fact that every particle of mass had the same acceleration. In the subsection on "rotation about a fixed axis" we took advantage of the fact that the acceleration of all particles were related due to the circlar motion of all particles about a single center. In general plane motion we seek to take advantage of the definition of the center-of-mass location of the body.

The acceleration of a typical mass element $\bar{A}_i$ may be written as $\bar{A}_{cm}$ + $\bar{A}_{i/cm}$ or, the words, the acceleration of a particle equals the acceleration of a reference point plus the acceleration of the particle relative to the reference point.

Examine Figure 4.34 in which the acceleration of a typical particle A at
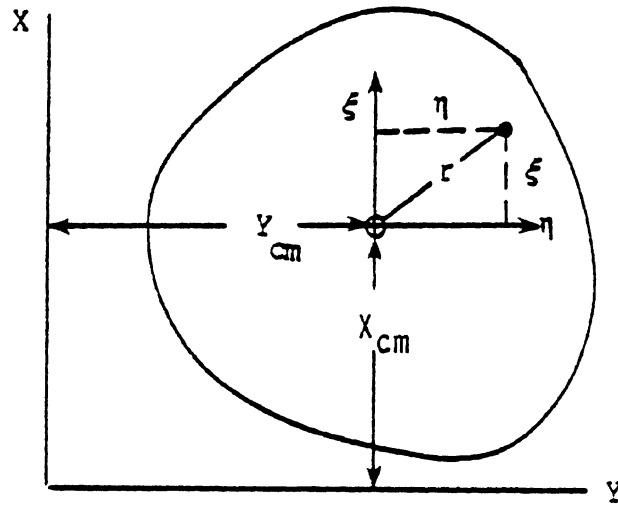
Figure 4.34   Relative Acceleration

coordinates $\xi($ $)$ and $\eta($ $)$ in a body fixed coordinate system is illustrated. The equations of motion of a particle of mass M_A at point A may be written:

$$F_{AX} - M_A(\ddot{X}_{cm} + \ddot{\xi}_A) = 0 \tag{4.54}$$

$$F_{AY} - M_A(\ddot{Y}_{cm} + \ddot{\eta}_A) = 0 \tag{4.55}$$

The summing of all equations of the type (4.54) produce:

$$\Sigma F_{AX} - \Sigma M_A(\ddot{X}_{cm} + \ddot{\xi}_A) = 0 \tag{4.55a}$$

$$\Sigma F_{AY} - \Sigma M_A(\ddot{Y}_{cm} + \ddot{\eta}_A) = 0 \tag{4.55b}$$

But since $\Sigma M_A \xi_A = \Sigma M_A \eta_A = 0$ due to the definition of the center of mass, it follows that $\Sigma M_A \ddot{\xi}_A = \Sigma M_A \ddot{\eta}_A = 0$ and Equations (4.55) reduce to:

$$\Sigma F_{AX} - \ddot{X}_{cm} M_A = 0 \tag{4.56a}$$

$$\Sigma F_{AY} - \ddot{Y}_{cm} M_A = 0 \tag{4.56b}$$

or

$$F_X - M\ddot{X}_{cm} = 0 \tag{4.57a}$$

$$F_Y - M\ddot{Y}_{cm} = 0 \tag{4.57b}$$

where F_X, F_Y symbolize the total of the X and Y components of external force on the body,

M symbolizes the total mass,

and $\ddot{X}_{cm}$, $\ddot{Y}_{cm}$ are the components of the acceleration of the center of mass

Equations (4.57) declare that the center of mass point of a rigid body in plane motion behaves as if it were a particle of mass M, with M equal to the total mass of the body, acted upon by all of the external forces of the system. The forces acting on the body produce a motion of the center-of-mass point which is totally independent of any ideas of rotation of the body. (This statement does not say that the forces are themselves independent of the rotation.)

The rotational degree of freedom of the body in plane motion has yet to have an equation of motion assigned to it. Several forms of such an equation may be written depending upon our choice of centers. The center concept might be one fixed in the ground-fixed coordinate system or fixed in the body-fixed coordinate system. Let us accept, in lieu of an acceptable proof that the "best" center to use, in terms of the simplest result, is a center is taken, the rotational degree of freedom equation of motion becomes identical in form to Equation (4.53) and reads:

$$M_{cm} - I_{cm}\ddot{\phi} = 0 \tag{4.58}$$

The equations of motion for a rigid body in plane motion are:

$$F_X - M\ddot{X}_{cm} = 0 \tag{4.57a}$$

$$F_Y - M\ddot{Y}_{cm} = 0 \tag{4.57b}$$

and

$$M_{cm} - I_{cm}\ddot{\phi} = 0 \tag{4.58}$$

where $I_{cm} = \Sigma \rho i^2 M_i$

$M = \Sigma M_i$

$\ddot{X}_{cm}$, $\ddot{Y}_{cm}$ = components of c.m. acceleration

F_X, F_Y = components of total force applied

M_{cm} = moment of the force system about

and $\ddot{\phi}$ = $\dot{\omega}$ = the angular acceleration

$\ddot{X}_{cm}, \ddot{Y}_{cm}$ components of c.m. acc.

M_{cm} moment of forces about c.m.

F_x, F_y components of force

$\ddot{\theta}$ angular acc.
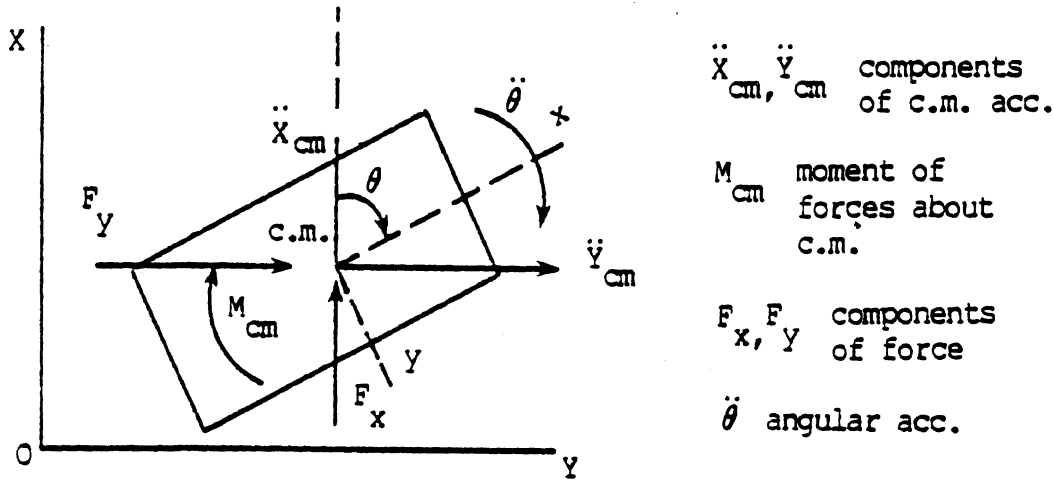
Figure 4.35   The Cartesian Equations of Plane Motion

4.4.6 The Force System Acting on the Vehicle. The three equations of motion (Equations 4.4.57a, 57b, and 4.58) involve a force system acting on the body of the vehicle.   There are two classes of forces—crash forces and tire-road forces.   Crash forces, when they exist, are large and brief, i,e., they are impulsive.   Tire-road forces may be ignored when crash forces are operating but otherwise are the modest level sustained force which control the motion (trajectory) of a vehicle after the instant of crash.   Tire-road forces are generated in the vehicle model at each of the four wheel positions. (Recall that the vehicle center-of-mass is considered to be at road level with the result that the tire forces do not cause any pitch or roll of the vehicle and thus do not disturb the static balance of normal forces at the wheels.

Tire-road forces depend upon:
a)   tire-road friction
b)   degree of wheel lock-up
c)   tire side stiffness
d)   tire normal force

The slip angle was defined earlier.
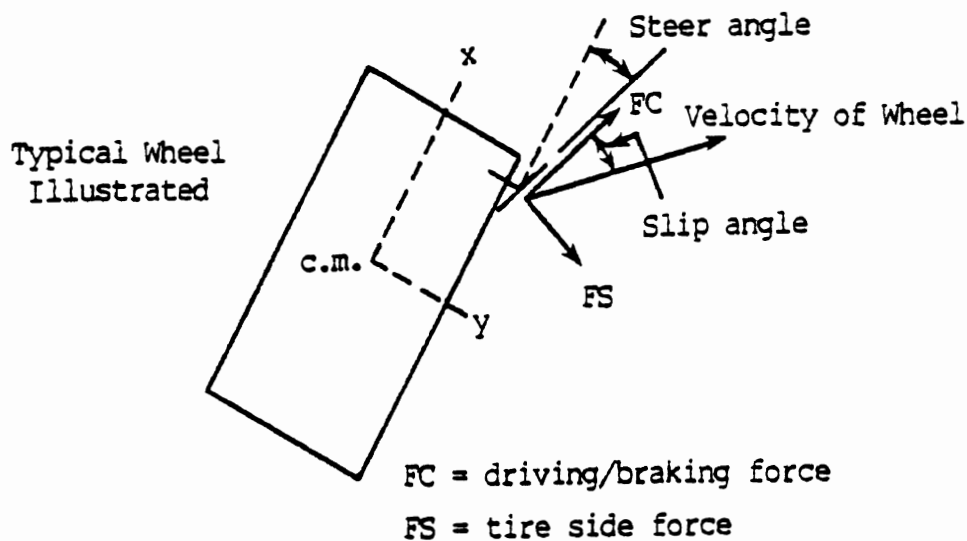
Figure 4.36   Tire Force

Figure 4.36 displays Crash2's tire force model based on slip angles. A maximum value of tire-road shear force is calculated and a component of tire longitudinal shear force is assigned at each wheel equal to the fraction of wheel lock-up times the maximum possible shear force which is the coefficient of friction times the normal load. This longitudinal force, FC, is limited to the maximum shear force times the cosine of the slip angle. The remaining component of shear force is assigned as the maximum allowable transverse component of tire force, FSMX. The actual tire side force FS will be some fractional part, up to 100%, of FSMX accordng to the "Fiala," third order equation illustrated in Figure 36.

Thus, at each wheel location a value for each of two components, FC and FS, of tire-shear force is determined.   The future calculations are eased by converting these components into two other components of force, FX and FY, in the body fixed coordinate system.

        4.4.7 Differential Equations. The three equations (4.57a, 4.57b, and 4.58 may be rewritten with the expressions (4.39) and (4.40) used for the components of acceleration of the   center of mass of the vehicle. The terms F_X and F_Y represent the components of total force parallel to the components of acceleration and thus are $(FX_1 + FX_2 + FX_3 + FX_4)$ and
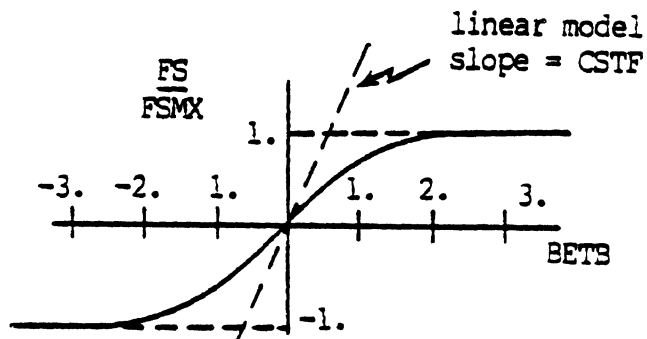
Figure 4.37 Tire Side Force Saturation Model

$(FY_1+FY_2+FY_3+FY_4)$ respectively. The expression for M_{cm} is $t(FX_2-FX_1+FX_4-FX_3)+a(FY_1+FY_2)-b(FY_3+FY_4)$. The differential equations of motion, in the vehicle-fixed coordinate system are thus:

$$M\,du/dt-v\dot{\phi} = FX_1+FX_2+FX_3+FX_4 \tag{4.59}$$

$$M\,dv/dt+u\dot{\phi} = FY_1+FY_2+FY_3+FY_4 \tag{4.60}$$

$$MK^2_{gy}\ddot{\phi} = t(FX_2-FX_1+FX_4-FX_3)+a(FY_1+FY_2)-b(FY_3+FY_4) \tag{4.61}$$

5 ENERGY CONSIDERATIONS

Summary. Conservation of energy is a useful principle in accident reconstruction. An overview finds that terms for kinetic energy, potential energy, energy loss while skidding, and the energy absorbed in plastic deformation of the vehicle may be involved. These terms are organized into an energy balance. Estimation of absorbed energy by the vehicle is reviewed. In particular, the data base and formulation are examined. In the final section, various considerations in the application of these techniques are discussed. These include such things as direct and induced damage, underride/override, narrow object impacts, and variations in structural stiffness.

The DAMAGE algorithm in the CRASH program uses estimates of the energy absorbed in crushing the vehicle during the impact to estimate the change in velocity of each vehicle. Vehicle damage is one of the primary forms of evidence available to the investigator. Knowledge of the structural characteristics of the vehicle allow vehicle damage to be related to the amount of energy absorbed. However, before the discussion of the basis for this estimation is begun, it is appropriate to review the general application of conservation of energy to accident reconstruction.

5.1 Conservation of Energy

Energy is defined as the capacity to exert to exert a force through a distance. If the force is constant, energy is given by:

$$E = Fs \tag{5.1}$$

where:

E = energy in foot pounds
F = force in pounds
s = the distance in feet

An application of this definition is shown in Figure 5.1. Here a mass, m, is raised from an elevation of s_1 to s_2. From the familiar Newton's law we know that the force exerted on the mass due to gravitational attraction(and which must be overcome in order to raise the mass) is given by F = mg where g is the acceleration due to gravity. Consequently, the energy required to move the mass a distance $(s_2 - s_1)$ against the force of gravitational attraction is

$$E = mg(s_2 - s_1)$$

This is the expression for the potential energy associated with a mass and is more commonly written with an h (for height) rather than the s.

$$\text{Potential Energy} = mgh \qquad (5.2)$$

Implicit is an initial elevation of h = 0.
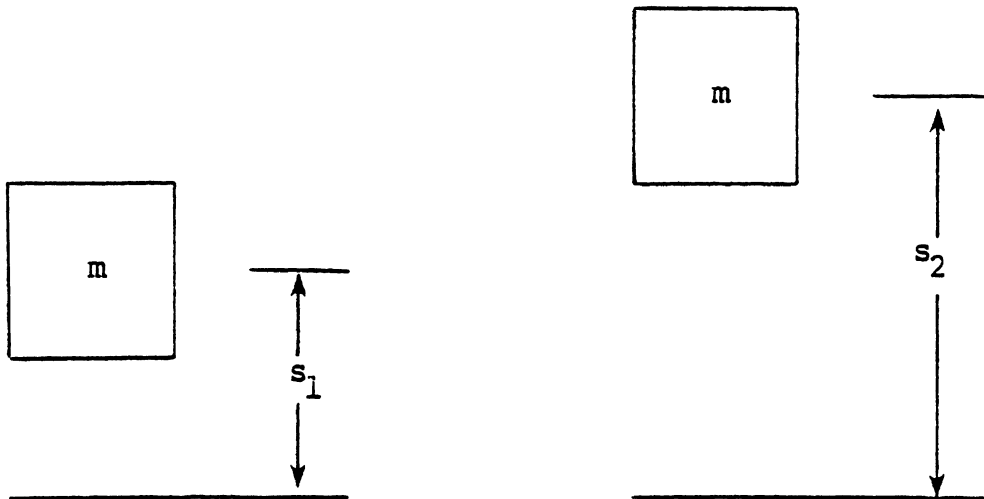
Figure 5.1   Potential Energy

If the force is not constant during the displacement, then the mathematical definition is more complicated, but the idea behind this notation is quite simple. Refering to Figure 5.2a, the energy associated with the action of a constant force through a distance, s, is shown. The product of f times s is equal to the area in the shaded rectangle. Figure 5.2b shows a situation which is only slightly more complicated. Here we have three different values of constant force acting through three different distances. The total energy is found simply by adding the energy associated with each force and the distance it acts through. That is $E_{Total} = f_1 s_1 + f_2 s_2 + f_3 s_3$. The general situation is shown in Figure 5.2c. Here the force is changing continuously as the displacement progresses. The approach is the same, however. It is only necessary that we take very small increments of displacement, s, so that the error in saying that the force is constant in that increment is sufficiently small. The energy computed for each
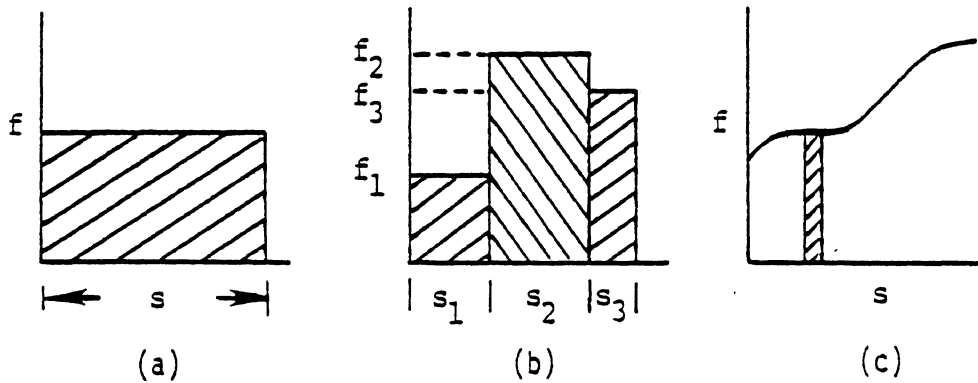
Figure 5.2 Energy Computation When the Force Varies

of the small increments is summed to get the total energy just as in Figure 5.2b. As in the first two examples, the answer we get is graphically equal to the area under the curve which shows how the force, f, varies with the displacement, s. This process of finding the area under a curve is called integration, and is indicated by the elongated "s" in the general definition for energy shown below.

$$E = \int Fds \qquad (5.3)$$

Having introduced the definition of energy, we now move on to the application of the energy balance to accident reconstruction.

Frequently used terms in the application of the principle of conservation of energy to accident reconstruction are shown below.

Kinetic Energy, E_v, $= m \, V^2/2$ (5.4)

Potential Energy, $E_h = mgh$ (5.5)

Energy Loss Skidding, $E_S = mg\mu s \, (LW)$ (5.6)

Energy Absorbed in Vehicle Damage, E_a

The expression of kinetic energy should be familiar to everyone. The expression for potential energy has already been developed. Examining

the expression for the energy loss associated with the skidding of a vehicle will show that it follows from our definition. The skid distance s is apparent. The remaining terms represent the retarding force acting on the vehicle. The coefficient of friction was defined in Chapter 4. The only new term is the wheel lock-up, LW, which expresses the braking force as a percent of the friction force resulting from full lock-up of the wheel. Discussion of the estimation of the energy absorbed in vehicle deformation will be deferred until the next section.

Conservation of energy is applied by writing an energy balance which includes the proper terms. The various energy terms are listed below.

Pre-Impact	Post-Impact
E_{V1}, E_{V2}	E_{a1}, E_{a2}
	E_{S1}, E_{S2}
	E_{h1}, E_{h2}

Subscripts 1 and 2 refer to vehicles 1 and 2. In the development of the equations for the DAMAGE algorithm, the energy balance is applied only to the impact phase. In this case an energy balance equates the sum of the kinetic energies for each vehicle before impact with the sum of their kinetic energies at separation and the energy absorbed in deformation of the two vehicles.

$$(mV_i^2/2)_1 + (mV_i^2/2)_2 = (mV_f^2/2)_1 + (mV_f^2/2)_2 + E_{a1} + E_{a2}$$

Here the subscripts i and f refer to pre-impact and separation (initial and final), while subscripts 1 and 2 still refer to the vehicles.

An energy balance can also be written for the post-impact trajectory of a vehicle. The familiar equation relating stopping distance and speed may be developed by equating the kinetic energy of the vehicle at the beginning of the skid with the energy loss of the skid as shown below.

$$m\,V^2/2 = mg\mu s\,(LW)$$

rearranging:

$$V = \sqrt{2g\,\mu\,s(LW)} \qquad\qquad (5.7)$$

This expression can be easily expanded to apply skidding on a grade. The energy balance must now include the kinetic energy of the vehicle at the beginning of the skid with the energy loss of the skid and the potential energy associated with the grade. This term is added if the grade is up and subtracted if the grade is down.

$$E_v = E_s + E_h$$

$$mV^2/2 = mgs\mu(LW) + mgh$$

rearranging:

$$V = \sqrt{2g[s\mu(LW) + h]} \qquad\qquad (5.8)$$

but, h = s*sinθ where θ is the angle of the grade, and if θ is small (less than 12–15°), then h = s(Grade %)/100 where the grade is measured as the elevation change for a given distance of roadway divided by the roadway length measured on a horizontal (expressed typically by Highway Engineers as a percentage). Substituting into eq. (5–8), we have:

$$V = \sqrt{2gs[\mu(LW) + (Grade\ \%)/100]} \qquad\qquad (5.9)$$

The important thing to notice in this expression is that the grade expressed as a fraction adds (or subtracts for a downgrade) directly from the term involving the friction coefficient. Taking the case where the wheel lock-up, (LW), is 1.0, we see that the effective friction of a 10% downgrade with a 0.70 coefficient is 0.60 (0.70 − 0.10). For a 10% upgrade, it would be 0.70 + 0.10, or 0.80. Even though the CRASH program is not formulated to handle elevation changes, this method provides a convenient way to incorporate the effect of roadway grades in the trajectory computation. This ends the overview of the use of an energy balance in accident reconstruction. The next section describes the data base and formulation of the procedures for estimating the amount of energy absorbed in crushing of the vehicle.

5.2 Estimating Energy Absorbed

Fortunately, one of the more permanent pieces of evidence is the damage to the vehicles. This permanent deformation is described as

"plastic" deformation to be distinguished from "elastic" deformation. If the deformation is elastic, the deformed body will return to its original dimensions when the force is removed. Plastic deformation is permanent and remains after the force is removed. Our definition of energy tells us that the amount of energy absorbed is related to the force and the distance it acts through. We are able to measure the distances involved. They are the crush measurements. If information was available on the forces required to produce the damage, then the amount of energy absorbed could be estimated. Remember that it was indicated earlier that the DAMAGE algorithm is able to compute the change in velocity of the vehicles if the energy absorbed by each is known.

Limited crash tests conducted with instrumented vehicles provide a basis for a first approximation of the vehicle force deflection characteristics. A sample of these data are shown in Figure 5.3. The data shown are from barrier impact tests. In such a test, virtually all of the vehicle's kinetic energy prior to impact is absorbed in deformation of the vehicle. The primary characteristic of these test data plots shown in Figure 5.3 is that the residual (permanent) crush increases approximately linearly with impact speed over the speed range of about 15 to 50 mph. One might wonder what kind of relationship between force and crush depth would produce this result. One possible (but not the only) answer is that the force must also increase linearly with crush. The equation for the force may be written in terms of the equation of the line relating the barrier test speed and the crush as follows:

$$F = (W/g) \, (b_0 b_1 + b_1{}^2 C)$$

where b_0 and b_1 are the slope and y-intercept from:

$$V_{barrier\ impact} = b_0 + b_1 C$$

This equation is taken from Figure 5.3.

One more step is necessary before this model has any application to real accident vehicles. Since the crush is seldom uniform across the vehicle, the model must also be able to address this situation. No data are available which would indicate how the stiffness varies across the width of the vehicle. In the absence of such information, it has been

$V = 3.0 + 1.35$ (Crush)

$W = 2500\#$

$V = 6.85 + 0.88$ (Crush)

$W = 4500\#$

RESIDUAL CRUSH, inches
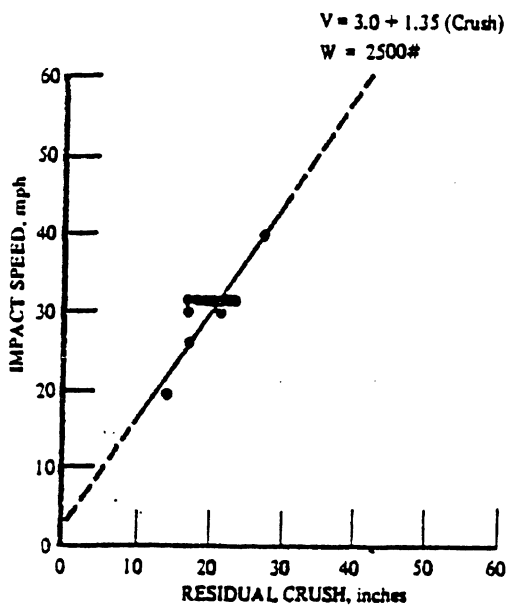
IMPACT SPEED, mph

Residual crush vs impact speed in full frontal barrier tests for '71-'74 Chevrolet Vegas
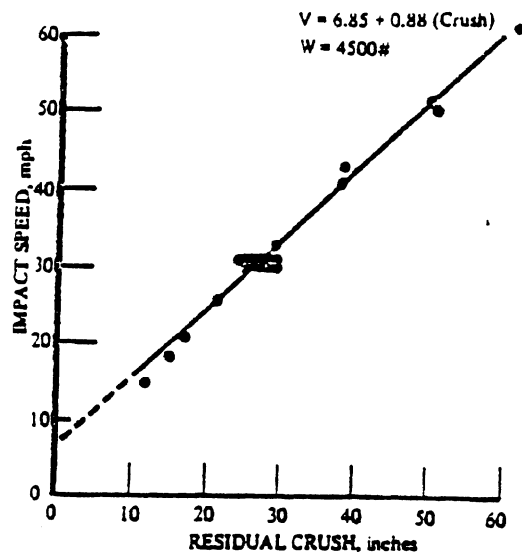
Residual crush vs impact speed in full frontal barrier tests for '71-'72 full size GM vehicles

Summary - Frontal Barrier Test Data

$V(mph) = b_0 + b_1 C(in.)$

GM Vehicles	b_0 mph	b_1 mph/in.	weight lbs.
71-72 Standard Full Size	6.85	0.88	4500
73-74 Standard Full Size	7.5	0.90	4500
73-74 Intermediate	7.5	0.90	4000
71-74 Compact	3.0	1.35	3400
71-74 Subcompact	3.0	1.35	2500

Vehicle Classification	Examples
Standard Full Size	Chevrolet Impala, Pontiac Catalina, Oldsmobile Delta 88, Buick LeSabre
Intermediate	Chevrolet Chevelle and Monte Carlo, Pontiac LeMans and Grand Prix, Oldsmobile Cutlass, Buick Century
Compact	Chevrolet Nova, Pontiac Ventura, Oldsmobile Omega, Buick Apollo
Subcompact	Chevrolet Vega

Note: The following vehicle classifications are not included in the above table:

Sports	(Chevrolet Corvette)
Compact Specialty	(Chevrolet Camaro, Pontiac Firebird)
Luxury Sedan	(Oldsmobile 98, Buick Electra 225, Cadillac Calais)
Personal Luxury	(Oldsmobile Toronado, Buick Riviera, Cadillac Eldorado)

Figure 5.3 Sample Data from Barrier Crash Tests

assumed that the force per unit width is uniform and is given by simply dividing the total force by the width of the vehicle. The resulting model is shown in Figure 5.4. The total energy is obtained by summing the area under the force curve for the entire deformed area. Mathematically this is expressed as:

$$E_a = \int\!\int F \, dC \, ds$$

Two integration symbols are needed since we now have a three-dimensional figure.
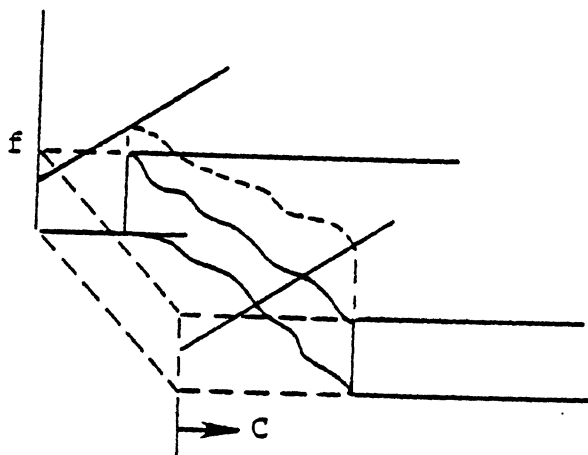
Figure 5.4 Force-Deflection Model

For simple damage patterns, this equation may be integrated and the results plotted. Such plots are shown in Figure 5.5. In these figures the energy absorbed is expressed as an "equivalent barrier speed" or "EBS." The EBS is defined as a vehicle speed such that its kinetic energy would equal the energy absorbed (as in the barrier test). The upper plot in Figure 5.5 is for '71-'72 full size GM vehicles, while the lower plot is for '71-'74 Chevrolet Vega. The model described was then applied to some angle and offset barrier tests where the true impact speed was known. The results of this comparison are shown in Figure 5.6. The accuracy seems to be approximately ± 3-5 mph.

This discussion has been limited to frontal damage. Much less
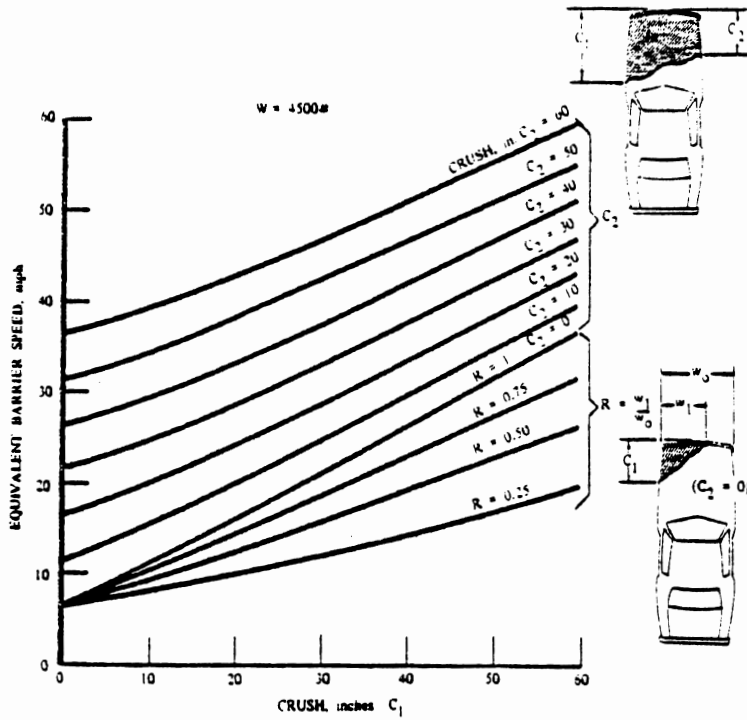
EBS vs crush for '71-'72 full size GM vehicles having angle barrier damage patterns
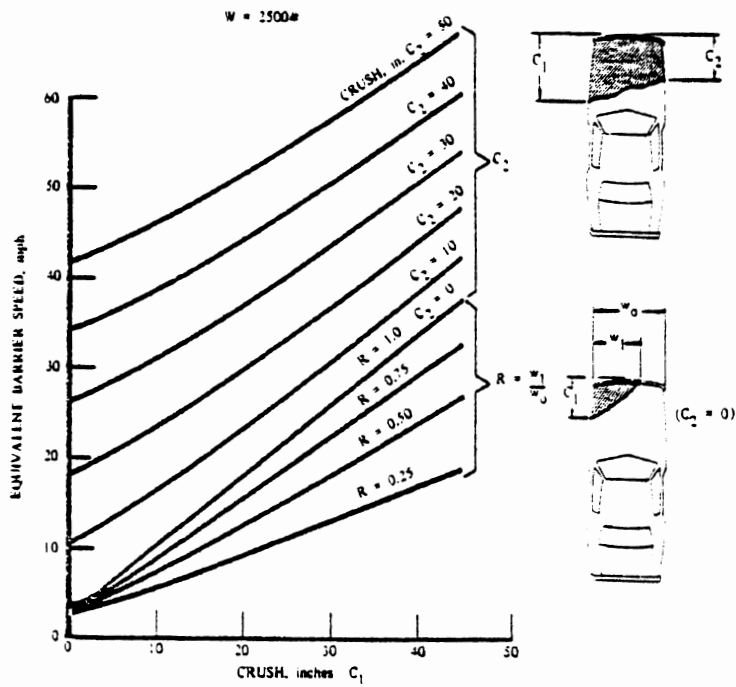
EBS vs crush for '71-'74 Chevrolet Vegas having angle barrier damage patterns

Figure 5.5   Relationship Between Crush and Energy

Comparison of Angle Barrier Test Data With Results
of Linear Force-Deflection Model

Test No.	Impact Speed	Test Weight	Left Crush	Right Crush	Estimated Speed	Error
			Chevrolet Vega			
1	30.6 mph	2390 lbs.	35.0 in.	6.5 in.	33.7 mph	+3.1 mph
			Standard Full Size GM Vehicles			
2	18.3	4715	0	25.0	19.4	+1.1
3	21.6	4873	29.0	0	21.0	-0.6
4	21.7	5032	38.0	2.0	25.7	+4.0
5	21.5	4730	1.0	28.0	21.2	-0.3
6	25.4	4536	39.5	0.5	27.3	+1.9
7	30.6	4820	37.0	2.0	25.7	-4.9
8	30.8	4808	43.0	5.5	29.9	-0.9
9	30.8	4505	40.0	0	27.5	-3.3
10	29.5	4573	36.0	0	25.3	-4.2
11	30.7	4561	3.0	49.5	33.2	+2.5
12	30.4	4835	4.0	40.0	27.8	-2.6

Average Error = -0.35 mph (0.6 km/h)

Comparison of Offset Barrier Test Data With Results
of Linear Force-Deflection Model

Test No.	Impact Speed	Test Weight	Left Crush	Right Crush	Estimated Speed	Error
			Compact R = 0.25			
1	29.4 mph	3611 lbs.	29.0 in.	1.5 in.	26.8 mph	-2.6 mph
2	29.1	3601	31.0	0	28.1	-1.0
3	30.7	3596	34.0	3.0	31.4	+0.7
			Standard Full Size GM Vehicles R = 0.25			
4	30.0	4233	42.2	6.2	31.6	+1.6
5	30.6	4192	40.0	0	28.9	-1.7
6	30.6	4333	41.0	3.0	29.7	-0.9
7	30.7	4395	43.5	6.5	31.8	+1.1
			Chevrolet Vega R = 0.25			
8	30.6	2543	35.0	0	32.0	+1.4
9	30.0	2388	29.0	3.0	28.8	-1.2
10	30.5	2373	34.0	4.5	33.7	+3.2
			Standard Full Size GM Vehicle R = 0.75			
11	30.9	4943	18.0	30.0	30.2	-0.7

Average Error = -0.01 mph (0.016 km/h)

Figure 5.6   Estimating Barrier Impact Speeds

information is available for side or rear damage. However, the same method has been applied, and the coefficients estimated as well as possible.

5.3 Considerations in Measuring Crush for Energy Estimation

Knowledge of the energy estimation procedure and the data it is based on can aid the investigator's measurement of crush and his assessment of the accuracy of the reconstruction. A number of specific topics will be discussed in this light in the following material.

Direct vs Induced Damage. Procedures developed for assigning a CDC specifically address direct, or contact, damage. Assignment of a CDC and crush measurements for energy estimation has a somewhat different objective. The CDC is intended to locate and describe the direct damage. For accident reconstruction, the emphasis is on defining the direction and magnitude of the applied force. It is not particularly important to the reconstruction if the description is not accurate so long as the right applied force is obtained.

Energy estimation procedures could probably be developed using direct contact damage only. In fact the coefficients were developed from tests where all the damage was direct contact damage. The resulting models were then tried for a handful of pole and offset barriers (only a fraction of the vehicle width contacts the barrier) where appreciable induced damage was produced. Use of the contact damage only produced energies which were appreciably lower than the known values. Use of the direct plus the induced damage produced results which were approximately correct. Guidelines at this time are primarily subjective. However, it is believed that they will improve the accuracy of the results, and it is always important that the reconstructions be carried out in as consistent manner as possible.

Underride/Override. The force-deflection model for estimating energy absorbed described earlier has no provision for variations in crush vertically. Underride or override types of damage are very difficult to assess. A simple average of the vertical crush measurements could be used if one felt the structural stiffness was uniform from top to bottom. Particularly for frame and body vehicles,

this is simply not likely to be true. The frame provides the major structural stiffness in these vehicles. Recognizing this, it is likely to be more accurate to weight the crush measurement in favor of the measurement of frame deformation. Of course, the actual measurements must be documented and preserved. More detailed models may someday be available. However, accident reconstruction is always an iterative process where one varies the parameters in line with their uncertainty in looking for the most consistent set of information. The "correct" crush measurements for estimating energy will be closer to those describing frame deformation than those describing only sheet metal damage.

Forces Applied at an Angle to the Normal. This topic will be discussed more completely in Chapter 9. However, it should be pointed out here that the force-deflection model described was developed from tests where the direction of force and the crush were always normal to the undeformed surface. Structural stiffness is likely to be different at different loading angles. Again the necessary information is lacking. Consequently, the energy estimates are more suspect as the angle to the normal of the applied force gets larger.

Narrow Object Impacts. Frame involvement is the important factor here too. The stiffness was assumed to be uniform across the entire width of vehicle. This seems reasonably sound as long as one or both of the frame rails is involved. If only the fender sheet metal outside of the frame rails is involved, the stiffness values in the program are likely to be too high. This situation is almost a sideswipe. Sideswipe collisions cannot be reconstructed with CRASH.

Other Variations in Stiffness. A few miscellaneous considerations will be mentioned here. A uniform stiffness for the entire length of the side structure is also assumed. Again it is likely that this is not exactly correct. One would expect that the structure in the area of the wheels will be stiffer, while the structure between the wheels would be relatively softer. Knowledge of possible problems like this should aid in the resolution of cases with apparent inconsistencies in the data. In the same fashion, station wagons and rear engine vehicles are likely to be appreciably stiffer than stored coefficients which presumably

apply to front-engine sedans. For the station wagons and rear-engine vehicles, a reasonable approximation might be to carry out the reconstruction as if the front structure was involved.

At the beginning of this section it was observed that accident reconstruction is an iterative process. Observed information is varied in accordance with its associated uncertainty in attempt to find a set of information which is as internally consistent as possible. The considerations presented in this section were not intended to suggest that "fictitious" data should be generated. Rather, this discussion was intended to provide the investigator with more complete knowledge of the limitations and the purposes of the measured data. In particular, this discussion should serve to better define the uncertainties which are involved in the application of the models for estimating the energy absorbed in vehicle deformation.

6 VEHICLE DYNAMICS

Summary The mechanics of the tire-vehicle system governing
the process of steering, braking, and accelerating are
discussed and explained so as to assist the accident
reconstructionist in obtaining an understanding of "why" and
"how" a motor vehicle responds to driver control and of the
circumstances under which the driver may lose his ability to
control the path of his vehicle.

6.1 Introduction

A course in accident reconstruction necessarily treats situations
in which vehicles become translating and rotating masses governed only
by the laws of physics as opposed to being also governed by the control
actions of the driver. The accident reconstructionist generally works
backward in time and space from the final resting place of the vehicles
involved in the accident to determine the crash and pre-crash conditions
that existed in a given accident event.

Clearly, the driver has little or no influence on the outcome of
the events which occur during a crash or after a crash. In the case of
the vehicle motions which take place prior to a collision, the control
actions of the driver can play a major role and the maneuvering limits
of the vehicle can be a large factor in setting up the sequence of
events which create the emergency.

In other words, we should differentiate between vehicle
trajectories which occur after the driver has lost his ability to
control the path of his vehicle and the trajectories which occur before
the driver has lost the ability to exercise control. In order to make
this differentiation, we find that the properties and behavior of the
pneumatic tire must be understood to a considerably greater level of
detail than is necessary to explain, predict, or reconstruct crash and
post-crash events. For example, unless the accident investigator has
some understanding of why the motor car turns when the steering wheel is
turned, he is not likely to understand why the same car can and will
steer differently at different times, as well as understand why
different cars respond differently to the same steering control actions
by a given driver. Accordingly, our objective, in these lectures, is to
give the student sufficient understanding of the factors influencing and

determining the maneuvering capability of the motor car such that he (she) will be able to recognize these factors in the field and make certain judgments as to the extent to which these factors may have led to the establishment of a given set of pre-crash conditions.

After establishing certain definitions, we shall discuss the characteristics of the pneumatic tire which determine the sublimit and the limit maneuvering characteristics of the motor vehicle. A substantial effort will be devoted to this topic. for it is the tire which ultimately provides the driver with the forces which allow him to control the vehicle. Following this tire mechanics discussion we shall investigate vehicle handling. Again, substantial effort will be devoted to examining the vehicles response to pure steering input. Vehicle handling will be completed with a brief examination of how braking can effect the turning process. Then, the braking process itself will be examined. We shall conclude this chapter by noting that it is possible to describe these various phenomena in mathematical terms and, consequently, it is feasible to use computers to determine when and why a vehicle will reach its maneuvering limit under a given set of conditions.

6.2 Definitions

To make the presentation and discussion more efficient, we should begin with some definitions. First, we need to have a consistent way of referring to the various motions of a motor vehicle and for identifying the various forces which act on the vehicle. These forces can be inertial forces associated with the acceleration of the mass of the vehicle in space as well as the forces that create this acceleration, namely, the external forces that act on the vehicle at tire-road contact and on the surface of the vehicle (as a result of the air pressures that are created by the flow of air around the body of the vehicle).

To facilitate these definitions, it is convenient to put a system of axes in the motor cars as shown in Figure 6.1. In the U.S., we have adopted the convention which places the so-called x-axis in the plane of symmetry of the vehicle, with the positive direction of the x-axis being forward, namely, towards the front of the vehicle. The y-axis is aligned perpendicular to the x-axis, with its positive direction out to

the right side of the vehicle, as seen from the driver's position. The z-axis is perpendicular to the plane formed by the x- and y-axes, with its positive direction being downward into the road. For our purposes, here, we shall not worry about the exact orientation of these x- and z-axis in the plane of symmetry, other than to note that the x-axis is essentially horizontal (i.e., parallel to a horizontal roadway), with the z-axis being a vertical axis that is normal to a horizontal roadway.

LATERAL VELOCITY (v)   PITCH VELOCITY (q)
  LATERAL FORCE (F_y)   PITCH
        Y   MOMENT
      axis   (M_y)

LONGITUDINAL
VELOCITY (u)

    X   ROLL VELOCITY (p)
  axis   ROLL MOMENT (M_x)

LONGITUDINAL
FORCE (F_x)

                                    YAW VELOCITY (r)
                                    YAW MOMENT (M_z)

                            Z
                          axis

                    VERTICAL VELOCITY (w)
                    VERTICAL FORCE (F_z)
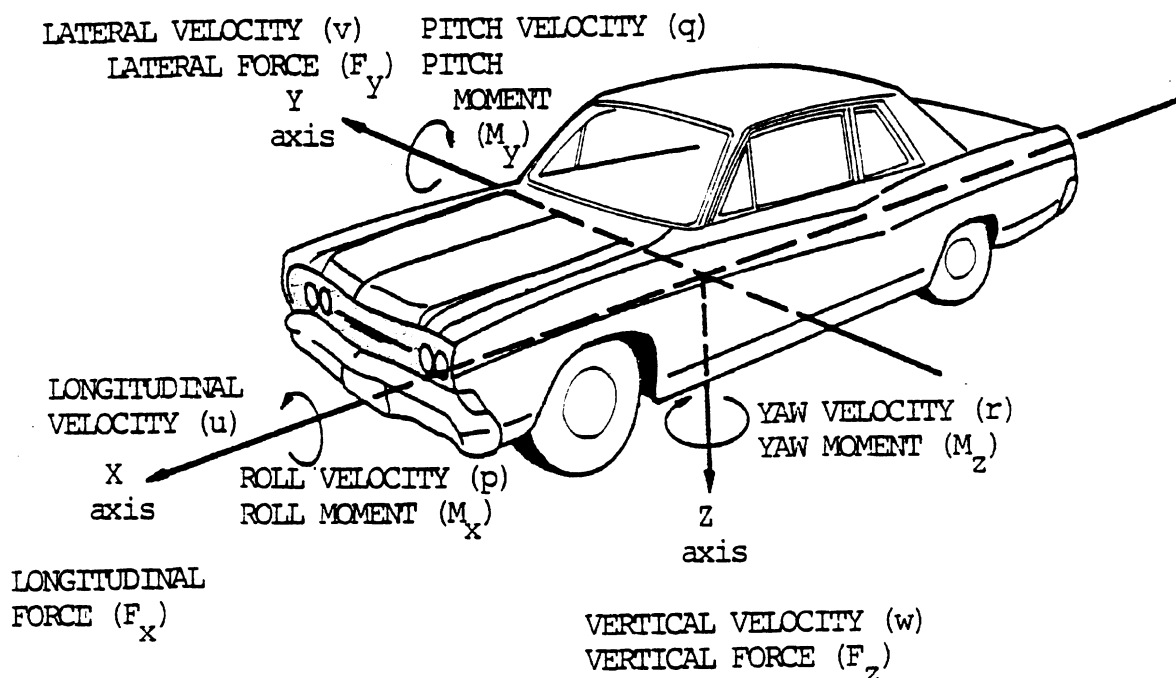
Figure 6.1 Directional Control Axis System

Since the body of the vehicle is attached to the running gear by an arrangement of rigid and compliant members that permit the body to move relative to the wheels, it is possible for the body to rotate about the x and y-axes and also to move vertically, as well as translate in the x and y directions along with the running gear. In addition, it is possible for the total vehicle to rotate about the vertical z-axis. The origin of this axis system can, under certain circumstances, have a

translational velocity in space which has a component along each of the three defined coordinates. We call the velocity component along the x-axis a "longitudinal" or forward velocity. The velocity component along the y-axis is called a "lateral" or "sideslip" velocity. The velocity component along the z-axis is alternatively referred to as the "vertical," "bounce," or "heave" velocity. An angular displacement or velocity about the x-axis is called a "roll" displacement or velocity. An angular displacement or velocity about the y-axis is called a "pitch" displacement or velocity. Finally, an angular displacement or velocity about the z-axis is called a "yaw" displacement or velocity.

Clearly, when a motor vehicle is traversing a straight, perfectly smooth and level road, its total translational velocity is aligned along the positive x-axis, that is, lateral and vertical motions are absent.

Similarly, the external forces acting on the vehicle can be defined in terms of their components along the same axis system used to define vehicle motion. If a component of force exists along the x-axis, we call it a longitudinal force. If this longitudinal force acts in the positive x direction, we may call it a "tractive" or "propulsive" force. If it acts in the negative x direction, we may call it a "resistive," "drag," "retarding," or "braking" force. If a component of force exists along the y-axis, we call this component a "lateral" or "side" force. If a component of force exists along the z-axis, we call it a "vertical" or "normal" force. If this force is aerodynamic in origin, we frequently call it a "lift" force.

In addition to defining what is meant by a longitudinal, lateral, or normal force, Figure 6.1 shows that moments acting about x, y, and z axes, respectively, are called rolling, pitching, and yawing moments.

Let us now consider the process by which a driver positions his car relative to the roadway. His three control devices are the throttle, the brake pedal, and the steering wheel. In essence, the throttle (through its control over engine torque) controls the propulsive force applied to the vehicle at the tire/road interface so as to determine the forward speed and forward acceleration of the vehicle. Figure 6.2 shows the system of components involved in this control activity, namely, the accelerator pedal which controls the air-fuel flow into an engine which,

in turn, creates a torque transmitted to a pair of driving wheels by means of an assemblage of shafts, gears, and drive axles, commonly referred to as the driveline. This torque controls the rolling speed of the tire and, as we will see, the tire provides the mechanism to apply propulsive force to the vehicle.
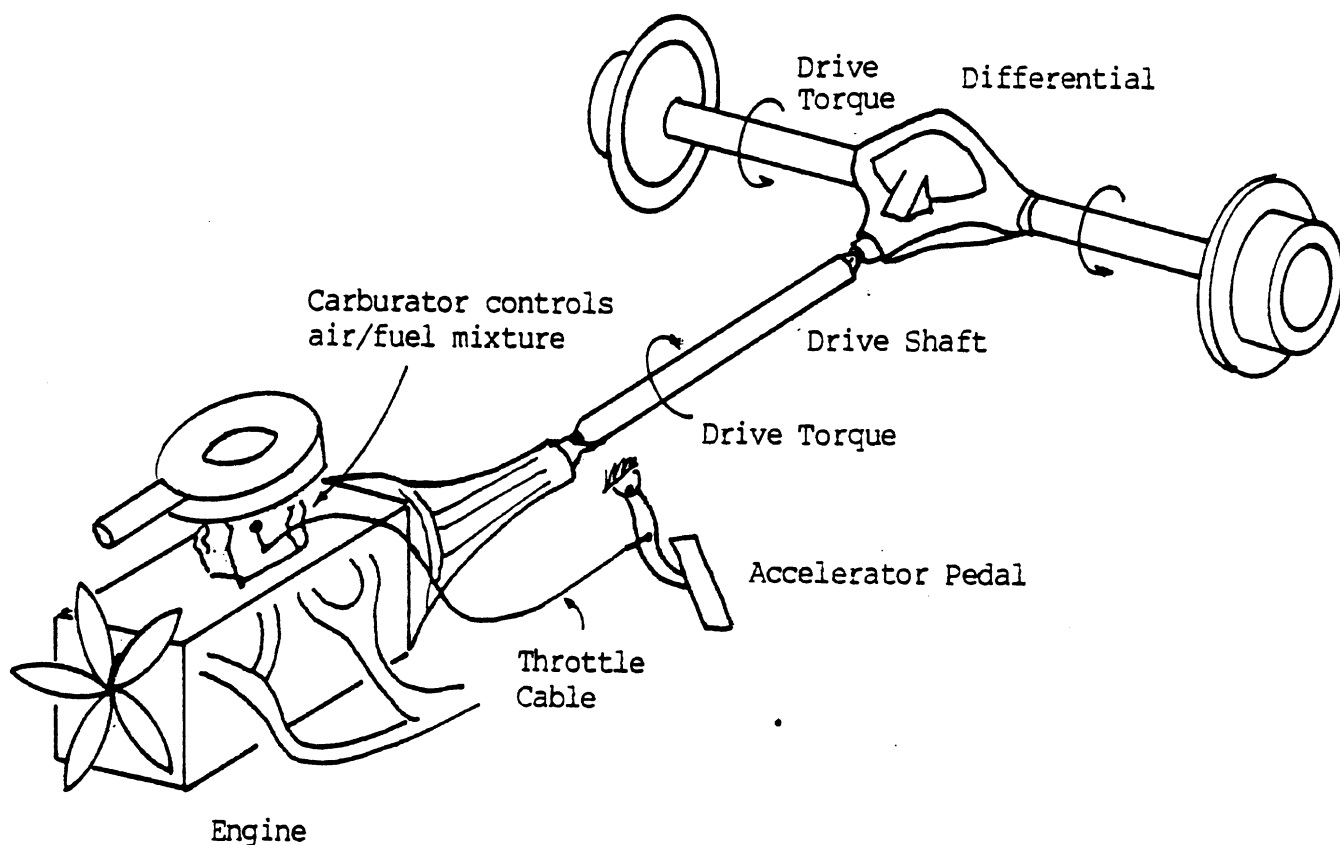
Figure 6.2 The Drive Train

The brake system, on the other hand, is the assemblage of components by which the force applied to the brake pedal by the driver is converted to a retarding force acting to decelerate or slow the vehicle. Figure 6.3 shows the combination of components by which the force on the brake pedal is converted to a torque at each of the wheels. This torque, again, controls the rolling speed of the tire which ultimately creates a retarding (or braking) force at the tire/road

interface.

Finally, we have the steering wheel which in combination with the steering system, diagrammed in Figure 6.4, provides a means for displacing the front wheels about an essential vertical axis and thereby cause the front tires to create a lateral force, and, more importantly, a yawing moment acting on the vehicle. This yawing moment causes an angular acceleration to exist about the vertical z-axis and, eventually, a steady yaw velocity and sideslip velocity are created which means that the vehicle is traversing a circular path having a fixed radius. We shall have much more to say about this turning process at a later point.

To begin to recognize the importance of the tire to vehicle' behavior, notice that all three of these control systems end at the tire. The forces which finally move (or stop) the vehicle are generated at the tire/road interface in all three axis. The purpose of all the other components of these three systems is to provide the driver with control over the tire.

Having identified throttle, brake-pedal, and steering-wheel displacement (and/or forces or torques) as the control devices or control inputs by which the driver positions his vehicle in space and time, we should define what is meant by "disturbance inputs." Disturbance inputs are forces applied to the vehicle over which the driver has no control. These forces derive from the environment in which the vehicle is operated. The environment has only two facets--the air through which the vehicle moves and the road surface which the tires traverse. If the air is moving relative to ground in an irregular fashion or if the vehicle is sheltered from a wind and then exposed to the wind, time-varying aerodynamic forces and moments will act on the vehicle so as to produce directional motions which have not been demanded by the driver through his control of the steering wheel. Similarly, if the road profile is irregular rather than smooth, the normal forces between tire and road experience a time-varying behavior that can influence the propulsive, braking, and steering control process. Further, if the road surface changes its frictional character (as the vehicle traverses the roadway), the external forces produced by the tires could change in a reasonably sudden manner, again producing
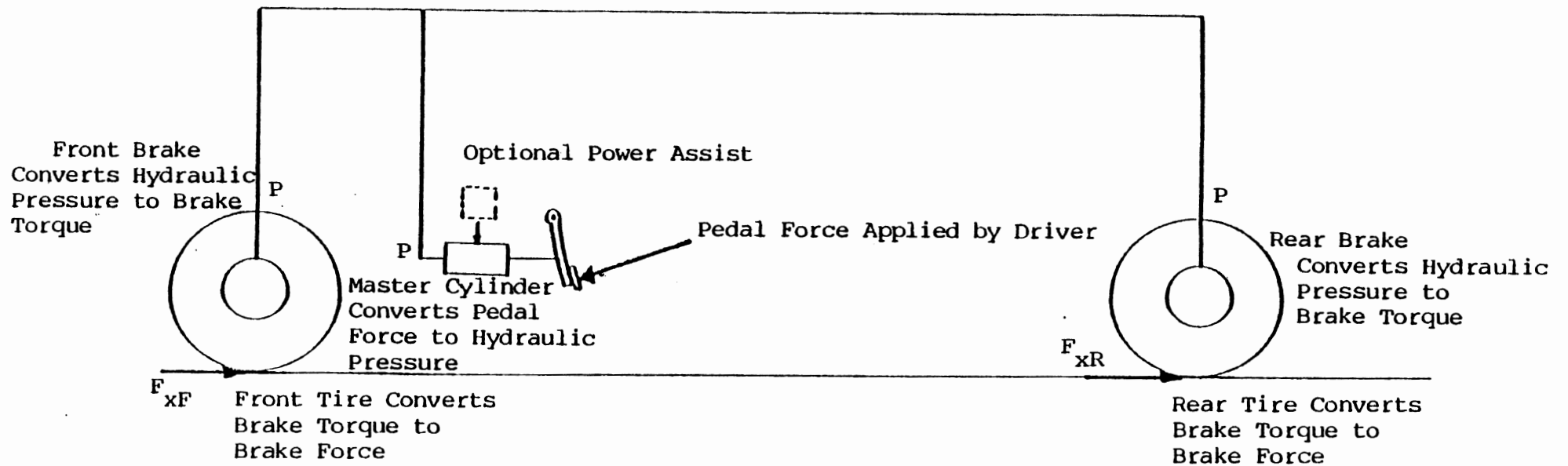
Figure 6.3 The Brake System
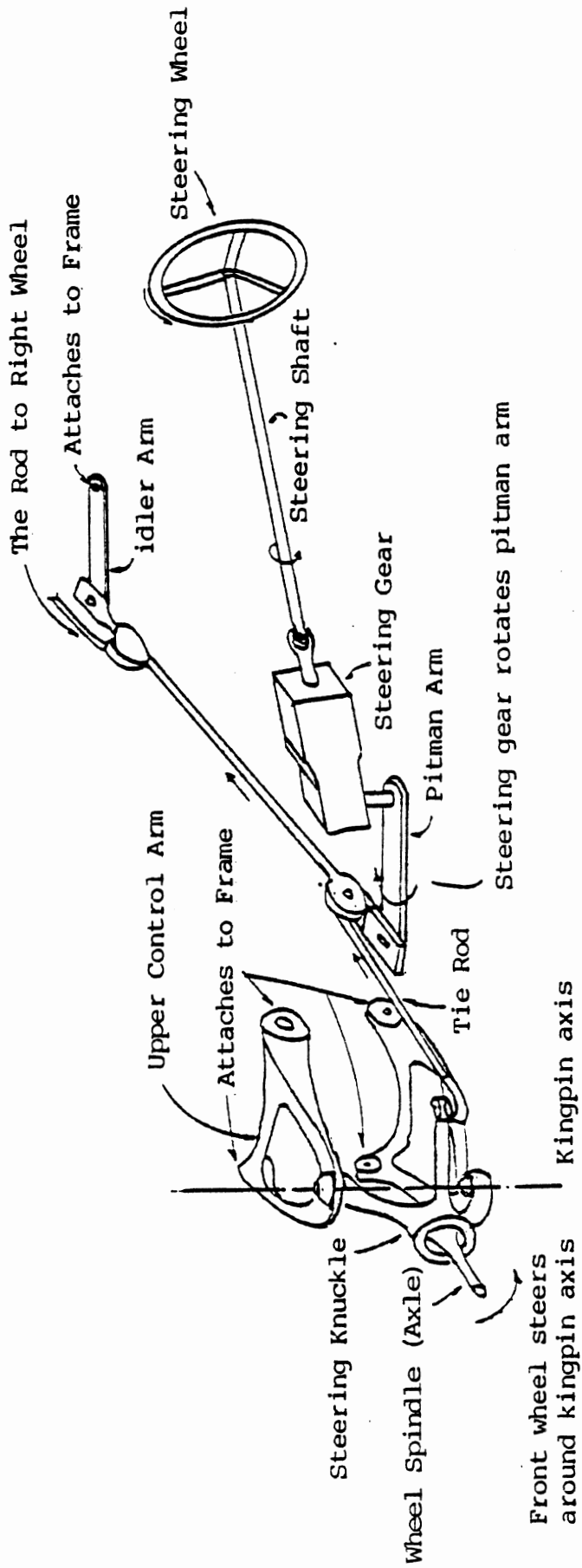
Figure 6.4 The Steering System

The Rod to Right Wheel

Attaches to Frame

idler Arm

Steering Wheel

Steering Shaft

Steering Gear

Pitman Arm

Steering gear rotates pitman arm

Upper Control Arm

Attaches to Frame

Tie Rod

Kingpin axis

Steering Knuckle

Wheel Spindle (Axle)

Front wheel steers
around kingpin axis

disturbance input over which the driver has no control.

Two final definitions of importance are the meaning of "steady state" and "transient" conditions. By "steady state" we mean that state in which all of the forces and moments acting on the vehicle remain fixed with time and a summation of all of the positive and negative forces and a summation of all of the positive and negative moments yield a zero net force and a zero net moment. That is, all forces and moments are in "balance." Under these conditions, the forces on the vehicle remain fixed with time until a control input is changed or a disturbance input has been encountered. For example, a vehicle that is dynamically stable will, after the driver has displaced his steering wheel, reach a steady state of turning equilibrium in which the vehicle traverses a circular path of constant radius. We shall find it very instructive later to look at this steady turning condition and to consider the factors that govern (1) the limit turning condition and (2) the manner in which the vehicle behaves if the driver should attempt too exceed the limit turning ability of his vehicle.

When the vehicle is approaching the state of equilibrium corresponding to a fixed control input, the forces on the vehicle are varying with time and the vehicle is said to be in a "transient" state. Although a detailed understanding of the transient state can only be obtained by solving the differential equations which describe the vehicle system, we shall briefly consider some factors which make vehicles take varying lengths of time to reach an equilibrium state following a driver's control action. We shall also want to consider to some depth the operating conditions and/or vehicle conditions which cause the transient state to be unstable. For example, if the driver has displaced the steering wheel, the vehicle does not achieve a state of steady turning equilibrium and, if left untended by the driver, will try to exceed its limit turning capability.

In order to consider the process of steering, braking, and accelerating in terms of the ability of the driver to control these processes (and thereby achieve a desired trajectory in space), we shall now begin to consider the various properties of the pneumatic tire that give the motor vehicle its unique set of characteristics.

6.3 Tire Traction and Compliance

The great success of the inflated, pneumatic tire derives mainly from its ability too provide a vertical compliance between the road surface and the wheel, simultaneously creating very little resistance to forward motion. In addition to this already remarkable capability, the tire provides a high level, frictional coupling between the vehicle and roadway so that forces of sufficient magnitude can be developed to provide the vehicle with acceptable levels of motivation, guidance, and stopping ability.

It is probably the pneumatic tire, with its unique combination of properties, which, more than any other single mechanical element, renders the modern highway/vehicle transportation system a reasonable operation. Moreover, it is the single element by which, ultimately, all control forces which guide the vehicle are generated. And so, an understanding of the mechanics of the pneumatic tire is the most basic element to an understanding of vehicle dynamics.

The two major characteristics of a tire which influence its behavior in producing vehicle control forces are:

1) Its compliance properties (how the tire generates forces through spring-like mechanisms.)
2) Its traction properties (how the tire forces are governed by the tire-road frictional coupling).

These two characteristics are active in all the different areas of tire force generation, that is, in longitudinal (braking and acceleration) force generation, lateral (turning) force generation, and the combination thereof. In each area, it is essentially true that the compliance properties govern the generation of forces in response to smaller driver-control inputs (virtually all normal driving activity) and that traction properties govern vehicle performance in reaction to larger control inputs (near the limits of vehicle performance).

Given the extreme importance of the tire, the vehicle dynamics community has expended a great deal of effort towards both defining the generic qualities of tire compliance and traction and measuring the compliance and traction properties of individual tires. Figures 6.5, 6.6, and 6.7 illustrate some laboratory and over-the-road "tire force

dynamomters" which are used to obtain the necessary measurements for these studies.

It has been found in such studies that tire compliance properties are relatively simple and generally are a function of the tire and its vertical load only. That is, compliances generally depend on tire construction, inflation pressure, wear state, and vertical loading and do not depend on roadway and environmental factors or speed. Conversely, tractive properties are quite complex and depend on a host of variables, including:

1) tire construction properties (carcass and tread rubber)

2) the normal load on the tire

3) inflation pressure of the tire

4) wear state, and in particular, the geometry of the tread profile

5) tread temperature

6) velocity

7) a variety of roadway properties which contribute to the friction mechanism

8) roadway contaminants (especially water) whose presence generally degrades friction.

9) Speed

In the following three sections we will examine how the tire generates lateral, longitudinal and combined forces, respectively, through compliance and traction mechanisms.

6.3.1 <u>Lateral Force Generation</u> Let us begin with an examination of tire compliance in the lateral (side force) direction. The simplified "tire model" of Figure 6.8 is a great help in comprehending lateral compliance. The model assumes that the tire width is reduced to zero so that it is simply a flat disc and the tread element has become a "deformable circle." In the top picture, in which the tire is rolling freely straight ahead, we see that the vertical load has deformed this
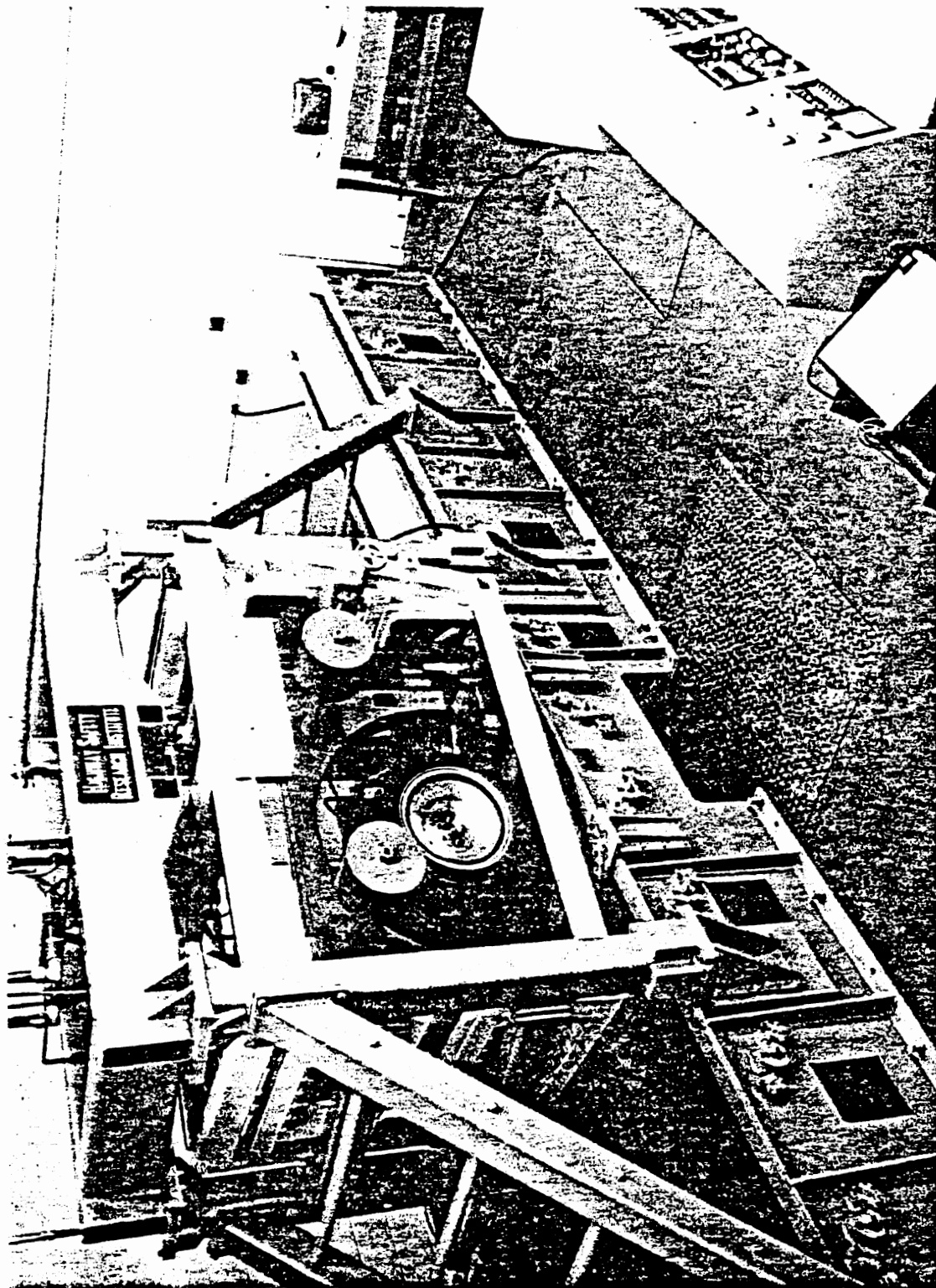
Figure 6.5 Flat Bed Tire Tester
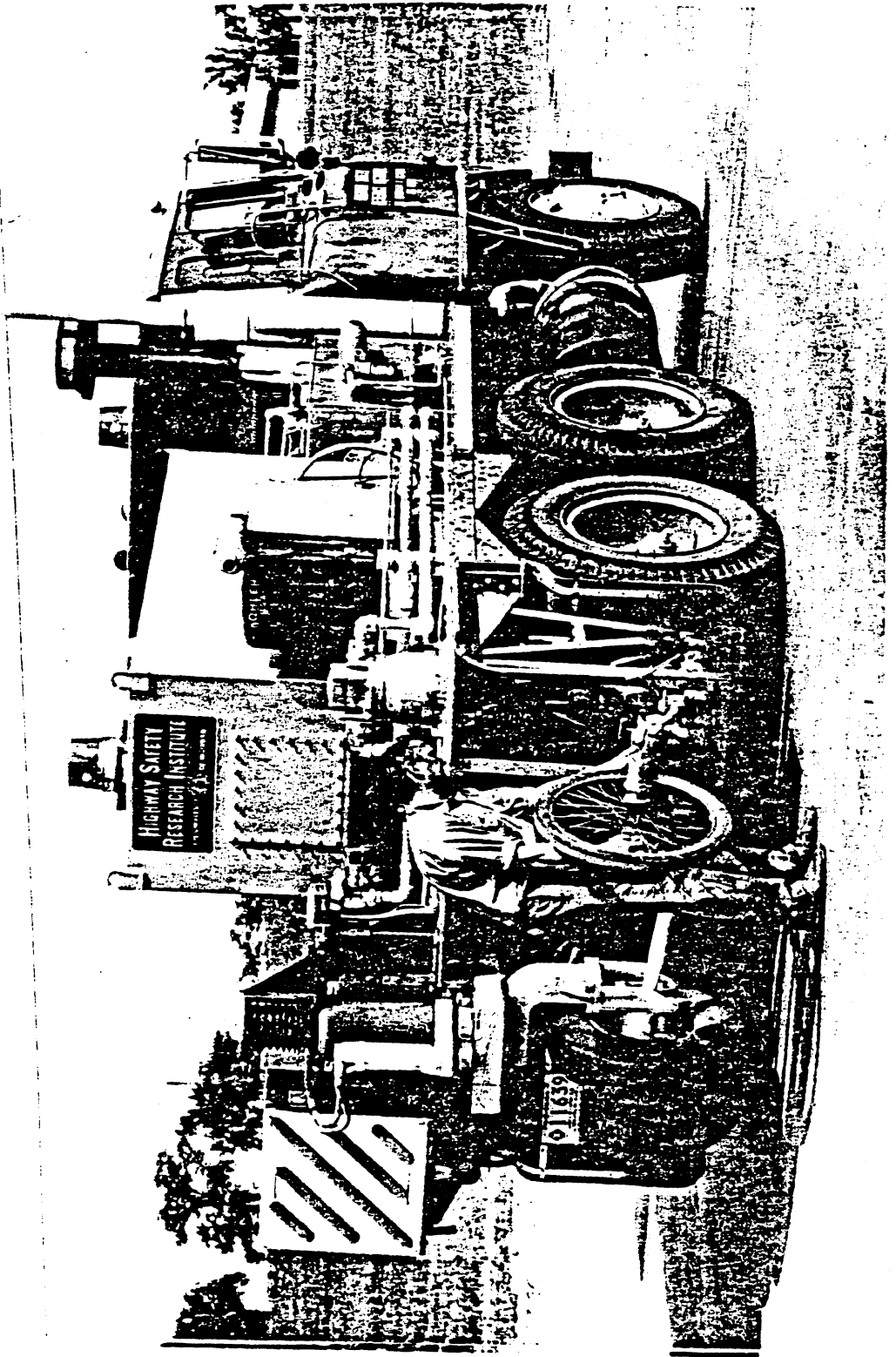
Figure 6.6 Mobile Passenger Car Tire Tester
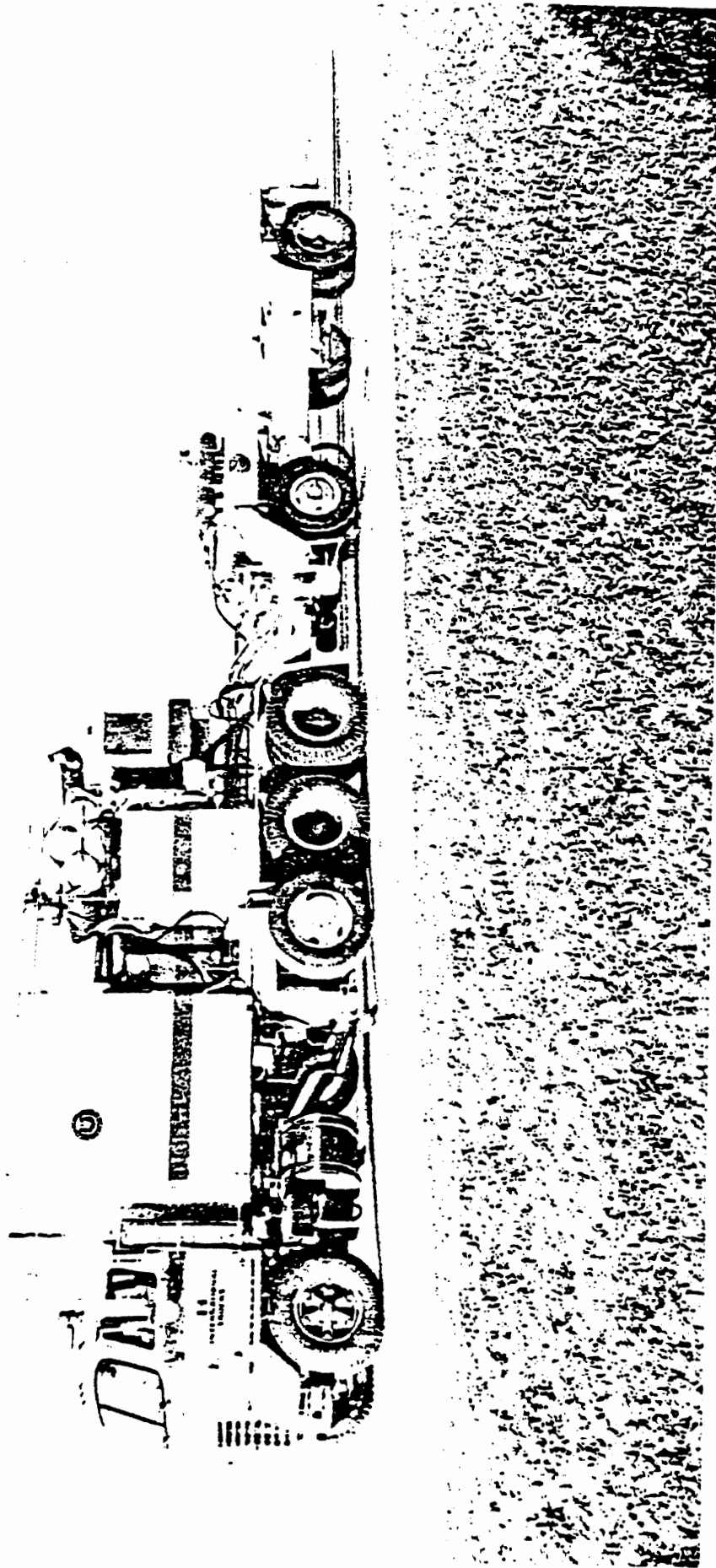
Figure 6.7 Mobile Truck Tire Dynomometer

circle so that the tire contacts the ground along a straight line of some length called the line of contact. The center point of this line is directly below the wheel center and is called the contact center. The first and last points of tire contact are shown at the ends of the line of contact. As the freely-rolling tire rolls forward, a given point on the tread circle first contacts the ground at the first point of contact. This point then remains fixed on the ground, but appears to move rearward on the line of contact as the tire continues to roll forward. Finally, the point, still at the same point on the ground, will have moved to the last point of contact and will lift off the ground. For the freely-rolling tire moving straight ahead, no side force is generated.

In the second picture, the sideslipping tire, the tire is not moving in exactly the direction it is pointed. The direction of travel differs from the direction the wheel is facing by an angle α called the slip angle. Nevertheless, let us assume for the moment that any point on the tread circle will still remain in a fixed position on the ground while it is in contact with the ground. (This is the essential property of the compliance regime which distinguishes it from the traction regime.) With this assumption, the figure shows that the actual line of contact is angularly displaced from the line of contact which the freely-rolling tire would have assumed and, therefore, as each point moves rearward along this line (and stays fixed to the road), it must progressively displace sideways. If we think of each little tread element as being connected to the wheel by a spring, then these springs must be stretched laterally as each element moves along the line of contact. Figuratively, it is the stretching of these springs which produces the compliant lateral forces.

The bottom picture of Figure 6.8 shows an overhead view of the contact patch area. Naturally, as each spring is stretched further, the lateral force which it produces gets larger. Thus, if the lateral force distribution shown in the figure were replaced by the total resultant force (also shown), we can see that this resultant force would have a center of action some distance behind the tire's contact center. This distance is called the "pneumatic trail" (and is considered positive

FREELY ROLLING TIRE

Direction of spin

Direction of
wheel heading

Last Point of Contact

First point of
contact

Line of contact   Y   Z   Contact center

X

Direction of travel

SIDE-SLIPPING TIRE

Direction of spin

Direction of
wheel heading

"Actual" line of contact

Line of contact for freely
rolling tire

First point of
contact

X

Y

Z   Contact center

Slip angle(α)

Direction of travel
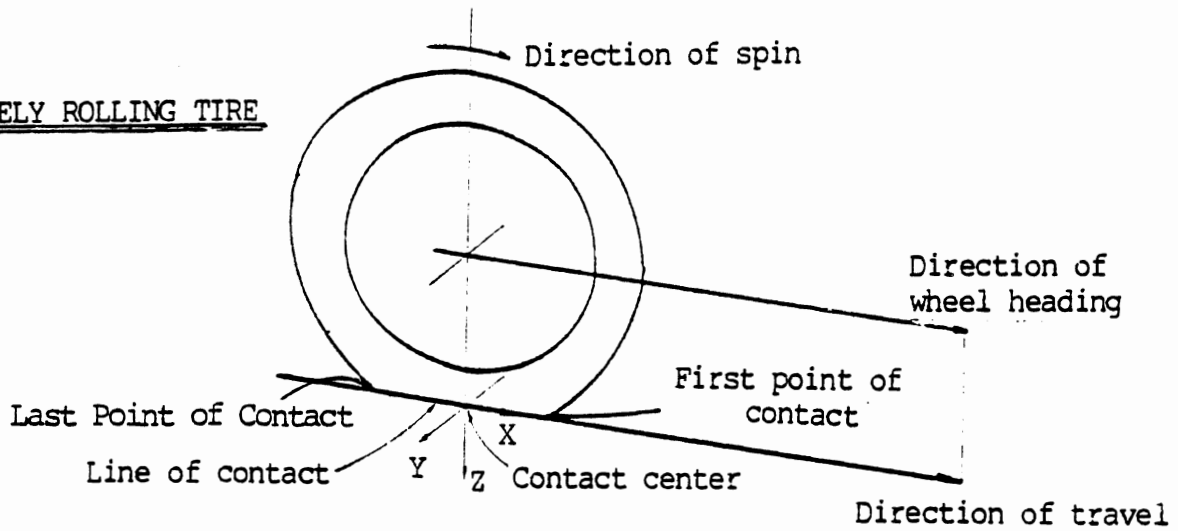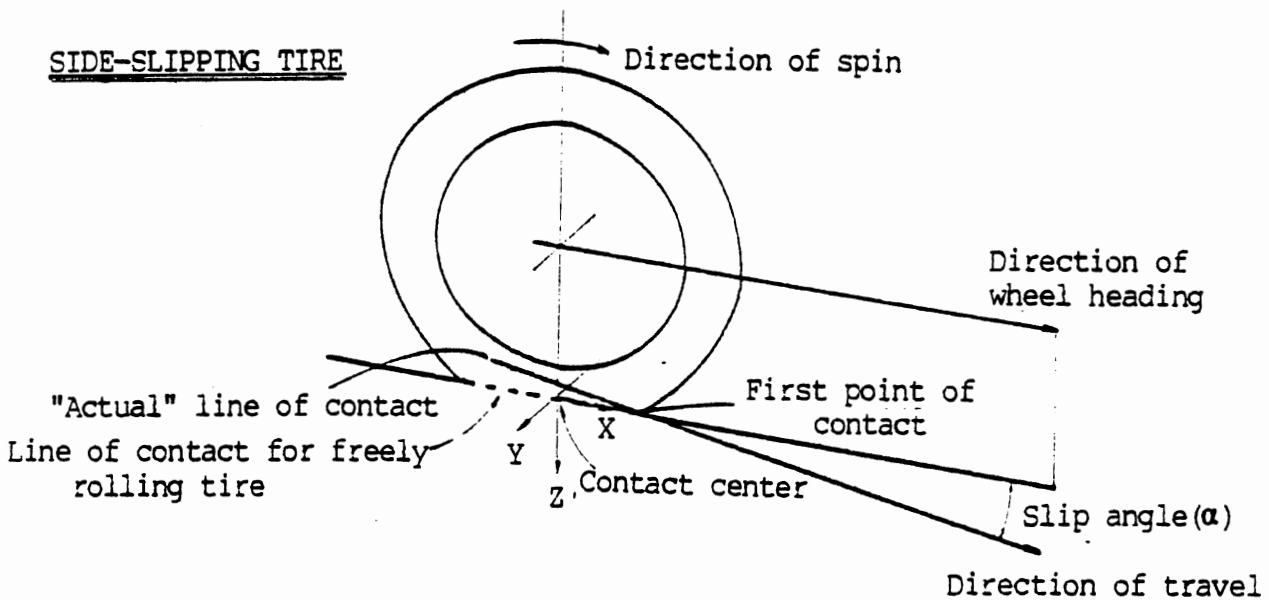
OVERHEAD VIEW OF THE
CONTACT PATCH OF THE
SIDE-SLIPPING TIRE

Pneumatic trail

Total resultant
side force

Side force
distribution

"Actual" line of contact

Direction of
wheel heading

X

α

Line of contact for freely
rolling tire

Contact center

Y

Direction of travel
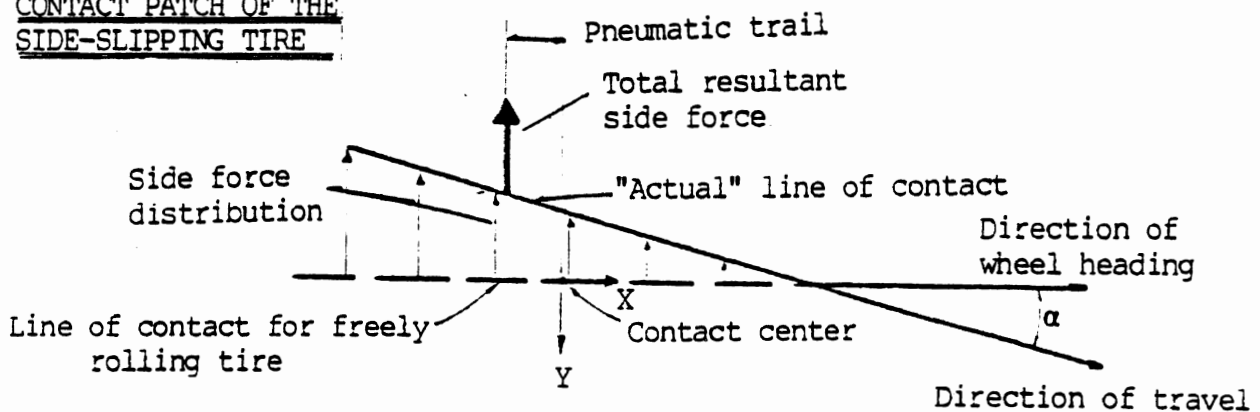
Figure 6.8 A Tire Model for Compliant Side Force Generation

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when the resultant force is behind the contact center.) Because the lateral force does act behind the contact center, the tire and wheel not only experience a side force, but also a "restoring" or "aligning moment" about the z axis of the tire. (Notice that, since the resultant force is behind the contact center, the aligning moment acts on the tire in a direction which would tend too "restore" the tire to a freely-rolling condition, that is, to "align" the tire direction with the direction of travel.) Lateral force and aligning moments developed in this way are the forces which determine the turning motion of a vehicle in normal driving. With respect to vehicle behavior, lateral force plays, by far, the major role (and later we will ignore aligning moment when examining vehicle handling.) However, the aligning moment is, in part, responsible for the steering torque which a driver feels and which tends to bring the steering wheel and, as a result, the vehicle, to a straight ahead condition should the driver release the steering wheel.

Let us now express the results of this model mathematically. If the spring elements of the tire are linear (force developed is in proportion to the distance stretched), which they generally are, then we can write the following equation:

$$F_y = C_\alpha \, \alpha \tag{6.1}$$

$$M_z = X_p \, F_y = X_p \, C_\alpha \, \alpha \tag{6.2}$$

where

F_y is the lateral force produced

M_z is the aligning moment produced

α is the slip angle

C_α is the tire's <u>cornering stiffness</u>

X_p is the tire's <u>pneumatic trail</u>

Equation (6.1) states that the lateral force produced is proportional to the tire slip angle. The proportionality constant, C_α, is called the tire's <u>cornering stiffness</u>. (C_α can be roughly thought of as the effective stiffness of the spring elements of the model.) <u>Cornering stiffness is the most important fundamental tire property for the normal driving regime.</u>

Equation (6.2) states that aligning moment is also proportional to slip angle. The proportionality constant $(C_\alpha X_D)$ is the product of cornering stiffness times pneumatic trail. For convenience, this product is often thought of as a single tire property, aligning moment stiffness (C_{Mz}) so that Equation (6.2) can be restated as:

$$M_z = C_{Mz} \, \alpha \qquad (6.3)$$

where

$$C_{Mz} = X_p C_\alpha \qquad (6.4)$$

Now, let us return and examine our original assumption, namely, that any point on the tread circle (while it is in contact with the ground) will remain fixed on the ground, even when a non-zero slip angle exists. This assumption is reasonably valid when, in our model, the force developed by a spring element does not become large enough too overcome the friction between the tread element and the road. In other words, if we continue to stretch a spring far enough, sooner or later enough force will develop so that the point on the tread can no longer stay fixed on the road and it will begin to slide sideways. This happens when and because the spring force has reached the maximum possible (saturation) friction force.

Consider Figure 6.9 where the overhead view of our tire model is again shown. In the figure, the model tire is shown experiencing larger and larger slip angles as we proceed from the top to the bottom picture. At the left of each picture, the maximum available level of friction is shown. In the top picture, α is sufficiently small that the spring forces never reach this level and the tire is operating within the compliance regime. In the second picture, α has increased to the point that the spring force has just reached this maximum level as the tread element leaves the ground. This is the limit of the "pure compliance" regime for our model. In the third picture, traction properties are beginning to have an effect. Spring elements are reaching the maximum level before leaving the contact patch. Therefore, near the rear of the contact patch, the tread elements are no longer fixed on the road surface and are sliding to maintain only the maximum available force level. As α increases, sliding begins earlier and earlier. In the last

picture, the limiting condition of our model is nearly reached.

What are the implications of this behavior? First, once sliding takes place, Equations (6.1), (6.2), and (6.3) no longer hold true. Side force is not developed in proportion to slip angle as it was for small angles and the tire is said too be operating in the nonlinear range. Figure 6.10 shows a typical relationship between α and side force for a bias-belted tire. The figure shows the reason for the terms linear and nonlinear range. For small α, where F_y is proportional to α, the relationship appears on the graph as a straight line (linear). At higher α, where the tire is sliding in the contact patch, the curve "rounds off" (nonlinear). The graph shows that the maximum total side force which the tire can produce will occur at around 10-15° of sideslip. (Our model was not sufficiently complete to demonstrate this behavior, but real tire data shows this to be so.) Radial tires differ somewhat from the bias tire shown. Radials tend to be stiffer (higher C_α), making the linear portion of the graph steeper. Thus, the maximum side force will occur at a smaller slip angle (5-10 degrees), but will be about the same value.

For real tires, sliding occurs over much (but not all) of the contact patch when peak side force is generated. Thus, it is clear that peak side force is largely determined by the tire-road friction mechanism. Generally, for frictional properties, the sliding force (F_y, in this case) is proportional too the normal (vertical) force (F_z, in this case) where the proportionality constant is called the coefficient of friction. In tire mechanics, this coefficient is usually represented as μ. Therefore, maximum side force can be approximated by the equation:

$$F_{yMAX} = \mu F_z \qquad (6.5)$$

We see that, as the load carried by a tire increases, the side force capability of the tire generally increases proportionately. (Eq. 6.5 is not exactly correct. Later in discussing limit handling, we will see the importance of this fact.)

Now refer to Figure 6.11 which is a repeat of Figure 9, but with the resultant side force and pneumatic trail also shown. As the side
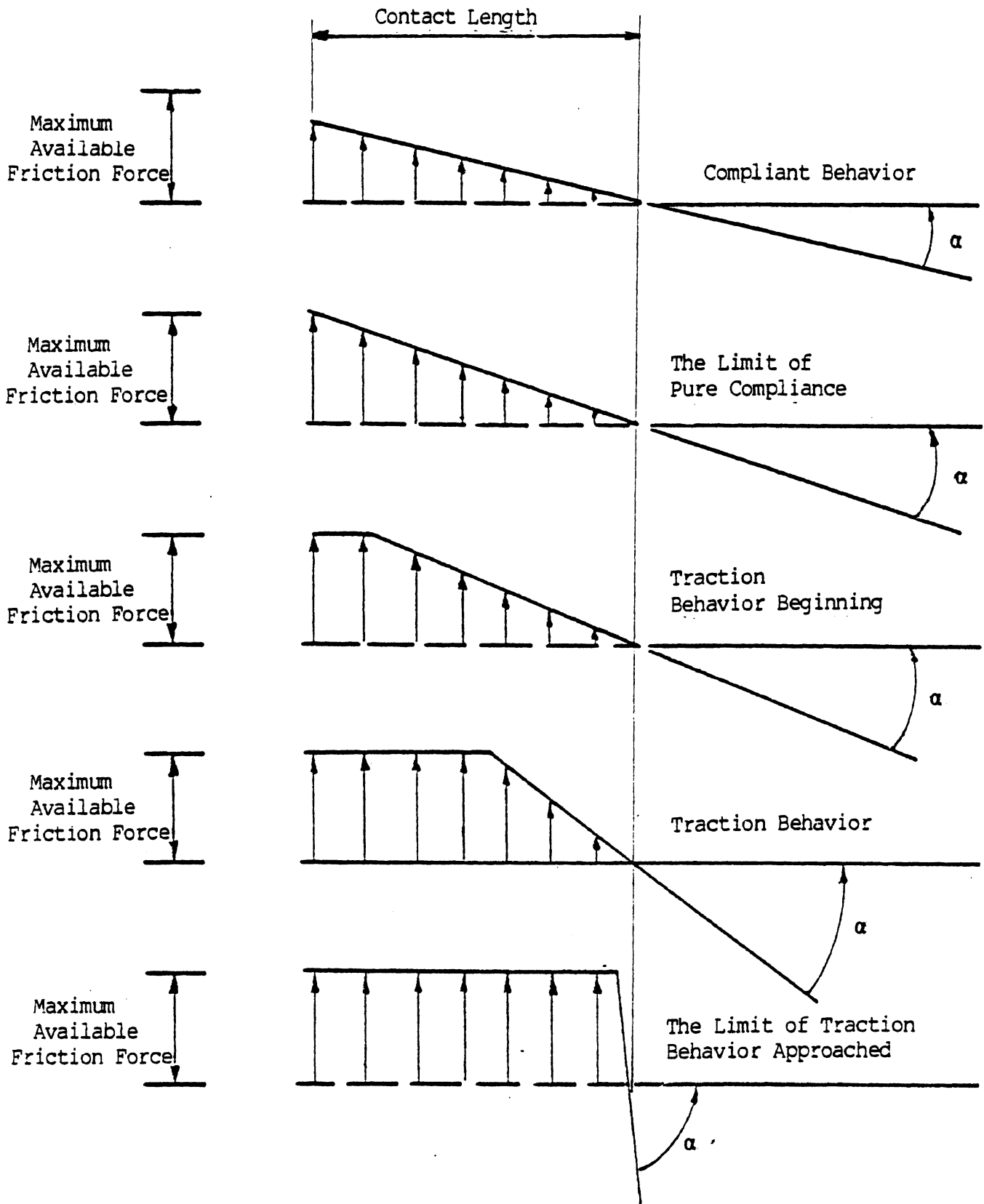
Figure 6.9 Behavior of the Model Tire in the Lateral Traction Regime

Side Force, F_y

Maximum
Total Side
Force

0

0

Proportional
or linear
Range

Non-linear Range

About $10-15^\circ$

About $3^\circ-5^\circ$
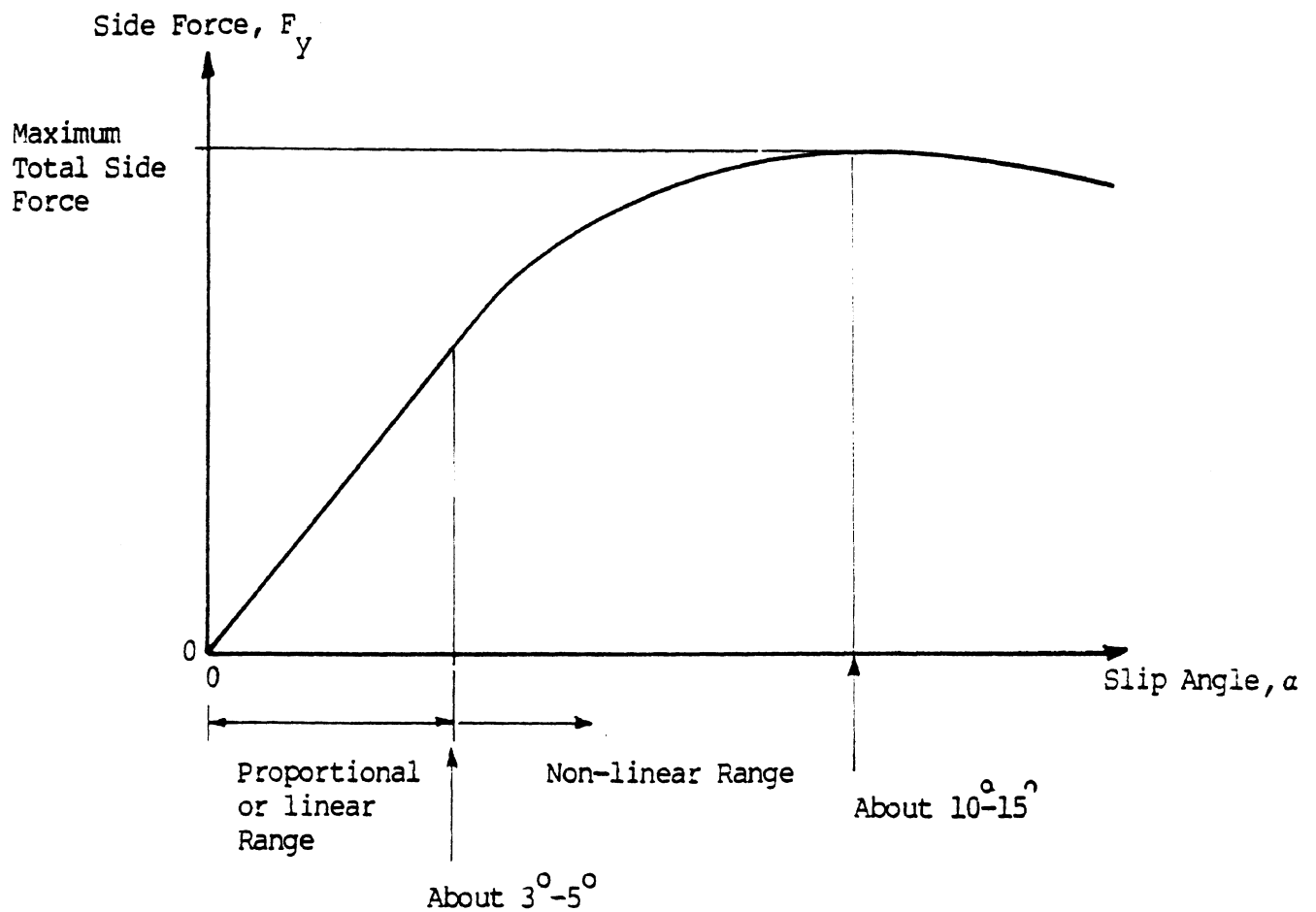
Slip Angle, α

Figure 6.10 Typical Relationship between Side
Force on Slip Angle for Bias Tires

force begins to saturate and the side force distribution "squares off,"
the center of action of the side force moves forward toward the contact
center and the pneumatic trail grows smaller. For our model, pneumatic
trail goes to zero at the limiting condition. For real tires, pneumatic
trail can actually cross over the contact center and become negative.
This means, in very severe maneuvers, the steering-wheel torque (which
usually tends to return the wheel too a neutral condition) experienced
by the driver can become very small or even reverse, tending too produce
more, rather than less, steer. In emergency situations, this phenomenon
can add to the confusion of the driver who has already been placed in a

demanding situation.

We have discussed the mechanisms by which a tire produces lateral forces in response to slip angle. Tires can also produce lateral force in response to camber angle. Camber angle is the angle at which the tire is inclined, as shown in Figure 6.12. The lateral force produced in response too camber angle is small (for passenger cars[1]) in comparison to slip angle forces. Further, the pneumatic trail associated with camber side force is small, thus producing very small aligning moments.

Earlier we noted that a variety of operating variables can have an effect on the compliance and traction performance of tires. The effect of many of these variables is relatively small and only of importance in detailed studies. However, certain of them may be of critical importance to lateral force mechanisms. First is the effect of vertical load. We have already noted that because of the nature of the friction mechanism, peak side force capability increases with vertical load. (Equation 6-5.) It is generally true also that cornering stiffness will increase with load in the normal loading range of a tire. At relatively high loads, however, these effects may become less pronounced. Next is the effect of inflation pressure. Over the normal range of inflation pressure used in passenger car tires, cornering stiffness (C_α) generally decreases as inflation pressure decreases. Very low pressure below 20 psi can result in dramatic reduction in C_α (Later we will see that this may be very important to vehicle stability.) Second, roadway contaminants (especially water) can dramatically reduce the maximum side force (traction) capability of a tire. At high speeds, and fairly large water depths, a tire may actually lose contact with the road and ride on a film of water (a process known as "hydroplaning"). A third factor, wear state, can also be important. A basic purpose of tread patterns molded into tires is to provide pathways for water to escape from the contact region. Badly worn tires reduce the size of these pathways and may dramatically degrade a

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[1] It is of interest to note, however, that side forces generated by camber angle, rather than by side slip, are the dominant side forces for motorcycles and bicycles.
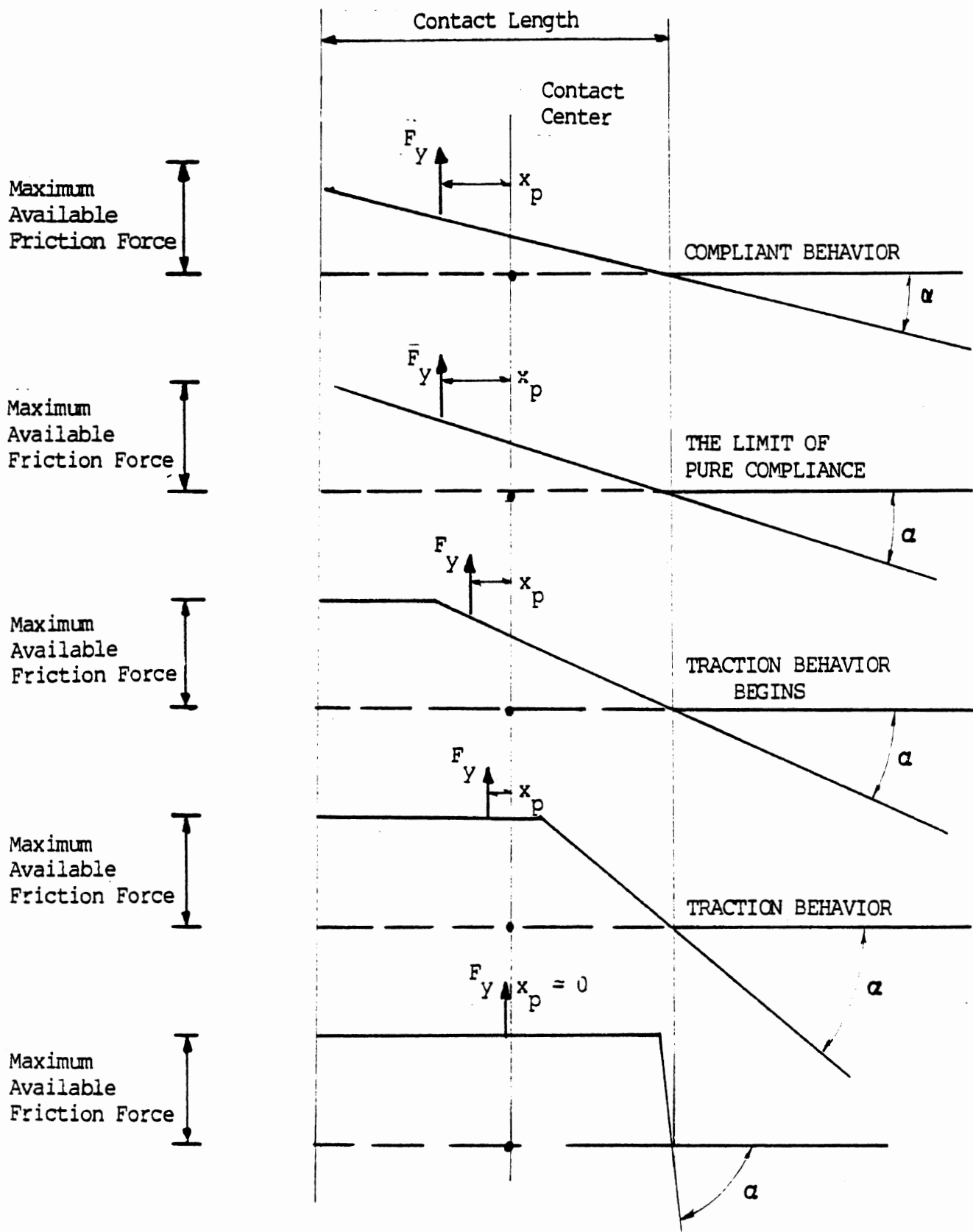
Figure 6.11   Pneumatic Trail in The Tractive Regime
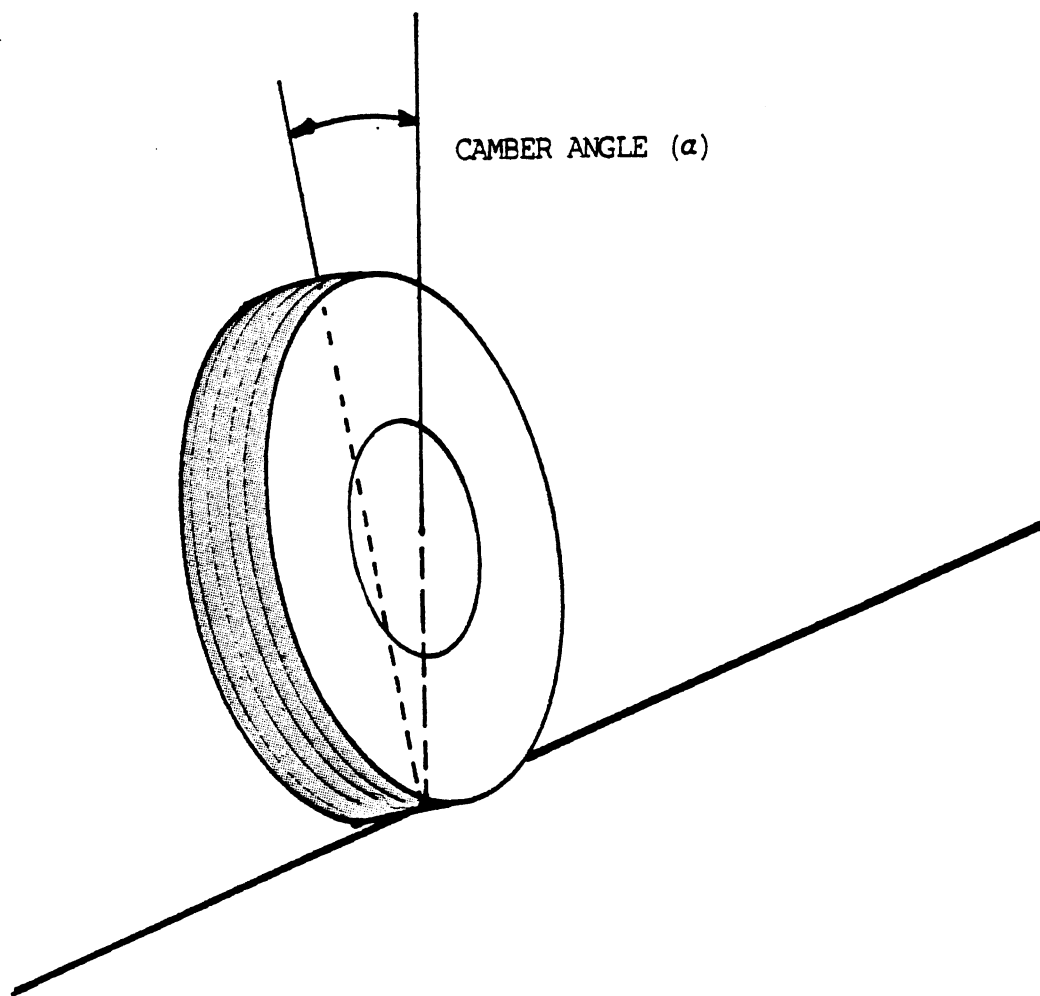
CAMBER ANGLE (α)

Figure 6.12 Tire at a non-zero Camber Angle

tire's wet traction. We will examine the importance of some of these effects later in the discussion of vehicle handling.

6.3.2 Longitudinal Tire Force Generation As was the case in lateral force generation, both compliance and traction mechanisms play a role in the generation of longitudinal forces (braking and acceleration) developed by the pneumatic tire. In the following discussion, we will concern ourselves primarily with brake force generation, remembering that accelerating forces are developed in essentially the same manner.

A tire model similar to the one employed previously appears in

Figure 6.13. Here we see the tire in the side view and again we model the tire as having spring-like elements which attach the tread elements to the tire carcass. In this case, the springs exhibit compliance in the longitudinal direction.

In the first tread area "close-up," the tire is freely rolling and no action is taking place to stretch the springs. Each tread element touches the ground at the first point of contact and stays fixed to the ground while traveling through the contact patch, finally departing the ground at the end of the contact patch. In this "freely-rolling state," the wheel is spinning at a rotational velocity (ω) so that the tread elements, located on the wheel at a radius (R), have an effective velocity which is equal to the translational velocity[2] (V). That is, in equation form

$$R \, \omega = V \tag{6.6}$$

Consequently, no spring stretching is required for the tread element too stay fixed on the ground.

In the second close-up view, a small brake torque has been applied to the tire causing its <u>spin velocity (ω) to slow down</u>, although it continues to travel (in translation) at the same speed (V). Consequently, the roadway is moving past the tire more rapidly than can be accounted for by the spin velocity of the tire. That is, $R \times \omega < V$. Now, when a tread element enters the contact patch and remains fixed to the ground, its spring element must stretch rearward to account for the difference in distance traveled due to the effective difference in velocities. The spring stretch results in the development of a brake force.

Thus, in the compliance regime, brake forces develop in proportion to the relative difference of the tire velocity too effective spin velocity. This difference is called longitudinal slip (S) and can be expressed in equation form as

$$S = 1 - (R\omega/V) \tag{6.7}$$

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[2] In these "closeups," it is more convenient to think of the tire as standing still (but spinning at a speed, ω) and the roadway passing by the tire. Thus, V is shown as the "relative roadway velocity."
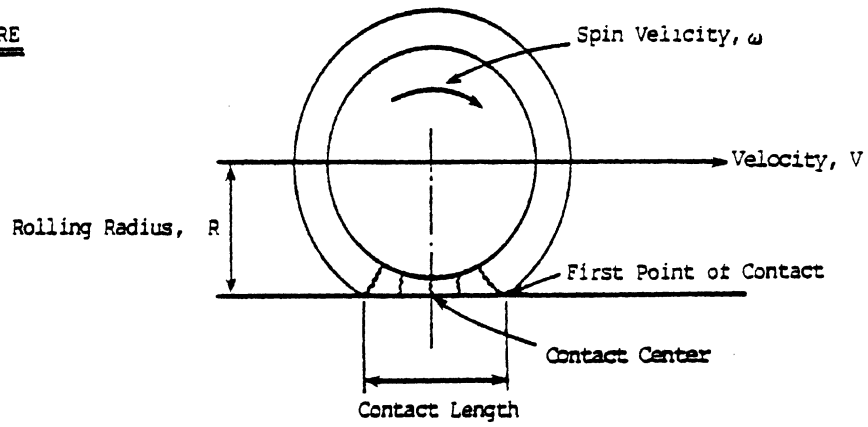
TIRE
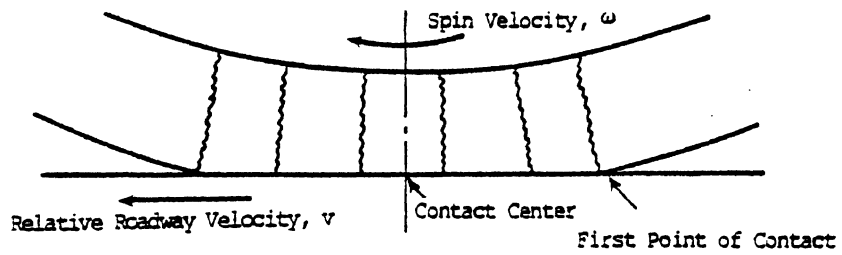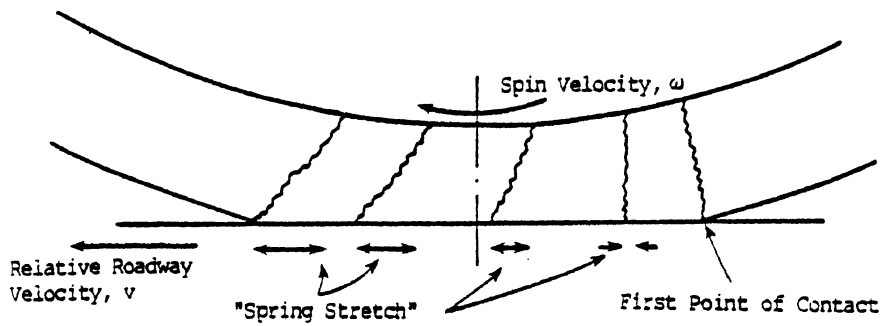
Figure 6.13 Tire Model for Compliant Brake Force Generation

Expressed in this manner, we can see that during braking, slip may vary from (0) at freely rolling (R x ω = V, therefore Rω/V = 1) to (1) when the wheel is "locked up" (ω= 0).[3]

In the compliance regime, brake forces develop in proportion to the amount of slip. That is

$$F_x = C_s S \tag{6.8}$$

where

F_x is the total brake force

and

C_s is the proportionality constant and is called the longitudinal stiffness of the tire.

Traction properties come into play in longitudinal force generation much as they do in lateral force generation. When slip becomes sufficiently large, tread elements in the rear of the contact patch begin to slide as the "spring force" becomes large enough to overcome the available friction limit.

Figure 6.14 illustrates the typical relationship between brake force and slip over the full range of positive slip (braking.) The figure shows a number of interesting points. First, the tire compliance properties dominate the braking process only at lower levels of slip as indicated by the linear portion of the graph, yet, rather high brake forces are available in this "linear" region. Virtually all normal level braking takes place in the linear or compliant range.

Second, rather shortly after the traction properties begin to dominate, maximum available brake force is obtained. The maximum brake force generally occurs at about .2 slip. At higher slip values, the level of braking force actually falls off, and as the graph shows, the braking force of a fully-locked wheel (S=1) is less than that of a wheel spinning only 20% slower than free rolling (S=.2).

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[3]To develop acceleration forces, slip must be negative. That is, R ω > V. In the extreme, during a "jack-rabbit start," the wheel spins very fast while V is nearly zero and slip becomes a very large negative number.
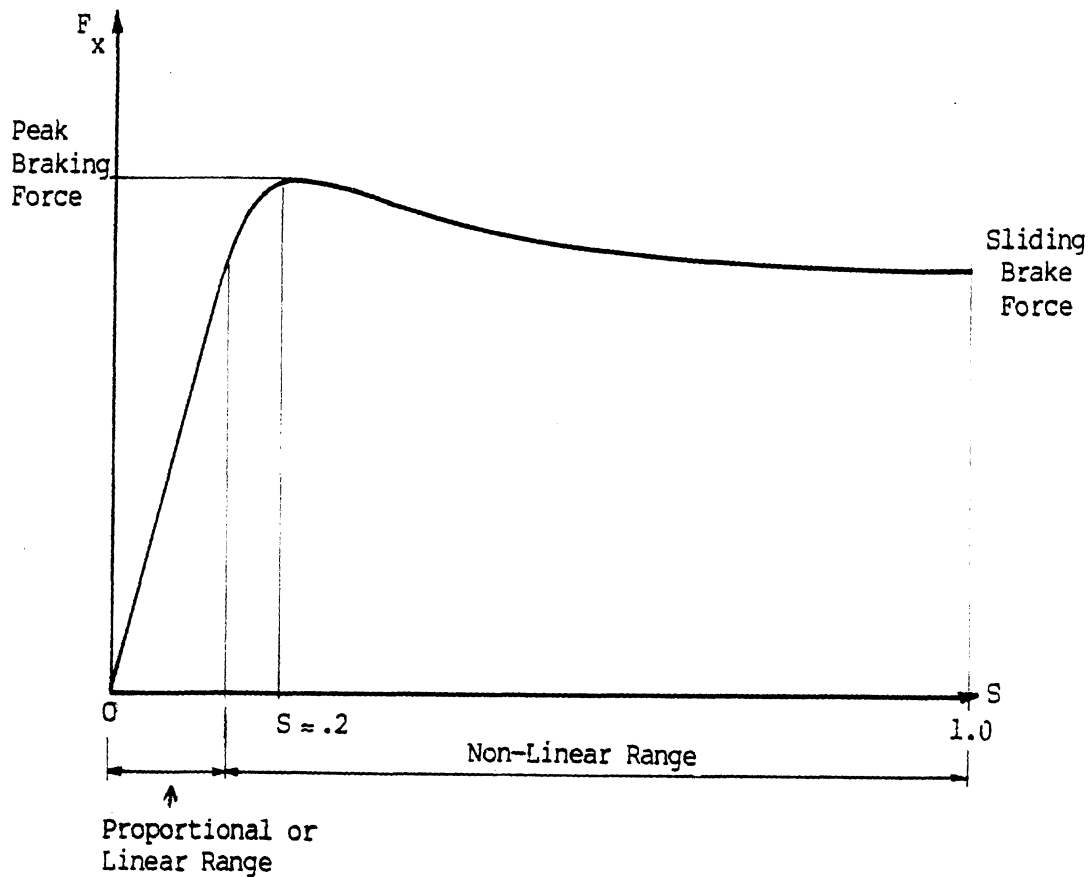
Figure 6.14 Typical Relationship between
brake force and longitudinal slip.

As was the case in lateral force generation, longitudinal force generation in the traction range is dominated by the friction process. Therefore, data of the type shown in Figure 14 is often displayed in terms of the apparent coefficient of friction rather than as brake force. That is recalling Equation (6.5), we may express brake force as

$$F_x = \mu F_z \qquad\qquad (6.9)$$

or, restated

$$\mu = F_x/F_z \qquad\qquad (6.10)$$

By dividing the brake force by the vertical load, we obtain an effective

friction coefficient. This process can be used to redisplay the data of Figure 6.14 as shown in Figure 6.15. In the figure we have designated the peak friction coefficient as μ_p and the sliding friction coefficient as μ_s. On clear, dry roads, passenger car tires generally have μ_p and μ_s values which may be typified as about 1.0 and 0.9, respectively. Note that the drop-off from peak to slide is not too severe under these conditions. On the other hand, on wet roads (without hydroplaning)μ_p and μ_s may be as low as 0.7 and 0.4, respectively. Notice that not only have both peak and slide values dropped off, but that the slide value on wet roads may be dramatically lower than the peak value on the same wet road.
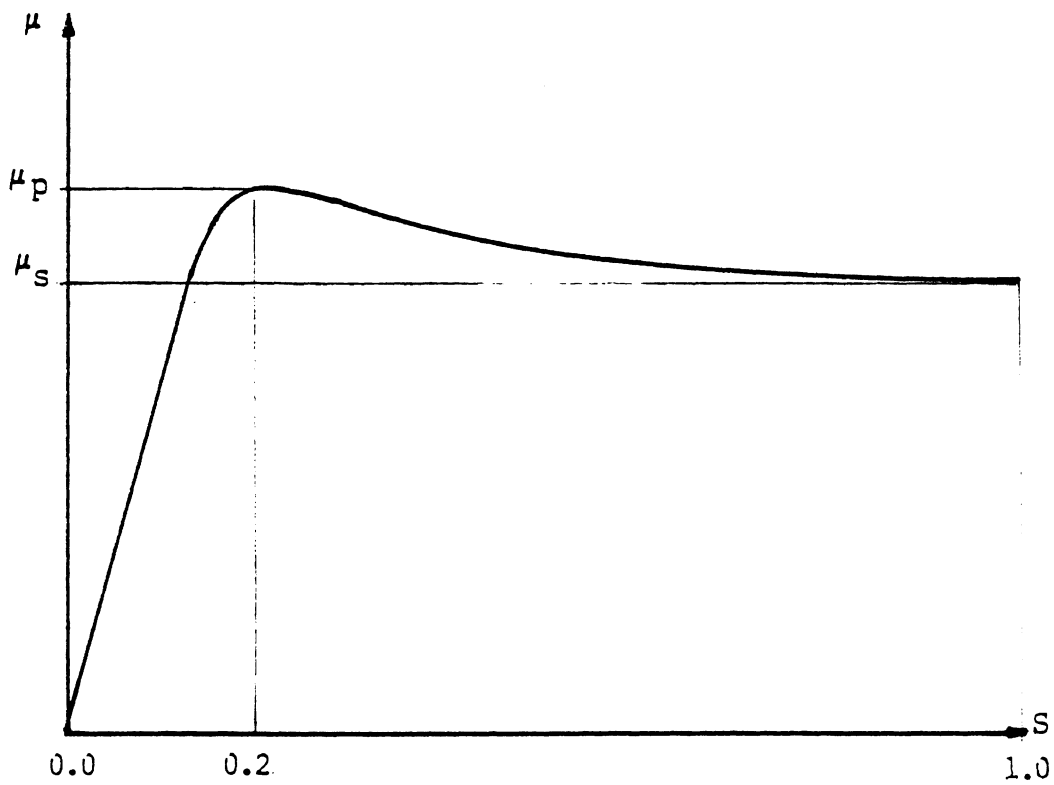
Figure 6.15 Typical Relationship between apparent
friction coefficient and longitudinal slip.

In practice, the tire cannot be effectively used in the area between peak and slide (except if the vehicle is equipped with an

antilock system). If the driver applies sufficient brake pedal force to attain peak brake force (or slightly more), the tire will proceed very quickly (well under a second) to lock up. This happens for the following reason. If sufficient torque is developed by the brake to ever-so-slightly exceed the brake torque required for maximum brake force, then the excess torque will further slow down the wheel so that slip will increase slightly above the slip required for maximum torque. As shown by Figure 6.14, this increased slip results in somewhat <u>less</u> brake force, implying even more excess torque available to further slow the wheel. Further slowing produces more slip--still less brake force-- more excessive torque--further slowing--etc., etc. This series of events takes place very rapidly, once the brake torque limit is exceeded, and results in virtually instantaneous lock up.

6.3.4 <u>Combined Tire Forces</u> In combined braking and turning maneuvers, the tire must produce both lateral and longitudinal forces simultaneously. When this is the case, the two force generation mechanisms interact, and the resulting forces are generally (but not always) both smaller than they would be if they were being generated individually.

Figures 6.16 and 6.17 illustrate the "cross-sensitivities" for a bias and a radial tire (in this case on a wet concrete surface.) In Figure 6.16 we see the effect of longitudinal slip on side force generation. The graphs show the relationship between side force and slip angle with each curve being obtained at a different amount of longitudinal slip. We see that small values of longitudinal slip (within the longitudinal compliance range) produce only moderate changes in side force behavior. <u>However, once longitudinal slip enters the traction range, the ability of the tire to produce side force is seriously degraded. When the tire is fully locked up in braking (S = 1.0), its ability to produce side force in response to slip angle is very poor.</u>

Figure 6.17 illustrates the effect of slip angle on longitudinal force generation. It can be seen that, again, when slip angle is sufficiently small to be within the compliance range, its effect on longitudinal force generation is small. Its effect on longitudinal
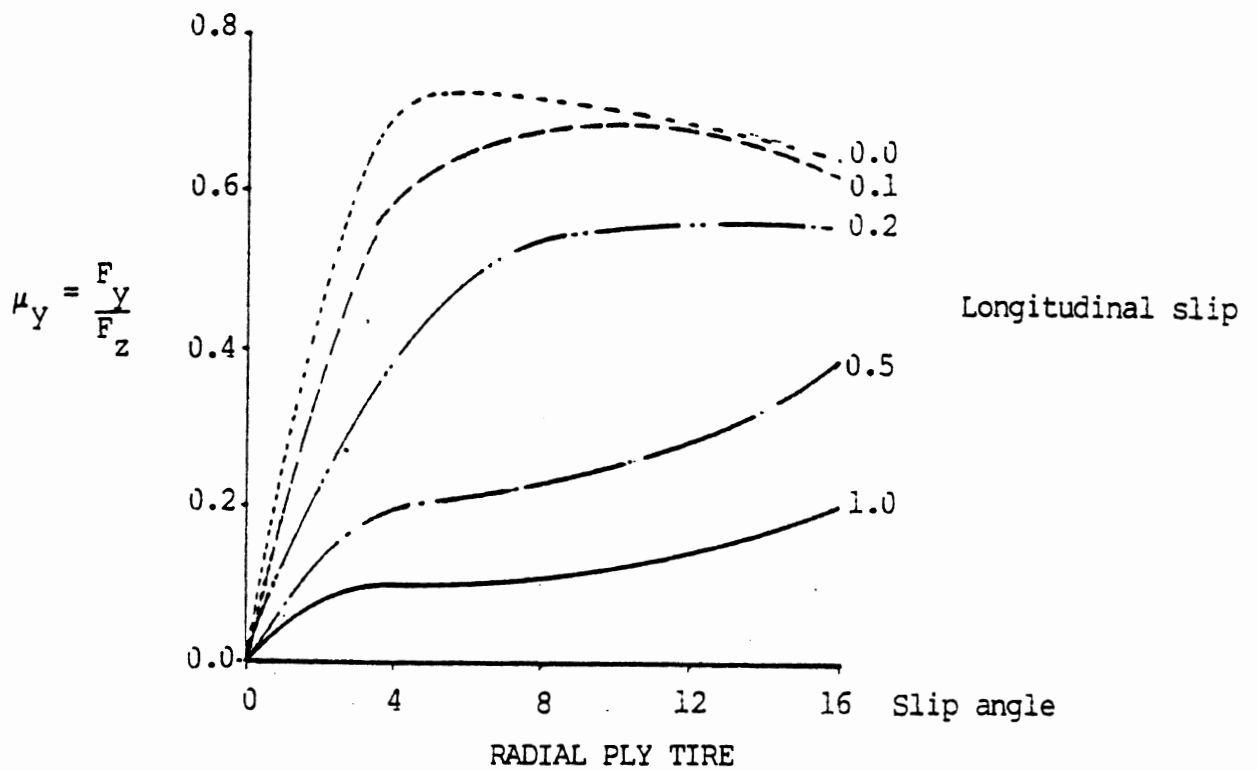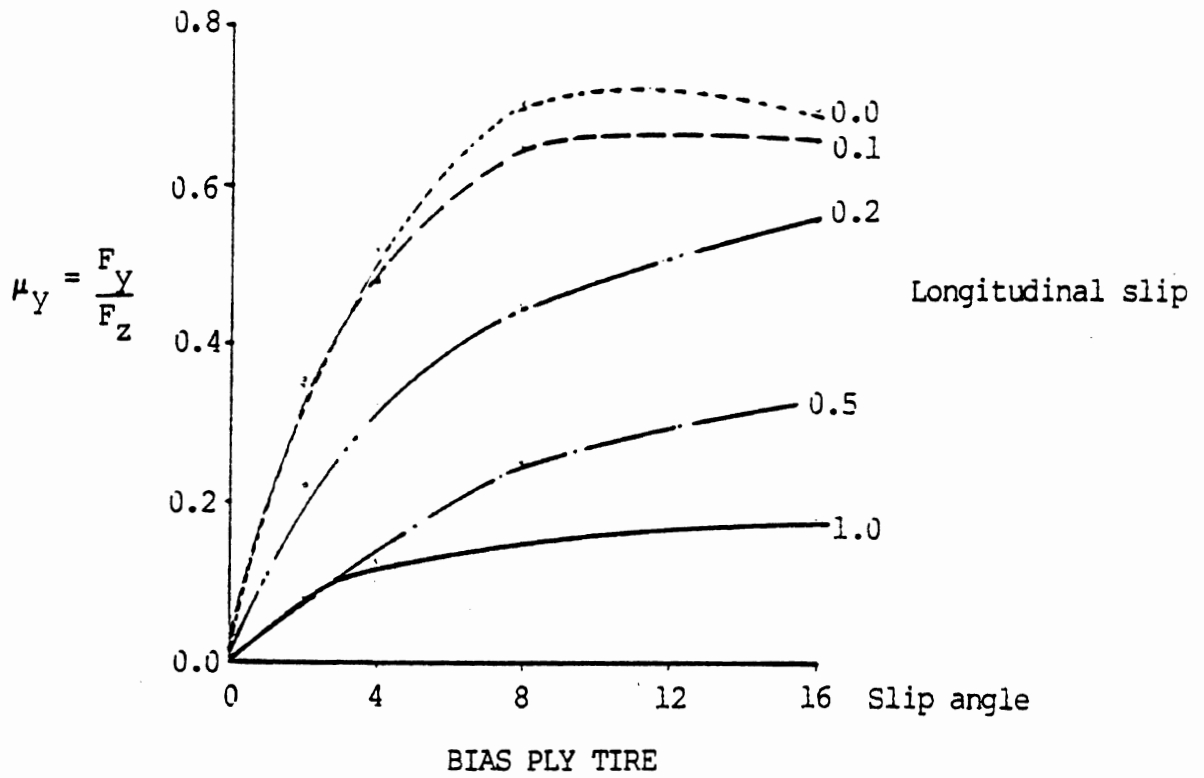
Figure 6.16 The Effect of Longitudinal Slip on Side Force Generation
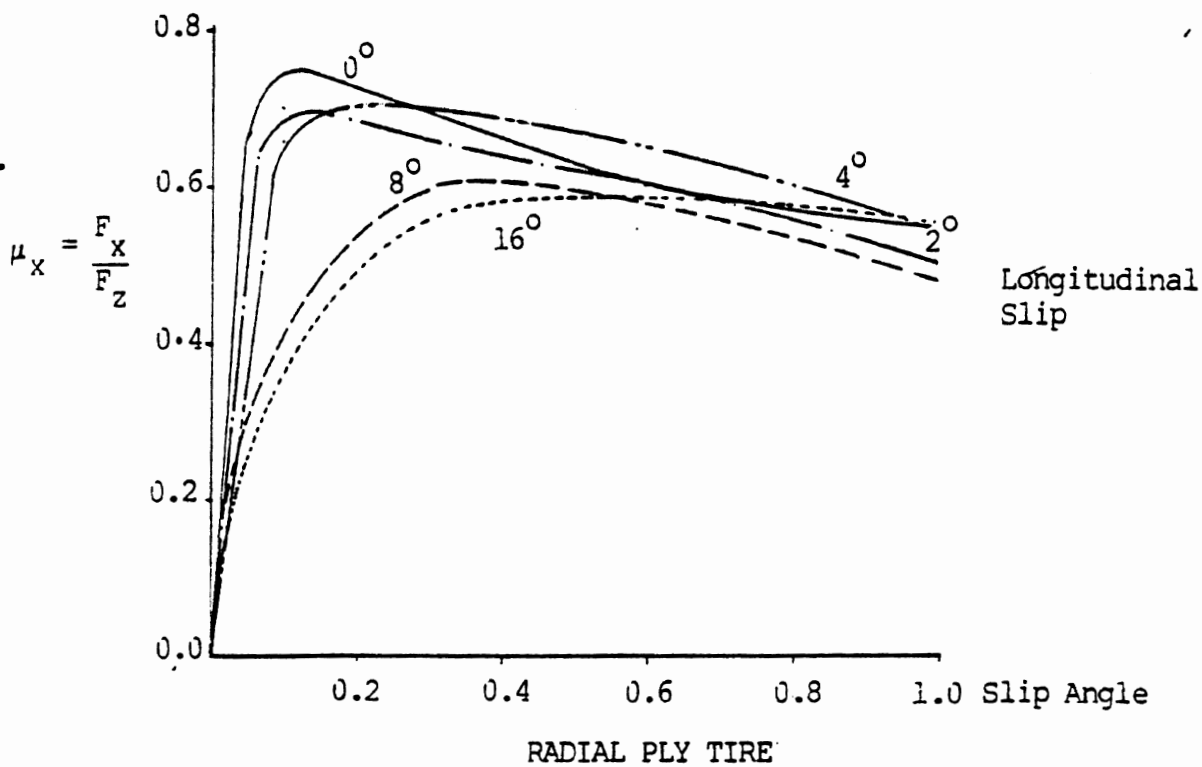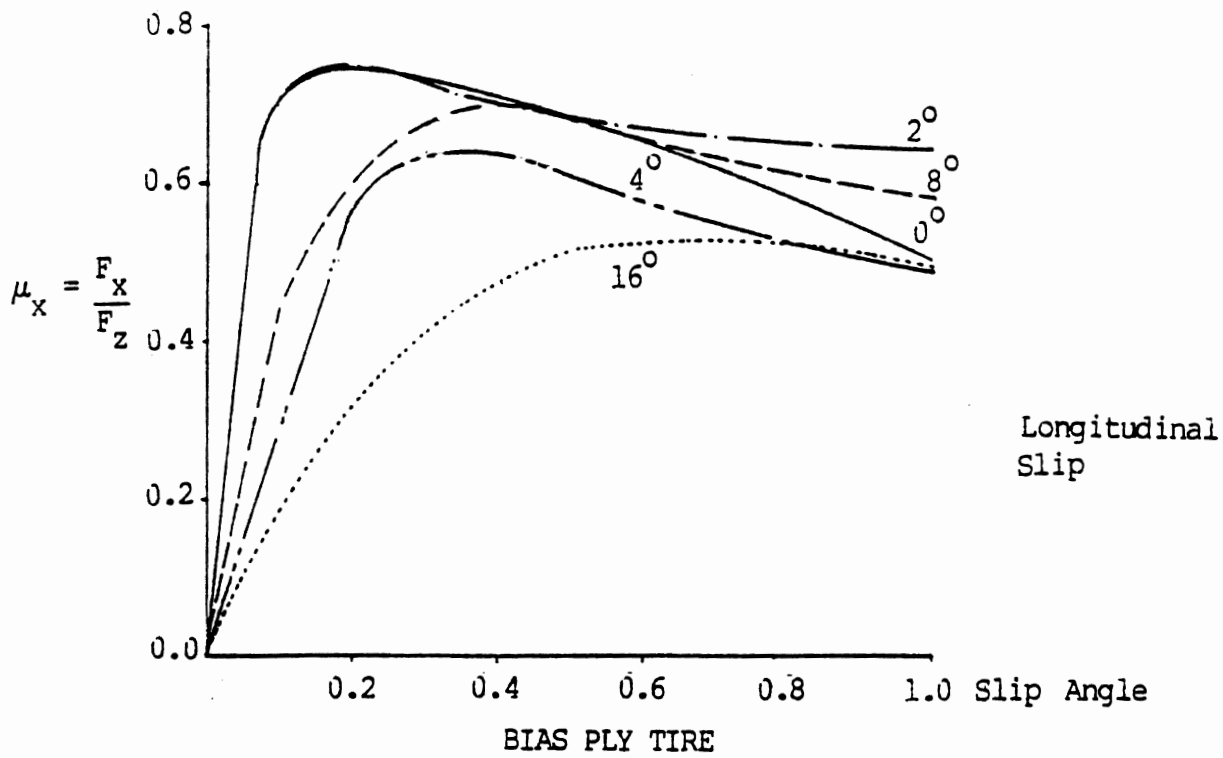
Figure 6.17 The effect of side slip on longitudinal force generation

force becomes greater at higher slip angles, but is generally not as severe as the effect which longitudinal slip had on side force.

The general implications of these cross-sensitivities in tire mechanics are:

1) <u>Severe braking can grossly degrade the handling response characteristics of the automobile.</u>

2) Severe turning also degrades braking capability, but generally not so completely as the reverse situation above.

We will examine these findings more deeply in later sections.

6.4 Vehicle Handling

For a four-wheeled, pneumatic-tired vehicle to experience a turning maneuver, a very complex mechanical process must take place. This process is illustrated, only in part, in Figure 6.18. The vehicle is a steady turn. To initiate the turn, front wheel steer angles have been introduced (δ_1 and δ_2). In making the turn, the vehicle experiences a lateral acceleration (A_y) which gives rise to centrifigal force (F_y). Tire side forces ($F_{y1}, F_{y2}, F_{y3}, F_{y4}$) must develop to counteract F_y and to produce a yaw moment balance. Thus, each tire develops a slip angle ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$). The radius of the turn (R) and the slip angles required establish the angular orientation of the vehicle relative to the turn. In this case, the vehicle is not traveling in the direction it is facing and so the vehicle has a body sideslip angle (β). Having a slip angle, the vehicle also has a sideslip velocity component (v). Its longitudinal velocity component (u) combines with v to produce the total velocity (V). The vehicle also has a rotational velocity (r) about the turn center. Because the centrifigal force (F_y) acts through the center of gravity (c.g.) which is above the ground plane at a height (h), the vehicle experiences a roll moment. This moment must be reacted by vertical tire loads. Therefore, the outside tires have larger vertical loads (F_{z1} and F_{z3}) than the inside tires (F_{z2} and F_{z4}). Since the front wheels are steered, the front wheel side forces (F_{y1} and F_{y2}) have a small component of forces acting longitudinally. For the vehicle to maintain speed, a drive thrust (F_{x3} and F_{x4}) is needed to balance these force components.

Surely, the reader has already decided that Figure 6.18 is not the way to be introduced to vehicle handling. It is far too complex even though it is actually quite incomplete. To begin, we will simplify this model quite a bit more. First we will concern ourselves only with "yaw plane" (overhead view) considerations, ignoring roll considerations. Second, we will collapse the vehicle into a two-wheeled "bicycle" model, assuming that the slip angles of the two tires on a given axle are very nearly the same. (This requires that the turn radius is large in comparison to the width of the vehicle, a condition which is nearly always true.)

(For clarity, Figure 6.19 is shown out of proportion. In fact, the turn radius, R, is much larger than shown and is very large relative to the vehicle wheelbase (ℓ). This mis-proportioning allows all of the angles in the figure to be shown much larger than they are. For the small angles which actually prevail, all forces (F_y, F_{yf}, and F_{yr}) are very nearly at right angles to the vehicle. The following discussion assumes that the angles are sufficiently small to assume that these forces are at right angles to the vehicle.)

The following discussion will deal exclusively with vehicle handling in the "normal driving range." That is to say, for the time being, we are assuming that <u>the tires are opperating in the compliant regime</u>.

Figure 6.19 illustrates some of the points noted above, viz.:

1) The vehicle is in a <u>steady</u> turn. That is,

     -Translational velocity (V) is constant
     -Rotational velocity (r) is constant
     -Turn radius (R) is constant

   These imply also that

   -Lateral acceleration (A_y) is constant

   -The center of the turn (A) is a fixed point

2) The lateral acceleration results in a centrifigal force acting on the c.g. which tends to "push" the vehicle away from the turn center.
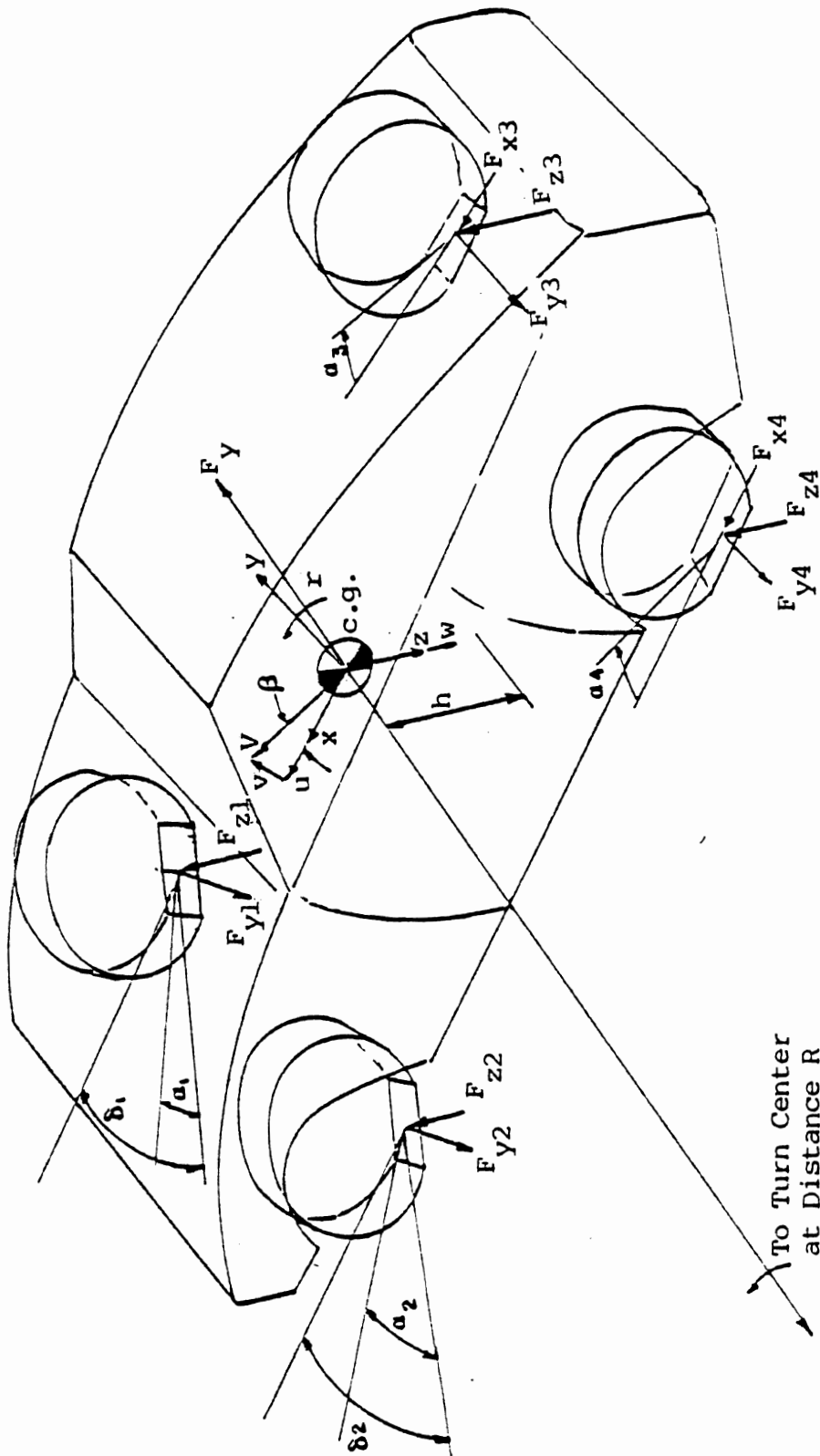
Figure 6.18 Vehicle in a Turn
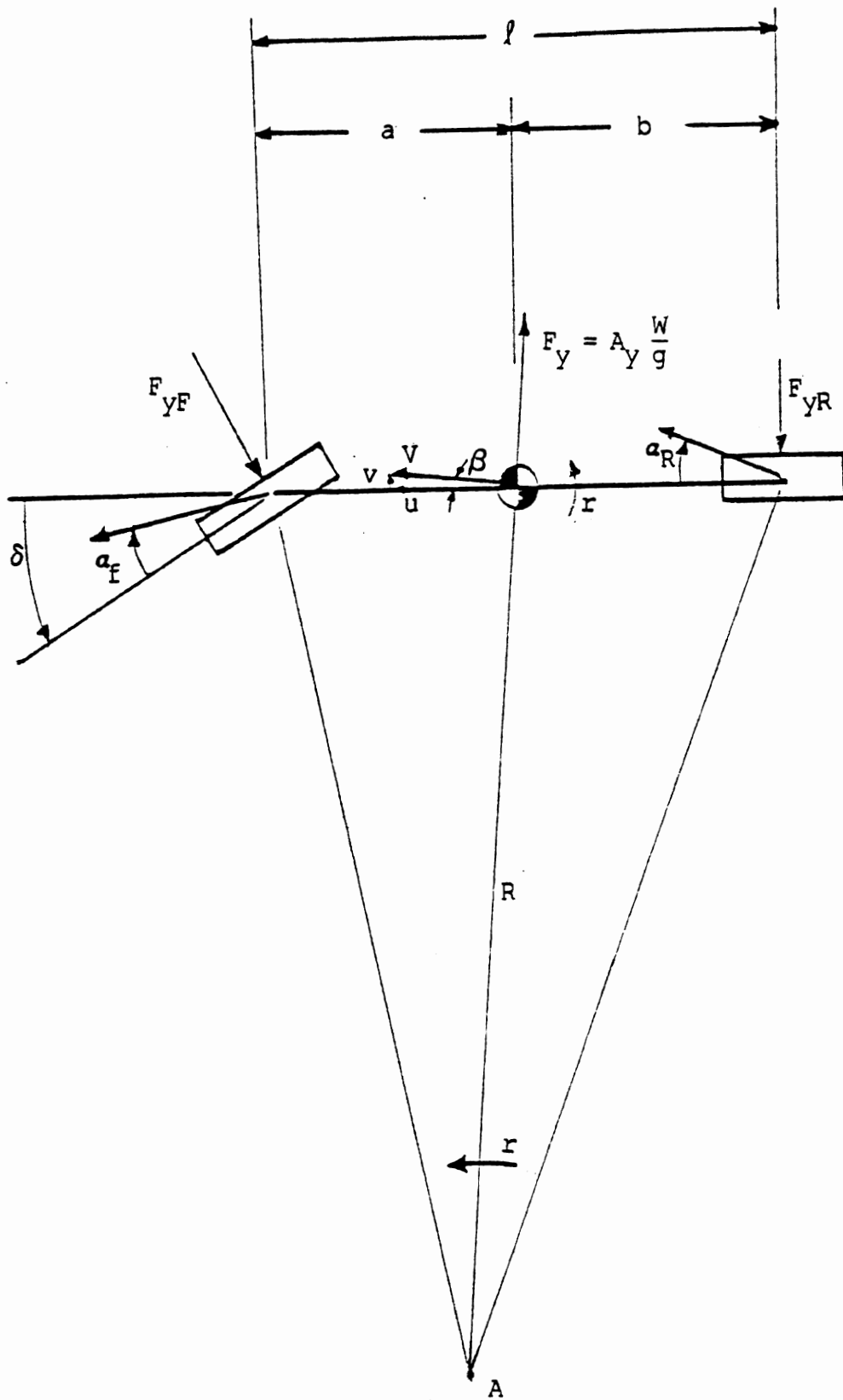
Figure 6.19 Yaw Plane View of a Bicycle Model Vehicle in a Turn

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3) Slip angles have developed at each tire so that the appropriate tire side forces are generated. These forces accomplish two functions:

- The total sum of the tire forces is equal (and in the opposite direction) to the centrifugal force. Thus, the tires "push" the vehicle toward the turn center in an amount which is equal to the centrifugal "push" away from the turn center. Thus, the vehicle is in a lateral "force balance," and has a constant lateral acceleration and turn radius. In equation form, this condition is stated:

$$F_{yF} + F_{yR} = A_y W/g \qquad (6.11)$$

- The front tire side force and the rear tire side force are of appropriate relative size (where the "appropriate" size is determined by the "moment arms," a and b) so that they produce an equal, but opposite moment about the vehicle c.g. Thus, the total moment is zero (the vehicle is in yaw "moment balance") and the vehicle will maintain a constant rotational velocity (r). Again, in equation form:

$$F_{yF} a - F_{yR} b = 0 \qquad (6.12)$$

therefore

$$r = constant$$

These two "force and moment balance" conditions are the essential requirements for "steady state."

4) The tire slip angles which the vehicle assumes in establishing steady-state also determine the geometry of the vehicle in the turn. The projection lines drawn at right angles to the direction of the tire velocities cross at the turn center. Thus, the third projection line which is drawn through the c.g. shows us the direction of the velocity of the center of gravity (V). Because this direction is not aligned with the longitudinal axis of the vehicle, the vehicle is said to have a body sideslip angle (β). Since β is non-zero, the velocity, V, is composed of two velocity components: longitudinal velocity (u) and lateral, or sideslip velocity (v).

In (3) above, we noted that the two essential conditions for a

steady turn are the tire side force-centrifigal force balance and yaw moment balance. Yaw moment balance concerns the relative size of the front and rear tire forces. The force balance requires that the magnitude of the total side force equals the centrifigal force. Let us then examine the source of the centrifigal force.

Centrifugal force arises from (and is proportional to) lateral acceleration, which in turn is determined by velocity and turn radius. We all know from experience that if we increase our speed in a given turn (constant radius), centrifigal force increases. Further, if we traverse two different turns at the same speed (constant velocity), we experience higher centrifigal force in the tighter turn (small R). In each case, the higher force is, of course, due to higher acceleration. In equation form, the relationship between lateral acceleration, velocity and radius is

$$A_y = V^2(1/R) \qquad (6.13)$$

The equation is written as "V square times <u>one over R</u>" for a specific reason. By convention, 1/R is called "path curvature" or simply, "curvature." It is often easier to relate vehicle behavior to curvature than radius. For example, the severity of a turn increases as curvature increases. (Severity, of course, decreases as radius increases.)

The relationship of Equation (6.13) is shown graphically in Figure 6.20. The top graph shows that, at constant speed, acceleration is proportional to curvature. The bottom graph shows that, at constant radius, acceleration increases with velocity and increases much more rapidly for higher velocities.

Having seen the source of lateral acceleration, recall now Figure 6.19 where it was shown that centrifigal force is simply lateral acceleration times the vehicle mass (which is W/g, the weight divided by the gravitational constant), that is

$$F_y = A_y W/g \qquad (6.14)$$

Thus, the tire forces must be larger for higher lateral accelerations and, the tires must develop higher forces for cars of greater weight. Remember, however, from the previous discussion concerning tires, that peak lateral force capability and cornering
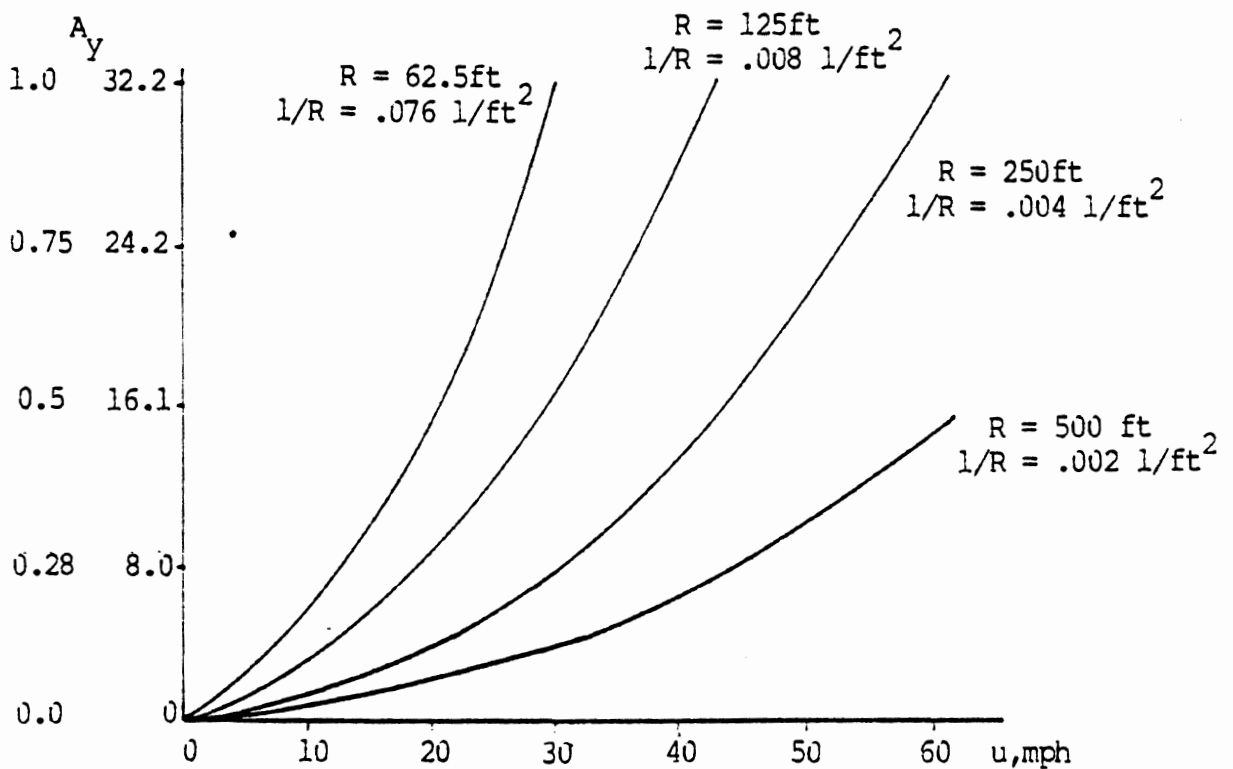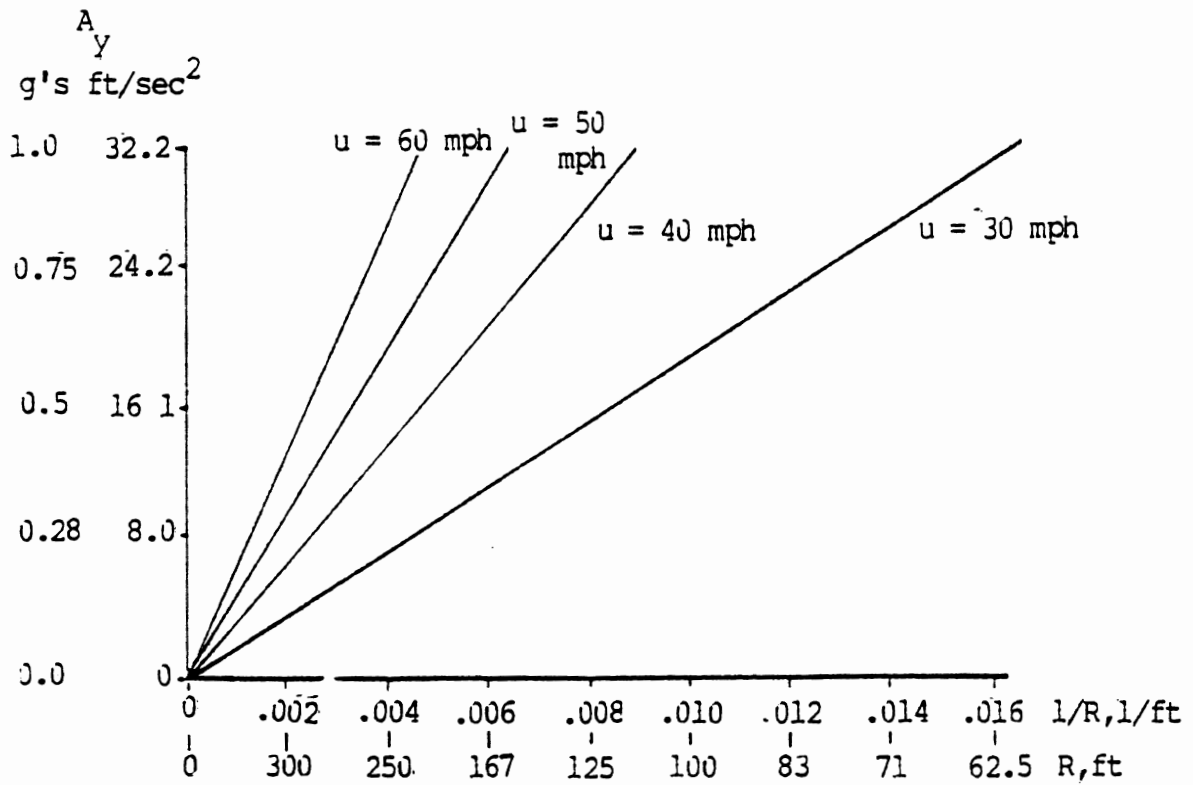
Figure 6.20 Lateral Acceleration as a Function of Curvature
(Radius) and Velocity

stiffness both tend to increase as vertical load on the tire increases. If the tires are properly sized for the vehicle, this increased tire capability tends to offset the increased demand placed on the tire so that vehicle weight has relatively little effect.

Because of this fundamental relationship between tire load (vehicle weight) and required tire side force, the ratio of load on the tire to cornering stiffness becomes of fundamental importance. The ratio is called cornering compliance and the vehicle is said to have a front cornering compliance and a rear cornering compliance[4], viz:

$$F_z/C_\alpha)_F$$

and

$$F_z/C_\alpha)_R$$

These two <u>cornering compliances can be considered the fundamental</u> <u>vehicle properties that govern vehicle handling in the normal driving</u> <u>range.</u> We will examine the implications of fore/aft distribution of compliance and the total level of compliance.

We will begin with a "baseline vehicle" which has equal cornering compliance front and rear. <u>This condition of equal compliance implies</u> <u>that the front and rear tires must obtain equal slip angles for the</u> <u>steady-state condition.</u> Figure 6.21 shows this vehicle in a steady turn of fairly low acceleration. The figure shows that the tires have developed equal slip angles ($\alpha_F = \alpha_R$), that the necessary steer angle (δ) to obtain this condition for the turn radius (R) and velocity (V) has been introduced "by the driver," and the vehicle has assumed a non-zero β which is necessary for the rear tire to assume its proper slip angle for the given turn radius and speed.[5] (In the following

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[4] Cornering compliance is a very useful concept and, in more detailed studies, can include many other tire, suspension, and steering system effects. For simplicity, we will consider only load and cornering stiffness, for these are the most fundamental factors, but one should be aware that in vehicle design, other factors are available to produce desirable levels of compliance.

[5] Notice that β is in the opposite direction from the way it was shown in Figure 19. (The vehicle is pointed "outside the turn" rather than inside.) The direction shown here is typical for low

discussion, Figure 6.21 will be shown as "background" in some of the subsequent figures so that the comparison to this "baseline" condition is more clear. The text will refer to the "foreground" vehicle of the figure.)

Now refer to Figure 6.22 where the same vehicle is negotiating the same radius turn but at a higher speed (greater lateral acceleration). Since the acceleration is higher, both slip angles are greater in response to the need for more side force, but they are still equal. The body sideslip angle, β has assumed a new value (the vehicle is now pointed inside of the turn rather than outside of the turn) necessary to obtain the larger rear tire slip angle. This change in β also produces an increase in front slip angle which is equal to the increase it produces in rear slip angle. Since the front and rear compliances are equal, this is exactly the condition required for equilibrium. So, very interestingly, the steering angle is exactly the same as it was in the previous figure when the vehicle was moving more slowly on the same radius curve. Because the vehicle has this property, that is, the same steer angle is required to travel a curve of a given radius regardless of speed conditions, it is said to be a "neutral steer" vehicle. The neutral steer property is a direct result of equal front and rear cornering compliance.

Interestingly, we may now "reuse" Figure 6.22 to show a different property of cornering compliance. Imagine that the vehicle of Figure 6.22 is no longer the baseline vehicle but is a vehicle with more compliant tires. This means that more slip angle is required to obtain a given side force. Front and rear compliances are still equal. Further, the vehicle is traveling at the same speed as the baseline vehicle was traveling in Figure 6.21. We see immediately that the effect of more cornering compliance (less stiff tires) is to require more slip angle (and consequently "more" β) in the same cornering situation. Changing to "softer" tires requires the same change in

accelerations. As a reference, consider a very low speed turn where A_y is almost zero, and therefore both slip angles are almost zero. In this case, the turn center is practically in line with the rear axle and β will be as large as possible in the direction shown in Figure 21. This is the "starting point" for β.

$$\frac{F_z}{C_\alpha}\Big)_F = \frac{F_z}{C_\alpha}\Big)_R$$

$$\alpha_F = \alpha_R$$
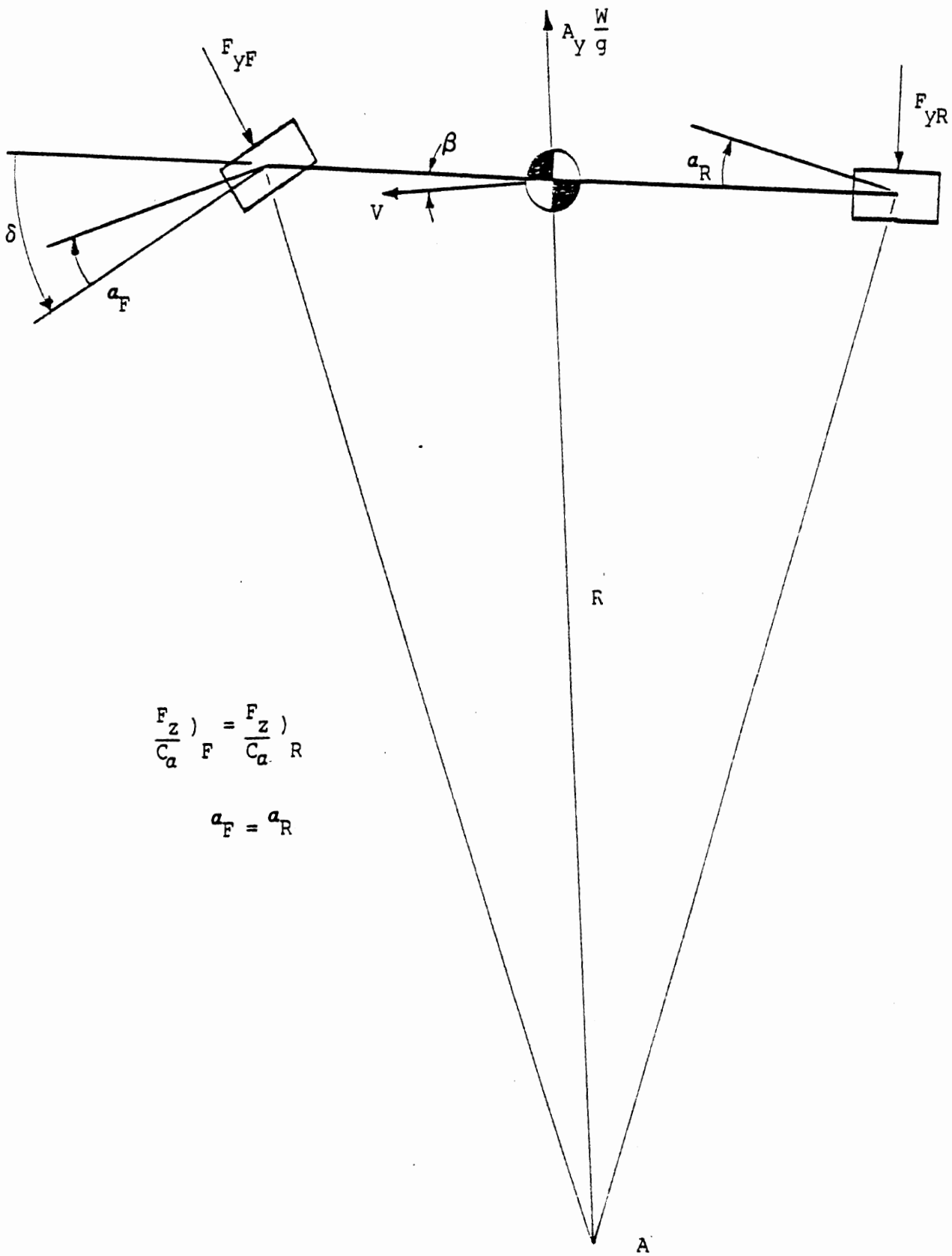
Figure 6.21   A. Front and Rear Compliance Larger than Baseline Vehicle
              B. Velocity Higher than Baseline Case
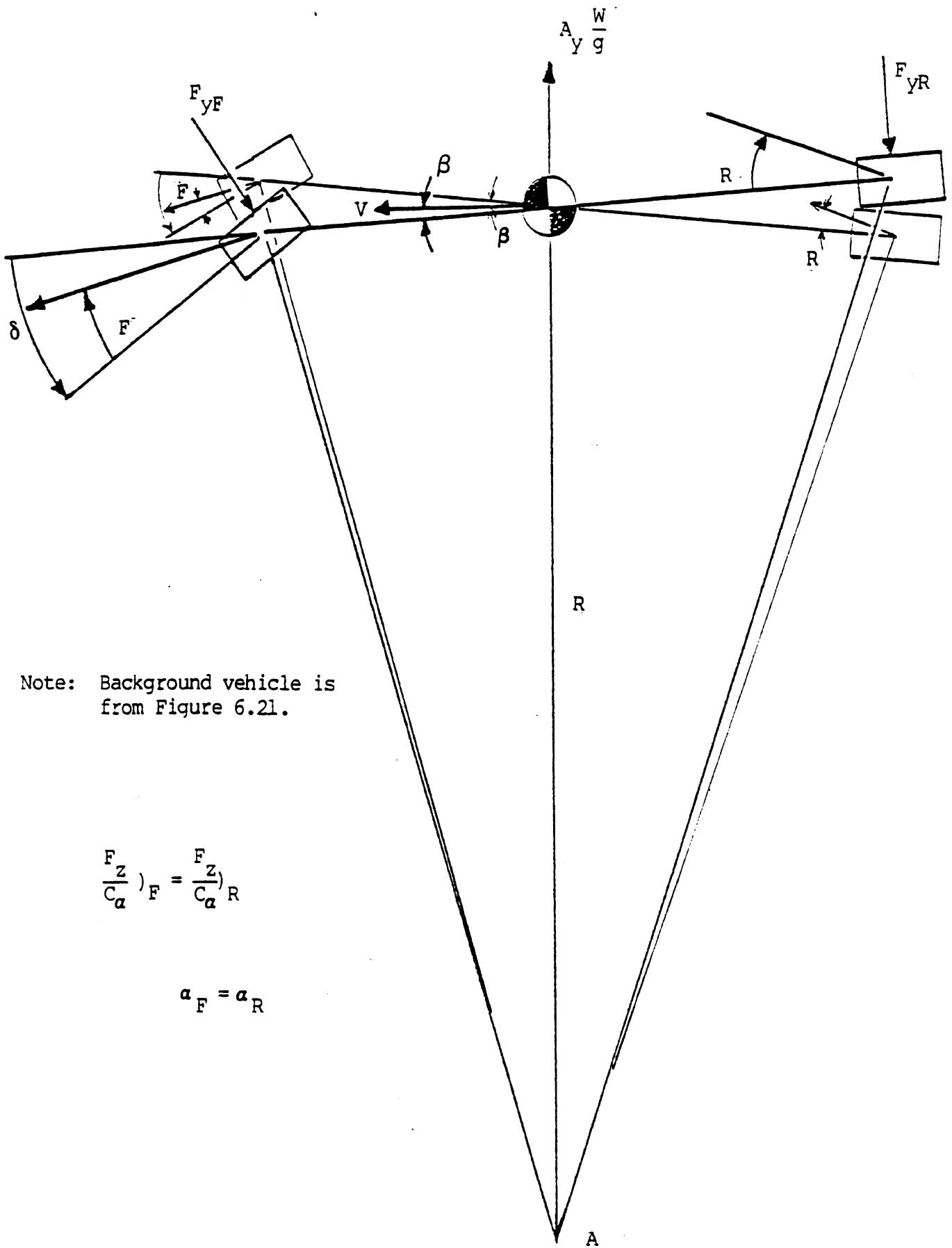
Note: Background vehicle is from Figure 6.21.

$$\frac{F_z}{C_\alpha})_F = \frac{F_z}{C_\alpha})_R$$

$$\alpha_F = \alpha_R$$

Figure 6.22 A. Compliance Larger than Baseline Vehicle
B. Velocity Higher than Baseline Vehicle

vehicle attitude that is required by higher speed.

Now let us examine the effect of having different cornering compliances front and rear. First, consider the vehicle in which front cornering compliance is greater than rear cornering compliance, that is, the rear tires are effectively "stiffer."

Figure 6.23 shows such a vehicle traversing the baseline curve at the baseline speed. We have obtained the difference in cornering compliance by increasing front compliance (less stiff tires) and reducing rear compliance (stiffer tires). Because the rear tires are stiffer, less slip angle is required there. Thus, β is different from the baseline case. (Note that it is _rear_ cornering compliance which determines β for a given turn condition.) Conversely, because the front tires are more compliant, more slip angle is required to obtain the required force. A larger _steering angle_ is therefore necessary to attain this slip angle. (Some of the additional slip angle required is due to the change in β.)

Figure 6.24 shows this new vehicle at a higher speed and greater acceleration.[6] As with the neutral steer vehicle, β has increased to provide the needed increase in rear slip angle, but unlike the neutral steer vehicle, _more steer angle_ is required to obtain the proper front-wheel slip angle _at the higher speed_. When front cornering compliance is higher than rear cornering compliance, more steer is required to travel a given curve as speed increases, and the vehicle is said to be understeer.

Finally, consider a vehicle where rear cornering compliance is larger than front cornering compliance. Figure 6.25 shows such a vehicle traversing the turn at high (foreground) and low (background) speeds. This figure shows that the higher speed condition requires a larger rear tire sideslip angle and thus a larger β. The critical point, however, is that front tire sideslip does not need as large an increase as the rear (because the front tires are stiffer). The change in β alone provides more increase in front tire slip than necessary and

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[6]The background vehicle of this figure is the vehicle of Figure 6.23
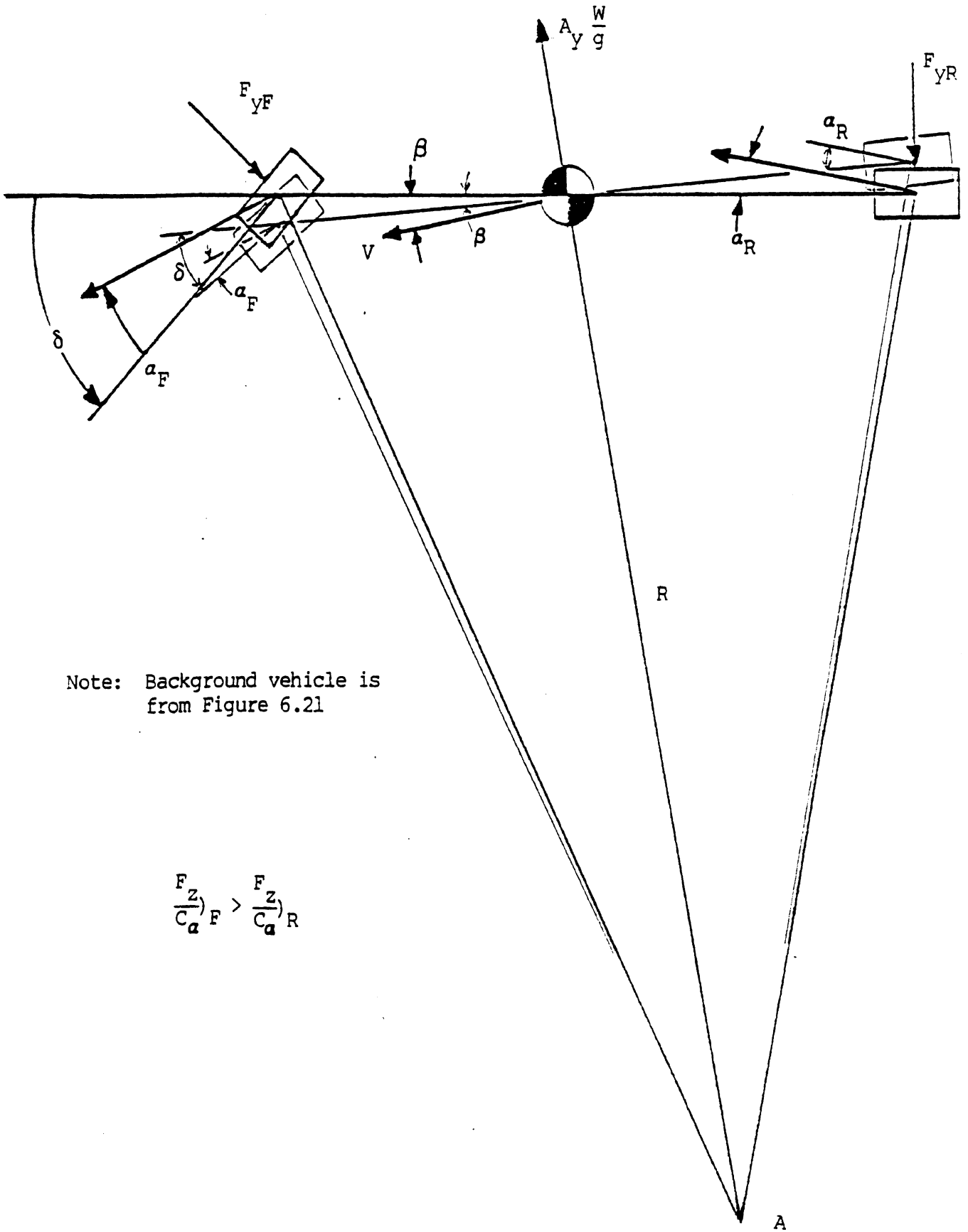
Note: Background vehicle is
from Figure 6.21

$$\frac{F_z}{C_\alpha})_F > \frac{F_z}{C_\alpha})_R$$

Figure 6.23 Understeer Vehicle
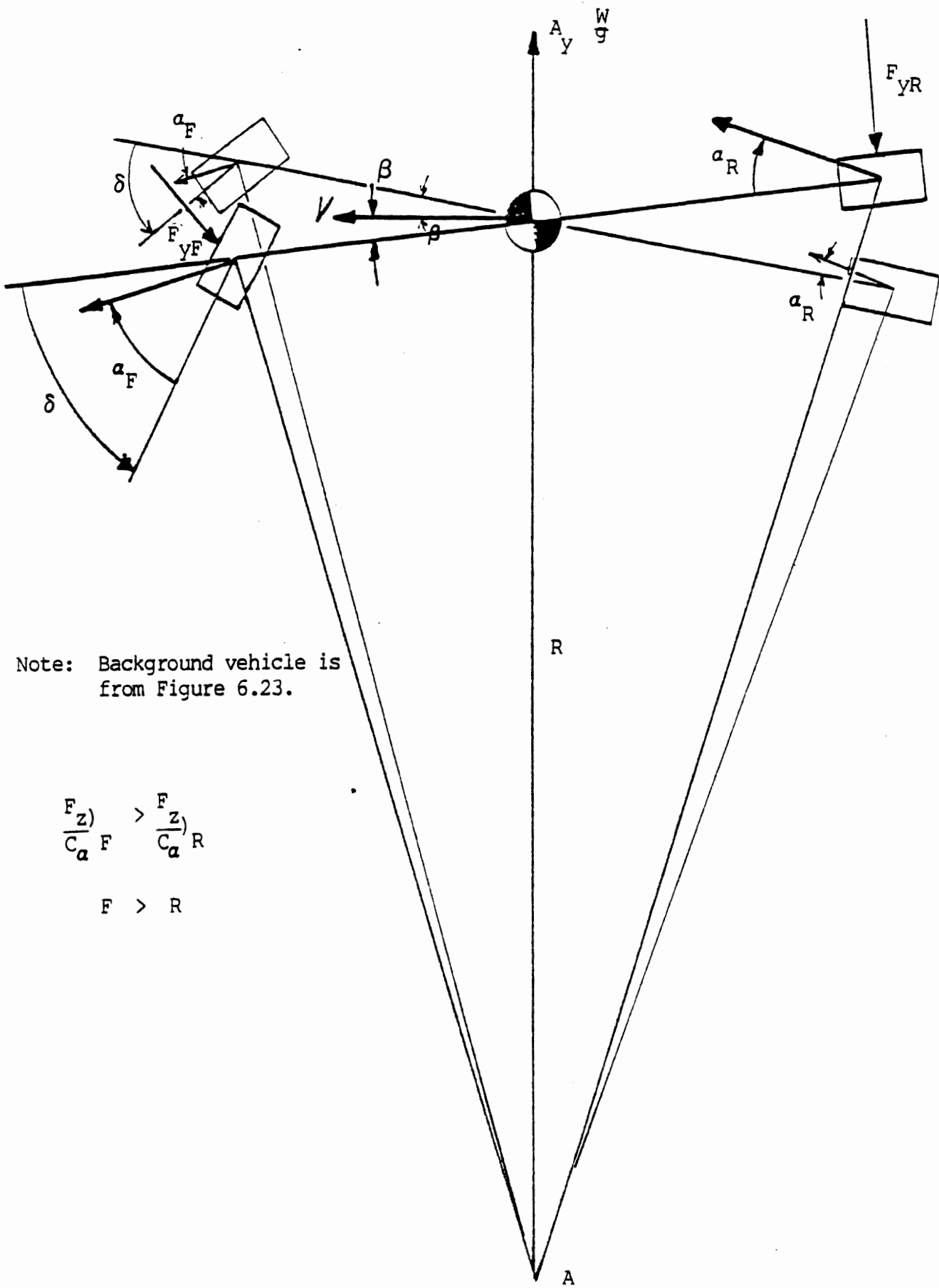
Note: Background vehicle is from Figure 6.23.

$$\frac{F_z)}{C_\alpha}_F > \frac{F_z)}{C_\alpha}_R$$

$$F > R$$

Figure 6.24 Understeer Vehicle at Higher Velocity

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thus <u>steer angle must be decreased</u> to maintain the same path. <u>When rear compliance is greater than front compliance, less steer angle is required to travel a given curve as speed increases and the vehicle is said to be oversteer.</u>

Most readers will have recognized the terms "understeer," "neutral steer," and "oversteer" introduced above. They have been introduced here in a manner intended to make their basic definitions most clear. Now let us examine their significance with respect to vehicle <u>stability</u>. We will begin this discussion with a presentation intended to provide some intuitive feel for the implication of understeer and oversteer on stability. Then, a very brief mathematical explanation of stability in the normal driving range will be presented.

All of the above discussion has been based on the assumption that the vehicle was in a "steady-state" condition. That is, the vehicle had established "equilibrium" by obtaining the "force and moment balance" conditions that were mentioned earlier. The notion of stability is involved with the vehicle's ability to reach a steady state condition when, for some reason, it is disturbed away from its existing steady-state. Thus, stability is closely related to <u>changing</u> vehicle response and whether changes naturally produce a response which can attain a new steady state or not.

Accordingly, begin with Figure 6.26 which illustrates the effect of changing tire slip angles. The vehicle begins with slip angles α_F and α_R such that it has a turn radius of R. The first change to consider is an <u>increase in front tire slip angle</u> to α_{F1}. The figure shows that the effect of this change is an <u>increase in turn radius</u> to R_1. The second change to consider is an <u>increase in rear tire slip angle</u> to α_{R2}. This change is seen to <u>decrease turn radius</u> to R_2.

Now consider an <u>understeer</u> vehicle (that is, front tire compliance is greater than rear) in a steady turn. What happens when the vehicle experiences a change due to a small disturbance which increases velocity? Recalling Figure 6.20, the increase in velocity <u>increases lateral acceleration</u> (consider this as the fundamental change caused by the disturbance) and therefore centrifigal force increases. To obtain the required "force balance," the tire side forces must increase meaning

Note: Background vehicle is traveling at slower speed.

$$\frac{F_z}{C_\alpha})_F < \frac{F_z}{C_\alpha})_R$$

$$\alpha_F < \alpha_R$$
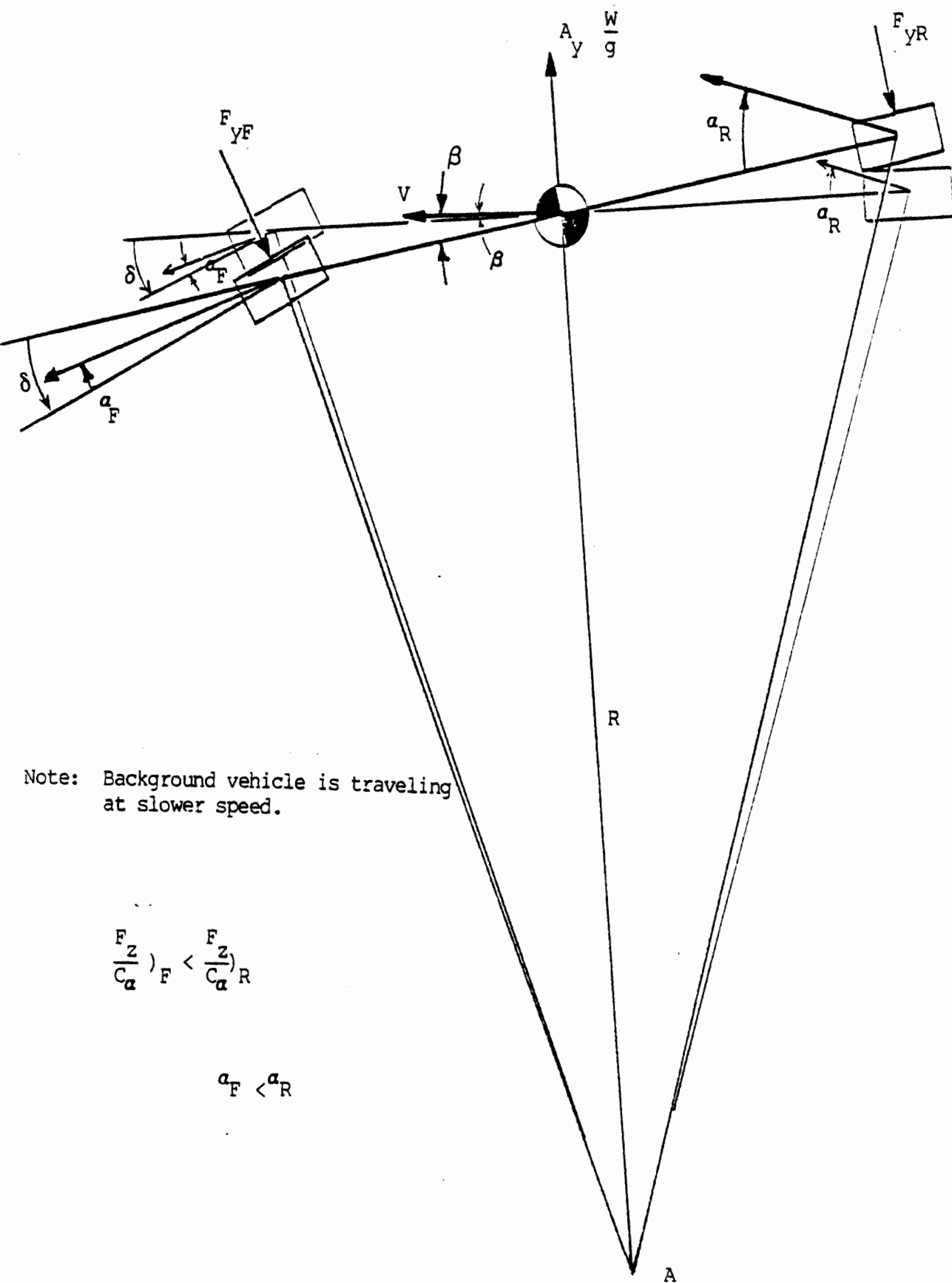
Figure 6.25 Oversteer Vehicle

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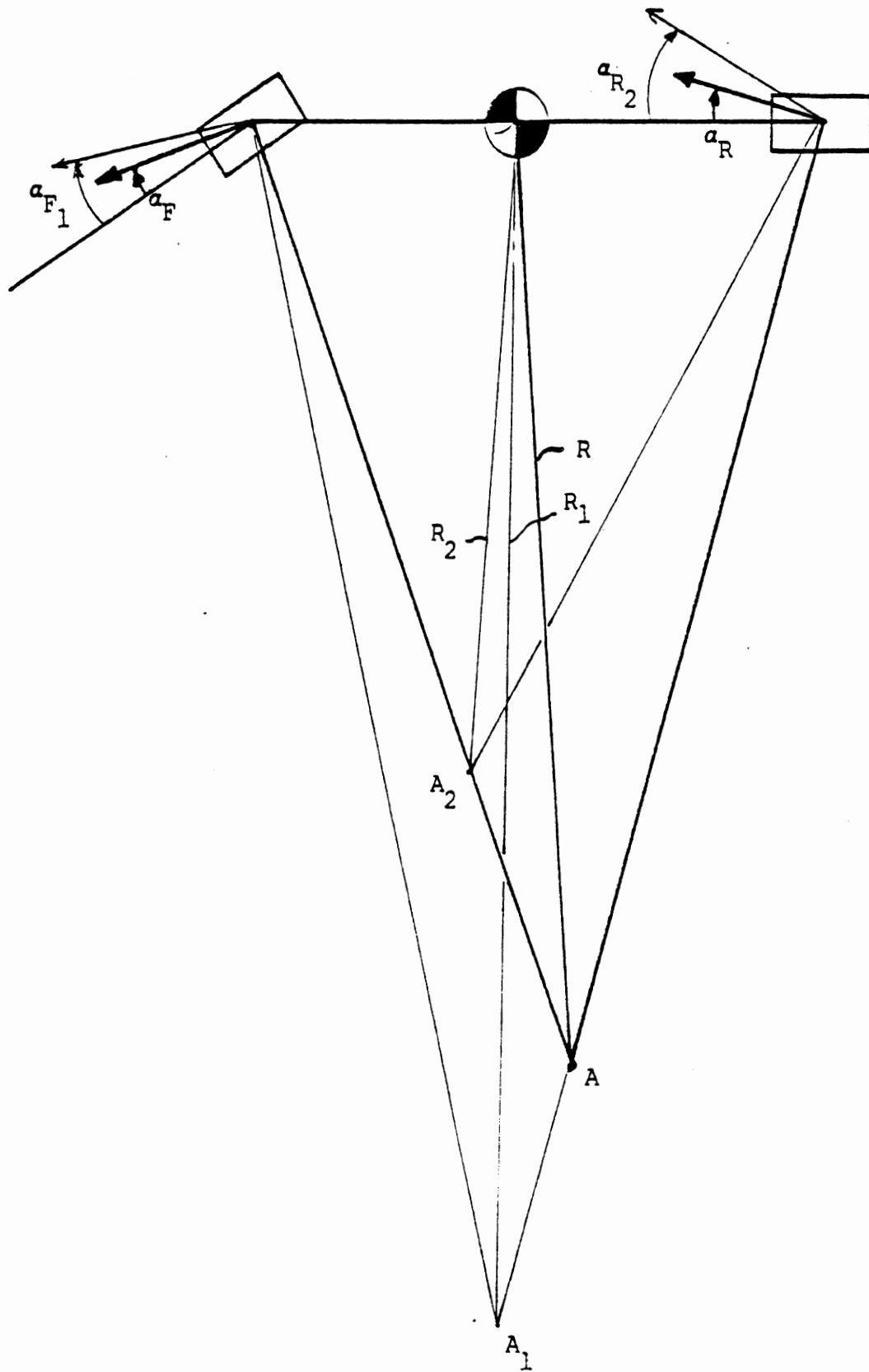
Figure 6.26 The Effects of Front and Rear Slip Angle
Changes on Turn Radius

that tire slip angles must increase. But since the front tires are more compliant, front slip angle increases more than the rear slip angle to obtain yaw moment balance. Figure 6.26 showed us that this proportioning of slip angle change will tend to increase turn radius (decrease curvature). Again, from Figure 6.20 , increasing turn radius tends to decrease lateral acceleration. But this is the opposite effect of the original fundamental change. The critical point to observe is that the understeer vehicle responds to the disturbance in a manner which tends to fundamentally reduce the severity of the disturbance. One might say that the vehicle is basically self-compensating or self-correcting--it is stable. A simple figure will help illustrate this basic stability quality. Figure 6.27 shows that when the disturbance first occurred, it implied a change in lateral acceleration. The vehicle then began to respond to the change, but this very response caused the lateral acceleration to move back toward the original condition. Clearly, the "response" of the vehicle and the "effect" will always meet at some intermediate "final condition." The vehicle is inherently stable.

As you probably foresee, the oversteer vehicle responds in just the opposite manner to a similar disturbance. When lateral acceleration increases due to the velocity increase, an increase in front and rear slip angles is again necessary to obtain a force balance. But, because the oversteer vehicle has more compliant rear tires, rear slip angle must increase more than front slip angle to obtain the moment balance. This proportioning of slip angle change will decrease turn radius (increase curvature) which in turn further increases lateral acceleration. That is, the fundamental response of the oversteer vehicle tends to exaggerate the severity of the original disturbance. Thus, the oversteer vehicle has the potential to be unstable.

Whether the oversteer vehicle is stable or not is more clearly shown by another simple diagram. The top diagram of Figure 6.28 illustrates a stable oversteer vehicle. The picture shows that the vehicle responds to the original change of acceleration by moving toward the new steady state. However, this response causes the effect of the disturbance (A_y) to increase further. Ultimately, this vehicle does

Attitude Response
to A_y Disturbance

Vehicle Attitude
(β, a_F, a_R)

Lateral Acceleration
(A_y)

Original
Steady state

Final
Steady state

A_y due to initial
disturbance

A_y Response to changing
vehicle attitude
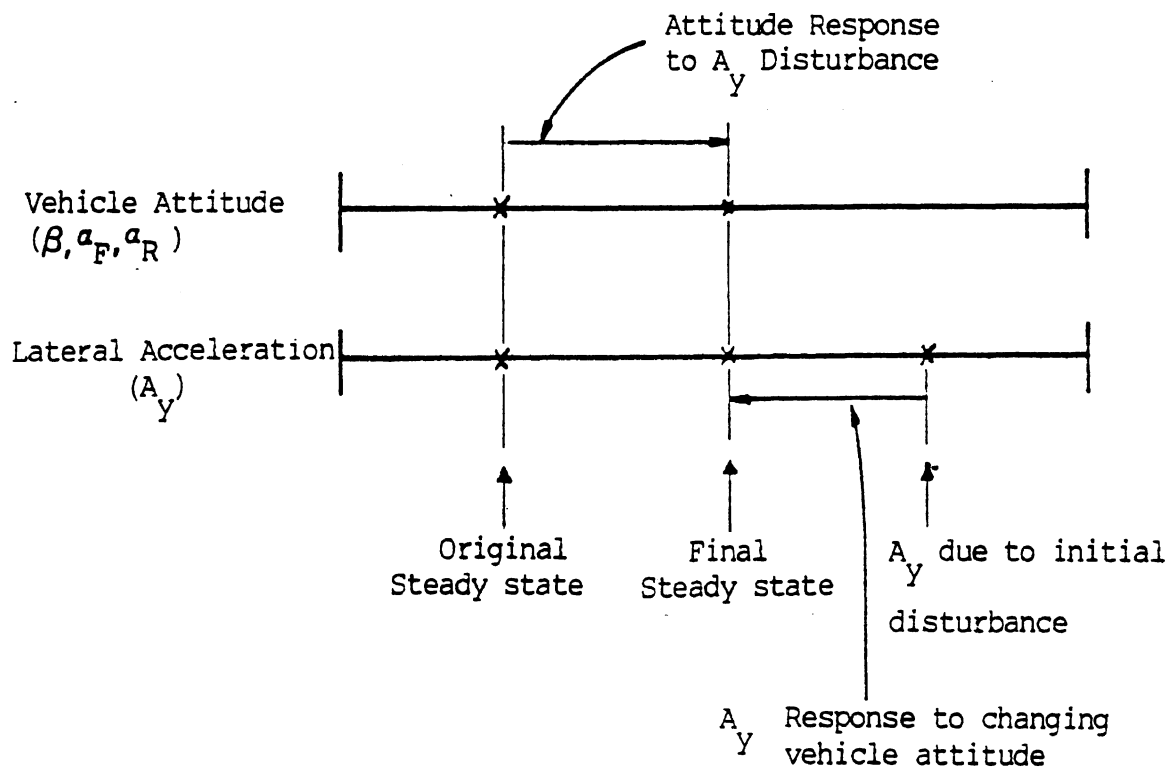
Figure 6.27   Understeer Vehicle's Response to a disturbance

reach a steady state because the vehicle's response is "stronger" than its effect on lateral acceleration. The vehicle attitude was able to "overtake" A_y to establish the steady-state response. The second diagram of the figure shows the other condition. For this oversteer vehicle the lateral acceleration "moves away" more strongly than the vehicle responds. The vehicle response continues to grow but can never reach steady state. The vehicle is unstable.

Whether an oversteer vehicle is stable or not depends on how severely oversteer it is and on the speed it travels. A simple equation (Eq. 6.15) which expresses, in part, the equilibrium condition of the model vehicle in a steady turn shows this point.

$$1/R = \delta / (\ell + KV^2) \tag{6.15}$$

where

Attitude response to
A$_y$ disturbance

Vehicle Attitude
$(\beta, \alpha_F, \alpha_R)$

Lateral Acceleration
(A_y)

Original
Steady State

Ay due to
initial disturbance

Final
Steady state

Ay response to
changing attitude

UNSTABLE

Attitude response to
Ay disturbance

Vehicle Attitude
$(\beta, \alpha_F, \alpha_R)$

Lateral Acceleration
(Ay)

Original
Steady state

Ay due to
initial disturbance

Ay response to
changing attitude

Figure 6.28   Oversteer Vehicle's Response to a disturbance

1/R is path curvature

δ is steer angle

ℓ is wheelbase

V is velocity

and   K is called the "understeer" coefficient and is simply the

difference of the front and rear cornering compliances,
i.e., $K = F_z/C_\alpha)_F - F_z/C_\alpha)_R$. For an understeer vehicle K is
positive.   K = 0 for neutral steer, and K is negative for an
oversteer vehicle.

For an understeer vehicle, since K is positive, the denominator of
Equation (6.15), $\ell + KV^2$, is always equal to or greater than ℓ. For an
oversteer vehicle, since K is negative, $\ell + KV^2$ is equal to or less than
ℓ. Most importantly for an oversteer vehicle, $\ell + KV^2$ can become zero
at a certain velocity (given K). When the denominator of Equation
(6.15) is zero, the solution to the equation is

$$1/R = \infty$$

That is to say, "steady state occurs at an infinite curvature" (zero
radius) or, in fact, does not exist at all. The velocity at which this
occurs is called the critical velocity. Oversteer vehicles are unstable
at the critical velocity and all higher velocities.

Having defined understeer and oversteer (through the tire
compliance oriented model), and explained its significance in a
technical context (stability issues), it is now valuable to discuss how
these properties relate to the in-use vehicle population and to "pre-
crash handling performance."

It can be said virtually without reservation that all modern
passenger cars, when maintained to manufacturer's specifications, are
understeer vehicles. (It has been suggested that the performance of
such cars as sports cars, often interpreted as neutral or oversteer, is
in fact understeer, but of relatively lower magnitude.) The reason for
this is obvious; the assurance of stable handling qualities in the
normal driving regime, that the understeer vehicle provides, is clearly
a highly desirable quality. Conversely, the potential for instability
in an oversteer vehicle is highly undesirable. (Neutral steer, by
implication, is also undesirable since it simply represents the fine
dividing line between these two. Thus, small in-use changes can cause
the neutral steer vehicle to become oversteer.) Too much understeer is
also undesirable, however. Large values of understeer can cause vehicle

response to be sluggish and to "plough" during cornering. (Large values of front and rear cornering compliance also produce a sluggish vehicle. Even if a vehicle has a reasonable level of understeer, if that understeer is arrived at through high rear compliance and even higher front compliance, response will be slow. Intuitively, the vehicle must travel through larger slip angle changes (β changes) to obtain steady state then it would if compliances were low.) Thus, a design compromise is implied between too much understeer and yet enough to ensure that the vehicle remains understeer throughout the range of reasonable in-use changes it will experience. Vehicles ranging from sports cars to luxury sedans indicate the range over which this compromise extends.

As implied by the definition of cornering compliance (F_z/C_α) the in-use factors which alter cornering compliance are tire and loading conditions.[7] Tire factors of importance are (1) tire type (radial versus bias construction and highway versus snow traction tread), (2) tire inflation pressure and, to a lesser extent (3) tire wear state.

Radial tires tend to have less compliance than bias tires, highway tread tires tend to be less compliant than snow tread tires, and higher inflation pressures tend to result in less compliance. To a lesser extent, severely worn tires tend to have less compliance than new tires.[8] In the worst set of circumstances, a vehicle could have heavily worn, fully inflated, radial, highway tread tires mounted in front and new, underinflated, bias, snow tires in the rear. Such a vehicle could be expected to be oversteer.

The situation can be further exasperated by loading condition. In the discussion of tire properties, it was noted that passenger car tires tend to increase their cornering stiffness (C_α) as the load that they

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[7] It was also noted that suspension and steering systems properties affect compliance. The types of changes which occur in these systems over time do not tend to have a strong effect on cornering compliance.

[8] This is a wholly different issue than the degraded traction performance of worn tires on wet roads. Here we are concerned with compliance, not traction. Further, the polarity of the effect is reversed. (If stiff tires are judged as "good," as they generally are, then worn tires are "better" in compliance.)

carry increases. This property helps to maintain a fairly constant cornering compliance over changing loads (a desirable quality). Unfortunately, at loads which approach the maximum design load of the tire, this quality begins to weaken. Thus, high loads tend to increase the cornering compliance of tires. Thus, in addition to the tire factors noted above, heavy rearward loads tend to make the vehicle more oversteer.

As noted at the outset, the entire discussion on vehicle handling to this point has been restricted to the "normal driving range" in which the tires operate in their compliant range. The effort expended may at first seem inappropriate since one might suggest that "accidents tend to occur in drastic situations where limit performance or tire traction properties seem more appropriate." To put the safety-relevance of normal range handling in context, several points can by made:

-Regardless of how drastic a maneuver will eventually become, virtually all maneuvers begin with the tires operating in the compliance regime. Further, experimental handling data exist which suggest that, in many cases, limit vehicle response motions are, in large part, determined by the way in which those motions were initiated in the compliance regime.

-Other data exist which suggest that average drivers rarely use the limit handling capability of the vehicle. All normal driving is of so very low a level relative to the limit capabilities of the vehicle, that drivers either tend to underrate the vehicle's ability, or are not prepared by experience to use it. Consequently, even in emergency conditions, a driver may not cause the vehicle to exceed the compliance handling performance range very significantly.

This discussion will now proceed into the area of limit handling performance in which the tires of the vehicle operate in the traction range. It will be seen that, to a large extent, the performance in this regime can be readily understood by drawing on insights developed in the previous discussions.

Passenger cars can generally be characterized as being "plow-out" or "spin-out" limited in handling response. "Plow-out" occurs when the vehicle's front tires reach their limit capability (before the rear tires do) and cannot produce the front tire side forces necessary to sustain the desired turn. The front tires "break-away" and the vehicle

departs from the desired path toward the outside of the curve. In doing so, the vehicle generally remains pointed in the direction of travel (β does not become very large) and so technically, it remains directionally stable but becomes uncontrollable. (Technically, uncontrollable means that no input, i.e., steer angle, is possible which will achieve the desired response, i.e., the desired path.)

Conversely, "spin-out" results when the rear tires are the first to achieve their maximum side force capability. Now the rear tires "break-away" and the rear of the car "comes around"--the car spins. In this case, the vehicle is directionally unstable and, for the average driver, is practically uncontrollable. (Theoretically, in marginal spin-out conditions steering inputs are available to maintain the desired path while experiencing very large β. These inputs must be constantly responsive to the vehicle motions in order to stabilize an otherwise unstable system.)

These two conditions, plow-out and spin-out limits, are analogous to the understeer and oversteer conditions taken to the extreme. Recall that for the understeer vehicle, when lateral acceleration increased the vehicle tended toward a larger turn radius because front tire slip angle increased more than rear tire slip angle to obtain the necessary side force. This condition is taken to the extreme when the front tires are at their peak. Clearly, if the front tires are already producing the greatest force available, their slip angles can increase to any angle they please without producing the necessary increase in force. Basically, this is what happens in plow-out, and the vehicle essentially follows a path prescribed by its momentum--off the outside of the curve.

A similar analogy, of course, holds between oversteer behavior and spin-out. In this case, with the rear tires already producing maximum side force, rear slip angle continues to increase in search of a non-existent increase of tire side force--the rear end "comes around." But what factors contribute to determining which tires obtain their maximum force capability first? The answer includes several vehicle, tire, and in-use factors.

First, if all other things were equal, understeer vehicles would tend to plow out and oversteer vehicles would tend to spin out, simply

because of the slip angle trends initiated in the compliance regime. Naturally, however, all other things are not equal, and, in fact, other factors are sufficiently powerful that understeer/oversteer (perhaps surprisingly) has relatively little importance in determining the limit condition if it is approached gradually. (As noted previously, limit vehicle response motions can be affected by how those motions were initiated in the compliance regime. This is particularly true if the maneuver is "quick" and high levels of momentum are developed in the linear and transitional operating regimes.)

The essential determinant embodied in vehicle design involves the interrelationship of vertical load on maximum tire traction capability. Earlier in the discussion of tire mechanics we noted that the friction process was such that maximum side force tended to increase with vertical load. In fact, the equation

$$F_{yMAX} = \mu F_z \qquad (6.16)$$

was presented. Equation (6.16) is not exact for real tires, and we will see that this difference is significant in determining which tires will saturate first.

We have avoided discussing the roll reaction of the vehicle to this point, but it now becomes significant because of the importance of vehicle roll on tire loading. When a vehicle experiences a turn, the centrifigal force produces a roll motion toward the outside of the turn with which we are all familiar. Correspondingly, weight is transferred from the inside tires to the outside tires - vertical load is increased on the outside tires and decreased on the inside tires.

Figure 6.29 shows how this load transfer affects the maximum available side force of the two tires of one axle. First of all, the figure is a graph of maximum available side force as a function of vertical load. The important point to note is that the graph is curved; beyond the linear range it is not a straight line as Equation (6.16) would suggest. A point is shown on the graph which corresponds to the two tires of an axle when they each support one-half of that axle's load. When the axle experiences roll in a turn, the load on the outside tire increases and the load on the inside tire decreases. The figure

shows that the average side force capability of the tires has decreased because of the load transfer. <u>The side force capability of the axle has decreased because of side-to-side load transfer.</u>
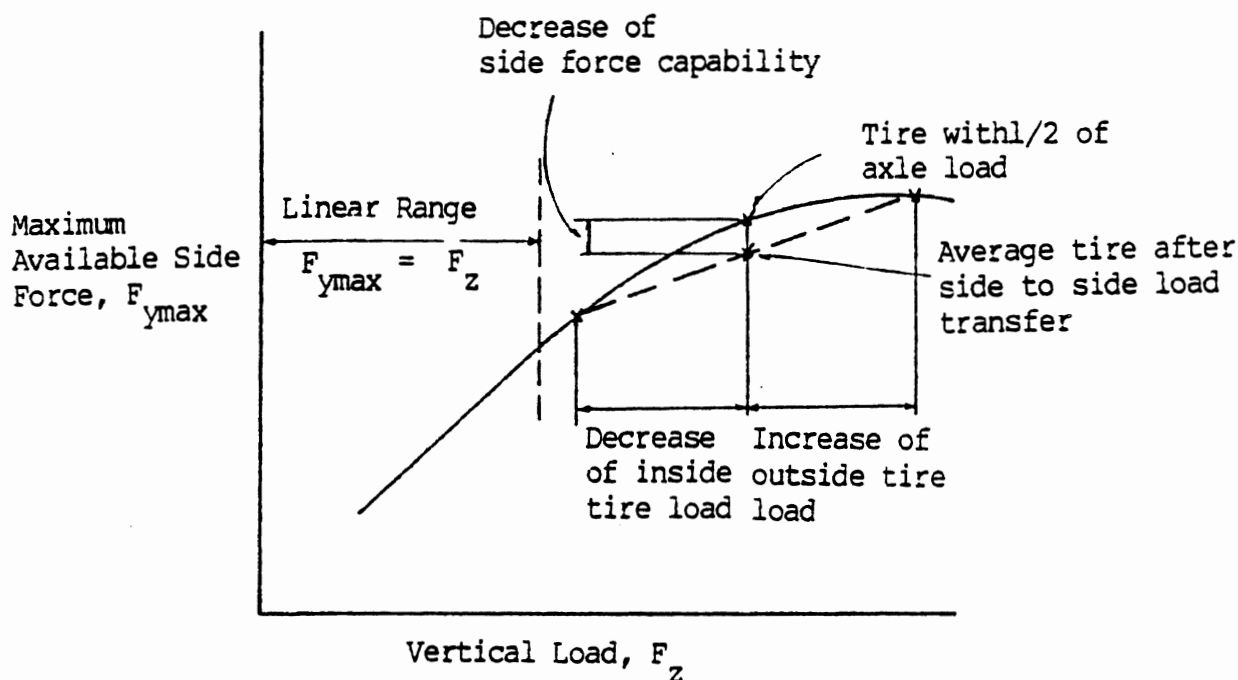
Figure 6.29 Side-to-side load transfer
decreases maximum side force capability.

Although the total side-to-side load transfer which occurs in a turn depends on lateral acceleration and c.g. height, the vehicle designer has available mechanisms within the vehicle suspension which allow him to determine how this load transfer is distributed front to rear. <u>If the vehicle is designed to have more side-to-side load transfer at the front, then the front tires will experience the greater loss of side force capability and the vehicle will tend toward plowout.</u> Of course, <u>more load transfer in the rear will lead toward spinout.</u> Most vehicles produced today are designed to be plow-out limited.

(It should be noted that while it is generally desirable to bias load transfer forward to produce plow-out limit response, too much bias is not desirable. In fact, a vehicle which is perfectly balanced for both axles to saturate simultaneously will have the maximum turning performance capability possible. This condition is considered to lie too close to the undesirable spin-out limit, however (as neutral steer is considered to lie too close to oversteer), and is therefore avoided in vehicle design.)

In-use factors can, of course, alter the initial design intent. Perhaps the most powerful in-use factor is tire wear state in combination with wet roads. Wet roads, of course, reduce the traction capability of all tires, but much more so for worn tires. Badly worn tires mounted on the rear will lead toward spin-limit performance on wet roads.

Another "in-use" factor can play a very important role in determining the plow- or spin-limit. That factor is the braking process. Recall that, in the section on combined force generation in tire mechanics, we observed that severe (high longitudinal slip) braking can drastically reduce the side force capability of the tire. Thus, if braking is involved, it may be the dominant factor in determining plow-out or spin-out limit by establishing if the front tires or rear tires are experiencing high longitudinal slip. The effect of braking on side force capability is so extreme that if both front wheels or both rear wheels lock up, the turning limit of the vehicle virtually goes to zero. That is, if the rear wheels lock, the vehicle may spin even if it was simply traveling straight ahead. If the front wheels lock, the vehicle will travel virtually straight ahead regardless of steering activity.

In Europe, the feeling among vehicle designers is that the latter situation is much more desirable than the former, and so, virtually all European vehicles are designed to lock front wheels first under virtually all conditions. This choice can lead to some loss in stopping capability in some conditions. In the U.S., design philosophy is generally geared toward optimizing stopping capability over all conditions, and so lock up may occur on either axle depending on tire and road conditions. The entire braking process will be examined in

more detail in the next section.

Traction capability can also be degraded by (negative) longitudinal slip generated by accelerating forces. Thus, rear-wheel drive vehicles increase their tendency toward spin-out in "power-on" cornering and front-wheel drive vehicles increase their tendency toward plow-out. This has led to a disturbing handling phenomenon in some front-wheel drive cars. During a power-on corner, some event may occur to cause the driver to suddenly release the accelerator. The quick elimination of front-wheel longitudinal slip can result in a very sudden and rather strong response away from plow-out.

6.5 The Braking Process for a Passenger Car

For a vehicle to slow down and stop, rearward forces must be applied which "decelerate" the vehicle. As with all other vehicle control functions, the primary forces to accomplish this task must be developed by the tires in contact with the road. (Of course, small deceleration forces result from wind drag and tire rolling resistance. These are so small that we can ignore them in this discussion. To appreciate how small these forces are, consider the distance required to coast to a stop from 60 mph as compared to the stopping distance under hard braking from 60 mph.)

In this discussion of the braking process we will first examine the elements of the vehicle braking system to see how these forces are developed and how they are distributed between the front and rear wheels. Later we will see how the vehicle reacts to braking forces and note the significance of their distribution.

The vehicle braking system starts at the brake pedal and ends at the tire-road contact patch. Its general function is to convert the input signal (driver applied pedal force) into braking forces capable of stopping the vehicle in a controlled manner. The major elements of the braking system are shown in Figure 6.30. When the driver applies a force to the brake pedal (which may be increased through power assist), this force is converted to hydraulic brake line pressure (P) by the brake master cylinder. This pressure is then sent to both the front and rear wheel brakes through small steel tubes and hoses. The wheel brakes

then convert the brake line pressure to brake torque (T) applied to the wheels. In the brake, a brake cylinder reconverts the hydraulic pressure to a brake application force. This force presses the brake shoes and linings against the rotating drum or disc. Friction between the lining and this "rotor" creates brake torque. The amount of brake torque developed per the amount of application force is called brake effectiveness. The direct result of the application of brake torque is a slowing of the spin velocity of the tire and wheel. As we found in our discussion of tire mechanics, the slowing of spin velocity results in longitudinal tire slip (S) and the generation of brake force (F_x). Up until maximum brake force of the tire is obtained, the amount of brake force generated is proportional to the brake torque applied. The proportionality constant is the rolling radius of the tire.

One can see that the level of brake force generated per the amount of applied pedal force (pedal force gain) is largely a function of brake effectiveness and the relative size of the brake cylinder and master cylinder. It is possible to design brakes with very high effectiveness. Unfortunately, high effectiveness also results in high sensitivity of the brakes to small internal changes. Highly effective brakes can be inconsistent or "grabby." At a conceptual level, the great value of power assist is to allow braking systems to have a high pedal force gain while still using brakes of fairly low effectiveness. (One of the properties of disc brakes, among others, which makes them desirable compared to drum brakes is their naturally low effectiveness and resultant consistency of performance.)

Not only do brake effectiveness and brake cylinder size determine pedal force gain, but their relative effect, front and rear, determines the relative distribution of front and rear brake force--brake force proportioning. Since the rolling radius of front and rear tires is generally the same and since the hydraulic pressure delivered to front and rear brakes is generally the same[9], brake effectiveness and cylinder size are the primary elements by which brake force distribution can be determined. It is significant to note that brake cylinder size

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[9]Later we will discuss "proportioning" valves which can modify the pressure delivered to the brakes.
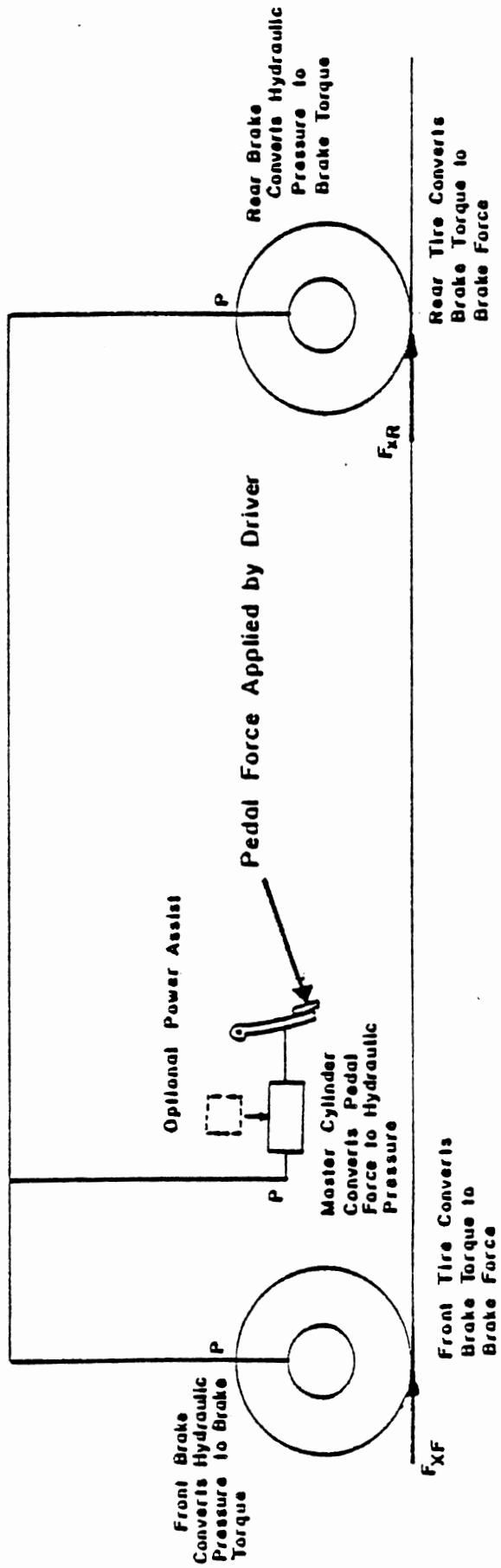
Figure 6.30 The Brake System

and brake effectiveness are essentially _fixed_ properties determined by design. Thus, _without the addition of other elements_ (proportioning valves, for example), _the fore/aft proportioning of brake force is fixed over the entire sublimit braking range._

Now we will examine the behavior of the entire vehicle during braking to determine the significance of brake proportioning. Figure 6.31 shows the side view of a vehicle traveling at steady speed (not braking.) The figure shows that the vertical load (the weight of the vehicle) is distributed between the front and rear wheels according to the location of the center of gravity. As was the case in vehicle handling, this "equilibrium condition" involves a "force and moment balance." The force balance implies: the total amount of vertical tire load "pushing up" on the vehicle is equal to (or balanced by) the force "pushing down" on the vehicle, i.e., the vehicle weight. The moment balance implies: the relative size of the two vertical tire forces are such that they produce equal and opposite moments about the c.g., i.e., the moments of these forces are in balance.

Figure 6.32 illustrates the same vehicle in braking. With the presence of _horizontal forces (braking forces) in addition to vertical forces, an additional force balance is implied._ In this case, the total of the front and rear braking forces balance the deceleration force ($A_x W/g$) acting at the c.g. (similar to the centrifigal force experienced in handling due to lateral acceleration). Through this force balance, the level of brake force determines the level of deceleration (A_x). Since the _brake forces are not in line with the vehicle c.g., they also enter into the moment balance._ The figure shows that _vertical load is transferred from the rear tires to the front tires to balance the additional moment applied by the brake forces._ We have already noted that properties of the brake system determine fore/aft _brake proportioning_ and that this proportioning _is a fixed quantity._ Figure 6.32 shows that vertical load proportioning is determined by the static vertical loads and by the level of deceleration. _As deceleration varies, vertical load proportioning is extremely important as the front or rear tires approach their maximum brake force capability._

In our discussion on tires, we noted that maximum available brake
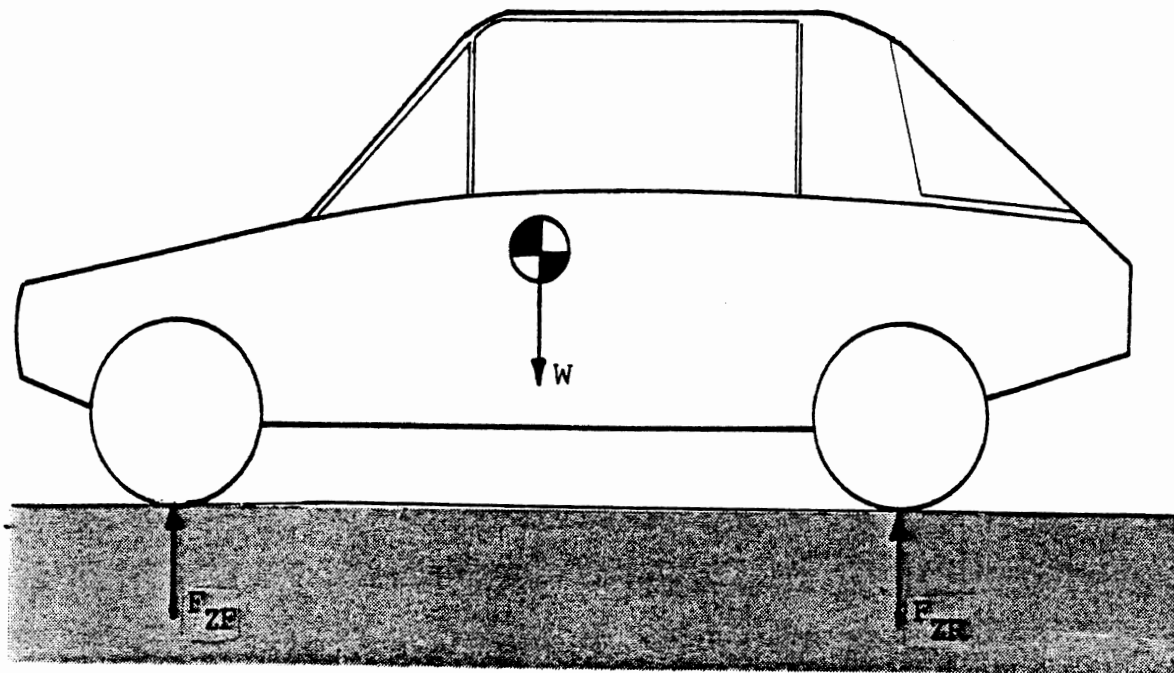
Figure 6.31 Vehicle at Steady Speed

force is generally proportional to the vertical load on the tire. That
is,

$$F_{xmax} = \mu_p F_z \qquad (6.17)$$

where μ_p is the peak braking friction coefficient. μ_p will generally be
the same for front and rear tires so that we can write Equation (6.17)
separately for both front and rear, that is

$$F_{xF} = \mu_p \, F_{zF} \qquad (6.18F)$$

$$F_{xR} = \mu_p \, F_{zR} \qquad (6.18R)$$

Dividing Equation (6.18,F) by Equation (6.18,R) gives us

$$F_{xF}/F_{xR} = F_{zF}/F_{zR} \qquad (6.19)$$

Equation (6.19) implies that, to reach maximum brake force at both front
and rear tires simultaneously, fore/aft brake force proportioning(F_{xF}/
F_{xR})must be equal to fore/aft vertical load proportioning(F_{zF}/F_{zR}).
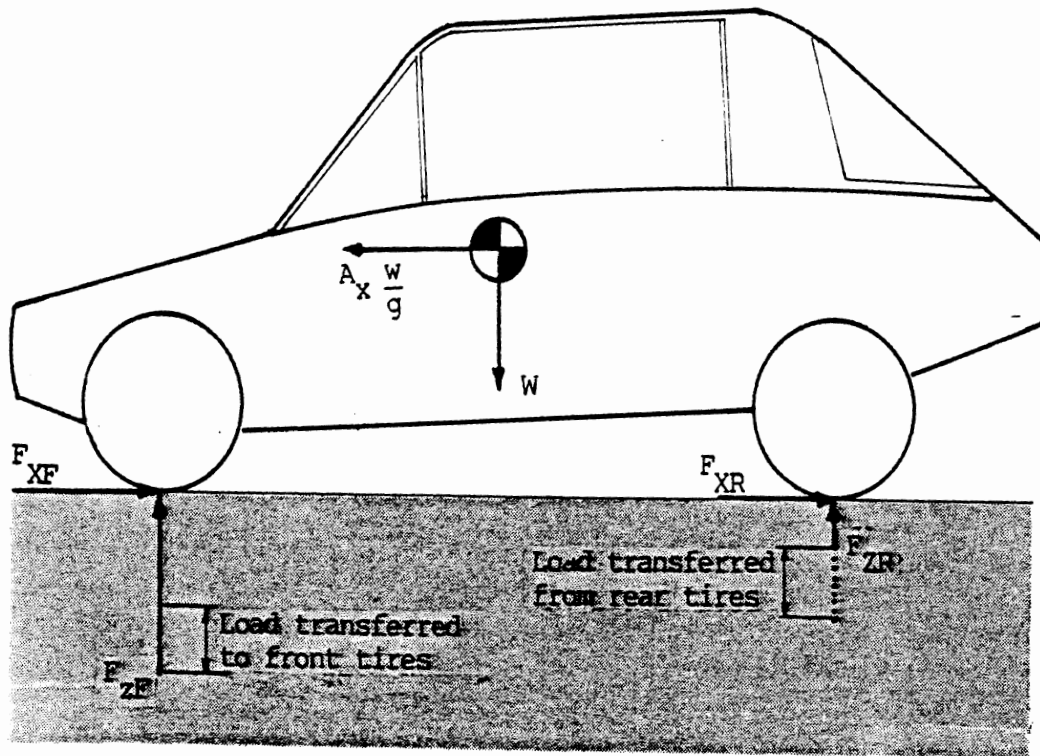
Figure 6.32 Load transfer during braking

Since vehicle deceleration increases with total brake force, it is intuitively clear that the simultaneous occurrence of maximum front and rear brake force is the condition required to obtain maximum deceleration. Although it is not so obvious, the maximum possible deceleration (in g's) is equal to the peak friction coefficient. (For example, if μ_p = 1.0, then, in theory, the maximum possible deceleration of the vehicle is 1.0 g, or 32.2 ft/sec^2.)

What does all this mean for a real vehicle? Figure 6.33 presents a graph which will help in the explanation. The graph shows the brake force and vertical load proportioning for a particular vehicle. This vehicle was designed with a brake system which resulted in 70% of the brake force occurring at the front wheels. As we learned above, this is a fixed quantity that does not vary with deceleration. We also see that

vehicle design factors (wheelbase and c.g. position) lead to a vertical load proportioning that varies from 50-50 to 80-20 (front to rear) as deceleration varies from zero to 1.0 g. These two force proportions are equal only at a vehicle deceleration of .67 g's. Consequently,this vehicle can only attain the maximum potential deceleration when μ_p = .67. If μ_p is less than .67, front brake force is proportionately larger than front vertical load at the potential maximum deceleration. Therefore, the front tires will reach their maximum capability first and the deceleration at this condition will be less than μ_p. When μ_p is greater than .67, rear brake force will be proportionately larger, and rear tires will reach their maximum capability first. Again, deceleration at this condition will be less than μ_p.

The "maximum deceleration" referred to in the above paragraph is, more precisely, maximum deceleration attainable without wheel lock. This is the most safety-relevant limit braking condition, for, as noted in the vehicle handling section, once wheel lock occurs the vehicle will either spin-out (rear wheels locked) or become unsteerable (front wheels locked). In many real emergency braking situations,the driver, of course, is not aware that tires have reached their maximum capability. He (she) may push still harder to obtain more braking. Since no additional brake force is attainable (actually brake force falls off, as in Figure 6.14), longitudinal slip increases rapidly to a value of 1.0 (lock up). The practice of "pumping the brake" during severe braking derives from the need to maintain rolling tires for handling quality. Strong application and release allows the tire to generate high braking force at least part of the time and also allows the wheels to "spin up" if lockup does occur.

Two "advanced technology" devices are available to improve braking performance. The device most commonly used on passenger cars in this country is the proportioning valve. Such valves are relatively simple and inexpensive devices which can be installed in the brake hydraulic lines to modify the brake line pressure delivered to the front or rear brakes. The intent of their use is to improve the "matching" of brake force and vertical load proportioning over a broader range of surface conditions. (Figure 6.33 showed that this match is not too good over a
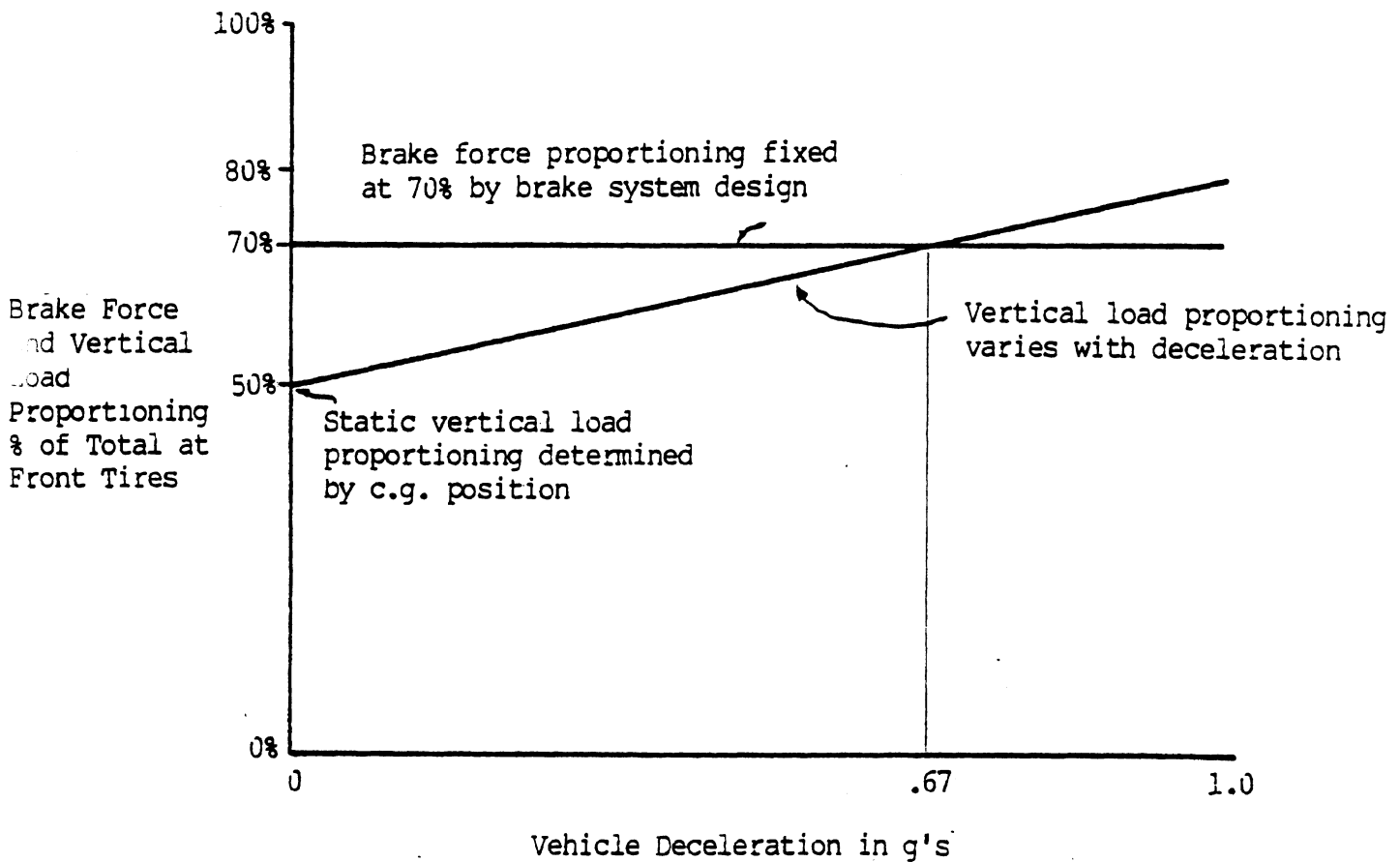
Figure 6.33 Brake Force and Vertical load
proportioning as a function of deceleration

broad range of surfaces when brake proportioning is fixed.)

Many passenger cars being manufactured today have proportioning valves. A variety of proportioning valves are used. Perhaps the most common is the rear brake limiter. With such a valve, at low brake line pressures the valve is not active and proportioning is still constant. At some higher pressure, however, the valve stops the increase of line pressure to the rear brake, maintaining it constant at the cut-off level. Since only the front brake receives higher pressures, front brake proportioning increases. Such a valve can result in a proportioning diagram like that of Figure 6.34. This figure shows the same vehicle as did Figure 6.33 (i.e., the same vertical load

proportioning). Notice that the fixed part (low acceleration) of the front brake proportioning curve has been lowered (through brake design changes) from 70 to 60%. This improves the proportioning match for low friction road surfaces. (μ_p as low as 0.2 implies an ice or hard packed snow surface.) The rear limiting valve has dramatically improved the match over a broad range of higher friction surfaces. Note also, that the brake proportioning curve is generally above the vertical load curve. This implies that front wheels will lock up first (generally more desirable) on almost all surfaces.
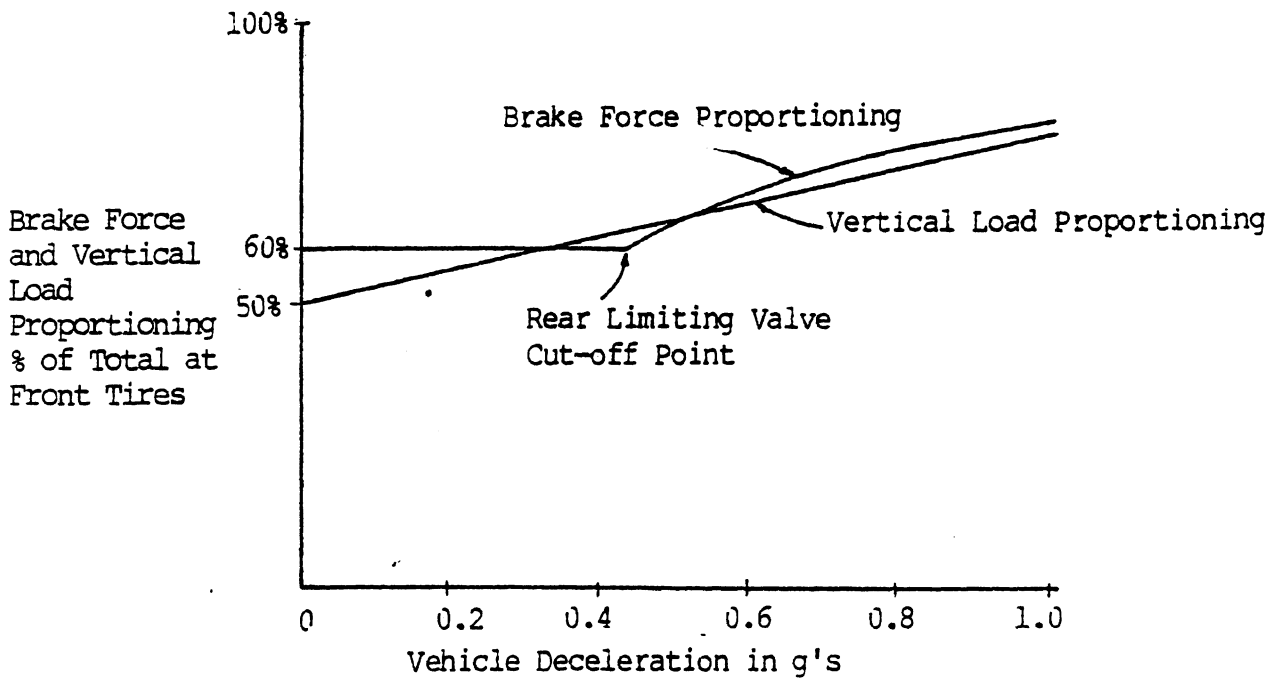
Figure 6.34 Proportioning valve improves the match of Brake force and vertical load proportioning.

Attaining good brake proportioning in practice is a good deal more challenging than implied by the above discussion. For instance, the

proportioning scheme of Figure 6.34 does not deal with changes in static load. If the vehicle of that figure were to take on two rear seat passengers and 200 pounds of luggage placed in the trunk, the "vertical load proportioning" would be shifted downward substantially and the proportioning match would not be so good. (Load changes can be particularly challenging with small passenger cars where changes in passenger and luggage loads represent relatively high percentage changes.) More complicated proportioning valves have been developed to deal with such problems, some of which "sense" tire load by effictively measuring suspension spring deflection.

Another advanced braking system concept is called "antilock." Antilock systems are rather complex and relatively expensive, but they hold very large potential for improving braking performance safety quality. In theory, antilock systems prevent any wheel from locking up by constantly monitoring wheel speed and modulating brake line pressure to that wheel's brake. If the driver applies sufficient brake pedal force to otherwise lockup a wheel, the antilock system senses the impending lockup and reduces the line pressure accordingly, still maintaining near maximum brake force at the tire. In practice, antilock system design is very difficult. A wide variety of problems relating to operating conditions, "false" signals, reliability, and other matters must be solved to obtain a viable antilock system. In theory, however, antilock represents the potential for a "perfect" proportioning scheme with the additional feature of insuring handling stability and controllability.

6.6 Studying the Vehicle Through Simulation

In the preceding discussion, we have conceptually broken the vehicle down into functional models (models of the tire and models separately concerned with turning and braking). Further, we have made these models extremely simple, eliminating many details and cross-influences in order to be able to understand the basic principles involved. More advanced vehicle studies require much more detailed models and models which can simultaneously experience the different areas of vehicle performance.

Vehicle models involving this level of complexity are extremely

difficult to manage "by hand" and to conceptualize in their totality. Fortunately, all the individual elements of the vehicle, of course, behave in accordance to the laws of physics. Thus, in every case, their behavior can be described in equation form, and so (if we are sufficiently clever to accomplish the task), mathematical models of appropriate complexity to deal with the problems for which they are intended can be developed for each of the relevant vehicle elements. Further, all these individual models can be brought together into a single formulation which is a complete vehicle model.

Such complete vehicle models may include thousands of individual equations. Thus, electronic computing capability is literally a requirement for their use. "Computer simulations" are generally formulated so that the vehicle "runs" on the computer in an analogous manner to the way it runs on the road. That is, inputs typically include steering angle and brake actuation levels and outputs are a history, over time, of the vehicle response.

A computer simulation of this type begins as a representation of a generalized vehicle. That is, the equations describing the vehicle include many generalized vehicle "parameters." (For example, in our discussion we have used such generalized parameters as vehicle weight (W), wheelbase (ℓ), tire cornering stiffness (C_α), etc.) In order to use a simulation, the model must be converted to a specific vehicle model by introducing specific numerical data for each of these parameters (that is, W = 3500 lbs, ℓ = 9.5 ft., etc.). This implies that to use a complex vehicle simulation, the user must be satisfied with a "typical vehicle" by using many "typical" parameter values derived from his experience, or a very large effort must be expended on the laboratory measurement of a specific vehicle's parameters if it is necessary to simulate that specific vehicle.

Perhaps the most valuable use of simulation is in "sensitivity" studies, whose purpose is too determine the effect of a given parameter on vehicle performance. In such studies, a typical set of vehicle parameters are used as a baseline condition. Then, in successive simulation "runs" the parameters of interest may be varied over the significant range and their resulting effect on the vehicle response

observed.   Observations made in this way can be extremely helpful in guiding the investigator toward a clear understanding of vehicle performance in what would otherwise be a hopelessly complex situation.
stop

7 TIREMARKS AND ROADWAY EVIDENCE AND ANALYSIS

Summary Tiremarks and roadway evidence are important in determining how vehicles moved into impact, moved about each other during impact, and moved from impact to final rest. While this determination may not always be possible using tiremarks, the potential always exists when they are present. Tiremarks must be searched out and interpreted with care. In law enforcement work they are viewed as the "thumb print of God" and are irrefutable when properly identified and analyzed to determine vehicle movement and speed. Skidmarks are marks left on the roadway by tires which are not rolling freely, or moving at angles on the roadway other than in the plane of the wheel (i.e., sliding sideways). They are indicative of hard braking, wheel lockup from either panic braking or from collision damage, or a combination of sliding and rolling (scuff marks).

The subject of vehicle handling and control, as well as tire interface with the roadway, is treated in greater depth in Chapter 4. However, a few remarks about vehicle braking at this point will help in terms of understanding how to analyze tiremarks. Most passenger cars have hydraulic braking systems for their service brakes and mechanically activated brakes for their parking brakes. The hydraulic service brake system may be either the expanding shoe one-drum type or disc and rotor type. The drum type brake operates by expanding brake shoes against the drums of the wheels by hydraulic cylinders. Disc brakes, primarily used on front wheels, operate by forcing pads attached to calipers against the disc (rotor) as it rotates in the plane of the wheel. The parking brake consists of a linkage of levers and cables to the rear wheels which moves the brake shoes by tensing the mechanical system. Parking brakes were once considered emergency brakes. This concept is now obsolete as they do not have the braking ability of service brakes. However, they can eventually bring the vehicle to a stop when activated.

When brakes are applied the vehicle has a tendency to pitch forward. This results in a transfer of some weight carried by the rear wheels to the front wheels. Auto manufacturers are aware of this characteristic and have designed service brakes so that the system on the front wheels, in terms of the size of their wheel cylinders and shoes, is larger than the rear wheel brake system. Characteristically, 55% of the braking capacity in a passenger car exists on the front wheels with 45% on the rear wheels. Because the vehicle's center of

gravity is above the roadway, and friction forces developed at the wheels during braking are at the roadway (below the level of center of gravity), a forward acting lever arm, or torque, is created to the front wheels about the center of gravity causing the front end of the vehicle to move downward. One might visualize a hypothetical vehicle with a very high center of gravity forced to rotate head over tail when braking is applied abruptly. During panic braking of a truck tractor (without its trailer), the rear wheels will lift off the ground.

When the brakes are applied, the kinetic energy of motion (as a result of the vehicle's mass and velocity) is transformed into heat by friction. The harder the brake shoes are forced against the brake drums, or calipers against the brake discs, the more rapidly the vehicle will slow down. This action continues to a point where the wheels are brought to a complete stop. This is referred to as wheel slip of 1.0, where all the energy of motion or kinetic energy of the vehicle is dissipated between the tires and the road surface in the form of heat. This is the condition of skidding.

Most drivers are not well versed with the effects of skidding, or how to best handle their vehicle when skidding. Skidding rarely takes place in normal day to day driving by typical motorists. It usually results in situations where panic braking occurs to avoid a collision. This may not happen many times in a driver's lifetime of operating motor vehicles. One reason that special emergency driving, or pursuit driving, courses are given to police officers is to give them the opportunity to learn how to handle a vehicle at its limits of steering, handling and braking.

The dipping down of the vehicle front end, or nose diving during braking, incidently is responsible for many of the override/underride crashes. It is also one reason why a standard has never been promulgated specifying the height that bumpers must set above the roadway.

When braking, but not skidding, the vehicle can be steered. Steering is a little more difficult under hard braking but nevertheless can be accomplished. In this situation the tread stock of the tires deforms or flattens out while in contact with the road so as to permit

2 - TIREMARK ANALYSIS

the creation of side thrust forces to the tire. Such side thrust forces are necessary to enable the wheels to turn while simultaneously braking.

During skidding with complete wheel lock-up, the tire-road interface is such that no side forces exist. The driver then loses his ability to steer the vehicle. A skidding vehicle obeys only Newton's laws of motion, i.e., a body continues in its line (vector) of travel until acted upon by an external force. When this condition occurs, the vehicle may slide sideways as easily as it slides frontwards, and no steering is possible. One might then visualize the vehicle as a rectangular box situated on four rubber pads at each corner of the box moving across a relatively non-slippery surface. The turn angle, or orientation, of these fixed rubber pads on the road surface will have no effect on the course of the vehicle or its stopping distance. One exception is that if a vehicle is rotating when it goes into a skid it will continue to rotate after wheels are locked-up and skidding. Many times this will account for the driver's insistence that loss of control of the car occurred while braking, or that the car did not respond to steering commands while trying to evade a crash. Curved skidmarks left in this manner indicate the vehicle was turning or rotating somewhat prior to wheel lock-up, or skidding. Smooth curved skidmarks left on the roadway in this manner indicate that the vehicle was rotating about its center of gravity, while this same center of gravity is moving forward down the roadway.

Small rubber particles, abraided from the tires and left on the roadway surface, combined with the heating up of the pavement surface itself, create the black or dark colored skidmarks discernible to the observer. The rubber tread stock of the tire is much softer than the harder, abrasive surface of the roadway. Hence the rubber is torn off in small particles as it skids along the pavement surface. The appearance of skidmarks is not grossly affected by tire pressure, except when the tire experiences vertical deflection greatly in excess of its deflection at rated load and pressure. Tire load and speed does influence the darkness of the skidmark. Heavier loads at lower speeds produce darker marks. On loose material surfaces, such as gravel and dirt, skidmarks are evidenced by a plowing through the loose material so

as to leave a groove or furrow in the surface equal in width to the tread stock of the tire in contact with the road. These may not be black because little, if any, rubber is abraided off the tire as it slides within this loose material.

Skidmarks in snow, of course, are characteristics of the snow being plowed aside similar to the loose material above, or packed down so as to create a slick glazed surface. Here the sliding tire tends to melt the surface of snow and create an icy, packed-down sliding mark which is somewhat more glazed or frosty in appearance than similar marks in which tires were not sliding while braking.

Skidmarks can be used to determine the speed lost by a vehicle while skidding. They also establish the path of the vehicle while skidding as well as its orientation. Perhaps the most important feature of skidmarks to investigators is that skidmarks are perishable. They are affected by weather, light (in terms of their ability to be observed) and by traffic. They can also be obscured, or made confusing, by movements of emergency vehicles at an accident scene. Skidmarks, and other road surface marks associated with the crash, will eventually be worn away by traffic passing over them. Many tire marks on the roadway are visible because they contain moisture within the abraided tire material. When sunlight dries up this moisture, the marks disappear. Similarly, on road and ice-covered roadways, daylight accompanied by sunshine will tend to melt away skidmarks.

One of the most difficult characteristics of skidmarks to discover is where the skidmark begins. If a tire is sliding so as to leave a mark characteristic of an eraser on a dry surface, the point at which the erasing begins is where tire started sliding. The beginning of the skidmark produces a faint mark relative to the heavier, darker skidmark in the middle of the skid. As the friction between the tire and roadway creates heat between tire rubber and roadway surface, the marks become gradually darker shadowing changing into heavy, solid, dark marks. If the tire has been laying down a treadprint on the pavement, then begins to slide, the point at which the discernable tread pattern of the tire ends and the smearing (marks) start indicates the beginning of the skid. The black smear type skidmark is the most common skidmark and the one

thought of by most people when skidmarks are discussed. A black smear-type skidmark is created when the tire tread black rubber material is heated by friction in sliding, and is smeared across the roadway pavement by the sliding tire in contact with the roadway. The deposited material may be either rubber from the tire, bituminous material consisting of tars in the pavement, or a combination of both. The point at which the black smear begins to be observable is referred to as the point of definite beginning, or the start of the skidmark. It is important to note that the wheel may have slid, or skidded, some distance before sufficient heat was generated to smear the pavement surface and be observable. In tests conducted by HSRI, calibrated test vehicles were skidded at various speeds. It was determined that wheel lock-up actually occurred some distance before the skidmark was actually discernible. Thus the area immediately ahead of the skidmark where a faint shadow of the skidmark can be observed is a part of the total length of the skidmarks. In conducting such tests in a range of 25 to 50 mph on light portland cement surface roadway, it was noted that approximately 10% of the skidmark was not discernible to the eye in this manner. For example, an observable skidmark 91 feet long may actually result from 100 feet of wheel lock-up. This is because it takes the tire some distance after lock-up to generate the necessary heat to make the black smear skidmark which is visible to the observer.

7.1 Tiremarks

7.1.1 Overlapping Marks. One problem area regarding skidmarks that requires interpretation by an experienced observer, is where skidmarks overlap. That is, where skidmarks from the rear wheels are superimposed on skidmarks from the front wheels so that the rear wheel marks are not distinguishable from the front wheel marks. This occurs primarily in straight skids where the tread width, or the lateral distance across the vehicle between the front wheels, is almost the same as the distance between the back wheels. An indication of skidmark overlap is where there is sudden change in the skidmark to a heavier, or more dense deposit of rubber which is the sum of rubber abraided from the back wheels with rubber abraided from the front wheels. The distance between the start of the skidmark and where it becomes darker is equal to the

distance of the wheelbase of the vehicle. It may also be distinguished by a slight off-set in the skidmark, or a slight widening of the skidmark.

It is less difficult to determine the terminal point of skidmarks. Skidmarks usually end when the vehicle making them comes to a stop, when the vehicle leaves the surface of the roadway on which they were imprinted, or when the vehicle's wheels making the skidmarks begin to roll again. When a vehicle has come to a stop by skidding but then is subsequently moved rearward, this can also be noted sometimes from observing the skidmark. Small mounds of mud, snow, dirt, or debris may be piled up ahead of each tire during the skid, since the tire tends to act as a squeegee across the surface of the roadway. This small loose material build up or mound, is characteristic of the termination of the skidmark.

Rear tire skidmarks can be distinguished from front wheel skidmarks because they are usually lighter than front wheel smears. This results from the decrease in weight supported by the rear wheels and transferred to the front wheels when braking (as mentioned earlier). Because rear wheel skidmarks are comparatively lighter than front wheel skidmarks, they often may be observed as thin open stripes in the smear pattern which correlate well with the circumferential grooved tread pattern of the tire.

When observing straight, overlapping skidmarks, care should be taken in their measurement. They do not represent the total length of the vehicle skid, but a continuation of skidding distance by both the front and rear wheels. When the front and rear skidmarks cannot be separated by visual examination the length of the skidding vehicle's wheelbase must be subtracted from total length of the measured combined skidmark of front and rear wheels.

7.1.2 Curved Skidmarks. Curved skidmarks indicate that the vehicle is rotating while skidding. All four vehicle wheel paths can usually be observed from curved skidmarks. Rotation during skidding can result from the driver initiating a turning movement as wheels are locked-up in braking and the vehicle continues in rotation from its turning path just before skidding begins. Curved skidmarks may also indicate unequal

braking force between the various vehicle wheels. If left side wheels are braking with greater force than right side wheels, the vehicle will tend to rotate counterclockwise in the direction offering most resistance to its movement. Badly adjusted brakes can result in this condition where braking on one side of the vehicle differs from the other side, resulting in the same type of curved motion.

An important consideration when interpreting curved skidmarks is the influence of an uneven, or non-level roadway surface. If the roadway is crowned so that it slopes downward away from the center of the roadway for drainage, as most roads do, this lateral roadway slope creates a tendency for the vehicle to move toward the edge of the road because of gravity while skidding. Should only the right front and rear wheels of one side of the vehicle go off on the shoulder, and the left side wheels remain on the roadway, the superior traction of the left wheels on the pavement will retard the vehicle greater than the wheels not on the roadway, creating a force which tends to pull the vehicle back onto the pavement. Similarly, variations in the coefficient of friction, roadway surface or skid factor, of the pavement surface can induce rotation of the vehicle. Skidding, for instance, where one side of the vehicle has its wheels on a vinyl traffic stripe (such as are used for pavement markings in pedestrian crossways, turn arrows, etc.) can result in braking forces being different on various wheels. On the more travelled center area of an asphalt surface traffic lane, for instance, the abrasive material may have been pushed down below the asphalt, making it smoother than on the outer surface. Wheels of a skidding vehicle on such a surface would not develop as much traction as the opposite side wheels in contact with the more abrasive surface next to the road edge. This results in the right side of the vehicle developing more skid friction force, which tends to turn the vehicle to the right.

Curved skidmarks may also be indicative of a half spin. A half spin is rotation of the vehicle 180° from its direction of travel and is caused by having one set of wheels lock-up prior to the remaining set. When rear wheels lock-up first before the front wheels, a condition is created which can cause the vehicle to spin around 180°. This can also

be the case when front wheels start rolling and the rear wheels remain locked-up causing the vehicle to similarly spin around and face the opposite direction.

Contrary to what one might intuitively think would happen, the vehicle is less stable in terms of directional control when rear wheels are locked-up before front wheels in a skid. Rear wheels will then lose the necessary tire side thrust forces first which are essential to directional control of the vehicle. When such side thrust forces are lost in the rear wheels, accompanied by a decrease in the maximum road tire tractive force as a result of skidding in the rear wheels, the vehicle will rotate 180°, or "switch ends". As indicated earlier, skidding wheels do not provide for optimal braking. Thus when rear wheels begin skidding while front wheels are not skidding when braking heavily, there is a reduction of rear wheel braking force compared to the front wheels. Thus the rear wheels will develop reduced braking force and lose lateral side thrust forces. This induces the vehicle to yaw, or to rotate so that the rear wheels will move ahead of the front wheels.

Vehicle steering systems are designed so that the tire area in contact with the roadway is slightly behind the steering axis of the front wheels. The steering axis is that imaginary vertical line which passes through the wheel assembly to the upper and lower central arm. This results in what is called positive caster. Positive caster enhances the directional stability of the vehicle since the imaginary steering axis passes through the road surface ahead of the center line of the wheel. The tire then tends to trail behind just as the caster on one wheel of a swivel chair "trails behind, when the chair is pushed into motion. This centering effect will tend to induce steering angle into a vehicle steering system as it spins around without driver control.

7.1.3 <u>Skip Skidmarks</u>. When skidmarks are not continuous but are intermittent, they may be made by a vehicle bouncing along on the roadway. In this situation the length of the skidmark and the length of the space between skidmarks is characteristically uniform and constant, and less than 3 or 4 feet apart. This condition can result when a
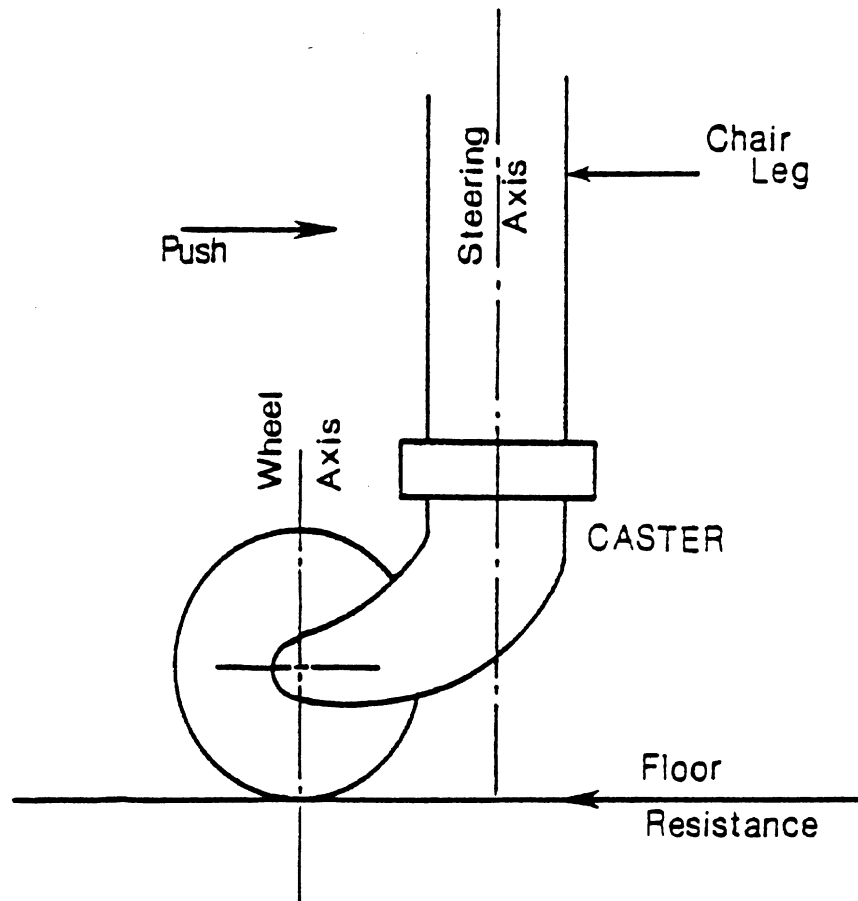
Figure 7.1   Caster Effect

skidding vehicle impacts a pothole, or   bump on the roadway which starts the vehicle bouncing.   The   initial,   part   of   the   skidmark   in such situations will usually be the longest segment of the entire skid path.

Skip skids should be measured for total length so that the gaps are included between the marks on the   pavement.   Consider the skips in such skidmarks as part   of   the   overall   skidmark.   Vehicle   braking is not reduced during the skip   portion   of   the   skid   by virtue of the wheels being off the ground   for   such   short   distances.   While actual braking does not occur during these   short   intervals   when the wheel leaves the ground, heavier braking does occur when   it returns back into the ground to compensate for the missed   distance.   Skip skids should be documented accurately   as   well   as   photographed   for   further   evaluation of the skidmarks when   all   available   information   regarding   the   accident is compiled.

7.1.4   Gaps in Skidmarks Many times skidmarks are observed in which there is a gap between the   termination   of the skidmarks on the roadway and a   reinitiation   of   the   skidmark   some   short   distance   down the

roadway. This will be the result of the driver pumping his brakes. Sometimes the driver may momentarily release his brakes because he believes that the conflict situation ahead, in which a collision appears imminent, has passed, only to re-apply them with even greater force when he realizes that his initial judgement was incorrect. This is a typical situation in collisions with pedestrians or bicyclists, where the slower movement of the person or bicycle can change abruptly from that which the driver anticipates. Unlike skip skids above, such gaps in skidmarks in this situation are not included in the overall measurement of the skidmark. They must be subtracted from the length between the start and stop of the total skid.

7.1.5 Variable Skidmarks. When the surface of the roadway is not continuous and constant, but rather consists of different materials with different friction characteristics, variable skidmarks may occur. This could be the situation, for example, when a vehicle is moving from a concrete surface, such as Portland cement, to a loose material surface such as gravel. Skidmarks associated with each surface represent different friction forces generated between the tires and roadway to slow down the vehicle. Here the coefficient of friction of the two types of roadway surface may be much different. As one would expect, each length of skidmark associated with each pavement surface should be measured individually, and not considered as an overall continuous skid on the same type of surface. This is important when attempting to sum up the potential speed loss indicated by the skidmarks for each pavement surface.

7.1.6 Acceleration Marks. Often, tire marks which appear as skidmarks are really acceleration marks. This is when a vehicle is accelerated rapidly from a stopped position, or when moving at slow speeds, so as to make heavy, dark tiremarks. These are acceleration marks and closely approximate skidmarks in appearance. They begin as heavy dark marks and slowly dissipate in intensity as the rotational velocity of the wheel begins to approach the linear velocity of the vehicle.

One characteristic often observed in acceleration marks that differentiate them from skidmarks, is their linearity. When rapidly

accelerating a vehicle under forward maximum acceleration, some steering may be necessary to keep the vehicle in a straight path. This is because the torque from each wheel, which is translated into linear motion to the vehicle, may not be equal at each rear wheel because of road-tire interface differences[1]. This small difference must then be corrected by the driver in steering. This results in a discernably curved, or "wavy", characteristic in the acceleration mark. The gradual decrease in intensity of the acceleration mark as the vehicle attains speed which reduces slip to the wheel with respect to the roadway, is the other important characteristic of an acceleration mark.

7.1.7 Tread Pattern Marks & Striations. Tire tread patterns (or lack thereof) are helpful in establishing driver actions relative to braking, steering, accelerating, or combinations of these, when they are observed in skidmarks. Tire patterns may consist of actual tread imprints of the tread design as if they were imprinted by a rubber stamp, tire rib marks, tread edge marks, and/or striation marks. In a recent study by HSRI[2], tire tread patterns, when observable in tiremarks, were examined so as to determine what information could be extracted from such pattern marks that would be helpful in determining driver actions and vehicle movement in a crash.

It was established that tread imprints (i.e., where an image of the tire tread is deposited) occur in straight-line braking and under longitudinal slip[3] conditions of between 5 and 20%. That is, it occurs in straight line braking where the wheels are retarded between 5-20% in angular velocity. Tread imprints also most probably result from skidmarks produced by front tires. This entire condition of tread prints in skidmarks is relatively rare since passenger car tire tread

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[1]For front wheel drive vehicles, this would then be true of the front drive wheels.

[2]"A Methodology for Determining the Role of Vehicle Handling in Accident Causation," D.F. Dunlap, L. Segel, F.L. Preston, P. Cooley, and B.C. Brown, February 1977, UM-HSRI-76-12-2.

[3]Slip here indicates the percentage difference in velocity between the wheel being braked and the velocity of the vehicle. One hundred percent longitudinal slip represents locked-wheel braking, while braking at less than 100% indicates the braked wheel is still partially rolling.

imprints are not easily discernible unless a tire loading condition exists which is equivalent to at least a 40% weight transfer to the front tires. Wheel lockup is certain to occur for any slip condition greater than 20% because of the brake force-slip ratio characteristic of the pneumatic tire.

Tire tread rib marks occur in straight skids. Tread edge marks are deposited by front tires when their loading is increased because vehicle weight is transferred to the front wheels as a result of braking. Striation marks appear as intermittent skidmarks, two to three inches in width, spaced at two to four inches apart on the pavement.

Striation marks are present in skidmarks when the tires are adjusted to a lateral slip condition; that is, when the vehicle is turning, either on a curved portion of the roadway or at a corner. As indicated earlier, these striation marks are at the same angle with the direction of the skidmark and result from both the lateral and longitudinal slip angle of the tire with respect to the direction of vehicle movement. This angle is independent of the speed of the vehicle. The width and spacing of striation marks is influenced by the interior tread pattern of the tire and by the edge pattern along the side of the tread. Even so, tires such as the ASTM[4] tire, with a simple circumferentially ribbed tread, and a featureless edge pattern, exhibit the same intermittent striation marks as do fully-treaded tires.

A discussion of characteristic striation mark angler relative to the direction of skidmarks is given in Section 7.3.

7.1.8 Point of Impact. An important characteristic in skidmarks which is indicative of the point of impact, is an abrupt bent offset to the parallel line skidmarks. This results from a skidding vehicle impacting another vehicle or fixed object so that its skidding path is altered by the impact. The vehicle is redirected from its path of travel up to the point of impact onto a new path towards its final rest. When skidding vehicles collide with pedestrians, cyclists, or objects whose mass is small relative to the vehicle striking them, observing such offsets in the skidmarks may be difficult. Where discernible, it

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[4]American Standards Testing Material

could be the most accurate indication of the point of impact available from the physical evidence. When measuring the total skidmark in which there is an offset, or abrupt change in direction, as a result of impact, a separate measurement should be made of the offset portion as well as the portion leading up to the offset. Each should be noted and recorded accurately. Measurement of the angle created by the offset is most helpful in establishing the redirected path of the vehicle from its path prior to impact.

7.1.9 <u>Tire Tread Imprints.</u> Imprints of the tire tread stock pattern on the roadway surface indicate that the wheel was rolling and not skidding. The mechanism here is much like an ink stamp in which a pattern in rubber is imprinted on a flat surface without smearing. The imprint may be the result of loose matter picked up by the tire as it rolled on the roadway, leaving its distinctive tread stock pattern, or by wiping aside wet or loose substances on a paved highway surface. It could also be the result of making an impression in the soft material such as sand, dirt, mud, snow or similar soft earth substances. The primary value of such information is to indicate the position path of the vehicle on the road surface, or off road area, and to indicate whether vehicle wheels while making these imprints were rolling rather than skidding. It may also indicate that a vehicle had failed to stop at a stop sign because brakes were not applied, or were not working for some reason.

Tire imprints are distinct from skidmarks in that they convey the tire tread pattern of the tire without any of the slick or smoothly worn features characteristic of skidmarks. In addition, the imprint pattern is uniform in contrast and noticeably similar to other imprint marks left by tires on other wheels of the same vehicle. They should be characteristic of the tread pattern on wheels and easily correlated with the wheels which left the imprint. Skidmarks are darker and more prominent at their beginning and end. Imprints will be continuous for a given surface with no beginning and end.

If the tire rolls through a surface covered with viscous substances such as oil, it will tend to lift little points of the sticky matter as it passes through it leaving a print characteristic known as a "stippled

pattern". One other noticeable characteristic of tire imprint marks is small bits of mud, or similar wet loose material, which clings to the side of the tire as it rolls. This material will tend to splatter, or squish out to the side of the tire as the tire comes more firmly in contact with the surface upon which it is rolling. Such splatter squish marks are made by forcing the material out from the tire tread pattern so as to be deposited adjacent to the tire tread imprint on the ground.

Evidence of steering may also be noted in tire imprints as a vehicle is turned. As indicated previously, a skidding vehicle denies the driver the ability to steer the vehicle. Where steering is possible through hard braking rather than skidding, the sliding wheels will show scuff marks rather than tread patterns indicative of tire imprints.

7.1.10 Scuff Marks. Scuff marks are marks left on the roadway surface by tires that are both sliding as well as rolling. The wheel may be rolling and slipping sideways at the same time. Scuff marks represent a combination of both tire imprints and skidmarks. They may result from lateral movement to wheels from a collision. They may also indicate that the vehicle sustained a blow-out or became deflated in the crash.

When a vehicle "spins out", or "slips out" while cornering, or is oriented in a direction different from its direction of travel, scuff marks may be deposited. Often they are in the form of light parallel grooves, referred to as striations, which run straight but are diagonal to the outline of the continuous scuff mark. This pattern may be similar to that of the milled edge of a coin. In tests conducted at HSRI with a mobile tire tester, it was established that the angle that such striations make with the path of the scuff mark indicates the yaw angle the vehicle makes with its direction of travel. Measuring the angle between the striation marks across the scuff pattern, with a line through the center of scuff pattern, will then indicate the angle at which the vehicle had yawed while the scuff marks were being deposited.

Perhaps the most important information one can obtain from scuff marks is when a vehicle is taking a curve at critical cornering speed. Critical cornering speed is the speed at which the vehicle is at the threshold of spinning out or slipping laterally, when in a curve. Tire

scuffs which occur are critical-speed scuff marks made by tires sliding a little as the vehicle traverses the curve, and are made by the outside edges of tires. The scuff mark left by the rear tire will fall outside the scuff mark made by front tire for that side of the vehicle tending to slip off the roadway as a result of centrifugal force.

Calculation of critical speed from cornering, or traversing a curve, is based on the centrifugal force acting on the vehicle because of the circular path it follows. This force is countered by lateral friction forces developed by the tires which keep the vehicle from sliding tangentially off the roadway. Such critical curve scuff marks are also called centripetal curve scuffs and represent the equal and opposite force to the centrifugal force acting on the vehicle to keep it on the roadway.

The centrifugal force acting on the vehicle is equal to the mass (m) of the vehicle times the square of its velocity (V^2) divided by the radius of the curve (r) over which it is waving:

$$F_{cf} = m \; V^2/r$$

The centripetal force necessary to counter centrifugal force and keep the vehicle on the roadway is a product of its mass (m) and acceleration (a):

$$F_{cp} = ma$$

By equating the two we can calculate the acceleration acting on the vehicle as:

$$a = V^2/r$$

The lateral forces which keep the vehicle from spinning out result from friction forces developed laterally to the tires. This friction force at the tires is the product of the roadway coefficient of friction (μ) and weight (W) of the vehicle:

$$F_f = (\mu) \; W$$

Substituting W = mg, and combining with the expression above gives:

$$(\mu)mg = mV^2/r$$

Which solved for velocity (V) becomes:

$$V = \sqrt{(\mu)gr}$$

Adjusting this expression for correct units for mass so that the radius (r) is in feet and the velocity (V) in mph alters the expression to:

$$V = \sqrt{15\mu r}$$

This is the equation which nomographs and slide rule type devices are based on. They can be used to calculate critical speed about a curve of radius (r) and roadway coefficient of friction (μ). For example, a vehicle travelling on a curved roadway whose radius is 500 feet (with roadway level), and whose average, dynamic coefficient of friction (μ) is 0.5 can achieve a speed of 61 mph before the vehicle begins to slide laterally off the roadway.

It is important to understand that this relationship can be used where critical speed scuff marks are noted and measured carefully. Where the roadway is banked, or superelevated, the safe speed which a vehicle can move around a curve is increased because the car is slightly tipped so as to have a tendency to slide toward the center of the curve. The angle which the roadway surface makes with the horizon contributes a component of the normal force of the vehicle on the roadway which tends to counteract the centrifugal force developed. This results in the expression being modified so that:

$$V = \sqrt{15r(\mu + e)}$$

Here the only difference with the previous expression is the inclusion of the superelevation term, "e". This is the tangent of the angle that the roadway makes with the horizon, and is the superelevation of the roadway curve. For roadways which slope toward the inside of the curve, the superelevation is positive. A negative superelevation exists when the slope is such that the roadway is highest on the inside of the curve. The latter, of course, is not a common occurrence on highways.

7.1.11 Scratches. Scratches on the roadway must be positively associated with the crash to be helpful. They are caused when stiff parts of the vehicle (usually melted), are pressed into or scraped along

the paved surface of the roadway. Where scratches are fresh and the result of rigid, metal parts scraping along the roadway surface, they tend to sparkle and be bright in the concrete and unmistakenly fresh in asphalt. Scratches are helpful in indicating the direction of movement of the vehicle from impact. When correlated with the parts of the vehicle in contact with the roadway which resulted in the scratches, they can also confirm the position of the vehicle on the roadway at impact.

Scratches resulting from a rollover may indicate where the vehicle initiated its rollover movement. A vehicle sliding along the pavement on its side, or top, usually leaves distinctive scratches made by its trim, bumpers, sharp sheetmetal edges, sheetmetal panels, door handles or other protruding parts. Scratches may sometimes have paint imbedded in them which can be observed with a magnifier. There are often corresponding marks in the object that covered them. Paint may come from most any exposed painted surface of the vehicle and is rubbed off and attached to the abrasive texture of the pavement surface. In violent rearend collisions, the sheet metal modesty trim panel across the rear of the vehicle, below the rear bumper, is often abruptly pressed down into the pavement, leaving scratches imbedded with the paint of the struck vehicle, and outlining the rear contour of the vehicle.

Scratches may also indicate the direction of impact as well as the relative force of the impact. This is true when scratches can be matched up with the undercarriage portion of the vehicle (engine, frame, wheel suspension, transmission, differential, exhaust system, frame rails, etc.). There are also some instances in which scratches on pavement prior to impact are indicative of some component failure on the vehicle. An example of this is when scratches occur from the rim of the wheel which sustained a flat tire, or blowout, before the crash. This situation may be confirmed by the distinctive pattern of a wobbly flat tire which is scrubbed, abraided and deformed as it is pressed between the rim and the pavement, and into the pavement, as it moves across the roadway.

Microscopic examination of scratches can sometimes tell the

direction in which they were made. This would be evident by the grooving, or pocketing of the component brought into contact on the pavement. A 10 power jeweler's or other magnifier is helpful in closely examining the characteristics of such scratches. Photographing scratches in their entirety as well as close-up views with a macro lens will permit the investigator to record such information for later study. Soft white metal trim from vehicles, when brought into contact with the pavement, will evidence small buildups along the road surface. Surface irregularities or roughness are also called apperities. On close-up examination, they can indicate the direction of the scratches.

7.1.12 Gouges. Gouges may be distinguished from scratches in that they are much deeper and wider, and tend to furrow, chip, or chop chunks of road surface material instead of sliding across more lightly. Examination of the undercarriage of the vehicle involved in the crash can indicate undercarriage corners, edges, rods, etc., in the structure (and its components) which may have gouged the pavement. These components will evidence a bright abrasive scratched type appearance sometimes containing powdery white stone material ground away from the pavement. They are characteristic in severe head-on collisions where the front end of one, or both vehicles are driven down with great force into the roadway as the rear ends of the vehicles come up off the ground. Gouges deposited in such crashes can be helpful in accurately establishing the point of impact in a crash.

When a colliding vehicle is found upright, with all tires inflated, and the undercarriage totally off the ground, with no evidence of rollover, it is doubtful that it was responsible for any gouges on the roadway. Like scratches, gouges may indicate a mechanical failure in one of the crash vehicles. The gouge may be caused by a separation of a steering linkage connection, connecting rod or other such undercarriage components, which contacted the ground causing a gouge.

Three typical types of gouges are grooved gouges, chip gouges, and chop gouges. The groove gouge is made by objects like bolts or other similar narrow diameter undercarriage parts as they are brought down hard into the surface of the pavement and scoop parts of it out as the vehicle moves so as to make a furrow or groove. If the groove is curved

or has a nonlinear pattern it could indicate how the vehicle moved about the other during impact. Chipped gouges result when hard vehicle parts strike the paving material with great force and chip or gouge out parts of the material. This may be accompanied by stones or other aggregate of the pavement being loosened or separated. Chopped gouges are similar to chipped gouges and result when broad rather than narrow sharp edges of vehicle parts such as cross frame members, transmission casing, or similar larger heavy structural components strike the pavement while moving. Chop gouges are usually followed by a broad rubbing scratch in the direction of vehicle movement showing that the part of the vehicle which caused the gouges remained in contact with the roadway.

7.1.13 <u>Ruts</u>. As the name implies, ruts are rolling wheel tread paths made in soft earth materials such as mud, wet dirt, clay, or snow. They indicate the direction and orientation of the vehicle after it has left the roadway, the final position of the vehicle off the highway, or the point of impact with off-road objects. They provide a history of movement of the car as it moves off-road to its position of final rest. When the vehicle is pushed laterally as a result of colliding with a fixed object, off-road ruts may also be formed.

7.1.14 <u>Furrows</u>. Furrows are similar to ruts. They result from a vehicle moving over soft earth which mounds up at the edges of the tires. The major difference is that furrows result from sliding wheels rather than rolling wheels, and are often a continuation of skidmarks from the pavement to the off-road surface of loose materials such as gravel or dirt. Like ruts, they indicate the path of the vehicle and its orientation while moving to final rest. These should be treated like skidmarks with the appropriate estimate of coefficient of friction characteristics of the material, and should be considered as part of the overall skid pattern of the vehicle when computing speed loss from skidding.

7.1.15 <u>Holes and Mounds</u>. Where the earth has been scooped up so as to provide a hole or gouge, with earth mounded up nearby, it may indicate the ends of ruts, furrows, skidmarks, scuffs, or gouges, made when vehicle wheels, or vehicle parts, move and scoop out broad areas in the earth. They may indicate a pivotal point at which the vehicle

started to spin, or a point at which the vehicle initiated a rollover. These should not be confused with gouged out portions of the earth resulting from the vehicle's undercarriage coming into violent contact with the earth (a result of its suspension being fully compressed and then rising so as to be fully extended). This is referred to as "bottoming out". Gouges and mounds of earth can be correlated with other physical evidence associated with rollover, or the vehicle moving over irregular terrain out of control.

7.1.16 Debris. Debris is not necessarily related to surface effects made by the vehicle directly in contact with the surface over which it is moving. However, inasmuch as debris associated with a crash is physical evidence similar in importance to skidmarks, gouges, scratches, etc., it should be considered with the same careful attention. Debris are large and small vehicle parts or pieces separated from the vehicle as well as undercarriage dirt and mud which drops onto the pavement, rust from vehicle metals which is shaken loose, paint, various vehicle fluids (radiator coolant, crank case oil, brake fluid, power brake fluid, automatic transmission fluid, battery acid, differential lubricate, etc.) and similar materials. Debris may also consist of portions of the roadway, whether it be permanent pavement or loose materials such as gravel, which is scooped up and spread in the crash area. In severe injury producing crashes, debris may include body fluids and matter, as well as portions of human bodies. Blood is perhaps the most common here, but may also be accompanied by other human matter. Pools of blood can show where an injured person lay or crawled to or was moved to after the accident.

Material carried with the vehicle, and within its passenger compartment, trunk or cargo area, is often dispersed about the crash site. This is an important consideration when attempting to interpret debris attached to or associated with only one crashed vehicle. Fluids are of particular significance. When a fluid container, such as a vehicle's crank case, is ruptured, crank case oil is dispersed forward with velocity. Such fluids were moving at the same speed as the vehicle and continue to move at that speed as they are released by the crash which causes the rupture. This results in fluid patterns which can help

determine vehicle movement at crash, and separation from the point of crash.

Fluids deposited may be "splattered" onto the ground, indicating contact at high velocity. They may also just pool onto the ground or pavement indicating that the bulk of the fluid was released after the vehicle came to rest. Fluid run-off is distinctive in that it follows the sloped grade of the pavement or terrain.

Dry undercarriage dirt and mud that falls from impact is similarly helpful. This material will tend to continue forward but be trapped by undercarriage components or sheetmetal edges such as wheel housings. It then falls to the ground and forms a distinctive pattern. Such patterns are helpful in determining point of impact and orientation of the vehicle at impact.

Debris in the form of fractured parts which separate from the vehicle at crash are deposited in a pattern consistent with the dynamics of the crash. This can be quite helpful in understanding basic vehicle directions and orientations at crash. Such parts, or automotive debris, because of their relatively light mass and great initial velocity, can travel great distances from the impact area. In head-on collisions, for example, parts from each vehicle can be propelled forward a considerable distance ahead of where the vehicles collided. In corner to corner head-on orientation crashes, such debris will move forward a distance in the direction in which the vehicles were travelling prior to the impact. Where the vehicles rotate away from each other as a result of being impacted at corners, and come to rest facing in opposite directions, debris locations will tend to confirm this rotation away from impact. Oftentimes debris from one vehicle will be carried onto the other vehicle. This may create some confusion.

A crashed vehicle at rest can also act as a canopy, shielding the roadway from falling debris. Debris patterns on the roadway will tend to outline the position and orientation of the vehicle which caught the debris and kept it from falling onto the roadway. This may outline the side of one vehicle with vivid contrast from the remaining roadway.

7.2 Speed Estimates From Skidmarks

Skidmarks are most important in relative importance among all physical evidence potentially available at a crash site. Skidmarks cannot only assist in establishing the precrash path and orientation of the vehicle, its point of impact, and its postcrash travel and orientation, but it can be of great help in estimating vehicle speed prior to skidding. Identifying skidmarks at a crash site is fundamental to reconstructing the crash.

Estimating speed from skidmarks is one area that has perhaps been written about most, and is of primary concern to accident investigators where punitive considerations in crashes must be considered. Since skidmarks are solid physical evidence of the crash, they can be used to estimate the speed of a vehicle at impact as well as its speed prior to skidding. By equating the relationship of kinetic energy to that of friction energy, one can arrive at a relationship in which speed is equated to the distance the vehicle skidded and the energy given up in the form of heat between tires and the surface of the roadway.

Here KE is:

$$KE = m V^2/2$$

Where (m) is the mass of the vehicle, and (V) its velocity.

Heat energy U_f is the distance (d) over which the forces, (F_f) of the skidding vehicle acted.

$$U_f = F_f d$$

Since force applied over a distance is what is produced we can equate these two expressions.

$$mV^2/2 = F_f d$$

The friction forces developed in skidding follow Couloms's law of friction. This law states that the ratio of the friction forces (F_f) and the normal force (N) for a body being slid across a surface is constant. This ratio defines the coefficient of friction,[5] (mg).

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[5]This is not precisely true since friction is more of a dynamic condition than a constant. It also varies between dynamic coefficient

$$\mu = F_f/N$$

Where the normal force is but the mass (m) of the vehicle, this becomes

$$\mu = F_f/m \quad \text{or} \quad F_f = \mu m$$

Substituting in the expression above:

$$mV^2/2 = F_f d$$

Gives:

$$mV^2/2 = \mu md$$

When solved for velocity (V) this becomes:

$$V^2 = \mu d/2$$

When adjusted for convenient terms of distance (d) in feet, and velocity (V) and miles per hour, with the inclusion of the gravitational constant to make the expression dimensionally correct, this expression becomes:

$$V = \sqrt{30 \, \mu d}$$

This is a common expression for deriving speed loss from skidmarks. Of course, all four wheels of the vehicle must be skidding, the ground must be level and must consist of the same surface material and texture (hence the same coefficient of friction). Where roadway grades are greater than 4% or the vehicle moves on dissimilar surfaces, or braking efficiency must be considered, the expression must be modified accordingly. Such formulas and derivation can be found in many texts dealing with accident reconstruction.

7.3 Striation Mark Angles with Skidmark Direction

The angle that a striation mark makes with the direction of a skid mark is dependent upon the prevailing conditions of longitudinal and lateral slip of the wheel being braked. This observed dependence is consistent with the recognition that the motion of the tire tread across

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of friction (friction of an object in motion) and static coefficient of friction (object at rest and put into motion). Pavement moisture, tire inflation, vehicle speed, ambience, road surface differences, and other considerations all affect the slipperiness of the pavement which we conveniently note as the surface coefficient of friction and keep as a constant in this relationship.

the pavement surface is the result of (1) the translational velocity of the wheel and (2) the rotational velocity of the tread about the spin axis of the wheel. A kinematic description of the motion is depicted in Figure 7.2 where:

$$\text{Tan } \theta = \frac{2}{(1-S_x) \text{ Sin } 2\alpha} - \text{Ctn } \alpha$$
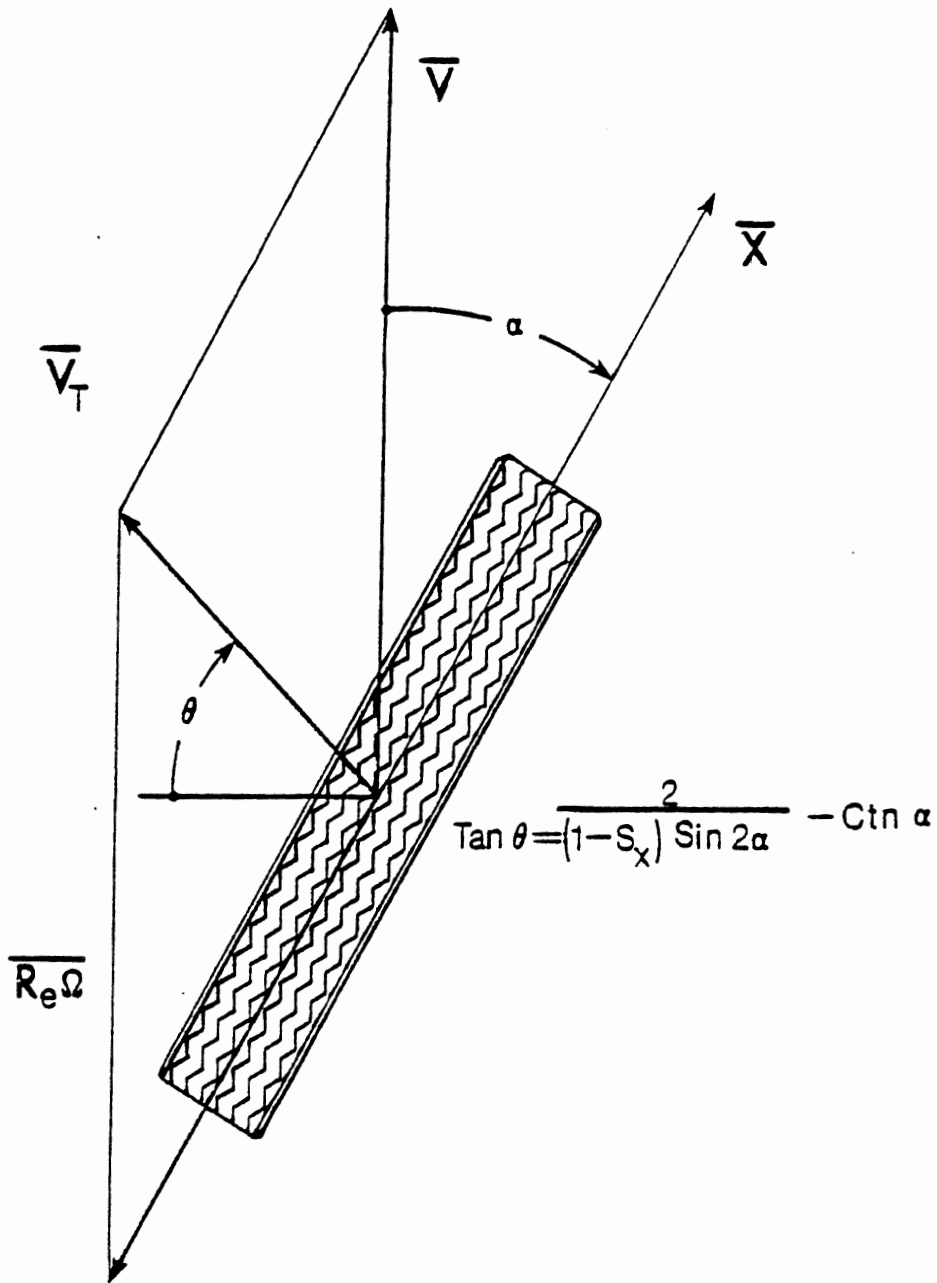
Figure 7.2 Tire Kinematic Motion

V = translational velocity of the wheel hub

x = a horizontal axis located in the centerplane of the wheel passing through the wheel hub (or axle)

α = lateral slip angle or steer angle of the tire with respect to the direction of translational motion

R_e = effective rolling radius of the tire

Ω = angular velocity of the tire

V_t = velocity of tread element along the road surface

θ = angle of a striation mark (or tread element velocity vector) with respect to a line which is perpendicular to a tangent to the skid mark.

From Figure 7.2 it can be shown that

$$V_t \cos \theta = R_e \Omega \sin \alpha \qquad (7.1)$$
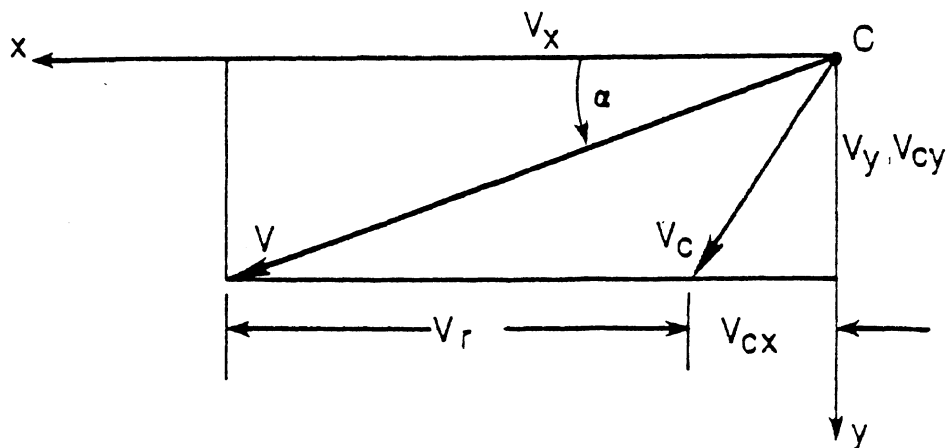
Figure 7.3 Tire Kinematic Conventions

$$V_t \sin \theta = V - R_e \Omega \cos \alpha \qquad (7.2)$$

On dividing, the following expression for the angle, θ, is obtained:

$$\tan \theta = V/R_e \, \Omega \sin \alpha - \text{ctn} \, \alpha \tag{7.3}$$

Examination of the kinematics of tire motion with braking applied (see Figure 7.3) shows that

$$R_e \Omega = V(1-s_x) \cos \alpha \tag{7.4}$$

where

$$s_x = V_{cx}/V_x \tag{7.5}$$

$\quad$ = longitudinal slip parameter

V_{cx} = longitudinal slip velocity

$$V_x = V \cos \alpha \tag{7.6}$$

Substituting and rearranging yields:

$$\tan \theta = 2/(1-s_x) \sin 2\alpha - \text{ctn} \, \alpha \tag{7.7}$$

This latter equation is the desired expression for the striation angle as a function of longitudinal and lateral slip. It may be noted that if $s_x - 0$, then $\alpha = \theta$ and the striation angle is equal to the slip angle. Similarly, if $s_x = 1$, $\theta = 90^\circ$ and the striation marks become collinear with the direction of the skid mark. The independence of the striation angle with velocity is clearly evident.

At the scene of an accident, the only information available to an investigator would be the angle of the striation marks and the direction of the skidmark. It is clear that knowledge of the striation mark angle is not sufficient to uniquely determine values for longitudinal and lateral slip. There are, however, some deductions that can be made at the accident scene from this type of evidence, viz.:

1. If the angle of striation marks are between 0° and 90°, <u>some steering action</u> is taking place.
2. If the striation marks are parallel to the direction of the skid mark, then <u>only braking action</u> is taking place.
3. If the striation angle is greater than 30° but less than 90° then a combination of steering and braking is occurring. (The maximum front tire steer angle for a typical passenger car is no more than 30°. Any striation mark angle greater than 30°, then, must result from a combination of steering and braking. Note that $\theta = \alpha$ when $s_x = 0$.)
4. If the striation angle is less than 90°, then the longitudinal

slip ratio, s_x, is not likely to be greater than 20%. (As noted earlier, incipient wheel lockup generally occurs for slip ratios greater than 10 to 20% which leads immediately to values of B of 90°.)

8 AN OVERVIEW OF THE OPERATION
OF CRASH2

Summary. A computer program is no more than a highly
organized set of instructions. CRASH2 is such a program.
After a brief introduction to computers, an overview of the
organization of CRASH2 is presented. Two basic types of
analysis may be performed: damage-based computations and
trajectory-based computations. Computer programs are
organized into sub-programs. These sub-programs may, in turn,
call other sub-programs. From the overall organization of the
CRASH2 program, one finds that the major functions are found
in the sub-programs named QUIZ, START2, and DAMAGE, along with
their supporting sub-programs. Most of the material in this
chapter focuses on the operation of these sub-programs.

The computer is a high speed calculating and data-manipulating
machine. Its speed allows us to perform complex and tedious tasks
quickly and efficiently. It has capabilities available in many common
scientific calculators, but also provides speed, useful devices such as
terminals, printers, and plotters, and memory to retain enormous amounts
of data indefinitely. The computer can perform time-consuming error and
consistency checks on the data in almost zero time compared with hand
checking. The computer is instructed in a method similar to our own
step-wise method of solving a problem. To allow a person to use the
computer without knowing the complex electronics of the machine,
programming languages such as FORTRAN were developed. The programming
language defines the set of human-understandable instructions which are
used to tell the machine how to solve a problem.

Basically the programs that the computer uses perform three
functions. It must get the data and instructions from the human and
retain them in its memory. It performs calculations on the data and
puts its results into memory. The program then displays the results of
the calculations to the human. These concepts are explained using an
analogy to the human thought process.

Suppose you are stopped on the street and asked the directions to
Maude's Restaurant? This request for information is referred to as
input in a computer program. If the question had been for directions to
Steve's Lunch, you would have dismissed the request as ridiculous, and
asked the passer-by to repeat the question. After an acceptable input
is received, you would begin to think about the location of the

restaurant and the best route for geting from your current location to the location of the restaurant. Some portions of your memory store the locations of all the places you are familiar with and a semblance of a street map of the locale. Another portion of your mind "executes" the computations to determine an acceptable route to follow. A computer is organized in a similar fashion. In this example, input information, knowledge of the area, and deductive capabilities were used to generate the requested information which was ultimately "outputted" to the "user."

Of course the computer must always follow the logic programmed into it. No creative thought is possible. The machine is also controlling mechanical devices, not a human body. The general concepts of input, computation, and output are similar to our own actions and thoughts.

Computer programs are divided into subprograms which perform narticular parts of the entire computation. To use the computer to solve a problem, we derive models for the problem. These models are stated in mathematical terms and finally in terms of data available about the problem. In accident reconstruction, the scene data is the input to a program which uses mathematical models to calculate the required information (such as change-in-velocity). The resulting data are displayed to the user. In solving the problem, the results are only as reliable as the available data, the accuracy of the model, and the accuracy of the program used to automate the model. The computer only does the tedious work given by the program. Occasionally the machine malfunctions, but generally it dumbly performs its task exactly as directed by the program. As a result, the human is responsible for the accuracy of the results!

8.1 Introduction to CRASH2

The CRASH2 program is organized into specialized subsections. Each subsection carries out a narrow range of functions under the control of a single section commonly called the MAIN program. An organization chart is illustrated in Figure 8.1.

The CRASH2 program may be used to perform two basic types of analysis: a damage-based computation and a trajectory-based computation.
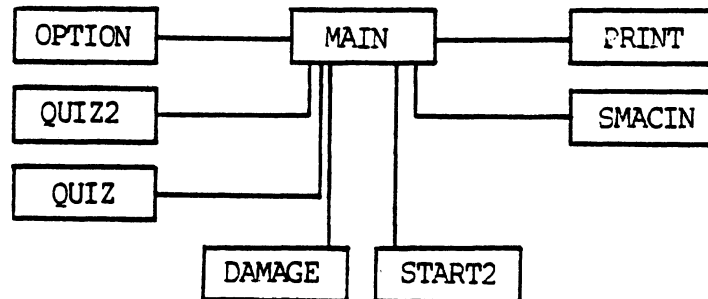
Figure 8.1 CRASH2 Organization

The damage computation is always done. Input information on the damage to the vehicle and various vehicle parameters are used to compute the velocity change of the vehicle during the impact phase. Pre- and post-impact speeds cannot be estimated using the damage-only analysis.

The trajectory analysis is based on information on the rest and impact positions of the vehicles. Different computations are performed for different post-impact motions of the vehicle. The available trajectory types are:

1) roll-out along a straight path

2) a non-rotating angular skid along a straight or curved path

3) a spinning (rotating) skid along a straight or curved path followed, optionally, by roll-out along a straight path.

If the trajectory simulation is answered "yes", an iterative trajectory computation is performed.

The trajectory computation produces estimates of the post-impact velocity of each vehicle. This information is used in a momentum balance to estimate the change in velocity during the impact phase and the pre-impact velocity without using the damage data. This computation is labeled "spinout trajectories and conservation of linear momentum." A single exception is the colinear collision where the damage-based estimate of the velocity change during impact is combined with

trajectory-based estimates of post-impact velocities to produce estimates of the pre-impact velocities. This is the result labeled "spinout trajectories and damage." With this introduction, an overview of the program organization follows.

The MAIN program always starts a session by communicating with the user through OPTION. This subsection presents the options (complete, abbreviated, rerun, batch, print, smac-cards, and end) to the user and processes the reply. The reply is a typed word on the terminal, the processing may inform the user that the option chosen was not appropriate, but when an appropriate response from the user is recognized, a variable, MENU is given an integer value and is stored in MAIN. The data stored in memories organized by MAIN are generally available to the other subsections. In this case, MAIN uses the integer value of MENU to select which subsection to turn to for the next operations. The subsections are, in effect, numbered. At the starting of a session, MAIN will pass control to the sub-section QUIZ after receiving a return value from option.

The subsections are generally organized into sub-subsections. This is the case of QUIZ. Broadly, the purpose of QUIZ is to put a logical set of prompting type questions before the user and to treat the replies. The replies to the questions of QUIZ fall into five (5) categories and QUIZ controls a set of sub-subsections designed to treat the five classes of reply, as shown in Figure 8.2. Section 8.2 will examine the detail of QUIZ itself along with its sub-subsections. At this time we merely observe that categories of questions posed by QUIZ to the user require five specialized treatments of the user's reply. The user may reply with a size alpha symbol, a simple yes/no, a rotation direction alpha symbol, a set of numeric values or an alpha-numeric CDC code. QUIZ "calls upon" the assistance of one of its sub-subsections to read and process the user's reply.

When the operation of QUIZ is concluded, control of the program is returned to MAIN. If in answering the questions presented from QUIZ, the user has answered Question 8 with a "yes," saying that both rest and impact positions are known, a flag IND is set equal to zero. IND is stored in a location created for it by MAIN and is used by MAIN to

decide between calling upon the subsection START2 or the subsection
DAMAGE. Since IND=0, in our example above, MAIN next calls START2 into
operation. Subsection START2 manages a collection of sub-subsections as
illustrated in Figure 8.3. Section 8.4 will examine the operations of
START2 and its management of the sub-subsections responsible to it. For
now let us merely remark that two of the three principal categories of
results from CRASH2 flow back through START2 to MAIN for storage. these
are: 1) the result labeled "SPINOUT TRAJECTORIES AND CONSERVATION OF
LINEAR MOMENTUM" and 2) the result labeled "SPINOUT TRAJECTORIES AND
DAMAGE."

```
                        ┌──────┐
                        │ QUIZ │
                        └──────┘
   ┌───────┬────────┬───────┬───────┬────────┐
┌──────┐ ┌──────┐ ┌──────┐ ┌──────┐ ┌──────┐
│VEHSIZ│ │YESNO │ │ROTAT │ │READ1 │ │VDICHK│
└──────┘ └──────┘ └──────┘ └──────┘ └──────┘
┌──────┐                    ┌──────┐ ┌──────┐
│VEHCHK│                    │READ2 │ │VDISCN│
└──────┘                    └──────┘ └──────┘
```
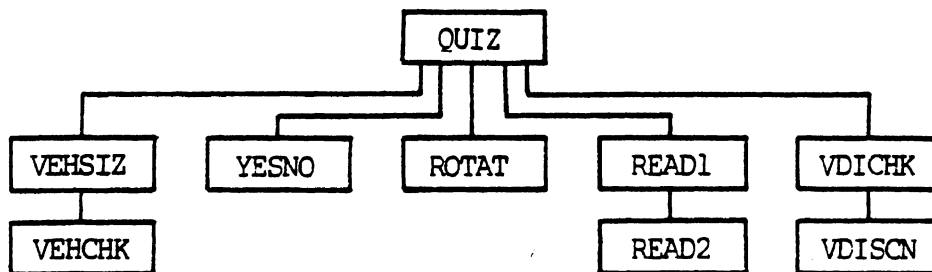
Figure 8.2 QUIZ'S Management of Sub-Sections of CRASH2

When the operation of START2 has been concluded, the sub-subsection
DAMAGE may or may not have been entered. If it was entered, an integer
flag variable JSPIN will have been set equal to the value 2. If START2
has been (nearly) concluded but DAMAGE has not been entered, JSPIN will
have been set to the value 1. As the control is passed back to MAIN,
JSPIN is tested and, if the value is 1, MAIN next calls upon the
principal subsection DAMAGE.

DAMAGE would have been called upon by MAIN instead of START2 if the
user had replied to Question 8 of QUIZ with a "NO." In any case, DAMAGE
is always called upon in a CRASH2 run and it is there that the
calculations are made and sent back to MAIN to ultimately appear as the
third principal output -- the output labeled "DAMAGE DATA ONLY."

```
                    ┌─────────┐
                    │ START2  │
                    └────┬────┘
        ┌───────────┬────┴─────┬──────────┐
   ┌────┴────┐ ┌────┴────┐ ┌───┴───┐ ┌────┴────┐
   │ PROXIM  │ │ OBLIQE  │ │ SPIN2 │ │ DAMAGE  │
   └─────────┘ └─────────┘ └───┬───┘ └─────────┘
                           ┌───┴───┐
                           │ USMAC │
                           └───┬───┘
                           ┌───┴────┐
                           │ RNGKT1 │
                           └───┬────┘
                           ┌───┴───┐
                           │ DAUX  │
                           └───┬───┘
                           ┌───┴───┐
                           │ TRAJ  │
                           └───────┘
```
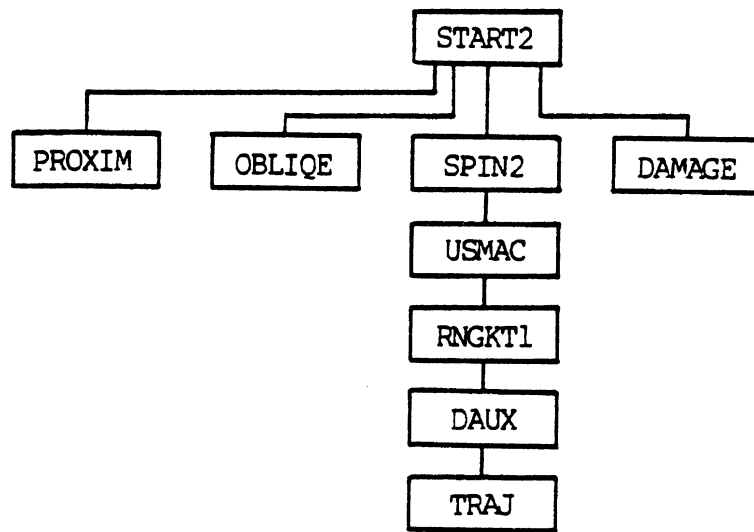
Figure 8.3 START2's Management of Sub-Subsections of CRASH2

With the operation of DAMAGE concluded, MAIN will bring its subsection "PRINT" into operation. As the user already knows, there are two classes of printed output from CRASH2. A short form of output is used when the user, originally presented with the question of OPTION, replies with "ABBREVIATED." A long form of output is used when the user originally replied with "COMPLETE." After a complete run of CRASH2, including the output, MAIN routinely calls upon OPTION again. The user is free now to nominate all of the choices except "batch" which is blocked. A response of END shuts the program down. A response of PRINT causes an immediate long form output of the last run. Responses of "COMPLETE" or "ABBREVIATED" start new runs. A response of "RERUN" is interesting. The control is passed from MAIN to QUIZ as before except the integer variable MENU is now 3 (before it was 1 for the complete form of questions and 2 for the short, or abbreviated, form of questions). In a regular run, MENU=1 or 2, the first operation of QUIZ is to put zero values or default values into all of the important variable storage locations, that is, to prepare for fresh data being fed in or selected from internal data sets by the user. In the rerun case these initialization steps are skipped and all data put into, selected by, or calculated on the last run remains in storage. QUIZ now presents the user with a request in the form "QUESTION NUMBERS?"

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If the user has responded to option with "RERUN" and is presented with the question 'QUESTION NUMBER?'and is unsure of what to do, he may reply with a question mark and a short explanation will appear informing him that a sequence of up to 12 question numbers may be returned and that these questions will then be presented to him. If new questions are asked, the resulting rerun of CRASH2 may be significantly different than the original run. For example, if originally the user answered Question 8 with a "NO," QUIZ would not present any questions in the range of 9 through 36. Now suppose the user responds with a selection of questions in this range. What will happen? This question is answered in the next paragraph. For now we note that, if questions the user was presented with before are picked for rerun, they will be asked again and the user may change any or all answers given on the original run. The rerun in this case will be an 'iteration' on the old run but not different in character.

Within QUIZ, as the last test after the treatment of the response to each question, the value of MENU is checked. If MENU=3, the rerun case, QUIZ does not proceed from one question to the logical next question but merely asks the user the questions named in his response. The RERUN option should be used only to change values entered in a prior run in preparation for a new run. To do otherwise has a high likelihood of aborting the entire run since the user must replace the logic of QUIZ with his own when answering a new series of questions.

The sections that follow will examine the three principal subsections: QUIZ, START2 and DAMAGE along with their supporting sub-subsections.

8.2 QUIZ Subroutines

This section begins with an overview of the operation and logic of the QUIZ subroutines. The overview is followed by a discussion of the treatment of five questions which typify the way CRASH2 handles user inputs.

8.2.1 Overview and Logic The subsection, or subprogram, QUIZ operates in two different modes, the RERUN mode and the regular mode. In the regular mode of operation a distinction is made between so-called

COMPLETE or ABBREVIATED operation. In all cases the purpose of QUIZ is to present selected questions to the user and to turn the user's responses into values of selected program variables. In the RERUN and the ABBREVIATED case the questions are presented in a brief or short form while in the COMPLETE case the questions are presented in an expanded or long form.

As previously noted the last step in handling the response to a question is to decide what to do next. If the user has set MENU=3 by selecting a RERUN option, the control logic of QUIZ causes a return to the next question on the user's list until there are no more whence the control is passed back to MAIN. If the user has set MENU=1 or 2 by selecting the COMPLETE or ABBREVIATED options, the control logic of QUIZ directs a sequence of questions which depend upon answers given. In Appendix C is a listing of all of QUIZ's questions. Chart 1 following is a self explanatory logic chart of QUIZ's control of question asking. A reading of Chart 1 with reference to the questions whose numbers are circled there will allow the user to take the place of the computer and prepare a listing of all the data needed by CRASH2 in its operation of the START2, DAMAGE, PRINT or SMACIN sections. Space is provided on Chart 1 for the user to enter this data by hand. A data summary sheet is found as the last page of the Chart series.

A special variation of CRASH2[1] is available to the student to assist in his gaining confidence in reading and using Chart 1. SAHP:CRASH contains a set of added routines which are slightly modified versions of some of CRASH2's subprograms. The program will print out the list of data entered by the user as well as most of the flags set in QUIZ. One minor modification is found in that values for DD, LL, and CC's will be output as reflected by DAMAGE's interpretation of the CDC code, if the user chooses not to enter these values.

The Subroutine QUIZ will present some of the 54 questions by typing them in a long form or a short form on the terminal. The long form is used for a "COMPLETE" run and the short form is used for "ABBREVIATED" or "RERUN" runs. After the user has typed an answer on the terminal for

---

[1] See documentation of SAHP:CRASH

the question just presented and has pressed the return key, QUIZ will "call" one of five different subroutines to act upon the user's input and to move the input data into the proper memory locations. These five subroutines are listed below along with the numbers of the questions which call each:

(a) VEHSIZ 2

(b) CDCCHK 3   4

(c) YESNO   5   8   11 12 14 17 18 19 21 24
            31 34 37 52

(d) READ1   6   7   9   10 13 15 20 22 25 26
            27 28 29 30 32 33 35 36 38 39
            40 41 42 43 44 45 46 47 48 49
            50 51 53 54

and

(e) ROTAT   16 23

The underlined Questions 2, 3, 5, 6, and 16 are typical of the others in their set. Following Chart 1, these notes will examine the primary and secondary subroutines used by QUIZ in processing these questions.

CHART 1 QUIZ--A Manual Version

USER'S NAME: =
DATE   : =
RUN NO.   : =

① ②

V#1

V#2

1. (M) INICAR
2. (S) UBCOMPACT
3. (C) OMPACT
4. (I) NTERMEDIATE
5. (F) ULLSIZE
6. (L) ARGE
7. (R) IGID
8. (B) ARRIER

JTYP(1) =

1. (M) INICAR
2. (S) UBCOMPACT
3. (C) OMPACT
4. (I) NTERMEDIATE
5. (F) ULLSIZE
6. (L) ARGE
7. (R) IGID
8. (B) ARRIER

JTYP(2) =

③ ④

VDI#1

VDI#2

JVDI(1,1) =
JVDI(1,2) =
JVDI(1,3) =
JVDI(1,4) =
JVDI(1,5) =
JVDI(1,6) =
JVDI(1,7) =

JVDI(2,1) =
JVDI(2,2) =
JVDI(2,3) =
JVDI(2,4) =
JVDI(2,5) =
JVDI(2,6) =
JVDI(2,7) =

⑤

YES

NO

JTYP(1) = 8

NO

⑥ JWSET(1) = 1

YES

w(1) =

JTYP(2) = 8

NO

⑦ JWSET(2) = 1

YES

w(2) =

⑧

YES

NO

⑨ IND = 0

IND = 1

XCR1   =
YCR1   =
PSIR1 =
SCR2   =
YCR2   =
PSIR2 =

⑩

XC10   =
YC10   =
PSI10 =
XC20   =
YC20   =
PSI20 =

⑪

NO

YES

JSKID(1) = 1

IFLAG(1) = 0

(See Note #2)

⑫

NO

YES

JSKID(1) = 0

(See Note #1)

⑬ IFLAG(1) = 1

XC11   =
YC11   =
PSI11 =

⑭

NOTE #1: If IFLAG(1) = 0, enter values of XCR1
                                          YCR1
                                          PSIR1

In places under 13 , that is   XC11 = XCR1
                                YC11 = YCR1
                                PSI11 = PSIR1

NOTE #2: If JSKID(1) = 0, enter values of XC10
                                          YC10
                                          PSI10

In places under 13 , that is   XC11 = XC10
                                YC11 = YC10
                                PSI11 = PSI10

Note #3 if IFLAG(2) = 0, enter values of   XCR2
                                             YCR2
                                             PSIR2
                                       XC12 = XCR2
                                       YC12 = YCR2
                                       PSI12 =PSIR2

        in places under 13, i.e.,

Note #4 if JSKID(2) = 0, enter values of XC20
                                           YC20
                                           PSI20
                                      XC12 = XC20
                                      YC12 = YC20
                                      PSI12 = PSI20

        in places under 13, i.e.,

CW

None

CCW

(23)

IRT(2) = 1

JIND(2) = 0,   IRT(2) = 0

IRT(2) = -1

YES   JIND(2) = 1

(24)

NO   JIND(2) = 0

(25) MU =

(26)

JROLL = 2

JROLL = 1

JTYP(1) = 8 — NO

Value =

LW(1,1) = Value/MU =
LW(1,2) = LW(1,1)
LW(1,3) = LW(1,1)
LW(1,4) = LW(1,1)

(29)

YES

JTYP(2) = 8 — NO

Value =

LW(2,1) = Value/MU =
LW(2,2) = LW(2,1)
LW(2,3) = LW(2,1)
LW(2,4) = LW(2,1)

(30)

YES

(31)

JTYP(1) = 8 — NO

LW(1,1) =
LW(1,2) =
LW(1,3) =
LW(1,4) =

(27)

YES

JTYP(2) = 8 — NO

LW(1,1) =
LW(1,2) =
LW(1,3) =
LW(1,4) =

(28)

YES

JTRAJ = 0

NO   YES   JTRAJ = 1   NO

(31)   JTYP(1) = 8   (32)   JSSET(2) = 1

YES

STEER(1,1) =
STEER(1,2) =
STEER(1,3) =
STEER(1,4) =

NO

JTYP(2) = 8   (33)   JSSET(1) = 1

YES

STEER(2,1) =
STEER(2,2) =
STEER(2,3) =
STEER(2,4) =

NO   YES

(34)   (35)   JBSET = 1   (36)   MU2 =

XBP(1) = -600.
YBP(1) = 600.
XBP(2) = 600.
YBP(2) = 600.

XBP(1) =
YBP(1) =
XBP(2) =
YBP(2) =

JTYP(1) = 8

37 YES / NO

VDI Col3, V#1 = F or B   YES / NO

38 JLSET(1) = 1
LL(1) =

39
CC(1,1) =
CC(1,2) =
CC(1,3) =
CC(1,4) =
CC(1,5) =
CC(1,6) =
JCSET(1) = 2
JCSET(1) = 4
JCSET(1) = 6

40 JDSET(1) = 1
DD(1) =

41 JLSET(1) = 1
LL(1) =

42
CC(1,1) =
CC(1,2) =
CC(1,3) =
CC(1,4) =
CC(1,5) =
CC(1,6) =
JCSET(1) = 2
JCSET(1) = 4
JCSET(1) = 6

43 JDSET(1) = 1
DD(1) =

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1. Name
   Date
   Run No.

2. JTYP(1) =
   JTYP(2) =

3. JVDI(1,1) =
   JVDI(1,2) =
   JVDI(1,3) =
   JVDI(1,4) =
   JVDI(1,5) =
   JVDI(1,6) =
   JVDI(1,7) =

4. JVDI(2,1) =
   JVDI(2,2) =
   JVDI(2,3) =
   JVDI(2,4) =
   JVDI(2,5) =
   JVDI(2,6) =
   JVDI(2,7) =

5. W(1) =
   JWSET(1)=

6. W(2) =
   JWSET(2) =

8. IND =

9. XCR1 =
   YCR1 =
   PSIR1 =
   XCR2 =
   YCR2 =
   PSIR2 =

10. XC10 =
    YC10 =
    PSI10 =
    XC20 =
    YC20 =
    PSI20 =

11. JSKID(1) =
    IFLAG(1) =

12,13. XC11 =
       YC11 =
       PSI11 =

14,15. JCURV(1) =
       XC21 =
       YC21 =

16,17. IRT(1) =
       JIND(1) =

18. JSKID(2) =

19,20. IFLAG(2) =
       XC12 =
       YC12 =
       PSI12 =

21,22 JCURV(2) =
      XC22 =
      YC22 =

23,24. IRT(2) =
       JIND(2) =

25. MU =

26. JROLL =

27,29. LW(1,1) =
       LW(1,2) =
       LW(1,3) =
       LW(1,4) =

28,30. LW(2,1) =
       LW(2,2) =
       LW(2,3) =
       LW(2,4) =

31. JTRAJ =

32. JSSET(1) =
    STEER(1,1) =
    STEER(1,2) =
    STEER(1,3) =
    STEER(1,4) =

33. JSSET(2) =
    STEER(2,1) =
    STEER(2,2) =
    STEER(2,3) =
    STEER(2,4) =

34,35. XBP(1) =
       YBP(1) =
       XBP(2) =
       YBP(2) =

36. MU2

38,41. JLSET(1) =
       LL(1) =

39,42. JCSET(1) =
       CC(1,1) =
       CC(1,2) =
       CC(1,3) =
       CC(1,4) =
       CC(1,5) =
       CC(1,6) =

40,43. JDSET(1) =
       DD(1) =

44. JLSET(2) =
    LL(2) =

45,48. JCSET(2) =
       CC(2,1) =
       CC(2,2) =
       CC(2,3) =
       CC(2,4) =
       CC(2,5) =
       CC(2,6) =

46,49. JDSET(2) =
       DD(2) =

50. JRSET(1) =
    RHO(1) =

51. JRSET(2) =
    RHO(2) =

53. ANG(1) =
    JASET(1) =

54. ANG(2) =
    JASET(2) =

8.2.2 Question Two Chart 2 establishes a route map for Question Two. The goal of Question 2 is to set integer values to the variables JTYP(1) and JTYP(2) and to go on to Question 3. The user (see Question Two) will have typed a response and QUIZ, ISN213^2 has called the Subroutine VEHSIZ to act on the input.

VEHSIZ. This Subroutine is designed to read and act upon an 80 element string of alpha-numeric characters. It then searches the string for a ? or a \$^{3} and, finding neither, searches for the first non-blank character. This character location marks the starting location, JVIS, of the first of two character strings which the user was prompted to enter. A search is made for the location of the first blank position (the end of the first string) and the location of the first string's last character is stored as JVIF. Further search finds the location of the second string and JV2S and JV2F are assigned these coordinates. The Subroutine VEHCHK is called to act upon the two character strings and to help provide integer values to J1 and J2, the vehicle size number codes. A flowchart for VEHSIZ follows.

VEHCHK. The call statements for this subroutine appear at ISN37 and ISN40 of VEHSIZ and are worded as, for example:

CALL VEHCHK (LINE, JVEH1, JV1S, JV1F, JCODE)

The known values of the array LINE and the integer values of JV1S and JV1F are transmitted to this routine so that here JSTART = JV1S and JEND = JV1F. This routine acts upon the character string found in locations LINE(JSTART) through LINE(JEND) and returns to VEHSIZ an integer value for the vehicle size variable JVEH1. A flowchart for VEHCHK follows.

---

^{2} The symbol ISN213 represents "Internal Statement Number 213. The executable statements of a subroutine are in effect, numbered in the order of their appearance. ISN213 refers to the 213th executable statement of QUIZ.

^{3} Symbols \$ and ? are explained by recognizing that VEHSIZ returns values of ICODE to QUIZ where ICODE=-1 signals that a ? has been read and ICODE=-2 signals that a \$ has been read. Examine the structure of QUIZ, ISN216-1217 for result.

CHART 2   QUIZ Route Map for Question Two

```
207 | is MENU = 1 |───YES──→ | present the long | 208,209
                             | form of Q #2    |
         NO │
            ▼
211,212 | present the short |
        | form of Q #2      |
                │
                ▼
213  | the user types the character    |
     | strings designating 2 vehicle   |
     | sizes                           |
     | Subroutine VEHSIZ is called     |
     | to act on the input and returns |
     | values for JC                   |
                │
                ▼
214  | is ICODE = 2  |───YES──→ | set  JTYP(1) = JV1 |
                                |      JTYP(2) = JV2 |
         NO │                   |      GO TO ISN 225 |
            ▼
215  | is ICODE = 0  |───YES──→ | write message '9001' |
                                |      GO TO ISN 207   |
         NO │
            ▼
216  | is ICODE = -1 |───YES──→ | repeat QUESTION #2 |
         NO │
            ▼
217  | is ICODE = -2 |───YES──→ | repeat QUESTION #1 |
         NO │
            ▼
218  | is ICODE = -999 |──YES──┐
         NO │                  │
            └──────────────────┼──→ | write message '9000' |
                                    |      GO TO ISN 207   |
```

Message '9000' is "****SYNTAX ERROR****"

Message '9001' is "****YOU MUST ANSWER THIS QUESTION****"

```
                        ┌──────────┐
                        │  START   │
                        └──────────┘
                             │
                             ▼
                        ╱─────────╲
          4  ──────────▶│    A    │
                        │ J = 1,80│
                        ╲─────────╱
                             │
          5                 ▼
                    ┌──────────────────────┐
                    │ set LINE(J) = 'blank'│
                    └──────────────────────┘
                             │
                             ▼
                            (A)

          7,8  ┌──────────────────────────────────────────────────┐
               │ read a string of 80 alpha-numeric characters from terminal │
               └──────────────────────────────────────────────────┘
                             │
                             ▼
                        ╱─────────╲
          9  ──────────▶│    B    │
                        │ J = 1,80│
                        ╲─────────╱
                             │
          10                 ▼                    YES
              ┌──────────────────────┐      ┌──────────────────┐
              │ is Line(J) = '?'     │─────▶│ set ICODE = -1   │─────┐
              └──────────────────────┘      └──────────────────┘     │
                             │ N                                     ▼
          11                 ▼                    YES          ┌──────────────────┐
              ┌──────────────────────┐      ┌──────────────────┐│ GO TO QUIZ,ISN214│
              │ is Line (J) = '$'    │─────▶│ set ICODE =-2    │└──────────────────┘
              └──────────────────────┘      └──────────────────┘
                             │
                             ▼
                            (B)

          13                 ▼
                        ╱─────────╲
             ──────────▶│    C    │
                        │ J = 1,80│
                        ╲─────────╱
                             │
          14                 ▼          YES        18
              ┌──────────────────────┐      ┌──────────────────┐
              │ is LINE (J) not 'blank'│───▶│ set JVIS = J     │
              └──────────────────────┘      └──────────────────┘
                             │                       │
                             ▼               19      ▼
                            (C)                 ╱─────────╲
          16                              ─────▶│    D    │
              ┌──────────────────────┐          │ K = J,80│
              │ set ICODE = '0'      │          ╲─────────╱
              └──────────────────────┘               │
                             │                        ▼
                             ▼              20  ┌──────────────────┐   YES
              ┌──────────────────────┐          │ is LINE (K) a 'blank'│────┐
              │ RETURN               │          └──────────────────┘        │
              │ QUIZ,ISN214          │               │ NO                   ▼
              └──────────────────────┘               ▼              24 ┌──────────────┐
                                                     (D)                │ set JV1F = K-1│
                                              22 ┌──────────────────┐   └──────────────┘
                                                 │ set ICODE =-999  │
                                                 └──────────────────┘
                                                     │
                                                     ▼
                                              23 ┌──────────────────┐
                                                 │ RETURN           │
                                                 │ QUIZ,ISN214      │
                                                 └──────────────────┘

                        (I)

                        20 - CRASH2
```

```
                    ┌───┐
                    │ I │
                    └─┬─┘
                      │
                 ┌────┴────┐
          ┌─────/    ⓔ     \
    25    │     \  L = K,80 /
          │      └────┬────┘
          │           │
          │     ┌──────┴──────────┐      YES   ┌──────────────┐   30
    26    │     │ is LINE (L) ≠ 'blank'├────────→│ set JV2S = L │
          │     └──────┬──────────┘           └──────┬───────┘
          │           │NO                            │
          │         ┌─┴─┐                      ┌─────┴─────┐      31
          └ ─ ─ ─ ─→│ E │              ┌ ─ ─ ─→/    ⓕ      \
                    └─┬─┘              │       \  M = L,80   /
          ┌───────────┴──────┐        │        └─────┬─────┘
    28    │ set ICODE = -999 │    32  │              │
          └─────────┬────────┘        │      ┌───────┴──────────┐  YES
                    │                 │      │ is LINE (M) a 'blank'├──────┐
          ┌─────────┴────────┐        │      └───────┬──────────┘        36 │
    29    │ RETURN, GO TO     │       │              │NO          ┌──────────┴──┐
          │ QUIZ, ISN214      │       │            ┌─┴─┐          │ set JV2F = M │
          └───────────────────┘       └ ─ ─ ─ ─ ─ →│ F │          └──────┬──────┘
                                           34      └─┬─┘
                                          ┌──────────┴───────┐
                                          │ set ICODE = -999 │
                                          └─────────┬────────┘
                                      35 ┌──────────┴───────┐
                                         │ RETURN, GO TO     │
                                         │ QUIZ, ISN214      │
                                         └───────────────────┘
```

```
          ┌────────────────────────────────────────┐
    37    │ with JVIS = J, JV1F = K-1 known         │
          │ Subroutine VEHCHK is called and         │
          │ returns values of JVEH1 and JCODE       │
          └────────────────────┬───────────────────┘
                               │
    38    ┌────────────────┐   YES   ┌──────────────────┐  45
          │ is JCODE ≠ 1   ├─────────→│ set ICODE = -999 │
          └───────┬────────┘         └────────┬─────────┘
                  │                           │
    39    ┌───────┴────────┐         ┌────────┴─────────┐  46
          │ set J1 = JVEH1 │         │ RETURN GO TO      │
          └───────┬────────┘         │ QUIZ, ISN214      │
                  │                  └───────────────────┘
          ┌───────┴────────────────────────────────┐
    40    │ with JV2S = L, JV2F = M known           │
          │ Subroutine VEHCHK is called and         │
          │ returns values of JVEH2 & JCODE         │
          └────────────────────┬───────────────────┘
                               │
    41    ┌────────────────┐   YES   ┌──────────────────┐  45
          │ is JCODE ≠ 1   ├─────────→│ set ICODE = -999 │
          └───────┬────────┘         └────────┬─────────┘
                  │                           │
  42,43  ┌───────┴────────┐         ┌────────┴─────────┐  46
          │ set J2 = JVEH2 │         │ RETURN, GO TO     │
          │ ICODE   = 2    │         │ QUIZ, ISN214      │
          └───────┬────────┘         └───────────────────┘
                  │
          ┌───────┴────────┐
    44    │   RETURN        │
          │ GO TO QUIZ      │
          │   ISN214        │
          └─────────────────┘
```

VEHCHK Flowchart

```
2  set JVEH = 0
        │
        ▼
   ┌──(A)──┐
6  ⟨ J = JSTART ⟩◄─────────────┐
   ⟨   JEND   ⟩                │
        │                      │
        ▼            YES       │
7  │ is LINE(J) = 'blank' │──────►(A)──►  9 │ set ICODE = 0 │──►  10 │ RETURN, GO TO │
        │ NO                                                        │ VEHSIZ, 38 or 41 │
        ▼                       YES
11 │ is LINE(J) a 'M' │────────────────►│ set JVEH = 1 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
12 │ is LINE(J) a 'S' │────────────────►│ set JVEH = 2 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
13 │ is LINE(J) a 'C' │────────────────►│ set JVEH = 3 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
14 │ is LINE(J) an 'I' │───────────────►│ set JVEH = 4 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
15 │ is LINE(J) a 'F' │────────────────►│ set JVEH = 5 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
16 │ is LINE(J) a 'L' │────────────────►│ set JVEH = 6 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
17 │ is LINE(J) a 'R' │────────────────►│ set JVEH = 7 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
18 │ is LINE(J) a 'B' │────────────────►│ set JVEH = 8 │
        │ NO                                    │
        ▼◄──────────────────────────────────────┘
        │                       YES
19 │ is JVEH = 0 │─────────────────────►  22 │ set ICODE = -999 │
        │ NO                                  │ GO TO VEHSIZ 38,41 │
        ▼
20 │ set ICODE = 1 │───────────────────►  21 │ GO TO VEHSIZ 38,41 │
```

The ISN's 2]3-2]8 of QUIZ and all of VEHSIZ and VEHCHK are seen to
be an elaborate means of writing integer values for the
vehicle #1 and vehicle #2 size code variables JTYP(1)
and JTYP(2) respectively

8.2.3 Question Three The goal of Question 3 is to determine that a correct Collision Deformation Classification (CDC), formerly the Vehicle Damage Index (VDI), code was entered for a single vehicle. Chart 3 establishes a route map for Question 3.

VDICHK. This subroutine, called from QUIZ, ISN233, has the purpose of locating the seven-element string of alpha-numeric characters entered by the user as a CDC code and, by calling the Subroutine VDISCN, confirming that each character is separately valid. The subroutine returns valid characters in an array LVDI and an integer value for ICODE. A flowchart follows.

VDISCN. This subroutine is designed to act upon a set of seven alpha-numeric characters in the array LINE with the set said to be located in columns JSTART=LFN through JEND=LEN. If the seven characters are numbered 1 through 7, then this subroutine checks that character #1 is found in the stored array of acceptable symbols LVDI(K,1) for K = 1,13. If it is , that character is loaded into IVDI(1) and the character #2 is checked against the array LVDI(K,2) for K = 1,13 and loaded into JVDI(2) if found. If, for example, character #2 is not so found, JCODE is set equal to 2 and the subroutine RETURNS to VDICHK, ISN22 where, since ICODE is not 8 but is 2, VDICHK will return to QUIZ, ISN234, prepared to trigger a message that an error exists in the 2nd column of the user's CDC..

ISN3 of Subroutine VDISCN holds a 13 by 7 array of allowable alpha-numeric symbols which may be used in a CDC designation. These are the values of the array LVDI (K,I) used in VDISCN, ISN13. In tabular form these are:

Row	Column						
	1	2	3	4	5	6	7
1	0	0	F	D	A	W	1
2	0	1	R	L	H	N	2
3	0	2	B	C	E	S	3
4	0	3	L	R	G	O	4
5	0	4	T	F	M	A	5
6	0	5	V	P	L	O	6
7	0	6	X	B	X	E	7
8	0	7	X	Y	X	E	8
9	0	8	X	Z	X	E	9
10	0	9	X	Z	X	E	0
11	1	0	X	Z	X	E	9
12	1	0	X	7	X	F	9
13	1	0	X	Z	X	E	9

A flowchart for VDISCN also follows.

CHART 3 QUIZ Route Map for Question Three

QUIZ ISN's

225	START	
226	is JTYP(1) = 8	YES → GO TO QUESTION #4
	↓ NO	
227	is MENU = 1	YES → 228,229 present the long form of Q #3
	↓ NO	
231,232	present the short form of Q #3	

The user types a VDI set
of 7 alpha-numeric characters

233 Subroutine VDICHK is called to act on the
input and returns with a seven element array
holding the characters and an integer value of
ICODE.

ISN	condition	YES action
234	is ICODE = 8	for K = 1 to 7, set JVDI(1,K) = LVDI(K) 247-249
235	is ICODE = -999	write message '9000' and GO TO ISN 225
236	is ICODE = -2	repeat Question #2
237	is ICODE = -1	repeat Question #3
238	is ICODE = 0	write message '9000' and GO TO ISN 225
239	is ICODE in range 1-7	244,245 write an advice stating the column number of the first error in VDI and repeat QUESTION #3

240,241 ↓ NO

write message '9001' and GO TO ISN 225

VDICHK Flowchart

ISN's

```
                    ┌──────────────┐
                    │    START     │
                    └──────┬───────┘
                           │
                           ▼
                    ╱──────────────╲          NOTE:  all returns are to
5,6  ┌──────────▶  ⟨      (A)       ⟩                QUIZ, ISN 234
     ┆              ╲   J = 1,80   ╱                 (see chart on Q #3)
     ┆                    │
     ┆                    ▼
     ┆            ┌──────────────────────┐
     ┆            │ set LINE(J) = 'blank'│
     ┆            └──────────┬───────────┘
     ┆                       │
     ┆                       ▼
     └─ ─ ─ ─ ─ ─(A)
                           │
                           ▼
7,8      ┌────────────────────────────────────────────────────┐
         │ read an 80 element alpha-numeric string into LINE(J)│
         └──────────────────────┬─────────────────────────────┘
                                │
                                ▼
                         ╱──────────────╲
9   ┌────────────────▶  ⟨      (B)       ⟩
    ┆                    ╲   J = 1,80   ╱
    ┆                           │
    ┆                           ▼                                    27
10┆        ┌──────────────────┐   YES   ┌────────────────────────┐
  ┆        │ is LINE(J) an '?' │───────▶│ set ICODE = -1, RETURN │
  ┆        └─────────┬─────────┘        └────────────────────────┘
  ┆                  │ NO                                         29
11┆        ┌──────────────────┐   YES   ┌────────────────────────┐
  ┆        │ is LINE(J) a '$' │────────▶│ set ICODE = -2, RETURN │
  ┆        └─────────┬─────────┘        └────────────────────────┘
  ┆                  │ NO
  └─ ─ ─ ─ ─ ─ ─(B)
                     │
                     ▼
              ╱──────────────╲ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─┐
13           ⟨      (C)       ⟩                     ┆
              ╲   J = 1,80   ╱                      ┆
                     │                              ┆
                     ▼                              ▼
         ┌────────────────────┐  YES             (C)      ┌──────────────────────┐
14       │ is LINE(J) a 'blank'│──────────────▶(C)──────▶│ set ICODE = 0, RETURN│
         └─────────┬──────────┘                          └──────────────────────┘
                   │ NO
                   ▼
         ┌──────────────────┐
15,16    │ set  LFN = J     │
         │      LEN = J+6   │
         └─────────┬────────┘
                   │
                   ▼
         ┌────────────────────────────────────────┐
21       │ CALL VDISCN(LINE,VDI,LFN,LEN,ICODE)     │
         └─────────────────┬──────────────────────┘
                           │
                           ▼
         ┌──────────────────┐  YES   ┌───────────────────────┐
22       │ is ICODE = 8     │───────▶│ set ICODE = 8, RETURN │
         └─────────┬────────┘        └───────────────────────┘
                   │ NO                    ⎛ NORMAL ⎞
                   ▼                       ⎝ CASE   ⎠
         ┌──────────────────┐
         │     RETURN       │
         └──────────────────┘
```

VDISCN Flowchart

ISN's

START

(A)

J = JSTART
JEND

5 →

6 |

LINE(J) ≠ 'blank'

(A)

8 set JCODE = 0 → set JBEGIN = J 10

9 RETURN

GO TO VDICHK,ISN22

(B)
I = 1,7   11

(C)
K = 1,13   12

Is Line (J BEGIN+I−1) = LVDI (K,I)   13

(C)

15 set JCODE = I

16 RETURN

GO TO VDICHK
ISN22

set IVDI(I) =
LINE(JBEGIN+I−1)   17

(B)

19 set JCODE = 8

20 NORMAL RETURN

GO TO VDICHK
ISN22

8.2.4 Question Five. We now examine in Chart 4 QUIZ, ISN274-293 which deals with asking the user a question, telling him he must answer it with a YES or NO and then determining which answer was given. Future progress in QUIZ depends upon the answer. If neither YES or NO the program may not continue, and, if YES or NO, it will take different paths in its next few steps. We shall look at the structure of QUIZ, ISN274-293 first, and subsequently at the Subroutine YESNO called upon at QUIZ, ISN281.

YESNO This subroutine is designed to accept a string of 72 alpha-numeric characters maximum and to first determine if the string contains a ? or $. If not, it determines whether the first character of the string is a Y or a N. A flowchart for YESNO follows.

8.2.5 Question Six. Question 6 is typical of a question set for which one or more numbers represent a proper answer. Question 6 requires a single number as an answer. Question 9 is similar except it requires six separate numbers as an answer. A QUIZ route map for Question 6 is shown in Chart 5.

READ1. This subroutine is designed to scan the eighty columns of a line of input, reading them into the matrix LINE(80). Special action is taken if any character in LINE is found to be the symbol $ or ?. Otherwise the subroutine is designed to find up to twelve starting positions and corresponding ending positions of character strings separated by spaces or columns. A flowchart for READ1 follows.

READ2. The user may note that READ1, ISN45 "calls" the Subroutine READ2 from a DO LOOP. The number of calls is determined by the integer NWORD determined at ISN29. Each "call" is made with the integer values of the variables ISTART and IEND corresponding to the column range of a user inputted number. Each call is expected to return a real number representation of the indexed number in the variable VALUE.

By assigning J, the integer value of JSTART determined by Subroutine READ1, to be the array index at which a number appears to start in the array LINE, and reading LINE(J=JSTART), LINE(J=JSTART+1),

CHART 4 QUIZ Route Map for Question Five

ISN's

275 START

276,277

is MENU = 1 — **YES** → present the long form of Q #5

NO

279,280 present the shore for of Q #5

The user types in YES or NO,
or any word starting with
the letter Y or the letter N and
        presses RETURN

281 the Subroutine YESNO is called
to act on the input and returns
an integer value for ICODE.

282 is ICODE = 2 — **YES** → GO TO QUESTION#6

NO

283 is ICODE = 1 — **YES** →

NO

284 is ICODE = 0 — **YES** → set JWSET(1) = 0
                                JWSET(2) = 0
                          GO TO QUESTION#8

NO

285 is ICODE = -1 — **YES** → repeat QUESTION #5

NO

286 is ICODE = -2 — **YES** → repeat QUESTION #4

NO

287 is ICODE = -999 — **YES** → write message '9000' and GO TO ISN 275

NO

YESNO Flowchart

1   START

5   (A) J = 1,72

6   set LINE(J) = 'blank'

     (A)

7,8   read 72 characters from terminal input

9   (B) J = 1,72

10   is LINE(J) = ?   YES   →   set ICODE -1, RETURN

     NO

11   is LINE(J) = $ YES   →   set ICODE = -2, RETURN

     NO

     (B)

13   (C) J = 1,72

14   is LINE(J) a blank   YES

     (C)

     16
     set ICODE = -999, RETURN

     22
18   is LINE(J) a Y   YES   →   set ICODE = 2, RETURN   NORMAL

     NO

     24
19   IS LINE(J) a N   YES   →   set ICODE = 1, RETURN   NORMAL

     NO

     16
     SET ICODE = -999, RETURN

CHART 5 QUIZ Route Map for Question Six

QUIZ ISN's

293   ```
       START
       ```

296,297

is MENU = 1 ──YES──▶ present the long
                     form of Q#6

NO ▼

299,300   present the short
          form of  Q. #6
▼

the user is expected to type a
character string representing
a real number

301   The Subroutine READ1 is called
      to act on the user's input and
      to return a floating point number
      for VALUE and an integer for ICODE

311

──YES──▶ is 1500<VALUE(1)<6000 ──YES──▶

302   is ICODE = 1

NO ▼          309                              314,15

303   is ICODE = 0 ──YES──▶ set        write message    set
                            JWSET(1)   '9002' and        JWSET(1) = 1
                            = 0        repeat Q#6        W(1) = VALUE(1)

NO ▼                                                    ▼

                                              GO TO Q#7

304   is ICODE = -1 ──YES──▶ repeat Q#6

NO ▼

305   is ICODE =-2 ──YES──▶ repeat Q#5

NO ▼

is ICODE = -999 ──YES──▶

NO ▼

write message '9000' and GO TO ISN 293

ISN'S

```
        ┌──────────────┐
        │      (A)     │
        │   J = 1,12   │
        └──────────────┘

        set  ISTART(J) = 0        4-8
             IEND   (J) = 0
             VALUE  (J) = 0

             (A)

        ┌──────────────┐
        │      (B)     │
        │   J = 1,80   │        9-11
        └──────────────┘

        set LINE(J) = 0

             (B)
```

NOTE: ISN12 is equivalent
to the loop shown here

```
        ┌──────────────┐
        │      (C)     │
        │   J = 1,80   │        12
        └──────────────┘

        read Jᵗʰ character in a line
        of input and place in memory
                as LINE(J)

             (C)
```

NOTE: RETURN is to
QUIZ, ISN 302
in case of Q#6

```
        ┌──────────────┐
        │      (D)     │
        │   I = 1,80   │        14-21
        └──────────────┘

                          YES
        is LINE(J) a '?'   ──────▶   leave loop, set ICODE = -1, return

        NO │
                          YES
        is LINE(J) a '$'   ──────▶   leave loop, set ICODE = -2, RETURN

        NO │
             (D)

             (I)      see next sheet
```

```
                              ┌───┐
                              │ I │
                              └─┬─┘
                                │
                        ┌───────▼───────┐
22,23                   │ set   JST  = 0│
                        │     ·NWORD = 0 │
                        └───────┬───────┘
                                │
                         ╱──────▼──────╲
24  ──────────┐         ╱     ┌───┐     ╲
              │         ╲     │ E │     ╱
              │          ╲──J = 1,80──╱
              │                │
              │                │                                              36,37
              │        ┌───────▼───────┐   YES    ┌──────────────────────────┐ YES
25            │        │  is JST = 1   ├────────► │ is LINE(J) a blank or comma├────►┐
              │        └───────┬───────┘          └──────────┬───────────────┘     │
              │                │ NO                          │ NO      38           │
       YES    │        ┌───────▼───────┐            ┌────────▼────────┐  YES        │
26  ◄─────────│────────┤ is LINE(J) a 'blank'│      │   is J<72       ├──────────► ╮│
              │        └───────┬───────┘            └────────┬────────┘            ││
              │                │ NO                          │ NO        39,40      ││
       YES    │        ┌───────▼───────┐            ┌────────▼──────────────┐      ││
27  ◄─────────│────────┤ is LINE(J) a ',' │         │ set IEND(NWORD) = J-1 │◄─────╯│
              │        └───────┬───────┘            │          JST = 0      │       │
              │                │ NO                 └────────┬──────────────┘       │
              │        ┌───────▼───────┐                     │                      │
28            │        │ set  NWORD = NWORD+1│               │                      │
              │        └───────┬───────┘                     │                      │
              │                │                             └──────────────────►  │
              │        ┌───────▼───────┐  YES  ┌─────────────────────────┐          │
29            │        │ is NWORD>12   ├──────►│ set ICODE = -999 /RETURN │          │
              │        └───────┬───────┘       └─────────────────────────┘          │
              │                │ NO                                                 │
              │        ┌───────▼──────────────────┐                                 │
30,31         │        │ set ISTART(NWORD) = J, JST = 1│                             │
              │        └───────┬───────┘                                            │
       YES    │        ┌───────▼───────┐                                            │
32  ◄─────────│────────┤   is  J<80    │                                            │
              │        └───────┬───────┘                                            │
              │                │ NO                                                 │
              │        ┌───────▼──────────────────┐                                 │
33,34         │        │ set IEND(NWORD) = J, JST = 0│                               │
              │        └───────┬───────┘                                            │
              │        '41    ┌▼┐                                                   │
              └─────────────◄─┤E│◄─────────────────────────────────────────────────┘
                              └┬┘
                               │
                        ┌──────▼──────┐  YES    ┌──────────────────────┐   42,43
                        │ is NWORD = 0├────────►│ set ICODE = 0, RETURN │
                        └──────┬──────┘         └──────────────────────┘
                               │ NO
                         ╱─────▼─────╲
45                      ╱    ┌───┐    ╲
                        ╲    │ F │    ╱
                         ╲ I =1,NWORD╱
                               │
                        ┌──────▼──────┐         ┌──────────────────────────┐  51,52
46                      │ CALL READ2  ├────────►│ if JCODE returns -999     │
                        └──────┬──────┘         │ set ICODE = -999 & RETURN │
                               │                └──────────────────────────┘
                              ┌▼┐
                         ◄────┤F│
                              └┬┘
                               │
                        ┌──────▼──────┐         ┌──────────────────┐   50
49                      │ set ICODE = NWORD├───►│  NORMAL RETURN    │
                        └─────────────┘         └──────────────────┘
```

etc. up to LINE(J=JEND) , we would have read that number. However, to allow freedom of format of the numbers in the array LINE, we must prepare here to read the number in a variety of formats, i.e., the number may by written as the integer 1527, as the floating point form of real number -1527.31E+03 or in another form. The algorithm here does just that and constructs the result as a floating point number and a code, ICODE. A flowchart for READ2 is shown.

On this flowchart, the value of SX is real while the value of X, SE, EX and NEX are integer. The function FLOAT(X) produces a real number equal to X. It is clear that RESULT is a real number

Example: Follow 15.0E+3 through the flowchart of the algorithm of READ2, character by character. Check the final results determined by the seven passes.

Use the flowchart of READ2 taking JSTART=1 and JSEND=7. Let the values of LINE by LINE(1)="1", LINE(2)="5", LINE(3)=".", LINE(4) = "0", LINE(5)="E", LINE(6)="+", and LINE(7)=3. Follow the route map through and confirm that SX=1.0, X=150, SE=1, EX=3, and NEX=1.

```
JCHAR = 1        IDIGIT = 1
                      X = 1
                    NEX = 0
                 JDIGIT = 1
                 JPLUSF = 1
                  JMINF = 1

JCHAR = 5        IDIGIT = 5
                      X = 15
                    NEX = 0
                 JDIGIT = 1
                 JPLUSF = 1
                  JMINF = 1

JCHAR = .        JPOINT = 1
                 JPLUSF = 1
                  JMINF = 1

JCHAR = 0        IDIGIT = 0
                      X = 150
                    NEX = 1
                 JDIGIT = 1
                 JPLUSF = 1
                  JMINF = 1

JCHAR = E         JEXP = 1
                 JPOINT = 1
```

READ2 Flowchart

ISN's
1,7-22

```
set J = JSTART,  JCODE  = 0   JXFP   = 0   EX = 0
                 JPLUSF = 0   JDIGIT = 0   SX = 1.0
                 JPLUSE = 0   RESULT = 0.0
                 JMINF  = 0   SE     = 1
                 JMINE  = 0   NEX    = 0
                 JPOINT = 0   X      = 0
```

23 set JCHAR = LINE(J)

24 is JCHAR a blank ──YES──▶

25
```
is  JPLUSF = 0
&   JMINF  = 0
&   JPLUSE = 0                    ──YES──▶
&   JMINE  = 0
&   JPOINT = 0
&   JXPP   = 0
&   JDIGIT = 0
```

NO │

24 NO

set JDONE = 1 ──YES──▶

28 is JCHAR a + ──YES──▶ 29 is JDONE = 1 ──YES──▶ set ICODE = -999
                                                      RETURN

NO │   NO │ 30   34,35

```
is  JPLUSF = 0
&   JMINF  = 0   ──YES──▶  set SX = 1.0
&   JEXP   = 0             JPLUSF = 1
```

NO │ 31   37,38

```
is  JPLUSE = 0
&   JMINE  = 0   ──YES──▶  set SE = 1
&   JEXP   = 1             JPLUSE = 1
```

NO │ 32,33

```
set ICODE = -999
    RETURN
```

40 is JCHAR a - ──────▶ 41 is JDONE = 1 ──YES──▶ 44,45
                                                      set ICODE = -999
                                                      RETURN

NO │   NO │ 42   46,47

```
is  JPLUS1 = 0
&   JMINF   = 0  ──YES──▶  set SX = -1.0
&   JEXP    = 0            JMINF = 1
```

NO │ 43   49,50

```
is  JPLUSE = 0
&   JMINE  = 0   ──YES──▶  set SE = -1
&   JEXP   = 1             JMINE = 1
```

NO │ 44,45

```
set ICODE = -999
    RETURN
```

(I)   (II)   (III)

```
                                            55,56
                                        YES   ┌──────────────────┐
┌──────────────────┐     YES   ┌──────────────┐──→ set ICODE = -999 │
│ is JCHAR a '.'   │─────────→ │ is JDONE = 1 │   │     RETURN       │
└──────────────────┘           └──────────────┘   └──────────────────┘
        │ NO                       │ NO    54                    57,59
        │                          ▼                         ┌──────────────┐
        │                  ┌──────────────┐  YES  ┌──────────┤ set JPOINT = 1│
        │                  │ is  JPOINT = 0│──────→           │    JPLUSF = 1 │
        │                  │  &  JEXP   = 0│                   │    JMINF  = 1 │
        │                  └──────────────┘                   └──────────────┘
        │                       │ NO   55,56
        │                       ▼
        │                  ┌──────────────┐
        │                  │ set ICODE = -999│
        │                  │    RETURN     │
        │                  └──────────────┘
        ▼                          61                    64,65
┌──────────────────┐  YES                         ┌──────────────┐
│ is JCHAR an 'E'  │──────┐                     ┌─→│ set ICODE = -999│
└──────────────────┘      │   62  YES           │  │    RETURN     │
        │ NO   61         ▼  ┌──────────────┐──┘  └──────────────┘
┌──────────────────┐ YES  └→│ is JDONE = 1 │           66,69
│ is JCHAR a 'D'   │────────└──────────────┘      ┌──────────────┐
└──────────────────┘          │ NO   63           │ set JEXP   = 1│
        │ NO                   ▼            YES ┌──│    JPOINT = 1 │
        │              ┌──────────────┐─────────┘  │    JPLUSF = 1 │
        │              │ is JEXP = 0  │            │    JMIN   = 1 │
        │              └──────────────┘            └──────────────┘
        │                   │ NO   64,65
        │                   ▼
        │              ┌──────────────┐
        │              │ set ICODE = -999│
        │              │    RETURN     │
        ▼  71,73       └──────────────┘
┌──────────────────┐           76
│ is JCHAR a digit,│  YES ┌──────────────────┐
│ i.e.,is JCHAR any one│──→│ set IDIGIT = JCHAR│
│ of 0,1,2,3,4,5,6,7,│   └──────────────────┘      74,75
│ 8 or 9           │          │  77  YES    ┌──────────────┐
└──────────────────┘          ▼  ┌──────────┤ set  ICODE = -999│
        │ NO          ┌──────────────┐──────┘  │    RETURN     │
        │             │ is JDONE = 1 │         └──────────────┘
        ▼             └──────────────┘
┌──────────────────┐       │ NO   78
│ set  ICODE = -999│       ▼         YES
│    RETURN        │  ┌──────────────┐──────────────────┐
└──────────────────┘  │ is JEXP =1   │                  │
                      └──────────────┘                  │
                          │ NO  79,83              85,88│
                          ▼                             ▼
              ┌──────────────────┐       ┌──────────────────┐
              │ set  X = X 10+IDIGIT│     │ set              │
              │    NEX = NEX+JPOINT │     │ EX = EX*10 +IDIGIT│
              │    JDIGIT = 1       │     │    JDIGIT = 2     │
              │    JPLUSF = 1       │     │    JPLUSE = 1     │
              │    JMINF  = 1       │     │    JMINE  = 1     │
              └──────────────────┘       └──────────────────┘
```

```
                JPLUSF = 1
                 JMINF = 1

JCHAR = +           SE = 1
                 JPLUSE = 1

JCHAR = 3        IDIGIT = 3
                     EX = 3
                 JDIGIT = 2
                 JPLUSE = 2
                  JMINE = 1
                 JPLUSF = 0
                  JMINF = 0
                 JPLUSE = 0
                  JMINE = 0
                 JPOINT = 0
                   JEXP = 0
                 JDIGIT = 0

RESULT =         1.0*FLOAT(150)*10.0**(1*3-1)
       =         1.0*150.*10.0²
                 15,000.  or 15.0E+3
```

8.2.6 Question Sixteen Question 16 requires the user to provide an answer of CW, CCW, or NONE. QUIZ, ISN537-561 has the task of prompting the user and finally setting values to the program variables JIND and IRT. Simply, if the answer is NONE, i.e., that the rotation direction during the skidding phase is none, the program sets JIND = 0 and IRT = 0. If the answer is CW, the program sets IRT = 3 and if the answer is CCW, the program sets IRT = 2. JIND will be discussed again under the discussion of the Subroutine START2 and SPIN2. A QUIZ route map for Question 16 is provided in Chart 6.

To determine the answer to Question 16, QUIZ, ISN543 calls the Subroutine ROTAT. This subroutine is designed to accept a string of up to 72 alpha-numeric characters and to determine if the line contains a ? or a $, or if the line is all blank or if the character string starts with a CC, CW, or an N. The flowchart for ROTAT follows.

CHART 6   QUIZ Route Map for Question Sixteen

ISN's
537

```
                    ┌─────────────────┐
                    │      START      │
                    └─────────────────┘
                             │
                             ▼                    538,539
                    ┌─────────────┐  YES  ┌──────────────────┐
                    │ is MENU = 1 ├──────▶│ present the long │
                    └─────────────┘       │ form of Q #16    │
                          │ NO            └──────────────────┘
                          ▼                         │
541,542   ┌──────────────────┐                      │
          │ present the short│                      │
          │ form of Q #16    │                      │
          └──────────────────┘                      │
                   │                                │
                   ▼                                ▼
          ┌───────────────────────────────────────────┐
          │ The user is prompted to type the          │
          │ character string CCW,CW, or NONE          │
          │ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─  │
          │ Subroutine ROTAT is called to act         │
     543  │ on the input and to return an             │
          │ integer value for ICODE                   │
          └───────────────────────────────────────────┘
```

544	is ICODE = 3	YES	set IRT(1) =1 (560)	
545	is ICODE = 2	YES	set IRT(1) = -1 (558)	GO TO Q#17
546	is ICODE = 1	YES	set IRT(1) = 0 JIND(1) = 0 (555,556)	GO TO Q#18
547	is ICODE = 0	YES		
548	is ICODE = -1	YES	REPEAT Q#16	
549	is ICODE = -2	YES	is JCURV(1) = 0 (553) YES	REPEAT Q#14
550	is ICODE = -999	YES	is JCURV(1) = 1 (554) YES	REPEAT Q#15
			set IRT(1) = 0 JIND(1) = 0 (555,556)	GO TO Q#18

```
          ┌───────────────────────────────────────────┐
          │ write message '9000' and RETURN to start   │
          └───────────────────────────────────────────┘
```

ROTAT Flowchart

ISN's

```
                    ┌─────────────┐
                    │    START    │
                    └──────┬──────┘
                           │
                    ╱──────▼──────╲
                   ╱      (A)       ╲
                   ╲    J = 1,72    ╱
                    ╲──────┬──────╱
                           │
        ┌──────────┐  ┌────▼──────────────┐
     5,6│          │  │ set LINE(J) = blank│
        └──────────┘  └────┬──────────────┘
                           │
                          (A)
                           │
                    ┌──────▼──────────────────────────┐
     7,8            │ read 72 characters from terminal input │
                    └──────┬──────────────────────────┘
                           │
                    ╱──────▼──────╲
      9            ╱      (B)       ╲
                   ╲    J = 1,72    ╱
                    ╲──────┬──────╱
```

ISN			
5,6	set LINE(J) = blank		
7,8	read 72 characters from terminal input		

			30,31
10	is LINE(J) a '?'	YES →	set ICODE = -1, RETRUN
	NO		32,33
11	is LINE(J) a '$'	YES →	set ICODE = -2, RETURN
	NO		

(B)

			16
13	(C) J = 1,72		
14	is LINE(J) a 'blank'	YES → (C)	set ICODE = 0, RETURN
	NO		19
18	is LINE(J) a 'N'	YES →	set ICODE = 1, RETURN * 'NONE'
	NO		22
21	is LINE(J) a 'C'	YES →	is LINE (J+1) a 'W'
	NO		YES 26
24	set ICODE = -999, RETURN	NO	set ICODE = 3

is LINE(J+1) a 'C'   → NO (to 24)   23

RETURN * 'CW'

YES ↓

set ICODE = 2   28

RETURN * 'CCW'

* NORMAL CASE

8.3 DAMAGE Subroutine

This discussion of the DAMAGE subroutine begins with an overview. Next the basic mathematical model is described, followed by a description of the computation of absorbed energy and the location of the impulse. A manual algorithm for these computations is also presented.

8.3.1 Overview The subroutine DAMAGE has the goal of producing values for the change in longitudinal and the change in lateral velocity of each vehicle involved during the impulsive phase of a crash. Figure 8.4 depicts this concept. In that figure a vehicle is illustrated in a pre-crash and post-crash state along with its velocity vectors. Delta V_x and Delta V_y are the outputs of DAMAGE.

V_0 = pre-crash velocity V_1 = post-crash velocity

Figure 8.4 |Delta V_x| and |Delta V_y|

The angular velocity Ω, which commonly is present in the post-crash instant is not calculated in DAMAGE.[4] The angular position or heading of the pre-and post-crash vehicle are identical, i.e., the vehicle is assumed to be not rotated during the crash instant. When the subroutine

---

[4] SAHP:CRASH does output the Delta Ω values.

START2 is explored, the angular velocity ω, and subsequent angular spin, or rotation, will be discussed.

A mathematical model is developed which argues that the linear impulse shared between two objects during the instant of impact may be evaluated in magnitude from the knowledge of the crush shapes of those objects (vehicles). If this model is accepted, the simple application of the principles of mechanics (physics) informs us that the vector 'linear impulse' applied to each vehicle equals each vehicle's 'change of linear momentum'. Linear momentum is defined as the product of a vehicle's mass and the linear velocity vector of that vehicle's center of mass. Linear impulse is defined as being the integral of the applied force-time pattern in magnitude and having the direction of the force vector. Figure 8.5 illustrates a vehicle which has an initial pre-crash linear momentum of $\overline{MV}_0$. and has applied to it a linear impulse $\hat{I}$. The second part of the figure illustrates the final linear momentum $\overline{MV}_1$ of the vehicle and the basic relationship of physics, that is $\overline{MV}_0 + \hat{I} = \overline{MV}_1$.

Figure 8.5   The Impulse-Momentum Relationship

Clearly, if $\hat{I}$ can be found from vehicle data after a crash, we may state that the vector Delta V equals the vector $\hat{I}$ divided by the mass of

the vehicle. Delta V_x and Delta V_y are the longitudinal and lateral components of Delta V and are treated as positive when directed in the forward and rightward directions respectively.

8.3.2 The Basic Mathematical Model of DAMAGE.

The DAMAGE subroutine is based upon the model of interaction of two particles of mass. Figure 8.6 illustrates the basic situation. In 8.6a two mass particles are pictured, each with a velocity along the line X-X, the velocity V_{10} is greater than the velocity V_{20}, so it is certain that mass #1 will overtake and strike mass #2. Both masses carry linear spring elements as shown. In 8.6b, the two particle's springs are just coming into contact and the origin of three displacement variables are shown. These displacements are X_1, the displacement of M_1, X, the displacement of the common end of the two spring elements, and X_2, the displacement of M_2. In 8-6c, the impact is underway, i.e., the mass #1 is still overtaking mass #2 and the springs are being deformed. In 8-6c the contact force between the springs is the common force F, i.e., F acts leftward on M_1 through the spring K_1 and F acts rightward on M_2 through the spring K_2.

Newton's Law of Motion applied to each mass produces the equations of motion   .

(8.1) $-K_1(X_1-X) = M_1 d^2X_1/dt^2$
and

(8.2) $K_2(X-X_2) = M_2 d^2X_2/dt^2$

Using the defined variable $\delta = X_1-X_2$ these two equations are reworked into the single equation

(8.3 $d^2\delta/dt^2 + (K_1K_2/(K_1 + K_2)) ((M_1 + M_2)/M_1M_2) \delta = 0$

Equation (8.3) is the well known, undamped simple harmonic oscillator equation having the general solution

(8.4) $\delta = (Amp) \sin(\omega t)$

> where: Amp is the amplitude of the oscillation and ω is the circular frequency of the oscillation in radians/second and

$$\omega = \sqrt{(K_1K_2/(K_1 + K_2)) ((M_1 + M_2)/M_1M_2)}$$

Figure 8.6   The Basic Model.   Two Oscillators Collide.

The time history of δ would be a simple sine wave of period $T = 2\pi/\omega$ and amplitude Amp if the interaction force was allowed to unload and enter the tensile region. This is not allowed in the basic model -- instead the force F is simply assumed to disappear after δ, the relative deflection, has reached the first extreme value, $\delta = $ Amp. The subsequent time history of δ is shown by the heavy curve in Figure 8.7.

Applying the known initial conditions,

$$(8.5) \qquad d\delta/dt)_{t=0} = V_{10} - V_{20}$$

the value of Amp is found:

$$(8.5a) \qquad Amp = (V_{10} - V_{20})\sqrt{(M_1 M_2/(M_1 + M_2))\ ((K_1 + K_2)/K_1 K_2)}$$

It is observed that $\delta = \delta_{max}$ = Amp when M_1 has been slowed and M_2 has been speeded until a common velocity, V_{com}, has been reached.

Figure 8.7   The Basic Deflection-Time History

The principle of conservation of momentum between the colliding particles may be applied, i.e.,

(8.6) $(M_1 + M_2)\, \overline{V}_{com} = M_1 \overline{V}_{10} + M_2 \overline{V}_{20}$,

to yield the common velocity.

During the collision process the two springs are known to deflect by the amounts $_1$ and $_2$ where

(8.7)] $\delta_1 = [K_2/(K_1+K_2)]\, \delta_{max}$,

(8.8)] $\delta_2 = [K_1/(K_1+K_2)]\, \delta_{max}$

(8.9) $\delta_1 + \delta_2 = \delta_{max}$

By writing

(8.10) $K_1 \delta_1^2/2 + K_2 \delta_2^2/2 = K_1 K_2^2 \delta_{max}^2/2(K_1+K_2)^2$
$$+ K_2 K_1^2 \delta_{max}^2/2(K_1+K_2)^2$$

we find that

(8.11) $(K_1 K_2/2(K_1+K_2))\, \delta_{max}^2 = K_1 \delta_1^2/2 + K_2 \delta_2^2/2$

where

   $K_1 \delta_1^2/2$ is the energy stored in spring K_1,

   and $K_2 \delta_2^2/2$ is the energy stored in spring K_2

44   CRASH2

Defining

(8.12) $E_1 = K_1 \delta_1^2/2$

and

(8.13) $E_2 = K_2 \delta_2^2/2$

we have that:

(8.14) $\Delta V_1 = V_{10} - V_{com} = (1/M_1)\sqrt{2(E_1+E_2)(M_1 M_2/(M_1+M_2))}$

and

(8.15) $\Delta V_2 = V_{com} - V_{20} = (1/M_2)\sqrt{2(E_1+E_2)(M_1 M_2/(M_1+M_2))}$

Fquations 8.14 and 8.15 predict the change in linear momentum $M_1 \Delta V_1$ and $M_2 \Delta V_2$ of masses M_1 and M_2 in the collision process described, and define the linear impulse to have been

(8.16) $\bar{I} = [\sqrt{2(E_1+E_2)(M_1 M_2/(M_1+M_2))}]\bar{i}$

The physics involved has been based upon the conservation of momentum between two bodies sharing an action-reaction type of common impulse. The fact that the impulse magnitude may be expressed by a function involving the sum of the energy absorbed by the two springs of the model DOES NOT indicate any use of a conservation of energy concept in the classical[5] sense!

CRASH2 would use this basic model for the calculation of Delta V's in the event that the vehicles shared a common, CENTRAL, impulse. In the field of mechanics the word CENTRAL modifies the word Impulse to describe an impulse vector whose line of action passed through the center of mass of the body to which it is applied. The impulse of a vehicle crash seldom passes through the center of mass of the vehicle,

---

[5]Editors Note: Energy dissipated is not considered to be conserved in the "classical" sense. However, a more general energy balance, including energy dissipated, is used elsewhere in this manual for accident reconstruction. While energy considerations are not discussed in this development, the assumption is made that the forces disappear once the maximum deflection is reached. The implication of this assumption is that the kinetic energy of the masses which was absorbed by the springs is not returned. For the development of this simple model, this assumption may be viewed as simply a physical constraint. For the more general application of this model, it is often helpful to view this assumption in the light of energy considerations.

and the basic model is modified to account for this fact. The modification is accomplished by multiplying the mass of the vehicle by a factor γ (gamma) where γ is calculated by equation 8.17. In Equation 8.17 the two symbols k_{gy} and h are, respectively, the radius of gyration of the vehicle in yaw and the center of mass offset of the impulse vector. Refer to Figure 8.8 for a visualization of these two terms.

(8.17) $\gamma = k_{gy}^2/(k_{gy}^2 + h^2)$

Figure 8.8   (a) Radius of Gyration in Yaw and
(b) Linear Offset of an I   impulse

If all of the mass of a vehicle were to be concentrated in the form of a hoop of mass M and centered at the vehicle's center of mass, that hoop would have the same angular momentum, given a rotation rate of radians per second as would the vehicle at the same rotation rate providing the radius of the hoop were k_{gy}. The linear offset of the line-of-action of an impulse from the center-of-mass is the perpendicular distance of the center-of-mass from that line of action

Given the γ_1 and γ_2, the Delta V values for CRASH2 vehicles are:

(8.18) $\Delta V_1 = (1/\gamma_1 M_1) \sqrt{2(E_1+E_2)(\gamma_1 M_1 \gamma_2 M_2/(\gamma_1 M_1 + \gamma_2 M_2))}$

and

(8.19) $\Delta V_2 = (1/\gamma_2 M_2) \sqrt{2(E_1+E_2)(\gamma_1 M_1 \gamma_2 M_2/(\gamma_1 M_1 + \gamma_2 M_2))}$

where E_1 and E_2 are the expressions for energy absorbed in vehicle #1 and vehicle #2 respectively. It remains understood that the directions of ΔV_1 and ΔV_2 are the directions of the mutual crash impulse (the direction of the principal force).

8.3.3 <u>Energy Absorbed and Impulse Location</u>. The basic mathematical model used to develop Equations 8.18 and 8.19 was seen to depend upon the idea of the mass of a vehicle being surrounded by a set of linear springs. CRASH2 effectively models the vehicle's energy absorption character by considering the vehicle as shown in Figure 8.9.

Figure 8.9 illustrates the plane view of a vehicle. The front surface is a distance XFF forward of the c.m. The rear surface is a distance XRR behind the c.m. The side surfaces are distances YSS left or right of the c.m. By considering the front, side or rear surfaces of the vehicle to be made up of arrays of linear springs extending $(XFF + (A/B)_{front})$ forward of the c.m., $(XRR + (A/B)_{rear})$ backward from the center of mass and $(YSS + (A/B)_{side})$ to either side of the c.m., and by giving these springs the appropriate spring constants, B_{front}, B_{rear} and B_{side}, the CRASH2 program arranges to calculate energy absorbed in the crush of the vehicle.

Imagine now that the crush shape of a vehicle is as shown in Figure 8.10.

In Figure 8.10 a triangular crush shape is shown on the front surface of a vehicle. The crush region has a total width of L. Over a small width of the crush, dL, a linear spring of constant B_{front} per unit width has been deformed by an amount δ where δ equals $(A/B)_{front} + C)$.

In this expression C is the crush dimension of the actual car and (A/B) front is the modeled extension of the linear spring beyond the vehicle's front surface.

The energy absorbed in the width dL is given as:

(8.20) $dE = (B/2)[(A/B) + C]^2 dL$

$= [(A^2/2B) + AC + (C^2 B/2)]dL$

$= (A^2/2B)dL + (AC)dL + (B(C/2)C)dL$

Figure 8.9   The Effective Linear Spring of CRASH2's Vehicle

The total energy absorbed is the sum of all the E's represented in the crush shape, and is seen to be

(8.21) $E = (A^2/2B)(L) + A(\text{Area of Crush}) + B(\text{First Moment})$

Figure 8.11 resketches Figure 8.10 but now illustrates Equation 8.21. The crush region has a centroidral coordinate of $\bar{x}$ from the front surface and an area called Area. The energy absorbed is modeled to depend upon the width(L), the area(Area) and the coordinate $(\bar{x})$.

Figure 8.10   A Crush Shape and a Spring Element

The model of basic energy absorbed is shown in Figure 8.11 and is written:

(8.22) $E = \text{Energy Absorbed} = GL + (A + B\bar{x})\text{Area}$

where   A = a system constant (equal to the force in the model spring when the spring is deflected an amount A/B)

B = a system constant (the spring constant per unit width of crush)

$G = A^2/2B$

L = width of crush

Area = area of crush (as seen in plan view)

$\bar{x}$ = centroidal coordinate of crush (from surface of vehicle.)

$$E = \frac{A^2}{2B} * L + (A+B*\bar{x}) \text{ Area}$$

Figure 8.11 Energy Absorption in Crush Region.

The second coordinate of the plane view area of crush is symbolized as D in Figure 8.11 and the linear offset of the impulse $\hat{I}$ is symbolized as h.

From geometry, the equation of the line of action of the impulse is

(8.23) $-x + (\cot\phi)y + (XFF-\bar{x}-D \cot\phi) = 0$

and the offset distance of the impulse from the c.m. is

(8.24) $h_{front} = |(XFF-\bar{x})\sin\phi - D\cos\phi|$

when the impulse is applied to the front of the vehicle.

For impulses applied to the side of the vehicle we have

(8.25) $h_{side} = |(YSS-\bar{y})\cos\phi - D\sin\phi|$

(note that $\bar{y}$ has replaced $\bar{x}$)

and for impulses applied to the <u>rear of the vehicle</u> we have

(8.26) $h_{rear} = |(XRR-\bar{x})\sin\phi + D\cos\phi|$

where XFF,$\bar{x}$,$\bar{y}$,YSS,XRR are absolute values.

CRASH2 modifies the energy previously calculated in 8.22 by multiplying by the factor

(8.27) $KK = (1 + \tan^2\phi)$ but ≤ 13.9

where ϕ is the angle between the impulse and the normal to the struck surface. This factor accounts for the fact that a greater than predicted force must be acting to create a given crush if the actual force is at an angle to the normal to the crushed surface. Thus the values of E found in 8.22 are multiplied by KK found in 8.27 before using the E values in 8.18 and 8.19.

<u>8.3.4 A Manual Version of DAMAGE</u> The subprogram DAMAGE may be exercised by a user aided only by a hand-held calculator by following the worksheets in Appendix D. These sheets follow the logic of CRASH2 directly. Their goal is to provide the student, who already has experience in field investigation of automobile crashes and in the preparation and use of data for the CRASH2 program, with sufficient understanding and experience to allow independent hand operation of the "Damage-Only" version of CRASH2. The materials needed, except for a calculator are included in Appendix D.

It is expected that the simplicity of the process of hand operation will materially aid the user in appreciation of the relative importance of the various data elements used by CRASH2 and with the manner that errors or uncertainties in the data may influence the results.

8.4 START2 Subroutine

8.4.1 Overview of START2 When the user of CRASH2 has answered Question 8 with a YES, a flag, IND, will be set which will cause MAIN to call the subroutine START2. START2 will in turn bring a number of other subroutines into action.

The situation with the user is this--he can supply answers to questions numbered 9, 10, 11, 14, 16, 18, 21, 23, 25, (27 or 28), and (29 or 30) at a minimum. This is equivalent to being able to sketch an impact (separation) position and heading, a rest position and heading, a ground tire friction value, and individual wheel lock-up ratios for both (or one, if only one) vehicles, as seen in Figure 8.12.

Figure 8.12 A Minimum Scene Sketch for Two-Vehicle Crash

Based upon this much information, or more, if the user can respond to other questions posed by QUIZ, START2 starts a process of estimating the linear (and angular) velocity of each vehicle at the instant of separation. The linear velocity estimate will be made in terms of two components US and VS, which are the longitudinal and the lateral components of the vehicle's velocity vector. The direction of the estimated velocity vector will be formed as one of the following:

a) along the line from the separation position to the rest position
b) along the line from the separation position to the end-of-skidding position given in answer to question 13 or 20
c) along the tangent to the circular arc at the separation position where the arc passes through the separation position, an intermediate point on the path, and either the end-of-skidding (if given) or rest position.

It is generally felt that cases b and c are better fits to reality than case a[6].

The linear velocity estimate made by START2 and its stable of subroutines is multiplied by the mass of the vehicle to produce an estimate of the vehicle's linear momentum at the instant of separation. This is done for both case vehicles. Linear momentum is a vector quantity and the vector sum of the two linear momentum vectors is the system momentum. It is known from the laws of physics that the system linear momentum is unchanged (conserved) during the crash phase when the only significant impulse applied to either body is the mutual, action-reaction impulse of the body-to-body contact. When this can be said to be the case, the original pre-crash linear momenta vectors may be calculated and the change in momentum (system impulse) also. Since the system impulse divided by the mass of one of the vehicles is the change in velocity (Delta V) of that vehicle, it appears that the individual vehicles precrash velocities as well as their Delta V's may be calculated. Figure 8.13 illustrates this paragraph.

This method is used providing the angle of intersection of the original heading lines, 1-1 and 2-2 in Figure 8.13, is not less than 15

---

[6]The direction of the estimated velocity vector is in fact altered from the direction stated above when there is an angular velocity at separation - see ISN 137, 138 in flow chart labeled SPIN2.

Figure 8.13 Application of the Conservation of System Momentum given an Internal to the System Impulse

degrees. When lines 1-1 and 2-2 are nearly parallel, it is not considered valid to carry out the construction above. When valid, the output appears under the heading "Spinout Trajectories and Conservation of Linear Momentum". When not valid due to a small angle between lines 1-1 and 2-2, the estimate of longitudinal change in velocity is made by the subroutine DAMAGE while the lateral velocity change is taken to be the lateral velocity component of the separation velocity. Output in this case is delivered under the heading "Spinout Trajectories and Damage".

When the user wishes to answer Question 31 with a yes, the secondary set of routines shown under SPIN2 in Figure 8.3 are called into the computation. SPIN2, which provides the estimated values of

velocity at separation now calls the the subroutine USMAC, supplying to USMAC the estimated linear velocity components and the yawing (angular) velocity along with all other required data in order that a mathematical model of the vehicle may be operated. In this model, the position and heading at separation, along with the longitudinal, lateral, and angular velocity estimates are used as initial conditions for a trajectory calculation. This calculation determines the tire force components at each wheel and the net longitudinal and lateral acceleration resulting from them as well as the angular acceleration due to the torque of the tire forces about the yawing axis. Numeric integration routines are used to solve the differential equations of motion as functions of time, producing velocity and displacement time histories. An example set of time histories of angular velocity and displacement is illustrated in Figures 8.14 and 8.15. The corresponding longitudinal, lateral, and total velocity magnitude time histories are illustrated in Figures 8.16, 8.17, and 8.18. The path of the center of mass is illustrated in Figure 8.19. (This set of data were generated by the routines USMAC, RNGRT1, DAUX, and TRAJ in a test run.)

Figure 8.14 Angular Velocity in Radians/Sec vs. Time in Seconds

Figure 8.15 Angular Displacement in Radians vs. Time in Seconds

In the mode of operation just introduced, the trajectory information developed by solving the governing equations of motion is compared with the known scene data. Based upon these comparisons, USMAC calculates an "improved" set of initial velocities, and runs a second, then a third, fourth, and fifth trajectory calculation--each time attempting to use an improved set of initial velocities. When the error criteria built into USMAC are satisfied, no further iteration is carried out, but in no case are more than five trajectory calculations made for each vehicle. Figure 8.20 illustrates the trajectories on a first through fifth iteration process by USMAC, overlaid on the true trajectory.

Figure 8.16 Longitudinal Velocity in Inch/Sec vs. Time in Seconds

Figure 8.17 Lateral Velocity in Inch/Sec vs. Time in Seconds

Figure 8.18 Total Velocity in Inch/Sec vs. Time in Seconds

Figure 8.19   North Displacement versus East Displacement (inches)

X

250

INCHES by INCHES

200

150

IC's True Path
U = 176.0, V = 44.0, $\dot{\psi}$ = 2.00

IC's Fifth Path
V = 174.3, V =45.1, $\dot{\psi}$ = 2.02

100

50

0

50   100   150   Y

Figure 8.20   Iterated Trajectories and True Trajectory

8.4.2 <u>Operations of START2 and SPIN2</u> Review of the operation of START2 and SPIN2 may be accomplished by study of the subroutine flow charts which follow. Recall that the entire purpose of START2 and SPIN2 is to transform scene data for each vehicle into an estimated velocity state at the instant of separation. This is entirely a geometric/ kinematic task and can be readily accomplished by hand. A fuller appreciation of the process will be had by actually carrying out, by hand, a process which parallels that of the computer program. A step-by-step guide for doing this is presented in Appendix E.

8.4.3 <u>Flowcharts.</u> This section presents annotated flowcharts for the subroutines: START2, SPIN2, USMAC, RNGKT1, DAUX, TRAJ, and OBLIQE.

Special notice should be given to the step in SPIN2 known as ISN 137. Recall that, if the vehicle was known to rotate (see QUIZ, Question 16 or 23), an estimated angular and linear velocity at separation will be found by the modified Marquand procedure (see SPIN2, ISN 73-134). Also, SPIN2 ISN 61-62 or ISN 69 will have produced an azimuth angle, GAMS, for this linear velocity vector. Now, if the vehicle was known to rotate, but the path from separation to end-of-skidding was taken to be straight, a correction to GAMS is made by ISN 137. This correction is that GAMS (in radians) is to be reduced by the amount (20 * PSISD / SSDOT). The CRASH2 program recognizes by this step that the path must be curved if there is an initial angular velocity, even though there is no scene data on its curvature.

```
                              - - - - -

START2's
 ISN's         ┌─────────────┐
               │    START    │
               └──────┬──────┘
                      │
               ┌──────▼──────────────────────────────┐
   5,6         │ set THETA1 = average of Veh #1 lockups│
               │     THETA2 = average of Veh #2 lockups│
               └──────┬────────────────────────────────┘
                      │
               ┌──────▼──────┐  YES  ┌──────────────────────────────────────────┐
   7           │ is IFLAG(1) = 1├────►│ DIST1 = [(XCR1-XC11)²+(YCR1-YC11)²]⁰·⁵    │
               └──────┬──────┘        └───────────────────┬──────────────────────┘
                      │ NO                                 │
               ┌──────▼──────┐        ┌───────────────────▼──────────────────────┐
   8,9         │ set DIST1 = 0│       │ S1D = [64.4·THETA1·MU·DIST1]⁰·⁵          │
               │     S1D = 0  │       └───────────────────┬──────────────────────┘
               └──────┬──────┘                            │
                      │◄────────────────────────────────────┘
                      │
               ┌──────▼──────┐  YES  ┌──────────────────────────────────────────┐
               │ is IFLAG(2) = 1├────►│ Dist2 = [(XCR2-XC12)²+(YCR2-YC12)²]⁰·⁵   │
               └──────┬──────┘        └───────────────────┬──────────────────────┘
                      │ NO                                 │
               ┌──────▼──────┐        ┌───────────────────▼──────────────────────┐
               │ set DIST2 = 0│       │ S2D = [64.4·THETA2·MU·DIST2]⁰·⁵         │
               │     S2D = 0  │       └───────────────────┬──────────────────────┘
               └──────┬──────┘                            │
                      │◄────────────────────────────────────┘
                      │
               ┌──────▼──────┐ YES  24-30              YES
   20          │ is JCURV(1) = 1├──►│ is d < 1.0 in ├──────────┐
               └──────┬──────┘      └──────┬────────┘          │
                      │ NO                 │ NO   31-32         │
               ┌──────▼──────┐      ┌──────▼──────────────┐ YES│
   21,22       │ set P1 = 0  │      │ is D²/8d + d/2>10000 in├──┤
               │     Q1 = 0  │      └──────┬──────────────┘    │
               └──────┬──────┘             │         34        │
                      │             ┌──────▼──────────┐        │
                      │             │ set JCURV(1) = 0│◄───────┘
                      │             └──────┬──────────┘
                      │     ┌──────────────┤
                      └────►│ path is straight│
                            └──────┬─────────┘
               ┌──────────────────▼─┐
   36,46       │ calculate (P1,Q1) as│
               │ center of circle thru│
               │ separation, intermed.│
               │ and end-of-rot. pt.  │
               └──────┬──────────────┘
                      │◄────────────────┘
               ┌──────▼──────┐
   47          │ CALL SPIN2  │
               └──────┬──────┘
               ┌──────▼──────┐  YES  ┌──────────────────────────────┐
   48          │ is JSTOP = 0├──────►│ ISN 51 thru 81 carryout same  │
               └──────┬──────┘       │ calculations for VEH #2       │
                      │ NO  set ISTOP = 1 └──────────┬───────────────┘
               ┌──────▼──────────┐                    │
   49,50       │ GO TO MAIN ISN15│◄──── NO  ┌──────────▼──────┐
               │     ABORT       │  set ISTOP = 2 │ is JSTOP = 0 │
               └──────┬──────────┘          └──────────┬──────┘
                      │                                 │ YES
                      │◄────────────────────────────────┘
                    ( I )
```

X axis diagram (scene data):

P_1 (sep. pt)
P_2 (inter. pt)
P_3 (end-of-ro

C (P,Q)
center of
arc

scene data

(I)

START2's
ISN's

```
set S1 = XC10        T1 = XC20
    S2 = YC10        T2 = YC20
    S3 = PSI10*57.3  T3 = PSI20*57.3
    S4 = US1/17.6    T4 = US2/17.6
    S5 = VS1/17.6    T5 = YS2/17.6
    S6 = PSID1       T6 = PSID2
```

94 | is JTYP(1) = 8 or JTYP(2) = 8 — YES

NO

95 | $\Delta\phi$ = Abs $(\psi_{10} - \psi_{20}).57.3$

96 | is $\Delta\phi > 360$ — YES → 99 | $\Delta\phi = \Delta\phi - 360$

NO

97 | is $\Delta\phi > 180$ — YES → 101 | $\Delta\phi = 360 - \Delta\phi$

NO

102 | CONTINUE

NO

104 | is $170 < \Delta\phi < 190$ — YES

NO   105

105 | is $-10 < \Delta\phi < 10$ — YES

NO

109 | CALL DAMAGE

106 | CALL OBLIQUE

110-112
```
JSPIN = 2
U10 = US1-DELVX1
U20 = US2-DELVX2
```

107
```
JSPIN = 1
```

CALL PROXIM

```
set PSISD1 = 0          PSISD2 = 0
    XCSP1f = XC10       XCSP2F = XC20
    YCSP1F = YC10       YCSP2F = YC20
    U10MPH = U10/17.6   U20MPH = U20/17.6
    V10MPH = 0          V20MPH = 0
    US1MPH = US1/17.6   US2MPH = US2/17.6
    VS1MPH = VS1/17.6   US2MPH = VS2/17.6
```

GO TO MAIN, ISN15

64 - CRASH2

SPIN2

6-31

Place values into registers

NO ── is JVEH = 2 ── YES

33-40

Place Veh #1 parameters
into registers

42-49

Place VEH #2 parameters
into registers

50-53

convert FEET to INCHES

55-56

Calculate Δy and Δx from separation to end-of-rotation

57 | is Abs $(\Delta x) < 0.001$ ──────────────→ set $\Delta x = 0.001$

NO

59 | is path from separation to end-of-rotation straight

58 & 60-67   NO   YES   58 & 69-70

JCV = 1

Find S1, GAMS
(see notes)

JCV = 0

Find S1, GAMS
(see notes)

72

is vehicle rotation flag IRT = 0,(vehicle did not rotate) ── YES

NO

73 | TEMP1 = PSI1 - PSIS
= heading at end-of-rotation minus
heading at separation.

74 | STMP1 = $\dfrac{TEMP1}{ABS\ (TEMP1)}$ +1 if TEMP1 is positive
-1 if TEMP1 is negative

(I)   (II)

CRASH2 - 65

(I)

is ABS (TEMP1) ≤ 2π →YES

NO ↓

76,77
$$TEMP1 = TEMP1 - 2\pi * STMP1$$
$$TEMP2 = TEMP1 - 2\pi * STMP1$$

$$TEMP1 = TEMP1$$
$$TEMP2 - TEMP1 - 2\pi *STMP1$$

Set SROT = 1 for CW or NO rot. direction
      = -1 for CCW   rot. direction

79
NO ← is SROT = STMP1 → YES

82
$$DPSI = TEMP2 + 2\pi \left[JIND(JVEH)*IRT(JVEH) \right]$$

80
$$DPSI = TEMP1 + 2\pi \left[JIND(JVEH*IRT(JVEH) \right]$$

83
Set ADPSI = ABS (DPSI)

is ADPSI ≤ 0.08 →YES→ (II)

NO ↓

85
NO ← is THETA ≥ 1 → YES

88-91
$$Set \quad a = (1 - THETA)*8.52*10^{-4}$$
$$b = .94 - .23*THETA$$
$$c = 40.64 - 8.64*THETA - \frac{S1}{ASPSI}$$
$$Find \ PP = [-b + (b^2-4ac)^{\frac{1}{2}}] / 2a$$

86
$$Set \ PP = 1.408\left(\frac{S1}{ASPSI} - 32.0\right)$$

Set P(1) = 0.70 PP
    P(2) = 0.85 PP
    P(3) = 1.00 PP
    P(4) = 1.14 PP
    P(5) = 1.30 PP

These values are used in CRASH2 as preliminary estimates of the ratio of linear to angular velocity of the vehicle at separation

(III)

(II)

The relationship between S1/ADPSI and SSD/PSID

The dotted line plots the relationship $\dot{S}_0/\dot{\psi}_0 = S1/ADPSI$ commonly used as a rule of thumb.

A set of $\alpha(I,J)$, $I = 1,5$ will be calculated for each P(J) above.

III

97

A

J = 1,5

98

is P(J) > 250   YES

NO

99

B

I = 1,5

102

C

I = 1,5

100

$$\alpha(I,J) = AA(I,1) + AA(I,2) * P(J)$$
$$+ AA(I,3) * P(J) * P(J)$$
$$+ AA(I,4) * P(J) * P(J) * P(J)$$

103

$$\alpha(I,J) = KKKK(I)$$

B

C

104

is Abs (THETA) < 0.001   YES

108   NO

is
$$\left(\frac{S1}{ADPSI(1 = 2 \ (DIST + \sqrt{DIST} \ \sqrt{(DIST+S1)} \ )/S1} \right)$$

LESS THAN OR EQUAL TO
$$\left\{ \frac{4 * \alpha(1,J) * K2 * THETA}{\alpha(5,J) * \alpha(5,J) * \alpha(2,J) * WB} \right\}$$

YES

109

Set BETA (J) = 10

NO

See Notes on next sheet
Assume $\dot{S}_{SEP}$ and $\dot{\psi}_{SEP}$ are known
and that SSD(J) = $\dot{S}_{SEP}$ and PSID(J) = $\dot{\psi}_{SEP}$

A

$$Set \ BETA(J) = \left\{ P(J) \left| \frac{PSID(J)}{SSD(J)} \right| - 1. \right\}$$

124,125

Set N = 1
BMIN = $\left| BETA(1) \right|$

IV

IV

II

126 D
J = 2,5

127
is | BETA (J) | < BMIN   YES→

128, 129
Set BMIN = | BETA (J) |
N = J

NO↓

D

131
is BMIN ≥ 10.   YES→

132,133
Set JSTOP = 1
RETURN

134,135 NO↓

Set $\dot{\psi}_{SEP}$ = PSISD = PSID(N)
$\dot{S}_{SEP}$ = SSDOT = SSD(N)

136
is JCV = 1, i.e., is path separation to end-of-rotation curved?

NO 137,138
GAMS = GAMS - 20.*PSISD/SSDOT
PSISD = PSISD *57.3

YES

140
Set PSISD = 0

141,142
$$TEMP1 = \left\{ \begin{array}{l} \sin^2(GAMS - PSIS) \\ + THETA^2*\cos^2(GAMS - PSIS) \end{array} \right\}^{1/2}$$

$$SSDOT = \left\{ 772.8*MU*S1*TEMP1 + S1D^2 \right\}^{1/2}$$

143,144
US = SSDOT*cos(GAMS - PSIS)
VS = SSDOT*sin(GAMS - PSIS)

145
is JTRAJ = 0, i.e., did user answer
Q #31 saying "DO NOT RUN A TRAJECTORY CALCULATION"   YES→

146 NO↓
CALL USMAC

145
RETURN

RETURN TO START2, ISN48

The user should be aware of the fact that SPIN2 is going to return estimated values of vehicle's longitudinal and lateral linear velocity and the estimated value of the vehicle's angular velocity, all for the moment of separation!

If no scene data is entered this will not happen! If scene data is entered and Q #31 is answered with a "NO", the velocities returned are estimated only! If Q #3 is answered "YES" the velocities returned are those producing the least error in an actual trajectory calculation!

ISN's

    1   - The Subroutine statement itself
   2-8 - Common, Equivalence Dimension and Real statements
  9-13 - system constants
 14-18 - variable changes
 19-24 - error weighting constants
 25-26 - system constants
 27-39 - vehicle #1 variable and dimension changes
 40-50 - vehicle #2 variable and dimension changes

---

51   Set INDXB = 1

TERRAIN FRICTION BOUNDARY LINE

Equation of line is:

$$Y = XI*X + FNUM$$
(see ISN 57,56 below)

52   given XBP(1), XBP(2), YBP(1), YBP(2)
calculate XBMXB

53   is XBMXB = 0   →   54,55   set   INDXB = 0
                                            XPP = XBP(1)*12

57,58   set   XI = (YBP(2)*12-YBP(1)*12)/XBMXB
                 FNUM = YBP(1)*12-XI*XBP(1)*12

ISN 59

Here a set of trajectory calculations will begin
On the first pass the IC's are XCP1, YCP1, PSI1
                        and U1, V1, PSID1
where the user supplied the positional data and
SPIN2 supplied the velocity data. The program
will calculate a trajectory, determine errors
based on given scene data and generate a new
set of initial velocity data. With this the program
returns, up to 4 times to ISN59 and repeats the
process. If a set of error calculations are satisfied
first, it may not return to ISN59.

(f)

Ⓘ

59-66
```
set    XCP1  = XSP
       YCP1  = YSP
       PSI1  = PSISP
       U1    = USP
       V1    = VSP
       PSI1D = PSISDP
       T     = 0.0
       ICOUNT = ICOUNT+1
```

> the variables on the left are the 'running variables', i.e., they change with time.

67-70
Calculate $DISTM = \left\{(X2P-XCP1)^2+(Y2P-YCP1)^2\right\}^{0.5}$
$X2PP = XCP1$
$Y2PP = YCP1$
$JROT = 0$

71 CALL RNGKTI, with INIT = 1

72 CALL RNGKT1, with INIT = 2

> distance from the 'running point' to the intermediate point.

> The Subroutine RNGKT1, discussed later will be repeatedly called and return the six variables above (XCP1, YCP1, PSI1, U1, V1 and PSI1D)
>
> This Subroutine, USMAC, gets this data at ISN 73 and must test it to decide if it should order time to continue and a new set be returned or order something else to happen

73 is JCURV(JVEH) = 0 — YES

↓ NO

74 Calculate $DISTT = \left\{(X2P-XCP1)^2+(Y2P-YCP1)^2\right\}^{0.5}$

75 is $DISTT \geq DISTM$ — YES

↓ NO

76,78
```
set    X2PP = XCP1
       Y2PP = YCP1
       DISTM = DISTT
```

79 is $T \leq 8.0$ — YES

↓ NO

80,81
```
set    JERR(JVEH) = 2
       RETURN
```

ⒾⒾ

(II)

82
is JROT = 1 → YES →

NO ↓

83 | is abs(PSI1D)>.01745 → YES →

NO ↓

84-87
```
set  X1PP = XCP1
     Y1PP = YCP1
     PSI1PP = PSI1
     JROT = 1
```
→

88 | is JROT = 0 → YES → GO TO ISN72 (above)

NO ↓

89 | Calculate
$$U1V1SQ = (U1 \cdot U1 + V1 \cdot V1)$$

↓

90 | is U1V1SQ ≤ 100. → YES → GO TO ISN72

NO ↓

91-93
```
set   XRPP = XCP1
      YRPP = YCP1
      PSIRPP = PSI1
```

CALCULATED
The reader notes that at this point, if reached, we have saved

INTER POINT
X2PP
Y2PP } based on nearest approach of running point
DISTM } to the intermediate point, if any.

END-OF-ROT. POINT
X1PP
Y1PP } based on running position data when the
PSI1PP } angular velocity fell below 1°/sec.

REST POINT
XRPP
YRPP } based on running position data when the
PSIRPP } linear velocity fell below 10 in /sec.

$$E(1) = d_1/D_1$$

$$E(2) = d_2/D_2$$

$$E(3) = \frac{(\Psi_3 - \psi_3)}{(\Psi_3 - \Psi_0)}$$

$$E(4) = \frac{(\Psi_4 - \psi_4)}{(\Psi_4 - \Psi_0)}$$

$$E(5) = d_5/D_5$$

SEPARATION $(XSP, YSP, PSISP = \Psi_0)$

$$Q = E(1) + 1/2E(2) + E(3) + 1/2E(4) + (1/4E(5) \text{ or } 0$$

ISN's
94-96 | calculate
$$E(1) = \left\{(XRP-XRPP) \cdot (XRP-XRPP) + (YRP-YRPP) \cdot (YRP-YRPP)\right\}^{0.5} \div$$
$$\left\{(XRP-XSP) \cdot (XRP-XSP) + (YRP-YSP) \cdot (YRP-YSP)\right\}^{0.5}$$

97-99 | calculate
$$E(2) = \left\{(X1P-X1PP) \cdot (X1P-X1PP) + (Y1P-Y1PP) \cdot (Y1P-Y1PP)\right\}^{0.5} \div$$
$$\left\{(X1P-XSP) \cdot (X1P-XSP) + (Y1P-YSP) \cdot (Y1P-YSP)\right\}^{0.5}$$

100 | calculate
$$E(3) = (PSIRP-PSIRPP) \div (PSIRP-PSISP)$$

101 | calculate
$$E(4) = (PSI1P-PSI1PP) \div (PSI1P-PSISP)$$

III

III

102 | is JCURV(JVEH) = 0 —YES→ E(5) = 0

NO ↓

103-104 | calculate
$$E(5) = DISTM \div \left\{ (X2P-XSP) \cdot (X2P-XSP) + (Y2P-YSP) \cdot (Y2P-YSP) \right\}^{0.5}$$

107 | calculate
$$Q = C1 \; Abs(E(1)) + C2 \; Abs(E(2)) + C3 \; Abs(E(3)) + C4 \; Abs(E(4)) + C5 \; Abs(E(5))$$
where $C1 = 1.0, C2 = 0.5, C3 = 1.0, C4 = 0.5, C5 = 0.25$

108 | Set NRUNS(JVEH) = ICOUNT

109-118 | The summary that is titled "VEHICLE #   TRAJECTORY ITERATION #" is next printed on the terminal. Iteration #1 is a result using the Marquard estimates for initial conditions at separation

Note QMIN was set at 5.0 during initialization. The following lines replace 5.0 with Q if Q 5 and on next iterations replace QMIN with Q whenever the Q of an iteration is less than the QMIN then stored

119 | is Q<QMIN —YES→ QMIN = Q

NO

121-125 | Set
US = USP
VS = VSP
PSISD = PSIDP 57.3
SSDOT = SSDOTP
GAMS = GAMSP

126 | is JVEH = 2 —YES→

NO ↓

127-129 | Set E1(1) = E(1)
E1(2) = E(2)
E1(3) = E(3)
E1(4) = E(4)
E1(5) = E(5)
Set QMIN1 = QMIN

131-133 | Set E2(1) = E(1)
E2(2) = E(2)
E2(3) = E(3)
E2(4) = E(4)
E2(5) = E(5)
Set QMIN2 = QMIN

IV

IV

144-177

E1G = GAMR-GAMRP	NOTE ERROR in ISN 148,149
E1L = (LR-LRP)/LRP	but not less than -.99
E2G = GAM1-GAM1P	NOTE ERROR in ISN 158,159
E2L = (L1-L1P)/L1P	but not less than -.99
E3 = E(3)	but not less than -.99
E4 = E(4)	but not less than -.99
E5G = GAM2-GAM2P	or zero if no intermed. point
E5L = (L2-L2P)/L2P	or zero if no intermed. point

178-185

$$A11 = \sqrt{1.+E1L}$$
$$A12 = \sqrt{1.+E2L}$$
$$A15 = \sqrt{1.+E5L}$$
$$A23 = \sqrt{1.+E3}$$
$$A24 = \sqrt{1.+E4}$$
$$A31 = E1G$$
$$A32 = E2G$$
$$A35 = E5G$$

189-194

$$SSD\phi TP_{new} = SSD\phi TP_{old} * \left[4\left(\frac{LR}{LRP}\right)^{\frac{1}{2}} + 2\left(\frac{L1}{L1P}\right)^{\frac{1}{2}} + \left(\frac{L2}{L2P}\right)^{\frac{1}{2}} \right] * \frac{1}{7}$$

$$PSISDP_{new} = PSISDP_{old} * \left[2\left(1 + \frac{\Psi_3 - \psi_3}{\Psi_3 - \psi_0}\right)^{\frac{1}{2}} + \left(1 + \frac{\Psi_4 - \psi_0}{\Psi_4 - \psi_0}\right)^{\frac{1}{2}} \right] * \frac{1}{3}$$

$$GAMSP_{new} = GAMSP_{old} + \left[4(GAMR-GAMRP) - 2(GAM1-GAM1P) + \underline{\hspace{3cm}} \right.$$
$$\left. (GAM2-GAM2P) \right] * \frac{1}{7}$$

$$USP_{new} = SSD\phi T_{new} * \cos(GAMSP_{new} - \psi_0)$$

$$VSP_{new} = SSD\phi T_{new} * \sin(GAMSP_{new} - \psi_0)$$

GO TO ISN59

ISN's

```
        ┌─────────────────────────────┐
        │ Set  SFX1  = 0.0            │
        │      SFY1  = 0.0            │
        │      SFN1  = 0.0            │
  7,14  │      SFX1T = 0.0            │
        │      SFY1T = 0.0            │
        │      SFN1T = 0.0            │
        │      SPSI1 = sin(PSI1)      │
        │      CPSI1 = cos(PSI1)      │
        └─────────────────────────────┘
                      │
                      ▼
        ┌─────────────────────────────┐
  15    │      CALL   TRAJ(JVEH)      │
        └─────────────────────────────┘
                      │
                      ▼
        ┌─────────────────────────────┐
        │ Set  SFX1 =  SFX1T          │
 16,18  │      SFY1 =  SFY1T          │
        │      SFN1 =  SFN1T          │
        └─────────────────────────────┘
                      │
                      ▼                        YES
        ┌─────────────────────────────┐───────────────────────┐
  19    │      is   JVEH = 2          │                       │
        └─────────────────────────────┘                       ▼
                    NO │                              23,24
                      ▼                        ┌─────────────────────────────┐
        ┌─────────────────────────────┐        │ Set  FIZ   = FIZ2           │
 20,21  │ Set  FIZ   = FIZ1           │        │      FMASS = FMASS2          │
        │      FMASS = FMASS1         │        └─────────────────────────────┘
        └─────────────────────────────┘                       │
                      │                                        │
                      ◄────────────────────────────────────────┘
                      ▼
```

```
        ┌──────────────────────────────────────┐
        │ DPSI1D  =  SFN1 ÷ FIZ                 │
        │ DPSI1   =  PSI1D                      │
 25,30  │ DU1     =  SFX1 ÷ FMASS + V1 * PSI1D  │
        │ DV1     =  SFY1 ÷ FMASS - U1 * PSI1D  │
        │ DXCP1   =  U1 * CPSI1 - V1 * SPSI1    │
        │ DYCP1   =  U1 * SPSI1 + V1 * CPSI1    │
        └──────────────────────────────────────┘
```

┌────────────────────────────┐
│ Here are the six first   │
│ order differential   │
│ equations of motion   │
└────────────────────────────┘

```
                      │
                      ▼
                ┌──────────┐
  31            │  RETURN  │
                └──────────┘
```

┌───┐
│ DER(1) = DXCP1   VAR(1) = KCP1   │
│ DER(2) = DYCP1   VAR(2) = YCP1   │
│ DER(3) = DPSI1   VAR(3) = PSI1D   │
│ DER(4) = DPSI1D   VAR(4) = PSI1D   │
│ DER(5) = DU1   VAR(5) = U1   │
│ DER(6) = DV1   VAR(6) = V1   │
└───┘

Subroutine TRAJ has the EQUIVALENCE statements and the same COMMON statements as subroutine USMAC and thus has available all values of variables known there.

ISN's

8-15

Set XC = XCP1 } c.m. coordinates
    YC = YCP1 }
    PSIC = PSI1   heading angle
    U = U1 } c.m. velocity components
    V = V1 }
    PSICD = PSI1D   angular velocity in yaw
    CSPSIC = CPSI1 } set in DAUX to be cos(PSI1)
    SNPSIC = SPSI1 }   sin(PSI1)

17-21

Set A = distance, c.m. to front axle   ) from
    B = distance, c.m. to rear axle   ) internal
    TRD2 = distance, c.m. to wheels, laterally   ) data
    FMASS = mass   ) tables
    FIZ = mass moment of inertia in yaw   )

Set SGNU = U ÷ Abs (U)

Set UD2 = Abs (U/2.)

Identify three new variables as shown

TRCPSC

c.m.

TRSPSC

TRPSCD = PSICD*TR2

33-35

Set Force and Moment Totals to Zero

TFX = 0

TRD2

c.m.

TMOM = 0

TFY = 0

I

ISN's

$$I$$

36   set AOMB = A

37   Set SGNI = 1

38-41   Calculate vertical tire loads
WW(1)) Static values
WW(2)) no bounce
WW(3)) no roll
WW(4)) no pitch

42   (A)   I = 1,4

43   set SGNI = -1.0*SGNI

44   is (I - 2) > 0   —YES→   set AOMB = -B   45

NO

46-47   set XW = XC + AOMB*CSPSIC + SGNI*TRSPSC
YW = YC + AOMB*SNPSIC - SGNI*TRCPSC
(the coordinates of the Ith wheel-print)

48   INDXB   — / + / 0

$+$ XPP = FNUM*XW/(YW - XI*XW)   51

52   YPP = YW*XPP/XW   (XPP,YPP) are coordinates of intersection of terrain boundary line with a ray from the origin and through the wheelprint

49,50   Set ISTOP = 15
RETURN

53,54   Set SX = U + SGNI*TRPSCD
SY = V + AOMB*PSICD   longitudinal and lateral velocity components of tire

55   RHOO = $\left[XW*XW + YW*YW \right]^{0.5}$ distance origin to tire

56   RHOP = $\left[XPP*XPP + YPP*YPP \right]^{0.5}$ distance origin to boundary along line thru tire

$$II$$

ISN's

(II)

57 | is RHOO > RHOP | — YES

NO

58 | set XMUS = MU | 60 | set XMUS = MU2 |

61,62 | $S = [SX*SX + SY*SY]^{0.5}$
 $XMU = XMUS*(1.0 - CMU*S)$ | velocity dependent <br> tire-road friction coef.

63,64 | Set XNUM = SY
     XDEN = Abs (SX) |

65 | is XDEN = 0 | — YES →   66 | is XNUM = 0 | — YES → | RATIO = 0 |

NO   69   NO

71 | RATIO = $\dfrac{XNUM}{XDEN}$ | | RATIO = 1.57 |

NOTICE:
 ISN 71 should be written
 RATIO = ATAN2 (XNUM,XD1)

72 | ALFA = RATIO - STEER (JVEH,I)*SGNU | ← ALFA is the slip <br> angle of Ith wheel

73 | F = XMU*WW(I) | max. tire shear force, Ith wheel

74 | TIF = -LW (JVEH,I)*XMUS*WW(I) | shear force component in <br> plane of the tire, Ith wheel

braking 75   driving

83 | FCOS = F*cos (ALFA) | ← ◇ TIF ◇ → | is (TIF - F) > 0 | — YES →

  0 ↓   free roll   NO
                    76   79

| FC = 0.0 | | FC = TIF | | FC = F |

84 | is (UD2 - 1.0) ≥ 1.0 | — YES →

NO   87

85 | set SMAL = UD2 | | set SMAL = 1 |

                               91

88 | is Abs (TIF) > FCOS | — YES → | FC = -FCOS*SGNU*SMAL |

NO   89

| FC = TIF*SGNU*SMAL |

(III)

III

Back to
ISN 42

is Abs(FC) < Abs(F) - 1.0

95
is Abs(U) > 0.5 → YES

NO
96
is Abs(V) > 0.5 → YES

NO

93
FS = 0.

97
$$FSMX = \left[F*F - FC*FC\right]^{0.5}$$

98
BETB = CSTF(JVEH,I)*ALFA ÷ FSMX ← this is the ratio of the maximum tire side force predicted by a linear model to the maximum possible tire side force

99
is Abs(BETB) > 3.0 → YES

NO

100   saturated tire
FS = FSMX*SIGN(1.0,BETB)

102   not saturated tire
FS = FSMX*BETB(BETB*BETB/27. - Abs(BETB)/3. + 1.)

linear model
slope = CSTF

$\dfrac{FS}{FSMX}$ 1.

-3. -2. -1.   1. 2. 3.

BETB

-1.

Set
    CSPSI = cos [STEER(JVEH,I)]
103-106   SNPSI = sin [STEER(JVEH,I)]
    FY = FS*CSPSI + FC*SNPSI
    FX = -FS*SNPSI + FC*CSPSI

Set TFX = TFX + FX
107-109   TFY = TFY + FY
    TMOM = TMOM + SGNI*TRD2*FX + AOMB*FY

Each pass thru the
loop adds the next
tire's contribution

A

Set SFX1T = TFX
111,113   SFY1T = TFy
    SFN1T = TMOM

114   RETURN

2,3

given M1,US1,VS1,ψ1 and
M2,US2,VS2,ψ2 calculate
A and B as the X-dir.
and Y-dir, components of
TOTAL SYSTEM LINEAR
MOMENTUM> (POST CRASH)

4

$$\theta = \psi_1 - \psi_2$$

If the original system momentum
had a component perpendicular to
the axis of VEH #2, it was.

$$M1 \cdot V10 \cdot \sin\theta = |\overline{ac}| - |\overline{be}|$$
$$= B \cos\psi_2 - A \sin\psi_2$$

SOLVE for U10

5

6

if the original system momentum
nad a component perpendicular to
the axis of VEH #1, it was

$$M2 \cdot U20 \cdot \sin\theta = |\overline{ab}| - |\overline{de}|$$
$$= A \sin\psi_1 - B \cos\psi_1$$

SOLVE for U20

Summary. This point marks the culmination of the readers
knowledge of accident reconstruction in general, and the CRASH
program as a particular tool. No two accidents are exactly
alike. Each case is special. This chapter attempts to
summarize the major considerations and the basic computations
which have been presented.

At this point if might be helpful to step back and take an overview
of the operation of CRASH. This is done in the first section of this
chapter. The following two sections discuss specific considerations in
each of the program's two major modes: the damage-based analysis and
the trajectory based analysis.

9.1 Overview

The major strength of the CRASH program is the capability for a
parallel analysis using damage information on the one hand and
trajectory data on the other. An overview of the CRASH flowchart is
shown in Figure 9.1. The damage analysis path is on the right and
trajectory on the left. Both result in estimates of Delta V for both
vehicles, but only the trajectory analysis produces estimates of the
pre-impact velocities.

Each analysis has it's relative strengths. Historically, the
trajectory analysis using conservation of linear momentum was applied
first. This approach works well for oblique, or intersection-type,
collisions. However, when the pre-impact velocities are with $\pm 10°$ of a
colinear (head-on) configuration, the momentum computation becomes very
sensitive to the measurement of this angle. It was pointed out in the
last chapter that the damage based estimates of the longitudinal
velocity are automatically used in this case.[1] The difficulty, of
course, lies in the need for complete information on the post-impact
trajectories of the two vehicles. Estimation of the velocities at
separation from these data is often difficult. The various computations
used in START2 are highlighted in Section 9.3.

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[1] Unless the third character of the CDC for vehicle 1 is an "R" or
"L" in which case the test for colinear collisions is inadvertently
bypassed.

Figure 9.1 Program Logic of CRASH2

2 - APPLICATION

As limited information on the structural stiffness of vehicles became available, the damage-based analysis was developed. This approach is best suited to the colinear configuration. Consequently the damage analysis nicely complements the trajectory-based computation. However, the damage computation also has limitations. As the collision forces act further from the center of mass (non-central collisions) and angles of 45° or more from the normal to the surface, this computation also becomes sensitive to the input information. These considerations will be discussed in the next section.

Before looking at considerations unique to the damage or trajectory analysis, it will be useful to summarize four general assumptions, or limitations which apply to all of the CRASH computations.

1) Pitch, roll, and elevation changes are not incorporated in either the impact or the trajectory portion. This means, for example, that weight transfer in braking does not take place.

2) Vehicles are assumed to have no lateral or angular velocity pre-impact. Abrupt avoidance maneuvers pre-impact should be investigated in light of this consideration.

3) Vehicles are assumed to come to a final common velocity prior to separation. Sideswipe collisions cannot be handled by CRASH. The same limitation applies to side impacts where the striking vehicle, slides along the struck vehicle as sometime occurs when the angle between the two vehicles approaches a sideswipe configuration.

4) Angular momentum and rotation kinetic energy are not considered during the impact phase. Rotation during impact is likely to be greatest in a side impact where the vehicle is struck in the front or rear quarter panel. Neglecting rotation probably does not appreciably affect the momentum or energy balance. However, a result of this assumption is that the positions of the vehicles of separation is assumed to be the same as at pre-impact. Rotation during impact could appreciably alter the post-impact trajectories.

9.2 Damage Analysis

Probably the primary considerations in the application of the damage routine are the measurement of crush, offset, and angle of

principle force.[2] These have already been discussed. In this section, collision forces which act at an angle to the normal and/or do not act through the center of mass will be covered. Frequently these two situations occur together.

A non-central collision is one in which the collision force does not act through the center of mass of the vehicle. The resulting moment will always produce rotation of the vehicle. The effect of rotation was mentioned in the first section of this chapter. For non-central collisions, only the centroids of the damaged areas of the respective vehicles are assumed to have a common final velocity prior to separation. A "reduced mass" is computed using the lever arm of the force and the radius of gyration. This mass reduction factor is called gamma and is computed as shown in Section 8.3.

Determination of the angle of principle force is important since gamma generally reaches 0.5 well before the lever arm reaches half the wheelbase. In effect this makes a full size car appear to weigh the same as a sub-compact in such collisions. Frequently, non-central collisions involve collision forces acting at an angle to the normal.

The problem with collision forces acting at angle to the normal to the underformed surface arises directly from the lack of data on vehicle structural characteristics. One might reasonably suspect that the stiffness of the structure would change when the structure is loaded at an angle to the normal. One might further speculate that the structure would be softer since they are primarily designed for at least longitudinal forces in the case of the front structure. No data are currently available to confirm or deny this speculation.

A second problem arises which is less difficult to deal with. Crush measurements are always made normal to the undeformed surface. When the collision force acts at an angle, the distance the force acts

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[2]Although the program will generate crush values from the CDC, this option should only be undertaken with caution. At a minimum, generated profile should be reviewed to ensure they are appropriate. Generated profiles are not symmetric for the right and left sides. Inappropriate CDC's sideswipe for example, will run but unreasonable results may occur.

through will always be greater by $1/\cos\theta$ than the measured crush.

A correction factor of $(1 + \tan^2\theta)$ is applied to the computed energy as was shown in Section 8.3. This correction would correspond to the $1/\cos\theta$ adjustment for the term involving the square of the crush $(1/\cos^2\theta = 1 + \tan^2\theta)$. This adjustment for crush measurement would be approximate since no adjustment is applied to the terms which are linear in crush. However, these terms contribute much less than the square term as the crush increases. Alternately, one may view this multiplication factor as providing for increased structural stiffness at angular loading.

Nevertheless this factor has a strong influence on the estimated energy. At 45° $(1 + \tan^2\theta)$ equals 2, and at 60° equals 4. The program limits this factor to 13.6 which occurs at 74.4°. The interaction of the mass reduction factor and the energy multiplication factor are shown in Figure 9.1. This figure was constructed for a sub-compact impacting the front center of a full size car at varying angles. Notice that initially the mass reduction term dominates so that the net effect is a reduction in energy. As the angle exceeds 45° the $1 + \tan^2\theta$ term begins to dominate, rising quickly to a multiplication of at degrees.

The point of this discussion is to describe some of the consideration in the application of the damage routine to non-central and/or angle impacts. No data are currently available to improve upon these factors. However, when inconsistent or unusual results occur one might ask whether part of the problem is with these terms. In general the DAMAGE routine is best suited for colinear impacts.

9.3 Trajectory Analysis

This section focuses on the non-iterative trajectory computations. These computations are aimed at the estimation of the post-impact, or separation velocities using the supplied trajectory data. The objective here is to highlight the various computations invoked in response to the answers provided by the user to the questions on lateral skidding, rotation, and curved path. Path description is discussed first.

9.3.1 Path Description The velocity at separation is the velocity

necessary to move the vehicle from its position at separation to the final rest position.[3] The non-iterative solution assumes this path is over terrain with a constant friction coefficient, and that no steering exists, i.e., the plane of the front wheel is parallel to the logitudinal axis of the car. Therefore, if the prompt for a trajectory simulation (Question #31) is answered no, the user will not be prompted for steer angles or a terrain friction boundary.

The program models the path from the point of separation (impact) to the rest position as a combination of two segments. One is an arc of a circle, the other is a straight line. A circular arc is used only if the user indicates the path was curved, i.e., Question 14 (or 21 for Vehicle 2) is answered "Yes". Regardless of the configuration of the actual path, any curvature is modeled as an arc of a circle. The total path may be modeled as (1) a straight path from the point of separation to the rest position, (2) a circular arc from separation to rest, or (3) a circular arc followed by a straight segment. The program never assumes a straight segment starting at impact followed by a circular arc ending at rest.

A typical path configuration consisting of both a circular arc and a straight segment is shown in Figure 9.2, along with the points necessary to describe the path. Coordinates of each of the four points are user inputs in general, give by the responses to questions requesting coordinates. There are, however, instances in which the coordinates of the point-on-curve and end-of-rotation are not applicable or are set internally in the program, depending on user responses to other questions. This will be discussed later.

The information on the path needed to compute the vector velocity at separation is the length of the path segments, and the angle (heading) of the path at separation. The length of a straight segment is simply the length of the line between the end points, and is found by methods of Analytic Geometry. If the straight line segment starts at the separation point, the angle of the velocity vector at separation is

---

[3]Two points should be made here. The program assumes the point of separation is identical to the point of impact, and positions to be discussed refer to the location of the center of mass of the vehicle.

Figure 9.3 Program Model of a Path Composed of
Both a Circular Arc and Straight Line

the angle the line makes with the x-Axis.

The geometry of a curved segment is shown in Figure 9.3. The ends of the arc are at SEP and EOS. The radius is determined from these points and the POC. Several computations are made relative to the arc. First the perpendicular distance (d) from the POC to the chord from SEP to EOS is computed. If this is less than one inch, the curvature is assumed to be negligible and the path used is the straight line chord from SEP to EOS. Secondly, an approximate radius is computed. If the approximate radius is greater than 833 1/3 ft., the chord is also used rather than the arc. The approximate radius used is the radius of a circle that would pass through SEP and EOS with a perpendicular distance from the center of the chord to the arc equal to d. It is approximate because POC will seldom be at a point half-way along the arc.

If both tests pass and the segment is treated as a circular arc, the coordinates (P,Q) of the center of the circle passing through all three points are calculated. The center is the intersection of the perpendicular bisectors of the cords SEP-POC and POC-EOS. The radius is then the distance from SEP to the center at P,Q.

Finally, the arc distance and direction of velocity at separation are determined as shown in Figure 9.4. The angles of the radii to the EOS and SEP are computed (ϕ_{EOS} and ϕ_{SEP} respectively), and the angle subtended by the arc is:

$$\phi = \phi_{SEP} = \phi_{EOS}.$$

The arc length S is then

$$S = R\phi$$

where ϕ is in radians. The angle of the separation velocity vector (ν_s) is equal to the angle between the radius R to the Y axis.

9.3.2 Types of Motion The section above has described two typed of path segments—circular arc and straight line—which are used singly or in combination to describe the total path.

Three types of motion are used to model the dynamics of a vehicle moving from separation. Over any one segment the motion is modeled by one of the following three types: (1) Rollout (2) Non-rotating-angular

Figure 9.3 Geometry of Curved Path

Figure 9.4 Computation of Length of Curved Path

skid (3) Spinning skid.

Each type will be described below. Then the application of each to the two types of path segments will be discussed. Finally, the method of computing velocity for each type will be described briefly.

Rollout. Rollout is motion of the vehicle in a longitudinal direction. The trajectory computation without simulation (non-iterative solutions) assume no steering input. The planes of the front wheels are assumed to be parallel to the longitudinal axis so the vehicle moves "straight ahead". More specifically, the angle between the velocity vector and the longitudinal axis of the vehicle is zero. This would be typified by a vehicle with the wheels in a straight-ahead position coasting to a stop—literal rollout. However, in the rollout mode as used in the CRASH2 program, braking may be present. The amount of resistance to free wheeling, is given by the Rolling Resistance for each wheel. This may range from 0.0 to 1.0. A value of 1.0 is equivalent to complete lock-up and skidding, either because the brakes are locked or because physical damage to the vehicle causes interference with wheel rotation. Thus, rollout motion includes the case of a "straight-ahead" skid. The expression for the velocity is the familiar:

$$V = \sqrt{2g\mu s}$$

where

        V is the velocity at the start of the skid

        g is the acceleration due to gravity

        μ is the coefficient of friction

        s is the stopping distance

Zero rolling resistance implies either that (1) the velocity at separation is zero, or (2) the velocity is non-zero but is followed by no deceleration (the vehicle never came to rest). Although the program will assume the first, i.e., separation velocity is zero, neither is consistent with the vehicle coming to rest at any position other than the impact position.

USER CAUTION - Entering an impact position and a different rest

position is inconsistent with rollout and rolling resistances of zero. Such a case should not be run. The rolling resistance should be non-zero on at least one wheel.

Non-Rotating-Angular Skid   A non-rotating-angular skid is motion along a path either straight or curved, with the longitudinal axis of the vehicle at a constant angle from the velocity vector. Figure 9.5 illustrates such a skid. The center of gravity of the vehicle is moving along the circular arc AA'. At any point on the path, the velocity vector is tangential to the arc as shown by the dashed lines. The angle between the velocity vector and the longitudinal axis of the vehicle, θ, is a constant in Figure 9.5. Several observations are helpful in differentiating this type of motion from the other two. Rollout is a special case of non-rotating-angular skid, the case when $\theta = 0$. In non-rotating angular skid, there is a lateral motion—a lateral component of skid. The heading of the vehicle of Figure 9.5 is changing, i,e., in going from the first to last position shown, the vehicle rotates about the fixed earth reference coordinate system. Nevertheless, it does not rotate relative to the velocity vector. An observer riding in the vehicle and always facing in the direction of motion would not sense any apparent rotation. The same expression for velocity that is used for rollout is used here, except that the friction coefficient is replaced with an "effective" friction given by the Mouk-Burgett formula:

$$\mu_{eff} = \mu(\sin^2\theta + (RR)^2\cos^2\theta)$$

This expression reduces to μ when $\theta=0$. RR is the rolling resistance or wheel lockup averaged over the four wheels.

Spinning Skid   A spinning skid is motion along a path such that the angle between the velocity vector and the longitudinal axis of the vehicle is constantly changing. This is the classical case of spinout and is depicted in Figure 9.6

9.3.3 Combinations of Types of Path Segments and Motions Used by the Program. The combinations of types of path segments and motions used by the program depend on the user responses to a series of questions. These questions are listed below with the numbers of the questions pertaining to vehicle #1. The question numbers for vehicle 2 are given

Figure 9.5 Non-Rotating-Angular Skid

Figure 9.6 Spinning Skid

in parentheses.

a. Rotational or lateral skidding?   11(18)

b. Did skidding of Vehicle 1 stop before rest?   12 (19)

c. Was path of Vehicle #1 curved?   14 (21)

d. Rotation Direction of Vehicle #1?   16 (23)

If question 11 is answered "Yes", either a non-rotating-angular skid or a spinning skid is used over a portion of the path. Which depends on the response to question 16. If 16 is answered "None", the non-rotating angular skid is used. If either "CW" or "CCW" is given, a spinning skid is used. Rollout is used only for the portion of the path from the end of the angular or spinning skid to rest.

If Question 11 is answered "No", indicating no angular or skid, the rollout starts at separation and rollout is used over the entire path. In this case the program sets the coordinates of the point at which angular or spinning skid stops at the impact point. If Question 16 (Rotation Direction) is answered "CW" while Question 11 is answered "No", the rotation direction will be ignored and rollout will be assumed from impact to rest. If it is answered "CCW", a division by zero will occur and interrupt the program.

If angular or spinning skid is indicated, the point at which it stops is dictated by Question 12. If Question 12 (did skidding of vehicle stop before rest?) is answered "Yes", the end of skidding will be at the user supplied coordinates of the end of skidding. If a "No"is given, the angular or spinning skid will extend from impact to rest.

If a curved path is indicated by the user, a circular arc will be used starting at the impact position and ending at the end-of-skid position. The circular arc will extend from the impact to rest positions if the user indicates the rotational and/or lateral skidding did not stop before rest.

An anomalous computation is made if the user indicates no rotational and/or lateral skidding and a curved path. In this case the program assumes non-rotational-angular skidding over the complete circle

which passes through the impact point, point-on-curve, and rest position, followed by rollout over the straight line from the impact to rest positions. This is a much longer path than is actually traversed by the vehicle and leads to an erroneous and large velocity at separation.

9.3.5 <u>Summary of Trajectory Computations without Simulation</u> The four sections above have attempted to briefly describe how the trajectory computations are performed and what models are used by the program in response to those questions which dictate the model selection. Table 9.1 presents a summary of the models used and when they are invoked. This should help the user understand the questions and prompts, and the consequences of the responses given.

1 GENERAL NOTES ON TABLE A

1. Rollout

Rollout means longitudinal travel of the vehicle with no lateral motion. This is the motion one would expect if the vehicle was rolling straight ahead. However, resistance to rolling is allowed. This is specified through the amount of rolling resistance entered. The total effective friction is then the entered friction coefficient times the average of the four individual wheel rolling resistances. The rolling resistance may be high enough to result in skidding and visible skidmarks. The rollout model is used, however, as long as there is no spinning and no lateral velocity.

It would be inconsistent to give a series of responses that involve rollout over some distance, and at the same time give rolling resistances of zero (0). This would imply that either the vehicle had zero velocity at the beginning of rollout, or it would never come to rest. The program assumes the former.

Rollout over a curved path is not allowed by the program. If no rotational or lateral skidding is given—invoking rollout computations—and a curved path is specified, an erroneous computation is made.

2. Non-Rotating -Angular Skid

Non-rotating-angular skid is vehicle motion in a direction that is

Table 9.1

SUMMARY OF COMPUTATION OF SEPARATION VELOCITY

IN TRAJECTORY RUNS WITHOUT SIMULATION

	Question	Responses			Computational Model
	Rotational and/or Lateral Skidding?	Skidding Stopped Before Rest?	Spinout Path Curved? [1]	Vehicle Rotate	
Question Numbers	11,18	12,19	14,21	16,23	
Response Combinations					
1	No	Not Applicable, No Prompt	No	No	Rollout over a straight path from impact position to rest position.
2	No	Not Applicable, No Prompt	No	Yes	Rollout over a straight path from impact position to rest position. [3]
3	No	Not Applicable, No Prompt	Yes	No	Incorrect results [2]
4	No	Not Applicable, No Prompt	Yes	Yes	Incorrect results [2,4]
5	Yes	No	No	No	Not-rotating-angular skid over straight line from impact to rest.
6	Yes	No	No	Yes	Spinning skid over straight path from impact to rest position.
7	Yes	No	Yes	No	Non-rotating-angular skid over circular arc from impact to rest position.
8	Yes	No	Yes	Yes	Spinning skid over circular arc from impact to rest position.
9	Yes	Yes	No	No	Non-rotating-angular skidding over straight line from impact to the end-of-skid position, followed by rollout from the end-of-skid position to the rest position.
10	Yes	Yes	No	Yes	Spinning skid over a straight line from impact to the end-of-skid position, followed by rollout over a straight line from the end of skid position to the rest position.
11	Yes	Yes	Yes	No	Non-rotating-angular skidding over a circular arc from impact to the end-of-skid position, followed by rollout over a straight line from the end-of-skid position to the rest position.
12	Yes	Yes	Yes	Yes	Spinning skid over a circular arc from impact to the end-of-skid position, followed by rollout over a straight line from the end-of-skid position to the impact position.

[1] If the spinout path is curved, the path is assumed to be an arc of a circule which passes through the impact point, the point on the curved path, and either the rest position or the end-of-skid position. There are two conditions under which the path is assumed to be a straight line even if Question 14 (or 21) was answered "Yes". These are (1) the perpendicular distance from the point on the curve to a straight line (chord) from impact to the end-of-skid point (or rest position) is less than 1.0 inch, and (2) if the radius of a circle which gives the same perpendicular distance--as in (1) but at the bisector of the cord--is greater than 833-1/3 feet.

[2] These combinations of responses will run, i.e., the program will execute, but the trajectory results will be erroneous. The assumed path from impact to rest will be composed of the entire circumference of the circle defined by the curved path plus the straight line segment from the impact position to the rest position. Non-rotating skidding will be assumed over the circle, followed by rollout over the straight segment.

[3] If a rotation direction of CCW is given, the program will attempt to divide by zero and be interrupted.

[4] Spinout error message may be returned.

at a constant angle to the longitudinal axis of the vehicle. Rollout is the special case where the angle is zero. The computation of velocity at separation is very similar to that for rollout except that the effective friction is a value between the tire-friction coefficient and the product of the tire friction coefficient and the average value of rolling resistance, depending on the angle of skid.

Non-rotational skid, which may be over either a straight or curved path, is invoked by responding "yes" to "Rotational and/or Lateral Skidding" (Questions 11 or 18) and "None" to "Rotation Direction" (Questions 16 or 23).

3. Spinning Skid

Spinning skid means spinning of the vehicle about its vertical axis because of rotational velocity impacted by the impact forces. The spinning may be over a straight or curved (circular) path. This computational regime is complex and involves the modified Marquard equations. It is invoked by responding "yes" to "Rotational and/or Lateral Skidding" (Questions 11 or 18), and either "CW" or "CCW" to "Direction of Rotation" (Questions 16 or 23). The computation may not succeed, in which case a note of "SPINOFF ERROR DETECTED" will be returned.

4. Rotation

Rotation is indicated by responding to "Rotation Direction" with either "CW" or "CCW". The difference in headings of the vehicle at impact and at rest (or at the end-of-rotation) is not used to determine whether or not rotation occurred.

Summary The velocity change, Delta V, computed by the CRASH2 computer program is a measure of the "severity" of the collision for each vehicle. This parameter is of central importance to many of the primary objectives of NASS, such as a statistical description of the accident environment, comparisons between specific accidents, and development of statistical models to relate collision, vehicle, and occupant variables to the probability of injury for the occupants.

Reconstruction of accidents to provide estimates of the velocity change, Delta V, of each vehicle is one of the more demanding tasks of the NASS Investigator. This chapter provides an introduction to the need for this information and presents examples of its eventual use. Section 1 of this chapter addresses the definition of collision severity, while applications are discussed in the second section.

10.1 Definition of Collision Severity

Collision severity does not have an easy definition. Loosely speaking it is part of a larger effort to quantify, or measure, the "potential for injury" of the collision. In order to clarify the role of collision severity measures, it is necessary to review the role of accident data in general.

The study of accidents can be broadly classified into two groups based on the objective of the investigation. These two groups are the study of accident causation and the study of vehicle crashworthiness. Accident causation studies seek to identify those things which cause accidents. Hopefully, an understanding of the causes of accidents would reveal changes to the roadway, vehicle or driver which would prevent accidents. Conversly, the study of vehicle crashworthiness focuses on the ability of the vehicle to prevent injury once a crash has taken place. Understanding the ways in which injuries are produced has led to design changes such as stronger door latches, the laminated windshield, head restraints, and a strong emphasis on occupant restraint systems.

Here is where collision severity comes in. Many factors influence the likelihood of injury in an accident. For example, old people are more susceptible to fractures than young people; collisions with rigid unyielding objects are more severe than collisions with yielding

objects. In fact, the severity of the collision is felt to be perhaps the single most important variable which must be taken into account when comparing the injuries which resulted from several different accidents. Restraint usage is the only other variable of comparable importance for this type of analysis.

Accident data fills several roles in the evaluation of vehicle crashworthiness. Knowledge of the collision severity pervades nearly all of these. As described in the preceding paragraph, the basic evaluation involves drawing conclusions with regard to the effectiveness of vehicle safety features from large groups of accidents where each case is still likely to be different from any other case. A measure of collision severity helps to quantify this difference.

Much of the development of vehicle safety features is carried out on the basis of laboratory tests. Impact sleds provide a deceleration pulse comparable to that in an actual accident for the evaluation of restraint systems. Barrier crash tests and staged car-to-car collisions provide information on the performance of the vehicle structure as well as the restraint system. Here the primary value of accident data is the comparison of actual injuries with the expected injuries indicated by the motion and deceleration of the test dummy. Accurate data on the severity of the real accidents is essential for this comparison.

Even more basic is the need for information on the frequency of the various accident types, and the distribution of collision severity for ch. What force level must seat backs, latches, and head restraints be able to withstand? How large a velocity change is a rear-impacted vehicle likely to sustain? How much energy should the rear structure be able to absorb before the rear seat area begins to be deformed? An accurate statistical description of the real world accident environment would provide objective answers to these questions. The statistical sampling procedures are critical to this need since their purpose is to try to ensure that a representative distribution of accidents is investigated. "Uninteresting" cases must be investigated in their proper proportion; the same is true for the "interesting" cases.

This discussion has tried to show that the need for information on collision severity is part of a larger problem. That larger problem is

to measure in an objective way all the various factors which make one collision different from another. If all the important factors can be objectively measured, then statistical techniques may be used to interpret the data. The definition of collision severity, then, is part of a general framework for analysis.

The objective is to understand why the injuries in one accident (or for one occupant) are different from those in another. The starting point is to think of all the factors which may have been different in the two crashes, such as the use of restraints, age of the occupant, make and model of car, the type of collision, and, of course, the collision severity. It is helpful to organize these variables into three groups which will be labeled collision variables, vehicle variables, and occupant variables. An example of this organization is shown in Table 10.1. Example variables were selected on the presumption that they have some influence on the injuries which result from a crash. Remember that the objective here is understanding the causes of <u>injuries</u> once a collision has occurred. Factors such as "driver had been drinking" or "brake failure" may have caused the collision in the first place, but the study of <u>accident</u> causes is usually kept separate from the study of <u>injury</u> causes. Consequently, a variable like "brake failure" would not be included in an analysis of vehicle crashworthiness.

Notice that the severity is only one of many variables which describe the circumstances of the collision. For convenience, all of the collision descriptors, except the severity, are said to describe the "type" of collision. Here collision type is a more detailed description than simply "head-on" or "rear." Within this framework, the measure of collision severity must distinguish, or measure, the potential for injury <u>among collisions of the same type</u>. The next step is to formulate a mathematical model using these variables. The outcome which we are trying to understand is the injuries, and it is called the dependent variable. Injuries are rated on the Abbreviated Injury Scale (AIS). In general, injuries of different types or to different body regions must be studied separately. All of the other variables, which are expected to explain the differences in injuries observed in the accidents

TABLE 10.1

GROUPING VARIABLES FOR
CRASHWORTHINESS ANALYSIS

Collision Variables	Vehicle Variables	Occupant Variables
Type	E.A.Column	seat location
...D.O.P.F.	laminated W.S.	sex
...damage area	side-door beam	age
...object struck	weight	height
...offset	head restr.	weight
	restraint type	
Severity		

investigated, are called the independent variables. This model is shown in Table 10.2. To save space, only the names of the three groups of independent variables are shown. The mathematical expression for this model is shown at the bottom of Table 10.2. This expression is read as "injury severity is equal to some mathematical function of the collision, vehicle, and occupant variables."

Using this model, the analysis may be described as follows: the "potential for injury" of the collision is determined by the values of all the independent variables, including the collision severity. The value of the model is determined by comparing this potential for injury with the actual injuries which were recorded. If there is good agreement between the potential, or predicted, injuries and the actual injuries, the statistical model will help identify the factors contributing to the severity of the injuries once a collision has occurred. Design changes might be developed from this information if the relationships in the statistical model follow the known physical laws which govern the forces on, and resulting motion of, the vehicles and occupants in a crash.

TABLE 10.2

CRASHWORTHINESS MODEL

Dependent Variables		Independent Variables
Injury Severity		Collision
		Vehicle
		Occupant

Injury Severity = F(collision, vehicle, occupant)

This section has described the need for information on collision severity. A definition for collision severity was presented in the context of a general framework for the analysis of vehicle crashworthiness. The next section describes some of the measures of collision severity which have been used, and presents some recent data on Delta V from the NCSS program.

10.2 Applications

Historically, many measures of collision severity have been used. Some of these are: repair cost, impact speed, maximum crush, energy(EBS), and Delta V. These are each discussed briefly.

About the earliest measure used as an indication of collision severity was the repair cost in dollars. While it was far from perfect, a recognition of the problem was indicated. Some police reports contained an estimate of "speed at impact." For a time, this seemed to be just the answer. Two things went wrong. Conceptually, it was realized that just because the speed at impact was high, the collision .as not necessarily harmful. It was recognized that the speed after the impact must be considered too. The second problem with police reported impact speeds was that they were often inaccurate. Frequently they came from witness statements rather than the officer. Even worse, they were

strongly influenced by the resulting injuries. People on the scene seeing how badly the people were injured presumed that the vehicle must have been speeding!

As deliberate accident data collection efforts were initiated, measurements of the vehicle damage were recorded. Maximum crush provided a more objective measure of collision severity, although it was recognized to be only an approximation of the amount of damage.

Within the last 5 years more analytical measures have come into use. The first of these is simply the energy absorbed in deformation of the vehicle. By this time the reader is completely familiar with the estimation procedure. Its advantages are that the evidence is relatively well preserved, its objective, and examination of only the vehicle of interest is necessary. This provides a considerable saving over measures which require investigation and reconstruction of the entire accident. The disadvantage is that the amount of energy absorbed by a vehicle in a collision is influenced by the stiffness of the struck vehicle or object. For this reason, it is a less general measure than Delta V. As the reader knows, the vector direction of the collision impulse is also incorporated in Delta V since it is a vector quantity.

In closing, a few examples of the information on Delta V which has been collected during the first year of the NCSS program are shown here. This information is of particular interest first because of the use of the CRASH2 program, and second, because of the sampling procedures used to select accidents for investigation. The sampling procedures are intended to produce distributions which are representative of the area selected.

Figures 10.1 through 10.4 present histograms of Delta V for: all vehicles, front damaged vehicles, side damaged vehicles, and rear damaged vehicles respectively. In viewing these figures, the large percentage of collisions which occur with Delta V values less than 20 mph is striking. It is also somewhat surprising that the Delta V values for rear damaged vehicles do not appear to be appreciably lower than those for frontals.

The final Figure, 10.5, shows the probability of an AIS 4 or

greater injury as a function of Delta V. This kind of information has been sought for many years. Even at speeds up to 50 mph, the probability is still below 0.20.

This chapter has tried to provide an overview of the need, definition, and use of measures of collision severity. It is important for the investigator to understand the central importance that this information plays.

Figure 10.1 Histogram of Delta V for All Vehicles

Figure 10.2 Histogram of Delta V for Front Damage Vehicles

Figure 10.3 Histogram of Delta V for Side Damaged Vehicles

Figure 10.4 Histogram of Delta V for Rear Damaged Vehicles

Figure 10.5 Probability of AIS 4+ by Delta V
for Frontal Damage Vehicles

APPENDICES

APPENDIX A

DEVELOPMENT OF EXPRESSION TO DETERMINE THE RADIUS
OF CURVATURE OF AN ARC FROM ITS CORD LENGTH
AND ARC TO CHORD OFFSET

A curved path of radius (r) whose chord length is (C) and medium off-set i (m) is shown in Figure A-1.

Figure A.1

This curved arc has a radius (r) as shown in Figure A.2.

The two triangles enclosed by the radius (r) which are formed from the chord and curved arc are right triangles.

By applying the Pythogorean theorum, which states that the sum of the square of the two sides of a right triangle is equal to the square of the hypotenuse, we obtain:

$$(c/2)^2 + (r-m)^2 = r^2 \tag{A.1}$$

When expanded this becomes:

$$c^2/4 + r^2 - 2rm + m^2 = r^2$$

Solving for the radius (r):

$$r = c^2/8m + m/2 \tag{A.2}$$

Figure A.2

APPENDIX B

DEVELOPMENT OF EXPRESSION FOR CRITICAL CURVE SPEED

Centrifugal force is that force acting on a body of mass (m) as it moves about an axis in a curved path of radius (r). The force is directly proportional to the product of the square of the velocity (V), mass of the object, and inversely proportional to the radius (r) of its curved path.

$$F_c = mV^2/r \qquad \text{(B.1)}$$

Friction forces developed laterally at all four wheels of a vehicle in a turn follow Coulomb's Law of Friction, which states that the ratio of the friction force (F_f) and normal force (N) of a body is constant. This constant is the coefficient of friction (μ) a characteristic of the surface.

$$\mu = F_f/N \qquad \text{(B.2)}$$

The normal force is the weight of the body on that surface, so that

$$\mu = F_f/W$$

Rearranging this expression gives

$$F_f = \mu W \qquad \text{(B.3)}$$

Equating the centrifugal force (F_c) to the friction force (F_f) is accomplished by equating equation (1) to (3) so that:

$$W/g \; V^2/r = \mu W$$

which rearranged is

$$V^2 = \mu r g \qquad \text{(B.4)}$$

When expressed in terms of velocity (V) in mph and radius (r) in feet, this becomes:

$$V^2 = 15\mu r \qquad \text{(B.5)}$$

or

$$V = \sqrt{15 \; \mu r} \qquad \text{(B.6)}$$

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APPENDIX C

QUESTIONS POSED BY CRASH2

1. Enter a descriptive title?   (80 characters maximum)

2. Enter size categories for Vehicle #1 and Vehicle #2.
   LEGAL CATEGORIES:

Minicar	Subcompact	Compact
Fullsize	Large	Rigid
Barrier	Intermediate	

3. Enter a seven (7) character vehicle damage index for Vehicle #1.
   NOTE: Refer to Appendix 1 in the CRASH Users Guide Form: 10LZEW2.

4. Enter a seven (7) character vehicle damage index for Vehicle #2.
   NOTE: Refer to Appendix 1 in the CRASH Users Guide Form: 06RWEW3.

5. Are any actual weights known?

   NOTE: A negative response will cause an automatic weight entry
   based upon the previously selected vehicle category. Obviously,
   accuracy is enhanced by providing an actual weight.

   (Answer yes or no.)

6. Enter the actual weight of Vehicle #2.

   NOTE: Include weight of occupants, cargo, and fuel.
   Form: 2497 (lbs.)

7. Are both rest and impact positions known?

   NOTE: A negative response limits program results to velocity
   change approximations based on damage data only.

   (Answer yes or no.)

8. Enter rest positions and heading for Vehicle #1 and Vehicle #2.

Form: XCR1 (ft.)	YCR1 (ft.)	PSIR1 (deg.)
XCR2 (ft.)	YCR2 (ft.)	PSIR2 (deg.)

9. Enter impact positions and headings for Vehicle #1 and Vehicle #2.

   Form: XC10 (ft.)   YC10 (ft.)   PSI10 (deg.)
         XC20 (ft.)   YC20 (ft.)   PSI20 (deg.)

10. Did rotational and/or lateral skidding of Vehicle #1 occur?

    NOTE: This refers to that portion of the trajectory during which
       the front and rear wheels do not run in the same tracks.

    (Answer yes or no.)

11. Did rotational and/or lateral skidding of Vehicle #1 stop before
    rest position was reached?

    NOTE: It is common in a skidding trajectory to have an abrupt
       change in motion as the wheels start tracking one another and
       the vehicle moves out to rest in a non-skidding fashion.
       The non-skid section must be a straight line.

    (Answer yes or no.)

12. Enter position and heading of Vehicle #1 at end of rotational
    and/or lateral skidding.

    NOTE: Response should define the position and heading of the
       vehicle at the point in the trajectory at which the front
       and rear wheels run in the same tracks.

    Form: SC11 (ft.)   YC11 (ft.)   PS111 (deg.)

13. Was the spinout path of Vehicle #1 between separation and rest
    curved?

    NOTE: Try to visualize the path of the vehicle c.g. If a
       prominent arc is present, answer affirmatively.

    (Answer yes or no.)

14. Enter an intermediate position of Vehicle #1 on the curved path
    between separation and rest.

    Form: XC21 (ft.)   YC21 (ft.)

15. Which direction did Vehicle #1 rotate?

    NOTE: Clockwise rotation turns from the X-axis toward the Y-axis.
       For the case of non-rotating angular skidding enter none.

    (Respond with: CW   CCW   None)

16. Did Vehicle #1 rotate more than 360 degrees between separation and rest?

NOTE: This is a rare occurrence and should be verified from tire mark data.

(Answer yes or no.)

17. Did rotational and/or lateral skidding of Vehicle #2 occur?

NOTE: This refers to that portion of the trajectory during which the front and rear wheels do not run in the same tracks.

(Answer yes or no.)

18. Did rotational and/or lateral skidding of Vehicle #2 stop before rest position was reached?

NOTE: It is common in a skidding trajectory to have an abrupt change in motion as the wheels start tracking one another and the vehicle runs out to rest in a non-skidding fashion. The non-skidding section must be a straight line.

(Answer yes or no.)

19. Enter position and heading of Vehicle #2 at end of rotational and/or lateral skidding.

NOTE: Response should define the position and heading of the vehicle at the point in the trajectory at which the front and rear wheels run in the same tracks.

Form: XC12 (ft.)   YC12 (ft.)   PSI12 (deg.)

20. Was the spinout path of Vehicle #2 between separation and rest curved?

NOTE: Try to visualize the path of the vehicle c.g. If a prominent arc is present, answer affirmatively.

(Answer yes or no.)

21. Enter an intermediate position of Vehicle #2 on the curved path between separation and rest.

Form: SC22 (ft.)   YC22 (ft.)

22. Which direction did Vehicle #2 rotate?

NOTE: Clockwise rotation turns from the X-axis towards the Y-axis. For the case of non-rotating angular skidding enter none.

(Respond with: CW   CCW   None)

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23. Did vehicle #2 rotate more than 360 degrees between separation and rest?

NOTE: This is a rare occu-rance and should be verified from the tire mark data.

(Answer yes or no.)

24. Enter the nominal tire-ground friction coefficient.

NOTE: Refer to Table 2 in the CRASH Users Guide for typical tire-ground friction values.

Form: MU

25. Rolling resistance may be entered as:

1 — The decimal portion of full rotational lock-up at each wheel.
2 — The level of longitudinal deceleration, in G units, produced by rotational resistance at the wheels.

(Answer 1 or 2)

26. Enter rolling resistances of wheels of Vehicle #1.

NOTE: Can be caused by braking, damage, engine braking, etc. Enter value for each wheel from 0.0 to 1.0. 1.0 = full wheel lock-up.

Form: RF   LF   RR   LR

27. Enter rolling resistances for wheels of Vehicle #2.

NOTE: Can be caused by braking, damage, engine braking, etc. Enter value for each wheel from 0.0 to 1.0. 1.0 = full wheel lock-up.

Form: RF   LF   RR   LR

28. Enter longitudinal deceleration, in g-units, produced by rotational resistance at the wheels of Vehicle #1.

NOTE: Enter a single deceleration less than friction value.

29. Enter longitudinal deceleration, in g-units, produced by rotational resistance at the wheels of Vehicle #2.

NOTE: Enter a single deceleration less than friction value.

Form: Decel

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30. Do you want the results checked by a trajectory simulation?

 NOTE: The separation velocities normally calculated by CRASH
      are used by a trajectory simulation to determine if the
      entered evidence matches the calculated speeds.
      If not, appropriate speed adjustments are made to obtain
      agreement with the evidence.
      (Answer yes or no.)

31. Enter the steer angles for each wheel of Vehicle #1.

 NOTE: Limit angles to plus or minus 90 degrees from straight
      ahead.

      Form: RF   LF   RR   LR (deg.)

32. Enter the steer angles for each wheel of Vehicle #2.

 NOTE: Limit angles to plus or minus 90 degrees from straight
      ahead.

      Form: RF   LF   RR   LR (deg.)

33. Is there a terrain boundary?

 (Answer yes or no.)

34. Enter a two-point friction boundary definition.

 NOTE: Key in the X and Y values of the end points.

      Form: XBP(1)   YBP(1)   XBP(2)   YBP(2)   (ft.)

35. Enter the friction coefficient of the non-origin side of the
    friction boundary.

 NOTE: Coefficient of the origin side has already been entered.

      Form: MU2

36. Are any actual damage dimensions known?

 NOTE: A negative response will produce damage data based on
      the submitted VDI.
      Obviously, providing damage measurements will enhance results.
      (Answer yes or no.)

37. Enter width of damaged area along side of Vehicle #1.

 NOTE: Use only the direct contact damage rather than the
      induced damage (sheet metal buckling induced by collision.)

      Form: L1 (inches)

38. Enter a profile of the extent of damage for Vehicle #1.

    NOTE: At two, four, or six points along the width of the dent measure the depth of the damage from the original side dimensions (entry sequence is from rear to front of vehicle).

    Form: C1   C2   C3   C4   C5   C6   (inches)

39. Enter distance along Vehicle #1 axis between the c.g. and the middle of the damaged region.

    NOTE: If this distance runs off to the rear of the vehicle enter it as a negative number.

    Form: D1 (inches)

40. Enter width of damaged area along end of Vehicle #1.

    NOTE: Use only the direct contact damage rather than the induced damage (sheet metal buckling induced by collision).

    Form: L1 (inches)

41. Enter a profile of the extent of damage for Vehicle #1.

    NOTE: At two, four, or six points along the width of the dent measure the depth of the damage from the original end dimensions (entry sequence is from driver to passenger side).

    Form: C1   C2   C3   C4   C5   C6   (inches)

42. Enter distance along Vehicle #1 axis between the c.g. and the middle of the damaged region.

    NOTE: If this distance runs off towards the driver side enter it as a negative number.

    Form: D1 (inches)

43. Enter width of damaged area along side of Vehicle #2.

    NOTE: Use only the direct contact damage rather than the induced damage (sheet metal buckling induced by collision).

    Form: L2 (inches)

44. Enter a profile of the extent of damage for Vehicle #2.

    NOTE: At two, four, or six points along the width of the dent measure the depth of the damage from the original side dimensions (entry sequence is from rear to front of vehicle).

    Form: C1   C2   C3   C4   C5   C6   (inches)

45. Enter distance along Vehicle #2 axis between the c.g. and the middle of the damaged region.

   NOTE: If this distance runs off to the rear of the vehicle enter it as a negative number.

   Form: D2 (inches)

46. Enter width of damaged area along end of Vehicle #2.

   NOTE: Use only the direct contact damage rather than the induced damage (sheet metal buckling induced by collision).

   Form: L2 (inches)

47. Enter a profile of the extent of damage for Vehicle #2.

   NOTE: At two, four, or six points along the width of the dent measure the depth of the damage from the original end dimensions (entry sequence is from driver to passenger side).

   Form: C1   C2   C3   C4   C5   C6   (inches)

48. Enter distance along Vehicle #2 axis between the c.g. and the middle of the damaged region.

   NOTE: If this distance runs off towards the driver side enter it as a negative number.

   Form: D2 (inches)

49. Enter the ratio of damage extents at edges of the damaged   ·
    areas of Vehicle #1.

   NOTE: When the entire front or rear has been crushed, measure the depth of penetration on each side and enter the ratio of the smaller over the larger.

   Form: RHO1

50. Enter the ratio of damage extents at edges of the damaged areas of Vehicle #2.

   NOTE: When the entire front or rear has been crushed, measure the depth of penetration on each side and enter the ratio of the smaller over the larger.

   Form: RHO2

51. Are the directions of the principal impact forces known more accurately than the entered VDI clock directions?

   NOTE: The clock direction limits principal force directions to 30 degree increments.
   If you can specify these angles more accurately, answer yes.

   (Answer yes or no.)

52. Enter the direction of principal force for Vehicle #1.

   NOTE: Enter angle as plus or minus degrees from straight ahead.

   Form: ANG1 (degrees)

53. Enter the direction of principal force for Vehicle #2.

   NOTE: Enter angle as plus or minus degrees from straight ahead.

   Form: ANG2 (degrees)

APPENDIX D

Manual Computation of CRASH2 Delta V
From Damage Data

APPENDIX D

Manual Computation of CRASH2 Delta V
From Damage Data

The CRASH2 program uses sets of formulae to calculate the magnitude of the linear impulse acting on each crashing vehicle. The direction of the impulse is determined when the user selects a direction for the principal crushing force--the impulse direction is the same as the force direction. The linear impulse magnitude divided by the vehicle's mass yields a "change in velocity" or a "Delta-V" of the vehicle's center-of-mass. This Delta-V is in the direction of the crushing force. The CRASH2 program takes account of the fact that the crushing force's line-of-action often does not pass through the vehicle's center-of-mass. The result of the force's offset is a change in the vehicle's angular velocity as well as its linear velocity. CRASH2 does not directly output a value of "Delta-Ω," but the user may properly estimate its direction and magnitude from the output.

The most commonly exercised portion of the CRASH2 program is the "Damage-Only" portion. At a minimum the program will operate with the designation of the vehicles' sizes and CDCs. The user should be aware of the fact that the program calculates a crush shape and crush location from the given CDC. The program calculates a width-of-crush, L, a center-of-crush distance from a vehicle axis, D, and a set of four crush-measurements, C_1, C_2, C_3, and C_4. The width-of-crush and the crush-measurements are always positive. The center-of-crush distance is positive if the center of the crush region is on the right-side of the vehicle's longitudinal axis for front or rear crushes or forward of the vehicle's lateral axis for right or left crushes. The user is always given the option of replacing the values of L, D, and the C's. If the user wishes he may insert a set of 2, a set of 4, or a set of 6 C values. The program automatically notes which he has done. If, when prompted for user values of L, D or C's, the user enters nothing, the program will use the previously CDC-derived values. Thus the program might be operated with the user's value for L, but with the CDC-derived values of D and the C's.

The program's formulae combine the values of L, D and C's with

values of the vehicle's dimensions, inertial properties, and elastic properties. The vehicle size category is used to allow internal table look-ups of the dimensions and inertial measures. The vehicles are modeled to have different elastic (energy-absorbing) properties on the front, the sides, and the rear (3 sets of values per vehicle). The vehicle size category supplied by the user is combined with the third CDC symbol to allow internal table look-up of the elastic parameters.

The program's formulae allow for the likely fact that more energy is absorbed by a vehicle with a given crush if the force causing that crush is delivered at an angle. The user supplies the required data for this "adjustment" in energy absorbed in the first two columns of the CDC. The user is also given the option to input a more exact value of the angle of the principal force later.

The basic calculations are made for each vehicle in the "Damage-Only" mode. A "mass-reduction" factor, γ, is calculated and an "energy-absorbed," E, magnitude is calculated. The γ_1 and E_1 of vehicle #1 along with its mass, M_1, are combined with the γ_2, E_2 and M_2 of vehicle#2 to produce the magnitude of the linear impulse on each.

$$ I = \sqrt{2(E_1 + E_2)[(\gamma_1 M_1)(\gamma_2 M_2)/(\gamma_1 M_1 + \gamma_2 M_2)]} $$

The γ calculation combines the offset of the line-of-action of the principal force, h, with the radius of gyration in yaw, k_{gy}^2 , of the vehicle as follows:

$$ \gamma = k_{gy}^2/(k_{gy}^2 + h^2) $$

The E (energy) calculation combines three measures of the crush geometry, its width, its area, and the first moment of its area about the surface, with two elastic parameters, A and B, to calculate a base energy figure which is then modified (increased) to account for the angle of the crushing force from the normal to the surface as follows:

$$ E = [(A^2/2B)(Width) + A(Area) + B(First\ Moment)][Correction\ for\ Angle] $$

The "centroid" of the crush region must be calculated to allow either the "offset," h, or the "first moment" to be calculated.

The centroid of a plane area is a defined location. If the plane area was to be thought of as a thin sheet of material, the "centroid" idea would be replaced by the "center-of-mass" idea. The CRASH2 program is organized to expect the crush area to be described by either one, three, or five quadrilaterals according to the following sketch:

Figure of one, three or five quadrilaterals

Each crush shape may be divided into a set of elementary sub-areas consisting of a sub-set of rectangles and a sub-set of right triangles. In each of the three cases the base of the rectangles and the right triangles is the same--this leads to the simplicity of the equations defining the centroid location.

An example calculation will be carried out for a case in which $l=30"$, $C_1=4"$, $C_2=6"$, $C_3=10"$, and $C_4=12"$. The centroid will be located in a temporary coordinate system.

The crush area is subdivided into 3 rectangular sub-areas and 3 right triangular sub-areas. The centroid of a rectangular area is defined to be at its center. The centroid of a triangular area is at the intersection of a pair of lines drawn at the 1/3 altitude levels, parallel to its bases. We have:

y

T₃

T₁ T₂ 2
R₁ R₂ R₃ 10

x

$$\bar{x} = \frac{5*40 + 15*60 + 25*100 + 6\ 2/3*10 + 16\ 2/3*20 + 26\ 2/3*10}{40 + 60 + 100 + 10 + 20 + 10}$$

$$\bar{y} = \frac{2*40 + 3*60 + 5*100 + 4\ 2/3*10 + 7\ 1/3*20 + 10\ 2/3*10}{40 + 60 + 100 + 10 + 20 + 10}$$

where

$\bar{x}$ is the temporary x-coordinate of the centroid, and
$\bar{y}$ is the temporary y-coordinate of the centroid.

or the x-coordinate of the centroid equals:

$$\bar{x} = \frac{\text{the sum of corresponding x-coordinates times the sub-areas}}{\text{the sum of the sub-areas}}$$

and similarly:

$$\bar{y} = \frac{\text{the sum of corresponding y-coordinates times the sub-areas}}{\text{the sum of the sub-areas}}$$

In the example, $\bar{x} = 17.78"$ and $\bar{y} = 4.42"$

The product, in this example, of $\bar{y}$ and the total area (4.42 * 240 = 1060.8 in³) is defined as the first moment of the area about the x-axis. For the remainder of the calculations, the coordinates of the centroid must be translated from the temporary coordinates to the vehicle

coordinate system.

As mentioned, the CRASH2 program must calculate a value for h--the perpendicular distance from the vehicle's center-of-mass to the line of action of the principal force. The principal force acts at the centroid of the crush region. The coordinates of this point (the centroid of the crush region) are calculated in the vehicle coordinate system. Then it follows that h is merely the moment of a unit force applied at this centroid, in the direction of the principal force, with the moment being expressed about the vehicle's center-of-mass. The sketch following illustrates this point. Notice that a unit force has Cartesian components of sine(ϕ) and cosine(ϕ) where ϕ is measured as shown.

$$h = \bar{y} * sin(\phi) - \bar{x} * cos(\phi)$$

(If the damage is frontal, the angle will be the compliment of the one shown, and the sine and cosine will be interchanged.)

Two crashing vehicles share the same but oppositely directed linear impulses. It is also true that at any instant, they share the same but oppositely directed force vectors. The crush is developed by a force system which, at maximum crush, assuming the crush to have occurred everywhere simultaneously, may be calculated as:

$$\text{Principle Force} = \frac{L (A + B(\text{Crush Area})/L)}{Cos (\phi)}$$

in the notation previously introduced.

While not demanding that the principal force maximums be equal on

the two vehicles, it seems reasonable that the two should be calculated and reviewed. Data inconsistency are indicated when the two values have ratios outside the 0.5 to 2.0 range!

In summary of this introduction, we find that the "Damage-only" calculations are directed at finding the "offset-distance" of the force to allow the calculation of , a mass-reduction-factor, and at finding the width, the area, and the first moment of the area of the crush about the surface to allow the calculation of the energy absorbed in crush. The energy calculated is increased to account for the angle of the principal force and this "corrected-energy" and the "reduced-mass" for each vehicle is entered into a calculation of I, the linear impulse shared between the crashing vehicles. This I, divided by the reduced mass of a case vehicle is the "Delta-V" of that vehicle.

The reason that the corrected energies and the reduced masses are believed to be related to the linear impulse is the topic of another lecture. This lecture now turns to the practical matter of operating CRASH2's Damage-Only routines--by hand.

The sheets that follow in this package consist of:
a) A set of instructions
b) A set of tables and figures
c) A set of working sheets

The working sheets will be replaced as they are used by the student in class or homework assignments. Please note that there are four different working sheets follow, one each for front, right, rear, and left damage, and that only one is to be used per vehicle.

The instructions that follow are thought to be adequate for students that have attended the lecture/workshop on this topic.

DAMAGE CALCULATIONS
(To be repeated for each vehicle)

1. Refer to Figure D.1. The case vehicle is to be identified as belonging to one family. Select a Work Sheet illustrating the family of the case vehicle. (Note: for illustration only, we shall refer to Work Sheet a, (W.S.a).

2. W.S.a has been selected. Enter the Vehicle Number and the CDC in the spaces designated.

3. Refer to Table D.1. Read from Table D.1 the numeric values of YSS, XFF, and XRR and enter on the figure of W.S.a. Read from the same table the values of m, k^2_{qy}, A, and B and enter these values into the Data Column. Enter the value of XFF here also.

4. Still referring to Table D.1, read a numeric value for Δ, the maximum crush dimension, based upon the 7th column of the CDC. Enter this value in the space designated (just above the CDC entry). When the damage extends across the entire width of the front or back, the crush is measured on each side, and ρ is entered as the ratio of the smaller to the larger.

5. Refer to Table D.2. Select the proper row of basic formulae according to Columns 3 and 4 of the case vehicle's CDC and carry out the calculations to produce numeric values. Enter the numeric values for the columns I, II, IV, V, VI, and VII onto W.S.a.

5. (Alternate) In the event that the user wishes to use non-CDC derived crush dimensions, refer to Table D.3. The user's values for D, L, and C are to be entered in the designated spaces on a copy of this Table. Formulae are presented to allow the remaining four column values to be calculated. Enter the numeric values for columns I, II, IV, V, VI, and VII onto W.S.a.

6. Carry out the calculations for Area, $\bar{x}$ and $\bar{y}$ on W.S.a and enter numeric values in the spaces designated.

7. The angle ϕ is the angle that the principal force makes with the normal to the original uncrushed surface. This angle is to be measured as positive clockwise from the normal to the force vector. The user may convert the CDC Col #1, #2 into an angle by multiplying the "hour angle" of those columns by 30. This angle is measured from the vehicle's +x-axis, however, and may need to have $90°$, $180°$, or $270°$ subtracted from it to produce the angle defined here. The angle ϕ may be positive or negative but, by definition, will not have a magnitude greater than $90°$. Enter onto W.S.a.

7. (Alternate) The user may wish to substitute a more refined value for on W.S.a.

8. The last three calculations of W.S.a may now be completed. The

values of k_{gy}^2, h, M, and Energy for this vehicle will be transferred to a <u>Final Worksheet</u> for velocity calculations. On the Final Worksheet, ANG is the angle of the principal force with respect to the positive x-axis. This angle is used to resolve the total Delta V into its vector components. Now proceed to carry out the calculations on the Final Worksheet.

(a) Front Crush   (b) Right Crush   (c) Rear Crush   (c) Left Crush

FIGURE D.1   The "Standard" Designation of Crush Geometry

NOTE: D is positive if measured to the right from the x-axis, or forward from the y-axis from the y-axis, otherwise D is negative.

TABLE D.1 Vehicle and Crush Data Used by CRASH2

| Size | Vehicle Dimensions | | | | Δ – Maximum Deflection From VDI Col. #7 | | | | | | | | | Force Factors | | Inertia Factors | |
	YSS	XFF	XRR		1	2	3	4	5	6	7	8	9	A	B	M*	k^2gy
Mini-car	30.4	76.0	83.8	Front	4.75	14.25	23.75	33.25	42.75	55.90	68.10	75.70	79.50	85.40	64.00	5.7	2006.
				Side	2.51	6.35	11.48	19.05	26.62	34.18	41.75	49.32	53.11	77.2	36.7		
				Rear	3.52	10.56	17.60	24.64	32.47	42.28	56.86	72.03	79.61	65.98	13.20		
Sub-compact	33.6	83.3	91.6	Front	5.21	15.62	26.03	36.44	46.89	61.27	74.64	82.97	87.13	94.89	71.11	7.9	2951.
				Side	2.77	7.02	12.68	21.05	29.42	37.78	46.15	54.52	58.70	140.4	66.7		
				Rear	3.85	11.54	19.24	26.93	35.50	46.21	62.15	78.73	87.02	65.98	13.20		
Compact	36.3	89.8	106.4	Front	5.61	16.84	28.06	39.29	50.51	66.05	80.46	89.44	93.93	154.6	69.52	9.18	3324.
				Side	2.99	7.59	13.70	22.74	31.78	40.82	49.86	58.90	63.42	173.3	57.1		
				Rear	4.47	13.41	22.34	31.28	41.23	53.68	72.19	91.45	101.08	78.18	15.64		
Inter-mediate	38.5	98.8	114.0	Front	6.18	18.53	30.88	43.23	55.58	72.67	88.52	98.40	103.34	233.7	49.9	10.99	3741.
				Side	3.18	8.05	14.53	24.12	33.71	43.29	52.88	62.47	67.26	143.0	50.4		
				Rear	4.79	14.36	23.94	33.52	44.18	57.51	77.35	97.98	108.30	85.51	17.11		
Full-size	39.9	101.8	121.9	Front	6.36	19.09	31.81	44.54	57.26	74.87	91.21	101.39	106.48	307.5	36.89	12.59	4040.
				Side	3.29	8.34	15.06	24.94	34.93	44.87	54.80	64.74	69.71	176.5	47.1		
				Rear	5.12	15.36	25.60	35.84	47.24	61.50	82.71	104.77	115.81	93.28	18.66		
Large	39.9	104.2	125.2	Front	6.51	19.54	32.56	45.59	58.61	76.64	93.36	103.78	108.99	307.5	36.89	13.74	4229.
				Side	3.29	8.34	15.06	24.94	34.93	44.87	54.80	64.74	69.71	176.5	47.1		
				Rear	5.26	15.78	26.29	36.81	48.52	63.16	84.95	107.61	118.94	93.28	18.66		

*To use user's mass, divide weight by 384.6.

TABLE D.2 Formulae Used by CRASH2 to Create Crush Geometry

VDI Col #3	VDI Col #4	Col I D	Col II L	C_1	C_2	C_3	C_4	Col IV area of crush $\div L$	Col V 1st moment of crush about surface $\div L$	Col VI Distance of centroid of crush from $\bar{L} \div L$	Col VII Distance of centroid of crush from surface
	D	0.0	2.0 YSS	Δ	$(2/3+1/3\rho)\Delta$	$(1/3+2/3\rho)\Delta$	$\rho\Delta$	$\frac{1}{2}(\rho+1)\Delta$	$\frac{3(\rho^2+\rho+1)}{18}\Delta^2$	$\frac{9}{54}\left(\frac{\rho-1}{\rho+1}\right)$	$\frac{1}{3}\left(\frac{\rho^2+\rho+1}{\rho+1}\right)\Delta$
	Z	.333 YSS	1.32 YSS	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
	Y	.333 YSS	1.32 YSS	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
F or B	C	0.0	.666 YSS	0.0	Δ	Δ	0.0	$\frac{2}{3}\Delta$	$\frac{5}{18}\Delta^2$	0.	$\frac{5}{12}\Delta$
	R	.666 YSS	.666 YSS	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
	L	-.666 YSS	.666 YSS	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
	D	0.0	.69 XFF+ .57 XRR	0.0	Δ	Δ	0.0	$\frac{2}{3}\Delta$	$\frac{5}{18}\Delta^2$	0.	$\frac{5}{12}\Delta$
	Z	-.5 XRR	XRR	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
	Y	.5 XFF	XFF	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
R or L	P	0.0	.37 XFF+ .57 XRR	0.0	Δ	Δ	0.0	$\frac{2}{3}\Delta$	$\frac{5}{18}\Delta^2$	0.	$\frac{5}{12}\Delta$
	B	-.79 XRR	.43 XRR	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$
	F	.69 XFF	.63 XFF	0.0	$.5\Delta$	$.5\Delta$	Δ	$\frac{1}{2}\Delta$	$\frac{11}{72}\Delta^2$	$\frac{7}{54}$	$\frac{11}{36}\Delta$

TABLE D.3 Formulae and Record of User Generated Crush Data

VDI Col #3	VDI Col #4	Col I D (user's)	Col II L (user's)	User Generated Crush Dimensions						Col IV area of crush ÷ L	Col V 1st moment of crush about surface ÷ L	Col VI Distance of centroid of crush from ₵ ÷ L	Col VII Distance of centroid of crush from surface
				C_1	C_2	C_3	C_4	C_5	C_6				

$$\text{Term 1} = \text{Col IV} = \frac{\text{Area of crush}}{L}$$

2 C's $\frac{1}{2}(C_1 + C_2)$

4 C's $\frac{1}{6}(C_1 + 2C_2 + 2C_3 + C_4)$

6 C's $\frac{1}{10}(C_1 + 2C_2 + 2C_3 + 2C_4 + 2C_5 + C_6)$

$$\text{Term 2} = \text{Col V} = \frac{\text{1st moment of crush about surface}}{L}$$

2 C's $\frac{1}{6}(C_1^{\,2} + C_2^{\,2} + C_1 C_2)$

4 C's $\frac{1}{18}(C_1^{\,2} + 2C_2^{\,2} + 2C_3^{\,3} + C_4^{\,2} + C_1 C_2 + C_2 C_3 + C_3 C_4)$

6 C's $\frac{1}{30}(C_1^{\,2} + 2C_2^{\,2} + 2C_3^{\,2} + 2C_4^{\,2} + 2C_5^{\,2} + C_6^{\,2} + C_1 C_2 + C_2 C_3 + C_3 C_4 + C_4 C_5 + C_5 C_6)$

$$\text{Term 3} = \frac{\text{1st Moment of Crush about Crush } ₵}{L^2}$$

2 C's $\frac{1}{12}(-C_1 + C_2)$

4 C's $\frac{1}{108}(-7C_1 - 6C_2 + 6C_3 + 7C_4)$

6 C's $\frac{1}{300}(-13C_1 - 18C_2 - 6C_3 + 6C_4 + 18C_5 + 13C_6)$

$$\frac{\text{Term 3}}{\text{Term 1}} = \text{Col VI}$$

$$\frac{\text{Term 2}}{\text{Term 1}} = \text{Col VII}$$

APPPENDIX D - 15

$\Delta =$ _____ $\left(\rho = \text{_____} \right)$

CDC = ⬭⬭Ⓕ⬭⬭⬭⬭

Principal Force Shown Acting at the Centroid of the Crush Region. The Force is at an Angle of ϕ from the Normal.

YSS = ___
XFF = ___
XRR = ___

x n
ϕ

L
$\bar{y}$

XFF

XFF

$\bar{x}$

h

c.m.

y

XRR

Veh # ___

YSS — YSS

Data:

$M =$ ___

$k^2_{gy} =$ ___

$XFF =$ ___

$A =$ ___

$B =$ ___

$Col\ I =$ ___

$Col\ II =$ ___

$Col\ IV =$ ___

$Col\ V =$ ___

$Col\ VI =$ ___

$Col\ VII =$ ___

Calculations:

$Area = Col\ II * Col\ IV =$ ___

$\bar{x} = XFF - Col\ VII =$ ___

$\bar{y} = Col\ I + Col\ II * Col\ VI =$ ___

$\phi = Angle\ of\ Force\ from\ n =$ ___

(+ as shown)

$h = \bar{y}\cos(\phi) - \bar{x}\sin(\phi) =$ ___

$\gamma_M = \dfrac{k^2_{gy}}{k^2_{gy} + h^2}\ M =$ ___

$Energy = Col\ II * \left[\dfrac{A^2}{2B} + A*Col\ IV + B*Col\ V \right] *$

$(1+\tan^2\phi) =$ ___

Note: The factor $(1 + \tan^2\phi)$ is evaluated as being equal to 13.9 for angles ϕ with magnitudes greater than $74.4°$.

FIGURE D.2a   A Worksheet for Front Crush

Principal Force Shown Acting
at the Centroid of the
Crush Region. The
Force is at an Angle
of ϕ from the Normal.

$\Delta =$ _____

CDC = ◯◯◯R◯◯◯◯◯

Calculations:

Area = Col II * Col IV = _____

$\bar{x}$ = Col I + Col II * Col VI = _____

$\bar{y}$ = YSS – Col VII = _____

ϕ = Angle of Force from n = _____
              (+ as shown)

$h = \bar{y}\sin(\phi) - \bar{x}\cos(\phi)$ = _____

$\gamma M = \dfrac{k^2_{gy}}{k^2_{gy} + h^2}$ M = _____

Energy = Col II*$\left[\dfrac{A^2}{2B} + A*\text{Col IV} + B*\text{Col V}\right]$*

$(1 + \tan^2\phi)$ = _____

Note: The factor $(1 + \tan^2\phi)$ is evaluated as being
equal to 13.9 for angles ϕ with magnitudes
greater than 74.4°.

Data:

M = _____

k^2_{gy} = _____

YSS = _____

A = _____

B = _____

Col I = _____

Col II = _____

Col IV = _____

Col V = _____

Col VI = _____

Col VII = _____

YSS = _____
XFF = _____
XRR = _____

Veh # _____

FIGURE D.2b   A Worksheet for Right-Side Crush

Principal Force Shown Acting at the Centroid of the Crush Region. The Force is at an Angle of ϕ from the Normal.

Calculations:

Area = Col II * Col IV = _____

$\bar{x}$ = -XRR + Col VII = _____

$\bar{y}$ = Col I + Col II * Col VI = _____

ϕ = Angle of Force from n = _____

(+ as shown)

h = $-\bar{y} \cos(\phi) - \bar{x} \sin(\phi)$ = _____

$\gamma_M = \dfrac{k^2_{gy}}{k^2_{gy} + h^2}$ M = _____

Energy = Col II* $\left[\dfrac{A^2}{2B} + A*\text{Col IV} + B*\text{Col V}\right]*$

$(1 + \tan^2 \phi)$ = _____

Note: The factor $(1 + \tan^2 \phi)$ is evaluated as being equal to 13.9 for angles ϕ with magnitudes greater than 74.4°.

Data:

M = _____

k^2_{gy} = _____

YSS = _____

A = _____

B = _____

Col I = _____

Col II = _____

Col IV = _____

Col V = _____

Col VI = _____

Col VII = _____

Δ = _____   (ρ = _____)

CDC. = _____

YSS = _____
XFF = _____
XRR = _____

FIGURE D.2c A Worksheet for Back Crush

Principal Force Shown Acting at the Centroid of the Crush Region. The Force is at an Angle of ϕ from the Normal.

Calculations:

Area = Col II * Col IV = _____

$\bar{x}$ = Col I + Col II * Col VI = _____

$\bar{y}$ = -YSS + Col VI = _____

ϕ = Angle of Force from n = _____ (+ as shown)

h = $\bar{y}$ sin(ϕ) + $\bar{x}$ cos (ϕ) = _____

$\gamma_M = \dfrac{k^2_{gy}}{k^2_{gy} + h^2}$ M = _____

Energy = Col II $*\left[\dfrac{A^2}{2B} + A * \text{Col IV} + B * \text{Col V}\right]*$

(1+tan$^2\phi$) = _____

Data:

M = _____

k^2_{gy} = _____

YSS = _____

A = _____

B = _____

Col I = _____

Col II = _____

Col IV = _____

Col V = _____

Col VI = _____

Col VII = _____

Δ =

CDC =

YSS = _____

XFF = _____

XRR = _____

Note: The factor (1 + tan$^2\phi$) is evaluated as being equal to 13.9 for angles ϕ with magnitudes greater than 74.4°.

FIGURE D.2d   A Worksheet for Left-Side Crush

FINAL WORKSHEET

Vehicle #1

$$k^2_{gy} = \underline{\hspace{3cm}}$$

$$h_1 = \underline{\hspace{3cm}}$$

$$\gamma_1 M_1 = \underline{\hspace{3cm}}$$

$$E_1 = \underline{\hspace{3cm}}$$

$$ANG_1 = \underline{\hspace{3cm}}$$

Vehicle #2

$$k^2_{gy} = \underline{\hspace{3cm}}$$

$$h_2 = \underline{\hspace{3cm}}$$

$$\gamma_2 M_2 = \underline{\hspace{3cm}}$$

$$E_2 = \underline{\hspace{3cm}}$$

$$ANG_2 = \underline{\hspace{3cm}}$$

ANG is the angle of the principal force from the + x-axis.

two vehicles: $I = \sqrt{2(E_1 + E_2)\left(\gamma_1 M_1 \gamma_2 M_2 / (\gamma_1 M_1 + \gamma_2 M_2)\right)}$

barrier: $I = \sqrt{2 E_1 \gamma_1 M_1}$

Vehicle #1

Vehicle #2
(omit if barrier)

$$\Delta V_1 = \frac{I}{17.6\,\gamma_1 M_1} = \underline{\hspace{2cm}} \text{ mph}$$

$$\Delta V_2 = \frac{I}{17.6\,\gamma_2 M_2} = \underline{\hspace{2cm}} \text{ mph}$$

$$\Delta V_{1x} = V_1 \cos(ANG_1) = \underline{\hspace{2cm}} \text{ mph}$$

$$\Delta V_{2x} = V_2 \cos(ANG_2) = \underline{\hspace{2cm}} \text{ mph}$$

$$\Delta V_{1y} = V_1 \sin(ANG_1) = \underline{\hspace{2cm}} \text{ mph}$$

$$\Delta V_{2y} = V_2 \sin(ANG_2) = \underline{\hspace{2cm}} \text{ mph}$$

$$\Delta \Omega_1 = \frac{I\,h_1}{\gamma_1 M_1 k^2_{gy1}} = \underline{\hspace{2cm}} \frac{\text{rad}}{\text{sec}}$$

$$\Delta \Omega_2 = \frac{I\,h_2}{\gamma_2 M_2 k^2_{gy2}} = \underline{\hspace{2cm}} \frac{\text{rad}}{\text{sec}}$$

$$F_{max1} = \frac{ColII_1 (A_1 + B_1 ColIV_1)}{\cos \phi_1} = \underline{\hspace{1cm}} \text{ lbs.}$$

$$F_{max2} = \frac{ColII_2 (A_2 + B_2 ColIV_2)}{\cos \phi_2} = \underline{\hspace{1cm}} \text{ lbs.}$$

$$\frac{F_{max1}}{F_{max2}} = \underline{\hspace{3cm}}$$

APPENDIX E

START2 and SPIN2
Step-by-Step Hand Calculations

APPENDIX E

START2 and SPIN2
Step-by-Step Hand Calculations
(Refer to Summary Sheet Attached to QUIZ Flowchart.)

START WORK ON VEHICLE NUMBER 1.

1. If IFLAG(1) = 0, set XC11 = XCR1
                          YC11 = YCR1
                          PSI11 = PSIR1
                          DIST1 = 0.
                          SID = 0.

2. If JSKID(1) = 0, set XC11 = XC10
                          YC11 = YC10
                          PSI11 = PSI10

3. If JCURV(1) = 0, set   P1 = 0.
                           Q1 = 0.
                   and go to Part 15 of this set to continue

4. Calculate the straight line distance from separation to end-of-skid.

$$D = \sqrt{(YC10-YC11)^2 + (XC11-XC10)^2}$$

5. Calculate the distance of the point-on-curve from the straight line through separation and end-of-skid.

$|d|$ = [(YC10-YC11)XC21 + (XC11-XC10)YC21 + (XC10)(YC11) - (XC11)(YC10)]/D

$$= \underline{\hspace{1.5cm}} \text{ ft.}$$

6. If d is less than 1/12ft., set JCURV(1) = 0 and go to PART 15 of this set to continue

7. Calculate the value of the term

$$(D^2/8d) + (d/2) = \underline{\hspace{1.5cm}} \text{ft.}$$

If this term is greater than 10,000/12ft. set JCURV(1) = 0 and go to PART 15 of this set to continue

8. Calculate the equation of the perpendicular bisector line which bisects the line from separation to point-on-curve, writing it in the form:

$$X + (B1)Y + C1 = 0$$

where

$$B1 = (YC21-YC10)/(XC21-XC10)$$

$$= \underline{\hspace{2cm}} ft.$$

$$C1 = -(XC10+XC21)/2 - [(YC21-YC10)/(XC21-XC10)](YC10+YC21)/2$$

$$= \underline{\hspace{2cm}} ft.$$

9. Calculate the equation of the perpendicular bisector line which bisects the line from point-on-curve to end-of-skid similarly.

$$B2 = (YC11-YC21)/(XC11-XC21)$$

$$= \underline{\hspace{2cm}} ft.$$

$$C2 = -(XC21+XC11)/2 - [(YC11-YC21)/(XC11-XC21)](YC21+YC11)/2$$

$$= \underline{\hspace{2cm}} ft.$$

10. Calculate the center coordinates (P1,Q1) of the circle passing through the three points: separation, point-on-curve, and end-of-skid. This is done by finding the intersection of the lines
$$X + B1Y + C1 = 0$$
and $X + B2Y + C2 = 0.$
find $\Delta = (B2 - B1)$
then $P1 = (B1C2 - B2C1)/\Delta = \underline{\hspace{2cm}} ft.$
$Q1 = (C1 - C2)/\Delta = \underline{\hspace{2cm}} ft.$

11. Calculate the radius of the circular arc which passes through the three points and has its center at X = P1 and Y = Q1.

$$RAD = \sqrt{(P1-XC10)^2 + (Q1-YC10)^2} = \underline{\hspace{2cm}} ft.$$

12. Calculate the angle of the tangent to the circular arc at the

separation point, measured from the + x-axis.

$$GAMS1 = tan^{-1}[(XC10-P1)/(Q1-YC10)] = \underline{\hspace{2cm}} radians$$

13. Calculate the arc length of the circular arc used above (this is the length of the skidding path).[1]

$$GAM = tan^{-1}[(XC11-P1)/(Q1-YC11)] = \underline{\hspace{2cm}} radians$$

$$S1 = RAD(GAM-GAMS1) = \underline{\hspace{2cm}} ft.$$

---

[1]To assure that the proper angle is calculated and used, it is necessary for the user to sketch the figure using the actual data and to "compensate" for the hand held calculators inability to properly handle angles in the second, third, and fourth quadrants.

14. The skidding path has a length of S1 from the separation point to the end-of-skid point, and the path tangent at the separation point is at an angle of GAMS1 from the +x-axis.

15. If JCURV(1) ≠ 0, S1 = (S1)(12.0) = _____ inches
(converting units of S1 from item 13)

GAMS1 = GAMS1 = _____ radians
(same value found in item 12)

If JCURV(1) = 0, Calculate

$$S1 = (12) \sqrt{(XC11-XC10)^2 + (YC11-YC10)^2} = \underline{\qquad} \text{ in.}$$

$$GAMS1 = \tan^{-1}[(YC11-YC10)/(XC11-XC10)]$$
$$= \underline{\qquad} \text{ radians}$$

16. Summary Plot and Record. (Example data used)


```
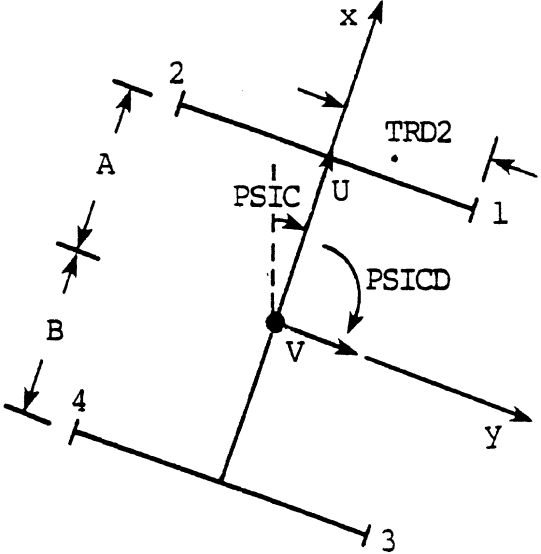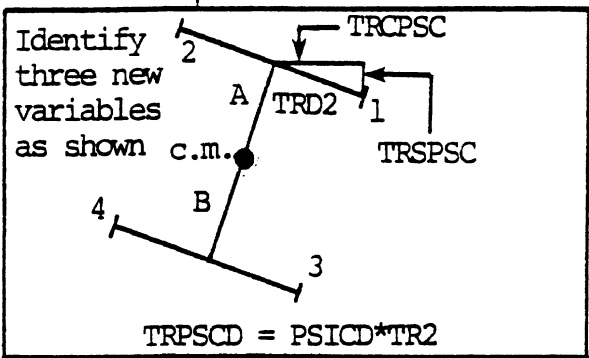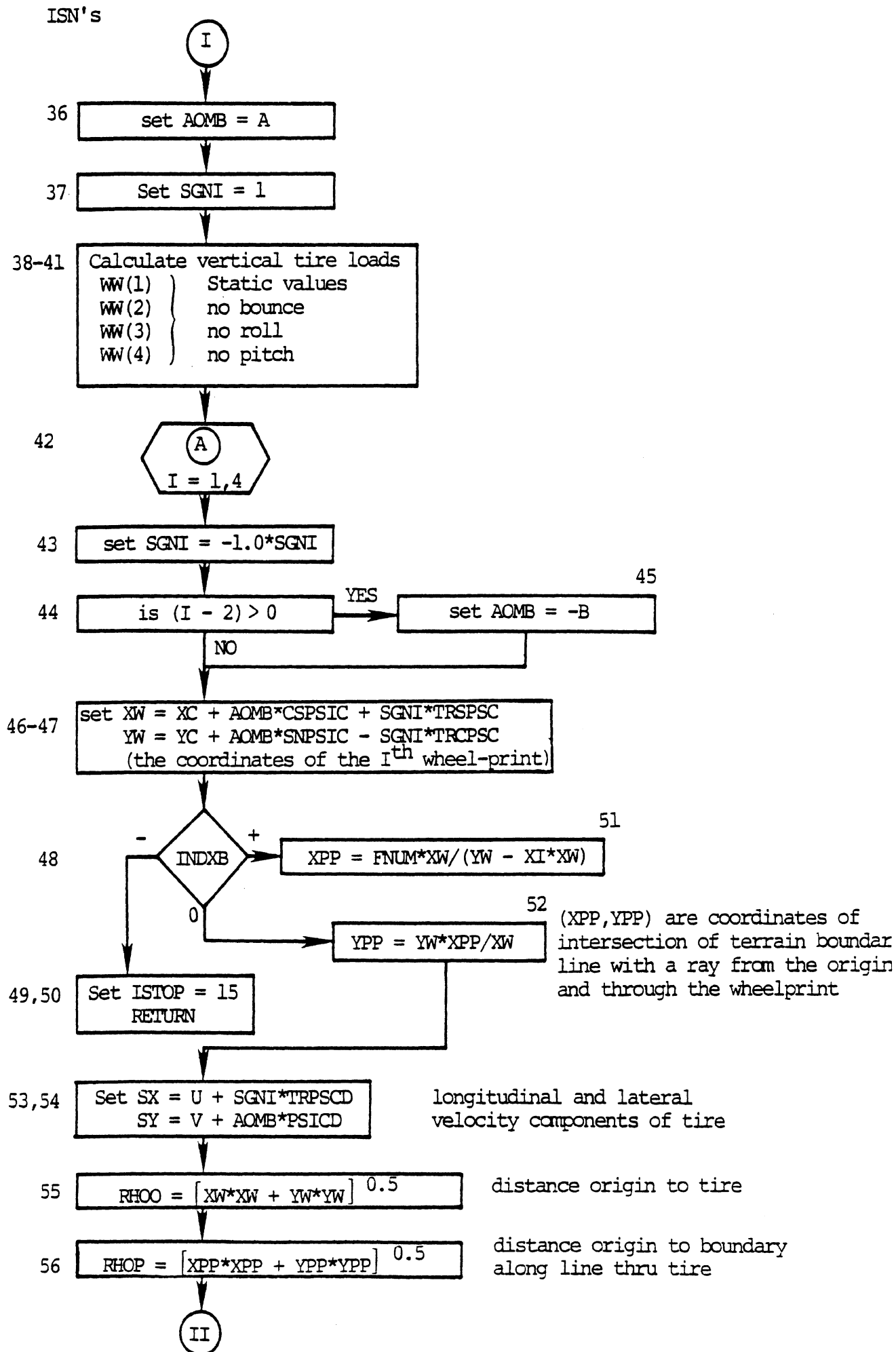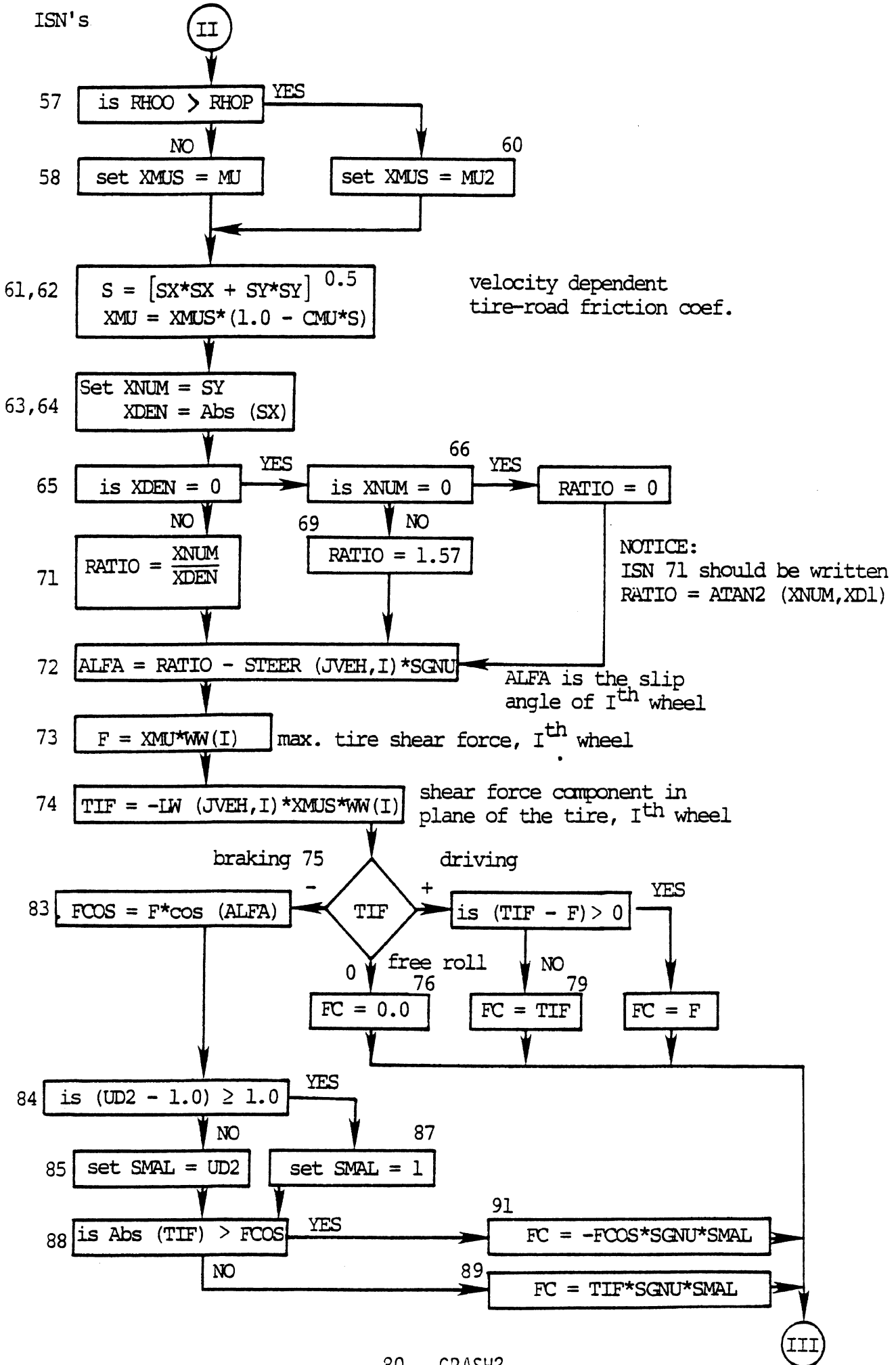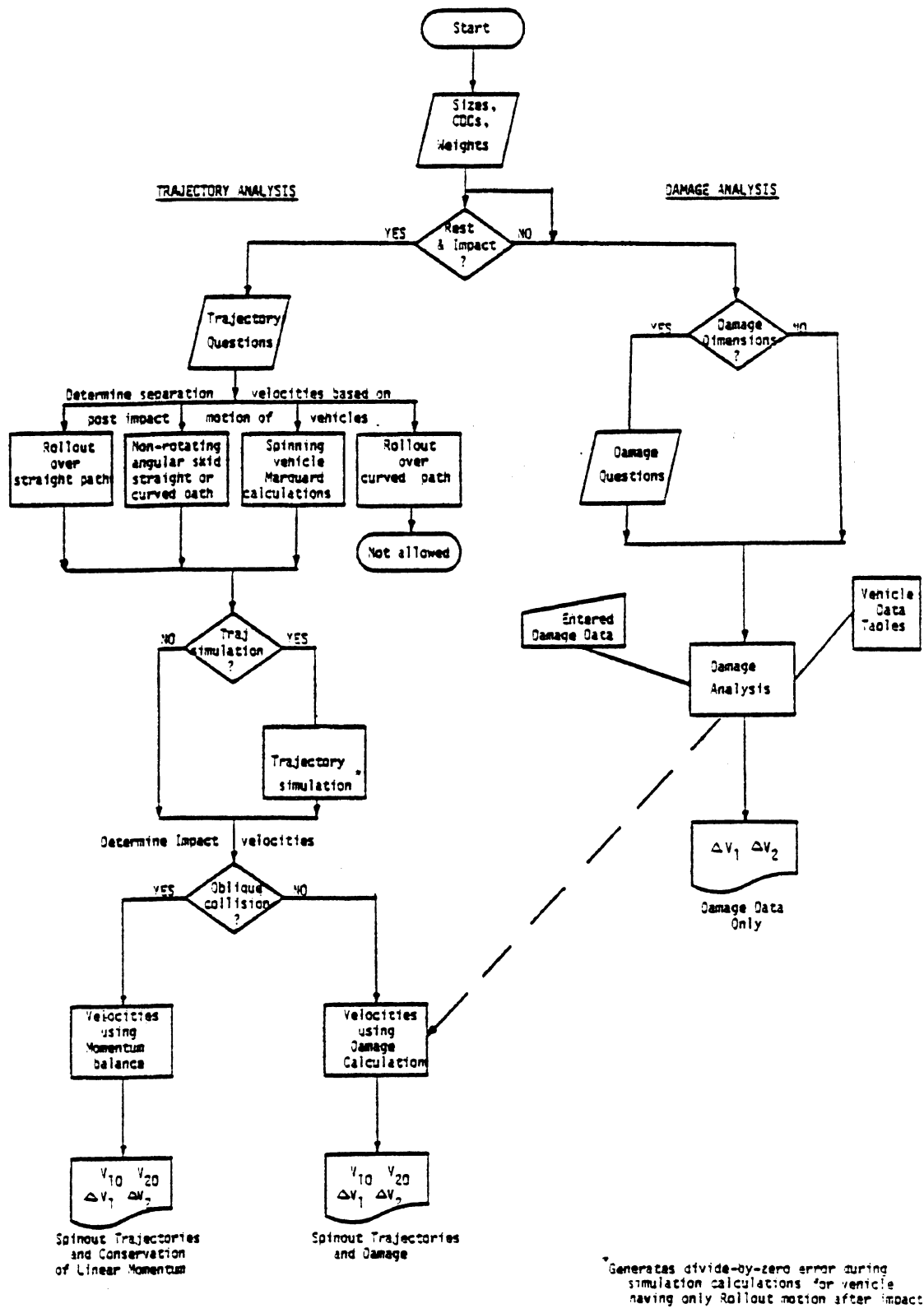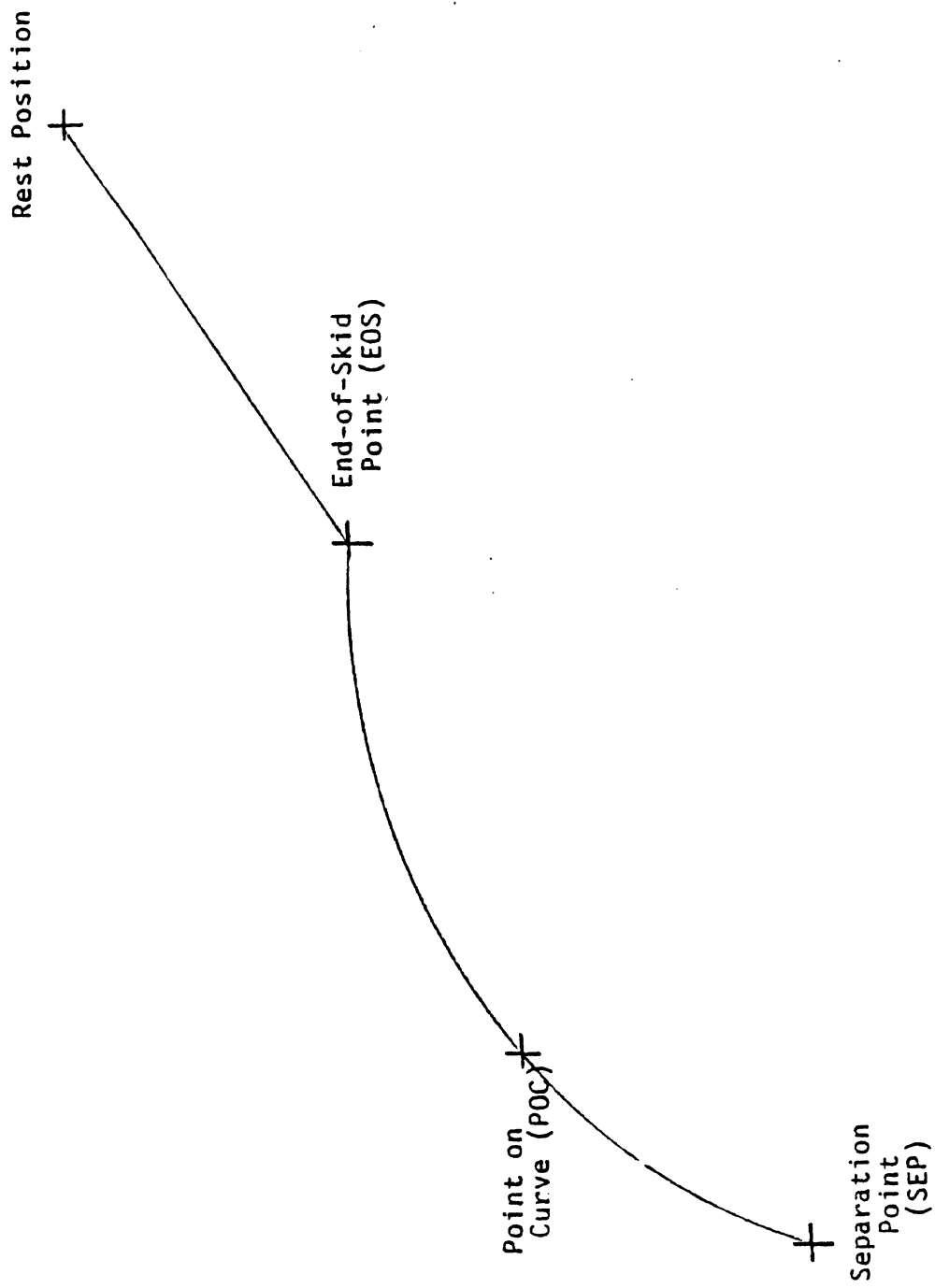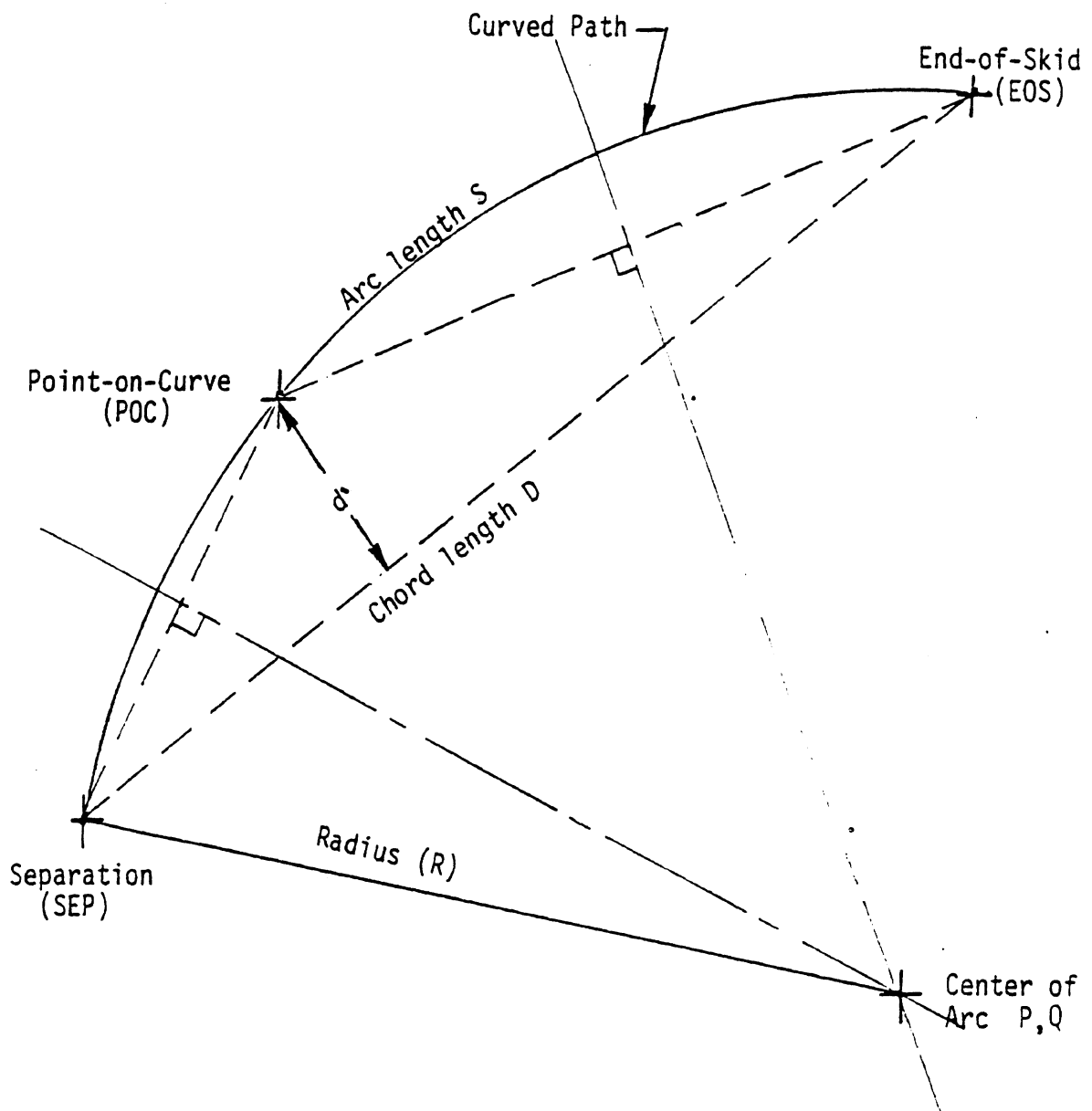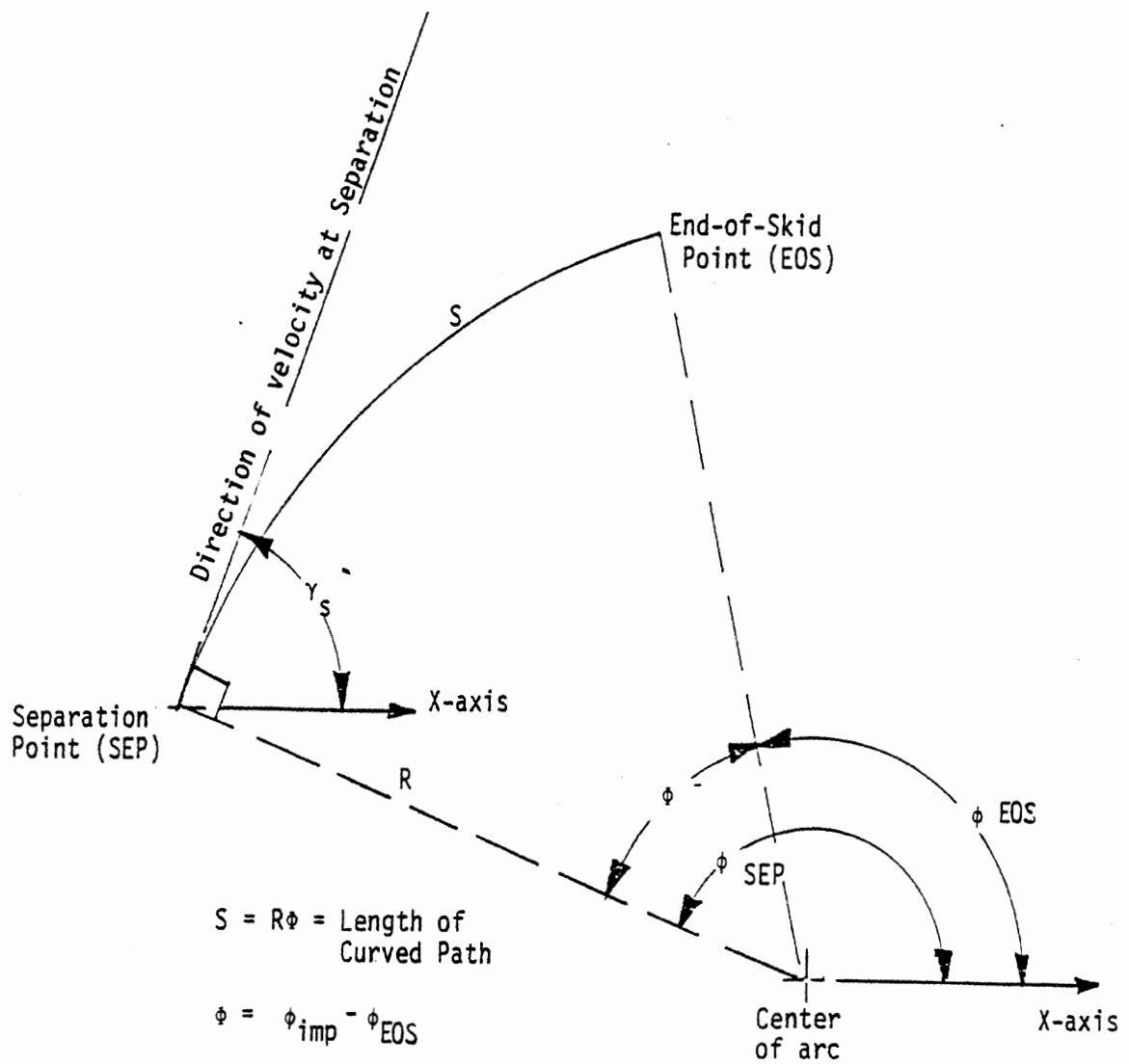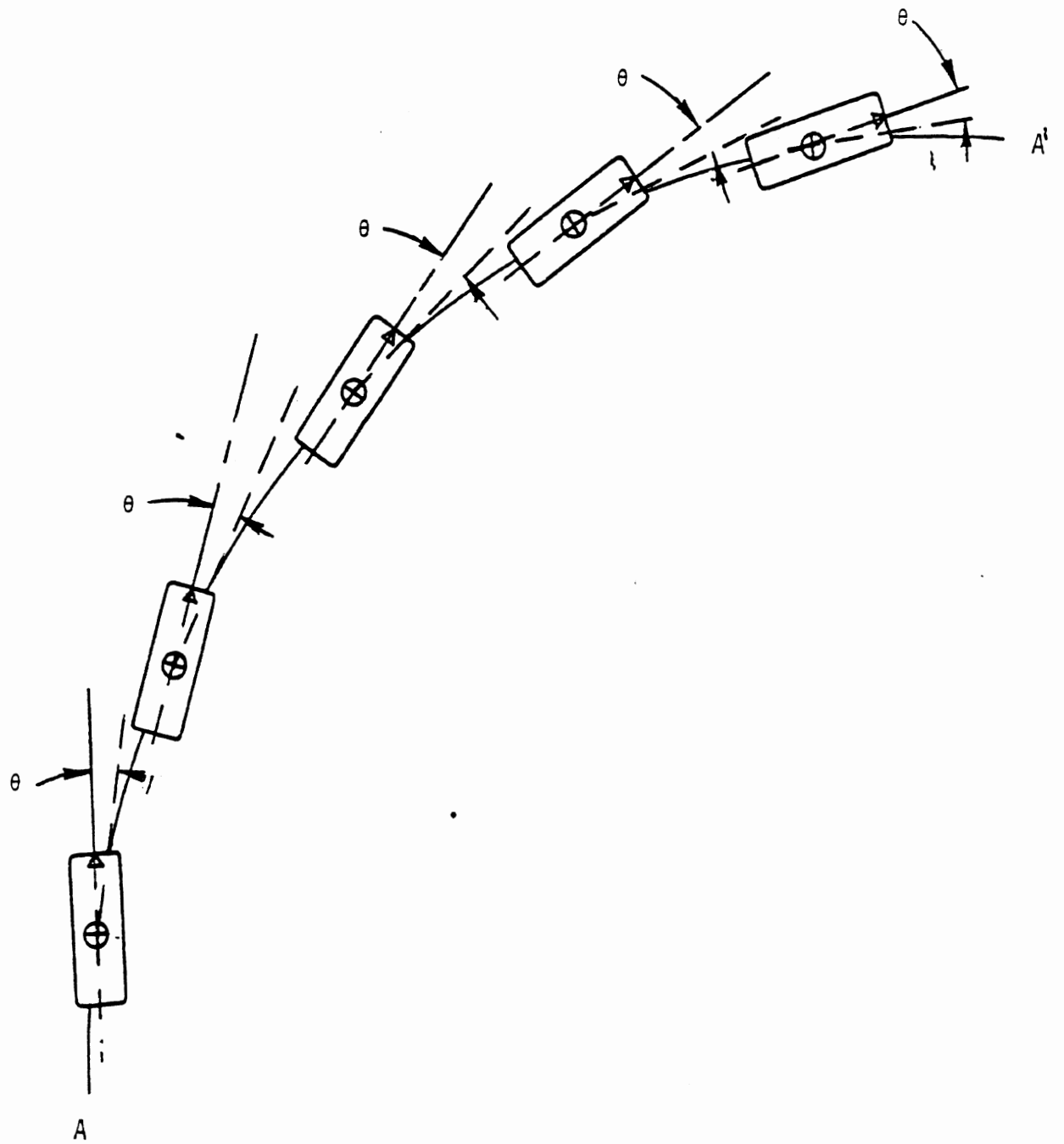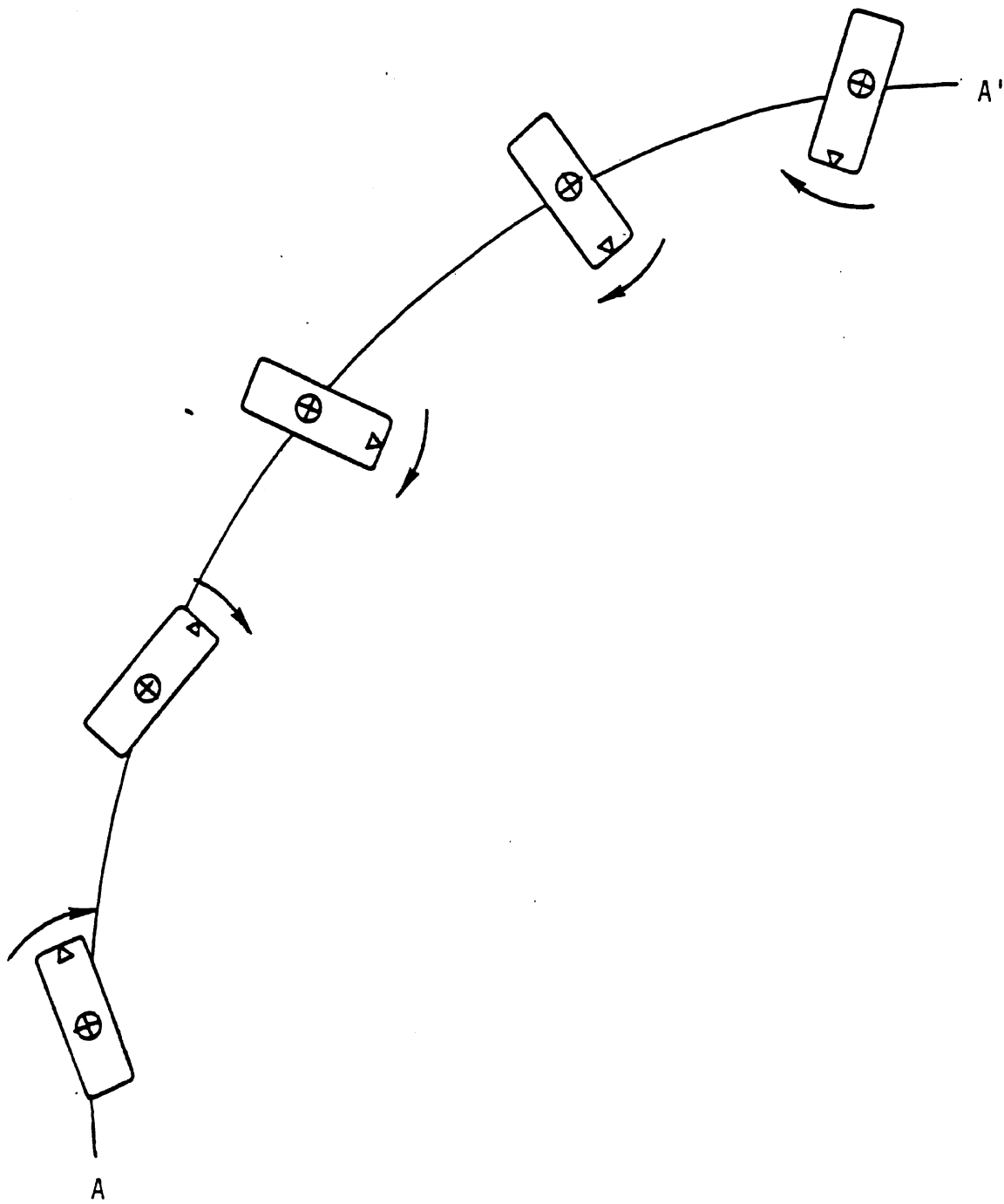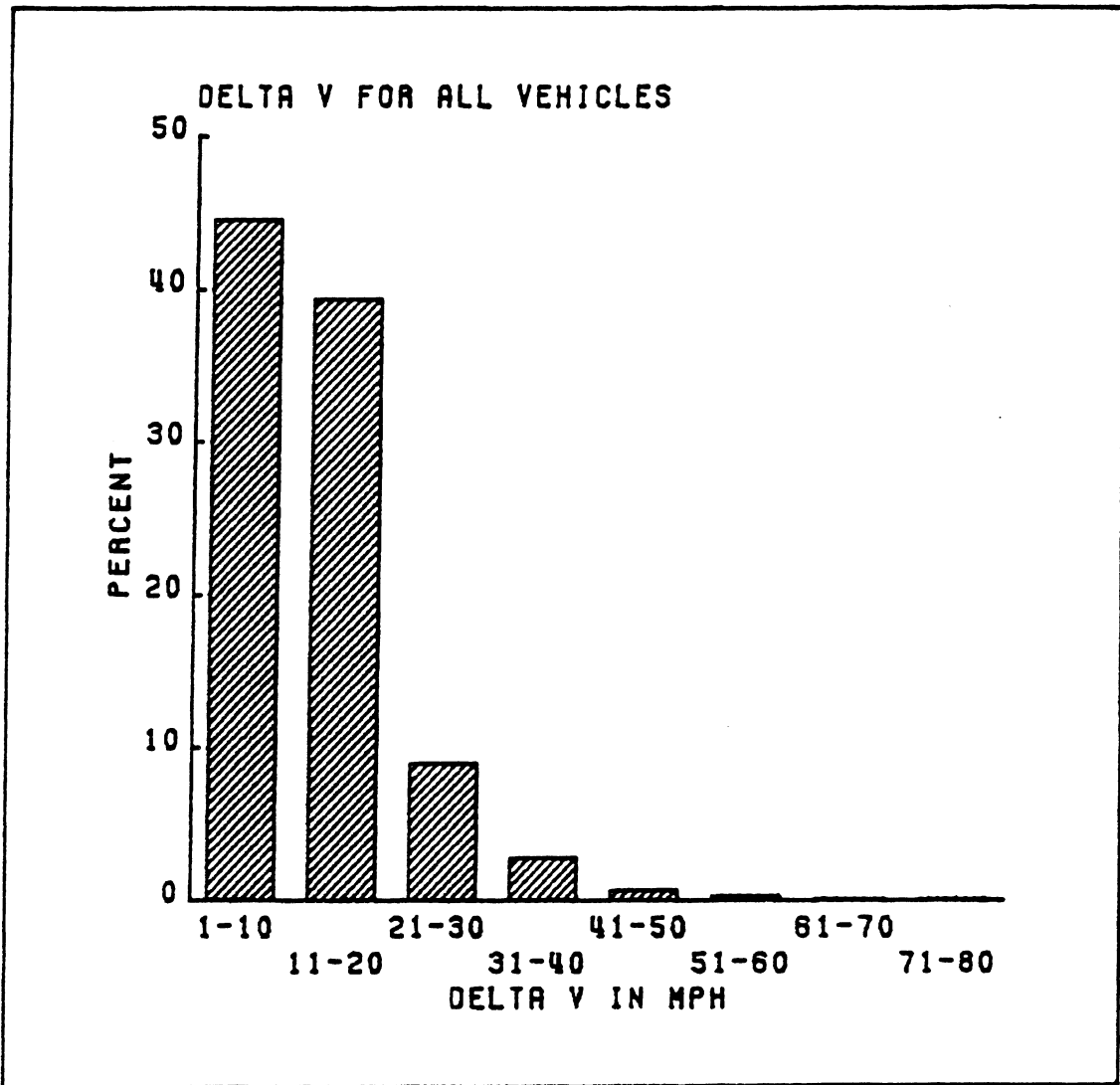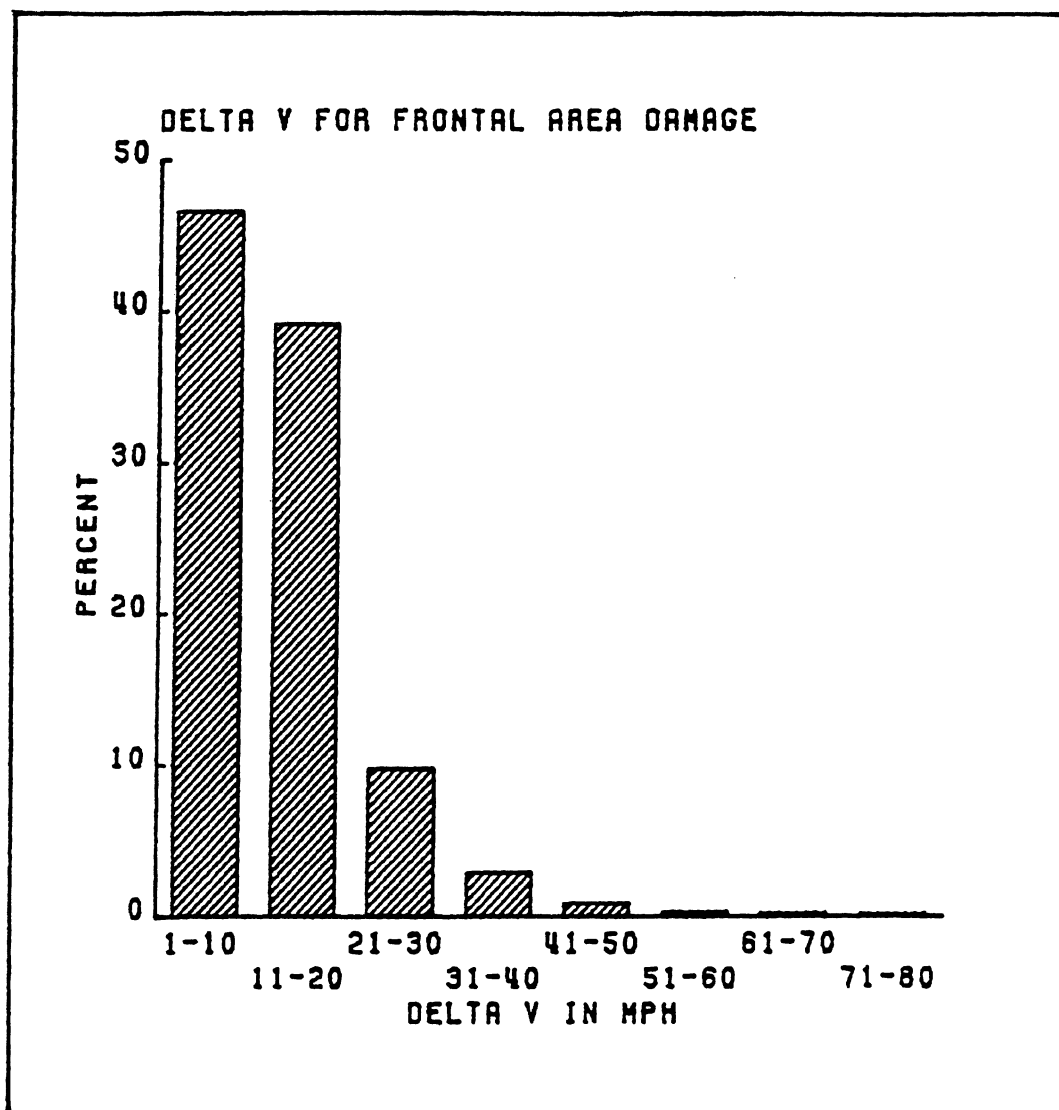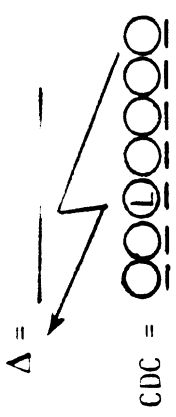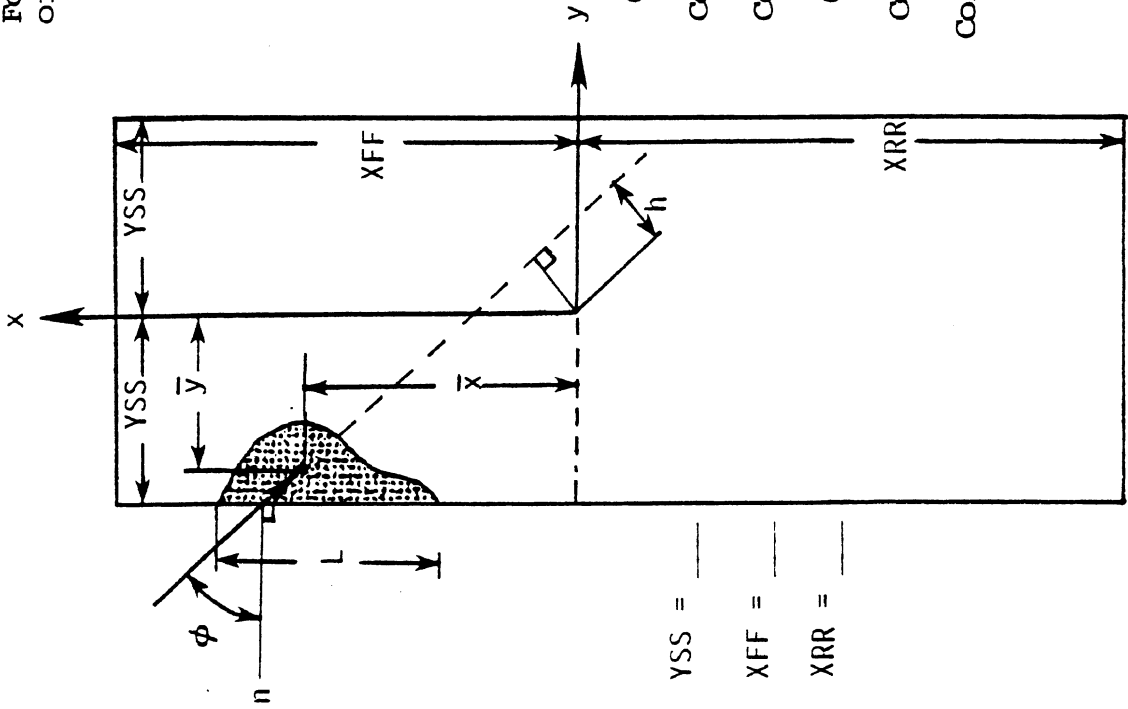XC10 = 12.05 ft.
YC10 =  4.36 ft.
XC21 = 13.86 ft.
YC21 =  5.40 ft.
XC11 = 15.64 ft.
YC11 =  7.95 ft.

|d| =  0.54 ft.
|D| =  5.08 ft.
P1  = 10.03 ft.
Q1  =  9.97 ft.
RAD =  5.96 ft.

GAMS1 =  0.346
S1       5.24 ft.

IFLAG(1) =  1
JSKID(1) =  1
JCURV(1) =  1
```

l_1 thru S(XC10, YC10) + E(XC11, YC11)
l_2 thru S(SC10, YC10) + P(XC21, YC21)
l_3 thru P(XC21, YC21) + E(XC11, YC11)
$l_4 \perp$ bisector of $\overline{SP}$
$l_5 \perp$ bisector of $\overline{PE}$
center, intersection of l_4 l_5
d, distance P to l_1
D, distance S to E on l_1
RAD, distance center to S, P or E

17. Calculate the roll-out-distance.

$$DIST1 = \sqrt{(XCR1-XC11)^2 + (YCR1-YC11)^2} = \underline{\hspace{2cm}}ft.$$

18. Calculate the speed at start of roll-out using:

$$THETA1 = [LW(1,1) + LW(1,2) + LW(1,3) + LW(1,4)]/4$$

as the average wheel-lock-up, MU as the coefficient of friction, and calculate speed as

$$SID = \sqrt{(64.4)(THETA1)(MU)(DIST1)} = \underline{\hspace{2cm}}ft/sec$$

19. Calculate the magnitude of the angular rotation from separation to end-of-skid.

$$ADPSI1 = |PSI1-PSI10| = \underline{\hspace{2cm}}rad$$

If ADPSI is less than 0.08 (4.5deg) skip steps below and go to item 24.

20. Calculate the value of PP1, an estimated ratio of linear to angular speed at separation, by following one of two routes:

ROUTE #1 (THETA1>1.0)

$$PP1 = (1.408)[((S1)(12)/ADPSI1)-32.0]$$
$$= \underline{\hspace{2cm}}in/sec$$

Route #2 (THETA1<1.0)

$$a1 = (1.0-THETA1)(8.52 \times 10^{-4})$$
$$b1 = 0.94 - (0.23)(THETA1)$$
$$c1 = 40.64 - (8.64)(THETA1) - (S1)(12)/ADPSI1$$
$$PP1 = [-b1 + \sqrt{b1^2-4a1c1}]/2a1$$
$$= \underline{\hspace{2cm}}$$

21. Enter TABLE E.1 5 times using $0.70PP1, 0.85PP1, PP1, 1.15PP1, 1.3PP1$ and read, for each entry value, the values of $a11, a12, a13, a14$ and $a15$. Record readings in Table E.2.

TABLE E.1

PP	$a11$	$a12$	$a13$	$a14$	$a15$
1.	2.573	0.876	0.847	0.872	1.240
2.	2.565	0.872	0.851	0.874	1.257
3.	2.558	0.868	0.856	0.875	1.274
4.	2.550	0.864	0.861	0.877	1.291
5.	2.543	0.861	0.866	0.878	1.307
6.	2.536	0.857	0.870	0.880	1.323
7.	2.529	0.853	0.875	0.881	1.339
8.	2.521	0.849	0.880	0.882	1.355
9.	2.514	0.845	0.885	0.884	1.370
10.	2.507	0.842	0.889	0.885	1.385
11.	2.500	0.838	0.294	0.887	1.401
12.	2.493	0.834	0.299	0.888	1.416
13.	2.486	0.830	0.303	0.889	1.431
14.	2.479	0.827	0.308	0.891	1.445
15.	2.472	0.823	0.312	0.892	1.460
16.	2.465	0.819	0.317	0.894	1.474
17.	2.458	0.816	0.321	0.895	1.488
18.	2.451	0.812	0.326	0.896	1.502
19.	2.444	0.808	0.330	0.897	1.515
20.	2.437	0.805	0.335	0.899	1.529
21.	2.430	0.801	0.339	0.700	1.542
22.	2.424	0.798	0.344	0.701	1.555
23.	2.417	0.794	0.348	0.703	1.568
24.	2.410	0.791	0.353	0.704	1.580
25.	2.403	0.787	0.357	0.705	1.593
26.	2.397	0.783	0.361	0.706	1.605
27.	2.390	0.780	0.366	0.708	1.617
28.	2.383	0.777	0.370	0.709	1.629
29.	2.377	0.773	0.374	0.710	1.641
30.	2.370	0.770	0.378	0.711	1.653
31.	2.364	0.766	0.383	0.713	1.664
32.	2.357	0.763	0.387	0.714	1.676
33.	2.351	0.759	0.391	0.715	1.687
34.	2.344	0.756	0.395	0.716	1.698
35.	2.338	0.753	0.400	0.717	1.708
36.	2.332	0.749	0.404	0.719	1.719
37.	2.325	0.746	0.408	0.720	1.729
38.	2.319	0.743	0.412	0.721	1.740
39.	2.313	0.739	0.416	0.722	1.750
40.	2.306	0.736	0.420	0.723	1.760
41.	2.300	0.733	0.424	0.724	1.769
42.	2.294	0.729	0.428	0.726	1.779
43.	2.288	0.726	0.432	0.727	1.788
44.	2.282	0.723	0.436	0.728	1.798
45.	2.276	0.720	0.440	0.729	1.807
46.	2.270	0.717	0.444	0.730	1.816
47.	2.264	0.713	0.448	0.731	1.825
48.	2.258	0.710	0.452	0.732	1.833
49.	2.252	0.707	0.456	0.733	1.842
50.	2.246	0.704	0.460	0.734	1.850
51.	2.240	0.701	0.464	0.735	1.858
52.	2.234	0.698	0.468	0.736	1.866
53.	2.228	0.695	0.472	0.738	1.874
54.	2.222	0.692	0.475	0.739	1.882
55.	2.216	0.689	0.479	0.740	1.890
56.	2.211	0.686	0.483	0.741	1.897
57.	2.205	0.683	0.487	0.742	1.904
58.	2.199	0.680	0.490	0.743	1.911
59.	2.193	0.677	0.494	0.744	1.918

61.	2.182	0.671	0.502	0.746	1.932
62.	2.177	0.668	0.505	0.747	1.939
63.	2.171	0.665	0.509	0.748	1.945
64.	2.165	0.662	0.513	0.749	1.952
65.	2.160	0.659	0.516	0.750	1.958
66.	2.154	0.656	0.520	0.751	1.964
67.	2.149	0.653	0.523	0.752	1.970
68.	2.144	0.650	0.527	0.752	1.975
69.	2.138	0.648	0.530	0.753	1.981
70.	2.133	0.645	0.534	0.754	1.987
71.	2.128	0.642	0.537	0.755	1.992
72.	2.122	0.639	0.541	0.756	1.998
73.	2.117	0.636	0.544	0.757	2.003
74.	2.112	0.634	0.548	0.758	2.008
75.	2.107	0.631	0.551	0.759	2.013
76.	2.101	0.628	0.554	0.760	2.018
77.	2.096	0.626	0.558	0.761	2.022
78.	2.091	0.623	0.561	0.762	2.027
79.	2.086	0.620	0.565	0.762	2.031
80.	2.081	0.618	0.568	0.763	2.036
81.	2.076	0.615	0.571	0.764	2.040
82.	2.071	0.612	0.574	0.765	2.044
83.	2.066	0.610	0.578	0.766	2.048
84.	2.061	0.607	0.581	0.767	2.052
85.	2.056	0.605	0.584	0.768	2.056
86.	2.051	0.602	0.587	0.768	2.059
87.	2.047	0.600	0.590	0.769	2.063
88.	2.042	0.597	0.594	0.770	2.066
89.	2.037	0.594	0.597	0.771	2.070
90.	2.032	0.592	0.600	0.772	2.073
91.	2.028	0.590	0.603	0.773	2.076
92.	2.023	0.587	0.606	0.773	2.079
93.	2.018	0.585	0.609	0.774	2.082
94.	2.014	0.582	0.612	0.775	2.085
95.	2.009	0.580	0.615	0.776	2.088
96.	2.004	0.577	0.618	0.776	2.090
97.	2.000	0.575	0.621	0.777	2.093
98.	1.995	0.573	0.624	0.778	2.095
99.	1.991	0.570	0.627	0.779	2.098
100.	1.986	0.568	0.630	0.780	2.100
101.	1.982	0.566	0.633	0.780	2.102
102.	1.978	0.563	0.636	0.781	2.104
103.	1.973	0.561	0.639	0.782	2.106
104.	1.969	0.559	0.642	0.782	2.108
105.	1.965	0.557	0.644	0.783	2.110
106.	1.960	0.554	0.647	0.784	2.112
107.	1.956	0.552	0.650	0.785	2.113
108.	1.952	0.550	0.653	0.785	2.115
109.	1.948	0.548	0.655	0.786	2.116
110.	1.944	0.546	0.658	0.787	2.118
111.	1.939	0.543	0.661	0.787	2.119
112.	1.935	0.541	0.664	0.788	2.121
113.	1.931	0.539	0.666	0.789	2.122
114.	1.927	0.537	0.669	0.790	2.123
115.	1.923	0.535	0.672	0.790	2.124
116.	1.919	0.533	0.674	0.791	2.125
117.	1.915	0.531	0.677	0.792	2.126
118.	1.911	0.529	0.679	0.792	2.126
119.	1.908	0.527	0.682	0.793	2.127
120.	1.904	0.525	0.684	0.793	2.128

121.	1.900	0.523	0.687	0.794	2.129
122.	1.896	0.521	0.690	0.795	2.129
123.	1.892	0.519	0.692	0.795	2.130
124.	1.889	0.517	0.694	0.796	2.130
125.	1.885	0.515	0.697	0.797	2.130
126.	1.881	0.513	0.699	0.797	2.131
127.	1.878	0.511	0.702	0.798	2.131
128.	1.874	0.509	0.704	0.799	2.131
129.	1.871	0.507	0.706	0.799	2.131
130.	1.867	0.506	0.709	0.800	2.131
131.	1.863	0.504	0.711	0.800	2.131
132.	1.860	0.502	0.713	0.801	2.131
133.	1.857	0.500	0.716	0.802	2.131
134.	1.853	0.498	0.718	0.802	2.131
135.	1.850	0.497	0.720	0.803	2.131
136.	1.846	0.495	0.722	0.803	2.130
137.	1.843	0.493	0.725	0.804	2.130
138.	1.840	0.491	0.727	0.804	2.130
139.	1.836	0.490	0.729	0.805	2.129
140.	1.833	0.488	0.731	0.806	2.129
141.	1.830	0.486	0.733	0.806	2.128
142.	1.827	0.485	0.735	0.807	2.128
143.	1.824	0.483	0.738	0.807	2.127
144.	1.821	0.481	0.740	0.808	2.127
145.	1.817	0.480	0.742	0.808	2.126
146.	1.814	0.478	0.744	0.809	2.125
147.	1.811	0.477	0.746	0.809	2.125
148.	1.808	0.475	0.748	0.810	2.124
149.	1.805	0.474	0.750	0.810	2.123
150.	1.802	0.472	0.752	0.811	2.122
151.	1.799	0.470	0.754	0.811	2.122
152.	1.797	0.469	0.756	0.812	2.121
153.	1.794	0.468	0.757	0.812	2.120
154.	1.791	0.466	0.759	0.813	2.119
155.	1.788	0.465	0.761	0.813	2.118
156.	1.785	0.463	0.763	0.814	2.117
157.	1.783	0.462	0.765	0.814	2.116
158.	1.780	0.460	0.767	0.815	2.115
159.	1.777	0.459	0.768	0.815	2.114
160.	1.775	0.458	0.770	0.816	2.113
161.	1.772	0.456	0.772	0.816	2.112
162.	1.769	0.455	0.774	0.817	2.111
163.	1.767	0.454	0.775	0.817	2.109
164.	1.764	0.452	0.777	0.818	2.108
165.	1.762	0.451	0.779	0.818	2.107
166.	1.759	0.450	0.780	0.819	2.106
167.	1.757	0.448	0.782	0.819	2.105
168.	1.755	0.447	0.784	0.820	2.104
169.	1.752	0.446	0.785	0.820	2.102
170.	1.750	0.445	0.787	0.821	2.101
171.	1.748	0.444	0.788	0.821	2.100
172.	1.745	0.442	0.790	0.821	2.099
173.	1.743	0.441	0.791	0.822	2.098
174.	1.741	0.440	0.793	0.822	2.096
175.	1.739	0.439	0.794	0.823	2.095
176.	1.736	0.438	0.796	0.823	2.094
177.	1.734	0.437	0.797	0.824	2.093
178.	1.732	0.436	0.799	0.824	2.091
179.	1.730	0.435	0.800	0.824	2.090
180.	1.728	0.434	0.801	0.825	2.089

181.	1.736	0.433	0.826	2.066
182.	1.734	0.432	0.826	2.065
183.	1.732	0.431	0.826	2.065
184.	1.730	0.430	0.827	2.064
185.	1.718	0.429	0.827	2.063
186.	1.716	0.428	0.827	2.063
187.	1.715	0.427	0.828	2.062
188.	1.713	0.426	0.828	2.062
189.	1.711	0.425	0.828	2.061
190.	1.709	0.424	0.829	2.060
191.	1.708	0.423	0.829	2.060
192.	1.706	0.422	0.829	2.059
193.	1.704	0.421	0.830	2.058
194.	1.703	0.420	0.830	2.058
195.	1.701	0.419	0.831	2.057
196.	1.700	0.418	0.831	2.056
197.	1.698	0.417	0.832	2.055
198.	1.697	0.417	0.832	2.054
199.	1.695	0.416	0.832	2.053
200.	1.694	0.415	0.833	2.053
201.	1.692	0.415	0.833	2.056
202.	1.691	0.414	0.834	2.058
203.	1.689	0.413	0.834	2.058
204.	1.688	0.413	0.834	2.057
205.	1.687	0.412	0.835	2.053
206.	1.686	0.411	0.835	2.052
207.	1.684	0.411	0.835	2.051
208.	1.683	0.410	0.836	2.056
209.	1.682	0.410	0.836	2.056
210.	1.681	0.409	0.836	2.059
211.	1.680	0.409	0.837	2.059
212.	1.679	0.408	0.837	2.058
213.	1.678	0.407	0.838	2.058
214.	1.676	0.407	0.838	2.058
215.	1.675	0.406	0.839	2.057
216.	1.674	0.406	0.839	2.057
217.	1.673	0.405	0.839	2.057
218.	1.672	0.405	0.840	2.056
219.	1.672	0.406	0.840	2.056
220.	1.671	0.405	0.840	2.056
221.	1.670	0.404	0.841	2.056
222.	1.670	0.404	0.841	2.056
223.	1.669	0.403	0.842	2.056
224.	1.668	0.403	0.842	2.056
225.	1.667	0.403	0.842	2.056
226.	1.667	0.403	0.843	2.057
227.	1.667	0.402	0.843	2.057
228.	1.666	0.402	0.843	2.057
229.	1.665	0.402	0.845	2.056
230.	1.665	0.402	0.845	2.058
231.	1.664	0.401	0.846	2.058
232.	1.664	0.401	0.846	2.059
233.	1.663	0.401	0.847	2.059
234.	1.663	0.401	0.847	2.051
235.	1.663	0.401	0.847	2.051
236.	1.662	0.400	0.848	2.050
237.	1.662	0.400	0.848	2.052
238.	1.661	0.400	0.848	2.053
239.	1.661	0.400	0.849	2.054
240.	1.661	0.400	0.849	2.051

```
241.  1.660  0.400  0.849  0.847  2.066
242.  1.660  0.400  0.849  0.847  2.069
243.  1.660  0.400  0.849  0.847  2.070
244.  1.660  0.400  0.850  0.848  2.072
245.  1.660  0.400  0.850  0.848  2.074
246.  1.660  0.400  0.850  0.848  2.075
247.  1.660  0.400  0.350  0.849  2.077
248.  1.660  0.400  0.850  0.849  2.079
249.  1.660  0.400  0.850  0.849  2.081
≥250. 1.660  0.400  0.350  0.350  2.083
```

22.

TABLE E.2
Alpha Values versus PP1

	0.70PP1	0.85PP1	PP1	1.15PP1	1.30PP1
$\alpha 11$					
$\alpha 12$					
$\alpha 13$					
$\alpha 14$					
$\alpha 15$					

23. Calculate the term:

$$Test1 = S1/[ADPSI(1 + 2(DIST1 + \sqrt{(DIST1)(DIST1+S1)}))]$$

Then calculate for each of the 5 cases of TABLE E.2 the term:

$$Test2 = [(4)(\alpha 11)(k_{gy}^2)(THETA1)]/[(\alpha 15)^2(\alpha 12)(WB)]$$

where

WB is the wheelbase, and

k^2_{gy} is the radius of gyration in yaw squared.

If, in any of the 5 cases, Test1 is less than Test 2, an assumption is made that the speed of the vehicle to be estimated in that case will be too small for the vehicle to be able to move from the separation to the end-of-skid point and preserve enough speed to yet move to the rest point. If all 5 cases produce values of Test 1 less than Test 2, it is required that the program be aborted here. If four or less cases have Test 1 less than Test 2, those cases are dropped and calculations continue.

24. For each case (of the 5) which survives the last step, the following equations will be solved for values of $\dot{S}$sep and $\dot{\Psi}$sep which are the values of linear and angular speed of the vehicle at separation. The equations to be solved are the following:

If ADPSI less than 0.08

$$\dot{\Psi}sep = 0$$

$$Temp1 = \sqrt{sin^2(GAMS1-PSI10) + (THETA1)^2 cos^2(GAMS1-PSI10)}$$
$$=\underline{\hspace{2cm}}$$

$$\dot{S}_{sep} = \sqrt{(772.8)(MU)(S1)(Temp1) + SID^2} = \underline{\hspace{2cm}}$$

If ADPSI greater than 0.08

$$Temp1 = (a14)(k^2_{gy})(ADPSI)(1 - THETA1(a13/a14)) = \underline{\hspace{2cm}}$$

$$Temp2 = (WB)(a12) = \underline{\hspace{2cm}}$$

$$Temp3 = (S1)(a15)/(2\ a11) = \underline{\hspace{2cm}}$$

$$DDDD = (Temp1/Temp2) + Temp3 = \underline{\hspace{2cm}}$$

$$BBBB = (SID)(ADPSI)/DDDD = \underline{\hspace{2cm}}$$

$$CCCC = (a11)(a14)(MU)(G)(ADPSI)^2/2DDDD = \underline{\hspace{2cm}}$$

$$\dot{\Psi}sep = [BBBB/2 + 1/2\sqrt{BBBB^2+4(CCCC)(DPSI)/ADPSI}] = \underline{\hspace{2cm}}$$

$$Temp4 = (a11)(MU)(G)(DPSI)/2\ \dot{\Psi}sep = \underline{\hspace{2cm}}$$

$$Temp5 = [|\dot{\Psi}sep|\ (k^2_{gy})\ (1 - (THETA1)(a13))]/a14 = \underline{\hspace{2cm}}$$

$$\dot{S}_{sep} = SID + 2(a14)(Temp4 - (Temp5/Temp2)) = \underline{\hspace{2cm}}$$

25. Each case passed through step 24 has produced a value of $\dot{\Psi}$sep and a value of $\dot{S}$ sep and was derived from the use of a value of PP. The value PP was an absolute value of the estimated ratio of the linear to angular speeds at separation. CRASH2 uses a logic which says-- the closer a result comes to an estimate, the better it is. Thus

we calculate for each case a

$$B = [PP \mid \dot{\Psi}sep/\dot{S}_{sep} \mid] - 1$$

and identify the case with the least absolute value of B and use the results from that case. At this point the velocity state of the vehicle at separation is known.

Figure

26-50 Repeat steps 1-25 for Veh #2.

51. Collecting all information entered by the user under prompting by QUIZ, plus tables of data, and using the procedures of START2 and SPIN2, we know the position and velocity state of each vehicle at separation as well as future trajectory data.

See the note in text relative to possible
adjustment of the value of GAMS1