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Technical Report

OBJECTIVE ANALYSIS INCONSISTENCIES IN GEOSTROPHIC WIND AND MOMENTUM TRANSPORT CALCULATIONS

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ABSTRACT

The basic NMC algorithm for analysis of the height field is investigated with special attention to inconsistencies in the implied geostrophic wind field. Two sources of error related to the analysis system are investigated analytically and empirically on idealized flows. Several experiments are made with real data to examine the influence of system-related inconsistencies on implied geostrophic mean zonal winds and meridional momentum transports with the latter subdivided into zonal wave number components. It is shown that the absolute magnitudes of these quantities are systematically increased by the analysis system but without special bias to any wave scale. It is also shown that system-related inconsistencies are greatly outweighed by inhomogeneity in the data network. The latter problem is particularly evident in geostrophic momentum transports. The results of this study are used to examine briefly other studies of the mean zonal wind and momentum transport which are based on NMC objective analyses of the height field.

1. INTRODUCTION

Over the past two decades meteorological synoptic analysis has become increasingly associated with a numerical or objective product. The change from more traditional methods has been occasioned by development of ever more sophisticated, high-speed computers which can assimilate and process observational data far more rapidly and efficiently than is possible by hand. Furthermore, the development of forecast models has evoked a need for fields of regularly gridded data which are obtained readily only through objective analysis. A residual benefit of routine objective analysis is the accumulation of an abundant library of meteorological fields which may be used in empirical research into atmospheric processes. Meteorological literature contains many such observational studies which would have been virtually impossible without recourse to objective analysis. However, the computer is a master at producting a regular and subjectively reasonable objective product, and it is all too easy to be mesmerized into equating these features with accuracy. The observation network from which analyses are produced is far from uniform. In consequence the objective analysis is not uniformly accurate. In addition, the mechanics of the analysis system may contain hidden bias and uncertainties which are not immediately apparent in the final product. Rather little attention has been directed to detailed investigation of these problems, particularly as they influence the results of empirical studies based on objective analyses. This study examines some of these aspects.

An entirely general approach is not feasible. A number of objective analysis schemes have been proposed, many of which are used operationally. Each deals with the observations in a different manner and so contains individual idiosyncracies. This study deals exclusively with the successive approximation technique originally devised by Bergthorssen and Döös (1955) and modified by Cressman (1959) for use at the National Meteorological Center (NMC). The model constructed for this study was designed to reproduce the one-level model used at NMC prior to 1965. NMC has subsequently introduced a multi-level analysis scheme, but the latter retains the basic features of the earlier version. The results presented here are consequently somewhat limited in scope, but they apply in general to the most widely distributed and most frequently used objective product and in particular to NMC analyses issued prior to 1965.

Several factors stimulated this investigation. Firstly there is the question of errors in the zonal wind predicted by numerical barotropic forecasts. This subject has been discussed in some detail by Bristor (1959) and Wiin-Nielsen (1960), but only from the aspect of the forecast models. Secondly, Bristor noted that the barotropic forecast predicts a jet which is displaced too far to the north, and it is common to many older objective analyses that they produce spurious jets in the Pacific. Thirdly, investigations by Wiin-Nielsen et al. (1963, 1964) of the poleward flux of angular momentum computed from objectively analyzed geopotential fields are notably different from studies not based on objective analyses (Holopainen; 1967 and 1968). Finally, an earlier study by the author (1968) has shown significant internal bias in the NMC analysis scheme with respect to objectively analyzed height fields. It seemed desirable to extend the investigation to byproducts of the height field or in particular to the implied geostrophic wind field. An understanding of system-related difficulties associated with this field is imperative for assessment of empirical studies which are based on gridded NMC data. Studies which are most directly related are computations of heat and momentum transport. In turn, these quantities are related to calculations of energy exchange since the conversion of zonal to eddy potential (kinetic) energy depends upon the transport of heat (momentum).

It is shown analytically that the analysis system places considerable artificial constraint on the geostrophic wind field and consequently on byproducts of this field. These constraints are partly alleviated by spatial averaging which is common to global flux computations. It is demonstrated empirically that internal inconsistencies in the analysis system help to explain errors in the location of the forecast jet but tend to reduce rather than amplify errors in the barotropically forecast zonal wind speed. It is also shown that inconsistencies in the analysis system aid in explaining the discrepancies in momentum flux computations, and that the existing inhomogeneity in the reporting network is potentially disastrous to transport computations based on objectively analyzed data.

2. ANALITICAL CONSIDERATIONS

2.1 The Analysis Model

It has been mentioned above that the analysis model constructed for this study duplicates the one-level model used at NMC prior to 1965. As such it employs a successive approximation technique in which an initial approximation to a variable specified at regular gridpoints is successively modified by recourse to observations of the variable at irregular points in the analysis area. In the particular system used here, a gridded array of the height field at 500-mb is modified in four successive scans. For an individual scan, the value at each gridpoint is influenced by all data which lie within a prescribed distance or scan radius. This distance is reduced with each successive scan so that the earlier scans capture the gross features while the later scans delineate the finer detail of the analyzed parameter. For a given gridpoint and an individual scan, the correction at the gridpoint depends on the type of data which may contain the height of the 500-mb surface and/or the speed and direction of the wind at the station. In this study, for the sake of simplicity, only those reports which include both observations are considered. One scan's correction at a gridpoint is then given by the difference between the value of the height at the gridpoint (either the initial approximation or the value from the preceding scan) and the value of the observed heights linearly extrapolated to the grid location by the implied height gradients which are obtained from the observed winds through the geostrophic approximation. This difference is weighted according to the distance separating the gridpoint from the observations in order to correct empirically for the inaccuracy of the extrapolation. Mathematically, the correction from a single report (δz_k) is expressed by:

$$\delta z_k = z_o + \frac{\partial z_o}{\partial r} d_k - z_g \qquad (1)$$

where z_o is the observed height at the station, $\partial z_o/\partial r$ is the height gradient derived from the wind report, d_k is the distance separating the observation from the gridpoint, and z_g is the height value at the gridpoint prior to the scan. The total correction for the gridpoint on a given scan is the weighted average of the individual corrections given by:

$$\delta z = \sum_{k=1}^{N} \frac{W_k \, \delta z_k}{N} \qquad (2)$$

where N is the total number of stations situated no farther from the gridpoint than the influence distance (scan radius (R)). The weighting factor is defined by:

$$W_k = \frac{R^2 - d_k^{\,2}}{R^2 + d_k^{\,2}} \qquad (3)$$

where R = 5.9, 3.6, 2.2, and 1.5 gridlengths for the four scans respectively. On the final scan, if $N \ngtr 3$, the correction is increased by replacing the denominator in (2) by the sum of the weights.

In addition to the smoothing implicit in the scanning technique, a nine-point smoother is applied to the grid fol-

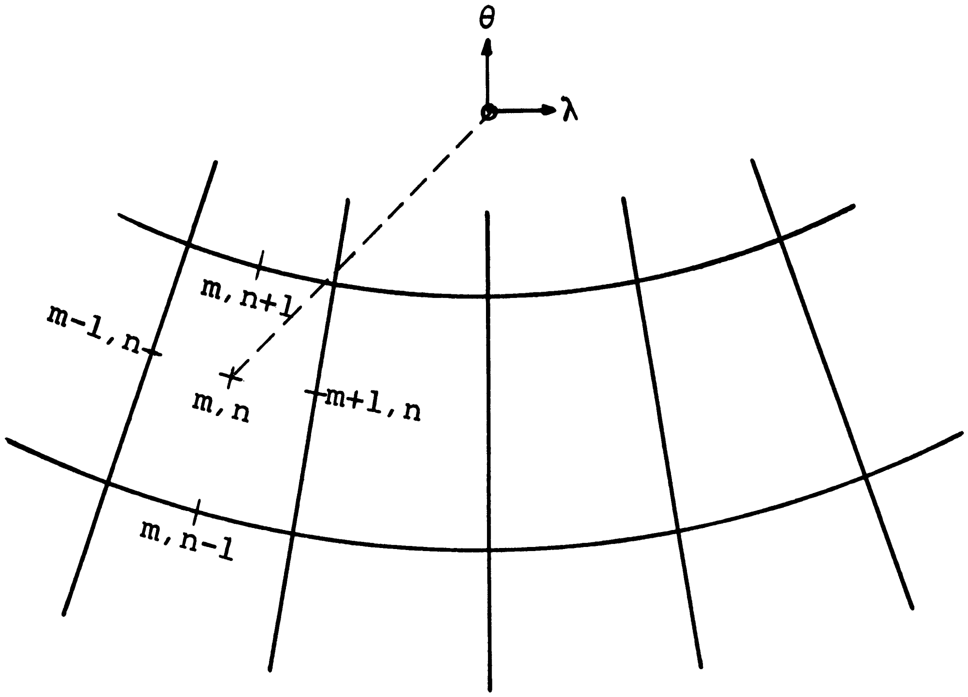


Figure 1

lowing the latter two scans. This smoother is identical to the one currently used at NMC (Shuman; 1967).

2.2 Geostrophic Winds

It was stated in the introduction that an earlier study has elucidated significant internal inconsistencies in height fields produced by the objective analysis scheme described above. In particular, the analysis system behaves erratically in sparse data areas and the analysis is strongly influenced by inhomogeneity in the reporting network. The purpose of this section is to extend the investigation to include the geostrophic winds implicit in the analyzed height field and further to investigate the effect in computations of mean zonal wind and meridional momentum transport. Since studies of the latter are generally computed on a geographic grid, the question of analysis error is formulated in this reference system. For simplicity, only the effect of a single station on a single scan is considered.

The corrected height (zl) for any scan at gridpoint (m,n) (Fig. 1.) is given by:

$$zl^{m,n}=z^{m,n}+W^{m,n}\left\{z_0-\frac{f_0}{g}\left[u_0(\theta^{m,n}-\theta_0)-v_0(\lambda^{m,n}-\lambda_0)\right]-z^{m,n}\right\} \quad (4)$$

or

$$zl^{m,n}=z^{m,n}+W^{m,n}(\delta z^{m,n}) \quad (5)$$

where θ is latitude (increasing north), λ is longitude (increasing east), f is the Coriolis parameter, g is the acceleration of gravity, and the zero subscript refers to the observation. The expressions $(\theta^{m,n} - \theta_o)$ and $(\lambda^{m,n} - \lambda_o)$ represent the actual distances separating the observation from the gridpoint in the meridional and zonal directions respectively. The implied zonal wind value (ul) following correction of the height values is found from the geostrophic approximation:

$$ul^{m,n} = \frac{g}{f^{m,n}} \frac{\Delta zl^{m,n}}{\Delta\theta^{m,n}} \quad \frac{g}{f^{m,n}} \left[\frac{zl^{m,n-1} - zl^{m,n+1}}{\Delta\theta^{m,n}}\right] \tag{6}$$

where $\Delta\theta^{m,n}$ is the distance of two latitudinal gridlengths surrounding gridpoint (m,n).

Letting

$$\hat{z} = \frac{g}{f} z \tag{7}$$

Then (ignoring the variation of the Coriolis parameter):

$$\begin{aligned} ul^{m,n} = u^{m,n} + \{[\hat{z}_o + v_o\,(\lambda^{m,n} - \lambda_o)]\ [W^{m,n-1} - W^{m,n+1}] \\ - u_o\,[W^{m,n-1}\,(\theta^{m,n-1} - \theta_o) - W^{m,n+1}(\theta^{m,n+1} - \theta_o)] \\ - [(W\hat{z})^{m,n-1} - (W\hat{z})^{m,n+1}]\}\ \frac{1}{\Delta\theta^{m,n}} \quad . \end{aligned} \tag{8}$$

If the change in the weighting function is approximated by

$$W^{m,n-1} = W^{m,n+1} + \epsilon_\theta^{m,n} \tag{9}$$

(8) can be rewritten as:

$$ul^{m,n} = u^{m,n} + W^{m,n-1}\,(u_o - u^{m,n}) - \frac{\epsilon_\theta^{m,n}}{\Delta\theta^{m,n}}\,(\delta\hat{z}^{m,n+1}) \quad . \tag{10}$$

In an analogous manner the implied meridional component (vl) is found from:

$$vl^{m,n} = v^{m,n} + W^{m-1,n} (v_o - v^{m,n}) + \frac{\epsilon_\lambda^{m,n}}{\Delta\lambda^{m,n}} (\delta \hat{z}^{m,n}) \qquad (11)$$

where

$$vl^{m,n} = \frac{g}{f^{m,n}} \left[\frac{z^{m,+1,n} - z^{m-1,n}}{\Delta\lambda^{m,n}}\right] \qquad (12)$$

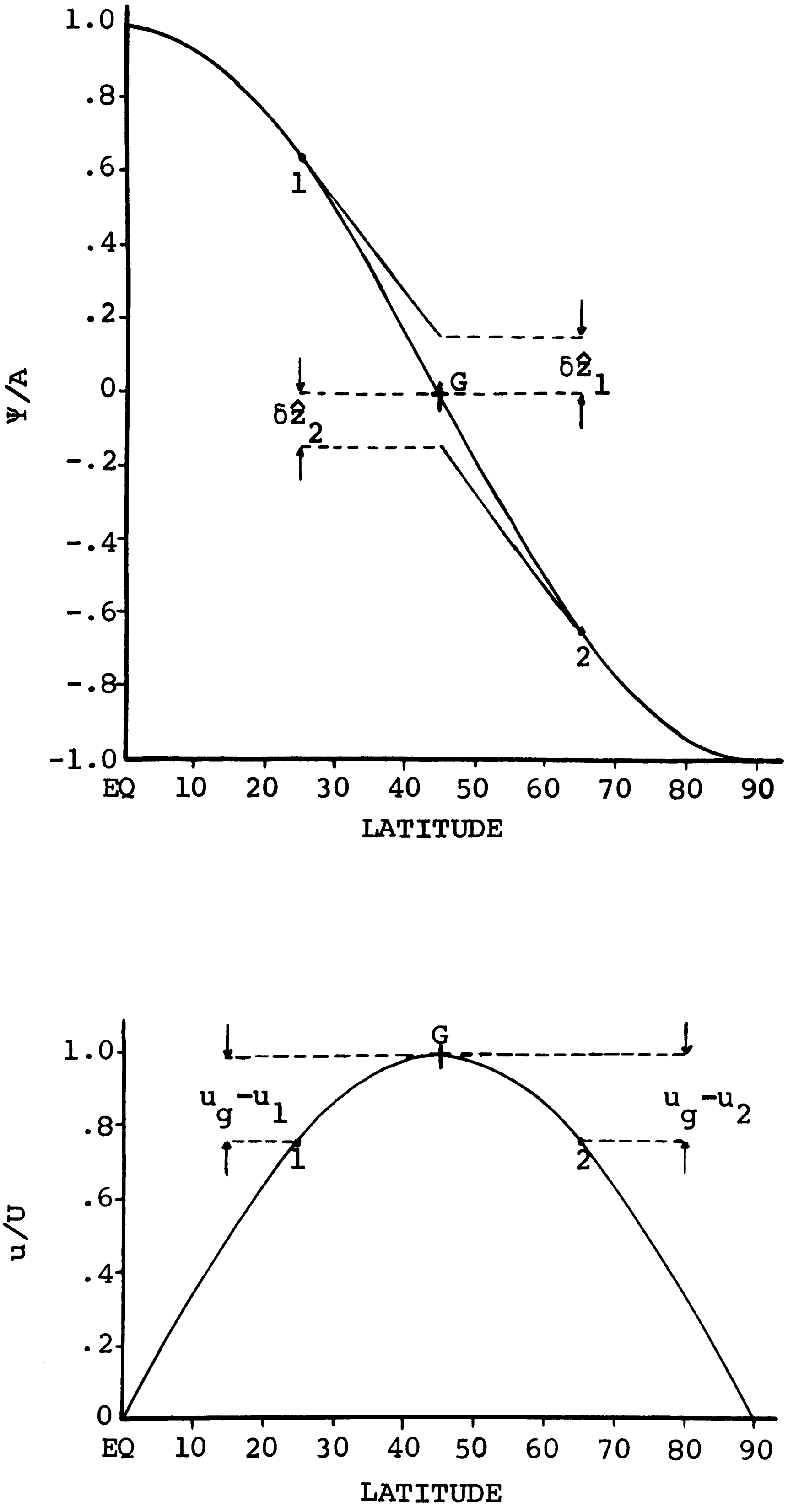
$$W^{m+1,n} + W^{m-1,n} + \epsilon_\lambda^{m,n} \qquad (13)$$

and $\Delta\lambda^{m,n}$ is the distance of two longitudinal gridlengths. Several points can be noted in (10) and (11).

i) An extrapolation error which is contained in $\delta\hat{z}$ and which is equivalent to that responsible for instability in sparse-data height field analysis is contained in the last terms of (10) and (11). It is present only because of truncation error associated with the weighting function.

ii) If the terms containing this extrapolation correction are excluded, the correction to a wind component at a gridpoint is the weighted difference between the value observed at the station and the value of the guess field at the gridpoint. This would not seem to be a felictous technique, particularly for the early scans. There is little reason to expect good correlation between wind vectors which are as much as 1500 kilometers apart. Note that the analogous correction to the height field from a station report of only the height is the weighted difference between the value observed at the station and the value of the guess field interpolated to the station from the surrounding gridpoints.

iii) Unlike the height field, the wind vector will not converge to the observed value when the station location and


1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.0
Ψ/A
1
2
G
δẑ1
δẑ2
EQ 10 20 30 40 50 60 70 80 90
LATITUDE
1.0
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.6
.4
.2
0
u/U
G
ug-u1
ug-u2
1
2
EQ 10 20 30 40 50 60 70 80 90
LATITUDE

Figure 2

the gridpoint are coincident. In this case $\epsilon_\theta = \epsilon_\lambda = 0$, but the weighting factor is not unity since it applies at an adjacent gridpoint. For the final scan, this weighting factor (on the NMC octogon) is 0.37. Consequently this term causes significant smoothing of the wind field even in dense-data regions.

iv) Both of the correction terms in (10) and (11) are biased in direction. For example, if observing stations are located equidistant north and south of a gridpoint, $W^{m,n-1}$ will be larger for the station located to the north and $Z^{m,n+1}$ will include a greater extrapolation distance.

The influence of the corrective terms can be examined by a simple example. For convenience the terms are defined as the weighted correction and the extrapolation correction (the second and third terms respectively in (10) and (11)). Let us define an idealized perturbation stream function which varies only with latitude:

$$\Psi = A \cos 2\theta$$

The meridional velocity component is zero while the zonal component is given by:

$$u = \frac{\partial\Psi}{a\partial\theta} = \frac{2A \sin 2\theta}{a} = U \sin 2\theta \qquad (15)$$

where a is the radius of the earth. We assume that an objective analysis is performed on this field with a uniform station distribution yielding accurate reports, and as a first approximation we shall ignore the direction bias discussed in (iv) above. An example of two station reports which provide the parameters $(u_o - u^{m,n})$ and $\hat{\delta z}^{m,n}$ is depicted graphically in Fig. 2. In this example the stations are located equidistant north and south of a gridpoint which is situated at the inflection point of the stream function profile.

At this point the extrapolation errors $\hat{\delta z}$ from the observations are of opposite sign (and in this idealized case equal in magnitude). However, ϵ_θ (10) also changes sign since it is positive for stations to the north of a gridpoint and negative for stations to the south. Thus in uniform data networks the effect of the extrapolation correction will be cumulative near points of inflection in the stream field or equivalently where the zonal wind has an extremum. The effect of the correction will always be to increase the relative magnitude of the wind. The bottom figure shows that the effect of the weighted correction at extrema of the wind profile is also cumulative and always leads to a decrease in the relative magnitude of the wind. Consequently the two corrective terms tend seredipitiously to offset one another. By similar reasoning it may be readily seen that where the stream function has an extremum the extrapolation corrections from stations north and south will tend to offset each other. In this instance $\hat{\delta z}$ is of the same sign, but ϵ_θ again alternates. Similarly the weighted correction will be reduced as the stream function extremum corresponds to an inflection point in the wind profile.

If the directional bias discussed in (iv) is not ignored it can be seen that an additional systematic error is introduced. Since the extrapolation is always greater to the north, the effect will be to shift the phase of the stream function and the wind profile to the south. Similarly with the weighted correction, the net effect will be a southerly shift in the wind profile. It should be anticipated, however, that these biases will be greatly outweighed by inhomogeneity in the reporting network.

The complexity of the real atmosphere makes it impossible to predict analytically the combined effects of the weighted and extrapolation corrections. One can, however, gain some insight into their relative contributions through a further idealized experiment. Following Wiin-Nielsen (1960), let us consider a channel flow described by a perturbation stream function of the following form:

$$\Psi(x,y) = A \cos \mu y \cos k\,(x+\alpha y^2). \qquad (16)$$

For convenience we define the flow in a region bounded to the north and south by solid walls at $y=\pm W$ and permit only the first mode in the y (meridional) direction with maximum amplitude in the middle of the channel. Thus

$$\mu = \frac{\pi}{2W}. \qquad (17)$$

The flow is considered to be continuous in the x (zonal) direction with wave number k. The parameter α is related to the slope of the trough and ridge lines in the flow. These are defined by the condition $v=0$ and so are parabolas of the form:

$$x + \alpha\, y^2 = 0. \qquad (18)$$

For $\alpha>0$ the parabolas are open toward the "west" which gives a positive tilt to the streamlines in the lower half and a negative tilt in the upper half of the channel. The flow is made meteorologically reasonable by adopting the following values for the parameters:

$W = 1/3 \times 10^7$; $2W \simeq 60^\circ$ of latitude

$kA = V_{max} = 20$ m sec.$^{-1}$

$\alpha = 1/4W$ corresponding to a slope of $\pm \frac{1}{2}$ at $y = \mp W$.

The influence of the analysis system on grid (as distinct from geographic) wind components can easily be computed from the equations given earlier if x is equated to λ, y to θ, and the geostrophic stream function $\hat{z}$ to Ψ. Computations can be made directly for any combination of observation, gridpoint, and analysis scan. With these parameters fixed, the corrections can be computed for variations in the zonal wave number. A number of such computations have been made and although the results are understandably erratic, they clearly demonstrate that the extrapolation correction is dominant in the outer reaches of the early (large) scan radii while the weighted correction becomes significant only in inner regions which are modified by later (smaller) scans. Consequently the net error of the analysis scheme taken as an entity is caused by the extrapolation correction. What is perhaps not so obvious is that variation in the zonal wave number has little influence on the relative error caused by the extrapolation and weighted corrections. In other words, errors caused by the analysis system do not appear to be related to the scale of the analyzed parameter. An example of corrections applying on the first scan at a gridpoint approximately four and one-half gridlengths southeast of a station located just south of a "low" is given by Table 1. This is included to show that the corrective terms are not inconsiderable (especially with respect to momentum transport) and to demonstrate the apparent independence of the zonal wave number. The weighted correction is given by WCOR and the extrapolation correction by ECOR.

TABLE 1.

CHANGE IN WIND COMPONENTS AND MOMENTUM TRANSPORT CAUSED BY SINGLE OBSERVATION AND ONE SCAN OF OBJECTIVE ANALYSIS

$m.\ sec^{-1}$ $m.^{2} sec.^{-2}$

K	WCORU	ECORU	U	U1	WCORV	ECORV	V	V1	UV	UV1
1	3.3	-3.2	-27.8	-27.7	2.0	-6.5	-4.3	-8.8	120	245
2	1.6	-3.2	-13.6	-15.2	3.8	-5.8	-8.3	-10.4	113	157
3	1.0	-3.8	-8.6	-11.4	5.2	-6.4	-11.6	-12.8	100	146
4	0.7	-4.5	-5.9	-9.8	6.2	-7.1	-13.9	-14.8	82	144
5	0.4	-5.1	-4.0	-8.7	6.7	-7.6	-15.0	-15.8	60	138
6	0.2	-5.6	-2.5	-7.9	6.6	-7.7	-14.8	-15.9	37	125
7	0.0	-5.9	-1.2	-7.1	6.0	-7.4	-13.4	-14.8	16	105
8	0.2	-6.0	-0.0	-6.2	4.8	-6.8	-10.8	-12.9	0	79
9	-0.3	-5.9	1.0	-5.2	3.2	-5.9	7.3	-10.1	-8	53
10.	-0.5	-5.7	2.0	-4.2	1.3	-4.9	-3.2	-6.8	-6	28
11	-0.6	-5.3	2.8	-3.1	-0.8	-3.7	1.2	3.3	3	10
12	-0.7	-4.8	3.4	-2.1	-2.8	-2.6	5.4	0.1	19	-0
13	-0.7	-4.2	3.8	-1.1	-4.5	-1.6	9.3	3.2	35	-4
14	-0.8	-3.6	4.0	-0.3	-5.9	-0.8	12.3	5.6	49	-2
15	-0.7	-2.9	3.9	0.3	-6.8	-0.2	14.3	7.2	55	2

2.3 The Influence of Zonal Averaging

Studies of the mean zonal wind obviously include the effect of zonal averaging and empirical studies of momentum and heat transport are generally simplified in this manner to give the flux across latitude circles. It is interesting to note that

The extrapolation error of (10) and (11) is somewhat reduced when the wind components are zonally averaged along the latitude segment which intercepts the circle of influence of the reporting station. These equations become:

$$\frac{1}{2M}\sum_{m=-M}^{M} ul^{m,n} = \frac{1}{2M}\sum_{m=-M}^{M}\{ u^{m,n} + W^{m,n-1}(u_o - u^{m,n}) + \frac{\epsilon_\theta}{\Delta\theta}[(\theta^{m,n+1} - \theta_o)\, u_o + \hat{z}^{m,n+1} - \hat{z}_o]\} \qquad (19)$$

$$\frac{1}{2M}\sum_{m=-M}^{M} vl^{m,n} = \frac{1}{2M}\{\sum_{m=-M}^{M}[v^{m,n} + W^{m-1,n}(v_o - v^{m,n})] - \sum_{m-1}^{M}\epsilon_\lambda [\frac{\hat{z}^{m+1,n} - \hat{z}^{m-1,n}}{\Delta\lambda} - m\, v_o]\} \qquad (20)$$

where (2M+1) is the number of gridpoints along the intercepted arc. A part of the extrapolation error (in each case that part associated with the opposite velocity component) is eliminated by the averaging process. In general, zonally averaged quantities should contain less analysis-related error than individual gridpoint values.

It is possible to express analytically the expression for the analysis system's influence on momentum transport, but this is not done here as the expression is long and cumbersome and it is not simple to isolate the important features. The interaction of the two corrective terms suggest that the distribution of reporting stations may be significant, but this can be investigated only empirically as is done in the following section.

3. EMPIRICAL RESULTS

The effect of the analysis system on real data has been tested for eight days of 500-mb data taken from standard NMC history tapes. A program (see Appendix) was constructed to investigate system-related changes in the mean zonal wind and zonally averaged momentum transports in the wave number regime. These parameters were chosen to afford comparison with empirical studies performed by other authors as will be discussed in Section 4.

The momentum transport across a latitude circle is obtained in the wave number regime by separating the geostrophic stream function into Fourier components along the latitude circle:

$$\hat{z}(\theta) = A_o(\theta) + \sum_{m=1}^{N} [A_n(\theta) \cos n\lambda + B_n(\theta) \sin n\lambda] + R(\theta) \qquad (21)$$

where N is the highest wave number considered (in this study N=15) and R (θ) is the residual due to truncation of the series. From the expressions

$$u = \frac{1}{a}\frac{\partial \hat{z}}{\partial \theta} \quad \text{and} \quad v = \frac{1}{a \cos \theta}\frac{\partial \hat{z}}{\partial \lambda} \qquad (22)$$

it is straightforward to show that the zonally averaged momentum transport (per unit mass) is given by:

$$\overline{uv} = \frac{1}{2a^2 \cos \theta} \sum_{n=1}^{N} n\{A_n \frac{dB_n}{d\theta} - B_n \frac{dA_n}{d\theta}\} + R^1(\theta) \qquad (23)$$

where the bar operator refers to the zonal average obtained from:

$$\overline{(\quad)} = \frac{1}{2\pi}\int_o^{2\pi} (\quad) \, d\lambda \qquad (24)$$

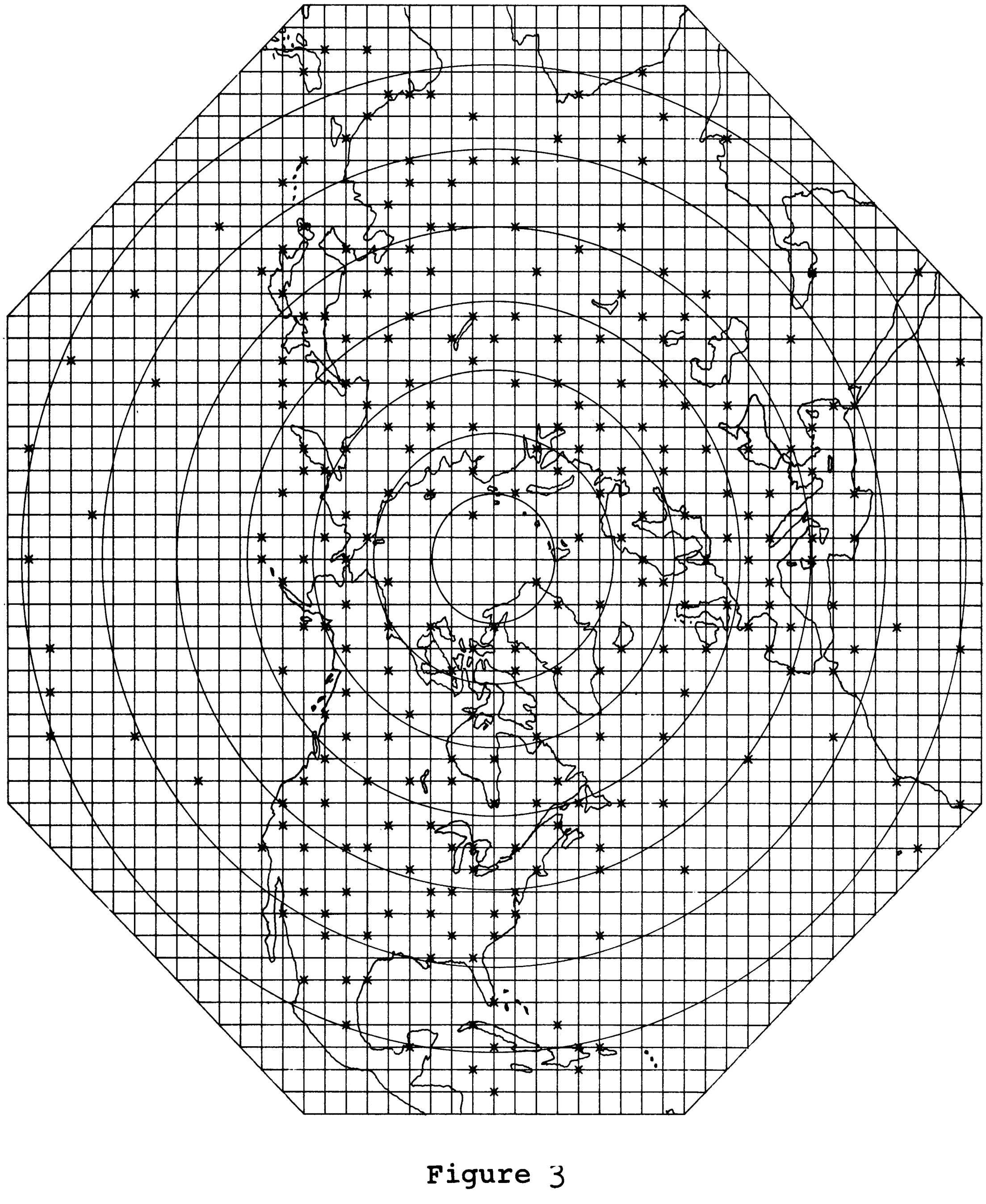


Figure 3

The transport by an individual wave number is given by:

$$\overline{uv}_n = \frac{n}{2a^2\cos\theta}\{A_n \frac{dB_n}{d\theta} - B_n \frac{dA_n}{d\theta}\} \qquad (25)$$

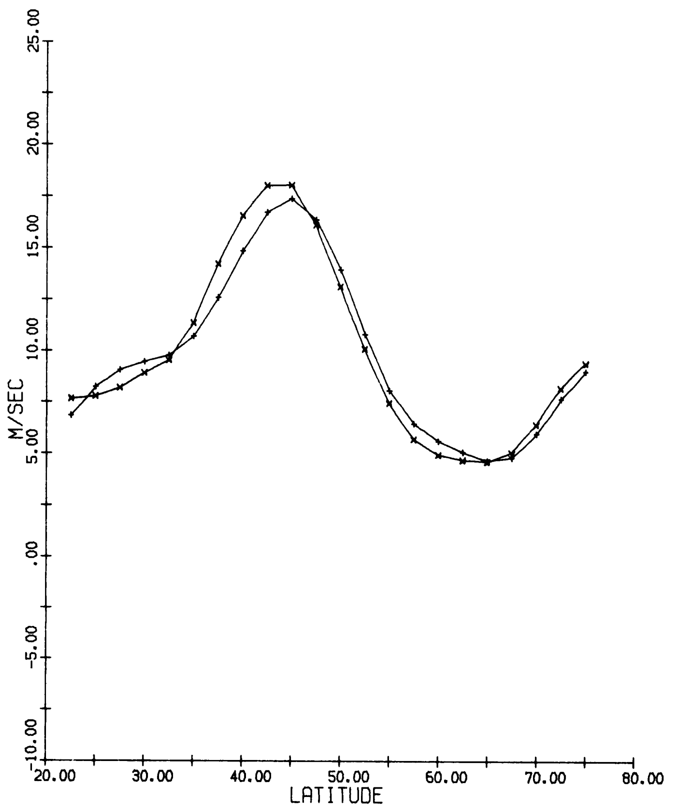
while the mean zonal wind is given by:

$$\overline{u} = \frac{1}{a}\frac{dA_o}{d\theta} \; . \qquad (26)$$

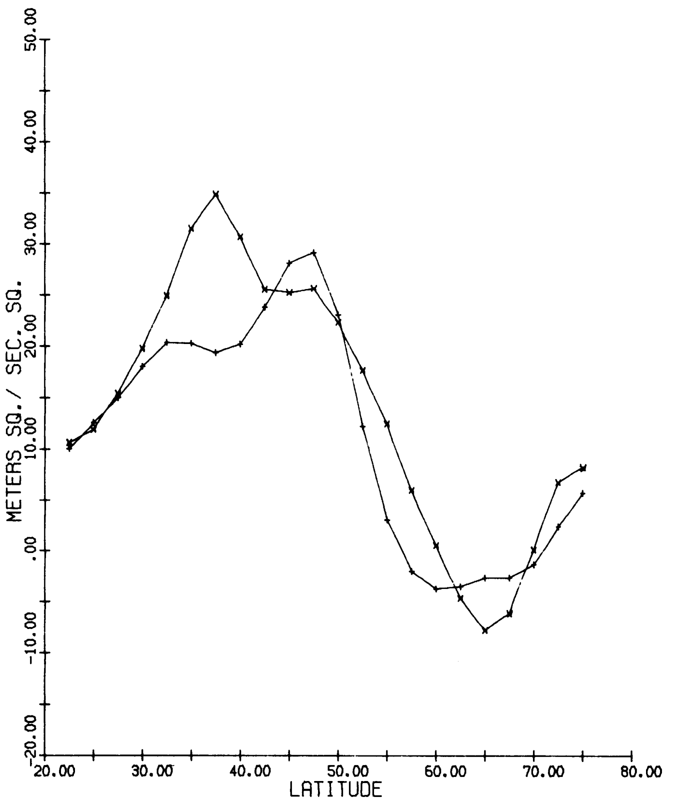
Four experiments have been conducted to investigate: the correspondence of the analysis system used here with the operational NMC model; system-related bias in a complete analysis; system-related bias in a partial analysis; and the effect of inhomogeneity in the reporting network.

i) Operational Analysis

The operational procedure at NMC is to perform 500-mb objective analysis every twelve hours (00 and 1200 GMT) using the forecast from the previous analysis as the first approximation. This procedure was simulated for the sixteen analyses of the eight day period used in this study. The normal twelve-hour forecast fields were taken from the history tapes and "station reports" of height and height gradient were abstracted from the corresponding final NMC analyses also available on tape. The station network which was used is depicted in Figure 3. It will be recognized that the total number of stations (258) is far smaller than the normal number of reports. However, it was indicated in earlier work that the quality of the initial approximation (the twelve-hour forecast) is such that analysis of the major features of the flow depicted on the 1977-point grid does not require the high data

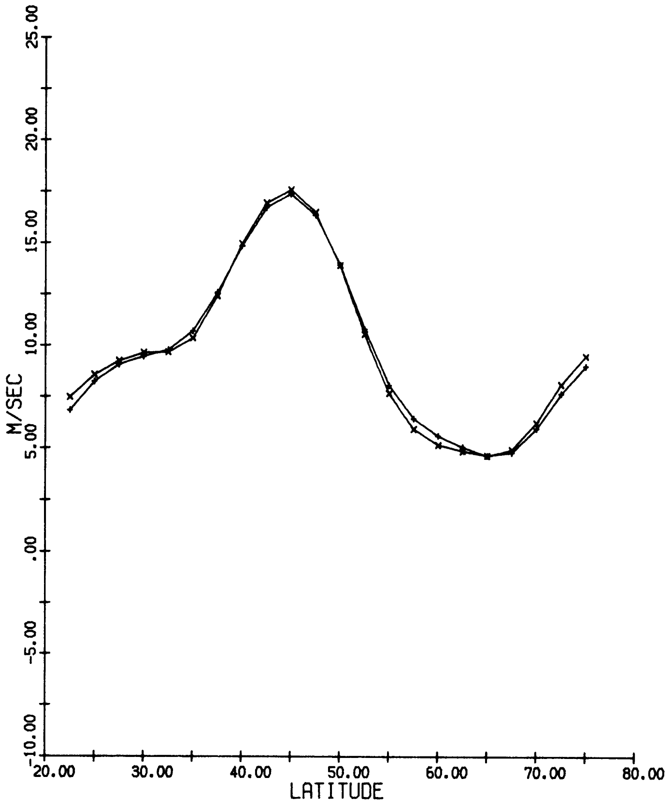


MEAN ZONAL WIND
+-ORIGINAL X-ANALYSIS

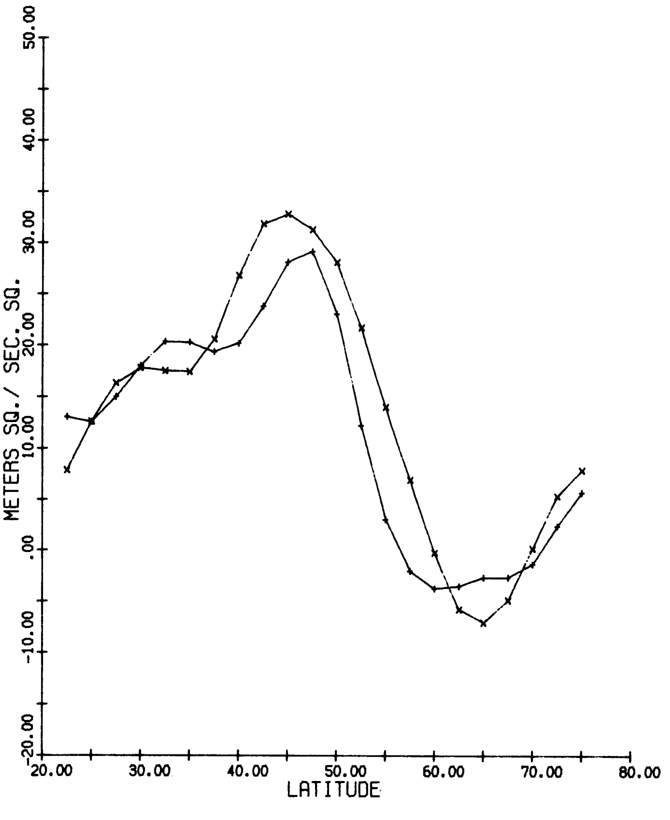


MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL X-ANALYSIS

Figure 4



MEAN ZONAL WIND
+-ORIGINAL   X-ANALYSIS



MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL   X-ANALYSIS

Figure 5

density prevalent over many land areas. In such areas the density has been reduced in the interests of economy. All stations in moderate density regions are retained in their approximate geographic locations.

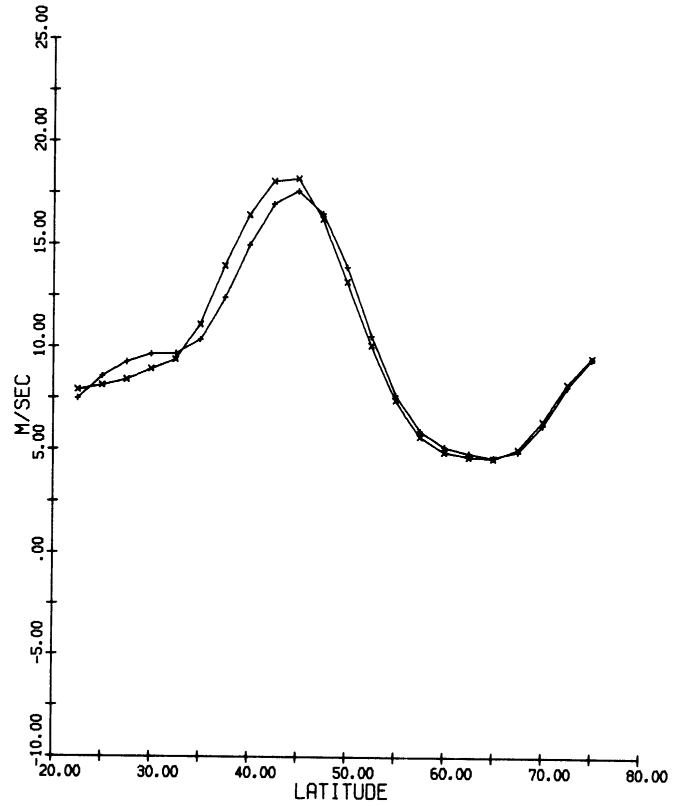
The averaged meridional profiles of the mean zonal wind and the total momentum transport (wave numbers 1-15) obtained from the sixteen analyses are presented in Figure 4. It can be noted that in the moderate and sparse data areas (middle and low latitudes) the analyses have increased the absolute magnitudes of the wind extrema. The change at high latitudes is very slight since the data density in this area restricts the effect of the large corrections which occur in the early scans. It is also apparent that the analyses have shifted the wind maximum to the south. This is in accordance with the implicit directional bias discussed in Section 2, but is actually caused by inhomgeneity in the reporting network as will be shown in (iv) below. The figure also shows that the analyses have increased the magnitude of the momentum transports and again displaced the positive (poleward) maximum to the south.

For purposes of comparison with the one-level model used here, the averaged mean zonal wind and momentum transport for the initial approximations and the final NMC analyses are depicted in Figure 5. As would be expected, the two analysis systems give very similar results in the dense-data high latitudes but are quite dissimilar in the middle and low latitudes. In particular there is no southerly shift in the zonal wind maximum, nor is the magnitude increased in the NMC product. It seems probable that the changes produced by the system used in this study are caused by the sparse-data instability which is related to the extrapolation error of the early scans. This deficiency has apparently been largely eliminated by modifications leading to the present NMC multi-level model. The discrepancies between the momentum transports resulting from the different analysis models are too complicated to afford simple explanation. In both cases the analysis has increased the poleward momentum flux in middle latitudes. The role of the initial approximation in producing this result is examined by the following experiment.

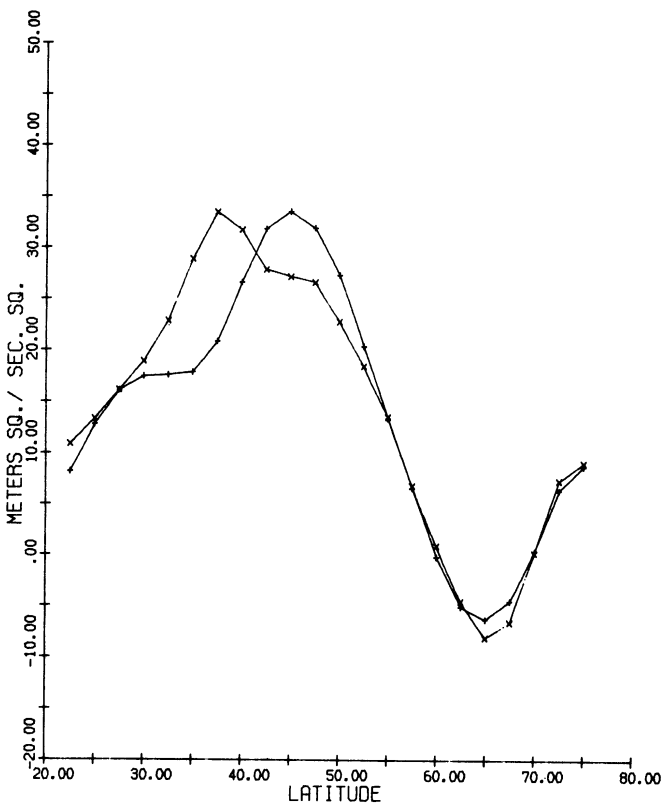
ii) Full analysis on Perfect Field

A means of investigating the inconsistencies of the analysis model without the complication of an inaccurate initial approximation is to perform the analysis on the field from which the "station reports" are abstracted. Ideally the initial field should remain unchanged although in practice it will be altered by the extrapolation to each grid-point of neighboring reports (5). The averaged results for sixteen such analyses possible with the available data are given in Figure 6. The increase in magnitudes and midlatitude phase shifts occur again in both the zonal wind and momentum transport. Figure 7 gives the momentum transport of Figure 6 subdivided into three groups of wave numbers. It is apparent that alterations are fairly evenly distributed (as a function of relative magnitude) over the entire spectrum. This is in full agreement with results obtained with the idealized flow in Section 2.

Figure 8 and 9 are presented to demonstrate the influence

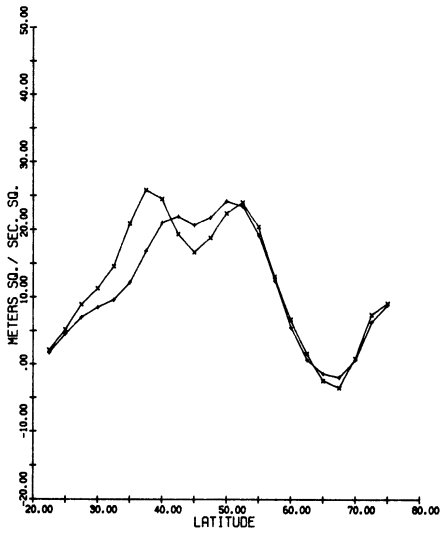


MEAN ZONAL WIND
+-ORIGINAL X-ANALYSIS

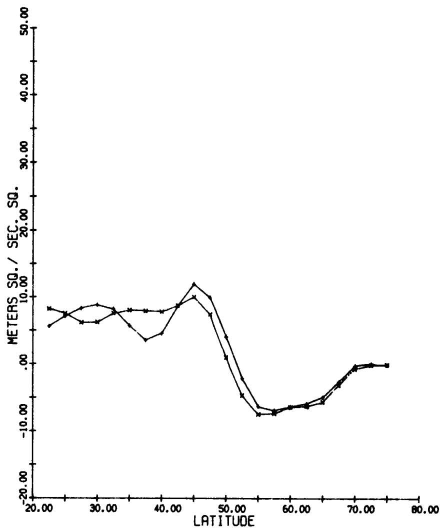


MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL X-ANALYSIS

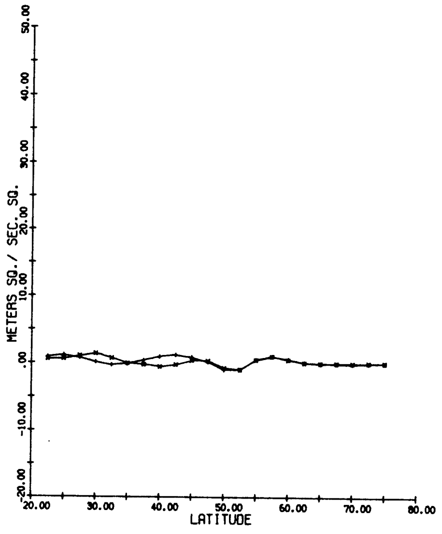
Figure 6



MOMENTUM TRANSPORT, WAVE NO. 1-5
+-ORIGINAL X-ANALYSIS

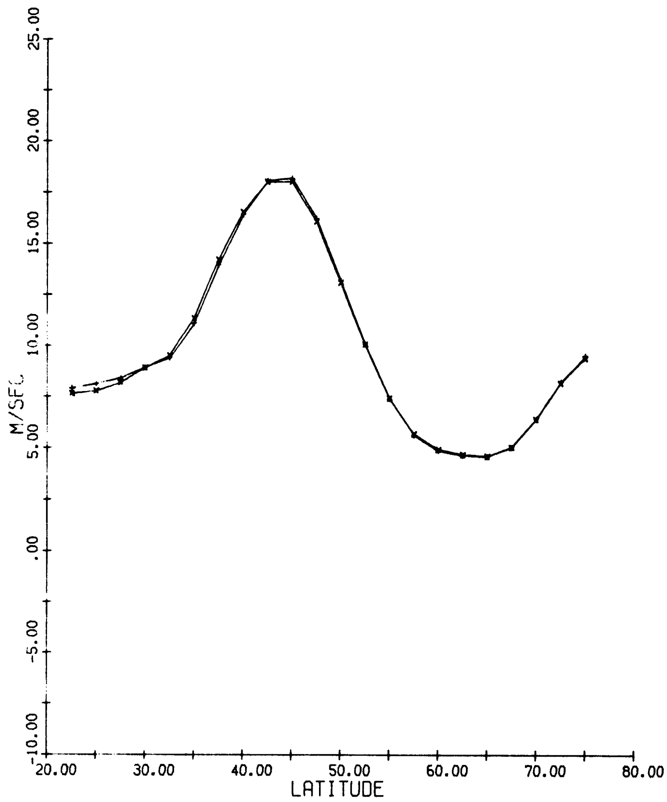


MOMENTUM TRANSPORT, WAVE NO. 6-10
+-ORIGINAL X-ANALYSIS

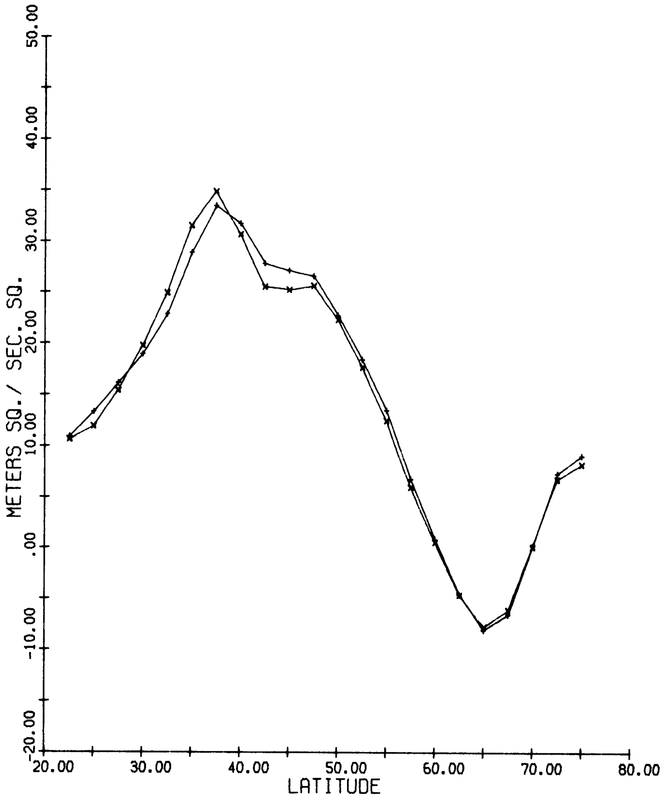


MOMENTUM TRANSPORT, WAVE NO. 11-15
+-ORIGINAL X-ANALYSIS

Figure 7

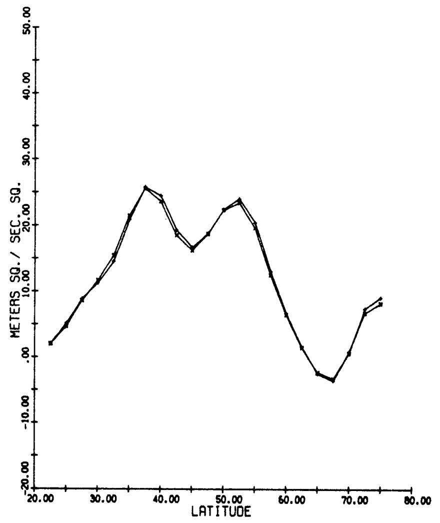


MEAN ZONAL WIND
+-ORIGINAL X-ANALYSIS

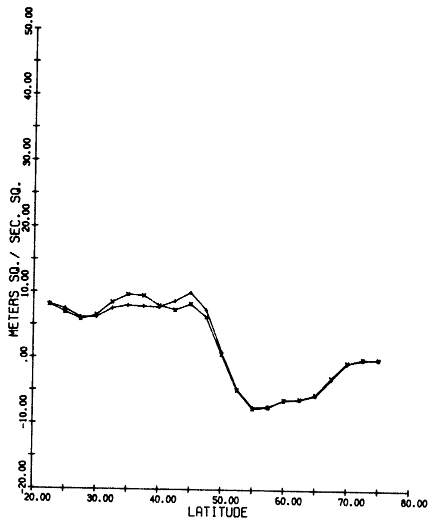


MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL X-ANALYSIS

Figure 8



MOMENTUM TRANSPORT, WAVE NO. 1-5
+-ORIGINAL X-ANALYSIS



MOMENTUM TRANSPORT, WAVE NO. 6-10
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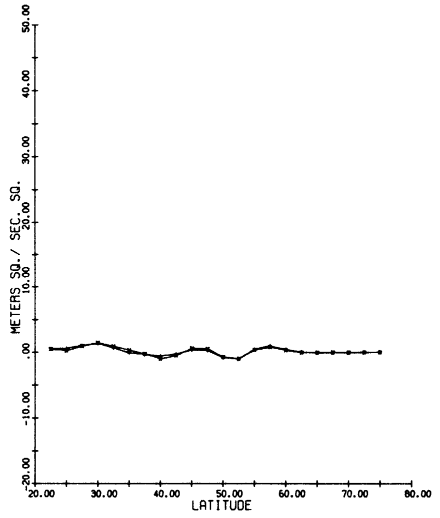
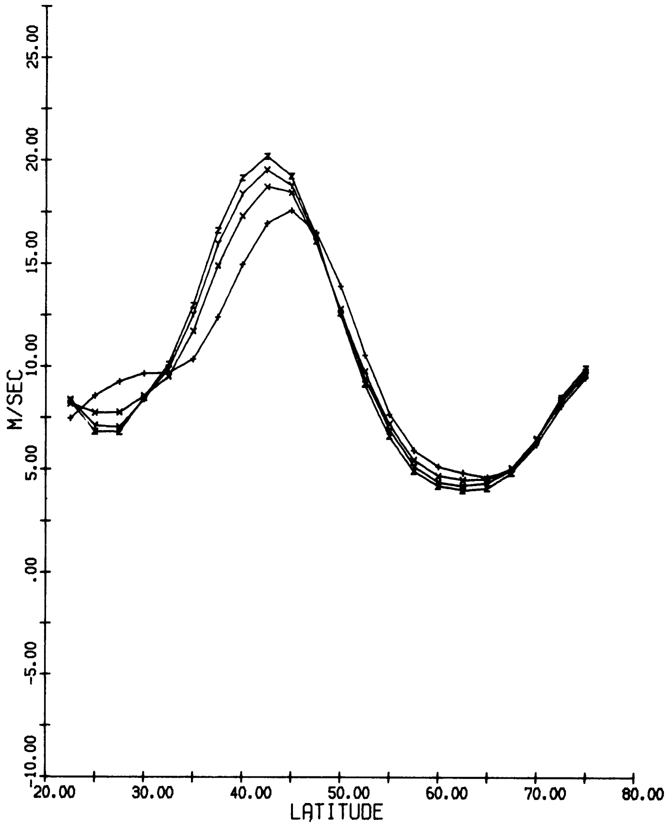
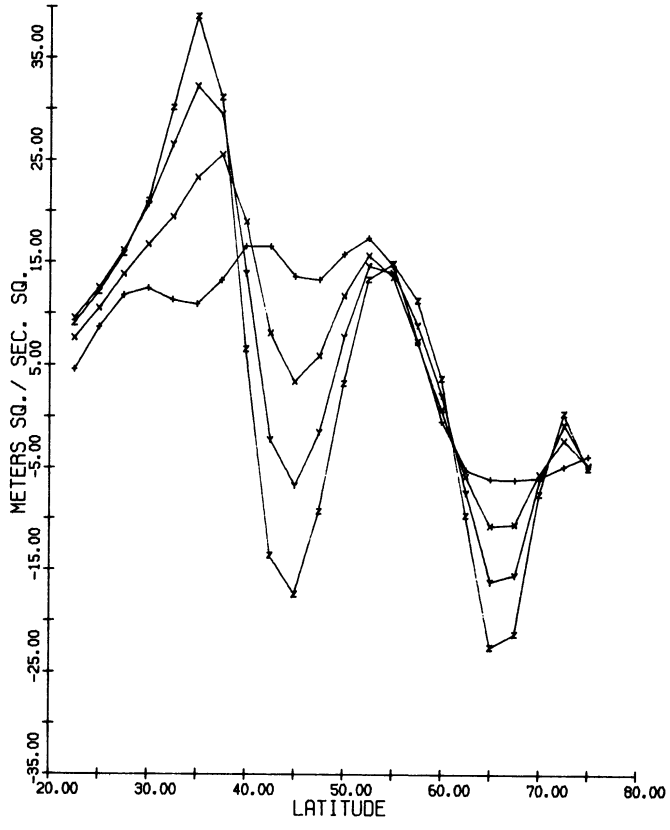


MOMENTUM TRANSPORT, WAVE NO. 11-15
+-ORIGINAL X-ANALYSIS

Figure 9

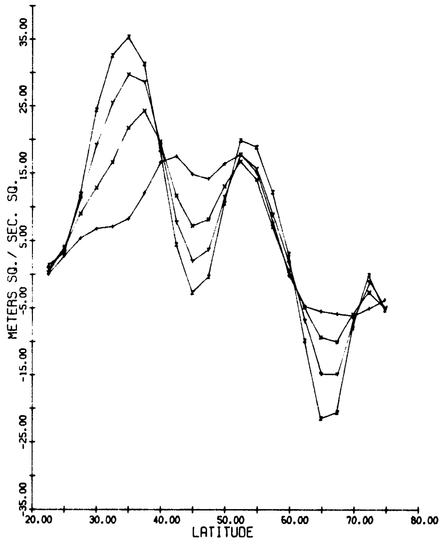


MEAN ZONAL WIND
+-ORIGINAL X-2 Y-4 Z-6

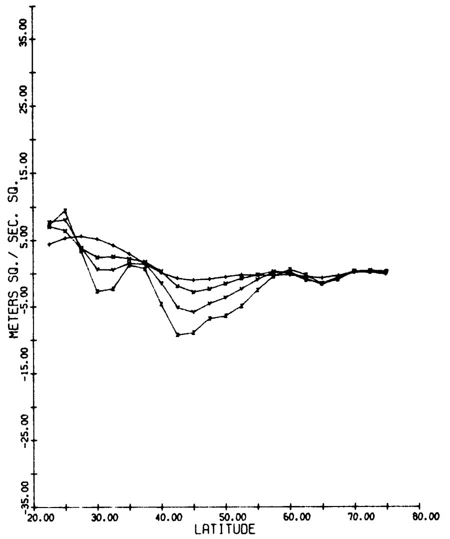


MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL X-2 Y-4 Z-6

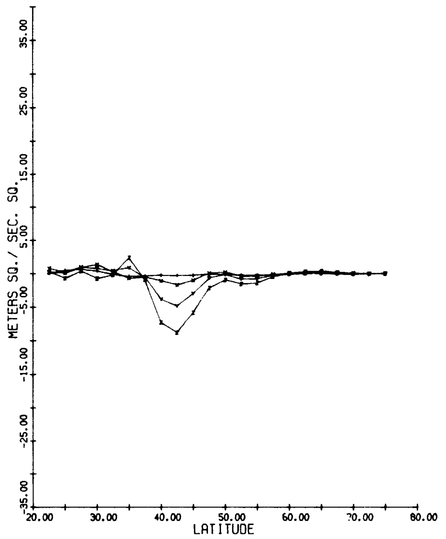
Figure 10



MOMENTUM TRANSPORT, WAVE NO. 1-5
+-ORIGINAL X-2 Y-4 Z-6

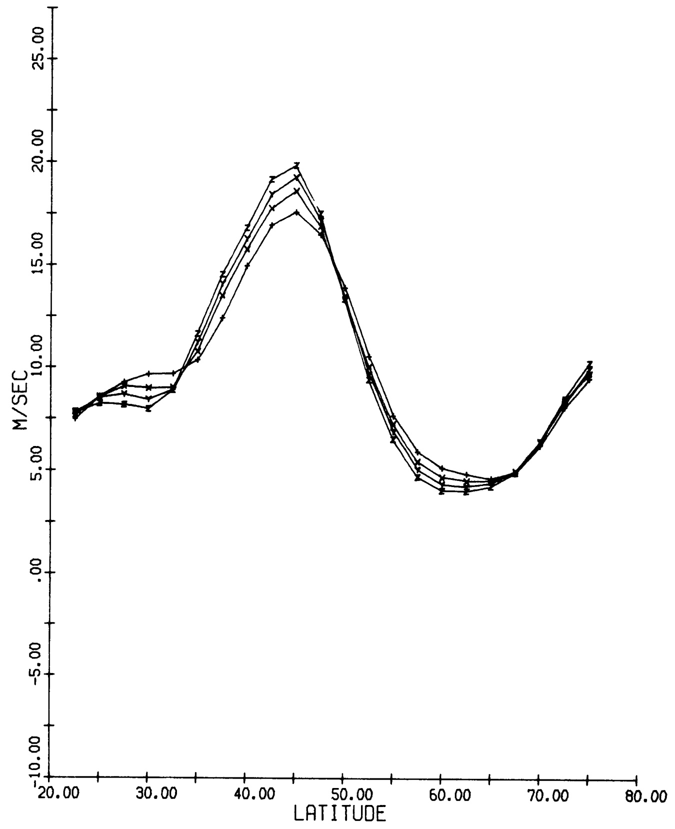


MOMENTUM TRANSPORT, WAVE NO. 6-10
+-ORIGINAL X-2 Y-4 Z-6

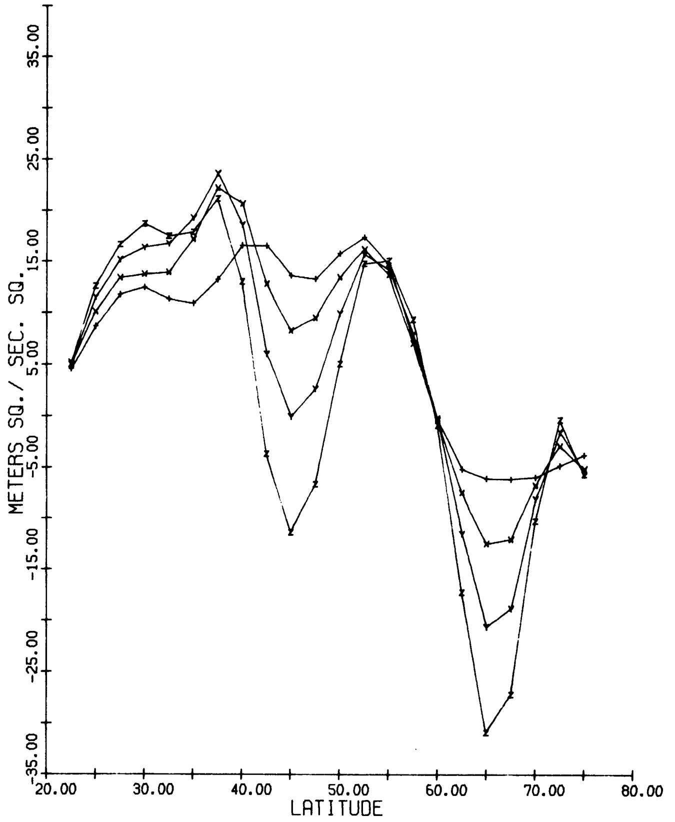


MOMENTUM TRANSPORT, WAVE NO. 11-15
+-ORIGINAL X-2 Y-4 Z-6

Figure 11



MEAN ZONAL WIND
+-ORIGINAL X-2 Y-4 Z-6



MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL X-2 Y-4 Z-6

Figure 12

of the initial approximation to the field on the final product. The figures compare the analyses performed here from initial fields of the NMC forecast and the NMC final analysis. Variations in the final product are practically negligible. It is interesting to note that almost the entire difference in the implied momentum transport is contained in the middle wave numbers. This is reported simply as a curiosity as there is no obvious explanation. It is the only instance encountered in this work where changes were found to be dependent on scale.

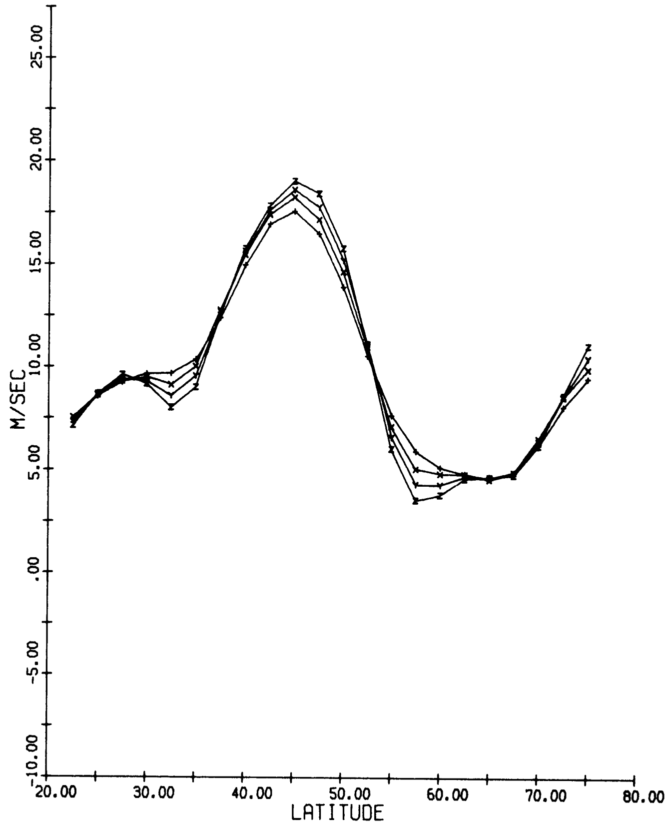
Partly to accentuate the inconsistencies in the analysis model and partly for reasons to be explained in Section 4 it was decided to perform repeated analyses on a single situation. The sixteen NMC analyses were averaged to form an initial approximation from which the initial "station reports" were abstracted. Six consecutive analyses were performed with new "reports" taken from the results of the preceding analysis. Results for the even analyses are given in Figures 10 and 11. Changes in the mean zonal wind are similar to those of Figure 6 which represents the average of the individual analyses. The changes are amplified with each succeeding trial although some damping is evident. Results for the momentum transport cannot be directly compared to the average of the individual analyses since the quantity in non-linear, and the initial approximation for the repeated-analysis experiment is not as smooth. In a sense this unevenness is fortunate in that it shows the catastrophic instability which can result. Unlike the modification in the zonal wind, changes in momentum flux are not damped with increasing trials. Again it is apparent that the effect is distributed throughout the wave number spectrum.

iii) Partial Analysis on Perfect Field

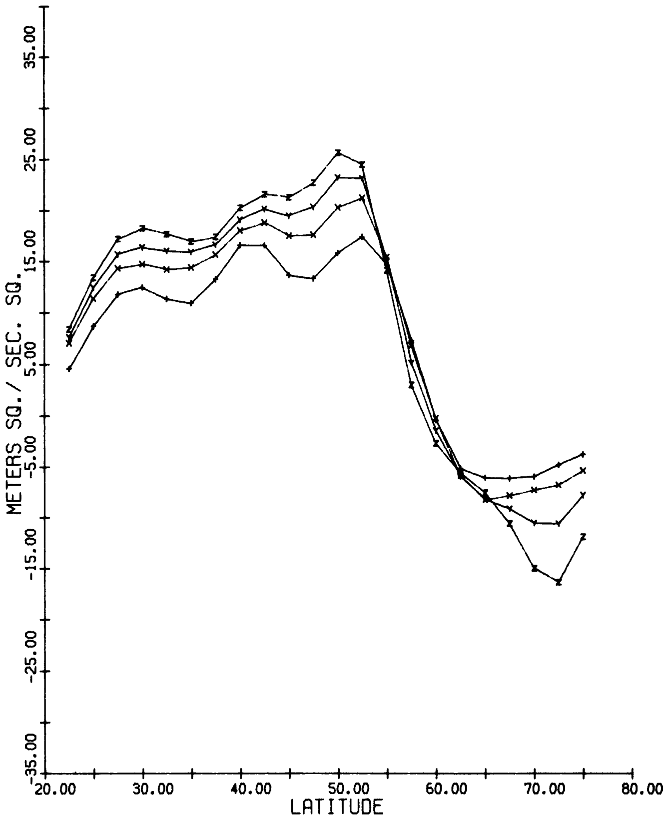
It has been repeatedly emphasized that the extrapolation correction is significant only in the early scans. In particular, for the first scan many individual gridpoint corrections are too large to be meteorologically reasonable. To investigate the influence of this scan on wind and momentum transport calcualtions it was decided to repeat the second part of (ii) above, but to begin each trail with the second scan. The results are given in Figure 12. (The division of momentum flux into wave number groups is not given as it is qualitatively similar to Figure 11.) It can be seen that the errors in the mean zonal wind have been significantly reduced in the low and middle latitudes. The effect on the momentum transport is more complex. In low latitudes the instability of the system has been greatly reduced; in the middle latitudes it has been slightly reduced; but in high latitudes it has been increased. This meridional dependence is obviously related to the northward convergence of the geographic grid with respect to the octagonal grid. But while it might be anticipated that abolition of the first scan would be most helpful in low latitudes it is not clear why it should be harmful in high latitudes. It is possible that this is merely coincidental. In any event it is probable that both first and second scans should be excluded in analysis at high latitudes.

iv) Analysis with Uniform Reporting Network

The influence of inhomogeneity in the reporting network was investigated by replacing the semi-real station distribution of Figure 3 by a perfectly uniform (with respect to the grid) array of 239 stations. For every row of the octagon stations were located at every fourth gridpoint with the stations in alternating rows staggered by two gridlengths. Repeated, complete analyses with this network were executed as in (ii) above. The results are given in Figures 13 and 14. It is apparent that the even distribution has reduced the errors in the mean zonal wind and removed the southward phase shift of the maximum. The shifts of Figures 3 and 6 can thus be attributed to inhomogeneity in the reporting network and not to the directional bias of the analysis system described in Section 2. Results for the momentum transport show that the instability is entirely caused by the uneven distribution of reporting stations which exists in the real case and which must systematically alter the slopes of analyzed troughs and ridges. Relative magnitudes with a uniform station network are uniformly increased suggesting that the analysis system contains a general bias towards increasing the magnitudes of the analyzed wind components. This result is again a manifestation of the error caused by the extrapolation correction. Because all of the experiments performed in this study show that the relative magnitude of the mean zonal wind is increased at the extrema it must be concluded (from the discussion in Section 2) that the error in the extrapolation correction outweighs the compensating error in the weighted correction in the real as well as the idealized case.

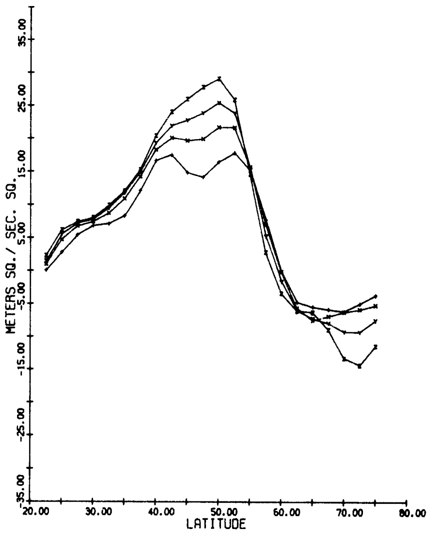


MEAN ZONAL WIND
+-ORIGINAL X-2 Y-4 Z-6

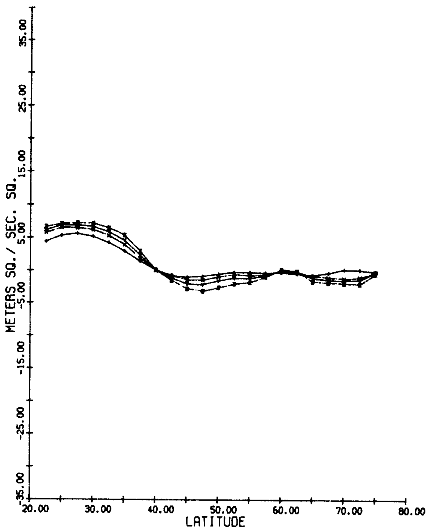


MOMENTUM TRANSPORT, WAVE NO. 1-15
+-ORIGINAL X-2 Y-4 Z-6

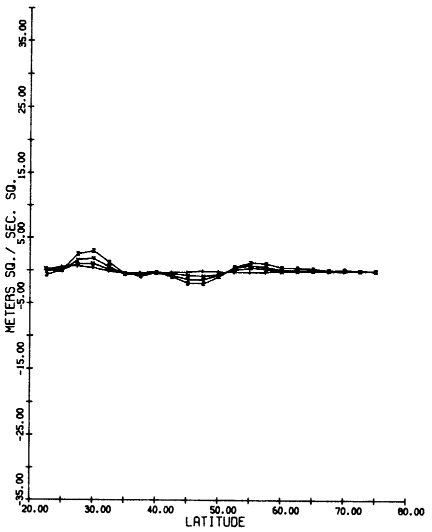
Figure 13



MOMENTUM TRANSPORT, WAVE NO. 1-5
+-ORIGINAL X-2 Y-4 Z-6



MOMENTUM TRANSPORT, WAVE NO. 6-10
+-ORIGINAL X-2 Y-4 Z-6



MOMENTUM TRANSPORT, WAVE NO. 11-15
+-ORIGINAL X-2 Y-4 Z-6

Figure 14

4. COMPARISON WITH OTHER STUDIES

The results of this study can be used to clarify the results of empirical studies of the mean zonal wind or momentum transport which are founded on objectively analyzed data. Bristor (1959) investigated the behavior of the barotropic forecast model and concluded that it generally gives midlatitude values of the zonal wind which are too high. He attributed this systematic error to the fact that the barotropic model is capable of only horizontal transport of angular momentum with consequent convergence in middle latitudes. This feature was studied theoretically by Wiin-Nielsen (1959) and his results substantiate Bristor's argument. The present study further validates Bristor's conclusions. The effect of the analysis system (which is used to produce the verification field for the forecast) is to increase the zonal wind values in middle latitudes and thus to mask the errors in the forecast. The errors indicated by Bristor's experiment are probably less than the real error. A related point mentioned by Bristor is that the forecast moves the jet too far to the north. From the results of the present work it appears that this argument may not be valid. It is clearly demonstrated in the figures of the preceding section that the existing distribution of stations forces the analysis system to shift the wind maximum to the south. Since the analysis model used here is equivalent to that used to produce verification charts in Bristor's study it is quite possible that the verification jet is too far to the south rather than the forecast jet too far to the north. It should also be noted that Bristor's most striking results are based on seventy-two hour forecasts where there have been six intervening objective analyses in producing the verification and therefore six southward shifts. This would cause the general effect indicated by the repeated analyses of the previous section (e.g. Figure 10).

It is typical of older objective analyses over the Pacific that they produce a strong, spurious jet not found in conventional analyses. This feature has also been attributed to momentum transport deficiencies in the barotropic forecast model. Theoretically, because of the lack of observations in the Pacific, the forecast which provides the first approximation is not modified in the final analysis (Holopainen; 1968). It has been found, however, that the first guess has rather little influence over the final analysis (even in data regions as sparse as the Pacific) with respect to features such as the jet (viz. Figure 8). It seems more probable that the spurious jet is caused by errors in the extrapolation correction of the analysis scheme, particularly as the jet is found at a distance approximately between the first and second scan radii from the majority of influencing stations (35°-40°N).

Holopainen (1967) presents a comparison of the annual mean poleward flux of angular momentum computed by various investigators. Of these, only Wiin-Nielsen et al. (1963, 1964) based their study on objective analyses of the geopotential. It is shown that their results differ markedly from other studies in that they give smaller northward flux in low latitudes and larger magnitudes of flux in the middle and high latitudes. It is suggested by Holopainen that the main source of the systematic error should be sought in the scheme of objective analysis or in the particular stream function employed by Wiin-Nielsen's group. A qualitative discussion of the former is possible here.

The figures of the preceding section demonstrate that the analysis model tends to amplify the peaks in the meridional profile of momentum transport. This helps to explain the high values obtained in middle and high latitudes by Wiin-Nielsen's study. The situation in low latitudes (below 25° N) is not elucidated by results of the present work which extends south only to 22.5° N.

The division of momentum transport into individual wave numbers has not revealed any further bias in the analysis system. The figures of Section 3 show that the relative effect on three groups of wave numbers is approximately the same. In addition, correlation coefficients have been computed for individual daily transports for all wave numbers before and after objective analysis (i.e. between first guess and final analysis) and these do not show preference for a particular scale of motion. Because of this, the conclusions reached by Wiin-Nielsen's group regarding the relative contributions of individual wave numbers in momentum transport are not invalidated by inconsistencies in the objective analysis model. The absolute magnitudes, however, are suspect.

5. SUMMARY

The objective analysis system used at NMC prior to 1965 has been studied with respect to its influence in specifying the geostrophic wind field. The consequence of using such objective analyses in empirical studies of the mean zonal wind and momentum transport has been discussed. It has been shown analytically that the formulation of the analysis model contains important inconsistencies in its treatment of the wind field. It is shown empirically that these inconsistencies lead to an increase in absolute magnitudes of the wind components, and although this bias is partially alleviated by zonal averaging, the effect is carried over into implied mean zonal wind speeds and momentum transports such that both are generally amplified in the analysis product. Inhomogeneity in the observation network has been briefly investigated and shown to have a strong influence on the momentum transport implied by objective analyses.

The meridional momentum transport has been subdivided into the contribution of individual wave numbers, and it has been found empirically that the analysis system does not show a preference for scale. It seems reasonable to conclude that analysis-related bias has no significant effect on studies which use objective analyses to investigate the relative importance of various scales of atmospheric motion.

The bias of the analysis system has been shown qualitatively to explain the apparent northward shift of the jet stream by the barotropic forecast model (Bristor), the spurious jet observed in older objective analyses of the Pacific, and the apparent overestimate of momentum transports computed from objective analyses (Wiin-Nielsen et al.).

Changes in the NMC analysis system which include vertical consistency checks and reduced scanning areas have undoubtedly reduced the problems discovered in this study. However, the basic algorithm (viz. (4)) remains the same. Since this has been shown to relate the observed and implied geostrophic wind in a rather unrealistic way, particularly in sparse data networks, it would seem of importance to seek better techniques for using the wind data in objective analysis of the height field and to exercise caution in founding empirical studies on winds derived from objectively analyzed height fields.

APPENDIX

The flow diagram for a general program which calculates the geostrophic mean zonal wind and momentum transport in the wave number regime is given in Figure 15. These quantities are computed for a 500-mb height field represented at gridpoints on the NMC 1977-point grid before and after an initial approximation has been processed by a one-level objective analysis routine (Section 2). The individual steps are described below.

1. On original entry the program takes exit A. On reentry exit B permits a second analysis to be performed with the same station observations but a new initial approximation. Exit C will be taken on reentry if the field from which the observations are abstracted is unchanged but the station distribution is new.

2. A reference field of gridpoint values is introduced. This field is assumed to represent the exact height of a 500-mb surface.

3. The distribution of station reports to be used in the objective analysis procedure is introduced. Each "report" is identified by its gridpoint location.

4. Station reports are created from the reference field. For each station introduced in step 3 the value of the height field and the gradient of the height field (corresponding to a wind report) are obtained from the reference field.

5. This step gives the option of using the reference field or any other field as an initial approximation for the analysis procedure.

6. Entry to subroutine (S) which computes the mean zonal wind and momentum transport in the wave number regime is

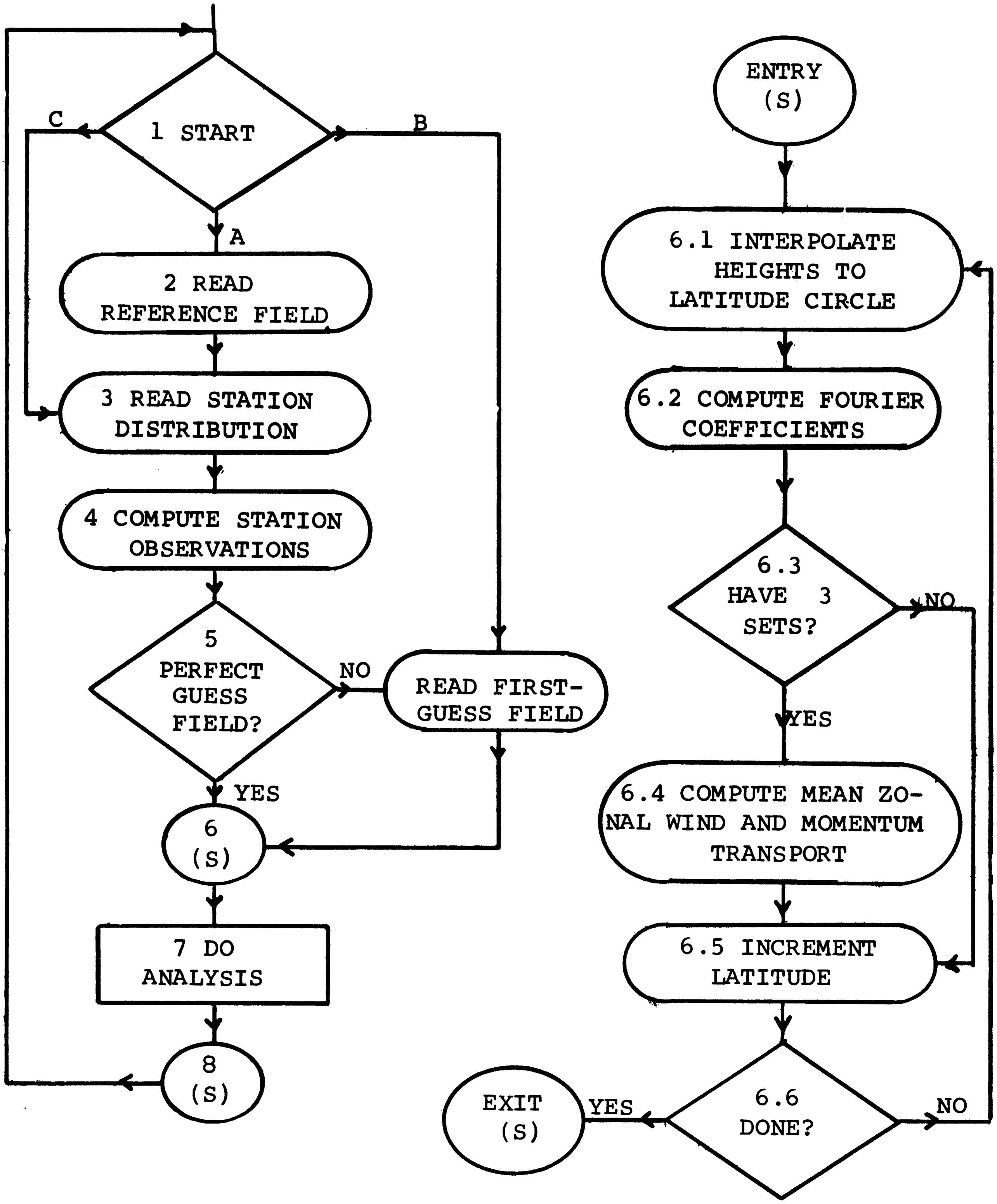

1 START
A
B
C
2 READ REFERENCE FIELD
3 READ STATION DISTRIBUTION
4 COMPUTE STATION OBSERVATIONS
5 PERFECT GUESS FIELD?
NO
READ FIRST-GUESS FIELD
YES
6 (S)
7 DO ANALYSIS
8 (S)
ENTRY (S)
6.1 INTERPOLATE HEIGHTS TO LATITUDE CIRCLE
6.2 COMPUTE FOURIER COEFFICIENTS
6.3 HAVE 3 SETS?
NO
YES
6.4 COMPUTE MEAN ZONAL WIND AND MOMENTUM TRANSPORT
6.5 INCREMENT LATITUDE
6.6 DONE?
YES
NO
EXIT (S)

Figure 15

optional at this point.

6.1 The gridpoint values of the height field are quadratically interpolated to a latitude circle for increments of 2.5 degrees of longitude at or south of 40°N and for increments of 5.0 degrees of longitude north of 40°N. The initial latitude circle is 20°N.

6.2 Sine and cosine Fourier coefficients are computed for the latitude circle.

6.3 This step tests the presence of three sets of coefficients which permits computation of the zonal wind and momentum transport.

6.4 Values are computed and printed for the middle latitude of the three latitude set.

6.5 The latitude counter is incremented by 2.5 degrees. The old coefficients are moved in storage to make room for a new northernmost set.

6.6 When the routine has completed computations for every 2.5 degrees of latitude between 20°N and 80°N a return is made to the main program.

7. The objective analysis scheme is executed.

8. Subroutine (S) is repeated on the product of the objective analysis.

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