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THE INTENSITY OF AIR-SHOWER CORES RELATIVE TO AIR SHOWERS

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The following discussion of the probability that an air-shower event in a detector of finite area be due to the passage of the shower axis through the detector supersedes the discussion in IV (a) and V (a) of the report "Air-Shower Cores" of August, 1951.

If we let a be the detector area and R the radius of the circle within which a shower axis must strike in order to display n electrons within the detector area, we have

$$P = \int_{N_m}^{\infty} af(N)dN \Big/ \int_{N_m}^{\infty} \pi R^2 f(N)dN$$

for the probability that an event with n or more electrons within a is due to a shower axis within a. The quantity f(N)dN is the experimentally determined frequency of occurrence of shower events with a total number of electrons N at the point of observation.[1,2,3] It has been found that

$$f(N)dN = KN^{-\gamma}dN,$$

where $\gamma = 2.5$ for $N < 10^6$ and 2.9 for $N > 10^6$. In setting up the denominator in the expression for P, it was assumed that the detector area is small (or circular). Actually, the region within which a given-size shower must strike is not circular for a rectangular detector; but a more exact

1 R. W. Williams, Phys. Rev. 74, 1689 (1948).

2 J. M. Blatt, Phys. Rev. 75, 1584 (1949).

3 J. Ise and W. B. Fretter, Phys. Rev. 76, 933 (1949).

treatment shows that the area of the region is not very different from the area of the circle.

Strictly speaking, N and R are related through an integral of the lateral-distribution function over the area of the detector. However a sufficiently good approximation is obtained by assuming that the average particle density within the detector is the same as the particle density at the center of the chamber; whereupon

$$\frac{n}{a} = \frac{0.454\ N}{R}(1+4R)\ \exp-4R^{2/3}\ ,$$

using the analytical expression of Bethe. [1]

For the case of a shower axis within the detector area, which must be considered in establishing the value for N_m, the above approximations are less valid. A graphical solution gives an average value of n/N = 1/200 for the case of axes passing through a rectangular area 25 by 80 cm.

In the experiment, n was chosen as 100. The calculated value of P for these particular conditions is found to be 0.1, which agrees with the cloud-chamber observation. The agreement may be taken as additional evidence favoring the assumed shower-size distribution and the assumed lateral distribution. The assumed lateral distribution is probably too steep near the origin for the smaller showers;[2] a correction for this would lower the calculated value for P. On the other hand, the shower cores are probably multiple, which would result in a higher calculated value for P.