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COLLEGE OF LITERATURE, SICNECE, AND THE ARTS
Communication Sciences Program

Technical Report

VARIANTS OF THATCHER'S ALGORITHM FOR CONSTRUCTING PULSERS

S. Hedetniemi

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RESEARCH PROGRESS REPORT

Title: "Variants of Thatcher's Algorithm for Constructing Pulsers," S. Hedetniemi, University of Michigan Technical Report 03105-29-T, 14 August 1964; Nonr-1234(21), NR 049-114.

Background: The Logic of Computers Group of the Communication Sciences Program of the University of Michigan is investigating the application of logic and mathematics to the design of computing automata. The detailed working out of the von Neumann growing automata design forms a part of this investigation.

Condensed Report Contents: The background material for this paper is John von Neumann's "Theory of Automata: Construction, Reproduction, Homogeneity" [Part II of The Theory of Self-Reproducing Automata, edited by Arthur W. Burks; to be published by the University of Illinois Press]. Within the framework of his cellular automata, von Neumann presents an algorithm for constructing a class of organs, called pulsers, which function as follows. When given an input pulse at time t, a pulser will, after a specified time delay, Δt, produce at time $t + \Delta t$, one arbitrarily specified output sequence. A pulser behaves in this manner like a one-input-one-output encoder.

James Thatcher, in his paper "Universality in the von Neumann Cellular Model" [to be published as an IBM Research Report], presents another algorithm for constructing pulsers which is far simpler than von Neumann's to apply, but is less efficient in the sense that the pulsers produced are larger in size and have longer time delays than those produced by von Neumann's algorithm.

This paper contains several algorithms which, while retaining much of the simplicity of Thatcher's algorithms, produce pulsers that are smaller in size and shorter in delay than those produced by either von Neumann's or Thatcher's algorithms.

For Further Information: The complete report is available in the major Navy technical libraries and can be obtained from the Defense Documentation Center. A few copies are available for distribution by the author.

1. THE PROBLEM

Design a simple and efficient algorithm for constructing pulsers for arbitrary (output) sequences of the form

$$\alpha = \alpha_1 \alpha_2 \ldots \alpha_p \qquad (\alpha_i = 0, 1) \quad , \qquad (1)$$

and in particular for those sequences of the form

$$\delta = 1k_1\ 1k_2\ 1 \ldots 1k_n\ 1 \qquad (k_i = 0, 1, 2, \ldots) \qquad (2)$$

where the k_i denote strings of k_i consecutive zeros. The algorithm will be simple if it is easy to understand and easy to apply. It will be more efficient than another algorithm if the pulsers it constructs are smaller in area and have shorter delays, between input stimuli and output sequences, than the pulsers constructed by the other.

2. THATCHER'S ALGORITHM FOR CONSTRUCTING PULSERS

An algorithm for constructing pulsers far simpler than the one given by von Neumann [1] is proposed by James Thatcher [2]; the details of Thatcher's algorithm are presented in this section.

For a sequence α having the form (1) construct, or iterate, p - 1 copies of the block of Fig. 1, followed by one copy of the column of Fig. 2.

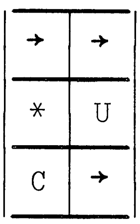


Fig. 1

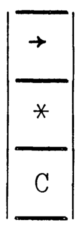


Fig. 2

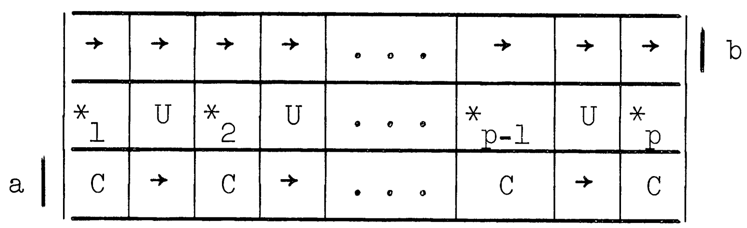


Fig. 3

The pulser so produced will have the form of Fig. 3, with input a and output b, and will contain p confluent cells (C) above each of which is an, as yet, unspecified cell marked with an asterisk and a number ($*_i$). The construction of the pulser for α is then completed by changing the i^{th} cell marked with an asterisk to either an unexcited cell (U) or an ordinary transmission-up cell (↑), depending on whether the i^{th} symbol of α, α_i, is a 0 or a 1, respectively.

3. VARIANT I OF THATCHER'S CONSTRUCTION

With Thatcher's construction the pulser for an arbitrary sequence of length p has at least p - 1 unexcited, and therefore 'unused', squares (U). It is, therefore, natural to ask whether an iterative scheme could be developed, along similar lines, which would make more efficient use of the cells within (the bounds of) the pulser, thereby producing a pulser with a smaller area and, perhaps, with a shorter delay. One such scheme is presented in this section.

Initial Step

For sequences having the form

$$\beta = 1k_i 1 \qquad (k_i = 0, 1, 2, \ldots)$$

where k_i denotes a string of k_i consecutive zeros, construct for the following values of k_i the corresponding pulsers with input a and output b:

$k_i = 0$,

	→	→	→	b
	↑	U	↑	
a	C	→	C	

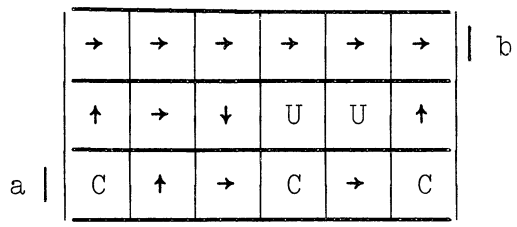
;

$k_i = 1$,

	→	→	→	→	→	b
	↑	U	U	U	↑	
a	C	→	C	→	C	

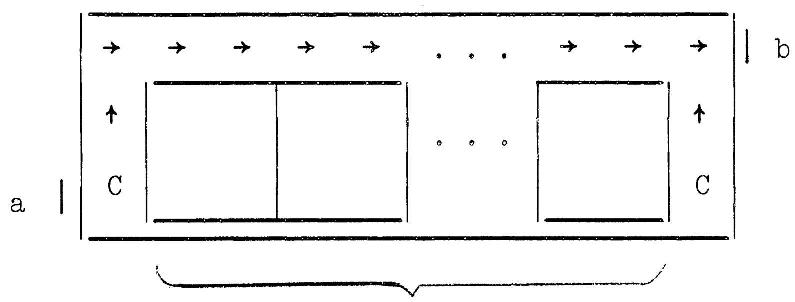
;

$k_i = 3$,



;

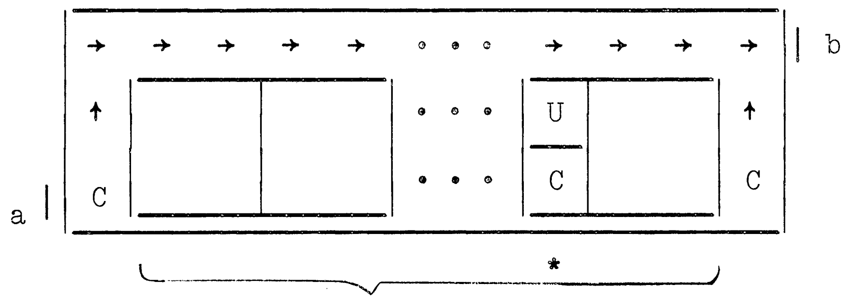
$k_i = 2m$,
$m \geqslant 1$



m blocks, each having

the form  ;

$k_i = 2m + 1$,
$m \geqslant 2$



m blocks, having the same form as above, the last two of which are separated by the column marked with the asterisk.

General Step

For sequences having the form

$$\gamma = 1k_1\ 1k_2\ 1$$

all one has to do is merge, or identify, the last column of the pulser $P(1k_1\ 1)$ with the first column of the pulser $P(1k_2\ 1)$. Notice that since all the pulsers constructed in the initial step have identical first and last columns, the last column of $P(1k_1\ 1)$ is the same as the first column of $P(1k_2\ 1)$, and thus the merging of these two pulsers can be accomplished quite naturally.

For sequences having the form (2) above,

$$\delta = 1k_1\ 1k_2\ 1 \ldots 1k_n\ 1 \quad ,$$

the process of merging generalizes very simply, i.e. $P(1k_1\ 1k_2\ 1 \ldots 1k_n\ 1)$ is the result of merging serially, or in sequence, $P(1k_1\ 1)$, $P(1k_2\ 1)$, ..., $P(1k_{n-1}\ 1)$ and $P(1k_n\ 1)$.

For sequences having the form

$$\epsilon = k_0\,\delta \quad ,$$

i.e. those having initial zeros, all one has to do is construct a path of delay k_0 before (to the left of) the first confluent square of $P(\delta)$. This can be done easily and in any of a number of ways.

4. END PULSERS — TYPE I

This section contains an algorithm for constructing a class of pulsers, called end pulsers. The algorithm, however, is not iterative. That is, it is not possible to merge the end pulser for a sequence $1k_1\ 1$ with the end pulser for a sequence $1k_2\ 1$ to obtain an end pulser for the merged sequence $1k_1\ 1k_2\ 1$. The construction of end pulsers only applies to sequences of the form $1k_i\ 1$. However, this construction can be applied to the end subsequences, $1k_n\ 1$, of sequences of the form (2). That is, if the end pulser for the sequence $1k_n\ 1$ is merged with the Variant I pulser for the sequence $1k_1\ 1k_2\ 1 \ldots 1k_{n-1}\ 1$, the resulting pulser for the merged sequence $1k_1\ 1k_2\ 1 \ldots 1k_{n-1}\ 1k_n\ 1$ will have a smaller area and a shorter delay than the corresponding Variant I pulser for the merged sequence.

The algorithm for constructing Type I end pulsers follows.

For sequences of the form $1k_n\,1$ construct for each value of k_n the corresponding pulser:

$k_n = 0$ (as in Sec. 3),

	→	→	→	b
	↑	U	↑	
a	C	→	C	

;

$k_n = 1$,

	→	→	b
	↑	←	
a	C	↑	

; $k_n = 2$,

	→	→	→	b
	↑	↑	←	
a	C	→	C	

;

$k_n = 2m + 1$,
$m \geqslant 1$

	→	→	→	. . .	→	→	b
	↑	←	←	. . .	←	←	
a	C	→	→	. . .	→	↑	

m columns, each having the form indicated;

$k_n = 2m$,
$m \geqslant 2$

	→	→	→	. . .	→	→	b
	↑	←	←	. . .	←	←	
a	C	→	→	. . .	→	C	

m - 1 columns, as above.

Notice (Table 1) that for $k_n \geqslant 2$ the width of $P_{\varepsilon}\,(1k_1\,1)$ is exactly half the width of $P(1k_n\,1)$.

Sequence $1k_n1$	Thatcher Pulser	Variant I Pulser	Type-I End Pulser
k_n even	$2k_n + 3$	$k_n + 2$	$\frac{k_n + 2}{2}$
k_n odd	$2k_n + 3$	$k_n + 3$	$\frac{k_n + 3}{2}$

Table 1 Widths of the various pulsers.

5. EXAMPLES

The following diagrams of a von Neumann, a Thatcher, a Variant I pulser and a Type I end pulser, for the sequence α = 10010001, illustrate some of the differences between the various algorithms.

von Neumann Pulser

	→	→	→	→	→	b
	↑	U	↑	U	C	
	↑	U	↑	→	↑	
	↑	U	↑	↑	←	
	↑	→	↑	→	↑	
	↑	↑	←	↑	←	
a	C	→	C	→	C	

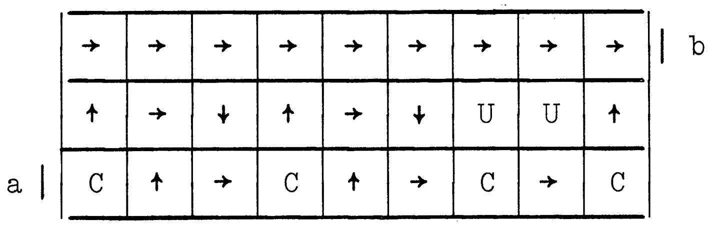
delay 13
area 35

Thatcher Pulser

	→	→	→	→	→	→	→	→	→	→	→	→	→	→	→	b
	↑	U	U	U	U	U	↑	U	U	U	U	U	U	U	↑	
a	C	→	C	→	C	→	C	→	C	→	C	→	C	→	C	

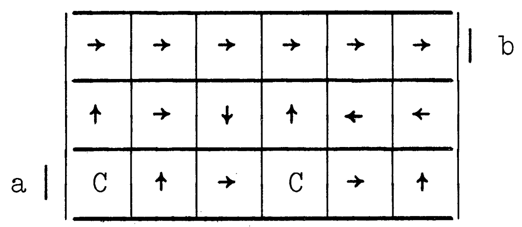
delay 19
area 45

Variant I Pulser



delay 13
area 27

Variant I Pulser with Type I End Pulser



delay 10
area 18

6. VARIANT II OF THATCHER'S CONSTRUCTION

We can modify the Variant I algorithm slightly and in so doing obtain, in a uniform manner, pulsers with shorter delays and sometimes (depending on the sequences) smaller areas. Figures 4 and 5 illustrate, for the sequence α = 100001, the basic differences between Variant I and Variant II pulsers (to be constructed).

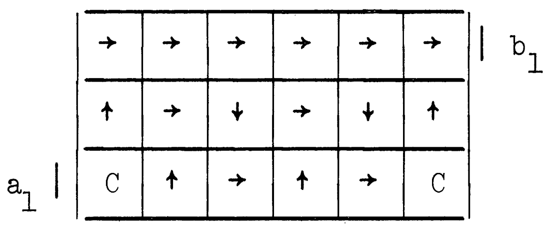


Fig. 4 Variant I

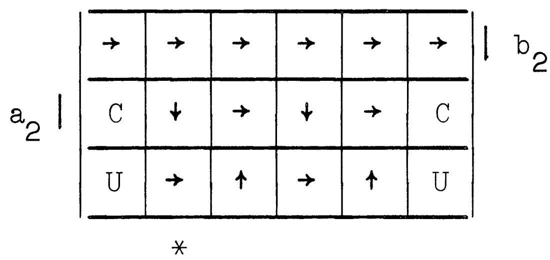


Fig. 5 Variant II

Notice (Fig. 5) that, by moving the input a_2 to the middle row, the delay of the Variant II pulser is decreased by one time unit. In addition it is possible to modify this pulser to produce the Variant II pulser for the sequence α' = 1000001 simply by changing the cell marked with an asterisk to a confluent cell. The modified pulser has the same area and same delay as the original Variant II pulser. Yet it is not possible to modify the Variant I pulser to produce a pulser for the sequence α' without increasing both the area and the delay. Thus for the sequence α' the Variant II pulser will have a smaller area and a shorter delay than the corresponding Variant I pulser.

The details of the construction of arbitrary Variant II pulsers follows.

Initial Step

For sequences having the form

$$\beta = 1k_i 1$$

construct for the following values of k_i the corresponding pulsers with input a and output b:

$k_i = 0$,

	→	→	→	b
a	C	→	C	
	U	U	U	

;

$k_i = 1$,

	→	→	C	→	→	b
a	C	↓	U	→	C	
	U	→	→	↑	U	

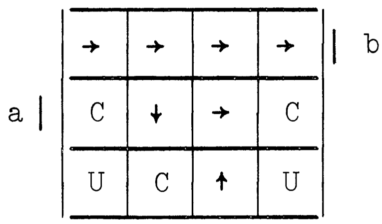
;

$k_i = 2$,

	→	→	→	→	b
a	C	↓	→	C	
	U	→	↑	U	

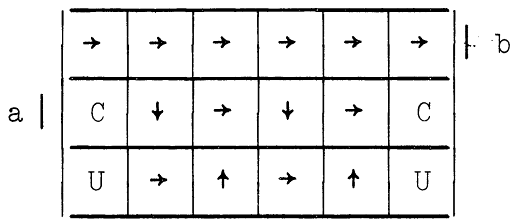
;

$k_i = 3$,



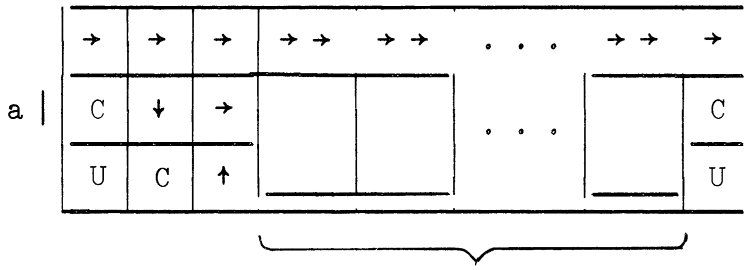
;

$k_i = 4$,



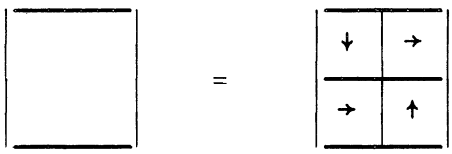
;

$k_i = 2m + 1$,
$m \geqslant 2$



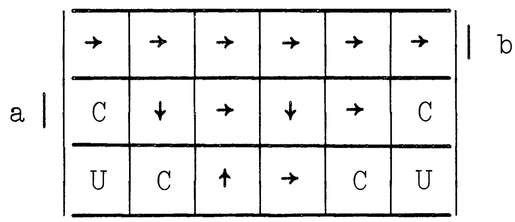
m - 1 blocks, each having

the form



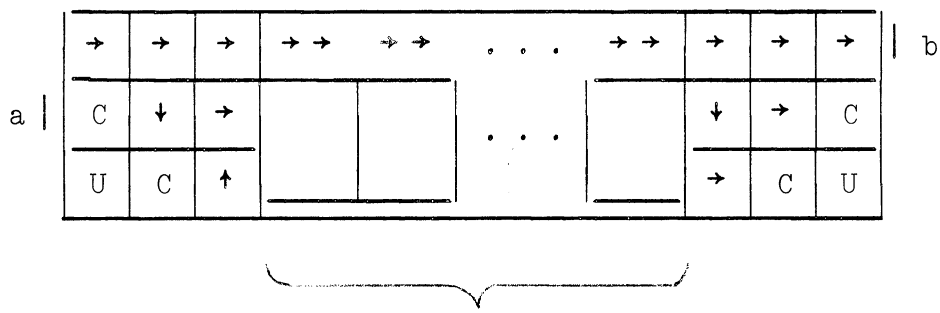
;

$k_i = 6$,



;

$k_i = 2m$,
$m \geqslant 4$



m - 3 blocks, having the

same form as above.

General Step

The general step, very much like that of the Variant I algorithm, involves a process of merging pulsers. Notice again that all the pulsers constructed in the initial step have identical first and last columns so that the merging of two pulsers can be accomplished very easily.

7. END PULSERS — TYPE II

In the same way that Variant I pulsers were modified to obtain Variant II pulsers, Type I end pulsers can be modified to obtain Type II end pulsers. That is, when constructing the Variant II pulser for the sequence

$$\alpha = 1k_1\, 1k_2\, 1 \ldots 1k_{n-1}\, 1k_n\, 1$$

merge $P(1k_1\, 1k_2\, 1 \ldots 1k_{n-1}\, 1)$ with one of the following class of pulsers, $P_\varepsilon(1k_n\, 1)$, depending on the value of k_n:

$k_n = 0$ (as in Sec. 6), a (row 2), b (row 1) ;

→	→	→
C	→	C
U	U	U

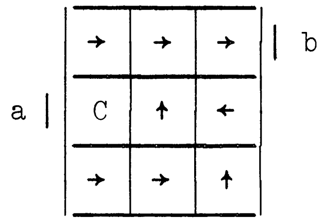
$k_n = 1$, a (row 2), b (row 1) ;

→	→
C	↑
→	↑

$k_n = 2$, a (row 2), b (row 1) ;

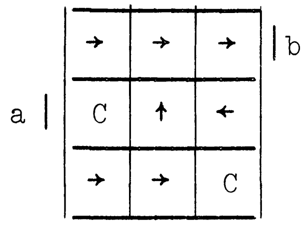
→	→
C	↑
→	C

$k_n = 3$,



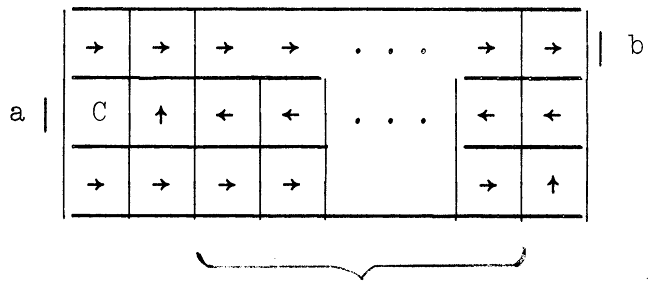
;

$k_n = 4$,



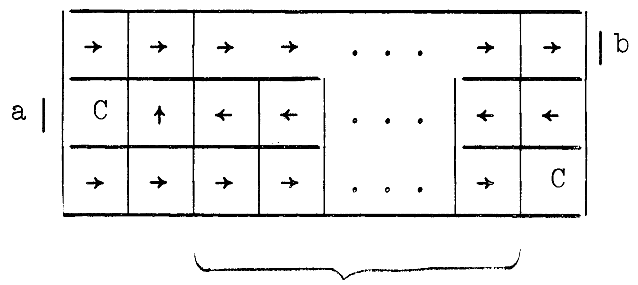
;

$k_n = 2m + 1$,
$m \geqslant 2$



m - 1 columns having the form indicated ;

$k_n = 2m$,
$m \geqslant 3$



m - 1 columns, as above .

For Variant II pulsers the width of $P(1k_n\ 1)$ is almost twice the width of $P_\varepsilon(1k_n\ 1)$, for $k_n \geqslant 5$. The construction of Type II end pulsers is also not iterative.

8. END PULSERS WITH ADJACENT CONFLUENT CELLS

By taking advantage of the fact that two confluent cells may be adjacent to each other, neither of which may feed or receive stimulations from the other, we can construct an even better class of end pulsers which can be merged with the ends of Variant II pulsers. For an end sequence $1k_n1$ construct for the values of k_n the corresponding pulsers:

$k_n = 0$, (input a at left of middle row; output b at right of top row) ;

→	→	→
C	→	C
U	U	U

$k_n = 1$, (a; b) ;

→	→
C	↑
→	↑

$k_n = 2$, (a; b) ;

→	→
C	↑
→	C

$k_n = 3$, (a; b) ;

→	→	→
C	↑	←
→	→	↑

$k_n = 4$, (a; b) .

→	→	→
C	↑	←
→	→	C

For $k_n \geqslant 5$ we present the construction somewhat differently. Let us letter the following six blocks:

a

→	→
C	C
→	C

b

→	→
C	↑
→	→

c

→
←
→

d

C
C
C

e

→
←
↑

f

→
←
C

For the following values of k_n construct the corresponding sequences of blocks:

k_n = 5 + 6m : $\underline{a}(\underline{cd})^{m}\underline{e}$

6 + 6m : $\underline{b}(\underline{dc})^{m}\underline{cf}$

7 + 6m : $\underline{b}(\underline{dc})^{m}\underline{de}$

8 + 6m : $\underline{ac}(\underline{dc})^{m}\underline{f}$

9 + 6m : $\underline{b}(\underline{dc})^{m+1}\underline{e}$

10 + 6m : $\underline{b}(\underline{dc})^{m+1}\underline{f}$

For example, the pulser for $1k_n 1$, where k_n = 11, is

	→	→	→	C	→	b
a	C	C	←	C	←	
	→	C	→	C	↑	
		$\underline{a}$	$\underline{c}$	$\underline{d}$	$\underline{e}$	

And as another example, the pulser for $1k_n 1$, for k_n = 15, is

	→	→	C	→	C	→	→	b
a	C	↑	C	←	C	←	←	
	→	→	C	→	C	→	↑	
		$\underline{b}$	$\underline{d}$	$\underline{c}$	$\underline{d}$	$\underline{c}$	$\underline{e}$	

.

9. SUMMARY

The following two tables provide comparisons of all of the algorithms for constructing pulsers, for the class of sequences of the form $1k_n 1$. [Notation: P_{ε}(CC) - End Pulser with Adjacent Confluent Cells; P_{ε}(II) - Type II End Pulser; P_{ε}(I) - Type I End Pulser; P(II) - Variant II Pulser; P(I) - Variant I Pulser.]

AREA

k_n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
<u>Pulsers</u>															
P_ε (CC)	9	6	6	9	9	9	12	12	12	15	15	15	18	18	18
P_ε (II)	9	6	6	9	9	12	12	15	15	18	18	21	21	24	24
P_ε (I)	9	6	9	9	9	12	12	15	15	18	18	21	21	24	24
P(II)	9	15	12	12	18	18	18	24	24	30	30	36	36	42	42
P(I)	9	15	12	18	18	21	24	27	30	33	36	39	42	45	48
von Neumann	6	12	12	15	18	21	24	27	30	33	36	39	42	45	48
Thatcher	9	15	21	27	33	39	45	51	57	63	69	75	81	87	93

DELAY

k_n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
<u>Pulsers</u>															
P_ε (CC)	6	5	5	6	6	6	7	8	7	9	9	9	10	11	10
P_ε (II)	6	5	5	6	6	7	7	8	8	9	9	10	10	11	11
P_ε (I)	7	6	7	7	7	8	8	9	9	10	10	11	11	12	12
P(II)	6	9	7	7	9	9	9	11	11	13	13	15	15	17	17
P(I)	7	9	8	10	10	11	12	13	14	15	16	17	18	19	20
von Neumann	6	8	8	9	10	11	12	13	14	15	16	17	18	19	20
Thatcher	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35

Notice that, oddly enough, although different in appearance (for $k_n \geqslant 4$) the von Neumann and Variant I pulsers, <u>for the sequences $1k_n1$</u>, have identical areas and identical delays. Figures 6 and 7 indicate the similarity of these two classes of pulsers for the case $k_n = 5$.

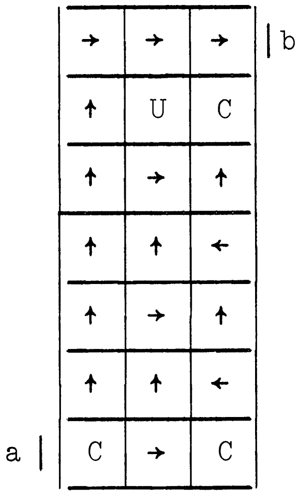


Fig. 6 von Neumann Pulser

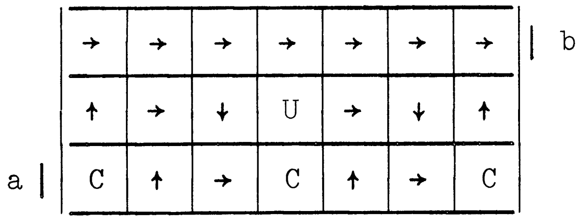


Fig. 7 Variant I Pulser

* * *

References

[1] von Neumann, John, "Theory of Automata: Construction, Reproduction, Homogeneity." Part II of The Theory of Self-Reproducing Automata, edited by Arthur W. Burks. To be published by the University of Illinois Press.

[2] Thatcher, James W., "Universality in the von Neumann Cellular Model," to be published as an IBM Research Report, September 1964.

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