ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

A CONSTANT-AMPLITUDE RANDOM FUNCTION GENERATOR

Technical Report No. 77

Electronic Defense Group
Department of Electrical Engineering

By: G. A. Hellwarth

Approved by: H. W. Farris

Project 2262

TASK ORDER NO. EDG-7
CONTRACT NO. DA-36-039 sc-63203
SIGNAL CORPS, DEPARTMENT OF THE ARMY
DEPARTMENT OF ARMY PROJECT NO. 3-99-04-042
SIGNAL CORPS PROJECT 194B

November 1957

TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	iii
ABSTRACT	iv
1. INTRODUCTION	1
2. THE RANDOM SAWTOOTH GENERATOR	2
2.1 General Description of Operation	2
2.2 Integrator Operation	4
2.3 Discussion of Laboratory Model	6
2.4 Limitations of the Sawtooth Waveshape	7
3. THE RANDOM TRIANGLE, COSINE, AND SQUARE WAVE GENERATOR	7
3.1 Design Considerations	7
3.2 Analysis of Generator Operation	8
3.2.1 Relation Between Integrator Input and Output Side Time	8
3.2.2 Integrator Input Requirements	11
3.2.3 Effect of Shaping on the Noise Signal Amplitude Probability Density	13
3.2.4 Relation Between Noise Signal Amplitude Probability Density and The Side Time Choice Probability Density	14
3.2.5 Output Wave Side Time Probability Density	16
3.2.6 Side Time Probability Density Using Gaussian Noise	18
3.3 Experimental Determination of the Output Power Spectra	20
4. GENERAL CIRCUIT DISCUSSION	22
4.1 Comments	22
4.2 Circuit Diagram	27
4.3 Oscillographs of Output Wave	27
APPENDIX A DETAILED CIRCUIT OPERATION OF THE FUNCTION GENERATOR	33
APPENDIX B CALIBRATION AND ADJUSTMENT	36
APPENDIX C SUGGESTIONS FOR A TRANSISTORIZED GENERATOR	38
DISTRIBUTION LIST	41

LIST OF ILLUSTRATIONS

		Page
Figure 1	Constant-Amplitude Random Functions	3
Figure 2	Block Diagram of Random Sawtooth Generator	5
Figure 3	Simplified Equivalent Circuit for Generating a Side	5
Figure 4	Block Diagram of Constant-Amplitude Random Function Generator	9
Figure 5	Operation of Integrating Circuit	10
Figure 6	Dual Noise Shaper Characteristics	12
Figure 7	Effect of Shaping Circuit on Rectangular Amplitude Probability Density Function	12
Figure 8	Relation Between Noise Voltage, e_s, and Generated Side Time by a Sample of e_s	15
Figure 9	Graph of Output Side Time Probability Density for Noise with a Rectangular Amplitude Probability Density	17
Figure 10	Output Wave Side Time Probability Density, r(t), for e_s Being Gaussian Noise	19
Figure 11	Cosine Wave Power Spectra as Noise Voltage, e_s, Is Varied	21
Figure 12	Cosine Wave Power Spectra Showing Effect of Changing Shaper Gain Ratio for Large Side Times	23
Figure 13	Power Spectra for the Three Constant-Amplitude Random Functions	24
Figure 14	Cosine Wave Spectra for Triangle and Sine Wave Noise Signals	25
Figure 15	Photograph of the Laboratory Generator	28
Figure 16	Three Output Modes at Steady State	28
Figure 17	Random Triangle and Cosine Wave Oscillograph	29
Figure 18	Random Sawtooth Generator Circuit	30
Figure 19	Constant Amplitude Random Function Generator Circuit	31
Figure 20	Generator Cosine Shaper and Power Supply	32
Figure 21	Transistorized Integrator	39

ABSTRACT

The equipment design discussed in this report resulted from the requirement for a generator capable of producing an audio frequency waveform with a constant peak-to-peak amplitude but randomly chosen paths between peaks. A mathematical analysis is given for various aspects of the several types of waveforms treated: sawtooth, triangle, and cosine. Probability density functions and power spectra describing the outputs of the generator are shown. The equipment is described in detail and results obtained from it are shown graphically.

A CONSTANT-AMPLITUDE RANDOM FUNCTION GENERATOR

1. INTRODUCTION

The constant-amplitude random function generator described in this report resulted from the need for an electronic generator which would combine some of the properties of random noise waveshapes with the known characteristics of periodic, constant amplitude waves. In this instance, a waveshape was desired which had as a main feature a constant peak-to-peak amplitude, but whose path between peaks was in some way randomly chosen.

At first inspection the simplest way to generate such a function would be that of amplitude clipping any noise-like waveshape. However, this was thought to be unsatisfactory because the wave spends too much time at the peak value, or clipping level, rather than traversing between peaks. The next criterion on the waveshape, then, was that it reverse its direction immediately after reaching each peak.

There appeared to be several common waveshapes that would accomplish this task. The most obvious were:

1. A sawtooth wave with the time length of its tooth randomly determined.
2. A triangular wave with the time lengths of both the positive and negative slopes randomly chosen.
3. A cosine wave with its half-cycle time length randomly chosen.
4. A wave with a random path between peaks, each path maintaining a slope of constant sign.

At this point, it becomes necessary to define some standard terms for discussion of the proposed waveshape.

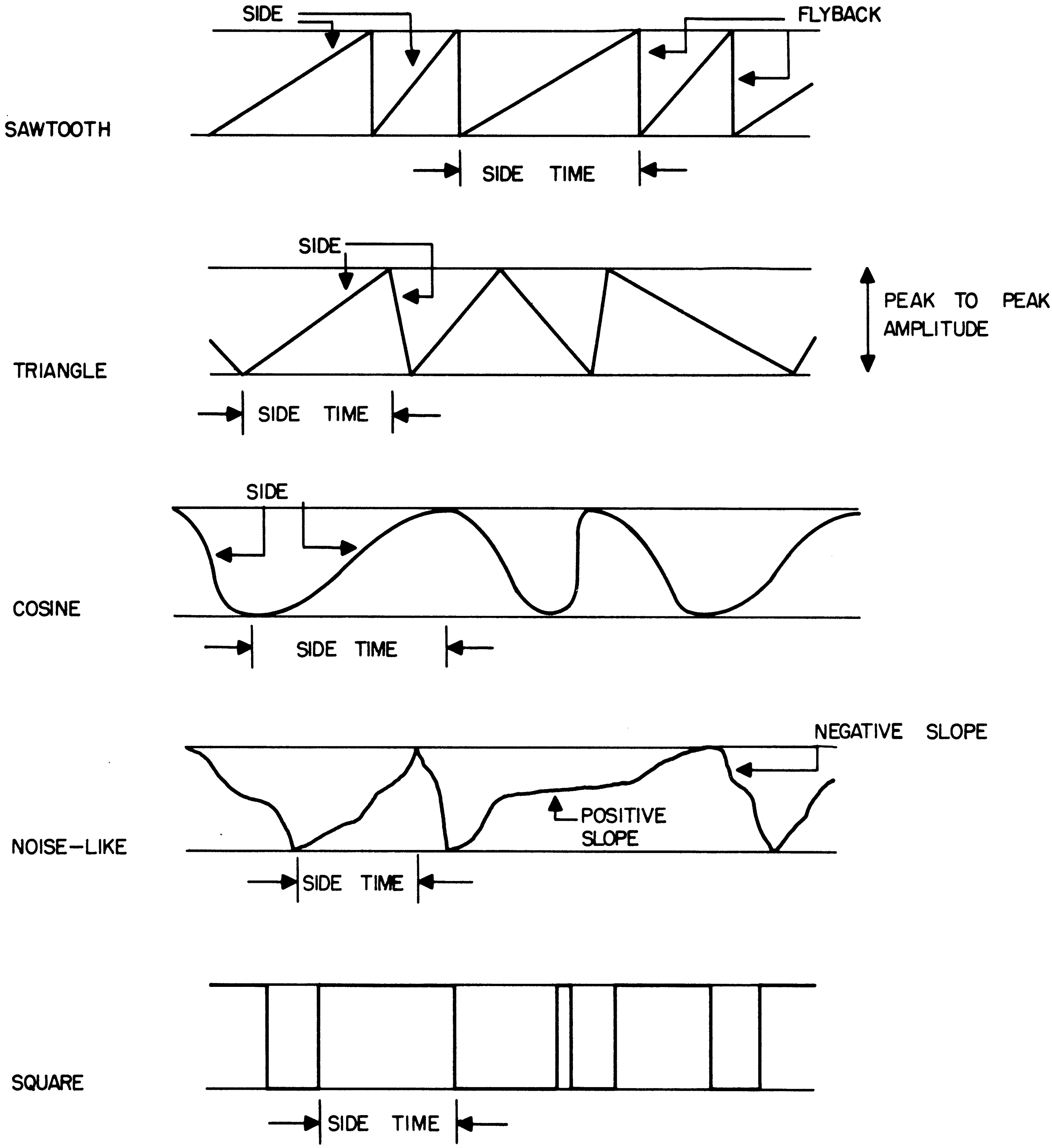
Given a constant amplitude waveshape, a single excursion of the wave between two peaks shall be called a "side". The time derivative or slope of a side is a constant for a sawtooth or triangle wave. A side of a cosine wave shall be the value of the cosine function between 0 and π, or π and 2π. The side time-length, or "side time", is defined as the time duration of a side's excursion from one peak to the next. The side time of a triangle or sawtooth wave is inversely proportional to the slope of the side. A constant side time triangle or cosine wave (not random) has a fundamental frequency which is the reciprocal of twice the side time. The fundamental frequency or repetition rate of a sawtooth wave is the reciprocal of the side time, assuming a zero flyback time (see Figure 1).

Perhaps the most evident reason for the introduction of this terminology is that the functions will actually be generated by randomly choosing each side time-length, resulting in the description of the wave being a probability distribution of side times. In practice, however, it was found to be much easier to measure the power spectrum than the side time distribution, so both frames of reference will necessarily be used.

2. THE RANDOM SAWTOOTH GENERATOR

2.1 General Description of Operation

The first type of random waveshape generated was the sawtooth, primarily because this type of generator was thought to be the simplest to design and build. The generator was to operate at audio frequencies and was to have as much flexibility as possible in controlling its side time-length distribution. The random



DIAGRAMS ARE INSTANTANEOUS AMPLITUDE VS. TIME

FIG.1 CONSTANT - AMPLITUDE RANDOM FUNCTIONS sawtooth block diagram is shown in Fig. 2, and a schematic circuit diagram is shown in Fig. 18. Essentially, the generator consists of a Gaussian noise source being sampled at the time of the sawtooth flyback, the noise sample being held and integrated until the output reaches some constant value. At this point the integrator is reset (causing the sawtooth flyback) and a new noise sample is taken and integrated, etc. The integrator is a constant-current pentode discharging a capacitor, resetting being due to a thyratron recharging the capacitor. The noise samples are capacitor-coupled into the pentode grid, a controllable grid bias determining the center side time (the center side time is that side time resulting from a zero noise sample, see Equation 4b).

2.2 Integrator Operation

A simplified integrator circuit as shown in Figure 3, is helpful in observing the generator operation during the formation of a single side.

The current sources in Figure 3 represent a pentode whose only plate load is a capacitor, C. Thus, during the length of time required to generate one side, a constant current is being fed into the capacitor to produce the ramp function of voltage across the capacitor. If g_m is the transconductance of the pentode; $I_o = g_m E_o$ is the pentode current resulting from the grid bias voltage, E_o; and $I_n = g_m E_n$ is the pentode current resulting from the noise sample voltage, E_n, then the capacitor voltage, E(t), will be:

$$E(t) = \frac{I_o + I_n}{C} t = \frac{g_m(E_o + E_n)}{C} t, \qquad 0 \leq t \leq T \tag{1a}$$

And since $(E_o + E_n)$ is constant during a side, the time length of the side is that time, T, required for the capacitor voltage to change by E_m volts, the peak-to-peak voltage of the sawtooth wave. Therefore,

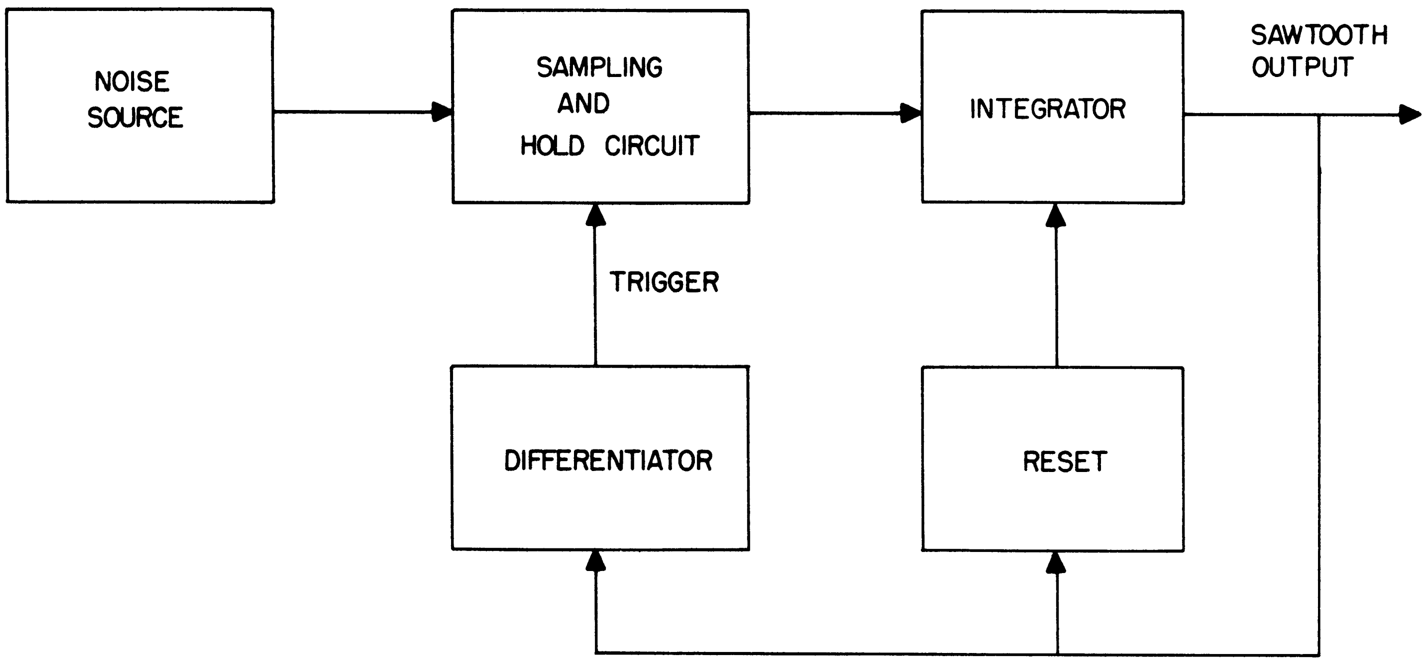


FIG. 2 BLOCK DIAGRAM OF RANDOM SAWTOOTH GENERATOR

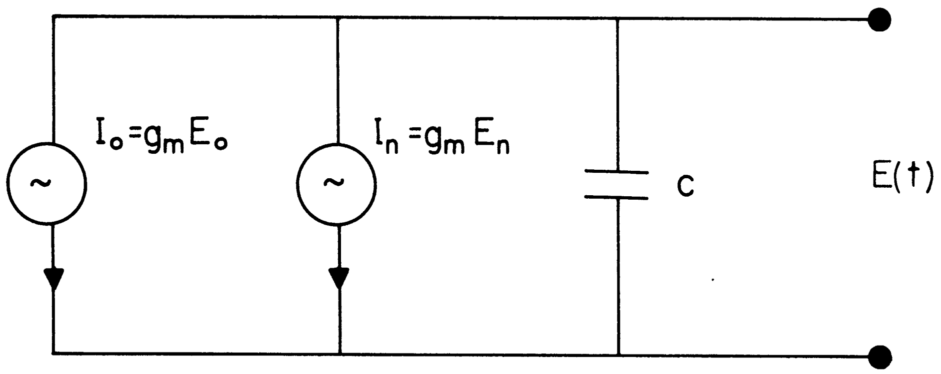


FIG. 3 SIMPLIFIED EQUIVALENT CIRCUIT FOR GENERATING A SIDE

2262 A-S7-60 PS 2-6-58 (10-23-57 SA)

$$T = \frac{C E_m}{g_m(E_o+E_n)} = \frac{C E_m}{I_o+I_n} . \qquad (1b)$$

Thus, Equation 1b gives the relation between the sample of noise, E_n, and the side time, T, produced by such a noise sample. The effect of varying the pentode grid bias, E_o, can also be seen. A quantity to note in Equation 1b is the transconductance, g_m, which in practical cases is never a constant, but is a function of E_n. By choosing differing types of pentodes in the generator circuit, one is able to obtain different relations between E_n and T according to the particular way a pentode transconductance varies with its input. Another tool available for controlling the side time, T, produced by a noise sample, E_n, is the nonlinear amplitude attenuating filter, which can be inserted between the noise source and the sampling circuit. This type of network will effectively introduce to Eq. 1b a nonlinear factor, a monotonic function of E_o, which multiplies E_o.

The circuit described in Figure 18 contains a sharp cut-off pentode and no amplitude attenuating network. Amplitude attenuating filters are discussed in greater detail in Section 3.2.2.

2.3 Discussion of Laboratory Model

The random sawtooth circuit is relatively stable and reliable. The most critical element appears to be the pentode, since its DC characteristics are used to determine the center side time. However, the output power spectrum is not seriously affected by slight changes in the pentode characteristics. A limit can be set on the maximum time length of any side by simply paralleling the integrating capacitor with a suitable resistance so that if the pentode were cut-off by a large noise sample, the resistor would discharge the capacitor at a predetermined exponential decay rate.

The center side time is controllable between 0.2 and 10 milliseconds by varying the pentode grid bias. Maximum deviations of side time from the center depend on the pentode grid bias setting, but 200:1 maximum to minimum side time ratios are achievable with the pentode grid bias near the middle of its range. A much more satisfactory way of changing the center side time would be to vary the size of the integrating capacitor, always leaving the grid bias in the middle of its range. This would yield a side time distribution which does not vary with the center side time setting.

2.4 Limitations of the Sawtooth Waveshape

The output power spectrum of the random sawtooth is very wide and contains a considerable band above audio frequencies. (There are no spectrum or side-time curves in the appendix for the sawtooth, since it became only a forerunner of the more significant triangle and cosine generator.) The higher frequency spectral power, caused chiefly by the rapid flyback of the sawtooth, requires a correspondingly wide bandwidth for any associated circuit to pass the wave and maintain its constant-amplitude character. It was then decided to try to generate one or more of the other waves described in Figure 1, waves which would not have the large amount of high frequency power resulting from discontinuities. In the triangle and cosine waves, the power spectrum was expected to be much more concentrated in the range corresponding to the side time distribution, resulting also in increased ability to control the output power spectrum.

3. THE RANDOM TRIANGLE, COSINE, AND SQUARE WAVE GENERATOR

3.1 Design Considerations

In planning the design of a random triangle generator, there were several problems of circuit choice and complexity that were not encountered in the sawtooth generator. The basic block diagram of the circuit, as shown in Figure 4, is similar to the diagram of the sawtooth, but the contents of the blocks are quite different.

The first design consideration resulted in choosing an operational-amplifier integrator circuit which could integrate positive and negative inputs. The output of the integration operates between constant peak-to-peak limits by the action of the amplitude discriminator circuit. When the integrator output reaches a certain magnitude (i.e., the pre-set peak value) the bistable discriminator circuit is "flipped", resulting in a trigger pulse to the sampler and a reversal of polarity of the integrator input signal. This action takes place at both positive and negative peaks, thus confining the integrator output between the peak values. The input to the integrator is the sum of two signals: (1) the constant-amplitude square wave output of the amplitude discriminator, E_o, and (2) the random amplitude square wave output of the noise sampling and holding circuit, E_n. The operation of the circuit is wholly dependent upon the successful generation of the integrator input signal.

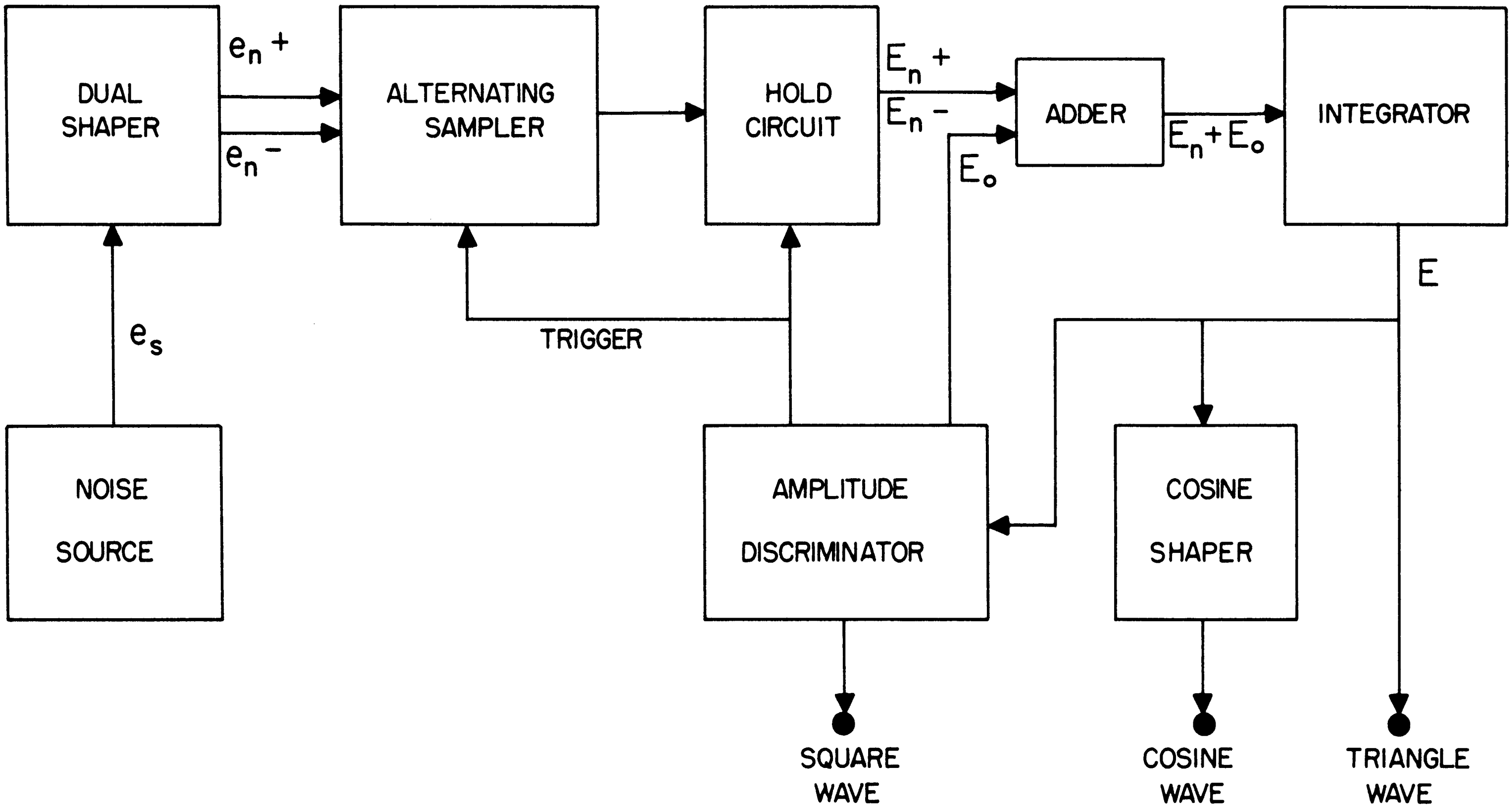
3.2 Analysis of Generator Operation

3.2.1 Relation Between Integrator Input and Output Side Time.

Given an integrator circuit as shown in Figure 5a, an equation of operation similar to Eq. 1 can be written. The time, T, required for E to change by E_m volts is given by Eq. 4a, since $(E_o + E_n)$ is constant throughout the interval T.

$$T = RC \frac{E_m}{E_o+E_n} \tag{4a}$$

The "center" side time, T_o, is again defined as that side time generated when the noise voltage sample, E_n, is zero.



2262 A-S7-61 PS 2-6-58 (SA 10-23-57)

FIG.4 <u>BLOCK DIAGRAM OF CONSTANT-AMPLITUDE RANDOM FUNCTION GENERATOR</u>

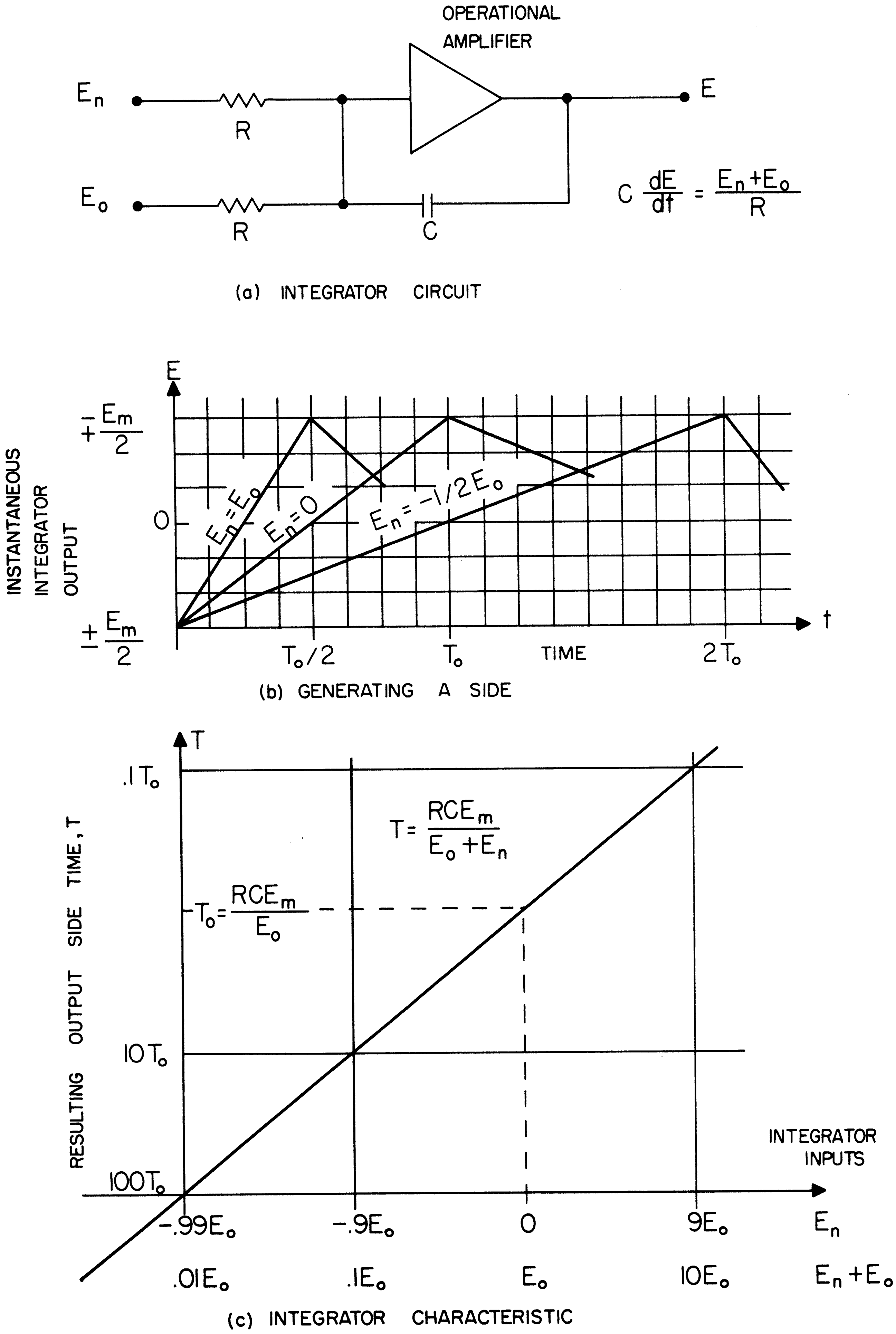


FIG. 5 OPERATION OF INTEGRATING CIRCUIT

That is:

$$T_o \triangleq RC \frac{E_m}{E_o} \quad . \tag{4b}$$

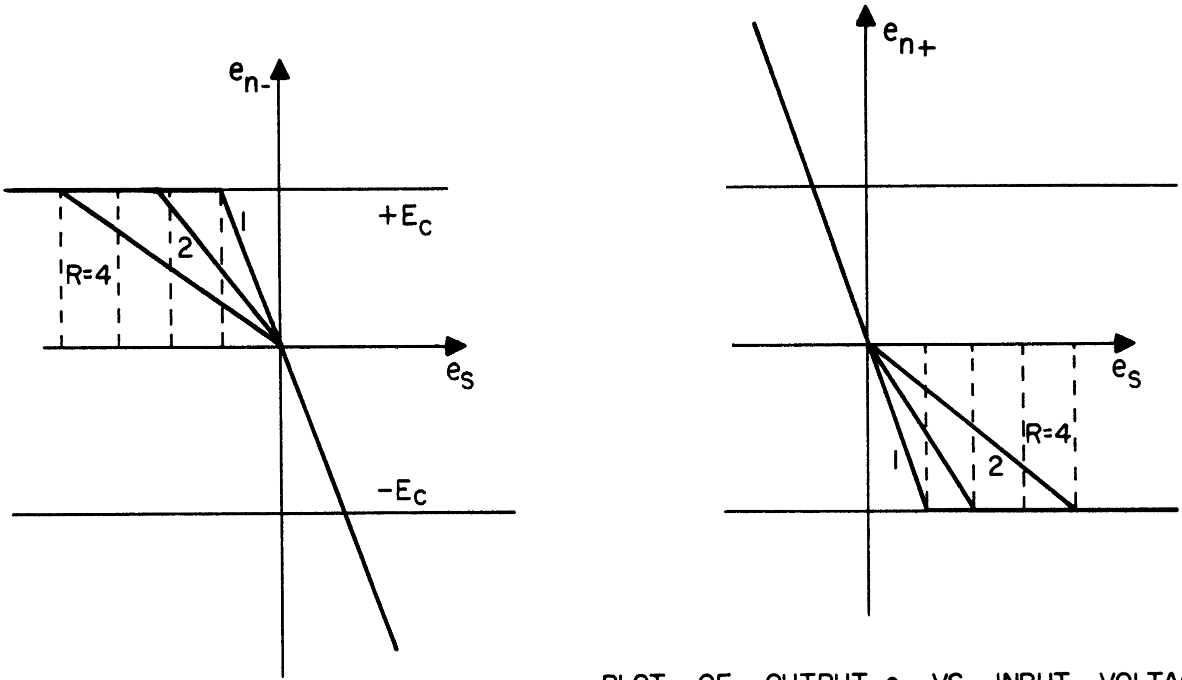
Figure 5b shows the expected ramp function of voltage vs. time obtained at the integrator output, when a constant is integrated. The differing times required for the output to change by E_m volts when the input is varied is indicated by the three sides pictured. Figure 5c is a quasi log-log plot of Eq. 4a.

3.2.2 Integrator Input Requirements. The first requirement to be noted is that at every proposed peak of the integrator output, the polarity of the integrator input signal (E_o+E_n) must be reversed. The signal E_o, which is a positive constant, is intended to produce the necessary changes of polarity at the integrator input by alternately appearing as +E_o or -E_o. However, in order that the total input signal change polarity as dictated by the sign of E_o, then Eq. 5 must hold:

$$\begin{aligned} E_n &> -E_o \quad \text{when } +E_o \text{ is integrated} \\ E_n &< E_o \quad \text{when } -E_o \text{ is integrated} \end{aligned} \tag{5}$$

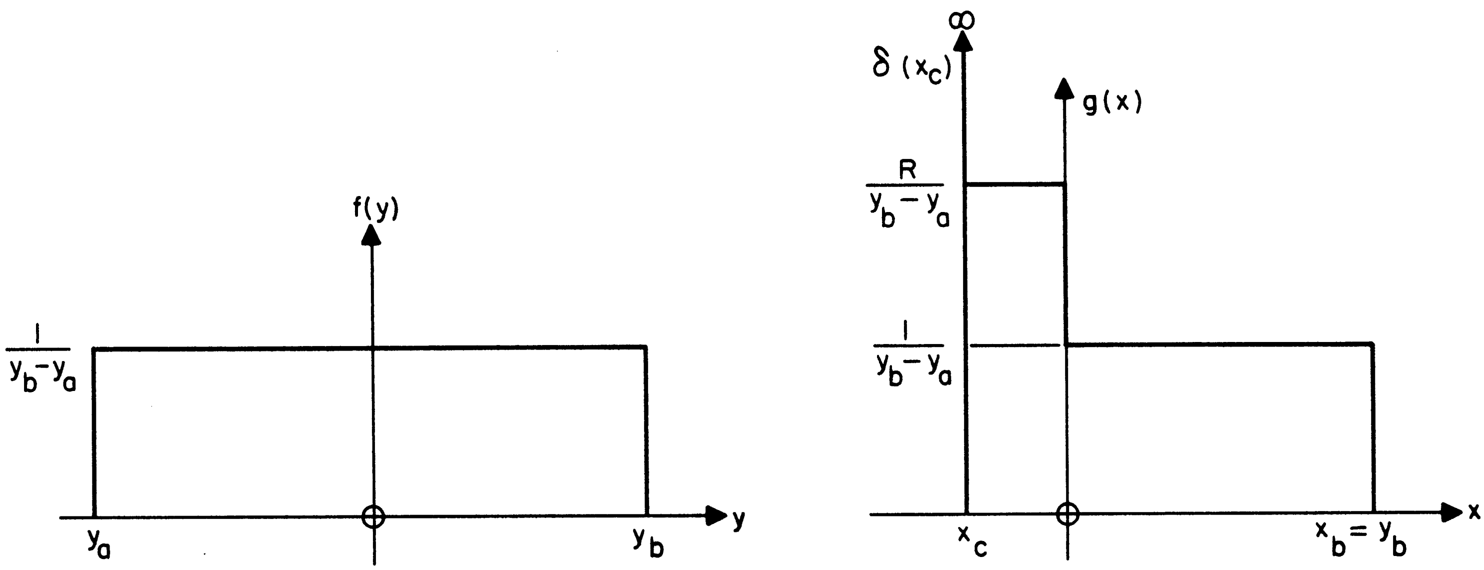
If side times shorter than $\frac{1}{2} T_o$ are to be desired, then there must be two separate sources of noise signal, E_n, from which the integrator receives its input. One source shall be defined as E_{n+}, and it will provide the input noise signal in conjunction with +E_o; the second source, E_{n-}, will provide the input with -E_o.

The dual noise shaper takes a single noise voltage input, e_s, and feeds this signal through two parallel shaping amplifiers (or attenuators)whose characteristics are opposite in polarity. These two shaped signals are then alternately sampled and held yielding E_{n+} and E_{n-}, respectively. The output vs. input amplitude characteristic of the shaper circuit is arbitrary, as long as the clipping levels maintain the limits on e_n as indicated in Figure 6. A relatively simple shaper characteristic was chosen for this generator: a fixed gain for noise signals of the same polarity as E_o, and a smaller, variable gain for signals



PLOT OF OUTPUT, e_n, VS. INPUT VOLTAGE, e_s, FOR DIFFERENT GAIN RATIOS, R, WITH A FIXED CLIPPING LEVEL, E_c

FIG.6 DUAL NOISE SHAPER CHARACTERISTICS



(a) BEFORE SHAPING

(b) AFTER SHAPING

FIG. 7 EFFECT OF SHAPING CIRCUIT ON RECTANGULAR AMPLITUDE PROBABILITY DENSITY FUNCTION opposite in polarity to E_o; the ratio of these two gains is designated as the shaper gain ratio, R. The amplitude characteristics of the two channels of the shaper circuit are alike except for polarity, as shown in Figure 6.

3.2.3 Effect of Shaping on the Noise Signal Amplitude Probability Density. The amplitude probability of the noise voltage is affected by the shaping circuit as follows. If f(y) is the amplitude probability density of the noise voltage, if the clipping level is x_c, and if the shaper gain ratio is R, then the amplitude probability density g(x) after shaping is given by

$$g(x) = 0, \quad x < x_c \tag{6a}$$

$$g(x) = \delta(x_c), \quad x = x_c \text{, where} \tag{6b}$$

$$\lim_{\epsilon \to 0} \int_{x_c-\epsilon}^{x_c+\epsilon} g(x)dx = \int_{-\infty}^{Rx_c} f(y)dy = \text{area of } \delta(x_c),$$

$$g(x) = Rf(y/R), \quad x_c < x < 0 \tag{6c}$$

$$g(x) = f(y), \quad x > 0 \tag{6d}$$

As an example, Figure 7 and Equation 7 show the shaper effect on a noise voltage with a rectangular amplitude probability density function:

$$g(x) = 0, \quad x < x_c \tag{7a}$$

$$g(x) = \delta(x_c), \quad x = x_c \text{, where} \tag{7b}$$

$$\lim_{\epsilon \to 0} \int_{x_c-\epsilon}^{x_c+\epsilon} g(x)dx = \int_{y_a}^{Rx_c} \frac{1}{y_b-y_a} dy = \frac{Rx_c-y_a}{y_b-y_a} = \text{area of } \delta(x),$$

$$g(x) = \frac{R}{y_b-y_a}, \quad x_c < x < 0 \tag{7c}$$

$$g(x) = \frac{1}{y_b-y_a}, \quad 0 < x < y_b \tag{7d}$$

Figure 6 and Equation 4 can now be combined to give the overall relationship between the initial noise signal, e_s, and the output side time, T. Included in the e_s vs T curves shown in Figure 8 is the shaper gain ratio as a parameter, assuming a fixed gain for the signals opposite in polarity to E_o. An arbitrary clipping level, E_c, is indicated:

$$T = \frac{E_o T_o}{E_o + E_c} \ , \quad e_s < E_c \ , \quad E_c > -E_o \tag{8a}$$

$$T = \frac{E_o T_o}{E_o + e_s} \ , \quad E_c < e_s < 0 \tag{8b}$$

$$T = \frac{E_o T_o}{E_o + Re_s} \ , \quad e_s > 0 \tag{8c}$$

The shaper gain ratio, R, can either indicate attenuation of noise voltages of one polarity, or amplification of noise voltages of the other polarity. However, the two actions are equivalent if the overall gain is properly adjusted, as shall be assumed in this discussion.

3.2.4 Relation Between Noise Signal Amplitude Probability Density and the Side Time Choice Probability Density. If a given noise voltage, e_s, has an amplitude probability density, $p(e_s)$, then from Eq. 8 and 9 the probability density, $q(T)$, of choices of side time can be calculated:

$$q(T) = p(e_s) \ \frac{de_s}{dT} \tag{9}$$

where

$$\frac{de_s}{dT} = \frac{E_o}{R} \ \frac{d}{dT}\left(\frac{T_o}{T} - 1\right) = \frac{E_o}{R} \cdot \frac{T_o}{T^2} \quad \text{which is} \tag{10}$$

valid for

$$E_c < e_s < 0 \qquad \text{and} \qquad 0 < e_s$$

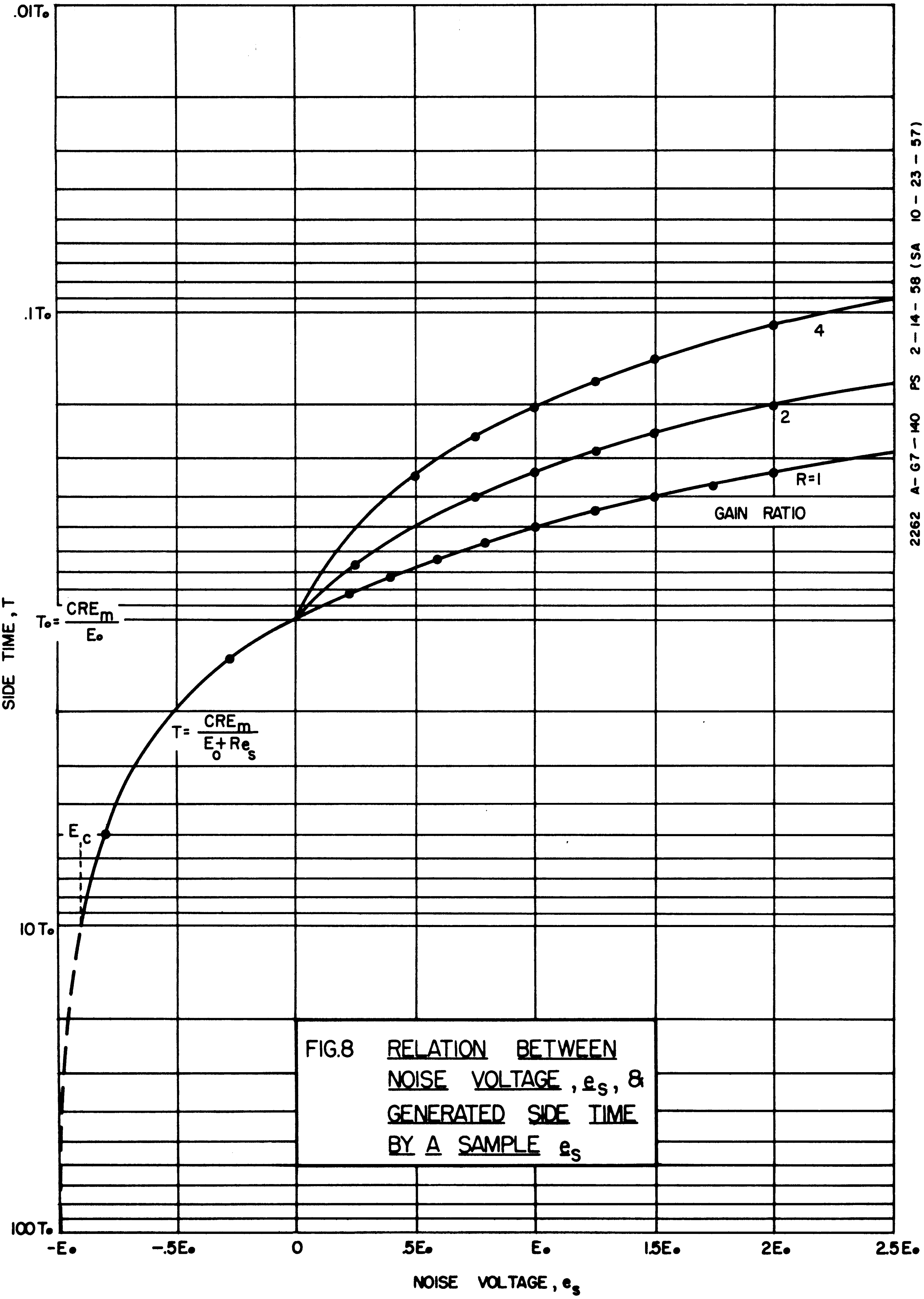


FIG.8 RELATION BETWEEN NOISE VOLTAGE, e_S, & GENERATED SIDE TIME BY A SAMPLE e_S

At $e_s = 0$, the gain ratio, R is not defined and at the clipping level, E_c, de_s/dT is an impulse function whose area is given by Equation 6b. Thus, the probability density of choices of side time, T, is known.

3.2.5 Output Wave Side Time Probability Density. To find the probability density, r(T), that the output of the integrator is generating a side time T, where $T_1 < T < (T_1 + dT_1)$, one weights the q(T) density with the factor, T, since each choice, T, of a side time lasts T seconds, and, therefore, occupies a portion of time proportional to T.

$$r(T) = r(T_1 < T < T_1 + dT_1) = \frac{T}{T_a} \cdot q(T) \qquad (11)$$

where $T_a = \int_0^\infty T \; q(T) \; dT$ maintains unity area under the density curve.

An example of interest is one where $p(e_s)$ is a rectangular distribution as defined in Figure 9a. Equation 12 gives the general relation for R(T). Excluding discontinuities;

$$r(T) = T/T_a \cdot q(T) = T/T_a \; p(e_s) \frac{de_s}{dT} = \frac{T}{T_a} \cdot \frac{1}{2E_o} \cdot \frac{E_o T_o}{T^2 R} \qquad (12)$$

Therefore, R(T) is found to be for this case:

$$R(T) = 0 \quad , \quad T < \frac{T_o}{1+R} \qquad (13a)$$

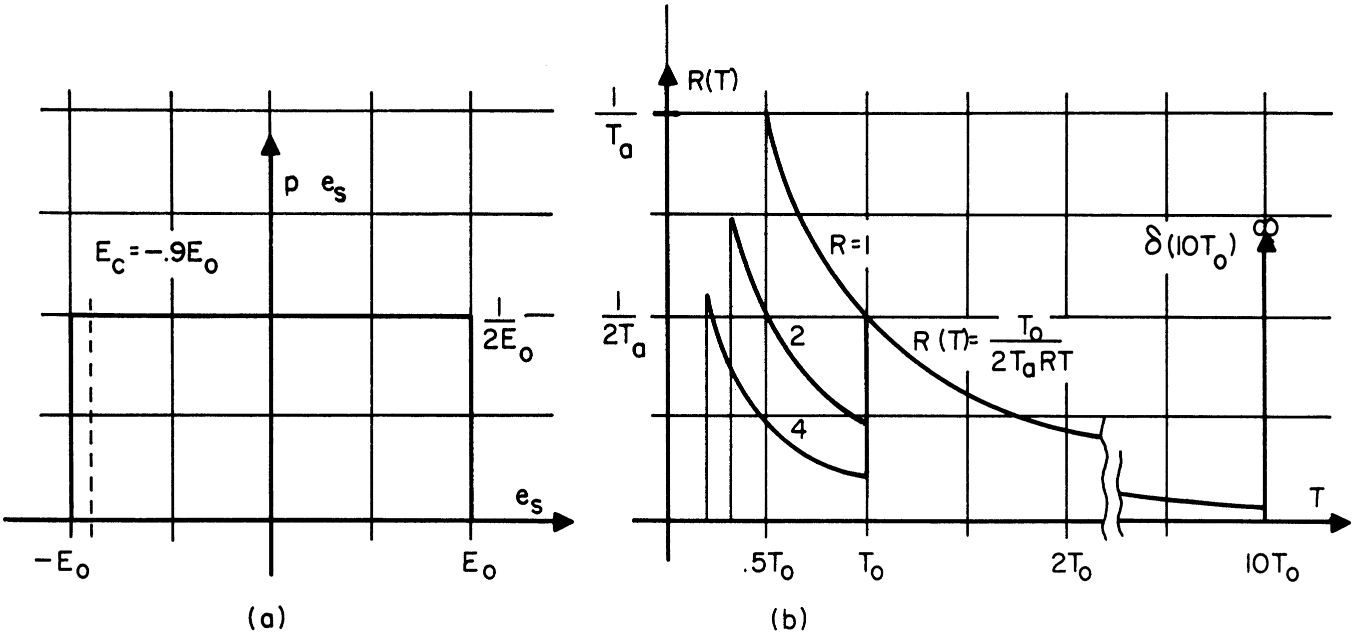
$$= \frac{T_o}{2RT_a T} \quad , \quad \frac{T_o}{1+R} < T < T_o \qquad (13b)$$

$$= \frac{T_o}{2T_a T} \quad , \quad T_o < T < 10\,T_o \qquad (13c)$$

$$= \delta(10\,T_o) \; , \quad T = 10\,T_o \quad , \text{ area} = 1/20 \text{ for } E_c = -.9E_o \qquad (13d)$$

$$= 0 \quad , \quad T > 10\,T_o \qquad (13e)$$

Equation 13 is plotted in Figure 9b.



NOISE SIGNAL AMPLITUDE PROBABILITY DENSITY, $p(e_s)$

OUTPUT WAVE SIDE TIME PROBABILITY DENSITY, R(T)

FIG.9 GRAPH OF OUTPUT SIDE TIME PROBABILITY DENSITY FOR NOISE WITH A RECTANGULAR AMPLITUDE PROBABILITY DENSITY

2262 A-G7-142 PS 2-14-58 (SA 10-23-57)

3.2.6 Side Time Probability Density Using Gaussian Noise. A second example of even more interest is the case where e_s is Gaussian noise, the amplitude probability density being

$$p(e_s) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot \exp\left[\frac{-(e_s)^2}{2\sigma^2}\right]$$

where σ, the standard deviation, will be some constant times E_o. Let $E_c = 0.9\,E_o$.

Then
$$r(T) = \frac{T}{T_a}\,q(T) = \frac{T}{T_a}\,p(e_s)\,\frac{de_s}{dT} = \frac{E_oT_o}{T_aTR}\,p(e_s) \tag{14a}$$

$$= \frac{E_o}{\sqrt{2\pi}\,\sigma\,T_a} \times \frac{\exp\left[-\left(\frac{T_o}{T}-1\right)^2 \frac{E_o^{\,2}}{2R^2\,\sigma^2}\right]}{RT/T_o}, \quad 0 < T < T_o \tag{14b}$$

$$= \frac{E_o}{\sqrt{2\pi}\,\sigma\,T_a} \times \frac{\exp\left[-\left(\frac{T_o}{T}-1\right)^2 \frac{E_o^2}{2\sigma^2}\right]}{T/T_o}, \quad T_o < T < 10\,T_o \tag{14c}$$

$$= \delta(10T_o),\ \text{area} = \frac{1}{\sqrt{2\pi}\sigma} \cdot \int_{-\infty}^{-.9E_o} \exp\left[-\frac{e_s^{\,2}}{2\sigma^2}\right] de_s, \quad T = 10\,T_o \tag{14d}$$

$$= 0, \quad T > 10\,T_o \tag{14e}$$

Figure 10 shows a plot of r(T) for four values of σ. For a particular shaper gain ratio, R, appropriate values of σ on either side of $T = T_o$ are chosen. The value of the standard deviation, σ, for Gaussian noise is just the rms value of the noise, so it would be relatively easy to determine σ/E_o in the generator circuit by measuring the unshaped noise voltage and calculating the effective gain of the following circuitry. In the laboratory model constructed, however, there are several variable gain controls affecting the noise sample amplitude with respect to E_o, so the value of σ/E_o is not so easily determined.

From inspection of Figure 10 and Eq. 14, it is seen that as the noise voltage, e_s, is increased, r(T) approaches a 1/T curve, since the Gaussian density function is being clipped for noise voltages opposite in polarity to E_o, and the

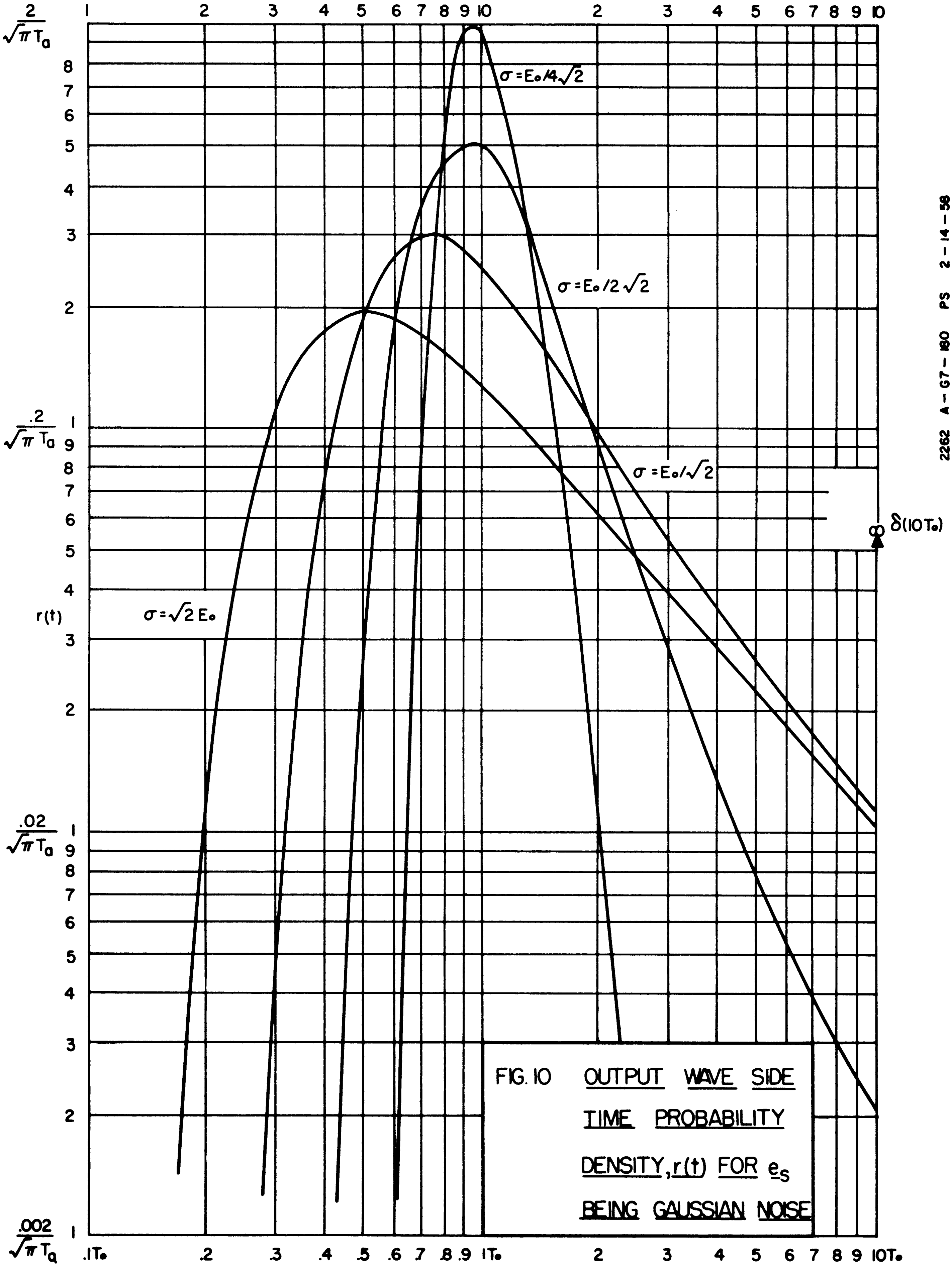


FIG. 10 OUTPUT WAVE SIDE TIME PROBABILITY DENSITY, r(t) FOR e_S BEING GAUSSIAN NOISE

SIDE TIME, T

2262 A-G7-180 PS 2-14-58

noise causing these large side times approaches a rectangular amplitude probability density, as analyzed for Figure 9. More important, however, is that the area represented by the impulse at the shaper clipping level, E_c, becomes larger and larger as the noise voltage, e_s, is increased. This means that the output triangular wave will have more and more side times equal to the maximum side time, which might be undesirable from the standpoint of the output power spectrum. One could arbitrarily set the noise voltage such that, say, 1/20 of the total area under the side time probability density curve, r(T), is contained in the impulse. This would be when $\sigma = E_o/1.645$, which would also appear to be a good value from the standpoint of obtaining a large side time density for large side times.

One could construct a shaper circuit which has a more complex characteristic to achieve different side time densities. This would, of course, depend on the application of the random function generator. For the purpose of this report previous discussion is considered to be sufficient with respect to calculating the output wave side time probability density, given an input noise voltage and shaper gain characteristic.

3.3 Experimental Determination of the Output Power Spectra

It would be desirable to know the relationship between the side time probability density, r(T) (previously determined), and the output power spectrum. Much time was spent in an attempt to relate these two functions, but without success. However, in spite of the advantages of having the mathematical equation, some practical insight into this relationship can be derived experimentally. When the noise signal, e_s, is Gaussian noise, the measured power spectrum for a cosine output (as shown in Figure 11) consists primarily of two relatively straight roll-offs from the center frequency, $f_o = 1/2T_o$. It would then seem necessary only to relate these roll-offs with the amplitude of the side time density on either side

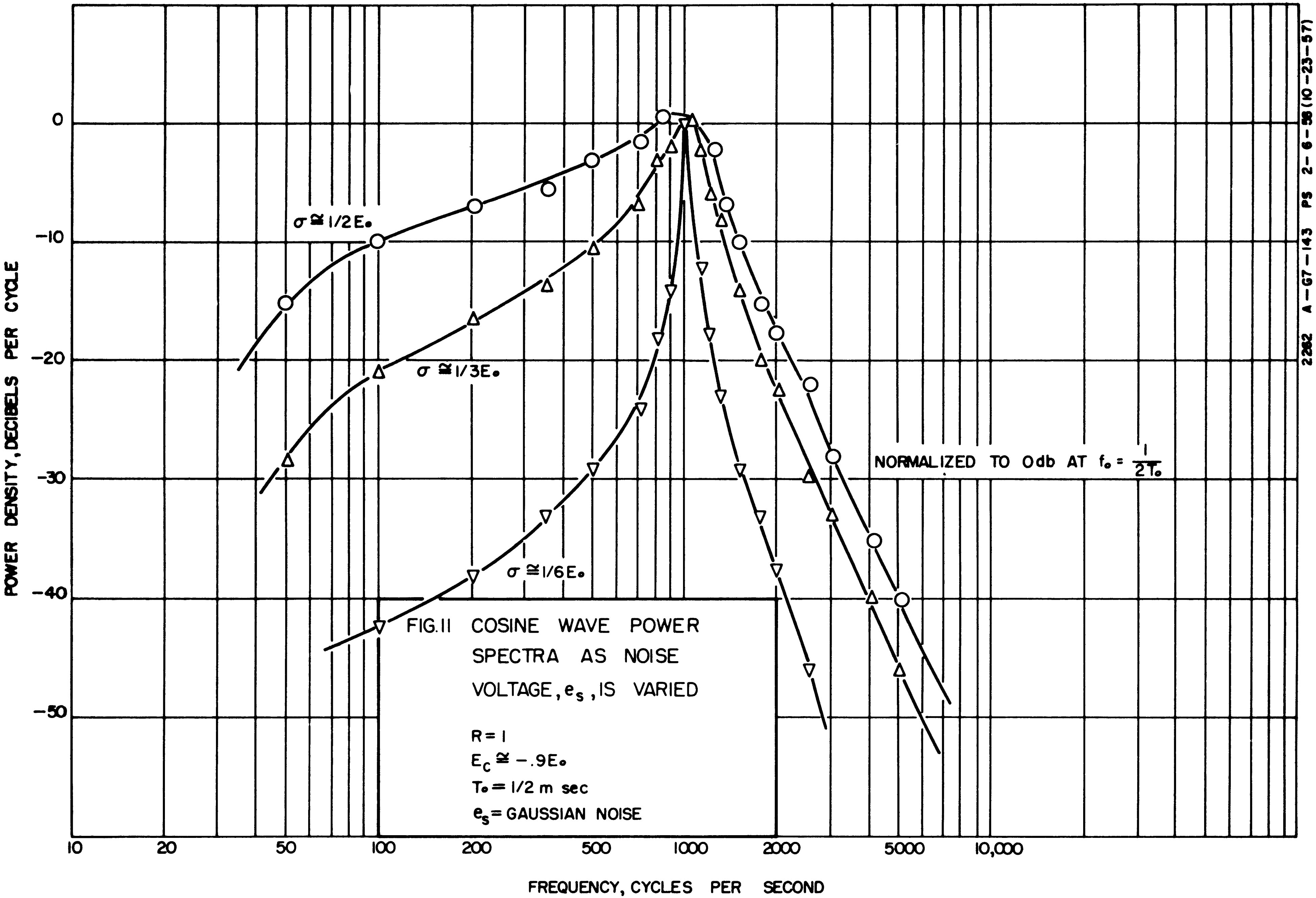


FIG. 11 COSINE WAVE POWER SPECTRA AS NOISE VOLTAGE, e_s, IS VARIED

of T_o. Unfortunately, the constructed laboratory generator was not built to provide easy measurement of the noise rms voltage, σ, the clipping level, E_c, and the shaper gain ratio, R. Also, the accuracies of the shaping, sampling, and hold circuits are not sufficient to guarantee a high degree of reproducibility of settings. Approximate values are stated, however, on the power spectrum measurements shown.

Figure 12 shows the effect on the output power spectrum when the shaper gain ratio is varied for large side times. It is noted that in varying the amount of long side times, the high side of the power spectrum is unaffected. Conversely, it can be experimentally shown that the high frequency side of the power spectrum only slightly affects the lower frequencies for the cosine wave output.

Figure 13 shows the power spectra for the three constant-amplitude random functions which the generator produces (i.e., the square wave, the triangle wave, and the cosine wave). Very little difference in the power spectra is noted between the triangle and cosine waves. Figure 14 is included to show the power spectrum obtained from a random cosine output when a triangle wave is used for the noise signal, e_s, the triangle having a rectangular amplitude probability density such as described in Figure 9. This curve is not easily related to the side time density, differing from the case using Guassian noise. Also included in Figure 14 is the power spectrum of a random cosine wave using a sine wave as the noise voltage.

4. GENERAL CIRCUIT DISCUSSION

4.1 Comments

The laboratory generator was designed and constructed as an experimental piece of equipment. Consequently, there are many potentiometers built in for

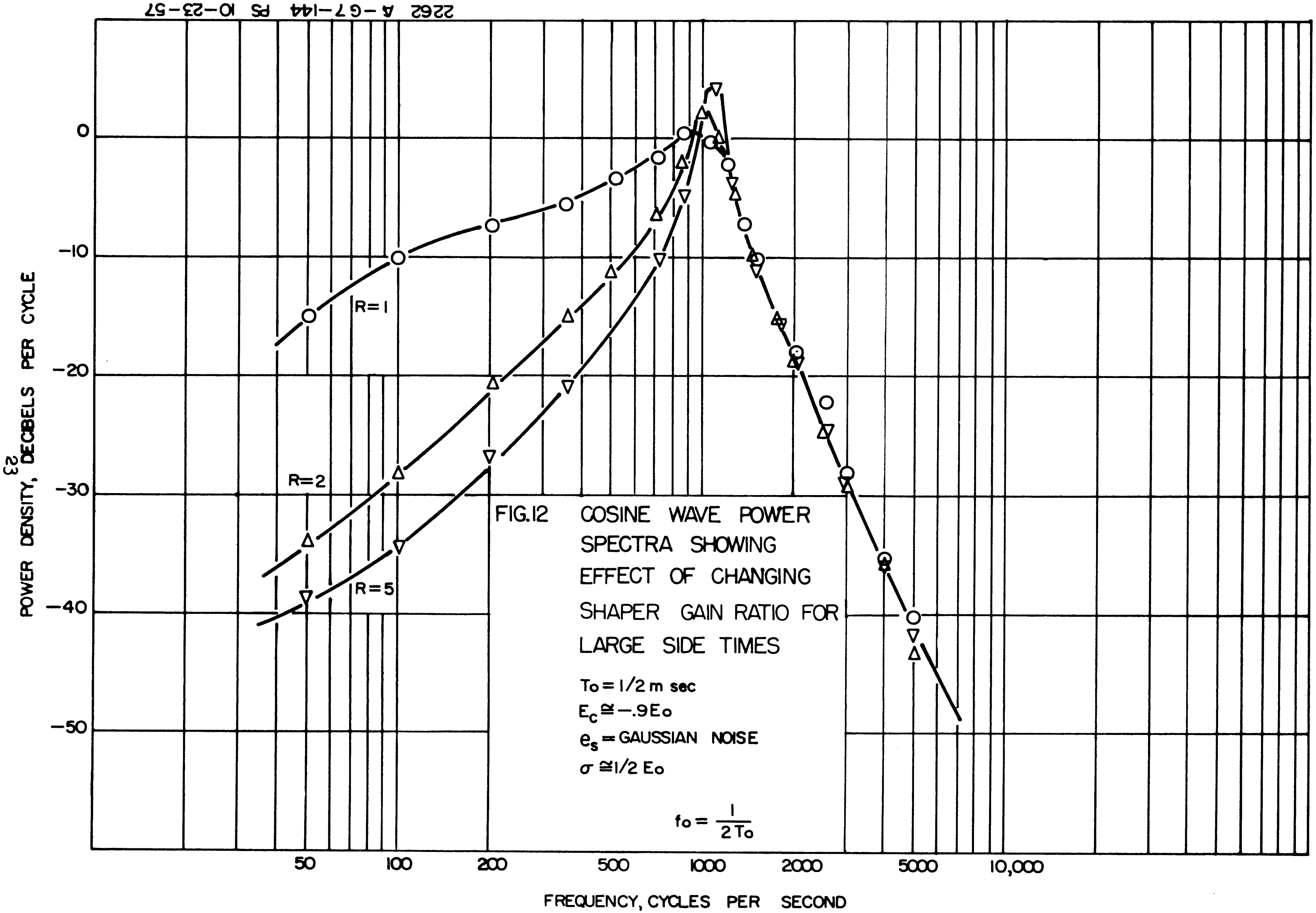


FIG.12 COSINE WAVE POWER SPECTRA SHOWING EFFECT OF CHANGING SHAPER GAIN RATIO FOR LARGE SIDE TIMES

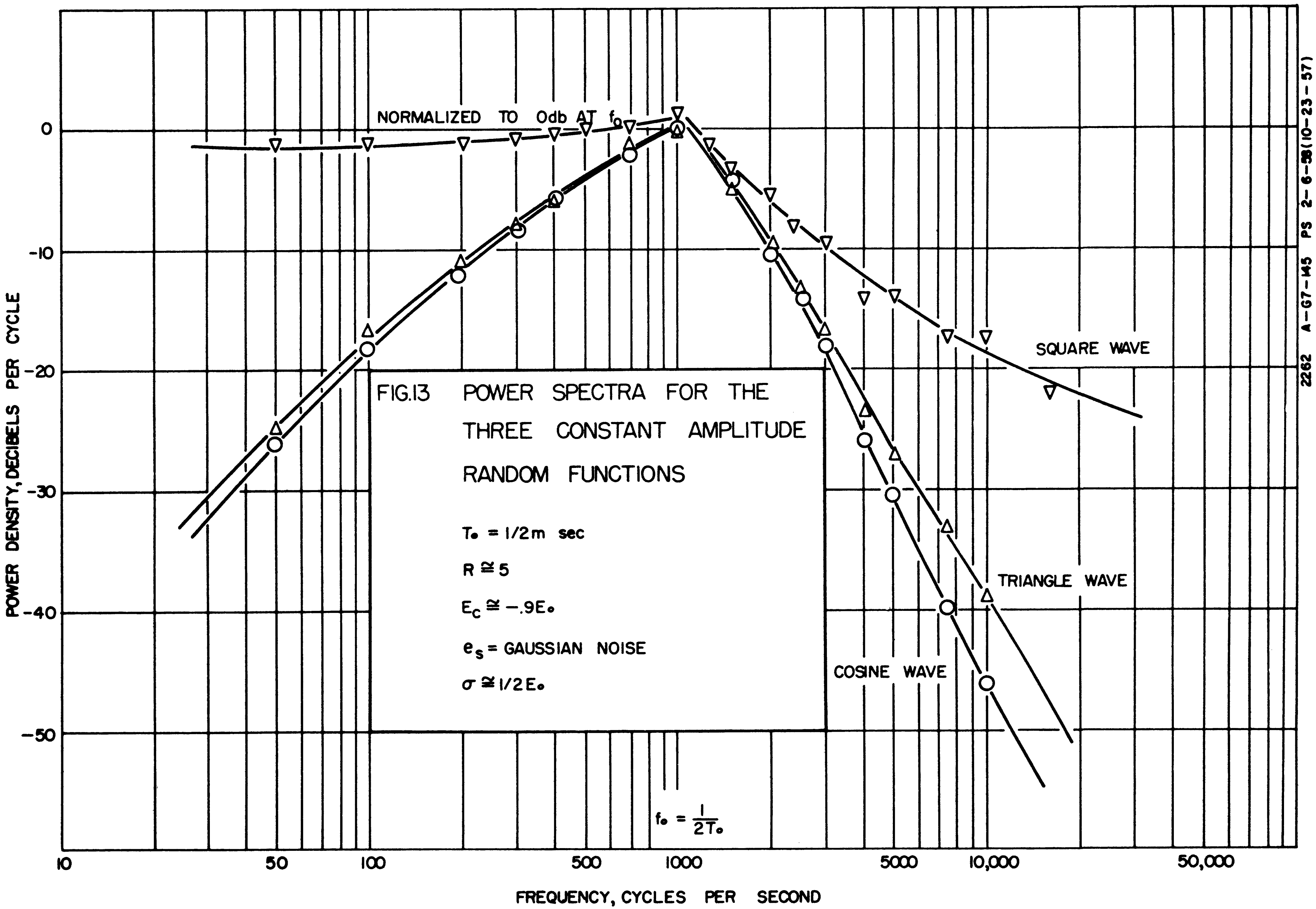


FIG.13 POWER SPECTRA FOR THE THREE CONSTANT AMPLITUDE RANDOM FUNCTIONS

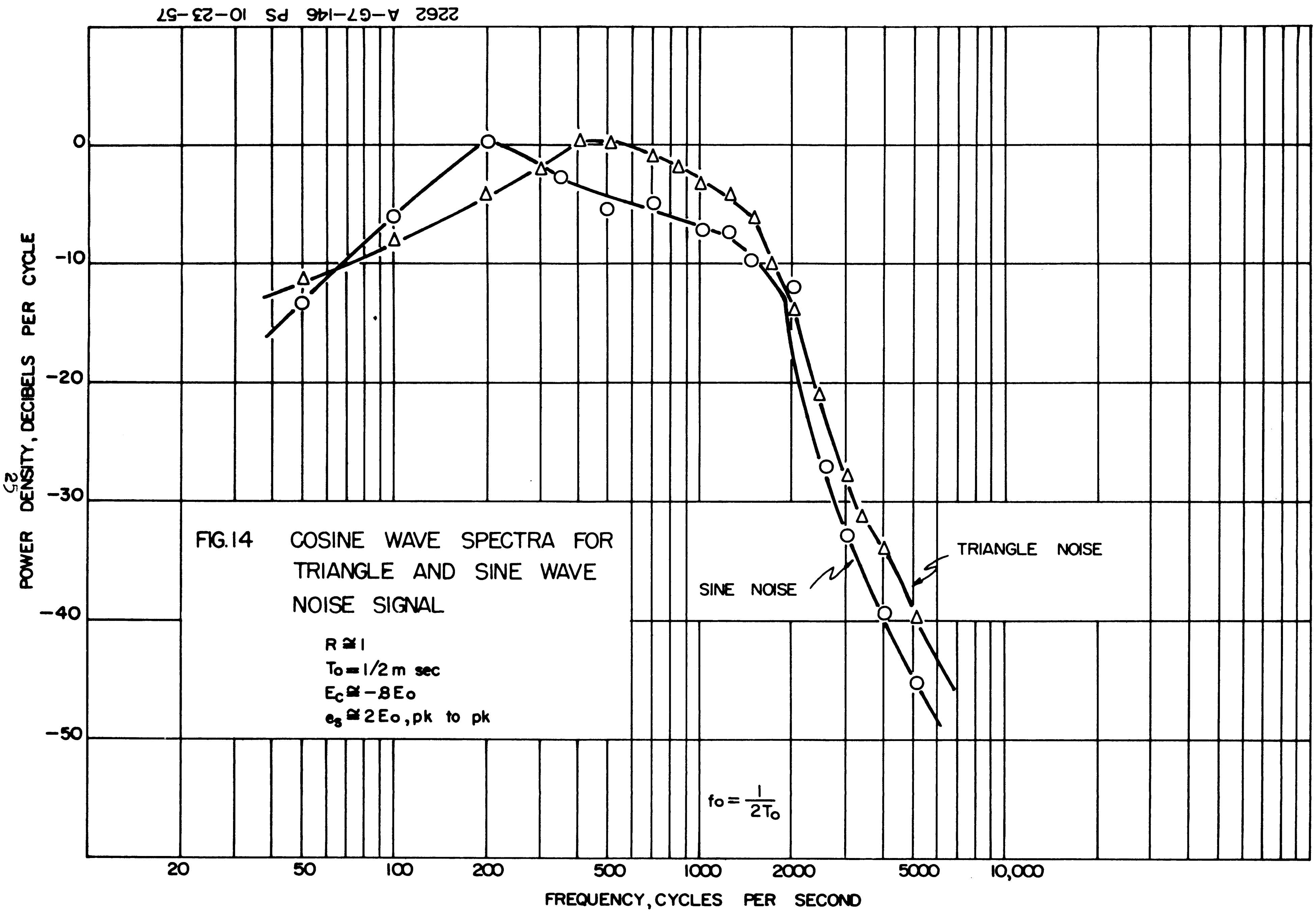


FIG. 14 COSINE WAVE SPECTRA FOR TRIANGLE AND SINE WAVE NOISE SIGNAL

adjusting parameters in the various parts of the circuit, most of the controls being either for gain or for DC bias. Adjustment and calibration procedures are given in Appendix B.

Perhaps the major problem encountered in the design and construction of the constant amplitude random function generator was insuring that the input to the integrator reversed polarity at the end of each side. In the circuit developed, this was done, as mentioned in Section 3.2.2, in the noise shaping circuit by the clipping diodes. Since the noise waveform in each channel of the shaping circuit is unbalanced, the shaping, sampling and hold circuitry must be direct-coupled in order to preserve the DC voltage level corresponding to the average value of the noise voltage before shaping. This requires that the two channels hold their DC potentials, with respect to each other, or the clipping levels, E_c, will vary; the tolerances on E_c are fairly small if side times as large or greater than 10 T_o are desired. Another problem in feeding the generator input properly is whether or not to capacitor couple the output of the hold circuit to the integrator input. A large capacitor was used in the circuit, but it is suspected that the small currents flowing through this coupling capacitor are sufficient to disturb the capacitor potential, since these currents are random.

A recommendation would be that the clipping level, E_c, be accomplished at the integrator input (see Appendix C), so that the circuit could more effectively guarantee reversal of polarity after each side. If the application of this function generator does not demand that the clipping level, E_c, be more nearly equal in amplitude to E_o than about -0.8 E_o, then the present circuit will operate reliably; in other words, this generator will easily produce side times five times as long as T_o (the center side time), but the probability that the integrator input polarity will not reverse after every side increases as one attempts to set the clipping level closer to $-E_o$ and obtain longer side times in the output. When the input polarity does not reverse after a side, the output will pass out of the constant peak-to-peak region and take about 0.1 second before the circuit resumes proper operation.

4.2 Circuit Diagram

Figure 19 shows the circuit of the laboratory model of the constant-amplitude random function generator. Figure 20 shows the cosine shaper and power supply. A general discussion of the circuit operation is given in more detail in Appendix A, while a calibration procedure is outlined in Appendix B.

4.3 Oscillographs of Output Wave

Several photographs of an oscilloscope displaying the output waveshape vs. time are included, each one a time exposure of many sweeps of the oscilloscope with each sweep beginning at a peak of the wave. Figure 16 shows the three output modes running at steady state; that is, without any noise voltage. Figure 17a shows three views of the triangle wave using different sweep rates on the oscilloscope; Figure 17 b shows three similar views of the cosine wave. On these photographs the side time distributions can be observed with respect to statistical variations in the choices of side times, the wide range of chosen side times, and the apparently continuous representation of side times when a large number of oscilloscope traces are recorded. Figure 17c and 17d differ from the above in that fewer traces are pictured in order to show statistical variations; all of these samples are at the same oscilloscope sweep speed.

Figure 15 shows the laboratory model with self-contained, regulated power supply.

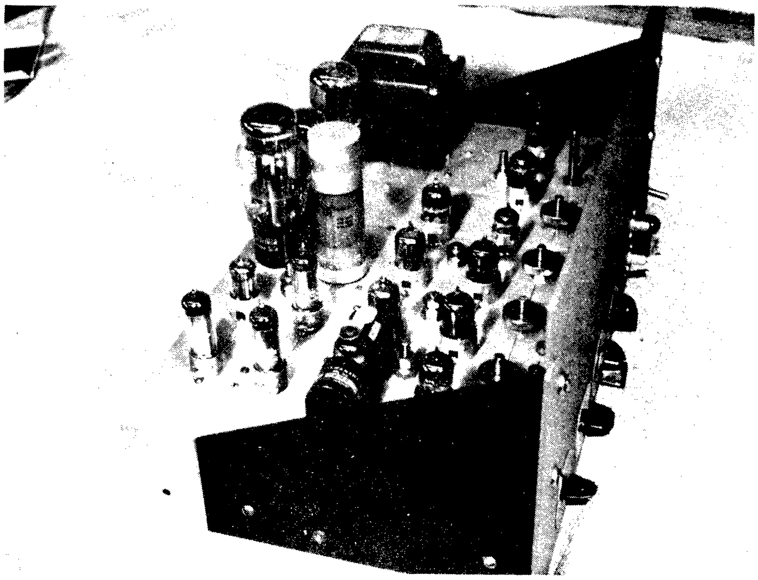
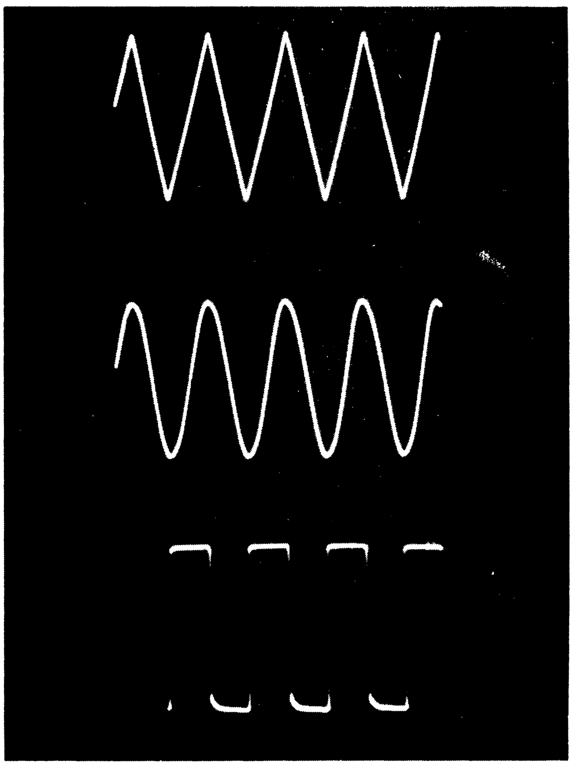


FIG. 15 PHOTOGRAPH OF LABORATORY GENERATOR

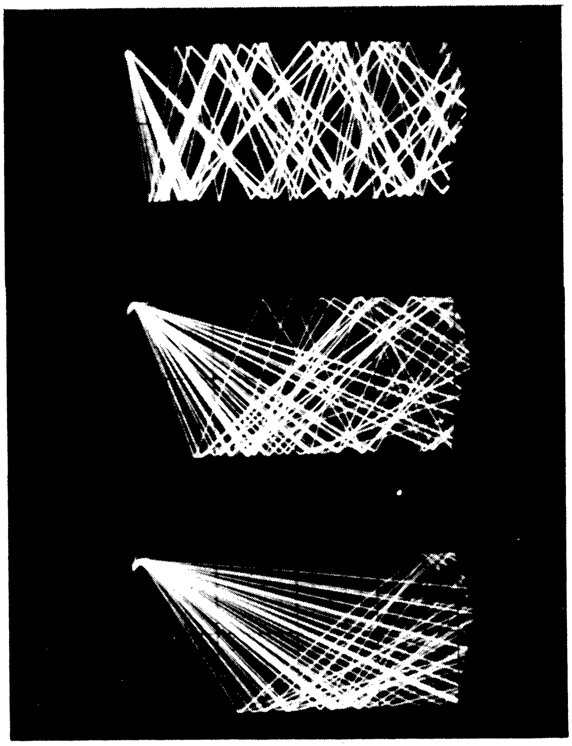


(a) TRIANGLE

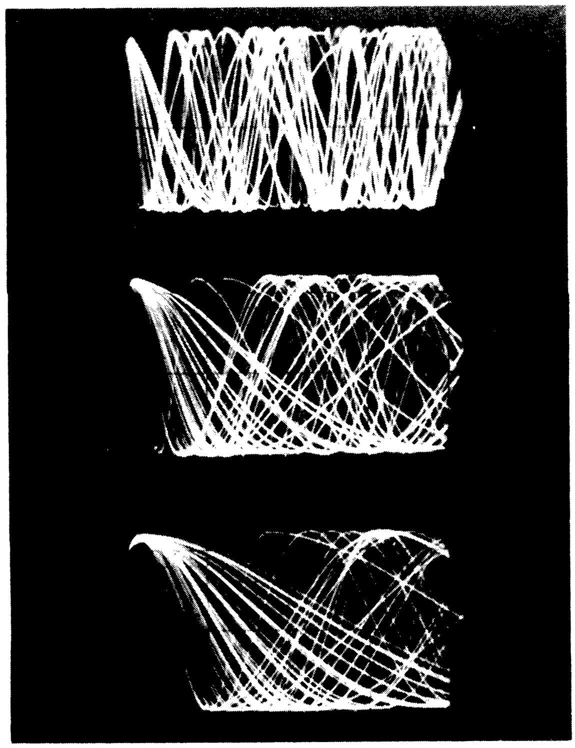
(b) COSINE

(c) SQUARE

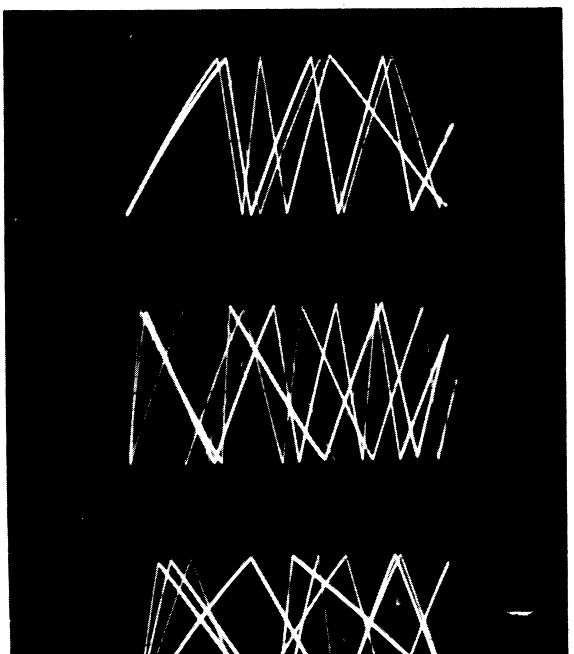
FIG. 16 THREE OUTPUT MODES AT STEADY STATE



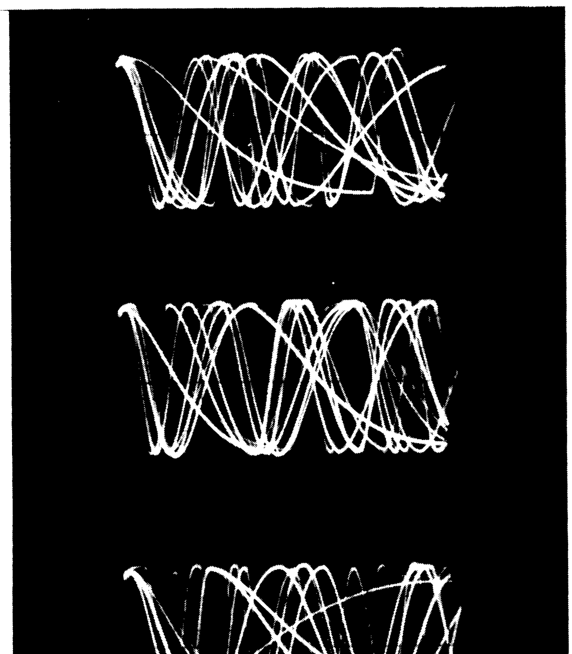
(a)
TRIANGLE



(b)
COSINE



(c)
TRIANGLE



(d)
COSINE

FIG. 17 RANDOM TRIANGLE AND COSINE OSCILLOGRAPHS

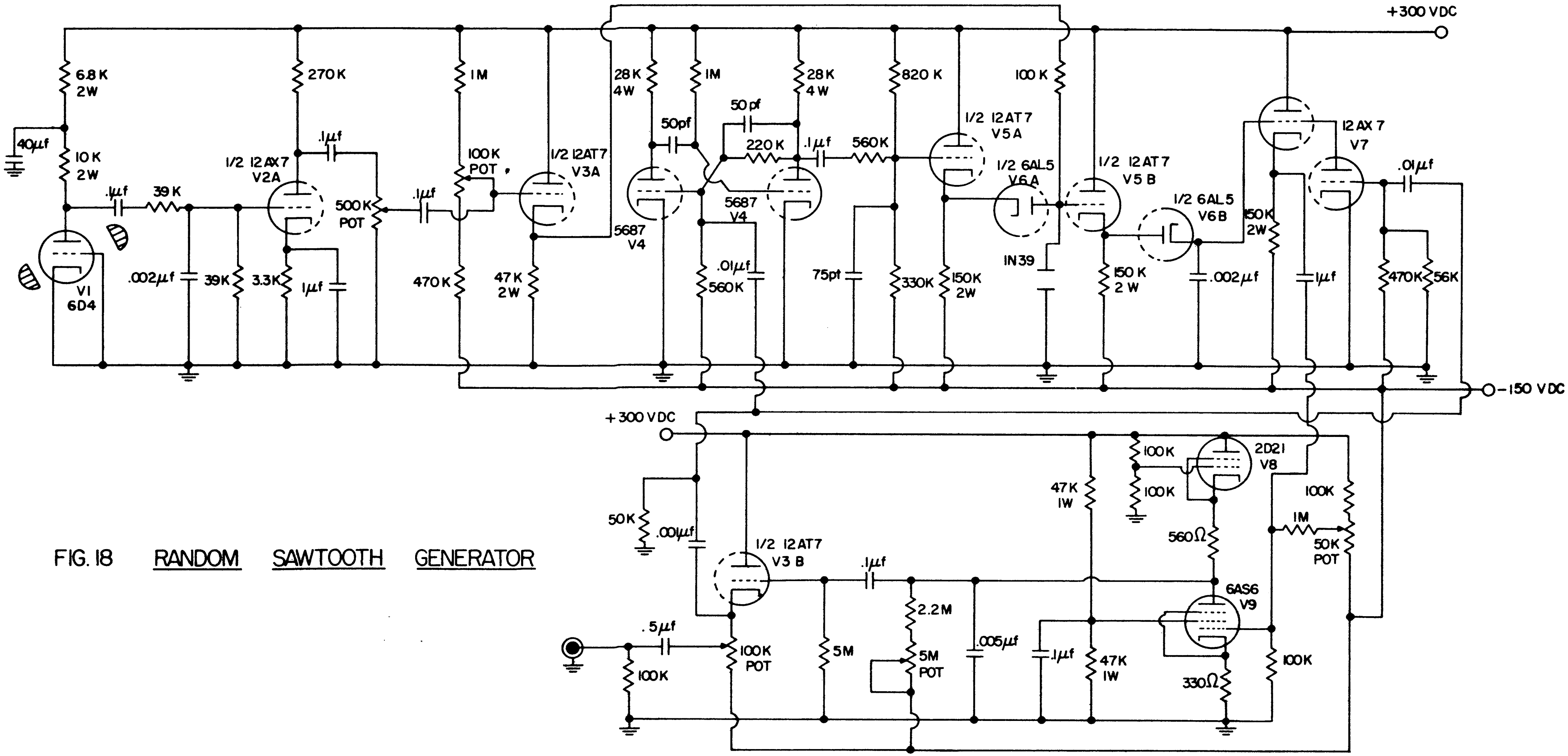
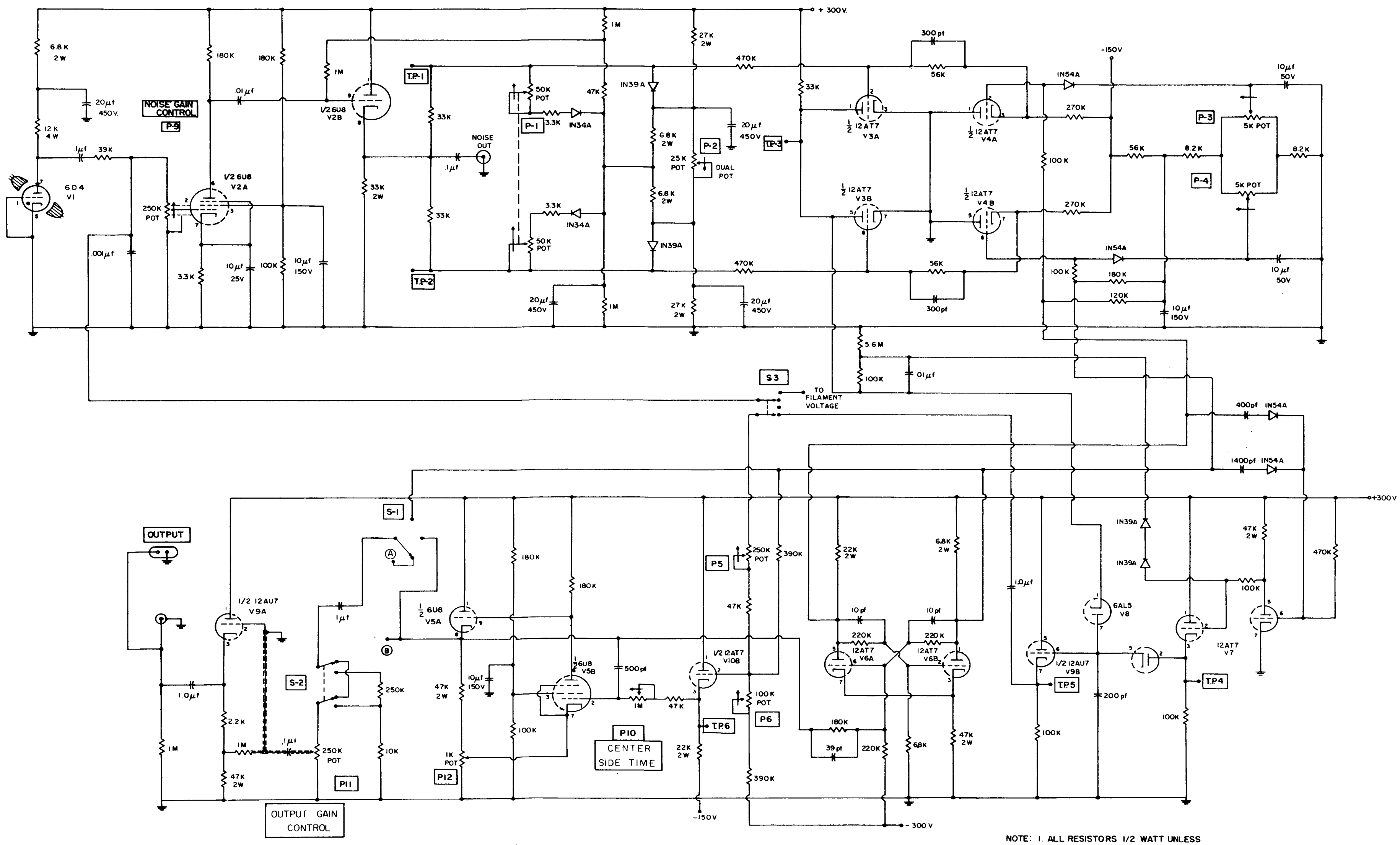


FIG. 18 <u>RANDOM</u> <u>SAWTOOTH</u> <u>GENERATOR</u>

2262 D-CM-13 REG 8-1-57

FIG. 19 CONSTANT AMPLITUDE RANDOM FUNCTION GENERATOR



NOTE: 1. ALL RESISTORS 1/2 WATT UNLESS OTHERWISE SPECIFIED.

2262 D-CM-13 REG 8-1-57

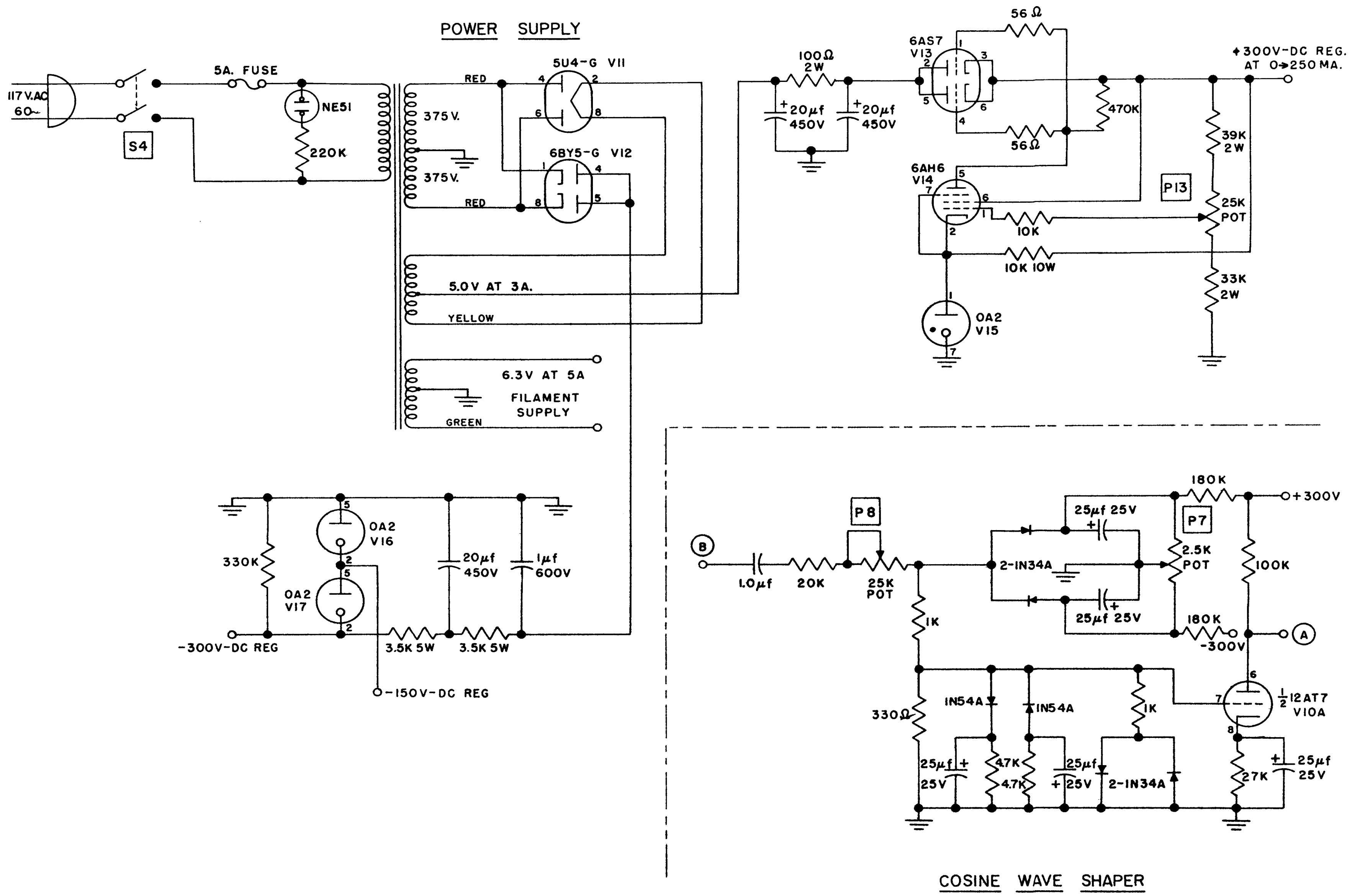


FIG. 20. GENERATOR COSINE SHAPER AND POWER SUPPLY

2262 C-CM-12 REG 8-1-57

APPENDIX A

DETAILED CIRCUIT OPERATION OF THE FUNCTION GENERATOR

The noise source, e_s, is a 6D4 gas triode (V1) operating as a diode in a magnetic field (see circuit of Figures 19 and 20). In order that the instantaneous noise voltage remain relatively constant during the 10 microsecond sampling gate, the noise is bandwidth-limited by a single RC lowpass filter with a time constant of about 40 microseconds. That is,

$$BW = \frac{10^6}{2\pi \times 40} = 4000 \text{ cps}$$

The filtered noise is then fed through a gain control (P-9) and amplified (V2) before being put into the dual shaping circuit.

The dual shaping circuit consists of four back-biased diodes. Two diodes (1N39A) effect the hard clipping of the noise, one diode in each channel, while a potentiometer (P-2) controls the bias on these diodes and therefore, the clipping level, E_c. A second pair of diodes (1N34A), each in series with a variable resistance, attenuates the instantaneous signals of the same polarity, in each channel, as those being clipped. A dual potentiometer (P-1) controls the amount of attenuation, thus controlling the shaper gain ratio.

The dual sampling circuit is made as follows: Two parallel triodes (V3) are operating with a common plate resistor in a normally cut-off state. The shaped noise voltage is direct-coupled to the grids of the triodes, but the grid bias is controlled by two cathode followers (V4). When a sample is taken, a 10 microsecond pulse is applied to the grid of one of the cathode followers, thus driving the corresponding grid of a sampling triode upward by a fixed amount plus an amount determined by the noise signal. With this arrangement, then, the two shaped noise signals are alternately sampled and fed out at the same point. When the grid biases are properly adjusted (P-3, P-4) on the sampling triodes, the plate (TP-3) will have 100-volt, 10 microsecond negative pulses appearing.

The voltage on the negative peaks of the sample is fed to the holding capacitor (200 pf) through a diode (V8), while another diode and pulse generator (V7, V8) sweep the holding capacitor voltage up to meet the level of the new sample, if the previously held sample was below the new sample. The trigger pulses for both the sampling tubes and the holding capacitor sweeping circuit are obtained by differentiating the two square wave outputs of the amplitude discriminator.

The amplitude discriminator circuit (V6) is a bi-stable multivibrator with a large amount of hysteresis, such that if the input is direct-coupled the multivibrator will flip into one state when the input rises to some specified value, and flop into the other state when the input decays to a second specified value. In this manner, the output of the integrator is contained within constant peak-to-peak limits, since it is the square wave output, E_o, of the amplitude discriminator circuit which is being integrated. (Of the two out-of-phase square waves available, only one is of the proper phase to be fed to the integrator. Similarly, proper arrangement of the pulses triggering the dual sampler is necessary).

A potentiometer (P-5) controls the effective amplitude of the sampled and held noise compared to the constant amplitude square wave, E_o, as the two signals are added and power amplified by a cathode follower (V10B). The center side time, T_o, is controlled by varying the resistor (P-10) at the integrator input. If a large change in T_o is desired, the feedback capacitor (500 pf) should be changed. The minimum side time this circuit will generate is about 50×10^{-6} seconds, it being limited by responses of the amplifiers.

The integrator operational amplifier (V-5) contains an internal positive feedback loop adjusted by a potentiometer (P-12) until a minimum voltage appears at the input grid of the integrator. Two front panel switches are provided. The first (S-1) is a function selector, giving either the triangle, cosine, or square wave mode at the output terminals. The second switch (S-2) is an output attenuator, the upper position giving a maximum output of about 100 volts, peak-to-peak, while the lower switch position yields about 4 volts. A potentiometer (P-11) continuously controls the output level.

The cosine shaping circuit consists of biased diodes arranged to attenuate gradually the peaks of a fixed amplitude triangle wave. Adjustment of the circuit is made by a potentiometer (P-8) controlling the input triangle amplitude and a potentiometer (P-7) adjusting the level of hard clipping at the waveshape peaks.

APPENDIX B

CALIBRATION AND ADJUSTMENT

The following procedure is suggested for making necessary potentiometer adjustments on the laboratory generator while setting desired parameters of operation.

1. Allow a 10 minute warm-up period. Rotate P-5 full counter clockwise to reduce the gain of the random signal, E_n, at the integrator input. Adjust noise gain control, P-9, to maximum and observe the noise voltage on an oscilloscope at TP-1 (test point).

2. With P-1, the shaper gain ratio control, and P-2, the clipping level control, set such that no shaping of the noise voltage is preeent, measure the rms voltage of the noise at TP-1. Adjust P-1 until the pattern on the oscilloscope shows the desired gain ratio. This is done because P-1 attenuates those noise voltages, in each channel, which produce long side times; when P-1 is used to change the shaper gain ratio, then P-2 must be adjusted also to maintain the same clipping level relative to the long-side-time producing voltages (see Figure 8). Check TP-2 on the oscilloscope to insure that the pattern is symmetrical with respect to that seen at TP-1. The shaper gain ratio and the clipping level relative to the noise voltage is now known.

3. Reduce P-9 to zero, and observing TP-3 adjust the DC biases for the sampling circuit, P-3 and P-4, such that the negative peaks of the samples seen at TP-3 are approximately +200 volts.

4. Adjust P-3 until no AC output is observed at the output of the hold circuit, TP-5. This completes the balancing of the dual sampling circuit.

5. Adjust the integrator input bias, P-6 such that the output waveform negative and positive going side times are equal.

6. Adjust P-10 to the desired center frequency, $f_o = 1/2T_o$.

7. Set the noise gain control, P-9, to maximum, and rotate P-5 clockwise until the gain of the random signal at the integrator input is such to cause deterioration of the output signal due to failure of the total integrator input to reverse polarity after each side. Turn P-5 slightly counter-clockwise until proper circuit operation is obtained. The clipping level is now approximately determined to be -.8 to -.9 times the constant amplitude square wave, E_o, and thus the amplitude relation between the different signals in the generator are known, and the triangle and square wave outputs are operating.

8. To adjust the triangle to cosine shaper, reduce P-9 to zero and adjust P-7 for symmetry and P-8 for general cosine similarity while observing the cosine wave output on an oscilloscope.

APPENDIX C

SUGGESTIONS FOR A TRANSISTORIZED GENERATOR

Since the construction of the random function generator, the author has observed some advantages that transistors offer over vacuum-tubes in implementing the circuit ideas of this generator. Some qualitative suggestions for transistorized circuitry will briefly be shown in this Appendix, along with the operational advantages which can be achieved. In general, the use of complementary symmetry with transistors enables the design of circuits balanced about ground potential in a push-pull, class B fashion.

An integrator is easily made by connecting a capacitor to the collectors of two transistors in parallel, one a pnp and the other an npn (as shown in Figure 21). In this configuration, one of the high-output impedance transistors will feed a current into the capacitor for integration of signals of one polarity, while the other transistor will be used to draw a current from the capacitor for integration of signals of the opposite polarity. The most attractive feature of this integrator for use in the function generator is obtained from the nature of the transistors and the integrator input arrangement, which is such that the input polarity can be reversed after each side with certainty, no matter what amplitude of noise signal is being integrated (see Section 4.1).

The input to the emitters is the square wave output of an amplitude discriminator sensing the integrator output, E(t) (see Figure 4). When the square wave is positive, the noise sample E_{n+} will determine the constant current flowing in the capacitor, while E_{n-} should be equal to $+E_o$ during this time in order for

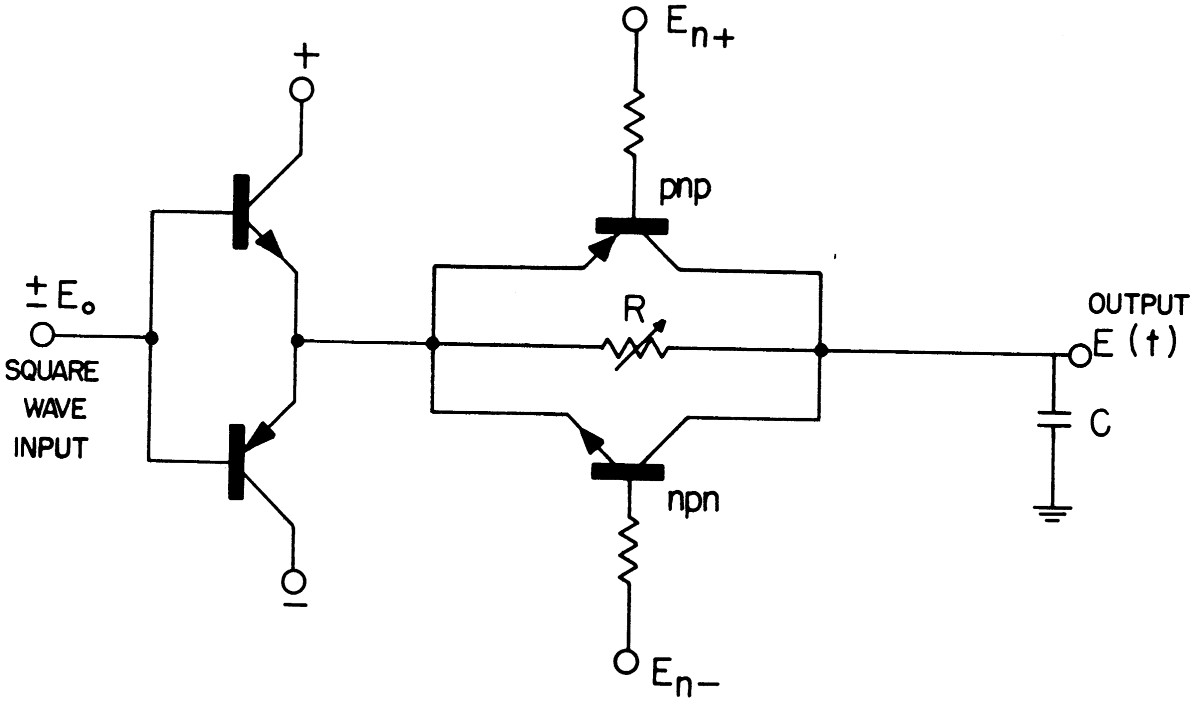


FIG. 21 TRANSISTORIZED INTEGRATOR

the current through the npn transistor to be negligible and to have the same polarity as the current through the pnp transistor. Should the amplifier providing the noise sample E_{n+} present a sample more positive than E_o, the transistor becomes a perfect clipper in that such a noise sample cannot possibly deliver a current of the wrong polarity to the integrating capacitor. Conversely, during integration of $-E_o$ and its corresponding noise sample E_{n-}, E_{n+} should always be equal to $-E_o$. The resistor R determines the longest side time that can be produced. Note that the output peak amplitude will necessarily be less than E_o.

In view of the previous discussion, the noise sampling circuit need only to provide a shaped noise signal with no clipping, thus eliminating any exacting requirements on the accuracy of the sampling circuit and its amplifiers. It is suggested that use of complementary symmetry again be employed in the construction of two separate sampling circuits, one providing E_{n-} alternately with the second circuit which supplies E_{n+}. Since the useful ranges of output voltage of these two noise samplers differ only in polarity (see Figure 6), the two sampling circuits would especially be suited by complementary design; that is, the npn and pnp transistors in one are replaced by pnp and npn transistors in the other, respectively, and the power polarities are opposite.

The amplitude discriminator, shaping circuit, noise source, and other parts of the circuit can be transistorized without any special changes from the vacuum-tube design. Thus it is believed that a transistorized constant-amplitude random function generator could be built with a much higher degree of reliability than was accomplished in the vacuum-tube circuit of this report.

DISTRIBUTION LIST

1 Copy	Document Room Stanford Electronic Laboratories Stanford University Stanford, California
1 Copy	Commanding General Army Electronic Proving Ground Fort Huachuca, Arizona Attn: Director, Electronic Warfare Department
1 Copy	Chief, Research and Development Division Office of the Chief Signal Officer Department of the Army Washington 25, D. C. Attn: SIGEB
1 Copy	Chief, Plans and Operations Division Office of the Chief Signal Officer Washington 25, D. C. Attn: SIGEW
1 Copy	Countermeasures Laboratory Gilfillan Brothers, Inc. 1815 Venice Blvd. Los Angeles 6, California
1 Copy	Commanding Officer White Sands Signal Corps Agency White Sands Proving Ground Las Cruces, New Mexico Attn: SIGWS-EW
1 Copy	Commanding Officer White Sands Signal Corps Agency White Sands Proving Ground Las Cruces, New Mexico Attn: SIGWS-FC
1 Copy	Commanding Officer Signals Corps Electronics Research Unit 9560th USASRU Mountain View, California
65 Copies	Transportation Officer, SCEL Evans Signal Laboratory Building No. 42, Belmar, New Jersey FOR - SCEL Accountable Officer Inspect at Destination File No. 22824-PH-54-91(1701)

1 Copy	H. H. Goode Engineering Research Institute University of Michigan Ann Arbor, Michigan
1 Copy	Document Room Willow Run Laboratories University of Michigan Willow Run, Michigan
12 Copies	Electronic Defense Group Project File University of Michigan Ann Arbor, Michigan
1 Copy	Engineering Research Institute Project File University of Michigan Ann Arbor, Michigan