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Research Study Pertaining to Low-Level Wind Structure

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OBJECTIVE

The object of the research is to analyze low-level wind structure as it pertains to dynamic wind loading.

ABSTRACT

TASK A: STATISTICAL DESIGNS

The problem of a statistical design is first described in its broad outlines. Statistical models of the wind field are then discussed briefly in terms of a sample space and a parameter space, in which a parameter point determines a probability distribution over the sample space. In analyzing the statistics of the wind field, it is pointed out that covariance functions which satisfy the wave equation may prove useful, since lagged correlations of wind components are at a maximum upwind. The Fourier transform of a specified covariance function is the spectral function σ which is expressed for the one-, two-, and three-dimensional cases. The covariance tensor is then analyzed briefly. The relation of the covariances between different wind components and the Reynolds stresses is brought out. The concept of a "ground ensemble" is introduced and its probable role in the creation of turbulence described. The prediction problem is illustrated by considering a specific covariance function, and finally the estimation problem is mentioned briefly.

An evaluation of certain existing wind data has been undertaken but not completed.

TASKS B, C, AND D

Work on wind-instrument analysis, data-reduction systems, and wind-tunnel studies has been deferred in order to mount an intensive attack during the summer of 1956 on the central problem of an optimal statistical design.

PURPOSE

The purpose of the research is to analyze, using the resources of mathematical physics, meteorology, aerodynamics, and statistics, the problem of the determination of the structure of the wind in the lower layers of the atmosphere as it pertains to dynamic wind loading of objects.

The research may be considered as consisting of four tasks, as follows:

Task A: To produce one or more statistical designs for field experiments which will reveal the wind-flow features which are significant for dynamic-loading problems.

Task B: To evaluate existing or possible wind-measuring instruments, such as anemometers, gustometers, and bivanes, to determine their suitability for field use in measuring the three-dimensional large-scale eddy structure of the atmosphere.

Task C: To recommend one or more systems for reduction to usable form of the data obtained by the sensing elements of the instruments.

Task D: To assess the suitability of the wind tunnel as a device for simulating eddy structure over specified terrain features.

PUBLICATIONS, LECTURES, REPORTS, AND CONFERENCES

There have been no publications or lectures during the reporting period.

Seven Monthly Reports have been submitted.

Conferences have been held as follows. On October 10 a conference was held at Princeton University on the various phases of the problem. In addition to those representatives of the Signal Corps, the following were present: Dr. John Tukey, Princeton University; Dr. Max A. Woodbury, George Washington University; Mr. Gerald C. Gill, Mr. Richard Legault, and Dr. E. Wendell Hewson, The University of Michigan. The main discussion was centered around the problem of statistical design, with consideration of instrumental factors.

Several other conferences with Dr. Max A. Woodbury and others have also been held at intervals.

FACTUAL DATA

TASK A: STATISTICAL DESIGNS by Max A. Woodbury

Introduction.—This study is intended to outline the steps necessary to provide a prediction of the dynamic wind loading on a moving structure. The particular problem to be considered here is the design and specification of the measuring system, or at least the specification of the criteria according to which the choices of instrumentation should be made.

The problem, as viewed in this report, is that of arriving at a specification of the possible (or probable) types of wind fields in the height range (above the ground) from 0 to 2000 ft. The time variation is considered to be of minor interest except and insofar as it affects the predictions of the dynamic-wind-loading effects. The spatial scale of variation in the wind velocity of interest in the study is to be limited to the range 15 to 1500 ft.

Statistical Models of the Wind Field.—The problem of specifying a model for the wind field is made considerably easier by the asumption that one desires to predict certain properties of the wind vector (u_1, u_2, u_3) at a specific time t only from data on u_1, u_2, and u_3 at times previous to t.

To provide an easily manageable statistical basis, let us introduce the notion of a sample space Φ and a parameter space Θ [1].* Thus for each point ϕ in the sample space Φ there corresponds a wind field, i.e., a specific vector function of (x_1, x_2, x_3, t) which determines the wind vector (u_1, u_2, u_3) at each time-space point. A parameter point θ in the parameter space Θ determines a probability distribution over Φ. That is, for any (measurable) subset Φ_0 of Φ, $P(\Phi_0/\theta)$ is defined and satisfies the usual laws of a probability distribution. [See, for example, A. N. Kolmogoroff's fundamental work for an exposition of these laws.]

Statistics of the Wind Field.—The structure of the wind fields will be studied later in terms of the sample space-parameter space framework. At the present, sufficient complications to engage our full attention will be available without considering the parameter space. We consider the wind vector $u = (u_1, u_2, u_3)$ as a function of the variables x_1, x_2, x_3, t, and ϕ. For simplicity we will consider here only $u(x, t, \phi)$, but suitable interpretations of x and u as vector quantities will allow generalization to three space dimensions. Consider

$$R(x, t) = \underset{\phi \text{ and } \Phi}{\text{Avg}} \left[u(x' + x, t' + t, \phi)\, u(x', t', \phi) \right]$$

*Numbers in brackets refer to the bibliography.

which results from an assumption of stationarity in space and time. In the atmosphere this will not be true in the vertical direction. A more realistic analysis will have to be gone into later since R will depend on both x_3 and $x_3 + x_3'$ on this account.

If the spatial pattern were a fixed array of wind vectors moving with a uniform pattern velocity, then one would find that R satisfied the wave equation with the appropriate velocity. It is possible that covariance functions R(x, t) which satisfy the wave equation will prove useful since it is known that lagged correlations of wind components are at a maximum upwind. That is, fluctuations from the mean wind appear to move with the mean wind and seem to be quite stable in character. As an example of such a function, consider

$$R(x,\ t) \;=\; R_0 \exp\left[-\beta\left|x - ut\right| - \gamma\left|t\right|\right] .$$

The Fourier transform of R(x, t) is the spectral function σ, a measure of the distribution of kinetic energy in various frequency and eddy size ranges. For the example at hand one has (for one space dimension),

$$\frac{4\beta\gamma}{(\beta^2 + \xi^2)\left[\gamma^2 + (\tau + u \cdot \xi)^2\right]} \;=\; \sigma_1\ (\xi,\ \tau) \quad .$$

For two dimensions,

$$\sigma_2 \;=\; \frac{1}{\beta}\,\sigma_1\ (\xi,\ \tau)$$

where ξ is interpreted as a 2-vector in the two-dimensional case.

For three dimensions,

$$\sigma_3 \;=\; \frac{2\pi}{\beta}\,\sigma_2 \;=\; \frac{2\pi}{\beta^2}\,\sigma_1$$

where ξ is interpreted as a 3-vector, etc.

Let us return now to the analysis of the covariance tensor. In line with the resolution of the wind components into mean wind and fluctuations is (using $u = u_1$, $v = u_2$, $w = u_3$, etc).

$$u \;=\; \bar{u} + u' \ ,\quad v \;=\; \bar{v} + v' \ ,\quad w \;=\; \bar{w} + w' \quad .$$

Here we note that the resolution is over a phase average given certain conditions such as

$$\mu(t_1 - t_2) \;=\; \int_{t_1}^{t_2} u(x_0,\ y_0,\ z_0,\ t;\omega)\ dt$$

$$0 = \int_{t_1}^{t_2} v(x_o, u_o, z_o, t;\omega)\, dt \quad .$$

[We assume that under practically all conditions $\bar{w} = 0$.] The above conditions are those given by the statement that the mean wind over the time interval (t_1, t_2) is $(\mu\ , 0, 0)$.

The Reynolds stresses are, of course, closely related to the covariances between different wind components at various times, so special attention is directed at the matrix

$$R = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{bmatrix}$$

at a single point which is assumed to be a function of height alone. The term $\overline{u'w'}$ is known to be negative for most cases in the range near the ground (see [2]), while $\overline{w'w'}$ generally increases markedly with height (it must be zero at the surface), and while $\overline{v'v'}$ seems to decrease with height. That which will be said now is to be considered as heuristic only. It may be modified later after further investigation.

It seems likely that turbulence is created by shearing stresses and that this explains its prevalence near the ground. The roughness of the ground plays a part, so it will most likely be necessary to consider an ensemble (phase space) of ground surfaces as playing a role in the statistics of turbulence. The "ground ensemble" will be presumably, a two-dimensional (stationary?) stochastic process and its spectrum together with the mean wind will undoubtedly provide the means for calculating the rate of introduction of turbulent eddies at low levels which will spread (by a diffusion-like process) in space and time and by a more general transformation to higher time frequencies and smaller-sized eddies.

<u>The Prediction Problem</u>.—At this stage, little can be said but we shall illustrate the problem by considering the covariance function used as an example above.

$$R(x, t) = \exp\left[-\beta\left|x - ut\right| - \gamma\left|t\right|\right]$$

If one could choose only one place for predicting the "wind" at a given place (one-dimensional case) at a specific time from data at a time t units previously, one would choose a point whose spatial separation x is such that $x - ut = 0$ or such that $x = 0$, depending on whether $|\beta ut|$ or $|\gamma t|$ is the smaller. When we consider two points the problem is already more complex, even for this simple case. It can be computed numerically, of course, in the general case.

A related problem of optimal prediction using a modification of the Wiener techniques has been considered in [3]. The predictability (squared multiple correlation) of the effect of gustiness on a moving structure is indicated as quite low. It is probable that, with the data at hand, equivalent results could be obtained with less computation by using direct correlation methods. It may be worthwhile to attempt this.

The Estimation Problem.—It is clear that wind-component correlations change over time and that one is dealing with essentially different statistical populations, e.g., contrast day and night, winter and summer conditions, etc. There is a problem of estimating the coefficients (correlation, regression, etc.) from data at the site and time of use. The computations may be very heavy—perhaps more than can be done in the field. Further development of these themes will have to await further work. They can only be touched on in this brief report which is written to indicate, in part, the present trend in thinking on this project.

REFERENCES

1. Blackwell, D. and Girshick, A., Theory of Games and Statistical Decisions. New York: John Wiley and Sons, 1954.

2. Sutton, O. G., Atmospheric Turbulence. London: Methuen and Company, Ltd., 1955.

3. Powers, J. B. and Kerr, R. E., Jr., "Numerical Determination of Optimum Filter Characteristics," North American Weather Instruments, Inc., Seventh Quarterly Progress Report (October 21, 1954 - January 20, 1955), U. S. Army Signal Corps Project No. 1052A.

TASK A, IN CONTINUATION

Since existing wind data of a suitable type may well provide important information needed for the establishment of an effective statistical design, an evaluation of certain wind data and analyses has been proceeding. These are the wind-structure measurements and statistical analyses conducted by Professor R. H. Sherlock and associates (see References 1 - 6 in First Quarterly Progress Report).

The purpose of this evaluation is to determine, by means of an analysis of such procedures as those used in calibrating the pressure-plate anemometers, whether bias exists in the data which might lead to erroneous statistical conclusions. This study has not yet been completed.

TASK B: WIND INSTRUMENTS

Although further consideration has been given to the problem of instrumentation for the measurement of large-scale eddies, active prosecution of this phase has been deferred until development of the statistical design provides additional guidance on the required response characteristics of such instruments.

TASK C: DATA-REDUCTION SYSTEMS
TASK D: WIND-TUNNEL STUDIES

Development of those phases has been postponed until Task A has been carried further.

CONCLUSIONS

The problem has been outlined in its fundamental aspects, but further study of statistical designs appropriate for the specification of low-level wind structures pertinent to dynamic wind loading is essential before a solution of the problem can be achieved.

PROGRAM FOR NEXT INTERVAL

It is planned to make an intensive attack on the central problem involved, that of an optimal statistical design, during the summer of 1956. Additional personnel skilled in statistical analysis will be employed, to work under the supervision of Dr. Max A. Woodbury. The basic difficulty in this problem is the lack of fundamental knowledge of the physical nature of atmospheric turbulence. Despite this difficulty, however, it is believed that such an intensive investigation may well lead to the development of an adequate statistical design.

IDENTIFICATION OF PERSONNEL

The following personnel have been employed during the reporting period: Professor E. Wendell Hewson, 288 hours; Gerald C. Gill, 84 hours; Harold W. Baynton, 291 hours; for consultation, Professor John W. Tukey of Princeton University, 8 hours, and Dr. Max A. Woodbury of George Washingtin University, 54 hours.