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Third Progress Report

RESEARCH STUDY PERTAINING TO LOW-LEVEL WIND STRUCTURE

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LIST OF NOTATIONS

$\Theta_n(s)$	The deviation of the rocket trajectory from the ideal path at a point at a distance s-p from the end of the launcher in the vertical direction.
$\Theta_b(s)$	Same for the horizontal direction.
s	Taken to be the point at which the deviation is measured, often the point at which "burnout" is assumed.
p	Effective launcher length.
n(q)	A unit vector (the normal)perpendicular to the trajectory lying in a vertical plane at the point a distance of q-p units from the end of the launcher.
b(q)	A unit vector (the binormal) perpendicular to the path and the normal vector. It lies in a horizontal plane.
	Denotes the transpose of a vector or a matrix.
u(q)	The column vector of wind components at the point q.
$\overline{}$	The mean value of the indicated random variable.
*	The complex conjugate (transposed for a matrix) of the indicated expression.
i	$\sqrt{-1}$
G_n	$G_n(s,q)$ the influence function of the wind at q on $\Theta_n(s)$.
G_b	Similarly defined for $\Theta_b(s)$.
R	The correlation tensor of turbulence (defined only for homogeneous turbulence).
Φ	The spectrum tensor of (homogeneous) turbulence.
$\vec{k}$	The wave number triple (k_1, k_2, k_3).
k and $\lvert\vec{k}\rvert$	$(k_1^2 + k_2^2 + k_3^2)^{1/2}$
k_1, k_2, k_3	The wave numbers (reciprocal wavelength) of turbulence.

LIST OF NOTATIONS
(Concluded)

δ_{ij}	Kroneker delta, 1 if i = j, 0 if 1 ≠ j.
$\eta'_n(k)$	$\int_p^s G_n(s,q)\ n'(q)\ e^{-ir(q)k}\ dq$
E(k)	The energy function for isotropic turbulence. The energy in a shell in wave number space on a sphere of radius k surrounding the origin. See Batchelor [5].
W(k)	The admittance function of the rocket dispersion power on turbulent energy.
μ_m	Absolute moment of order m.
α_m	Normalized moment of order m.
w	Vector of predictors for wind.
$\Lambda(k)$	$\frac{1}{2\pi} \int_p^s \overline{u(q)\ w'}\ e^{-ikq}\ dq$. Cross spectrum vector of predicting variables.
$\lambda'_n(s)$	$\int_{-\infty}^{\infty} \Gamma_n(s,k)\ \Lambda(k)\ dk$
$\hat{u}$	Predicted wind.
$\hat{\theta}_n$	Predicted mil error in the direction of the normal to the trajectory.
f(r)	Longitudinal correlation function of isotropic turbulence.
g(r)	$f(r) + \frac{r}{2} f'(r)$. Lateral correlation function of isotropic turbulence.

ABSTRACT

TASK A: STATISTICAL DESIGNS

The effect of homogeneous and homogeneous isotropic turbulence on the dispersion of an unguided ballistic rocket at burnout is expressed as an integral, over the wave number domain, of a quadratic form involving the turbulence spectrum tensor (matrix). In addition, the reduction in dispersion possible by using a set of predicting elements in aiming the rocket launcher is computed for the same cases. The integrals involve only one real variable in the isotropic case.

TASKS B, C, AND D

Work on wind-instrument analysis, data-reduction systems, and wind-tunnel studies has been postponed until further progress has been made on the central problem of an optimal statistical design.

OBJECTIVE

The object of the research is to analyze low-level wind structure as it pertains to dynamic wind loading.

PURPOSE

The purpose of the research is to analyze, using the resources of mathematical physics, meteorology, aerodynamics, and statistics, the problem of the determination of the structure of the wind in the lower layers of the atmosphere as it pertains to dynamic wind loading of objects.

The research may be considered as consisting of four tasks, as follows:

Task A: To produce one or more statistical designs for field experiments which will reveal the wind-flow features which are significant for dynamic-loading problems.

Task B: To evaluate existing or possible wind-measuring instruments such as anemometers, gustometers, and bivanes, to determine their suitability for field use in measuring the three-dimensional large-scale structure of the atmosphere.

Task C: To recommend one or more systems for reduction to usable form of the data obtained by the sensing elements of the instruments.

Task D: To assess the suitability of the wind tunnel as a device for simulating eddy structure over specified terrain features.

PUBLICATIONS, LECTURES, REPORTS, AND CONFERENCES

There have been no publications or lectures during the reporting period.

Three Monthly Reports have been submitted.

Early in May a meeting was held in Washington, D. C., with the purpose of discussing the forthcoming summer conference at The University of Michigan, Ann Arbor, on low-level wind turbulence pertinent to dynamic wind loading. The following were in attendance: Ben Davidson, E. Wendell Hewson,

H. A. Panofsky, Irving A. Singer, and Max A. Woodbury. Detailed plans for the summer conference were made.

On May 18 and 19, further discussions on the summer program were held between Dr. Max A. Woodbury and Professor Hans A. Panofsky of Pennsylvania State University.

A good deal of time was devoted in May and June to completing detailed preparations for the July conference.

The conference was held as scheduled, with the following scientists participating.

Ben Davidson: Meteorology, New York University
Raymond J. Deland: Meteorology, Pennsylvania State University
A. Nelson Dingle: Meteorology, University of Michigan
Geoffrey Keller: Astronomy, Ohio State University
Vi-Cheng Liu: Aerodynamics, University of Michigan
Howard E. Reinhardt: Statistics, University of Michigan
Leo J. Tick: Statistics, New York University
Max A. Woodbury: Statistics, New York University
E. Wendell Hewson: Meteorology, University of Michigan.

The conference succeeded in making substantial progress with the problem. Some of the results of the analyses made during the conference are set forth below; further results will be presented in the Final Report.

FACTUAL DATA

TASK A: STATISTICAL DESIGNS, by Max A. Woodbury

Introduction.—The objectives of the study have been outlined and discussed in [1].*

The basic problem is the response of a rocket in the burning phase (between launcher and burnout) to impressed wind forces. Basic work on this problem has been done by S. M. Robinson [3] based on the now standard work of Rosser, Newton, and Gross [2]. In [4], Hunter, Shef, and Black discussed an extension of Robinson's work. The graph of a rocket response function (frequency domain) which they prepare seems to be a highly idealized version. A response function based on assumed characteristics (yaw wavelength, diameter, mass and thrust, time to burnout, distance from center of lift to center of gravity, etc.) of a rocket and Robinson's equations shows little resemblance

*Bracketed numbers refer to references, numbers in parentheses to equations.

to Hunter's graph. These computations will be presented in a later report. Since the equations presented by Robinson are complicated and not adapted for use as a response function, the general analytical form of the response of the rocket to homogeneous turbulence is not available except in particular cases of numerical calculation. The terms need to be collected as functions of the wave number λ (here we use k) rather than of p and s, the dimensionless lengths (introduced in Rosser, Newton, and Gross [2]) of the effective launcher length and to burnout, respectively.*

In this paper it is assumed that the rocket response functions are known. The lateral and vertical (pitch and yaw) responses are kept separate so the results can apply more generally than to the symmetric rocket alone. In equations (1n) and (1b) the response of the rocket to the wind (using the response to a "unit impulse" of wind) is calculated. The mean wind effect is given in (2n) for the normal component, and a similar formula for the binormal (lateral) component can readily be written down. Since the effect of the mean wind is 90 percent of the total wind effect, it will require special attention not given here. Much work remains to be done in this area. If time delays of a few minutes are required between measurement of the wind and use of this measurement in aiming the launcher, a "mean wind" is practically the only useful predicting quantity.

Statistical properties now come into play in (3) and (4) where the correlation function and the turbulence spectrum for the homogeneous case are considered. It will be hard to define a turbulence spectrum in the non-homogeneous case since it will differ from one place to another. In (5) the mean-square dispersion of the rocket at burnout is related to the spectrum for the homogeneous case. Simplifications which result when the turbulence is isotropic allow all integrals to be one dimensional. Consequently the resulting calculations become more nearly feasible. The final outcome in the homogeneous case is set forth in (10) and for the isotropic case in (11) and (12')

Approximations to the filter function $W_n(k)$ of the rocket response to turbulence for small wave numbers are obtained from properties (moments) of the response function $G_n(s,q)$, in the remainder of the section.

In the last section, the effect of predictability in allowing a reduction in the mean-square dispersion is exploited.

The least-square predictors for u(q) and Θ_n are given in (17) and (18n) in the time domain and the latter is translated into the turbulence wave domain [using (19)] and the results appear in (20). In (21) the mean-square dispersion which can be removed by prediction is computed in the wave number domain for the homogeneous case. For isotropic turbulence the explicit

*The significance of the notations used in this report is brought out in the list of notations in the front.

predictor is not written down, but it can be obtained by using the cross spectral function matrix for the isotropic case (25) in (22n). The mean-square error reduction due to prediction is available in (26) in the turbulence wave number domain.

General Development of Statistical Dispersion Formulae.—In reference [6] Shaffer uses the angular deviation at burnout θ_b of the actual rocket trajectory from the course it would have pursued in the absence of wind, using a standard calculation method (ideal path) as a measure of the effect of wind. The response function of the rocket to a unit impulse of wind can be used to compute the total effect of the wind. In particular

$$\theta_n(s) = \int_p^s G_n(s,q)\,[n'(q) \cdot u(q)]dq\,, \qquad (1n)$$

where θ_n is the angular deviation of the rocket trajectory from the standard path in the vertical plane, s is the distance along the path to "burnout," p is the effective launcher length, n is the unit normal vector to the trajectory (it lies in the vertical plane containing the ideal path) at the point "q" which is at a distance q-p along the ideal path from the end of the launcher, and u(q) is the wind vector at "q." The horizontal deviation is given by

$$\theta_b(s) = \int_p^s G_b(s,q)[b'(q) \cdot u(q)]dq\,. \qquad (1b)$$

Here b is the binormal unit vector and u(q) is again the wind vector at "q." The same function G appears in both cases due to the axial symmetry of the rocket. If one considers the wind vector u(q) as a random variable as in the statistical theory of turbulence, then $\theta_b(s)$ and $\theta_n(s)$ are random variables also. The mean value of the random variable $\theta_n(s)$ is given by

$$\overline{\theta_n(s)} = \int_p^s G_n(s,q)[n'(q) \cdot \overline{u(q)}]dq\,, \qquad (2n)$$

where $\overline{u(q)}$ is the mean wind vector. Except over sloping terrain, it is usually assumed that the vertical component of the mean wind is zero.

To compute the variance of $\theta_n(s)$ and $\theta_b(s)$ as well as their covariance (we shall assume hereafter that the mean wind components have been removed from the wind vector), we need

$$\mathrm{Var}[\theta_n(s)] = \int_p^s \int_p^s G_n(s,q_1)\left\{\overline{u'(q_1)\; n(q)\; n'(q)u(q_2)}\right\} G_n(s,q_2)dq_1\, dq_2, \qquad (3)$$

where, owing to the assumption of homogeneity of the turbulence, there is no dependence of R on $\vec{x}$ but only on $\vec{x} + \vec{r} - \vec{x} = \vec{r}$, where $\vec{r}$ and $\vec{x}$ are vectors. By $u_i(\vec{x})$ is meant, of course, the i-th component (i = 1,2,3) of the wind vector at the space point $\vec{x}$. This assumption of homogeneity is quite reasonable when $\vec{r}$ is without a vertical component over flat terrain, but not in general.

To compute this, the value of

$$\left\{\overline{u'(q_1)\ n(q_1)\ n'(q_2)\ u(q_2)}\right\}$$

is needed, where the bar indicates average value. This can be shown to be

$$n'(q_2)\ R(q_1,\ q_2)\ n(q_1)\ ,$$

where

$$R(q_1,\ q_2) = R_1\left\{u(q_1),\ u(q_2)\right\}$$

and R is the correlation tensor defined by

$$R_{ij}(\vec{r}) = \overline{u_i(\vec{x}),\ u_j(\vec{x} + \vec{r})}\ .$$

The specification of the turbulence tensor of the spectrum in the homogeneous case as $\Phi_{ij}(\vec{k})$—$\vec{k}$ being wave number—then gives one

$$R_{ij}(\vec{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\vec{r}\cdot\vec{k}}\ \Phi_{ij}(\vec{k})\ dk_1\ dk_2\ dk_3\ , \tag{4}$$

which, when it is substituted in an expression for $\overline{\Theta_n{}^2} = \text{Var}\ \Theta_n$, gives

$$\overline{\Theta_n{}^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \eta'_n(\vec{k})\ \Phi(\vec{k})\ \eta_n(\vec{k})^*\ dk_1\ dk_2\ dk_3\ , \tag{5nn}$$

where * denotes the complex conjugate and

$$\eta_b(\vec{k}) = \int_p^s e^{-i\vec{r}(q)\cdot\vec{k}}\ b(q)\ G_b(s,q)\ dq \tag{6b}$$

and

$$\eta_n(\vec{k}) = \int_p^s e^{-i\vec{r}(q)\cdot\vec{k}}\ n(q)\ G_n(s,q)\ dq \tag{6n}$$

[$\eta_b(\vec{k})$ is defined so that $\overline{\Theta_b{}^2}$ can be computed from a similar definition]. Also, please note that $\Phi(\vec{k})$ is the turbulence tensor [$\Phi_{ij}(\vec{k})$].

Statistical Dispersion Formulae in the Isotropic Case.—The above development is for homogeneous turbulence. If the turbulence is isotropic and the path is straight, it may as well be taken for convenience that:

$$n = (0,0,1); \qquad b = (0,1,0); \qquad r = (q,0,0),$$

$$\Phi_{ij} = \frac{E(|\vec{k}|)}{4\pi|\vec{k}|^4}(|\vec{k}|^2\delta_{ij} - k_i k_j) \tag{7}$$

(See Batchelor [5], p. 49). The assumptions above merely facilitate computations.

From (6n) and (5nn) it can be seen that

$$\eta_n(\vec{k}) = (0,0,1)\Gamma_n(s,k_1) , \tag{8'}$$

where

$$\Gamma_n(s,\vec{k}) = \int_p^s G_n(s,q)e^{iqk_1}dq . \tag{8"}$$

Also

$$\eta_b(\vec{k}) = (0,1,0)\ \Gamma_b(s,k_1) , \tag{9'}$$

where

$$\Gamma_b(s,k_1) = \int_p^s G_b(s,q)e^{iqk_1}dq . \tag{9"}$$

By substitution of (6n) into (5nn) and using the above values of n, b, and r, it also follows that

$$\overline{\theta_n{}^2} = \iiint \Phi_{33}(\vec{k})|\Gamma_n(s,k_1)|^2 dk_1 dk_2 dk_3 \tag{10nn}$$

$$\overline{\theta_n\theta_b} = \iiint \Phi_{32}(\vec{k})\ \Gamma_n(s,k_1)\Gamma_b{}^*(s,k_1)dk_1 dk_2 dk_3 \tag{10nb}$$

$$\overline{\theta_b{}^2} = \iiint \Phi_{22}(\vec{k})|\Gamma_b(s,k_1)|^2 dk_1 dk_2 dk_3 , \tag{10bb}$$

where $\overline{\theta_n\theta_b}$ is the covariance of θ_n and θ_b. This is seen to be zero since Φ_{32} is an odd function of k_3 integrated over a symmetric range in the isotropic case. The above formulae apply in the homogeneous case.

The use of the expression for Φ in (7) using $E(\vec{k})$ gives

$$\overline{\theta_n^2} = \iiint \frac{E(|\vec{k}|)}{4\pi|\vec{k}|^4} (|\vec{k}|^2 - k_3^2)|\Gamma_n(s,k_1)|^2 dk_1 dk_2 dk_3 ,$$

which reduces to

$$\overline{\theta_n^2} = \int_0^\infty E(k)\, W_n(\mathbf{k}) dk , \qquad (11nn)$$

where $W_n(\mathbf{k})$ may be considered as a filter function with the value

$$W_n(k) = \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\pi} (1 - \cos^2 \phi \cos^2 \theta)|\Gamma_n(s,k \sin \phi)|^2 \cos \phi d\theta d\phi . \qquad (12)$$

The change from rectangular to spherical coordinates results in k now being treated as a scaler. The expression (12) can be simplified since

$$\frac{1}{4\pi}\int_0^{2\pi} (1 - \cos^2\phi \cos^2\theta)\, d\theta = \frac{1}{4} (1 + \sin^2 \phi) .$$

A further change in the variable of integration gives

$$W_n(k) = \frac{1}{4}\int_{-1}^{1} (1 + t^2)|\Gamma_n(s,kt)|^2 dt . \qquad (12')$$

<u>The Behavior of $W_n(k)$ for Small Wave Numbers</u>.—The behavior of W(k) for small wave numbers is of interest in the study of the effect of turbulence since for large k, W(k) decreases to small values and a measure of the "cut-off" point is often indicated by the first few terms in a power series expansion in k, the wave number. Since

$$\Gamma_n(s,k) = \int_p^s e^{ikq}\, G_n(s,q)\, dq ,$$

the expansion of e^{ikq} in a power series will allow calculation of the desired result. The results of the expansion and integration give us

$$\Gamma_n(s,k) = \mu_o + \mu_1(ik) - \frac{\mu_2}{2} k^2 - \ldots , \qquad (13)$$

where the moments of G_k,

$$\mu_m = \int_p^s q^m\, G_n(s,q)\, dq , \qquad (14)$$

are the coefficients.

If the series in (13) is used in the expression (12') for W(k), the first few terms of the resulting series turn out to be

$$W(k) = \mu_o^2 \left\{ \frac{3}{4} - \frac{4}{15} (\alpha_2 - \alpha_1^2)k^2 + \frac{1}{70} (\alpha_4 - 4\alpha_3 \alpha_1 + 3\alpha_2^2)k^4 \ldots \right\}, \quad (15)$$

where $\alpha_m = \mu_m/\mu_o$. Another form,

$$W(k) = W(0) \left[1 - \frac{16}{45} (\alpha_2 - \alpha_1^2) + \frac{2}{105} (\alpha_4 - 4\alpha_3\alpha_1 + 3\alpha_2^2) k^4 \ldots \right], \quad (15')$$

will be useful in determining an approximate "cutoff" point of the "filter function" W(k). This point is the zero of

$$1 - \frac{16}{45} (\alpha_2 - \alpha_1^2)k^2 , \qquad \text{i.e.,}$$

$$k_o \approx \frac{3\sqrt{5}}{4} \Big/ \sqrt{\alpha_2 - \alpha_1^2} , \quad (16)$$

provided the magnitude of the term in k^4 is small at $k = k_o$, which will generally be the case since a typical value of the term is about 0.06.

It will be useful to compute the "rough cutoff point" for some functions which appear reasonably like the values given by Hunter. Two examples are given below.

If $G(u) = \frac{1}{\sqrt{u}}$, then

$$\mu_o = 2(s^{1/2} - p^{1/2}) \qquad \alpha_1 = \frac{1}{3} (s + \sqrt{sp} + p)$$

$$\alpha_2 = \frac{1}{5} (s^2 + s\sqrt{sp} + sp + p\sqrt{sp} + p^2)$$

$$\alpha_2 - \alpha_1^2 = \frac{1}{45} [4(s - p)^2 - \sqrt{ps} (\sqrt{s} - \sqrt{p})^2] .$$

If $p = \lambda$ and $s = 20\lambda$, λ being wavelength, then $\alpha_2 - \alpha_1^2 = 31\lambda^2$ approximately and

$$k_o = \frac{1}{\sqrt{11}\lambda} .$$

If λ = 20 feet, then $1/k_o$ = 66 feet.

As the second example, if $G(u) = 1/\alpha$

$$\mu_o = \ln s/p$$

$$\alpha_1 = \frac{s - p}{\ell n\ s/p}$$

and

$$\alpha_2 = \frac{s^2 - p^2}{2\ell n\ s/p} .$$

If $p = \lambda$ and $s = 20\lambda$, then

$$\alpha_2 - \alpha_1{}^2 = 24\lambda^2$$

so that

$$k_o = \frac{1}{2.92\lambda}$$

and if $\lambda = 20$ feet, $1/k_o = 58$ feet.

The Effect of Prediction in Reducing Dispersion.—Because it is assumed that the launcher can be moved so as to correct the aim for any predicted effects of wind on the path of the rocket, it can also be assumed that the reduction of dispersion by prediction is equal to the variance accounted for by the predictor in the statistical sense.

Since, as we recall,

$$\Theta_n = \int_p^s G_n(s,q)\ [n'(q) \cdot u(q)]\ dq , \tag{1n}$$

the problem is to predict the wind vector $u(q)$ along the trajectory from the available predictors which are denoted by a vector w whose transposed representation is

$$w' = (w_1, w_2, \ldots, w_n)$$

and where the components are random variables whose correlation with the wind vector $u(q)$ is known along the path before burnout, $p \leqq q \leqq s$.

From multivariate analysis we know that

$$\hat{u}(q) = (\overline{u(q)\ w'})\ (\overline{w\ w'})^{-1}\ w \tag{17}$$

is the least-squares predictor for $u(q)$ so that the least-squares predictor $\hat{\Theta}_n$ for Θ_n is

$$\hat{\Theta}_n = \int_p^s G_n\ (s,q)\ [n'\ (\overline{u(q) . w'})\ (\overline{w\ w'})^{-1}\ w]\ dq . \tag{18n}$$

From the reciprocal relation of (8") we have

$$G_n\ (s,q) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Gamma_n(s,k)\ e^{-iqk}\ dk \tag{19}$$

so that

$$\hat{\theta}_n = \int_{-\infty}^{\infty} \Gamma_n(s,k) \left\{ \frac{1}{2\pi} \int_p^s n'[\overline{u(q)\ w'}]\ e^{-ikq}\ dq \right\} \overline{ww'}^{-1}\ w'\ dk \tag{20n}$$

$$= \left\{ \int_{-\infty}^{\infty} \Gamma_n(s,k)[n'\Lambda(k)]\ dk \right\} \left\{ \overline{ww'}^{-1}\ w' \right\}$$

$$= \left\{ n' \int_{-\infty}^{\infty} \Gamma_n(s,k)\Lambda(k)\ dk \right\} \left\{ \overline{ww'}^{-1}\ w' \right\}$$

and

$$\hat{\theta}_b = \left\{ b' \int_{-\infty}^{\infty} \Gamma_b(s,k)\Lambda(k)\ dk \right\} \left\{ \overline{ww'}^{-1}\ w' \right\}, \tag{20b}$$

hence

$$\overline{\hat{\theta}_n^{\,2}} = n'\ \overline{\lambda_n'(s)}\ \overline{ww'}^{-1}\ \overline{\lambda_n(s)}\ n\ , \tag{21nn}$$

where

$$\lambda_n'(s) = \int_{-\infty}^{\infty} \Gamma_n(s,k)\Lambda(k)\ dk \tag{22n}$$

and

$$\Lambda(k) = \frac{1}{2\pi} \int_p^s \overline{[u(q)\ w']}\ e^{-ikq}\ dq\ . \tag{23}$$

From Bessel's inequality we know that

$$\overline{\hat{\theta}_n^{\,2}} \leqq \overline{\theta_n^{\,2}}\ .$$

The multiple correlation squared is the ratio

$$\overline{\hat{\theta}_n^{\,2}} / \overline{\theta_n^{\,2}}$$

and hence, from the above, at most unity. This ratio specifies the effect in reducing the dispersion. For isotropic turbulence we recall that

$$\Phi_{ij} = \frac{E(\vec{k})}{4\pi|\vec{k}|^4} \left(|\vec{k}|^2\ \delta_{ij} - k_i k_j \right) \tag{7}$$

and, from [5], that

$$R_{ij} = \overline{u^2}\left(\frac{f(r) - g(r)}{r^2} r_i r_j + g_r \delta_{ij}\right), \qquad (24)$$

where $g = f + \frac{r}{2} f'$, $f(0) = g(0) = 1$. If we consider the information given by the wind at the launcher as w, the predicting variable, and we normalize by taking $\overline{u^2} = 1$ in expression (24), then

$$\overline{ww'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \overline{u(q)w'} = \begin{bmatrix} f(q) & 0 & 0 \\ 0 & g(q) & 0 \\ 0 & 0 & g(q) \end{bmatrix}.$$

In this case

$$\Lambda(k) = \frac{1}{2\pi}\int_p^s \begin{bmatrix} f & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g \end{bmatrix} e^{-ikq}\, dq \qquad (25)$$

$$= \frac{1}{2\pi}\begin{bmatrix} \psi(k) & 0 & 0 \\ 0 & \rho(k) & 0 \\ 0 & 0 & \rho(k) \end{bmatrix},$$

where $\rho(k) = \int_p^s g(q)e^{-ikq}\, dq$ and ψ is similarly defined. We have then,

$$\Gamma_n(s,k) = \int_p^s e^{ikq}\, G_n(s,q)\, dq \qquad (8'')$$

and

$$\overline{\hat{\theta}_n^{\,2}} = \left| \int_{-\infty}^{\infty} \Gamma_n(s,k)\rho(k)dk \right|^2, \qquad (26)$$

which is the desired result of this development.

Acknowledgment.—The assistance of Howard E. Reinhardt in developing this analysis is gratefully acknowledged.

TASK A: IN CONTINUATION

During the reporting period Harold W. Baynton completed his preliminary evaluation of the wind data obtained earlier by Professor R. H. Sherlock and his associates (see Second Progress Report and References 1-6 in First Progress Report).

TASK B: WIND INSTRUMENTS

Activity in this phase has been postponed until development of the statistical design provides additional guidance on the required response characteristics of such instruments.

TASK C: DATA-REDUCTION SYSTEMS
TASK D: WIND-TUNNEL STUDIES

Development of these aspects has been deferred until Task A has been carried further.

CONCLUSIONS

The dynamic wind loading by homogeneous and homogeneous isotropic turbulence has been expressed analytically and the possible utility of a set of predicting elements explored.

PROGRAM FOR THE NEXT INTERVAL

The next interval will be devoted to completing the analyses commenced at the summer conference and to related tasks.

IDENTIFICATION OF PERSONNEL

The following technical personnel have been employed during the reporting period: H. W. Baynton, 28 hours; Ben Davidson, 168 hours; R. J. Deland, 168 hours; A. N. Dingle, 40 hours; E. W. Hewson, 216 hours; G. Keller, 16 hours; V.-C. Liu, 48 hours; H. A. Panofsky, 12 hours; H. E. Reinhardt, 248 hours; L. J. Tick, 168 hours; and M. A. Woodbury, 186 hours.

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