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Phase Report

THE PLASTIC DEFORMATION ACCELEROMETER

H. H. Hicks, Jr.

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ABSTRACT

This report describes a shock-measuring instrument called a Plastic Deformation Accelerometer. The instrument is an economical and effective means of measuring acceleration, and is also rugged and simple. Apparently its accuracy is comparatively rather poor, but is adequate for many types of application. Satisfactory calibrations of the instrument have been made, but the theoretical analysis of its characteristics and limitations is incomplete.

OBJECTIVE

The objective of this report is to describe the Plastic Deformation Accelerometer, its development, characteristics, and sources of error.

INTRODUCTION

The material of this report had its origin in research done by The University of Michigan for the U. S. Army Ordnance Corps. Early in 1954, the University began an investigation for the U. S. Army Ordnance Corps of the shock environment resulting from a ballistic impact on an armored vehicle. It became necessary during the course of this investigation to measure the shocks occurring simultaneously at many points on a vehicle being tested Some of these shocks are very severe. Electrical instrumentation based on crystal and strain-gauge pickups was generally suitable for the measurement of these shocks, but this system proved to be too expensive to be applied at the many points where shock measurements were needed. Electrical instruments should not be used in the vicinity of the impact since they may be broken, and their applicability is thus further limited; and the necessity of having electrical power available is at best an inconvenience and may even preclude some types of tests.

These considerations along with other less important ones pointed to the necessity of obtaining supplementary instrumentation means for the shock-measurement work. A consideration of the known possibilities led to the conclusion that a mechanical peak-reading instrument, wherein the operation is based on the plastic deformation of a replaceable part, was well suited to the overall requirements of the shock-environment study.

The body of this report describes the development of this instrument, called the Plastic Deformation Accelerometer (abbreviated PDA).

It should be borne in mind that the development of the PDA was undertaken on the basis of very limited funds and time. Accordingly, many desirable refinements in both the analysis and the construction of the instrument could not be undertaken. The instrument is useful in its present form, but can undoubtedly be considerably improved and refined.

PRINCIPLE OF OPERATION

The essential operation of the Plastic Deformation Accelerometer is the measurement of the force developed by a seismic mass that is subjected to accelerative displacement. The force is measured by means of the impression made in a soft-metal recording anvil by a hard, conically shaped punch. A device which incorporates the essential features of these ideas is shown in Fig. 1. In this simplified PDA the conical punch forms part of the seismic mass, and the soft-metal recording anvil is attached to the PDA frame, which in turn is attached to the device, etc., the acceleration of which is to be measured. Acceleration applied to the frame of the PDA is communicated to the seismic mass through the anvil. The resulting accelerative force drives the punch into the anvil and the relation between this force and the indent diameter in the anvil constitutes the basic usefulness of the PDA.

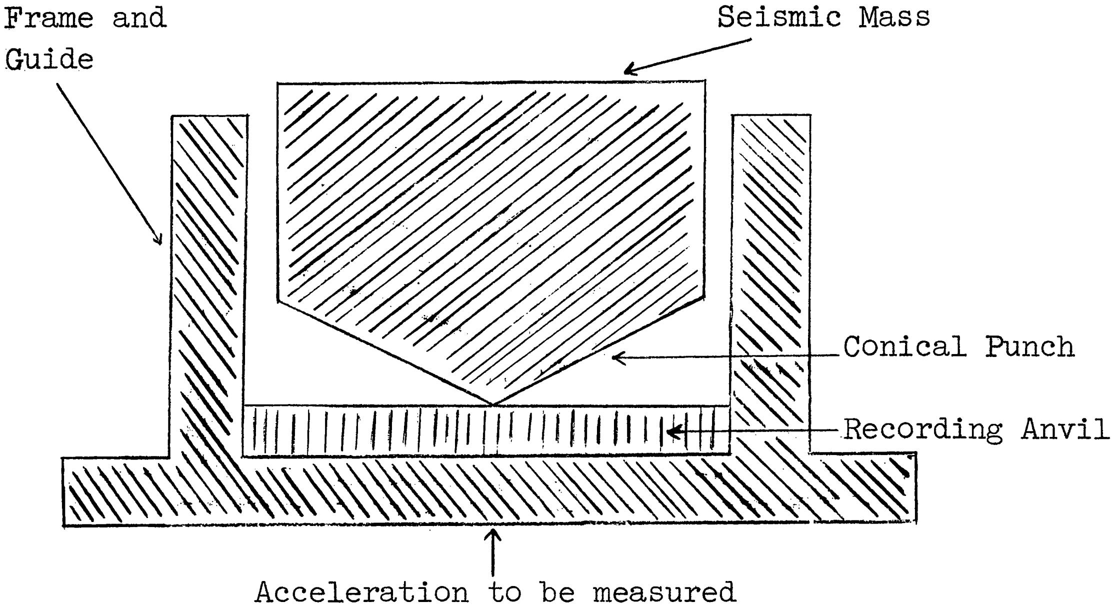

Fig. 1. Section of Simplified Plastic Deformation Accelerometer.

In the practical application of the PDA principle, several factors prevent the attainment of an exact and predictable relation between the applied acceleration and the size of indentation made in the anvil. Some of these factors are fairly well understood, some only qualitatively recognized, and some merely guessed at. These factors are considered in some detail in the Appendix of this report.

The measurement range of the PDA (maximum measurement attainable) depends on the size of the seismic mass, the length of the conical punch, and on the hardness of the metal anvil. The range depends slightly on the sharpness (included angle) of the conical punch. The sensitivity (degree of resolution in measurement) is generally in inverse proportion to the measurement range of the instrument and is also influenced slightly by the sharpness of the punch.

The relation between the imprint diameter made by the conical punch and the force on the punch is of basic importance to the theory of the PDA. The assumption is made that the load-bearing ability of the soft anvil is proportional to the area of contact between the punch and the anvil. Using this assumption as a basis, it is easily apparent that a parabolic relation exists between force and imprint diameter (i.e., the force is proportional to the square of the imprint diameter). The validity of the assumption is apparent in the static test results (Fig. 4), and is also observable in the dynamic calibrations of the PDA (Fig. 9).

DEVELOPMENT OF THE PLASTIC DEFORMATION ACCELEROMETER

The necessity for the development of a low-cost, expendable accelerometer, discussed in the introduction to this report, led to an effort to determine the most promising approach to produce the needed instrument.

A continuously reading instrument was obviously desirable, because of the value of the time base in the record, but no method of supplying the time base was discovered that could satisfy the requirements of simplicity, ruggedness, and economy (expendability). The possibility of using an electrical instrumentation system was then still being considered, but the failure to find a satisfactory time base led to the abandonment of the continuously recording meter, and the adoption of a peak-reading meter. It appeared probable that there was a good chance of developing a satisfactory peak-reading meter of a purely mechanical type. The mechanical meter was therefore selected for development.

It was then necessary to devise or select a means for recording the output of the meter.[1] Several schemes for doing this were considered, and some promising ideas were developed.[2] Unfortunately, due to the restriction of the program, it was not possible to evaluate these ideas by experiment,

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[1] It was assumed that a force-mass type of accelerometer would be used, but since acceleration is a space-time concept, it is conceivable that it could be measured directly by some other means, for example, motion pictures.

[2] A peak-reading accelerometer could be made using the mass-spring combinations in conjunction with a ratchet device for retaining the spring extension or compression at the point of maximum value (corresponding to maximum acceleration). It was felt, however, that this device could not be made sufficiently rugged to withstand the expected shocks. Another idea concerns the use of a scribe or stylus attached to the accelerometer mass. In this application, the scribe writes on a card referenced to the structure being tested and thus makes a record of the maximum acceleration. But it was decided that this device would be too complicated, and basically too insensitive for the intended application.

or even by extensive analysis. A decision was made, however, to develop the mechanical type of accelerometer around the idea of a permanently deformed part which could be easily replaced after using. It was believed that the requirements of simplicity, ruggedness, and economy could be achieved on this principle, and various schemes for using this principle were considered. Three arrangements using the permanent deformation (plastic deformation) principle were experimentally tested. These arrangements involved:

1. A steel ball pressed into a flat lead plate;

2. A lead ball crushed between two flat steel plates; and

3. A hard conical punch, pressed into a flat lead plate.

Experimental results of this test are shown in Figs. 2, 3, and 4 of this report. It is apparent from these figures that the conical punch pressed into the lead plate gives the most regular and presumably the most reproducible results of the methods tested. A particular advantage is that a very light impression made by a conical punch is sharply defined, while a light impression made by a ball is only faintly visible, if at all.

It was quickly apparent that reading the diameter of the conical punch mark was much easier and more reliable than reading the depth. The hole diameter is, furthermore, as good a measure of penetrative force as is the depth. A micrometer stage laboratory microscope was used to measure the static test hole diameters.

It was therefore decided to use the conical punch pressed into a lead plate as the accelerometer recording element. The next step was to adapt the principle to a practical instrument.

The first attempt to design an accelerometer on this principle resulted in the instrument shown in Figs. 5 and 6. This instrument (called PDA-M1) was fabricated out of built-up parts so that it could be produced as economically as possible, because it was not reasonably certain that the instrument would be practically successful.

PDA-M1 was designed to be able to measure a maximum acceleration of 600 times gravity. This acceleration, it was calculated, would be sufficient to imbed the 1.33-oz conical punch almost completely into the soft, pure lead recording anvil. Dynamic calibration of PDA-M1 was made on a drop test stand (Figs. 7 and 8), and the indentation in the lead anvil, resulting from a given drop height, was compared with the acceleration measured by a reference accelerometer. The reference accelerometer was of the barium titanate crystal type and it was used with the necessary recording equipment. The test stand (Figs. 7 and 8) was designed to test 12 PDA's at one time and simultaneously to compare their readings with those of a single reference accelerometer. It was found, however, that consistent results could not be obtained unless the PDA was mounted on the test stand very close to the reference accelerometer. This behavior was apparently due to buckling of the test-stand plate when it was dropped, and therefore it was necessary to check the PDA's one at a time against the reference accelerometer. An inherent design factor that prevented consistent calibration results was the effect of air compression in the body of the PDA. A few calculations revealed the fact that with PDA-M1 the force due to compressed air could possibly equal or exceed the force needed to penetrate the lead anvil, this being especially true at low acceleration. An adequate remedy was found in venting the body of the accelerometer to avoid the entrapment of air.

PDA-M1 was used for field tests with moderate success. The principal limiting factor for field use was its lack of structural integrity owing to the built-up type of construction. The instrument was able to withstand large accelerations applied in its measuring direction, but it often came apart during the field tests. This difficulty was apparently caused by lateral accelerations or by accelerations applied opposite to its direction of measurement.

The mechanical failure of the PDA structure resulted in the loss of many of the component parts during the field tests.

Figure 9 is the calibration curve for PDA-M1. This curve represents data taken by three different instruments on separate occasions. The variation from the mean value appears to average about 15%.

A micrometer stage microscope was satisfactory for reading the accelerometer punch marks in the laboratory and was used for calibrations, but for field use a portable reading instrument was desired. A convenient instrument for this purpose is a device called a Pocket Comparator.[3] This device is an optical comparator which consists of a magnifying eyepiece and a reticle.

The reticle is engraved with various geometrical markings including linear scales. In use, the reticle is placed against an object to be measured or compared, and the reticle and the object are simultaneously viewed through the magnifying eyepiece. An easy and accurate comparison between the reticle scale and the object is made, thus providing a means for measuring the object.

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[3] National Tool Company, 11200 Madison Avenue, Cleveland 2, Ohio.

The minimum linear dimension measurable with the Pocket Comparator is about 0.005 inch. This resolution is nearly sufficient for measuring the larger punch marks in the lead anvil, but for small impressions corresponding to low accelerations a more accurate gauge is needed.

This was provided by combining the Pocket Comparator with a standard 5X microscope objective so as to form a compound microscope. With this arrangement the minimum linear dimension measurable is about 0.001 inch. This is sufficiently accurate for all practical PDA work. A schematic drawing of the optical comparator used as a compound microscope is shown in Fig. 10 and a photograph of the instrument is shown in Fig. 11. This arrangement is particularly convenient since the basic optical comparator can be used with or without the compound microscope.

The results of the field use of PDA-M1 indicated a need for some modifications in the design of the instrument. It was essential to have a more rugged instrument and desirable to have an instrument capable of measuring higher peak acceleration. It was also desirable to redesign the instrument for increased convenience in use. These design objectives were fairly well realized in the second model of the instrument, designated PDA-M2.

This instrument (Fig. 12) includes a one-piece steel body machined from hexagonal stock. The seismic mass is reduced to allow for a larger peak acceleration measurement. The same punch (60°) is used as with PDA-M1. It is possible to remove the entire instrument from the mounting base by simply unscrewing the body from the base. This feature greatly facilitates replacement of the soft-lead anvils where the instrument is in a place not easily accessible.

No attempt was made to calibrate PDA-M2 since the operational range was quite high (designed for maximum acceleration of 10,000 g's), and since no comparison equipment was available for the calibration at the time it was needed. It was assumed that the maximum range was inversely proportional to the amount of the seismic mass and the readings were interpreted on this basis. The ratio of the seismic masses of the two instruments (PDA-M1 to PDA-M2) is fifteen to one. The calibration chart for PDA-M1 (Fig. 9) is used for PDA-M2 by multiplying the "g" scale by a factor of fifteen.

PDA-M2 functioned satisfactorily in the field tests, and proved to be extremely rugged. The instrument was not broken when located almost directly on the opposite surface of a test armor from the point of ballistic impact. A maximum acceleration of about 5000 g's was indicated on this test, and it appeared that the instrument would be able to withstand and record considerably more severe shocks.

The field tests using PDA-M2 proved the serviceability of the instrument, but it was evident that improvements of the original basic design could still readily be made. A very desirable feature, it was thought, would be the ability to change the range of the instrument depending on the expected severity of the shock to be measured.

Accordingly PDA-M3 was designed as a modification of PDA-M2. The primary feature of PDA-M3 which the earlier models lacked is the provision for a variable seismic mass. The physical dimensions of the seismic mass in PDA-M3 are such that an instrument range variation of 10 to 1 is possible by changing from an unweighted hollow steel seismic mass to one weighted with lead. The design was also more convenient to use (Figs. 13 and 14).

Only a single item of PDA-M3 was made. It was planned to test and calibrate this modification extensively in the laboratory before using the PDA on further field tests. It was hoped that PDA-M3 could become a standardized model. These tests were never performed because modifications and curtailment of the parent program made it necessary to drop the accelerometer development. The present status of the PDA development therefore includes a fairly well calibrated but inadequately designed instrument (PDA-M1), a better designed but poorly calibrated instrument (PDA-M2), and a still better designed but completely uncalibrated instrument (PDA-M3).

Recently, some limited time was made available to consider the PDA further. It was decided to use this time to try to answer a basic problem relating to PDA performance instead of attempting what would probably be an inadequate calibration of PDA-M3. The basic problem referred to is the question of the validity of using a single calibration curve for instruments of different ranges. The difficulties of calibrating a high-range instrument have been noted, but no particular difficulty has been encountered in calibrating the low-range instrument. It was therefore planned to build a special test instrument which would operate at lower ranges than PDA-M1 and which would serve as a check on the validity of extrapolating the PDA-M1 calibration data to different acceleration ranges.

Accordingly, a PDA-M1 was equipped with a special seismic mass which weighed 4.52 oz instead of the regular seismic mass weighing 1.33 oz. This instrument will be referred to as PDA-M1*. The procedure for calibrating the modified instrument was the same as had been used for PDA-M1 (page 4).

At this point, a new difficulty arose. The original lot of pure lead, which had been used to make the first anvils, was exhausted, and a new supply of lead was obtained. It was suspected that the new lot of lead might be different from the original lot and to check this point it was decided to recalibrate the PDA-M1 using anvils from the new lot of lead. It was found that the original calibrations could not be duplicated, and it appeared that the new anvils were harder than the old ones. Heat treatment or aging of the new anvils did not affect their relative hardness and it seemed likely that the source of the difficulty was probably the slight impurities present in the lead. It was reasoned that the relative hardness of very pure lead could be given a large proportional change by small changes in the trace impurities. This theory was not pursued any further, for it was decided to try to avoid the difficulty by using 40-60 lead-tin alloy for the anvil material instead of pure lead. The idea here was that a small composition change of this alloy would not be as likely to affect its hardness as it would a very pure metal. The lead-tin alloy was harder than the pure lead, so that a direct calibration comparison could not be made. However, it was expected that when the relative hardness factors of the lead and the lead-tin alloy were taken into consideration, a good comparison of results would be obtained.

The changing of the seismic mass of the accelerometer from 1.33 oz to 4.52 oz is expected to increase the force resulting from a given acceleration by $4.52/1.33 = 3.40$ times. This would increase the area of the anvil imprint by 3.40 times also, according to the assumptions made concerning the load-sustaining ability of the anvil.

The diameter of the imprint is correspondingly increased by a factor $\sqrt{3.40} = 1.85$. All this occurs at a given acceleration. The effect of the anvil hardness was investigated by making a static-load calibration with anvils made of the lead-tin alloy. A comparison of the results was made with the static-load calibration of the original anvils (Fig. 4), and it was found that the load-carrying ability of the lead-tin alloy averaged 1.55 times that of the pure lead (Fig. 4) for a given imprint area. For given forces, the diameter of the lead impression is $\sqrt{1.55} = 1.25$ times that of the lead-tin alloy.

Combining all these factors and assuming all assumptions used are correct, the calibration curve for PDA-M1* should be computable from that of PDA-M1 by multiplying the imprint diameter given for the latter by the factor 1.48 which is the ratio of the seismic mass change to the anvil hardness change. These results are displayed graphically in Fig. 15, and it is seen that the extrapolation is very poor. The computed curve for PDA-M1* would fall closer to the experimental curve if the hardness factor were greater. It is possible that the lead-tin alloy anvils are much harder to dynamic forces than they are to static forces. This would account for the discrepancies noted, but actually the cause of the discrepancies has not been determined.

APPENDIX

FACTORS AFFECTING ACCURACY OF THE PDA

It was mentioned in the introduction to this report that the PDA was developed to serve the needs of a relatively small program, and consequently its development could not justify the commitment of extensive funds or time. The result is a "practical" device whose theory and operating limitations have not been extensively studied and consequently are not well understood.

The calibration data of the PDA show a deviation of perhaps 15% from the mean value. This percentage of error, while in no way invalidating the usefulness of the instrument, is greater than usually considered acceptable. At the present stage of development, there is need for a more basic understanding of the operation of the PDA so that the sources of error may be discovered and minimized.

Observations of the development, calibration, and operation of the PDA have suggested certain factors which in theory should exert considerable influence on the magnitude of the errors. Four such factors are currently recognized, and they are:

1. Failure of a force equilibrium to be established between the seismic mass and the recording anvil of the PDA;

2. Variation of the strength of the recording anvil with rate of deformation;

3. Effect of hammering of seismic mass on anvil; and

4. Frictional restraints on seismic mass.

These factors will be taken up in turn and qualitatively discussed. Sufficient information on which to base firm analytical conclusions is not available.

The first of these factors, the one concerning the establishment of force equilibrium, is a matter relating to the fundamental instrumentation problem. That is, the problem is not peculiar to the PDA, but applies to any inertia type of accelerometer. The essential source of the error lies in the failure of the seismic mass to follow exactly the motion of the body under observation. This failure is in turn made necessary by the requirement of obtaining "output" from the accelerometer. In the case of the PDA, the "output" is a hole punched in the soft recording anvil.

The magnitude of error due to the first factor can be analytically evaluated for simple cases. An accelerometer having a linear force-displacement characteristic subjected to a sinusoidal acceleration is an example of such a simple case. The PDA, however, has an approximately parabolic force-displacement relation and this occurs in only one direction. No recording force is generated in the opposite direction. In addition, the accelerations usually measured by the PDA is not cyclic, but occurs as an impulse of unpredictable shape. Consequently, there is no practical way to make an analytical evaluation of the measurement error. Nevertheless, the basic principles of the simpler cases are applicable to the PDA. These principles indicate that certain conditions and design features will tend to minimize the measurement errors, because these conditions, etc., tend to reduce the relative motion needed to get output. Errors will be minimized when:

1. The accelerative displacement of the body being observed is large compared to the penetration distance of the punch into the recording anvil; and

2. The punch penetrates the recording anvil only a slight amount to establish equilibrium between the acceleration force and the resistance of the anvil. This is encouraged by having a very blunt cone for the punch, and by using a hard (i.e., large Young's modulus) material for the recording anvil.

A large value of Young's modulus (this property controls the elastic deformation which occurs prior to plastic deformation) is desirable in an anvil material to minimize the elastic deformation. A high-tensile-strength anvil material is desirable since this allows the anvil to develop the necessary penetration resistance with only slight penetration. It should be noted, however, that there are disadvantages connected with the use of a hard-strong anvil. Obtaining and machining punch materials significantly harder than the anvil are likely to be difficult. Another problem concerns the accuracy of measurement of the smaller impression in the hard anvil. These problems were avoided on PDA-M1 by the use of soft anvils, and of course by inviting the type of error now being discussed.

In calibrating PDA-M1, an attempt was made to disclose an error of the force-equilibrium type by varying the rate of change of the acceleration. A given peak acceleration would be obtained by dropping the calibration stand a short distance on a hard pad, or the same peak acceleration could be obtained with a long drop on a soft pad. The essential difference is in the distance over which the peak acceleration is attained, or the time during which it is attained.

Calibrations made in this way exhibited no significant or systematic difference attributable to the force-equilibrium factor. There is necessarily a condition, however, where this factor will cause a significant error, but it is apparently outside the range of conditions to which the PDA has thus far been calibrated. This matter needs further experimental and analytical investigation.

The second error-producing factor of the PDA is based on the resistance to stress that a plastic material exhibits as a function of the rate of deformation. There are no theories or experimental data available that show how these variables are related. The most closely related information is probably that connected with the creep strength of materials, but this information applies to long-time duration phenomena, whereas the PDA processes occur in a relatively very short time. A thorough investigation of the stress-strain rate properties of plastic materials applicable to PDA use is needed. This would, however, involve a considerable program of investigations into these properties of materials.

Calibration errors attributable to the rate of deformation of the recording anvil would tend to be disclosed by the type of calibration made, but there were no systematic indications of this type of error.

On the basis of present knowledge, it cannot be predicted that an error attributable to the rate of deformation actually exists under any conditions, but the properties of true plastic materials strongly suggest this error by analogy. It is conceivable, of course, that a dynamic resistance to flow would become evident at extremely high rates of deformation, but this condition is almost certainly beyond the useful limits of the instrument.

A third PDA error factor is due to the effect of hammering. This effect may occur when a vibratory condition is encountered. Under this condition, it is expected that the repeated blows will drive the punch more deeply into the recording anvil than is necessary to support the accelerative forces attributable to the momentary body motion.

This type of error factor would cause the acceleration indicated by the PDA to be too high. The elasticity of the recording anvil is apparently a major factor in affecting this type of error, since once an impression has been made in the anvil, the inherent elasticity of the anvil material will arrest the motion of the seismic mass before additional plastic deformation can occur. Reduction of this error apparently calls for an anvil material possessing a high yield strength and a low value of Young's modulus.

According to the above ideas, the problem of arresting the hammering of the seismic mass is a matter of energy absorption. It can be shown that the elastic energy storage capability of a cylindrical piece of material is given as follows:

$$W_m = \frac{T^2 V}{2E} \quad ,$$

where W_m = maximum elastic energy storage,

T = yield stress of material (tensile or compression per unit cross section),

V = volume of material, and

E = Young's modulus (assumed constant).

It is assumed, from the discussion of the first error factor, that the contribution of yield stress and Young's modulus to the merit of the accelerometer is as follows:

$$M = kTE \quad ,$$

where M = merit factor for reducing errors due to first factor, and

k = constant.

On combining the expressions for error factors into a single product, the following is obtained

$$M' = \frac{kTE \cdot T^2 V}{2E} = \frac{kV}{2} T^3 \quad .$$

This relation indicates that high yield strength is the controlling factor in selecting a material that will minimize the first and third error factors. If there is no need to minimize the third error factor, then the best anvil material can be chosen on the basis of the largest values of E and T (both values should be large).

The effect of hammering is probably not very pronounced in the applications for which the PDA has thus far been used. In this application, the acceleration is characterized by a large initial impulse followed by smaller vibrations.

The fourth error factor mentioned at the beginning of this section was the effect of friction on the accuracy of the PDA. Friction will tend to move the seismic mass of the PDA so that not all the accelerative force is applied to the seismic mass through the recording anvil. This will reduce the imprint in the anvil and cause an error.

An analytical solution of this error factor adds the element of friction coefficient to the variables affecting the first error factor. It appears doubtful whether an exact analytical solution can be obtained, but a good idea of the contribution of friction can be gotten in another way.

If a friction error of, say, 10% is tolerable, then the friction force on the seismic mass may be allowed to be 10% of the acceleration force. This situation would occur (for the case where the friction coefficient between the seismic mass and the guide is 5%) when the sideways force on the seismic mass is two times the force in the direction of the punch. This means that the applied accelerations could be about 60° away from the accelerometer axis and the accelerometer would show an error of only about 10% for the measurement of the axial component of the acceleration.

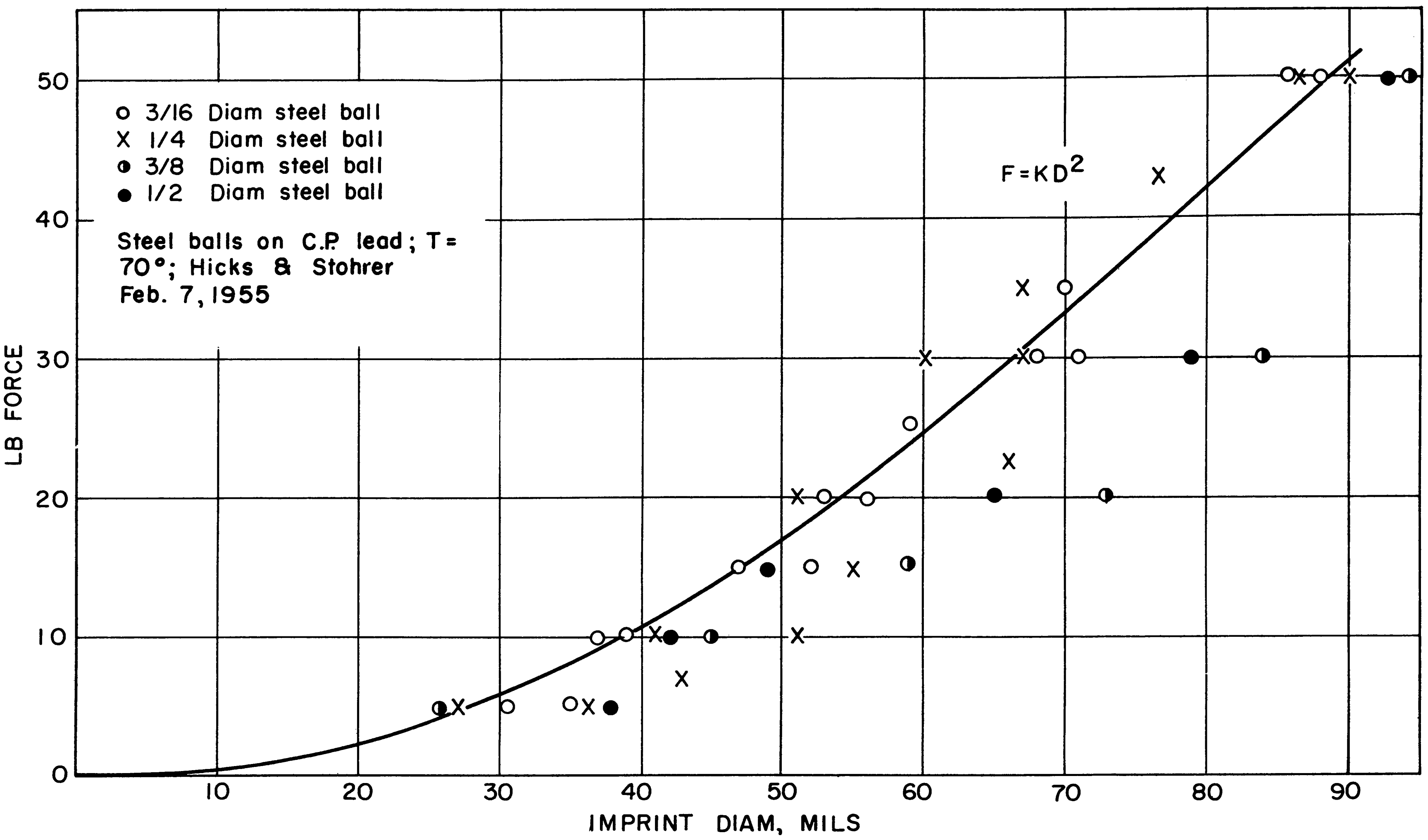


Fig. 2. Steel-ball—lead-plate calibration.

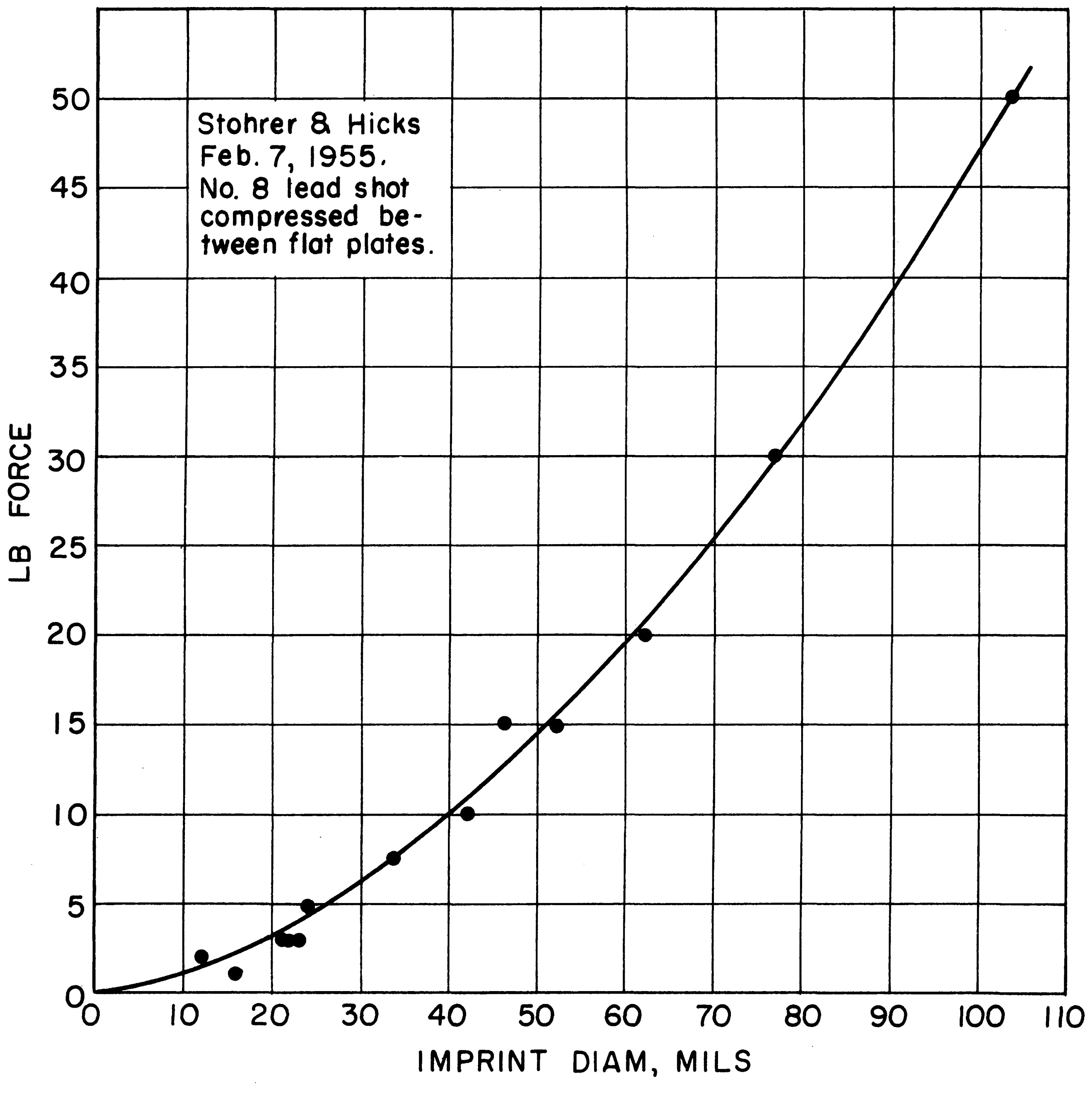


Fig. 3. Lead-ball—flat-plate calibration.

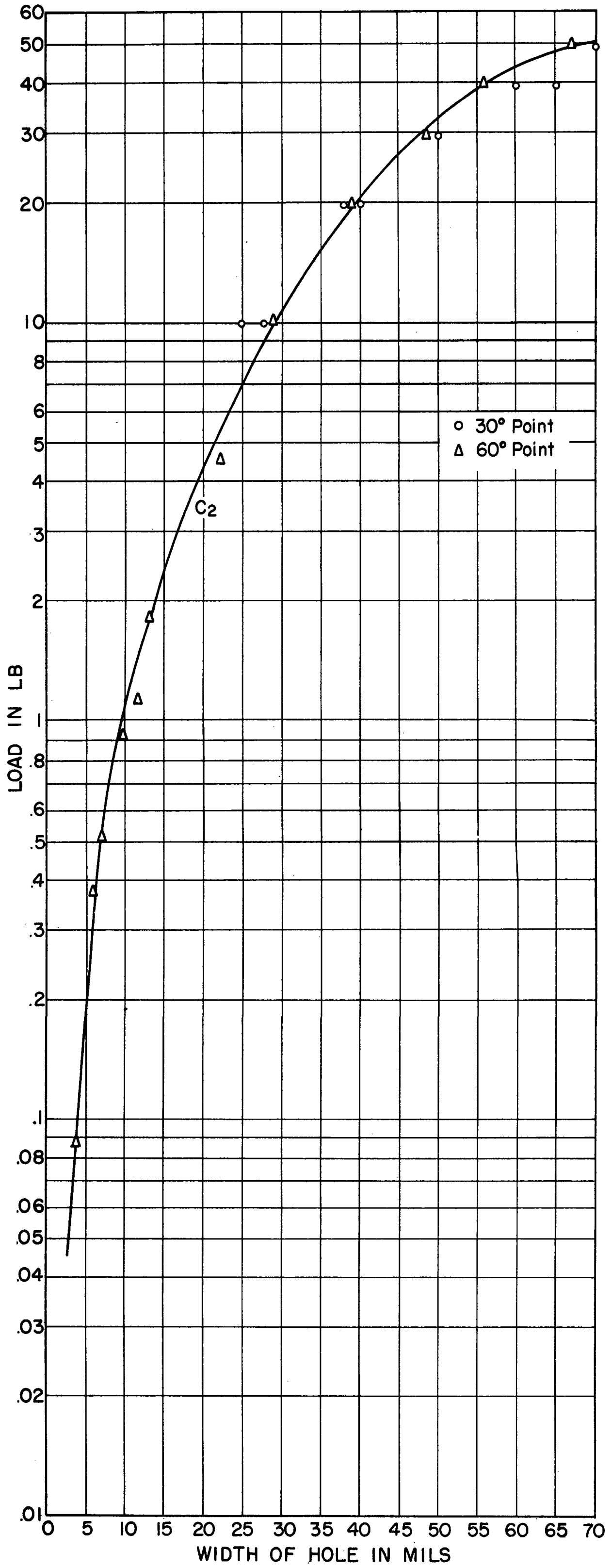


Fig. 4. Static tests of impression made in lead anvil with 30° and 60° conical point.

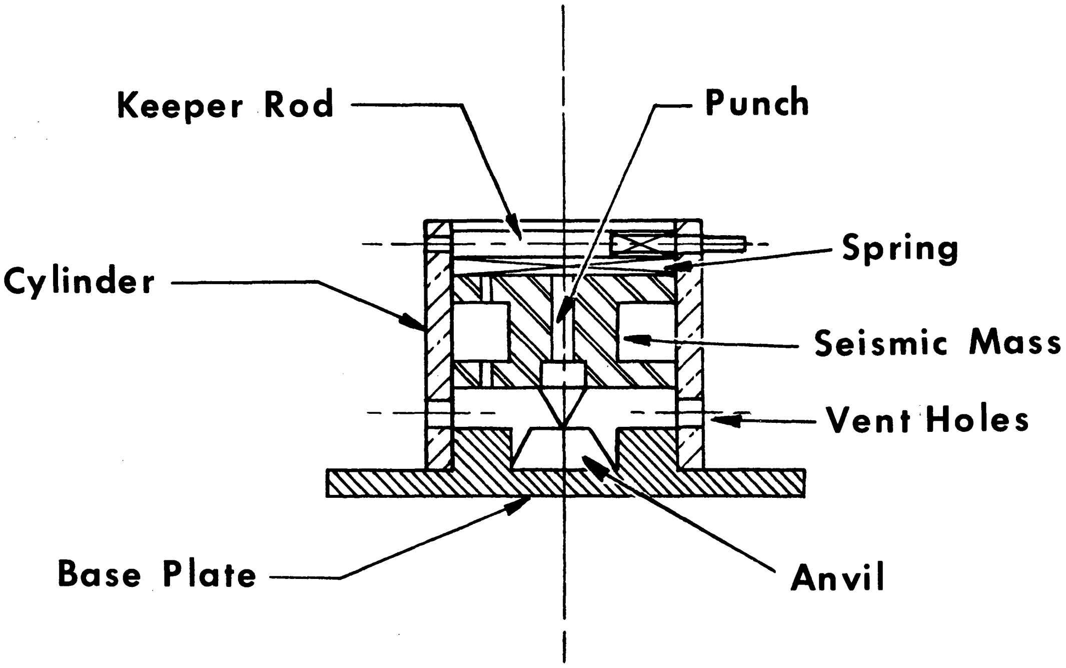


Fig. 5. Cross-section assembly of PDA-M1.

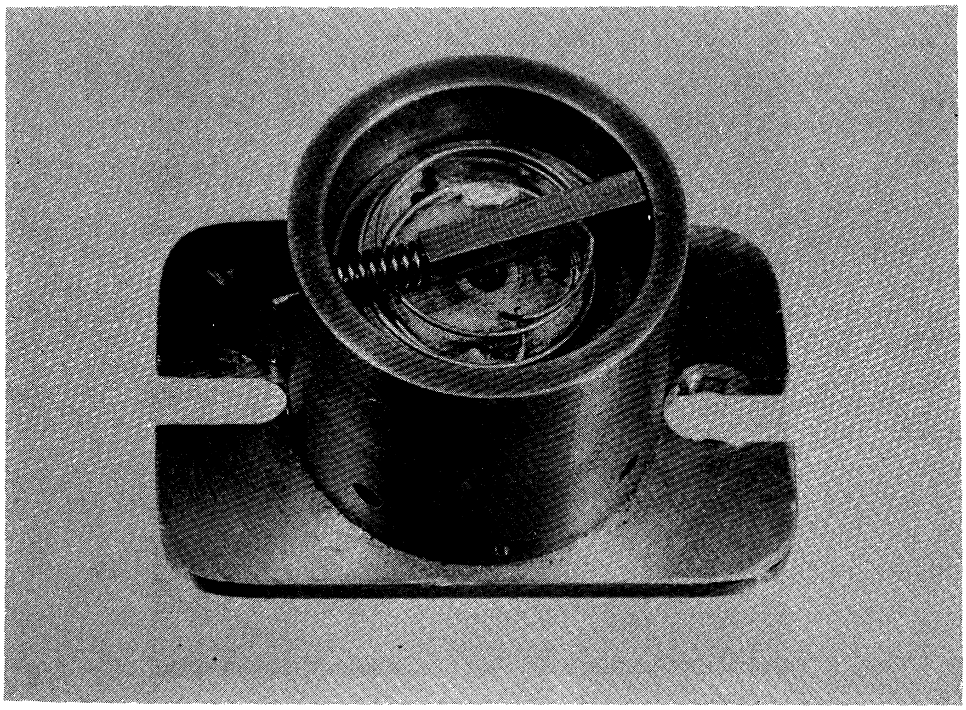


Fig. 6. Photograph of PDA-M1.

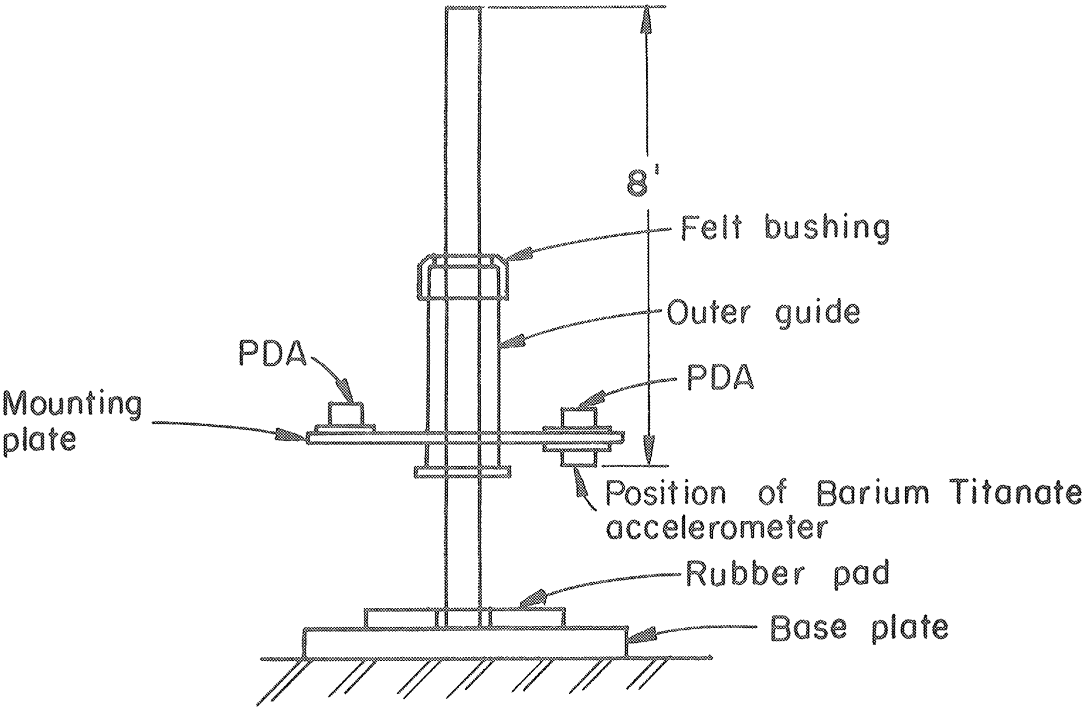


Fig. 7. Accelerometer calibration test fixture.

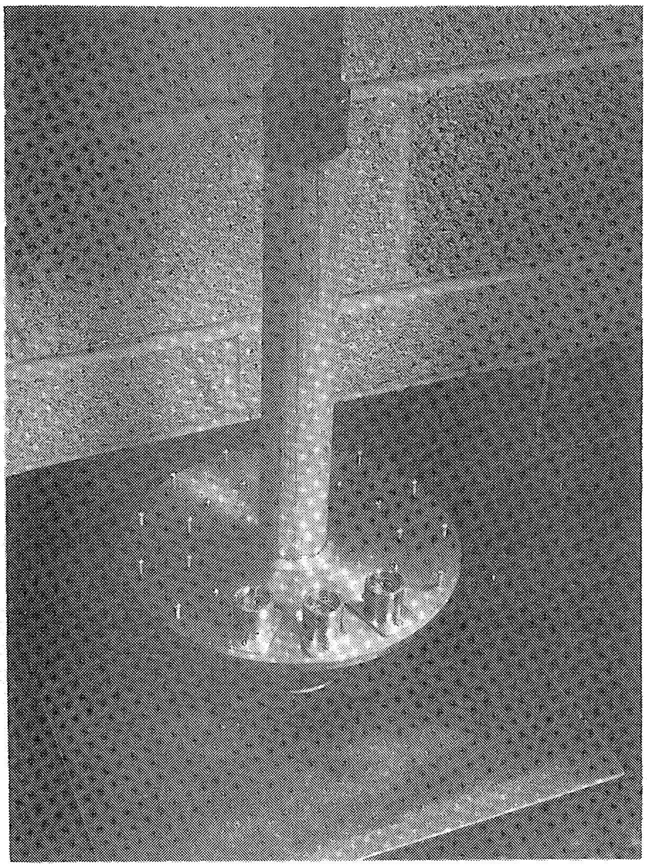


Fig. 8. Accelerometer calibration test fixture showing accelerometer in place.

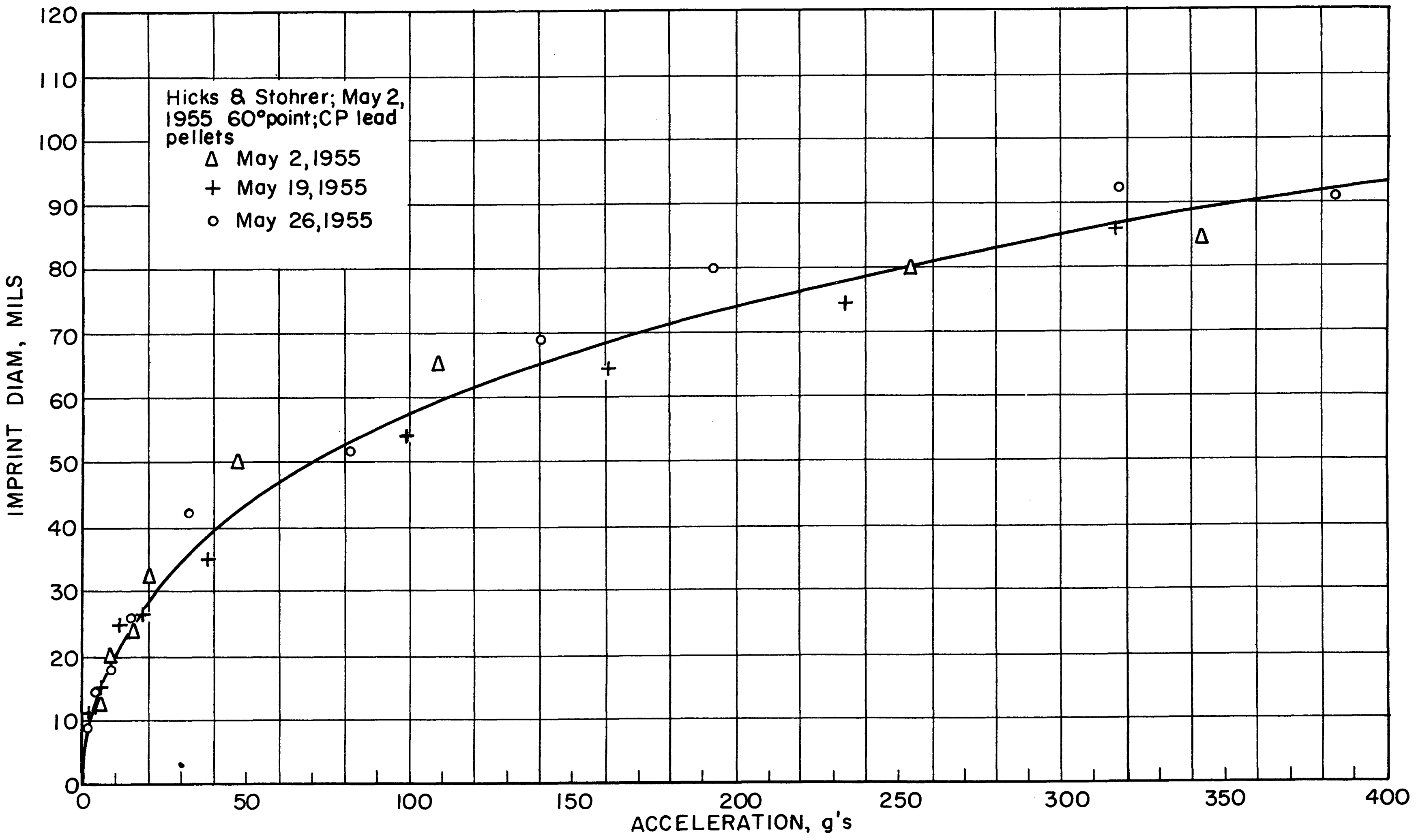


Fig. 9. Calibration curve for PDA-M1.

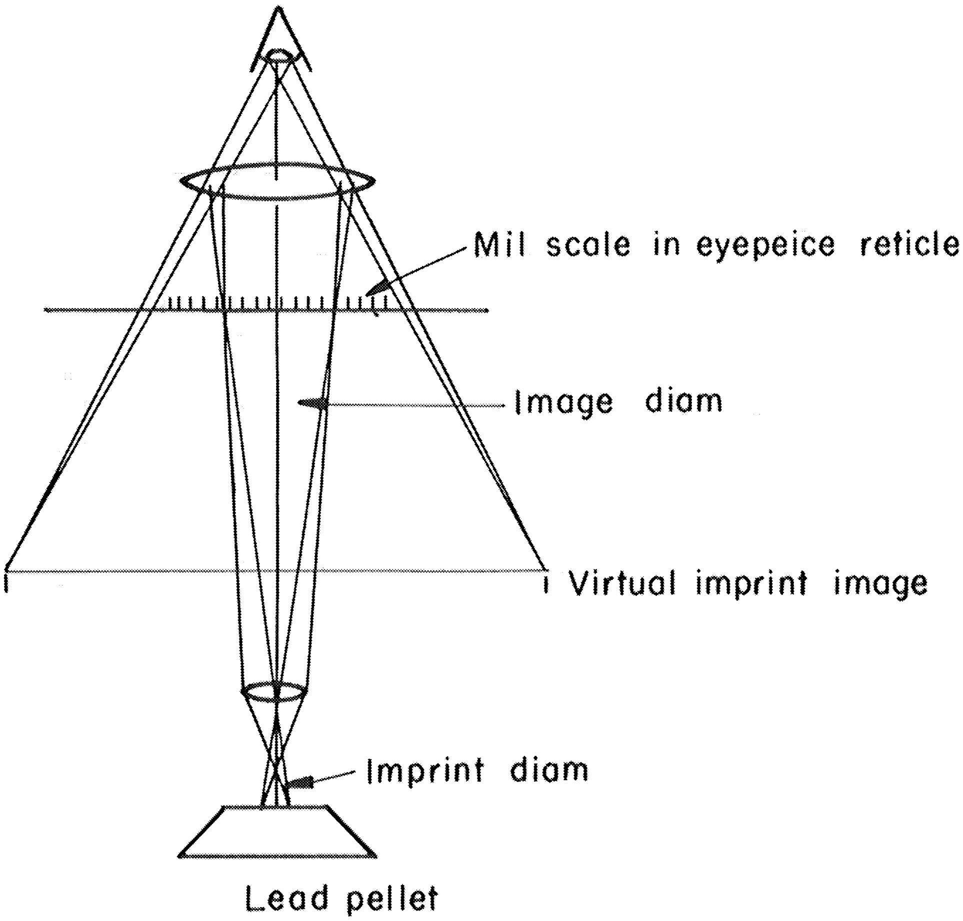
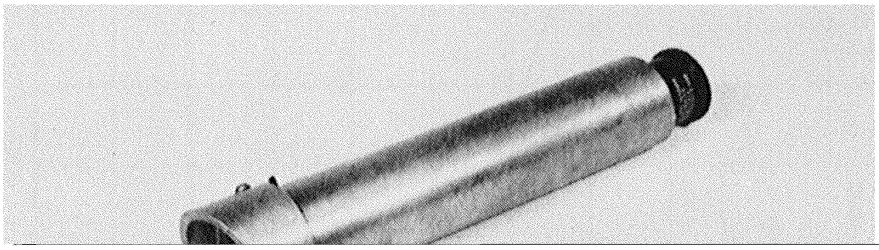


Fig. 10. Schematic drawing of optical comparator.



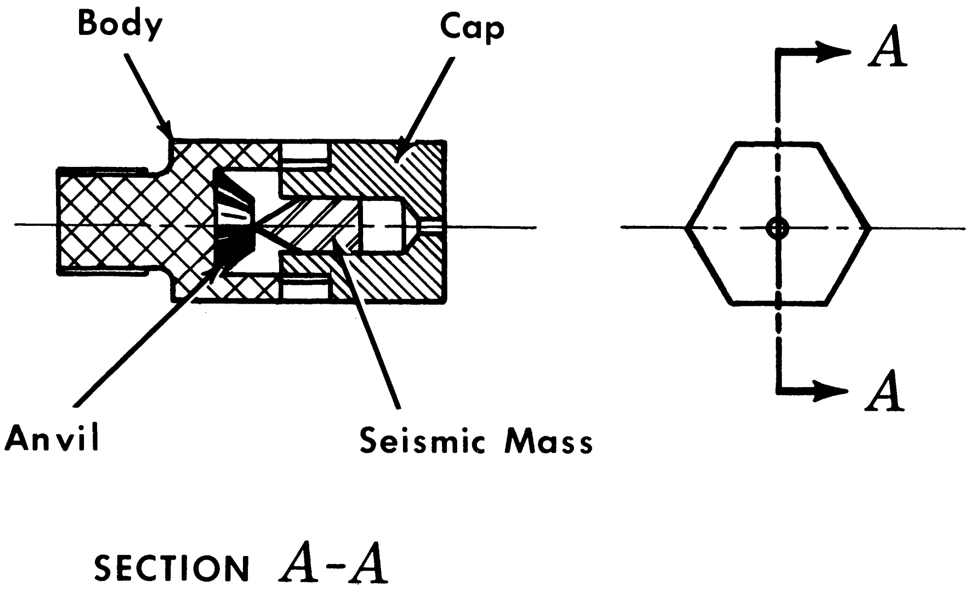


Fig. 12. Sectional view of PDA-M2.

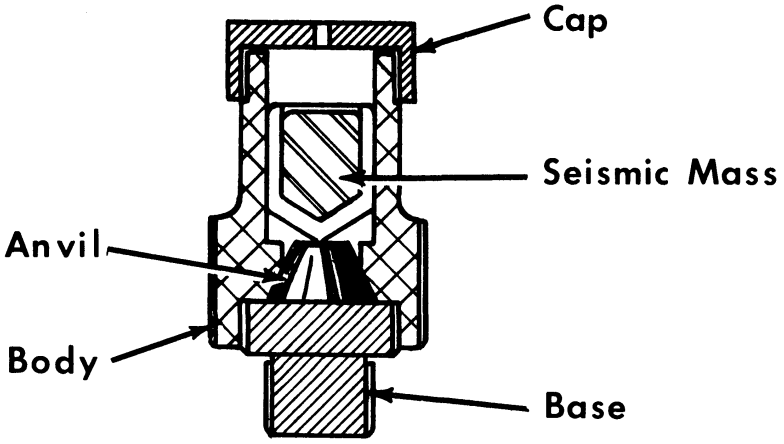


Fig. 13. Sectional view of PDA-M3.

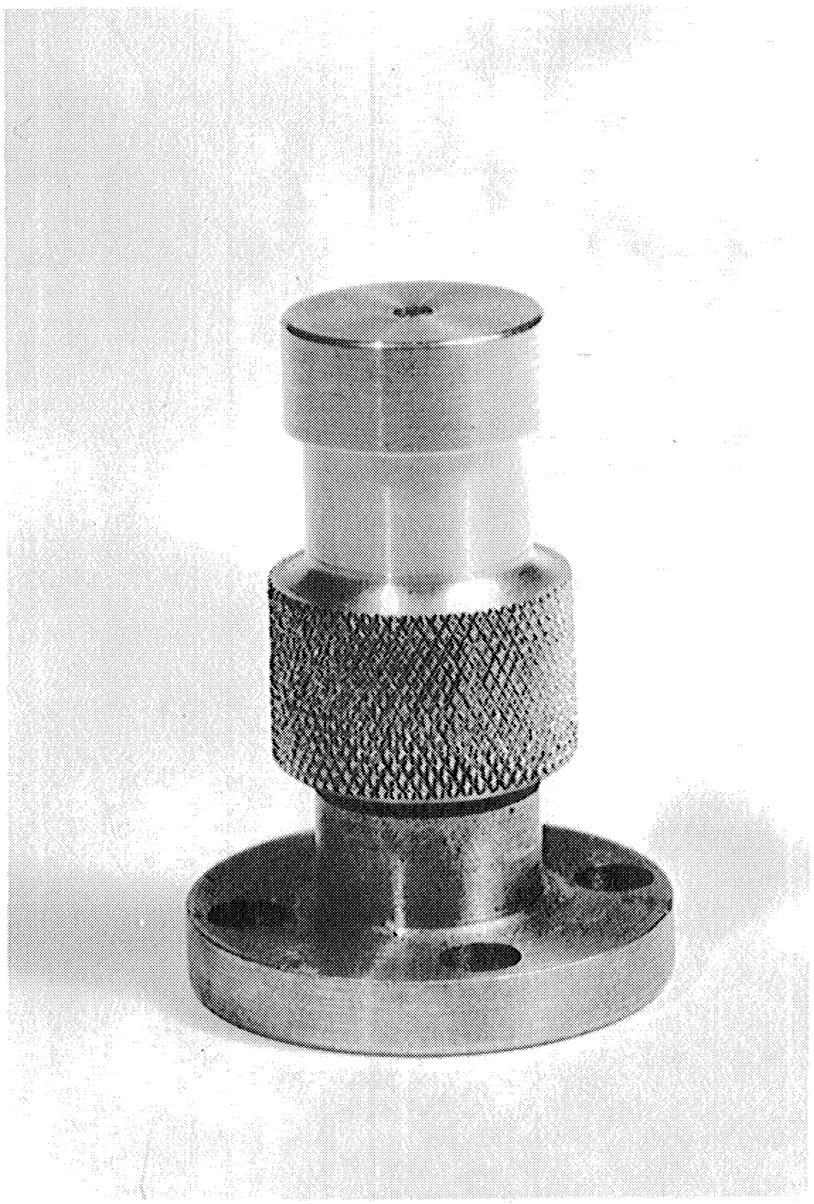


Fig. 14. Photograph of PDA-M3.

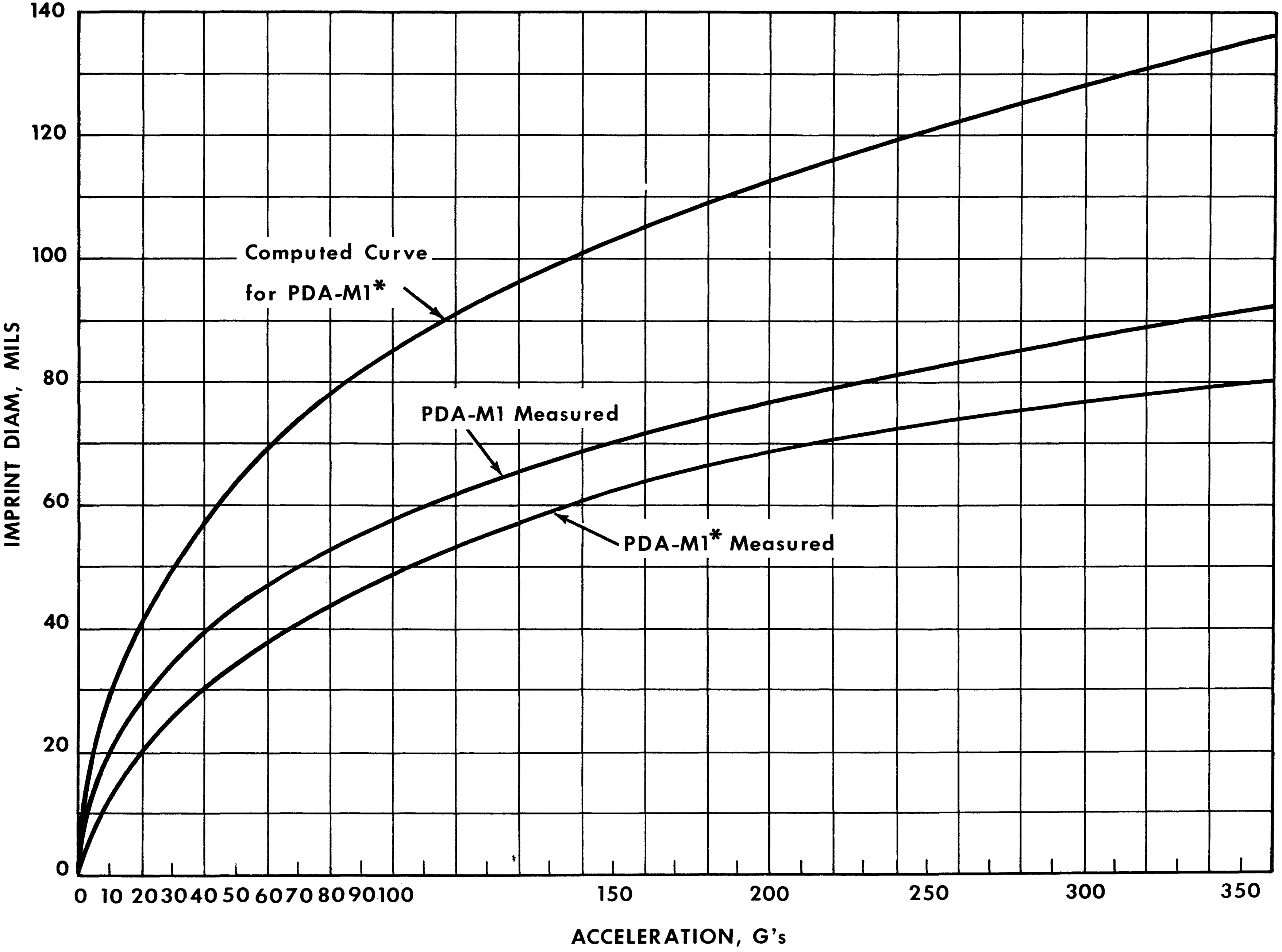


Fig. 15. Comparison of PDA-M1 with PDA-M1*—computed and measured results.